Viscosity from black-hole physics

D.T. Son (INT)

Works by DTS
G. Policastro
A. Starinets
C. Herzog
P. Kovtun
A. Buchel
J. Liu

Viscosity : introduced by C. L. M. H. NAVIER (1822)

(Navier - Stokes equation)

$$\rho \frac{\partial \vec{v}}{\partial t} + \rho(\vec{v}\cdot\vec{\nabla})\vec{v} = -\vec{\nabla}P + \underset{\substack{\uparrow \\ \text{viscosity}}}{\eta}\nabla^2\vec{v} \qquad \vec{\nabla}\cdot\vec{v}=0$$

From real-world experience : <u>ANY</u> finite-temperature interacting system can be described, at largest time and length scales, by <u>hydrodynamic</u> equations :

- local
- few variables

Finite-T QFT also behave hydrodynamically

Hydrodynamic variables : effective degrees of freedom

- densities of conserved charges $\underbrace{T^{00},\ T^{0i}}_{\text{any theory}},\ \underbrace{j^0}_{\text{model dep.}}$

  a large lump of charge dissipate slowly (diffusion)

- Phases of condensates that breaks global symmetries

  example: superfluid ^4He $\quad \vec{v}_s = \frac{1}{m}\vec{\nabla}\varphi$

- Unbroken $U(1)$ gauge fields

  magnetohydrodynamics

- ??

Structure of hydrodynamic theory

Goal not yet achieved : an effective Lagrangian for hydrodynamics (perhaps on CTP contour)

Instead : equations of motion

e.g. conservation law $\quad \partial_\mu T^{\mu\nu} = 0$

+ constitutive equations

e.g. $$T_{ij} = P\delta_{ij} - \frac{\eta}{\epsilon+P}\left(\partial_i T^{0j} + \partial_j T^{0i} - \frac{2}{3}\delta_{ij}\partial_k T^{0k}\right) - \frac{\zeta}{\epsilon+P}\delta_{ij}\,\partial_k T^{0k} + \xi_{ij}$$

noise to describe fluctuations

$$\langle \xi_{ij}\,\xi_{kl}\rangle = \dots$$

Clumpsy, but works for simple applications

Idea : use gauge/gravity duality to investigate the hydrodynamic regime of field theory

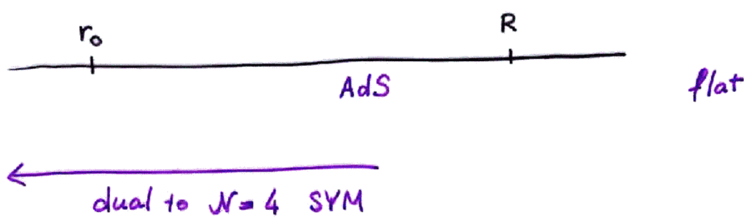
finite-T QFT $\Longleftrightarrow$ black hole with translationally invariant horizon

"black brane"

Example :

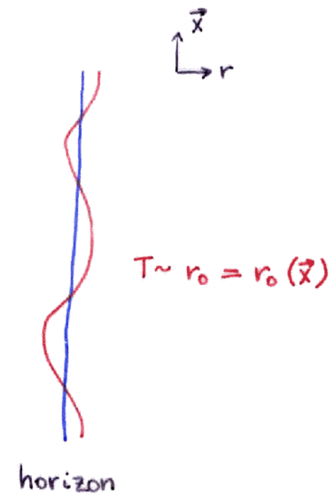
$$ds^2 = H^{-1/2}\left(-f\,dt^2 + dx^2 + dy^2 + dz^2\right) + \frac{1}{H^{1/2}}\left(\frac{dr^2}{f} + r^2 d\Omega_5^2\right)$$

$H = 1 + \frac{R^4}{r^4}$ $\qquad f = 1 - \frac{r_0^4}{r^4}$ $\qquad r_0 \ll R$



Hawking temperature $T = \frac{r_0}{\pi R^2}$

Dynamics of flat horizons :



Generalizing black hole thermodynamics $M, Q \ldots$

to black brane <u>hydrodynamics</u>

$T = T_H(\vec{x})$, $\mu = \mu(\vec{x})$ ....

