2d Vacuum Defects in 4d YM Theory

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Lattice data are mostly due to group of M. Polikarpov (ITEP, Moscow)

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I. Language

will consider pure SU(2) case (mostly)

lattice ≡ Euclidean space time

Explicit UV cut off provided by the lattice spacing a

Λ_{QCD} (or σ) emerges through RG

Specific for the lattice:
UV divergences are tractable

e.g. $\langle \alpha_s (G^a_{\mu\nu})^2 \rangle = c\,\frac{N_c^2 - 1}{a^4}(1 + a\alpha_s + \dots)$

Both calculable and measurable

Percolation

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Introduce probability p for a link to be open

Probability to find a cluster of length L:

$$W(L) = p^{L/a} \cdot N_L$$

number of trajectories

$$N_L \simeq 7^{L/a}$$ cubic lattice, 4d

If $p > p_{cr} \simeq 1/7$

there exists infinite, or percolating cluster (phase transition at $p = p_{cr}$)

For a given link

$$\theta_{perc} = (p - p_{cr})^{\alpha}, \quad \alpha > 0$$

Percolation vs. field th.

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Start with action $S_{cl} = L \cdot M$

then, obviously, $e^{-M \cdot a} \Rightarrow p$

Evaluating Feynman propagator

$$D(x, x') = \sum_{paths} \exp(-S_{cl}(x, x'))$$

gives physical mass (propagating)

$$m^2_{phys} = \frac{\#}{a} \left(M(a) - \frac{\text{“}\ln 7\text{”}}{a} \right)$$

physical mass $\equiv$ fine tuned mass (tuning of action and entropy)

One loop $\Rightarrow$ spectrum of finite clusters

$$P(L) = \frac{\#}{L^3} \exp(-\,const\; m^2 \cdot a \cdot L)$$

related to α

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Confinement

defined as $\langle W \rangle \sim \exp(-\sigma \cdot A)$

no theory in non-Abelian case,

full understanding in $U(1)$, Z_2 cases

"Compact" $U(1)$:

$$\mathcal{L} = -\frac{1}{4e^2} F_{\mu\nu}^2 + (\text{no action for the Dirac string})$$

Radiative mass of the Dirac monopole

$$M_{mon} = \frac{\#}{e^2} \int_a d^3 r \, \vec{H}^2 = \frac{\#}{e^2 \cdot a}$$

If $e^2 \geq e^2_{crit}$

$$\frac{\#}{e^2_{crit} \cdot a} = \frac{\ln 7}{a}$$

monopoles condense $\equiv$ confinement

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Z_2 gauge th.

Links $Z_\mu(x) = \pm 1$

Plaquette $= \prod_{i=1,2,3,4} Z_i$

Action $S = \beta \cdot A_{neg}$

Since entropy is exponentially large there is percolation at β_{cr}

Wilson loop

$$W = \prod_{contour} Z \equiv \prod_{area} (\text{Plaquettes})$$

$$\langle W \rangle \overset{assume}{=} \langle \text{Plaquette} \rangle^{A/a^2}$$

$$\langle \text{Plaquette} \rangle = \theta_{pe}(-1) + (1-\theta_{pe})(+1)$$

string tension $\sigma = 2\,\theta_{pe}\, 1/a^2$

where θ_{pe} – probability to belong to infinite cluster

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Conclusions to I:

1) Confinement is well understood in $U(1)$, Z_2 cases

2) In both cases, confining field configurations are infinite clusters (cancellation of random large numbers for $\langle W \rangle$)

3) In both cases, there are dual formulations in terms of excitations which provide confinement

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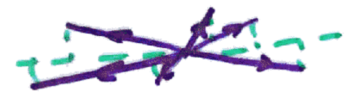
II. Monopoles, vortices in SU(2)

Projection on $U(1)$ or Z_2

Analogy:



replacing original momenta with "closest" collinear configuration:

a) choose $\vec{n}$ such that $\max \sum_i |\vec{p}_i \vec{n}|$

b) replace $\vec{p}_i \Rightarrow \vec{n}\,(\vec{p}_i \cdot \vec{n})$

In $SU(2)$ case:

a) find $\max \sum_{x,\mu} |A^3_\mu(x)|^2$ using gauge inv.

