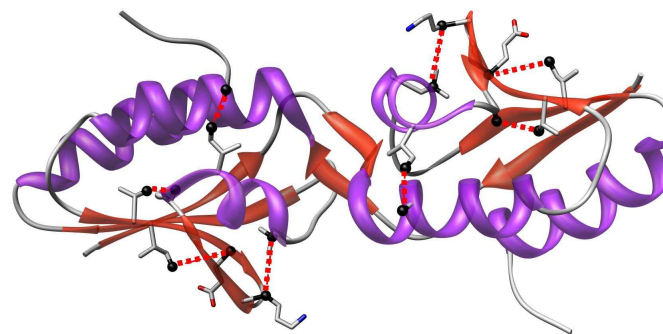
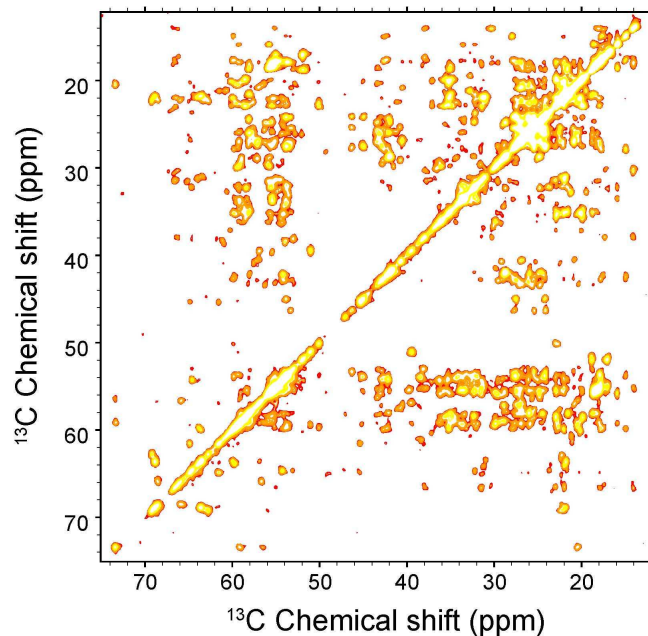


# CMRR and TSAR Recoupling

## Advanced Control for Structure Determination in Solid-State NMR



Massachusetts  
Institute of  
Technology



Crh 2x10.4 kDa - PAR 900 MHz



Dr. Gaël De Paëpe



Massachusetts  
Institute of  
Technology

# Acknowledgment

---



## MIT, Cambridge (MA):

Jozef Lewandowski

Matt Eddy

Marvin Bayro

Robert Griffin



## CEA-INAC, Grenoble (FR)

Mathilde Giffard

Sabine Hediger

Michel Bardet

Guillaume Gerbaud



## IBCP, Lyon (FR):

Antoine Loquet

Simon Megy

Carole Gardiennet

Anja Böckmann



## CERM, Florence (IT):

Moreno Lelli

Anusarka Bhaumik

Claudio Luckinat

Ivano Bertini **CERM**

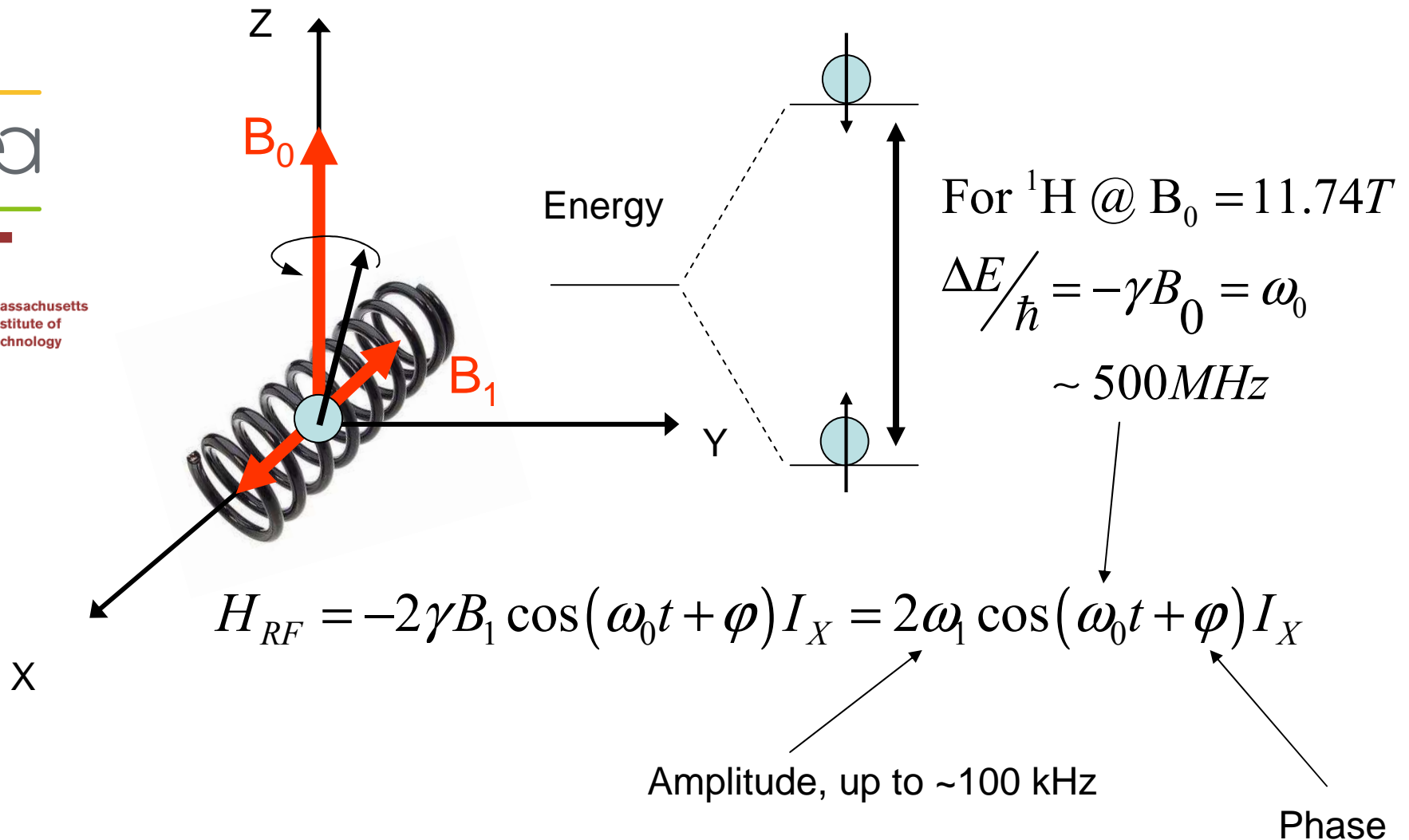
## Bruker, Billerica (MA):

Jochem Struppe

Werner Maas

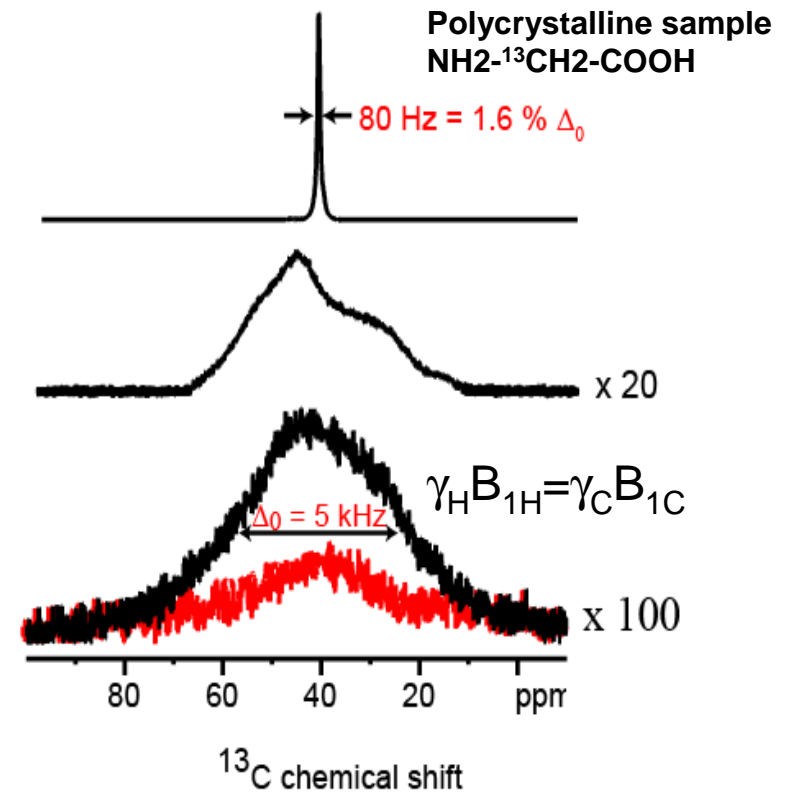
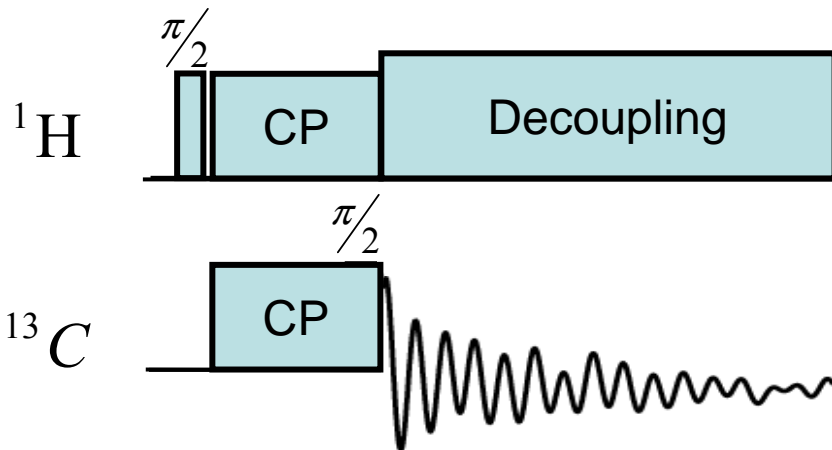
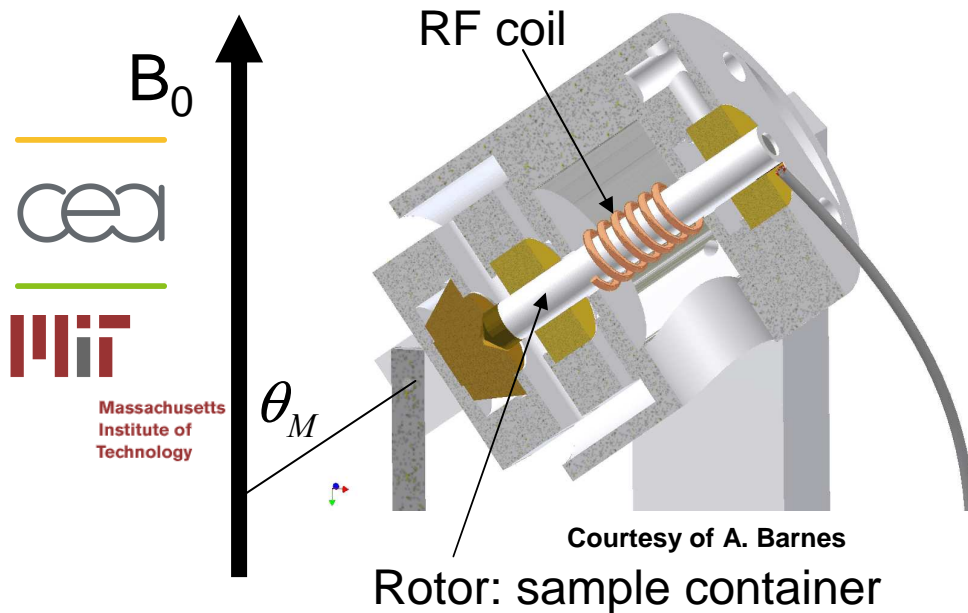