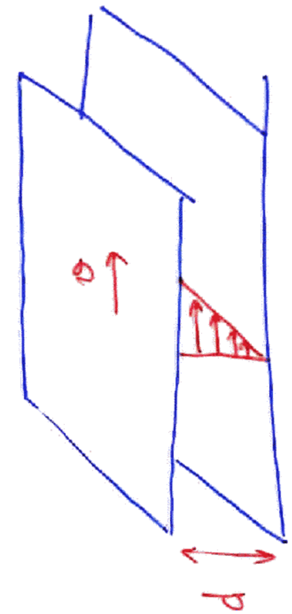
Event horizons behave as vicous fluids

$$S = \frac{\text{Area of horizon}}{4G}$$ Bekenstein Hawking

$\mathcal{N}=4$ SYM: $S(g^2N \to \infty) = \frac{3}{4} S(g^2 N \to 0)$

What is viscosity from the point of view of gravity?

Viscosity: textbook definition



$$F = \eta A \frac{v}{d}$$

Viscosity : Kubo's formula

$$\eta = \lim_{\omega\to 0} \frac{1}{2\omega} \int dt\, d\vec{x}\, e^{i\omega t} \langle [T_{xy}(t,\vec{x}),\ T_{xy}(0,\vec{0})] \rangle$$

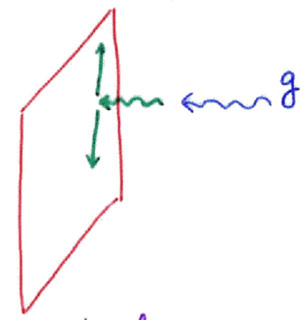
$$= -\lim_{\omega\to 0} \lim_{\vec{q}\to 0} \frac{1}{\omega} \operatorname{Im} G^R_{xy,xy}(\omega, \vec{q})$$

retarded Green's function of T_{xy}

Similar relations exist for other kinetic coefficients (diffusion constants, conductivities ...)

Gravity counterpart of Kubo's formula:

AdS/CFT "dictionary"



Coupling: $h_{\mu\nu} T_{\mu\nu}$

bulk graviton

boundary stress energy

1997 Klebanov: absorption of a graviton falling at right angle to the black brane

$\kappa = \sqrt{8\pi G}$

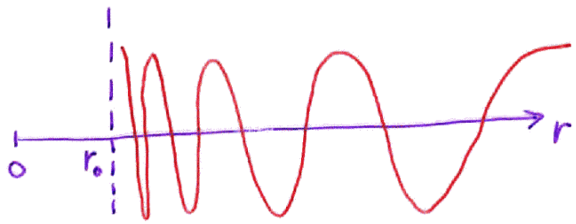
$$\sigma_{abs} = -\frac{2\kappa^2}{\omega} \operatorname{Im} G^R(\omega)$$

$$= \frac{\kappa^2}{\omega} \int d^4x\, e^{i\omega t} \langle [T_{xy}(x),\ T_{xy}(0)] \rangle$$

Viscosity = absorption cross section of low-energy gravitons

$$\eta = \frac{\sigma_{abs}(0)}{2\kappa^2} = \frac{\sigma_{abs}(0)}{16\pi G}$$

Absorption cross section can be found classically



← incoming waves

$$\text{"} \Box h_{xy} \text{"} = 0$$

$$h''_{xy} + \frac{5r^4 - r_0^4}{r(r^4 - r_0^4)} h'_{xy} + \omega^2 \frac{r^4(r^4+R^4)}{(r^4-r_0^4)^2} h_{xy} = 0$$

The computation of σ_{abs} is made easy by 2 theorems, valid for a wide class of back ground:

- Equation for h_{xy} is the same as of a minimally coupled scalar
- For a minimally coupled scalar

$$\lim_{\omega \to 0} \sigma_{abs}(0) = \text{Area of event horizon}$$

Das, Gibbons, Mathur

Consequences of 2 theorems:

$$\eta = \frac{\sigma_{abs}(\omega \to 0)}{16\pi G} = \frac{A}{16\pi G}$$

$$s = \frac{A}{4G}$$

$$\Rightarrow \frac{\eta}{s} = \frac{1}{4\pi}$$

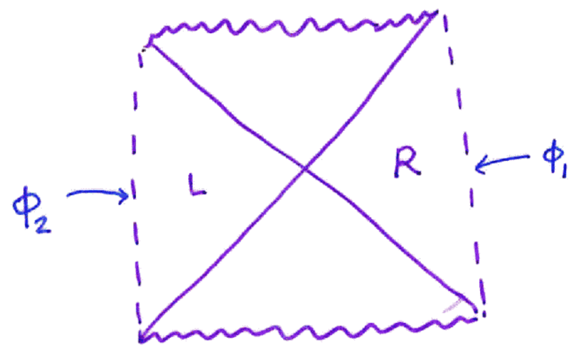
Caveats:

- Role of asymptotically flat region $r \gg R$
- Asumptions of Das, Gibbons, Mathur

More systematic way:

real-time finite-T AdS/CFT correspondence

Maldacena
C. Herzog, DTS



Penrose diagram of AdS black hole:
2 boundaries

$$Z[\phi_1, \phi_2] = e^{iS_{cl}[\phi]}$$

classical action

$\phi \to \begin{cases} \phi_1 & \text{R-boundary} \\ \phi_2 & \text{L-boundary} \end{cases}$

specific b/c on horizon

field-theory partition function on CTP contour

sources ϕ_1 on upper contour
ϕ_2 on lower contour

Differentiating $\ln Z[\phi_1, \phi_2]$

find 2×2 Schwinger-Keldysh propagators

$$\langle OO \rangle = \begin{pmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{pmatrix}$$

Retarded propagator

$$G^R(q) = G_{11}(q) - e^{-\frac{\beta}{2}q_0} G_{12}(q)$$

Example : $\mathcal{N}=4$ SYM, finite T

$$\langle T^{00}T^{00}\rangle_R(q) = \frac{\pi^2}{8}N^2T^4 \frac{5\vec{q}^{\,2} - 3q_0^2}{q_0^2 - \frac{\vec{q}^{\,2}}{3} + \frac{2i}{3\pi T}q_0\vec{q}^{\,2}}$$

Position of the pole :

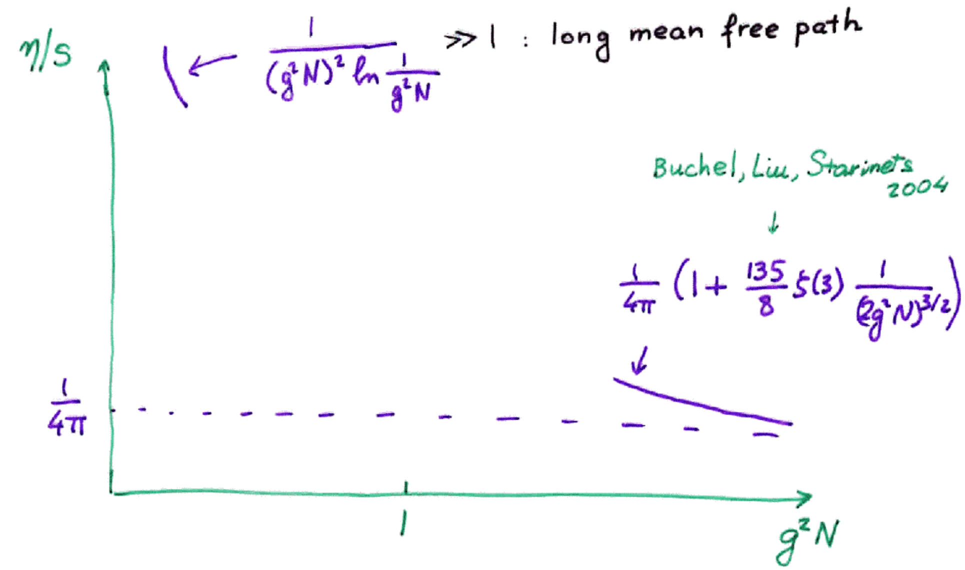
$$q_0 = \frac{|\vec{q}|}{\sqrt{3}} - \frac{i}{6\pi T}\vec{q}^{\,2}$$