b) replace, "project" $A^{\pm}_\mu(x) \equiv 0$

Similar procedure in Z_2 case

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Lower-dimensional defects

By construction, monopole trajectories, vortex surfaces, are infinitely thin 1d, 2d defects (on the dual lattice)

No theory because of the projections

However, remarkable observations:

(a) Percolating clusters exist, in the both cases

(b) one can argue that these clusters are responsible for

$$\lim_{R\to\infty} V_{Q\bar{Q}}(R) = \sigma R$$

(removal of clusters destroys confinement)

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(c) Absence of fractal dimensions

$$\mathcal{L}^{perc}_{mon} = c\,\Lambda^3_{QCD} V_4\,, \quad A^{perc}_{vort} = c\,\Lambda^2_{QCD} \cdot V_4$$

lattice volume

Alternatively, probability to belong the perc. cluster:

$$\Theta_{link} = (a \cdot \Lambda_{QCD})^3 \quad , \quad \Theta_{pl} = (a \cdot \Lambda_{QCD})^2$$

as is needed for confinement:

$$\sigma = 2\,\Theta_{plaq} \frac{1}{a^2} \sim \Lambda^2_{QCD}$$

Looks as if close to the 2nd order phase transition.

Amusing physics of powers $(\Lambda_{QCD} \cdot a)$, with no logarithmic "background"

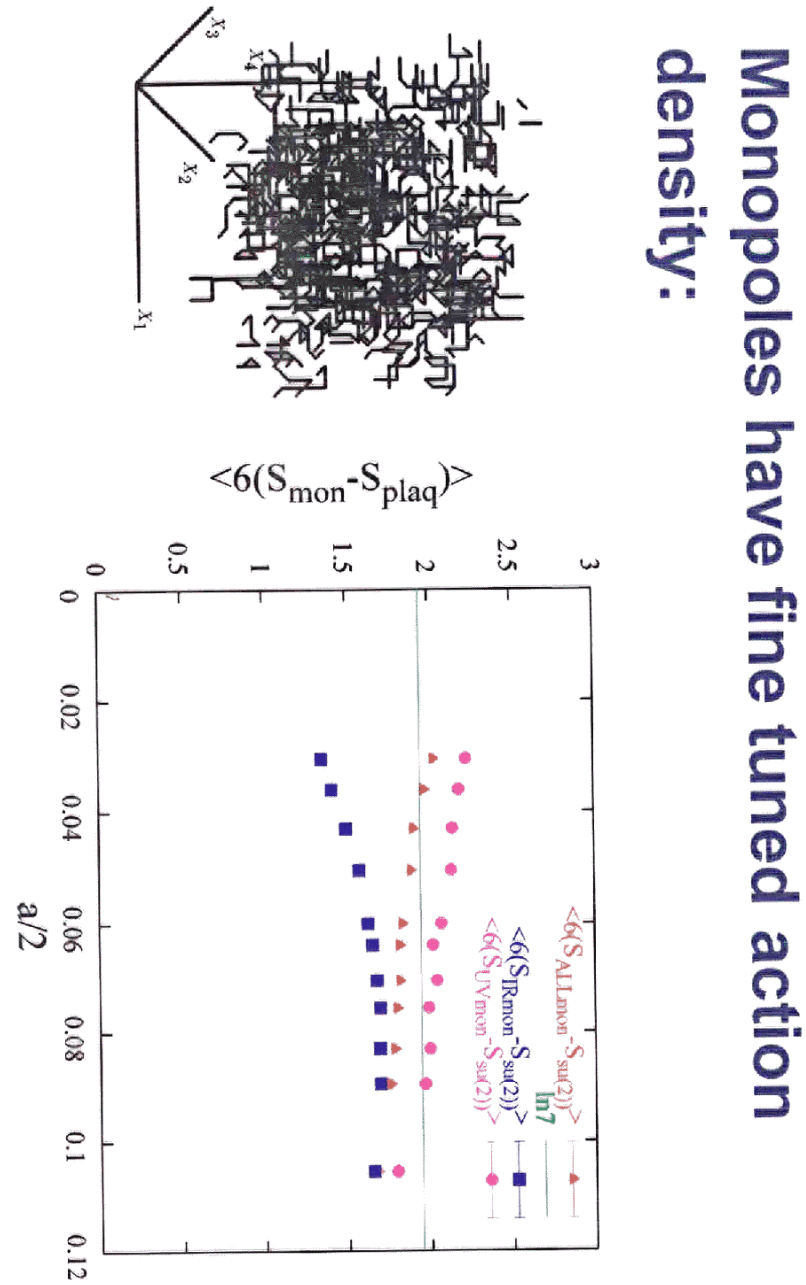
Conclusions to II

Painstaking search for "confining fluctuations" revealed some percolating clusters, defined rather algorithmically, which exhibit, however, remarkable, SU(2) invariant, scaling laws