# High Field Nuclear Magnetic Resonance



- RF excitation of the nuclei at their Larmor Frequencies

# High Resolution Solid State NMR



Dipolar & Chemical Shift Anisotropy broadening

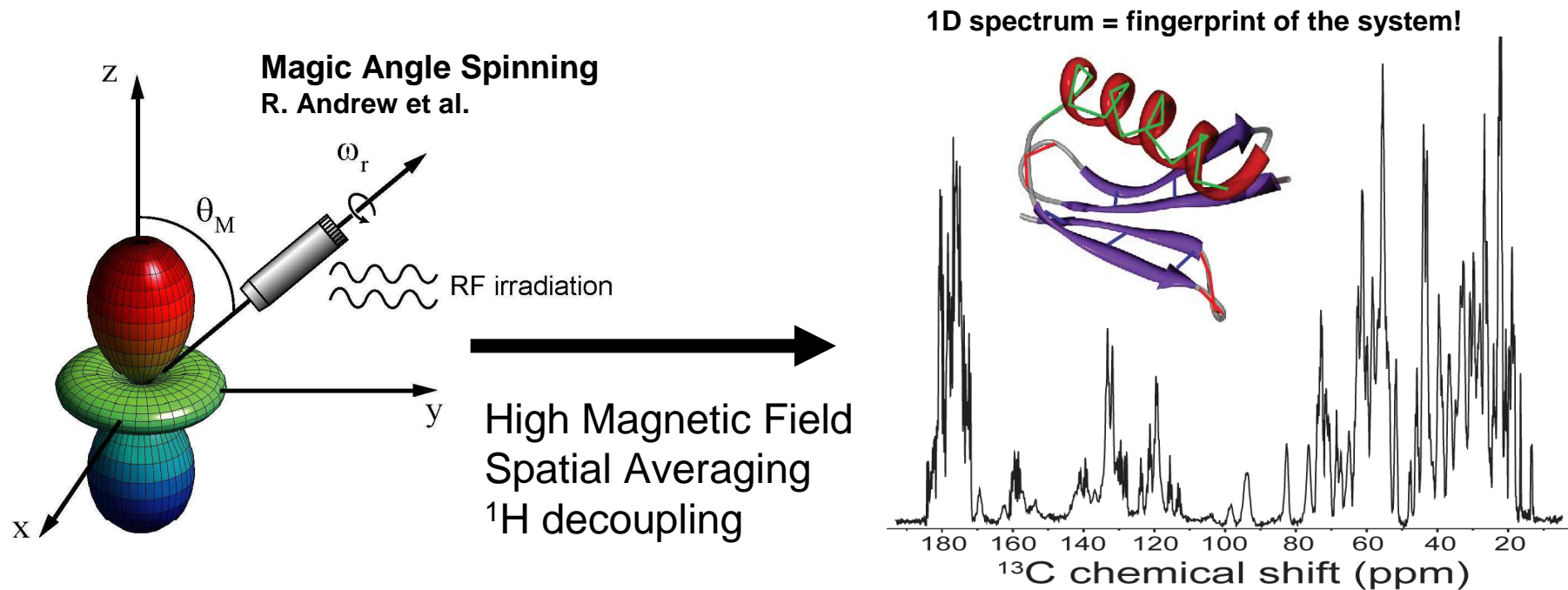
removed if  $3 \cos^2(\theta_M) - 1 = 0$

$$\hat{H} = \sum A_{l0}^{\lambda} \tau_{l0}^{\lambda}$$

• Tremendous progress over the last 50 years..  $\lambda$

Key authors: Abragam, Andrew, Griffin, Hartmann, Hahn, Mehring, Pines, Schaefer, Waugh, et al.

# Structure Determination in Solid State NMR



- **MAS provides high resolution:**  $H_{\text{detection}} = H_{\text{CS}} + \cancel{H_{\text{CSA}}} + \cancel{H_{\text{DIP}}}$
- **In order to recover the distance information, we need to design RF pulses that can interfere with MAS averaging:**

$$H_{\text{mixing}} = \cancel{H_{\text{CS}}} + \cancel{H_{\text{CSA}}} + H_{\text{DIP}} \longrightarrow \text{Recoupling sequences in multi-dimensional experiments}$$

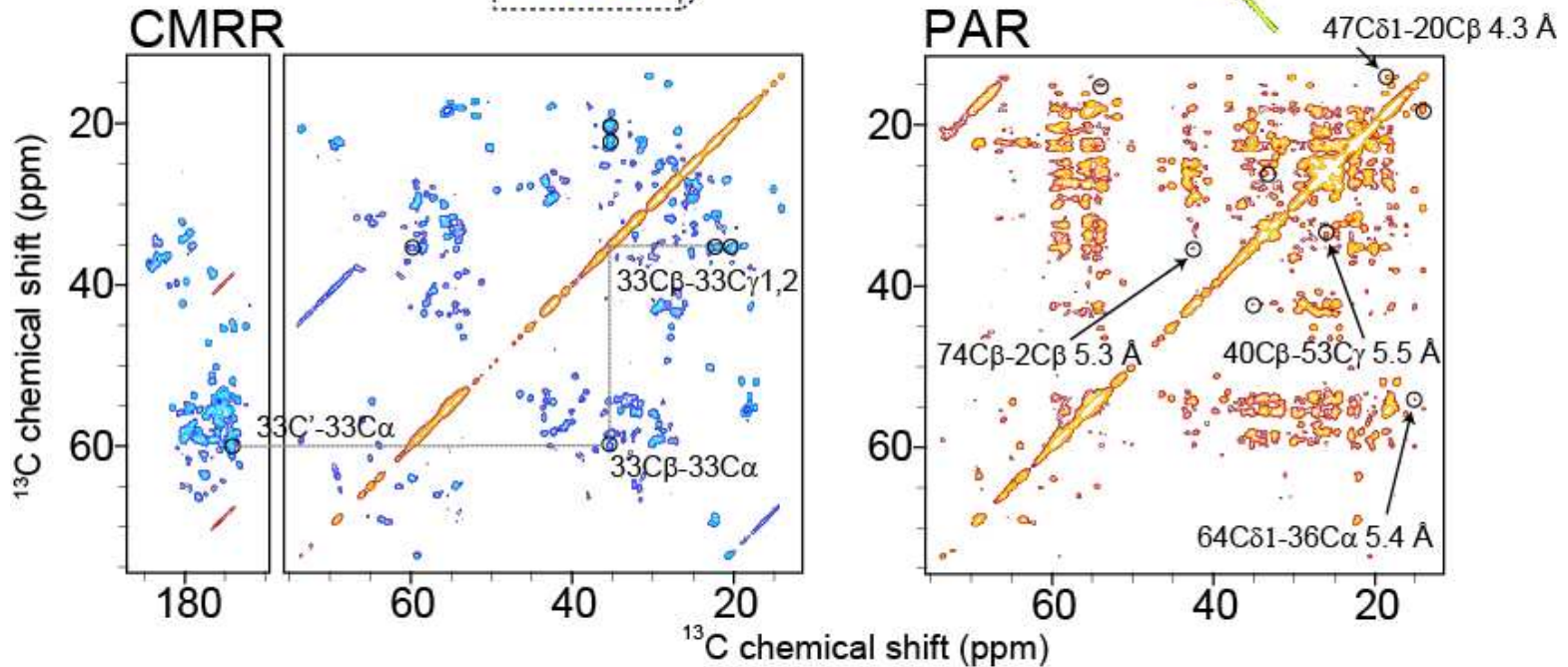
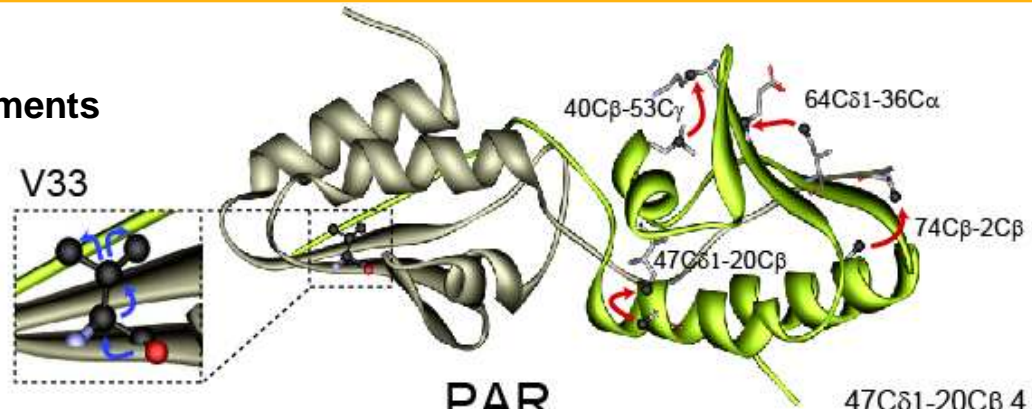
Key authors: Schaefer, Griffin, Levitt, Tycko, Nielsen, Khaneja, Baldus etc.

# New methods for structure determination



Massachusetts  
Institute of  
Technology

$^{13}\text{C}$ - $^{13}\text{C}$  correlation experiments  
750/900 MHz  $^1\text{H}$  Freq.  
20 kHz MAS freq.



- **Who's who** → *assigning the resonances* -- **Part I - CMRR**
- **Structurally relevant restraints** – **Part II – TSAR mechanism**

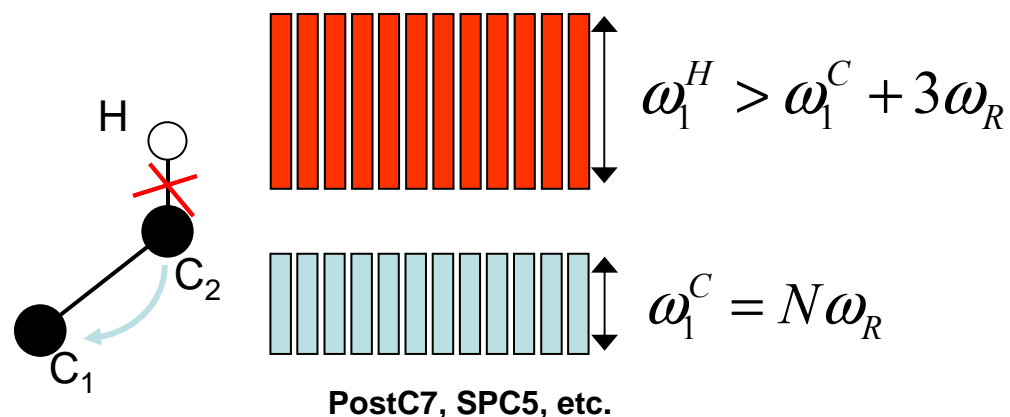
## Part I – One-bond transfer dipolar recoupling sequence

*A lot of dipolar recoupling sequences have already been reported...*

*In practice, they necessitate concurrent  $^1\text{H}$  decoupling...*



Massachusetts  
Institute of  
Technology



- @ 20 kHz  $\omega_r/2\pi$ , the  $^1\text{H}$  decoupling field should be at least 150 kHz

→ Beyond probe and sample limits!!