Sound velocity $= \frac{1}{\sqrt{3}}$

Sound attenuation $-\frac{2i}{3}\frac{\eta}{\epsilon+P}\vec{q}^{\,2} \Rightarrow \frac{\eta}{\epsilon+P} = \frac{1}{4\pi T}$

$\frac{\eta}{s} = \frac{1}{4\pi}$

Finite-temperature $\mathcal{N}=4$ SYM



Conjecture :

$$\frac{\eta}{s} \geq \frac{1}{4\pi}$$

for relativistic QFT, with zero chemical potentials

Quark-gluon plasma !

Results:

	η	s	$\frac{\eta}{s}$
D3	$\frac{\pi}{8}N^2T^3$	$\frac{\pi^2}{2}N^2T^3$	$\frac{1}{4\pi}$
M5	$\frac{2^5\pi^2}{3^6}N^3T^5$	$\frac{2^7\pi^3}{3^6}N^3T^5$	$\frac{1}{4\pi}$
M2	$\frac{2\sqrt{2}\pi}{27}N^{3/2}T^2$	$\frac{8\sqrt{2}\pi^2}{27}N^{3/2}T^2$	$\frac{1}{4\pi}$

Herzog

$\frac{\eta}{s} = \frac{1}{4\pi}$ also for deformations of $\mathcal{N}=4$ SYM

- $\mathcal{N}=2^*$

Buchel, Liu

- Klebanov-Tseytlin
- Maldacena-Nunez

$\Big\}\ \mathcal{N}=1$

Buchel: $\frac{\eta}{s} = \frac{1}{4\pi}$ for all metrics of the form

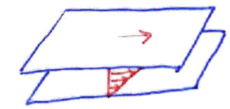
$$ds^2 = -\Omega_1^2(y)\,dt^2 + \Omega_2^2(y)\left(dx^\alpha dx^\alpha + g_{mn}(y)\,dy^m dy^n\right)$$

with $R^\alpha_\beta = \delta^\alpha_\beta R^t_t$

How about non-relativistic systems?

(relativistic with $\mu - m \ll m$)

Restoring $\hbar$, c



dimension of η:

$F = \eta A \frac{v}{d}$

$$[\eta] = \frac{\text{force}\cdot\text{length}}{\text{area}\cdot\text{velocity}} = \frac{\hbar}{\text{volume}}$$

$$[s] = \frac{1}{\text{volume}}$$

$$\Rightarrow \left[\frac{\eta}{s}\right] = \hbar$$

Water under normal conditions

$$\frac{\eta}{s} = 380\,\frac{\hbar}{4\pi}$$

$\frac{\eta}{s} \geq \frac{\hbar}{4\pi}$ even for non-relativistic system?