Strategy:

assume that SU(2) inv. objects are detected

look for further SU(2) inv. properties



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III Lattice strings

Selftuning of the monopoles

Measure non-Abelian action of monopoles

Expect:
$$S_{mon} \sim \Lambda_{QCD} \cdot L$$

(Bulky, non-pert. field)

Instead find rather similarity to the Higgs case

Data

$$m^2_{Higgs} = d \cdot \Lambda^2_{UV} - M_0^2$$

For monopoles:

$$m^2_{mon} \approx \frac{\#}{a}\left(M_{rad} - \frac{\ln 7}{a}\right) \sim 0$$

measured directly, non-Abelian

role of M_0^2 (known now)

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Pointlike facet of monopoles

$$M_{rad} \gtrsim 6\ \text{GeV} \quad \text{(at present lattices)}$$

Free path:

$$L_{free} \sim 2\ \text{fm} \sim (100\ \text{MeV})^{-1}$$

For finite clusters:

$$N(L) \sim \frac{1}{L^3}$$ spectrum in length

$$r^2(L) \sim L \cdot a$$ size of the cluster

Both predictions work perfect

as if saw vacuum loop

Monopoles vs. Asymptotic Freedom

No new particles are allowed

"particle" $\Rightarrow$ UV divergence

$$\langle |\varphi_M|^2 \rangle \sim \Lambda_{UV}^2 \sim a^{-2} \quad \text{not allowed}$$

$$\langle |\varphi_M|^2 \rangle \sim \Lambda_{QCD}^2 \quad \text{is allowed}$$

magnetically charged field

Can derive: $\langle |\varphi_M|^2 \rangle = \text{const} \cdot a \, \rho_{mon}^{tot}$

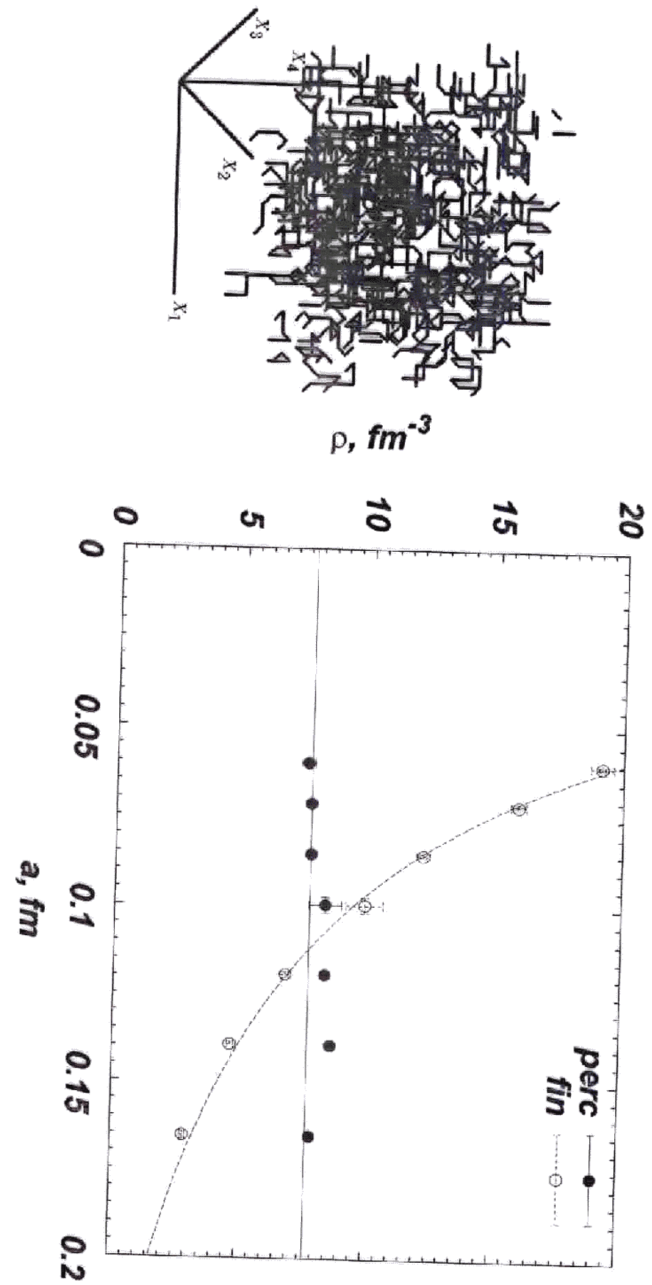
where $L_{mon}^{tot} \equiv \rho_{mon}^{tot} \cdot V_4$