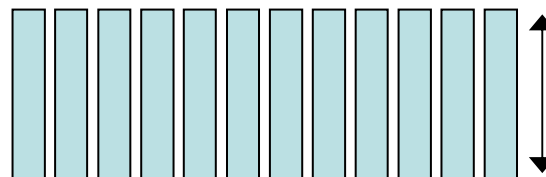
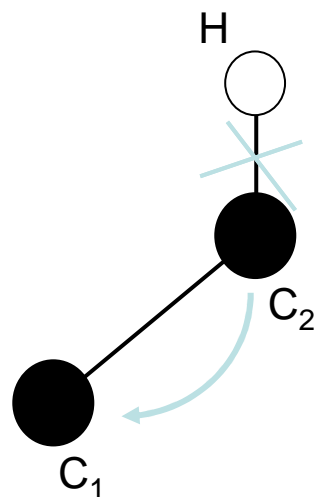
# CMRR recoupling with $^1\text{H}$ decoupling



Massachusetts  
Institute of  
Technology



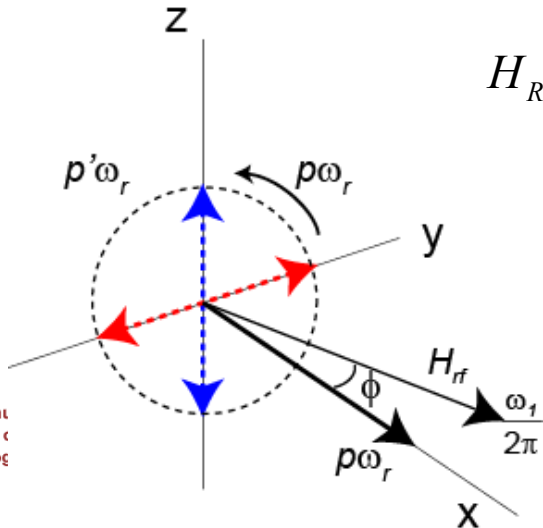
*Can we design a recoupling sequence that does not require  $^1\text{H}$  irradiation?*



$$\omega_1^C = N\omega_R$$

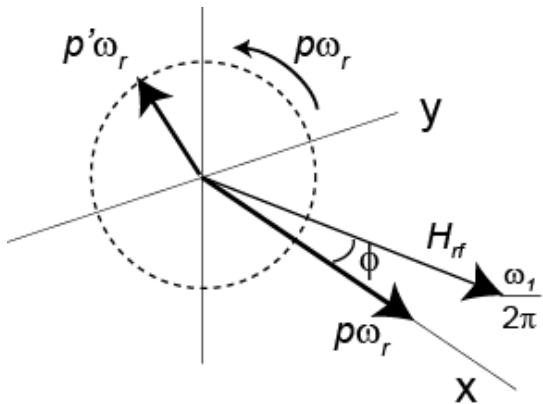


# Generalization of the second averaging principle



$$H_{RF} = \underbrace{\omega_1 \cos \phi(t)} \sum_i S_x^i + \underbrace{\omega_1 \sin \phi(t)} \sum_i S_y^i + \underbrace{\Omega(t)} \sum_i S_z^i$$

$$\begin{cases} \omega_1 \cos(\phi) = p\omega_R \\ \omega_1 \sin(\phi) = p'\omega_R \sin(p\omega_R t) \\ \Omega(t) = p'\omega_R \cos(p\omega_R t) \end{cases}$$



$$\begin{cases} \phi = \arctan\left(\frac{p' \sin(p\omega_R t)}{p}\right) \\ \omega_1 = p\omega_R \left[ 1 + \left(\frac{p'}{p} \sin(p\omega_R t)\right)^2 \right]^{1/2} \\ \Omega = p'\omega_R \cos(p\omega_R t) \end{cases}$$

$$\longrightarrow U_{RF} = \exp(-ip'\omega_R t C_Z) \exp(-ip\omega_R t C_X)$$

# Interaction Frame

$$U_{\text{RF}} = \exp(-ip'\omega_R t C_Z) \exp(-ip\omega_R t C_X)$$

Hamiltonian  $H(t) = H_{\text{RF}}(t) + \sum_{\lambda} A_{I0}^{\lambda} T_{I0}^{\lambda}$

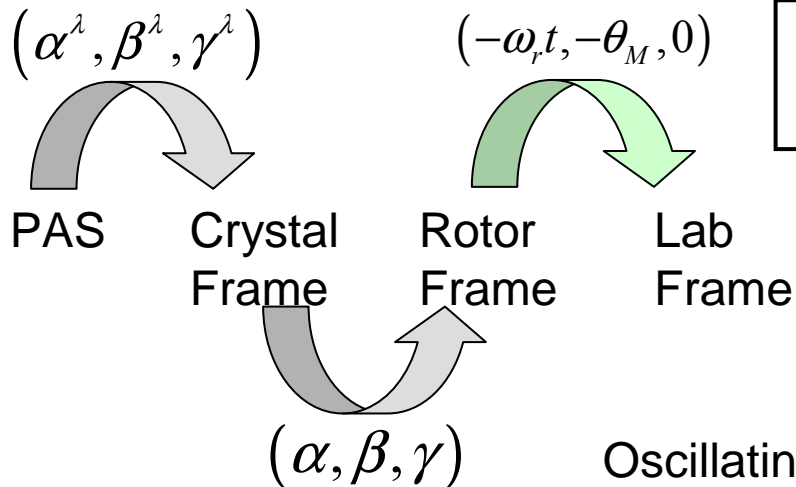
RF pulses  $\rightarrow$   $H_{\text{RF}}(t)$

Chemical shifts, Dipolar interactions etc.  $\rightarrow$   $\sum_{\lambda} A_{I0}^{\lambda} T_{I0}^{\lambda}$

Interaction Frame Hamiltonian  $H'_{\text{int}}(t) = U_{\text{RF}}(t,0)^{\dagger} H_{\text{int}}(t) U_{\text{RF}}(t,0)$

Homonuclear dipolar interaction:

$$H_{\text{dip}} = \omega_{AB} [2C_{AZ}C_{BZ} - C_{AX}C_{BX} - C_{AY}C_{BY}] = \sqrt{6}\omega_{AB} T_{20}^{AB}$$



$$[T_{20}^{AB}]' = \sum_{q'q} T_{2q'}^{AB} e^{-i(q'p'+qp)\omega_r t} d_{0q}^2\left(\frac{\pi}{2}\right) d_{q'q}^2\left(\frac{\pi}{2}\right)$$

$$\omega_{AB} = \sum_{m=-2}^2 \omega_{AB}^{(m)} \exp(-im\omega_r t)$$

Oscillating terms at the frequencies:  $(m + pq + p'q')\omega_R$

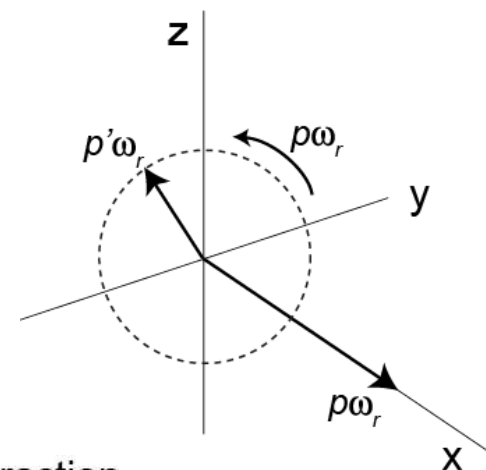


# First order Average Hamiltonian Theory

$$\overline{H'_{\text{int}}^{(1)}} = \frac{1}{T} \int_0^T dt_1 H'_{\text{int}}(t_1)$$

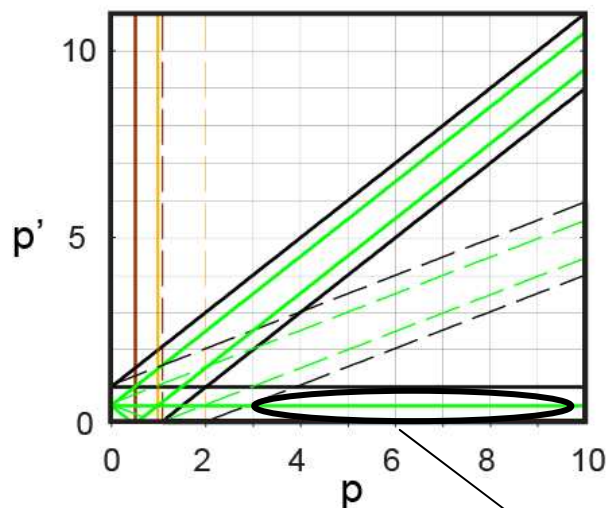
First order recoupling if:  $m + p \cdot q + p' \cdot q' = 0$

MAS →  $m$   
 first →  $p \cdot q$   
 second averaging →  $p' \cdot q'$

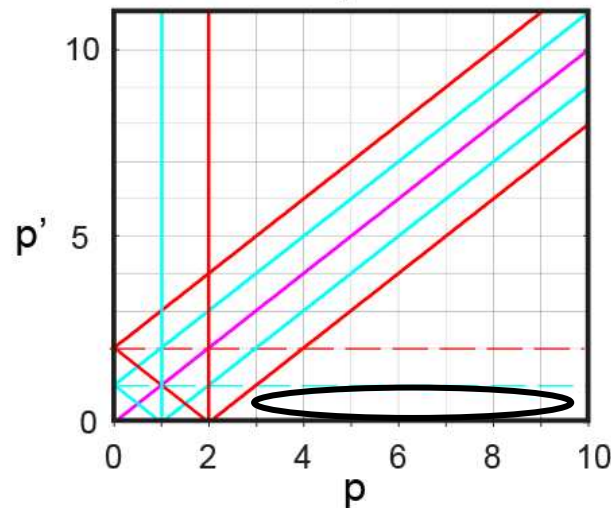


Massachusetts  
Institute of  
Technology

Homonuclear dipolar interaction



CS interaction  
heteronuclear dipolar interaction



**CMRR:  $p'=1/2$  and  $p$  from 3.5 to 8**

- This new scheme can be seen as a very versatile recoupling toolbox

# DQ CMRR recoupling without $^1\text{H}$ decoupling

$$p = 5$$

$$p' = 1/2$$

$$\overline{H'_{\text{CMRR}}^{(1)}} = \frac{3}{8\sqrt{2}} \frac{\mu_0}{4\pi} \frac{\gamma_C^2}{r_{AB}^3} \sin(2\beta) \left[ \exp(-i\gamma) T_{2-2}^{AB} + \exp(i\gamma) T_{2+2}^{AB} \right]$$

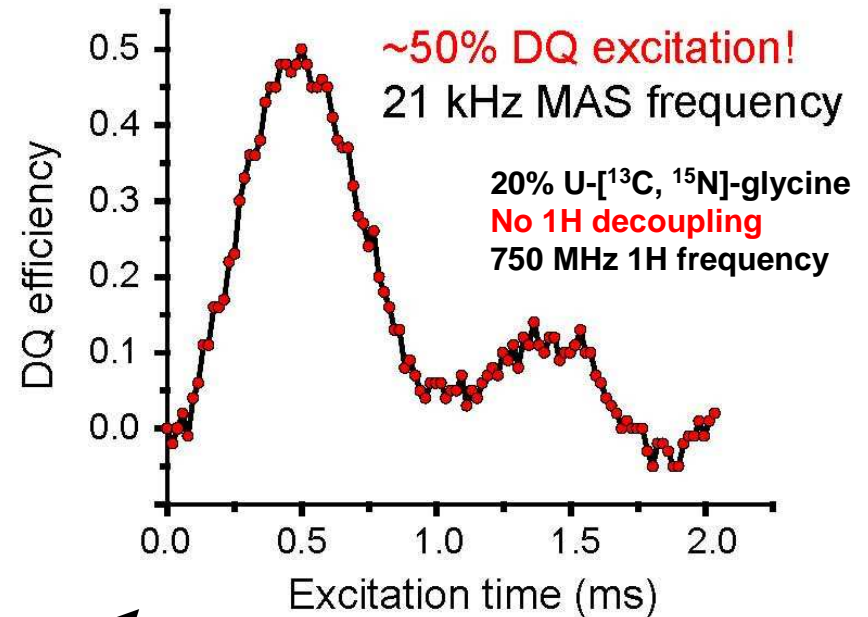
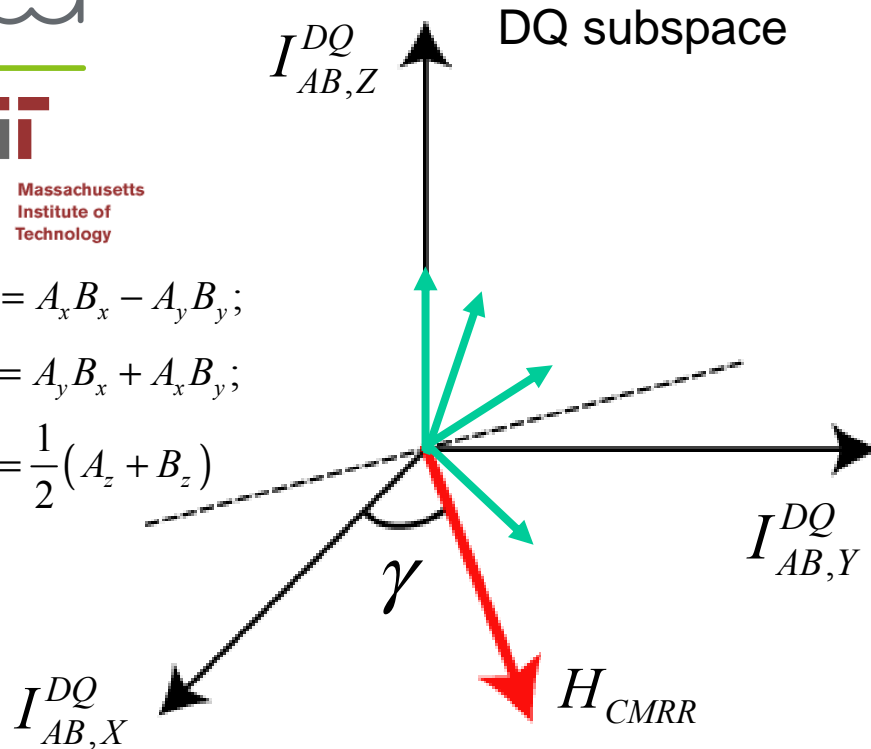