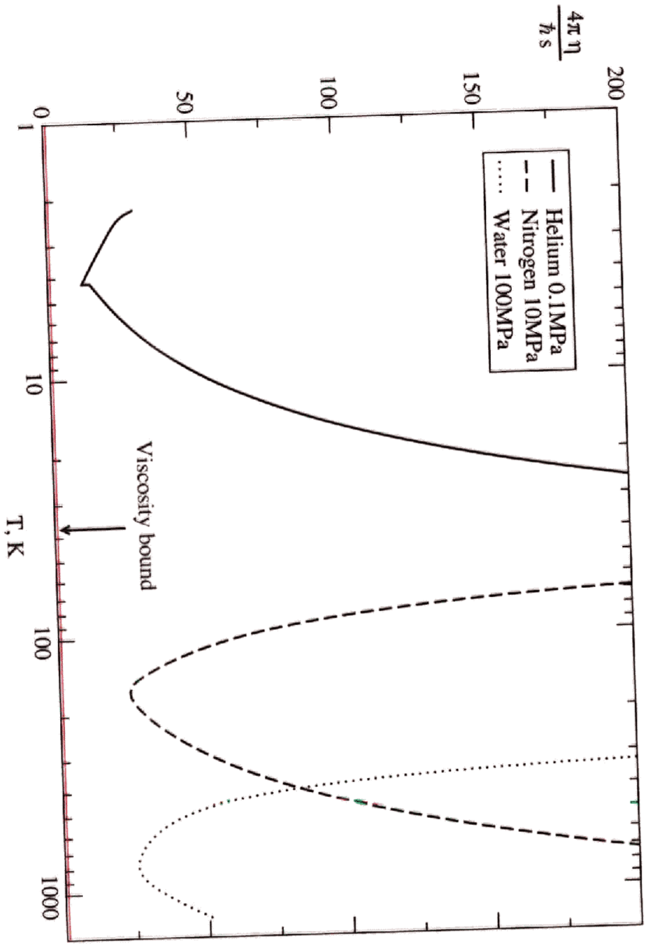


Figure 1: The viscosity-entropy ratio for some common substances.

A viscosity bound conjecture

$$\frac{\eta}{s} \geqslant \frac{\hbar}{4\pi k_B}$$

does not contain c, G

c.f. Entropy bound $S \leqslant \frac{c^3}{\hbar G}\,\frac{A}{4}$

Bekenstein bound $S \leqslant \frac{c}{\hbar}\, 2\pi R M$

Applications :

- Quark-Gluon Plasma
- Trapped atomic gases

Counter-examples ?

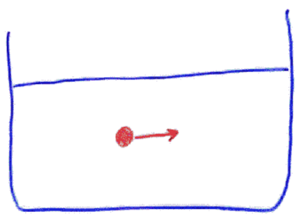
- Ideal gas ?
  - viscosity diverges when interaction is turned off

$\eta \sim \rho v \ell_{mfp}$ ↑ mean free path

$s \sim \frac{\rho}{m}$

$\frac{\eta}{s} \sim m v \ell_{mfp} \gtrsim 1$

- Superfluids ?
  - Landau: non-zero "normal-component" shear viscosity, measurable by moving an object inside a volume of superfluid



drag $\sim \eta$

The Quark-Gluon Plasma

probably created at RHIC

$T \sim O(\Lambda_{QCD})$: strong-coupling regime

Viscosity: needed for hydrodynamic description of the expanding fireball

Our conjecture is $\eta \geqslant \frac{s}{4\pi}$

Some indication of a small viscosity at RHIC

Would be interesting to see if $\frac{\eta}{s}$ satisfies the bound

Shuryak, Teaney: $\frac{\eta}{s}$ close to saturating the bound

Trapped atomic gases

Dimensionless coupling constant

$$\underbrace{n}_{\text{density}}\ \underbrace{a^3}_{\text{scattering length}}$$

- Strong coupling regime $na^3 \gg 1$ has been achieved by Feshbach resonance
- observed to expand hydrodynamically

It would be interesting to compare η/s in the strong coupling regime with $\frac{1}{4\pi}$

Outlook:

- Why $\frac{\eta}{s} = \frac{\hbar}{4\pi}$ for such many theories?

  field-theory arguments?

- Are there theories with gravity duals with $\frac{\eta}{s} \neq \frac{\hbar}{4\pi}$

  chemical potential?

- Is the viscosity bound correct? Proof?

  under which assumption?

- Effective Lagrangian formulation of hydrodynamics: insight from gravity?

  example where gravity is useful:

  hydrodynamics with anomalous currents

  (GEORGE NEWMAN, to appear)