Allowed: $\rho_{mon}^{tot} \sim \Lambda_{QCD}^3 + \Lambda_{QCD}^2 / a$

Data: $\rho_{mon}^{tot} \approx \frac{1.6}{a\,(fm)^2} + \frac{1.5}{(fm)^3}$

Globally, monopoles live on a 2d surface
$(\rho \sim a^{d-1})$

Length of IR monopole cluster scales, $\rho = \frac{L}{V_4}$



Z_2 side

Excitations: closed surfaces on the dual lattice

Data:

$$A_{tot} = (\text{const})\, \Lambda^2_{QCD} \cdot V_4$$

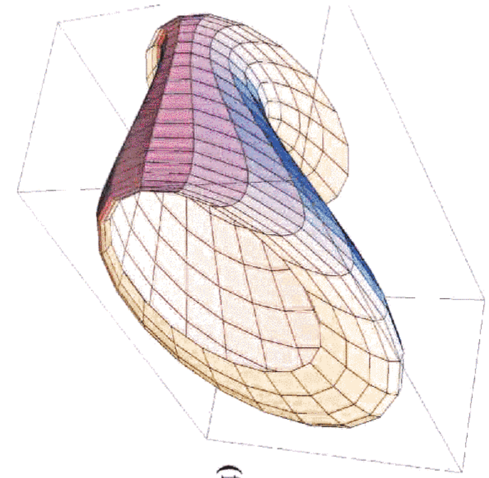
$$S^{tot}_{vort} = 0.54\, \frac{A_{tot}}{a^2}$$

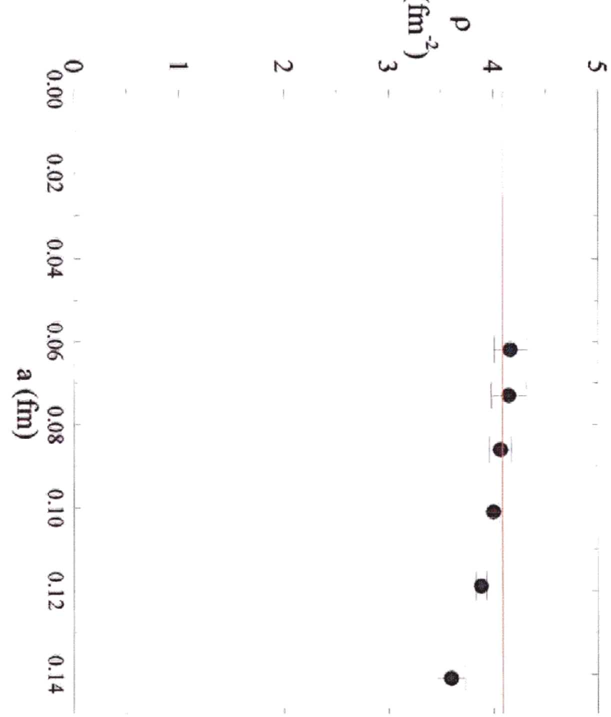
Moreover, the monopole trajectories fall on the same surface

U(1) and Z_2 defects are unified

"Monopoles" are not spherically symmetric, rather their field is confined to the surface

P-VORTEX density, Area/(6*V_4), scales:

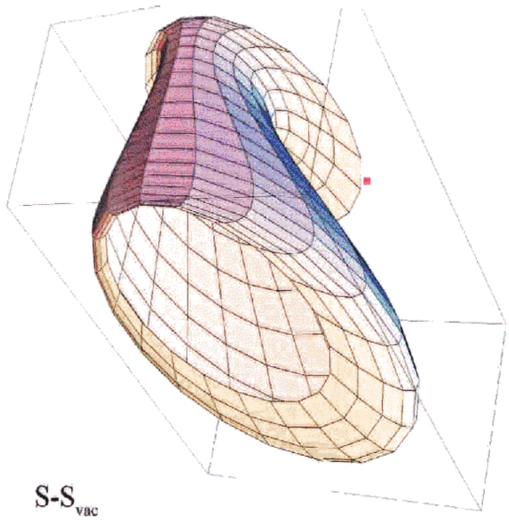


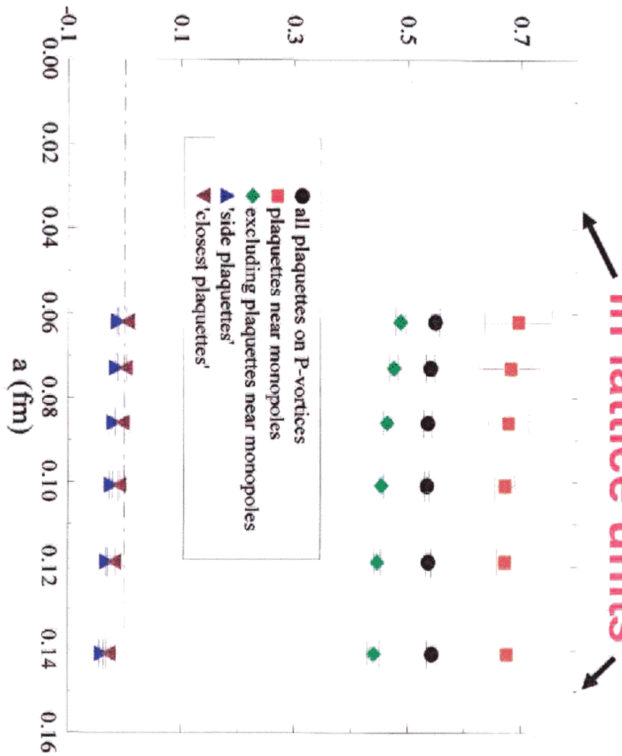


P-VORTEX has UV divergent action density:

$(S-S_{vac})=Const./a^2$

In lattice units





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Closed strings on the dual lattice

(a) UV divergent action

(b) UV divergent entropy

(c) total area is in Λ_{QCD}^{-2}

(a) - (c) equivalent to strings of tension $\sim \Lambda_{QCD}^{2}$

(d) strings condense

(e) there are excitations living on the strings ("monopoles")

(f) alignment of fields with geometry

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Lattice string = dual string

(a) Rather obvious (or wrong)

(b) no other symmetry can command self tuning

(c) confinement is simple in terms of the string

(d) lives on the dual lattice

(e) contribution to local matrix elements is to be dual to gluons (pert. th.)

Concentrate on

$$\langle \alpha_s (G^a_{\mu\nu})^2 \rangle_{string} \sim \Lambda^2_{QCD} \Lambda^2_{UV}$$

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Strings and power corrections

$$\langle \alpha_s G^2 \rangle = \frac{\#}{a^4} \left(1 + a_1 \alpha_s + \ldots (a \cdot \Lambda_{QCD})^2\right)$$

power correction

There is a kind of theorem that $(a \cdot \Lambda_{QCD})^2$ correction is calculable pert. Pert. th. can fail only on $(a \cdot \Lambda_{QCD})^4$ terms.

Explicit calculation exists up to α_s^{10} and one can watch how pert. series eats up the $(a \cdot \Lambda_{QCD})^2$ term

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3d defects

Very recently, there is evidence obtained that chirality breaking is due to 3d defects

de Forcrand et al.
Koroveth ..

The corresponding action cannot diverge in UV strongly

Could be

$$(G+\tilde{G})^2 - (G-\tilde{G})^2 \sim \frac{1}{a}$$

while G^2 is not enhanced

But this is speculative, for orientation only

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Conclusions

I believe that there is quite strong evidence that a "dual string" has been observed (satisfies AF constraints)

Then, suggests existence of a dual description for SU(2) YM.

Probably, 3d defects are coming