Massachusetts  
Institute of  
Technology

$$I_{AB,X}^{(DQ)} = A_x B_x - A_y B_y;$$

$$I_{AB,Y}^{(DQ)} = A_y B_x + A_x B_y;$$

$$I_{AB,Z}^{(DQ)} = \frac{1}{2} (A_z + B_z)$$



Powder averaging

$$A_z = \left( \frac{A_z + B_z}{2} \right) + \left( \frac{A_z - B_z}{2} \right) \longrightarrow - \left( \frac{A_z + B_z}{2} \right) + \left( \frac{A_z - B_z}{2} \right) = B_z$$

De Paëpe, Lewandowski, Griffin JCP (2008)

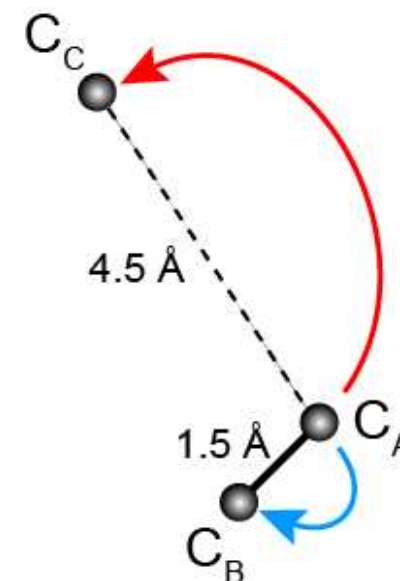
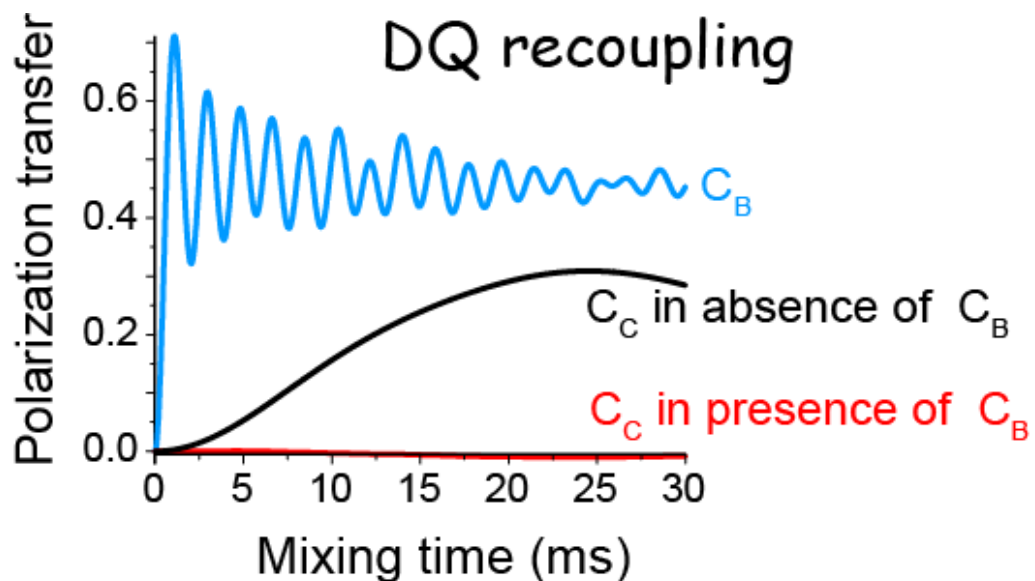
# CMRR and dipolar truncation

$$\overline{H'_{\text{int}}^{(1)}} = \omega'_{AB} \sin(2\beta) \left[ \exp(-i\gamma) T_{2-2}^{AB} + \exp(i\gamma) T_{22}^{AB} \right] \\ + \omega'_{AC} \sin(2\beta) \left[ \exp(-i\gamma) T_{2-2}^{AC} + \exp(i\gamma) T_{22}^{AC} \right] \\ + \omega'_{BC} \sin(2\beta) \left[ \exp(-i\gamma) T_{2-2}^{BC} + \exp(i\gamma) T_{22}^{BC} \right]$$

Non-commuting terms!



Massachusetts  
Institute of  
Technology

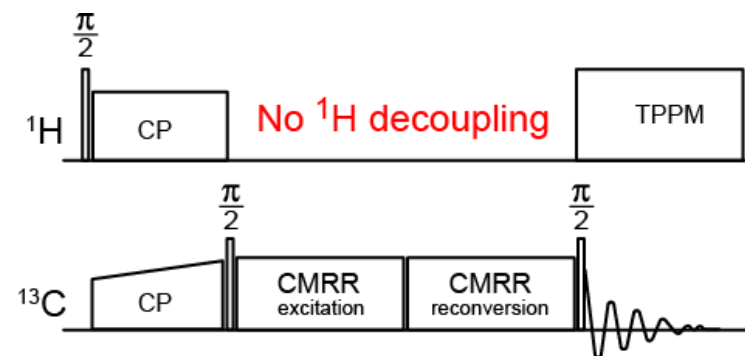
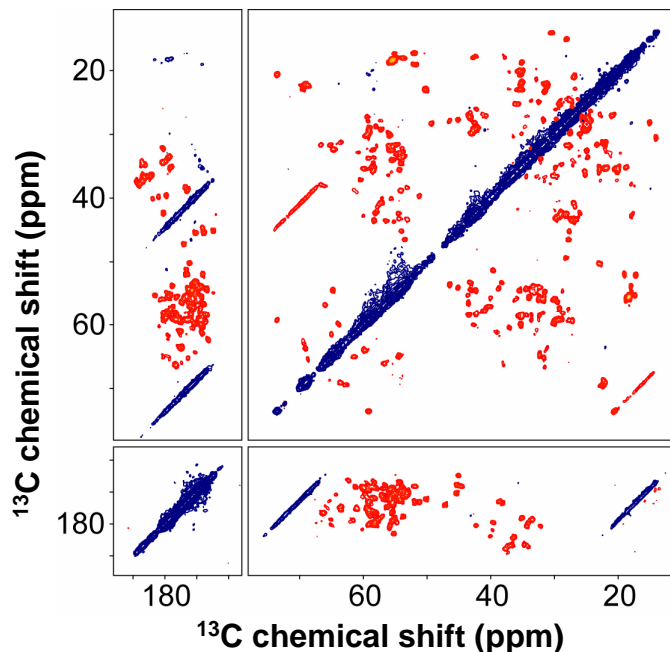


*SPINEVOLUTION, Veshtort et al., JMR (2006) 178.*

- Spin dynamics dominated by one-bond couplings
- very useful for assignment!

# DQ-CMRR recoupling without $^1\text{H}$ decoupling

*Red cross peaks = one-bond transfer only!*



$^{13}\text{C}$ - $^{13}\text{C}$  recoupling without  $^1\text{H}$  decoupling  
20 kHz MAS CM5RR\* on [U- $^{13}\text{C}$ ,  $^{15}\text{N}$ ]-Crh  
750 MHz, 15 hours

\*De Paëpe, Bayro, Lewandowski, Griffin, JACS (2006)  
De Paëpe, Lewandowski, Griffin, JCP 128 (2008)

Properties/advantages:

- Single channel irradiation
- Attenuation of r.f. sample heating
- Efficient at high  $B_0$ , high  $\omega_r/2\pi$

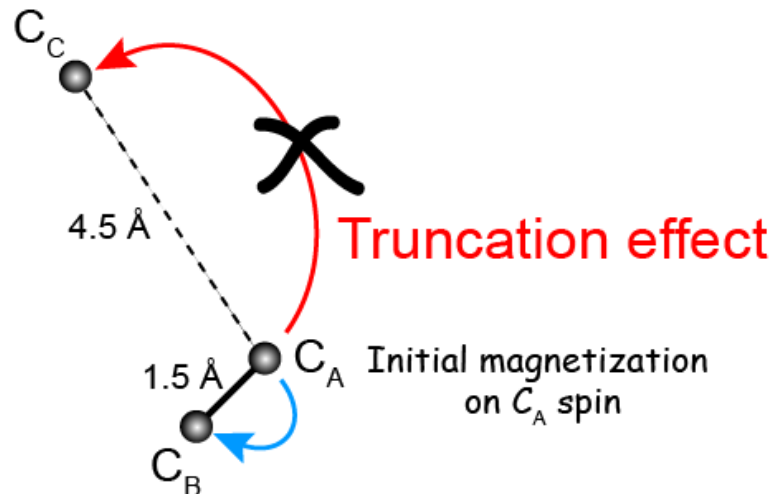
# Challenges to obtain long distance transfer

---

- DQ/ZQ broadband  $^{13}\text{C}$ - $^{13}\text{C}$  dipolar recoupling yields truncation



Massachusetts  
Institute of  
Technology

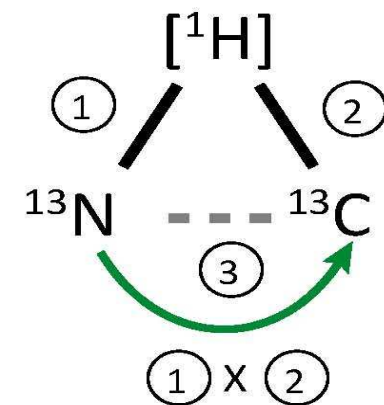
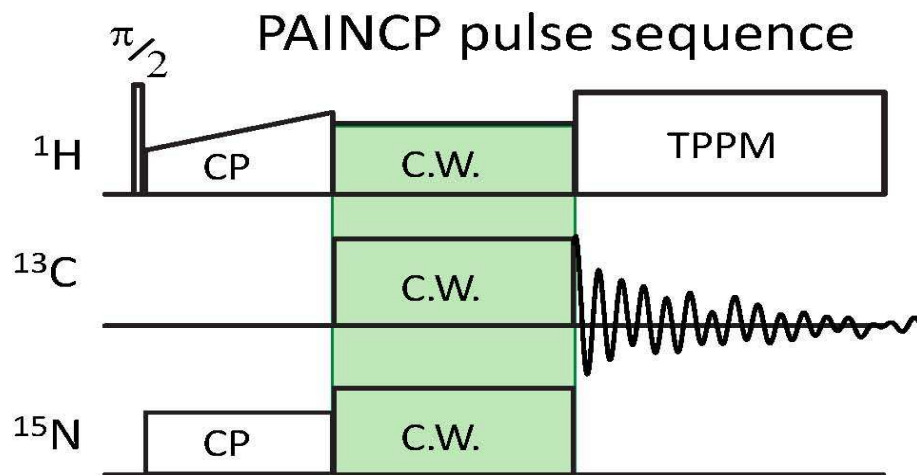
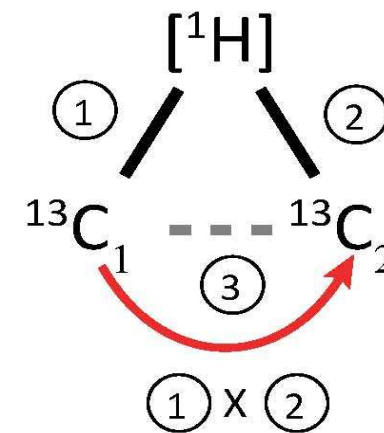
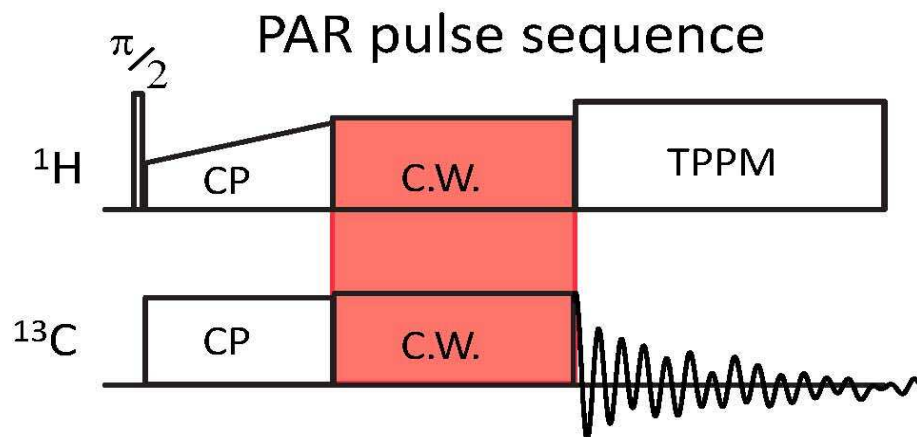


→ useless for long distance transfer!!

## Part II - TSAR mechanism: use of assisting spins!



Massachusetts  
Institute of  
Technology





# Third Spin Assisted Recoupling - Principles

Assisting Spin: e.g. 1H's

$$H_{\text{RF}} = \omega_{1C} A_X + \omega_{1C} B_X + \omega_{1H} H_X$$

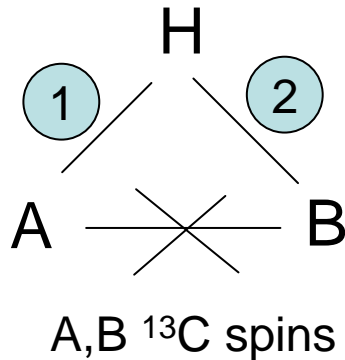
$$= \omega_R (p_C A_X + p_C B_X + p_H H_X)$$

$$U_{\text{RF}} = \exp\left\{-i\left(\underbrace{p_C \omega_R t A_X}_{\text{red}} + \underbrace{p_C \omega_R t B_X}_{\text{red}} + \underbrace{p_H \omega_R t H_X}_{\text{red}}\right)\right\}$$

$$\omega_{ij} = \sum_{m=-2}^2 \omega_{ij}^{(m)} \exp(\underbrace{-im\omega_r t}_{\text{green}})$$



Massachusetts  
Institute of  
Technology



$$H_{\text{int}} = \omega_{AH} 2A_Z H_Z \textcircled{1} + \omega_{HB} 2H_Z B_Z \textcircled{2}$$

$$\xrightarrow{\exp(-i\omega_r X t)} \begin{cases} X_1 = m_1 \pm p_C \pm p_H \longrightarrow \textcircled{1} \\ X_2 = m_2 \pm p_H \pm p_C \longrightarrow \textcircled{2} \end{cases}$$

Average Hamiltonian Theory – Magnus Expansion

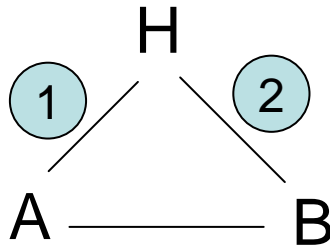
$$\overline{H'} = \overline{H'_{\text{int}}^{(1)}} + \overline{H'_{\text{int}}^{(2)}} + \overline{H'_{\text{int}}^{(3)}} + \dots$$

~~$$\overline{H'_{\text{int}}^{(1)}} = \frac{1}{T} \int_0^T dt_1 H'(t_1) \quad \text{zero if } X \neq 0$$~~

**Let's calculate the second order!**

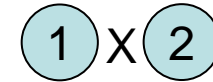
De Paëpe et al., J. Chem. Phys. (2008).

## Second order Average Hamiltonian Theory - TSAR term



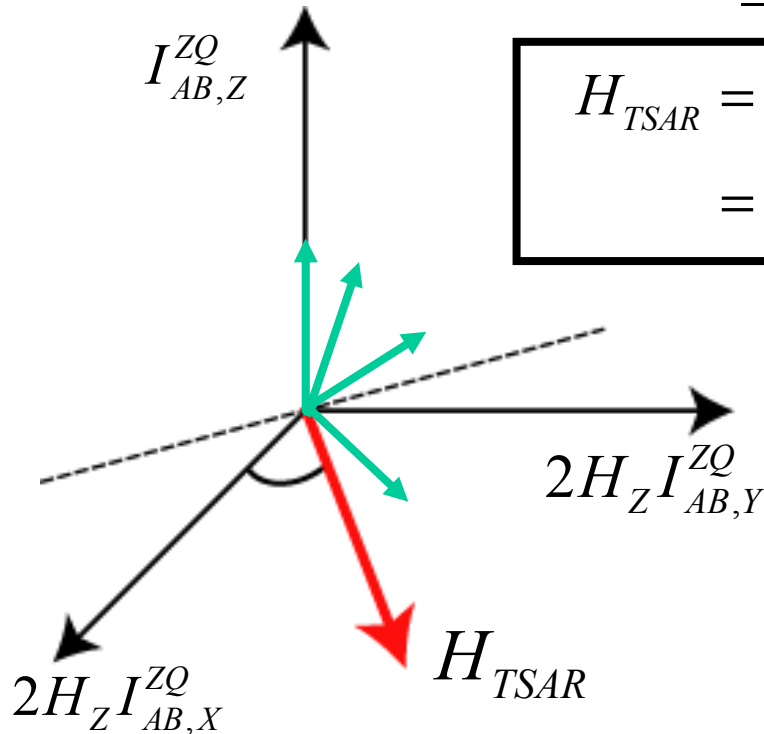
$$\overline{H'_{\text{int}}(2)} = \frac{1}{2iT} \int_0^T dt_1 \int_0^{t_1} dt_2 [H(t_1), H(t_2)]$$

Second order TSAR Hamiltonian



$$H_{TSAR} = 2\omega_{TSAR} A^+ B^- H_Z + 2\omega_{TSAR}^* A^- B^+ H_Z$$

$$= \text{Re}(\omega_{TSAR}) 2I_{AB,X}^{ZQ} H_Z + \text{Im}(\omega_{TSAR}) 2I_{AB,Y}^{ZQ} H_Z$$



$$\omega_{TSAR} = \frac{1}{\omega_r} \left[ \frac{\text{Re}(\omega_{AH}^1 \omega_{HB}^1) \lambda(1, p_C, p_H) + i \text{Im}(\omega_{AH}^1 \omega_{HB}^1) \sigma(1, p_C, p_H)}{\omega_{TSAR}^1} + \frac{\text{Re}(\omega_{AH}^2 \omega_{HB}^2) \lambda(2, p_C, p_H) + i \text{Im}(\omega_{AH}^2 \omega_{HB}^2) \sigma(2, p_C, p_H)}{\omega_{TSAR}^2} \right]$$

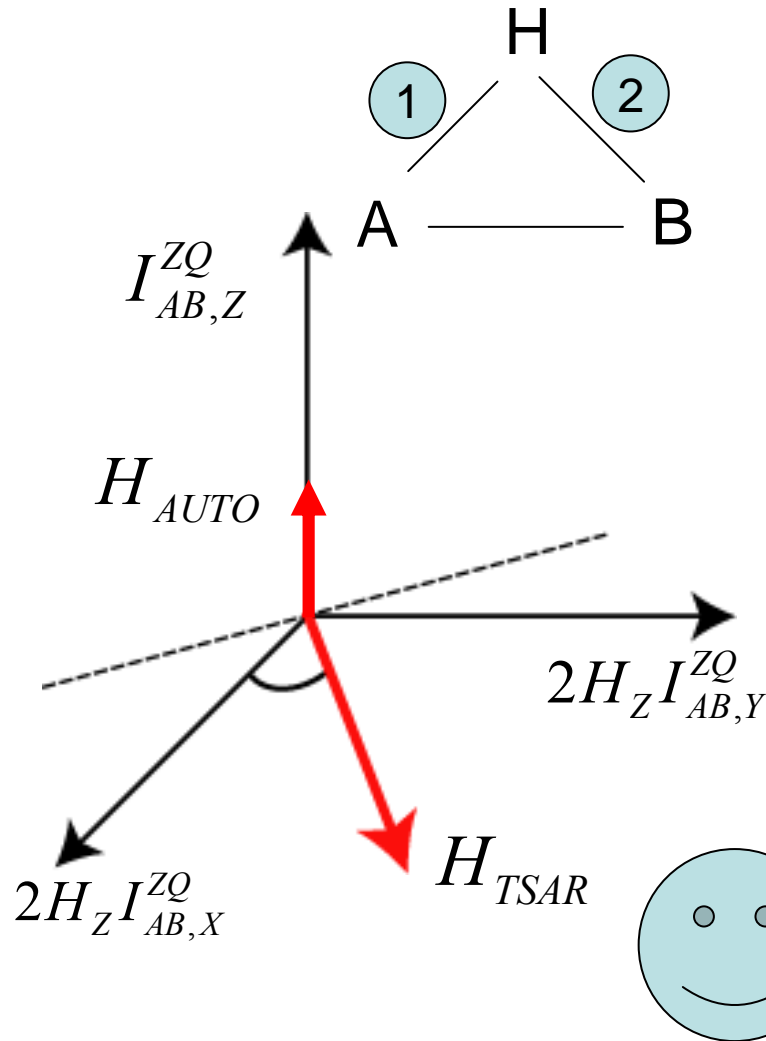
$$\lambda(m, p_C, p_H) = \left( \frac{-(p_C + p_H)}{m^2 - (p_C + p_H)^2} + \frac{-(p_H - p_C)}{m^2 - (p_H - p_C)^2} \right)$$

$$\sigma(m, p_C, p_H) = \left( \frac{m}{m^2 - (p_H + p_C)^2} - \frac{m}{m^2 - (p_H - p_C)^2} \right)$$

•TSAR subspace:

→coupled basis between a fictitious ZQ spin and an assisting proton spin.

# Second order Average Hamiltonian Theory - TSAR term



$$\overline{H'_{int}}^{(2)} = \frac{1}{2iT} \int_0^T dt_1 \int_0^{t_1} dt_2 [H(t_1), H(t_2)]$$

Second order TSAR Hamiltonian

① X ①

② X ②

$$H_{AUTO} = \omega_{AUTO} I_{AB,Z}^{(ZQ)}$$

$$\omega_{AUTO} = \frac{1}{\omega_r} \left[ \begin{aligned} &(\omega_{C_1H}^1 \omega_{HC_1}^{-1} - \omega_{C_2H}^1 \omega_{HC_2}^{-1}) \chi(1, p_C, p_H) \\ &+ (\omega_{C_1H}^2 \omega_{HC_1}^{-2} - \omega_{C_2H}^2 \omega_{HC_2}^{-2}) \chi(2, p_C, p_H) \end{aligned} \right]$$

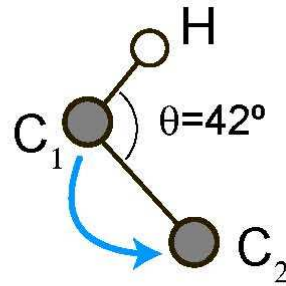
$$\chi(m, p_C, p_H) = -\frac{1}{2} \left( \frac{(p_H + p_C)}{m^2 - (p_H + p_C)^2} - \frac{(p_H - p_C)}{m^2 - (p_H - p_C)^2} \right)$$

$$\chi(m, p_C, p_H) = 0 \quad \text{if} \quad p_H = \sqrt{p_C^2 - m^2}$$

- Auto-cross terms leads to longitudinal off-resonance contributions!
- But these off-resonance contributions can be minimized!

# Analytical versus numerical simulations

Homonuclear case



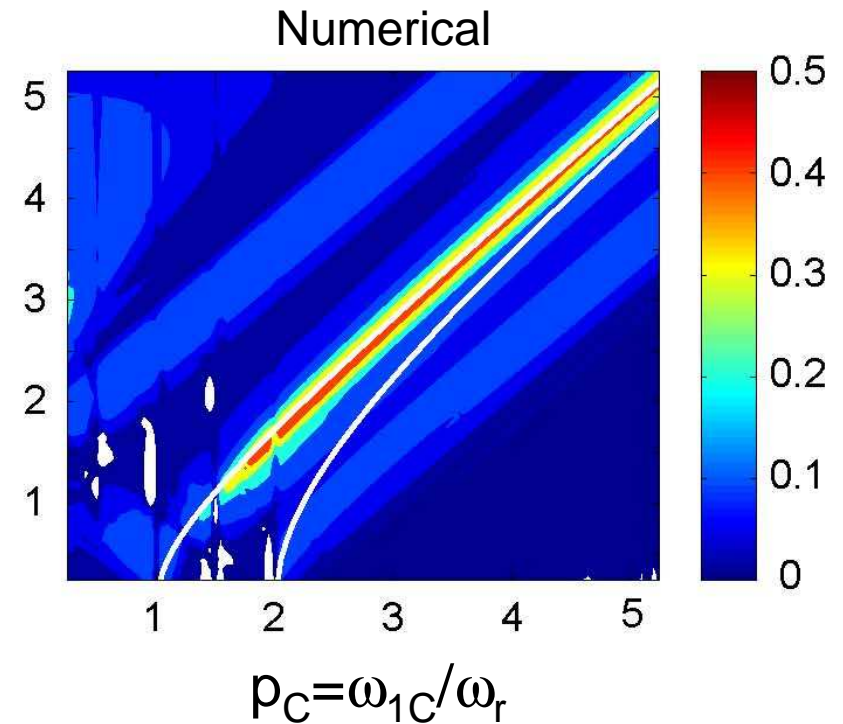
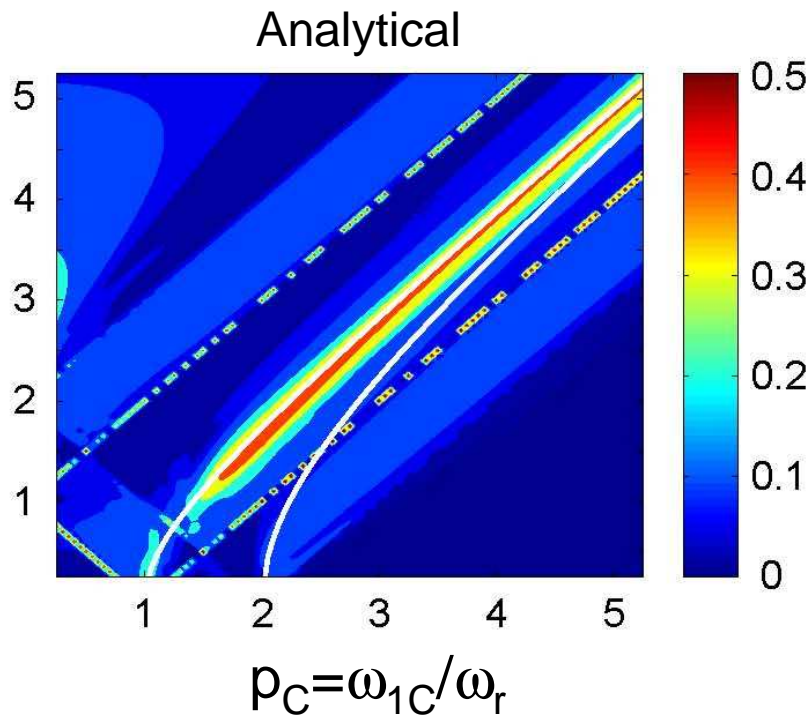
$$p_H = \sqrt{p_C^2 - 1}$$

$$p_H = \sqrt{p_C^2 - 4}$$



Massachusetts  
Institute  
Techno

$p_H = \omega_{1H}/\omega_r$



- Second order AHT explains the numerical maps!

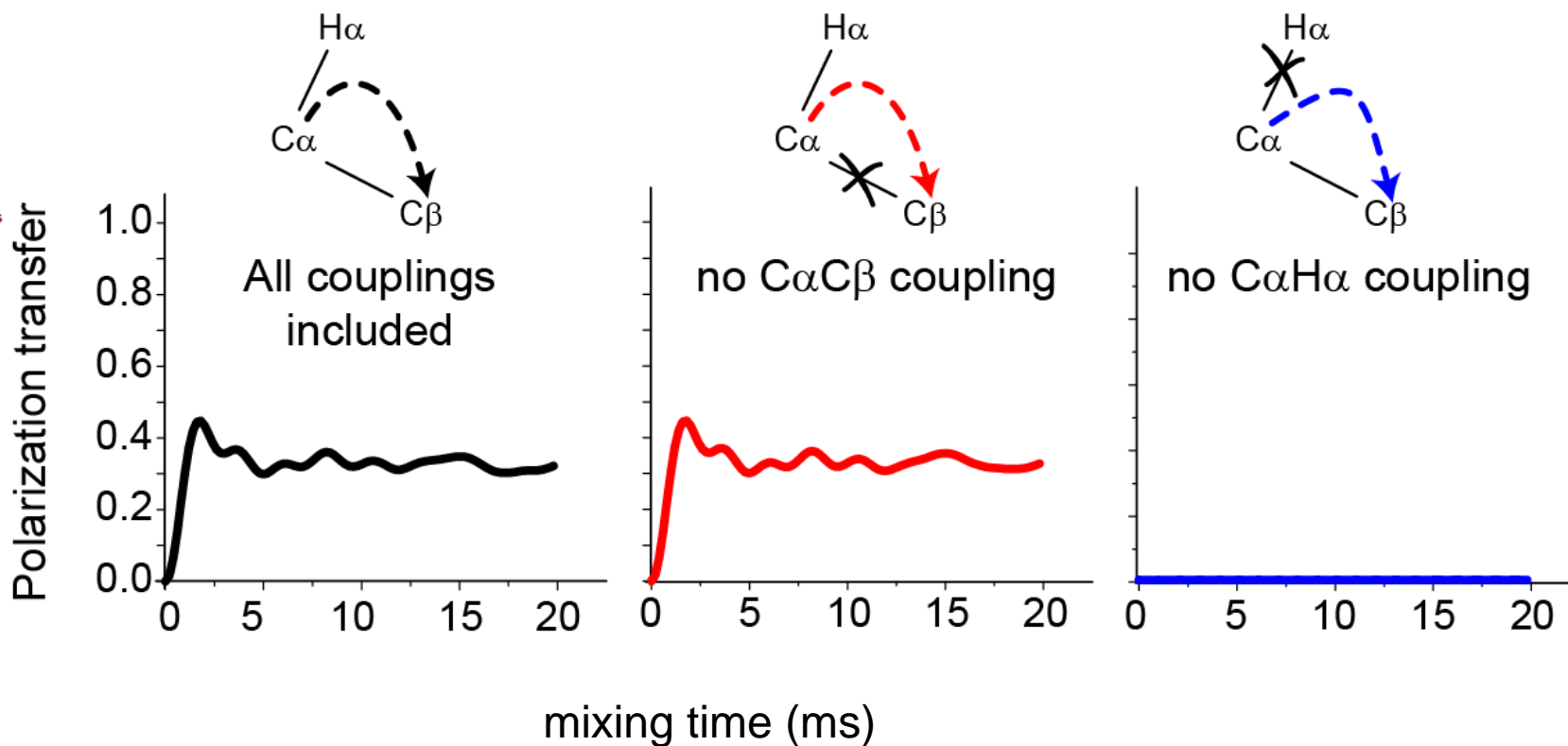
# Insight in the PAR experiment

Homonuclear case

$\omega_r/2\pi=20$  kHz  $\omega_0/2\pi=750$  MHz

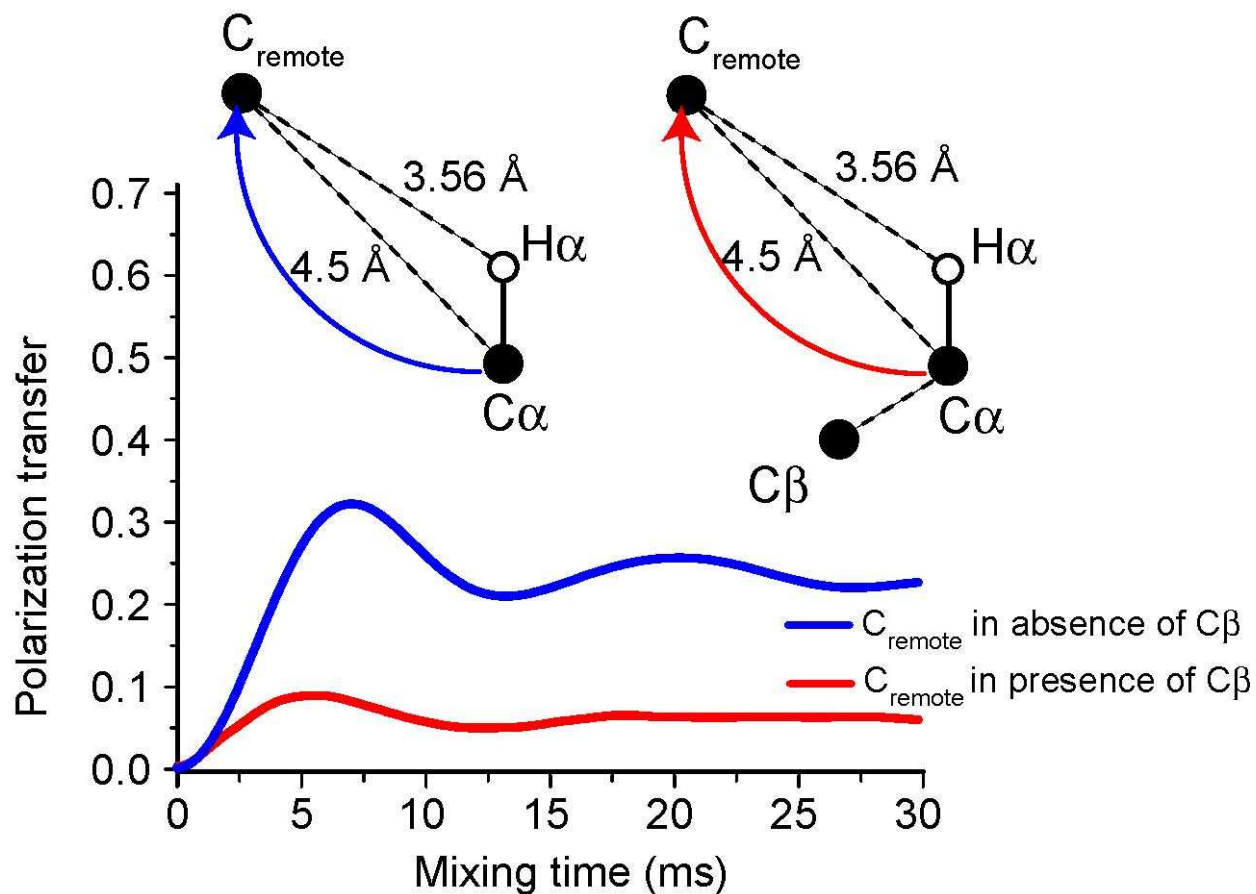


Massachusetts  
Institute of  
Technology



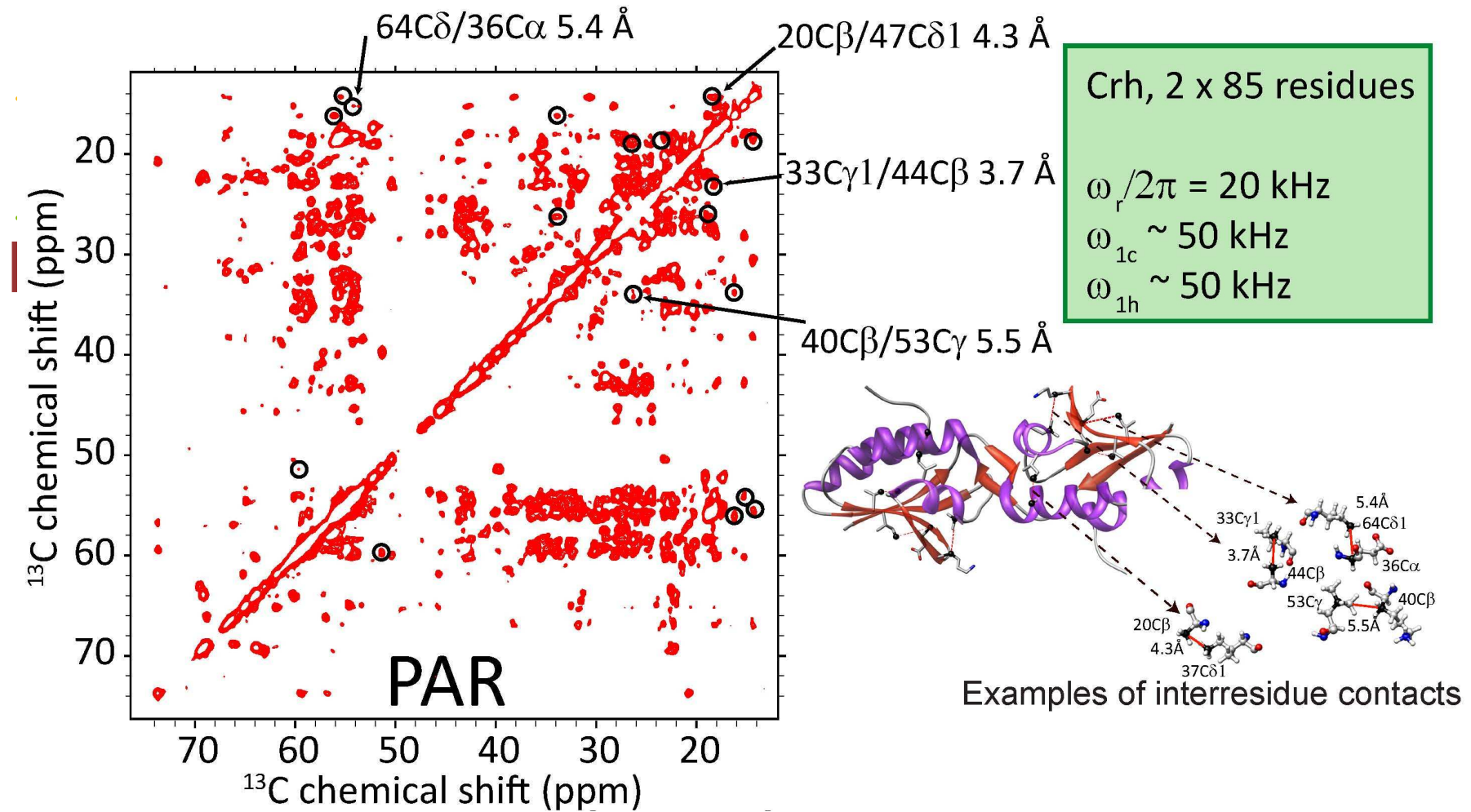
- PAR recoupling does not rely on CC coupling!

# PAR versus dipolar truncation



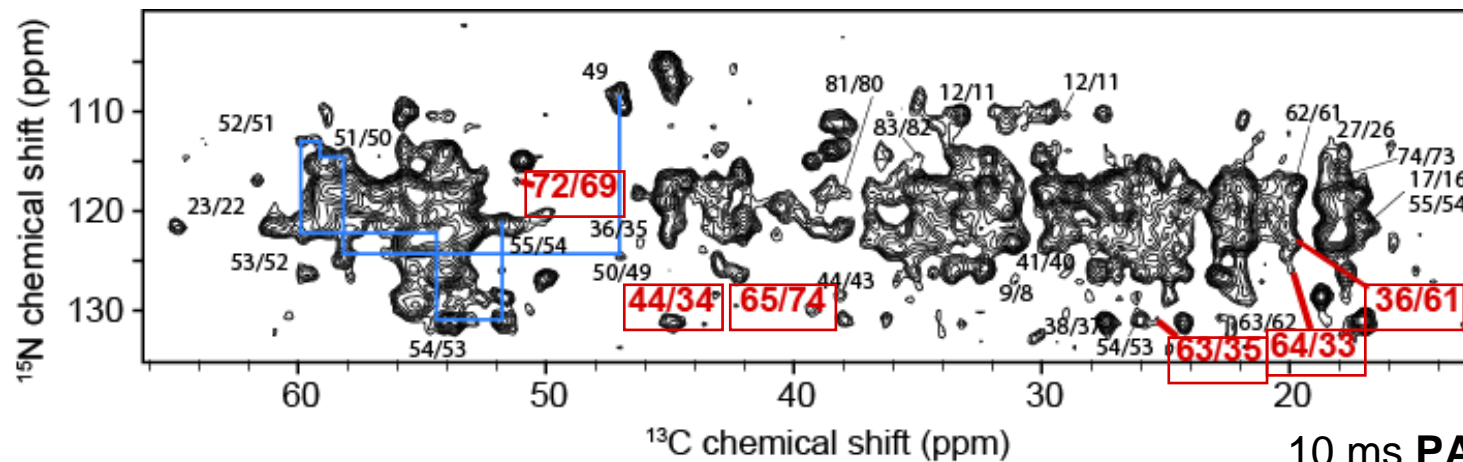
- Long distance transfer ( $\sim 4.5 \text{ \AA}$ ) in presence of directly bonded carbon: **reduction of the dipolar truncation!**

# PAR on [U-<sup>13</sup>C,<sup>15</sup>N]-Crh at 750 MHz, 20kHz MAS

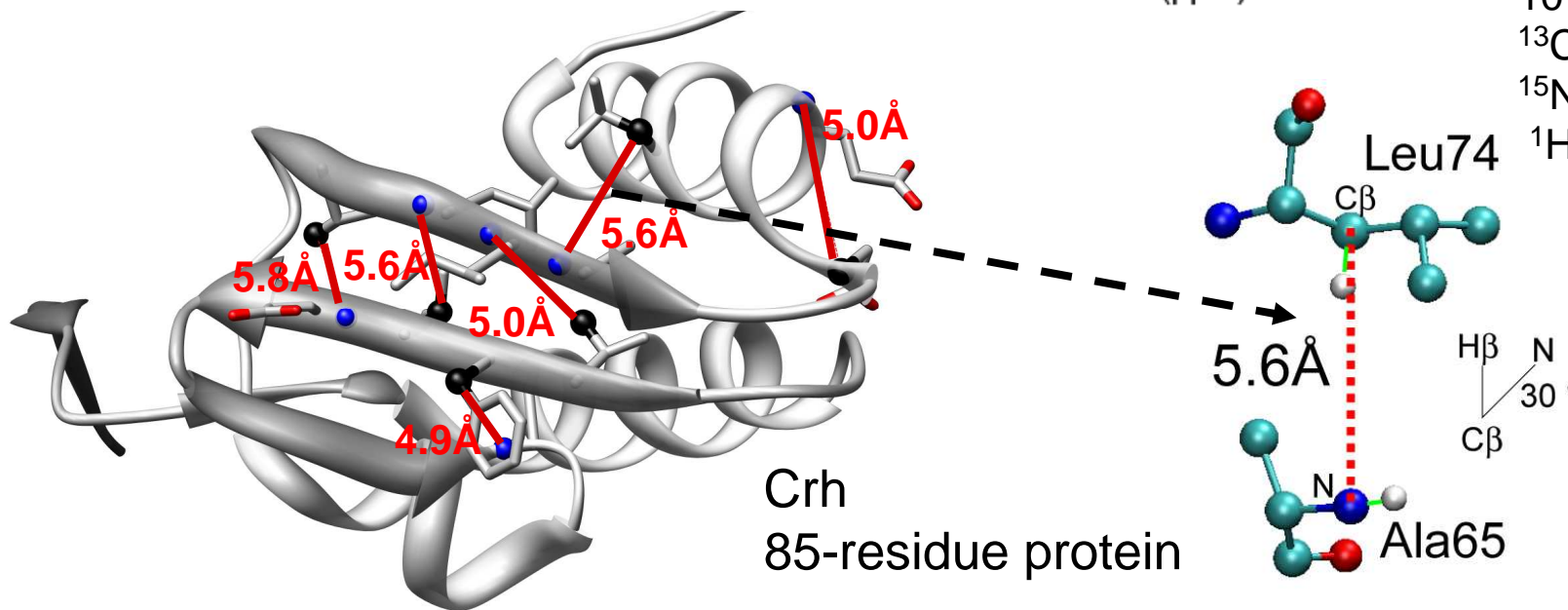


- Long distance transfer in **uniformly labeled protein!**

# PAINCP on [U-<sup>13</sup>C,<sup>15</sup>N]-Crh at 750 MHz, 20kHz MAS



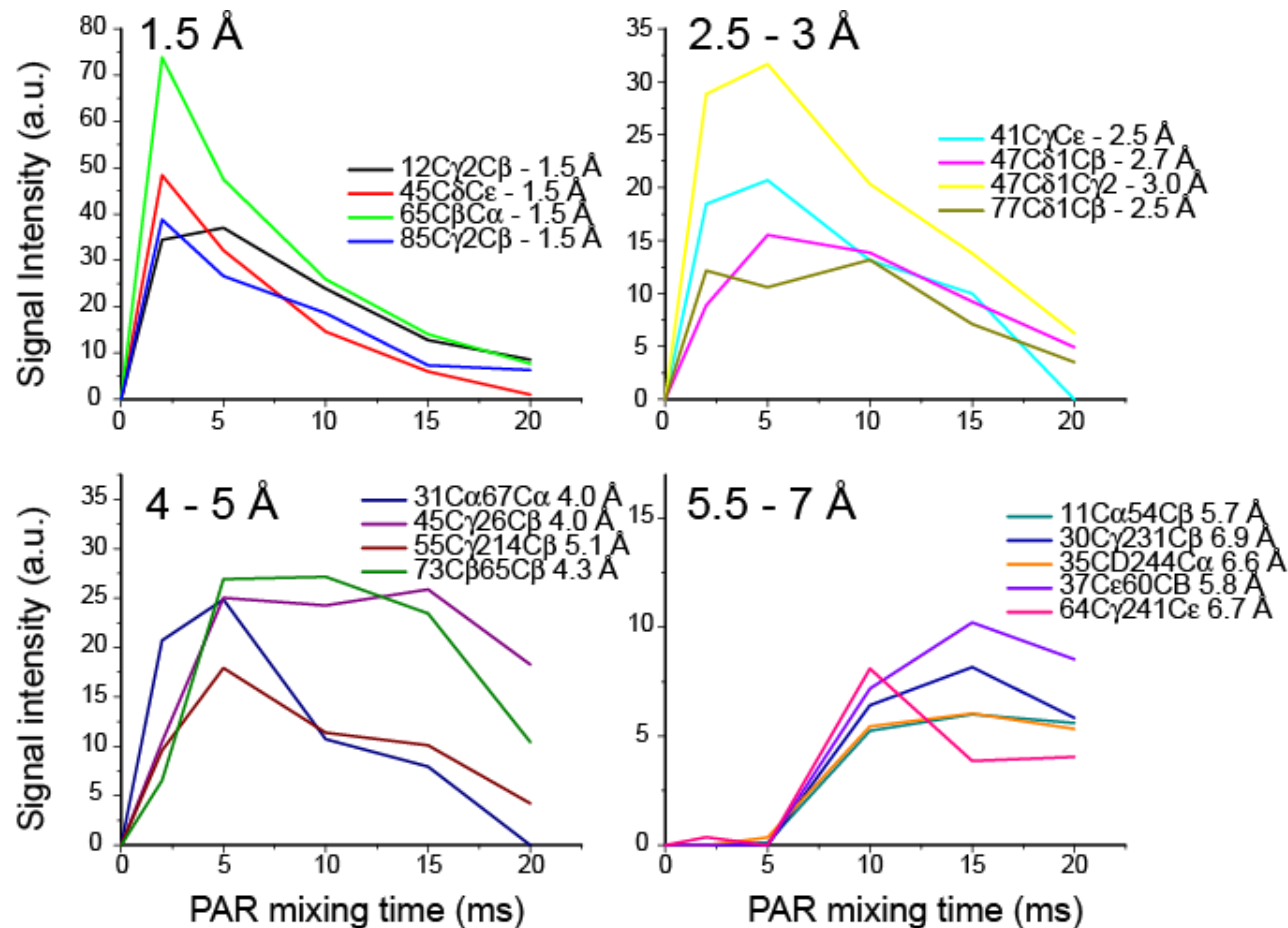
10 ms **PAINCP**  
<sup>13</sup>C rf - 15 kHz  
<sup>15</sup>N rf - 15 kHz  
<sup>1</sup>H rf - 53 kHz



- >5Å <sup>15</sup>N-<sup>13</sup>C contacts between secondary structure elements



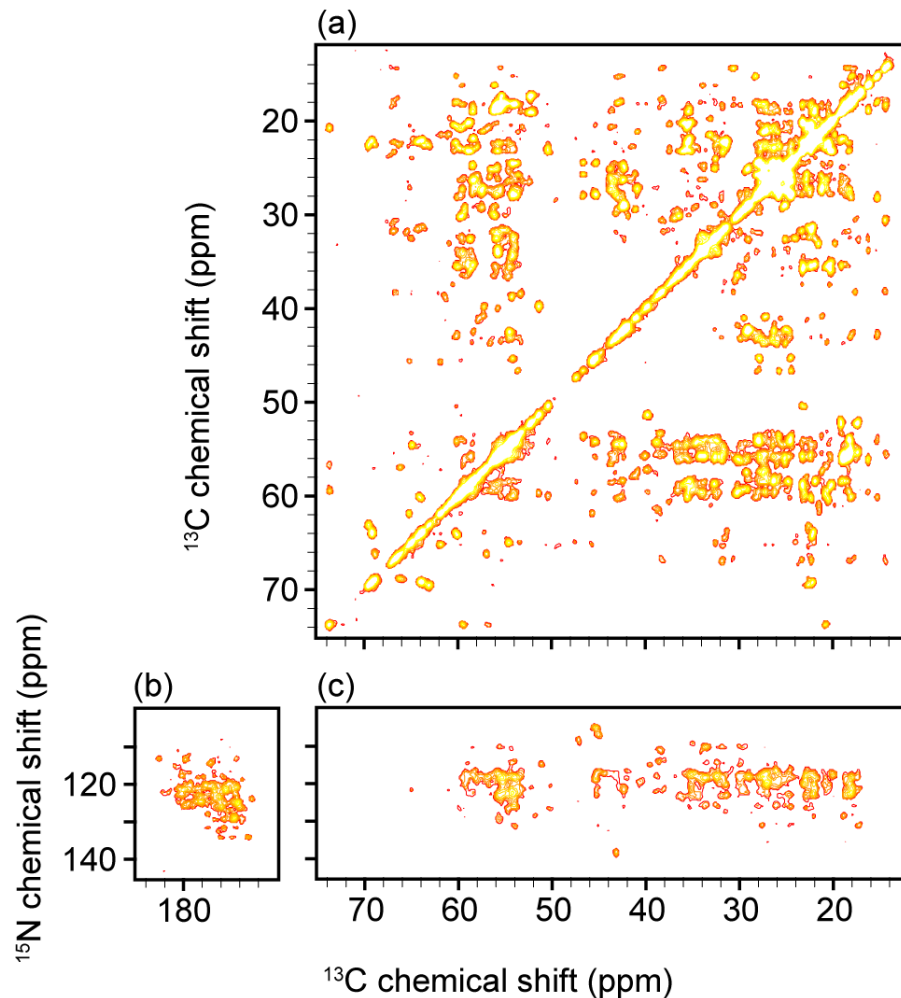
# Structure determination by SSNMR



PAR 900 MHz  
 2, 5, 10, 15 & 20 ms  
 PAR irradiation  
 $\omega_r/2\pi = 20$  kHz  
 $\omega_{1C}/2\pi \sim 50$  kHz  
 $\omega_{1H}/2\pi \sim 50$  kHz

- Observed buildups can be categorized into different distance classes
- Upper bond distance can be estimated

# De novo structure determination – Crh dimer (2 x 10.4 kDa)



**PAR & PAINCP @ 900 MHz**

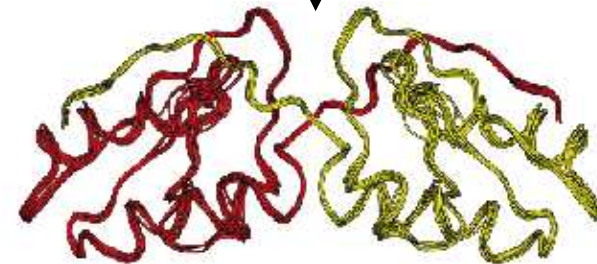
Collaboration with A. Böckmann et al. at IBCP, France

Crh assignment (e.g. CMRR)  
+  $^{13}\text{C}$ - $^{13}\text{C}$  PAR (~15 ms)  
+  $^{15}\text{N}$ - $^{13}\text{C}$  PAINCP (~15 ms)

ARIA

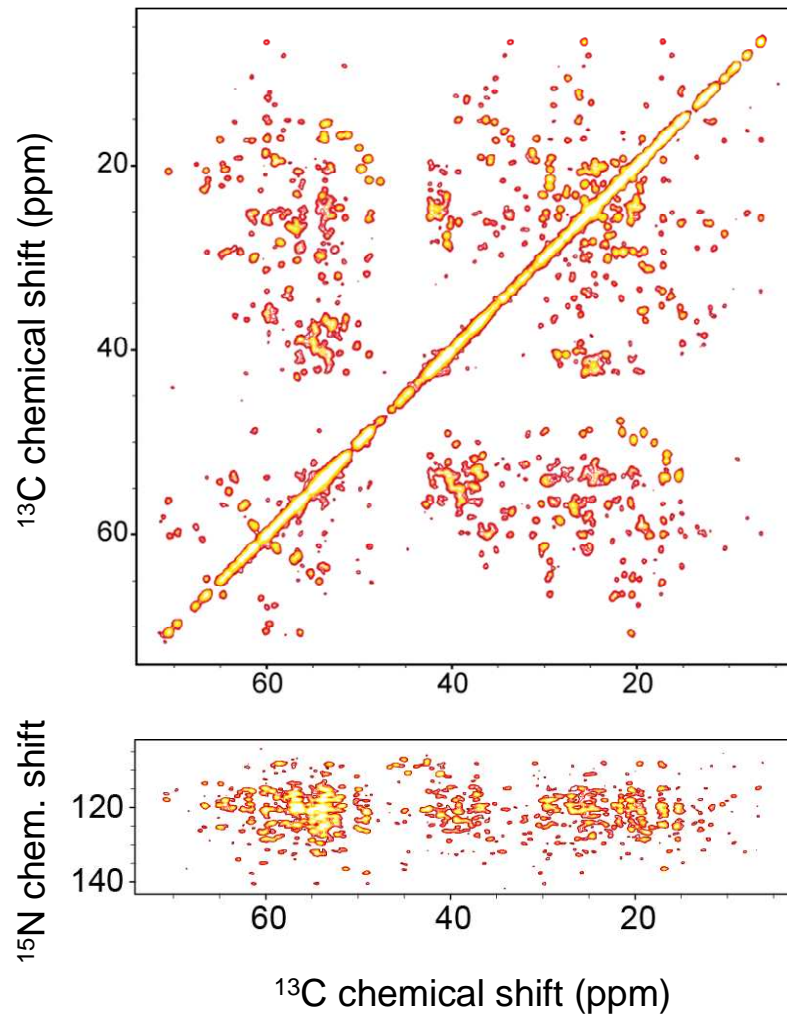
795 unambiguous CC constraints  
(**269 long range**)  
400 unambiguous NC constraints  
(**130 long range**)

TALOS  
+ XPLOR-NIH



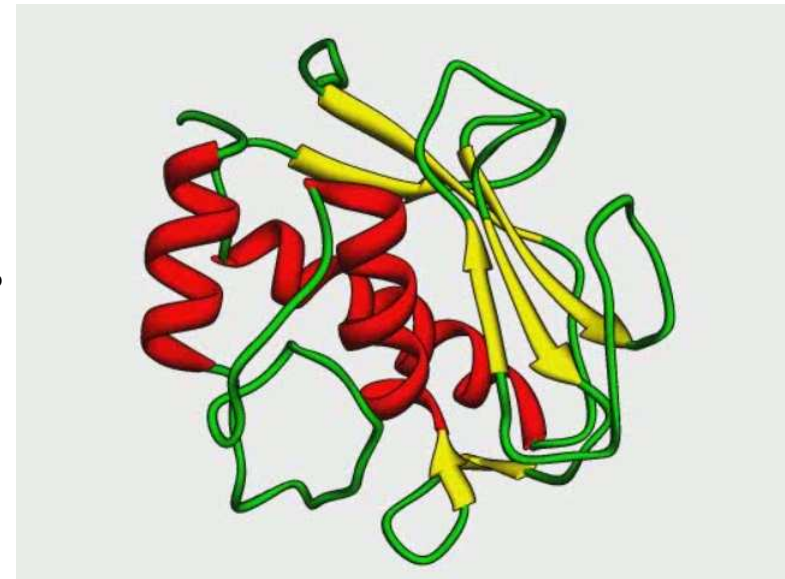
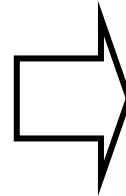
RMSD 0.58 Å

# De novo structure determination – MMP-12 (17.6 kDa)



$$\omega_r / 2\pi = 20 \text{ kHz} \quad \omega_{1H} / 2\pi = 900 \text{ MHz}$$

PAR + PAINCP



**MMP-12 protein (17.6 kDa)**

Collaboration with I. Bertini et al. at CERM, Italy

- Application to larger and larger systems...

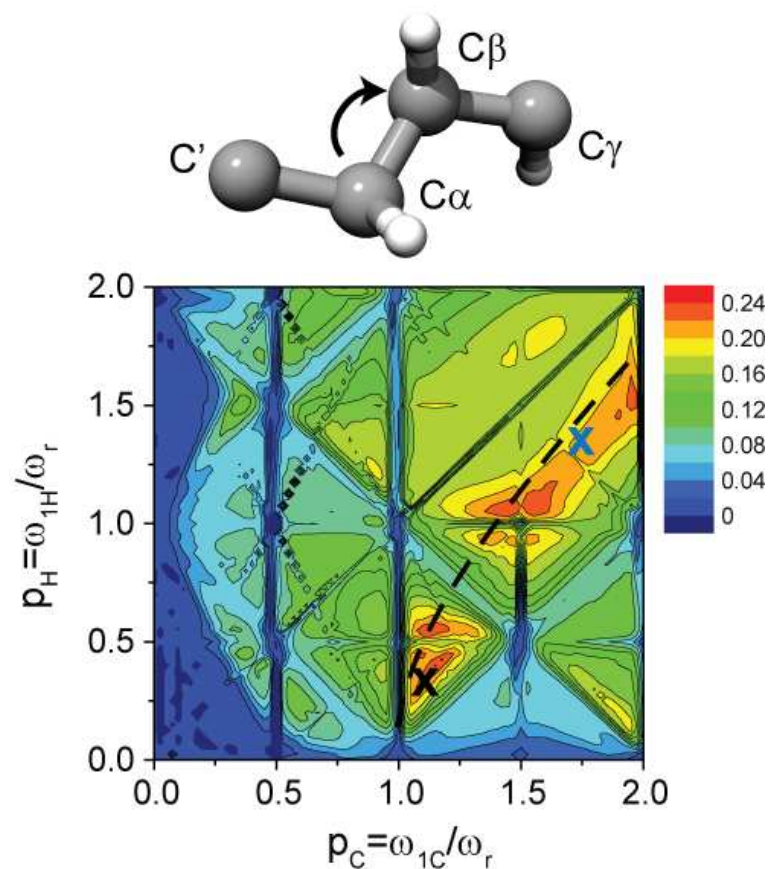
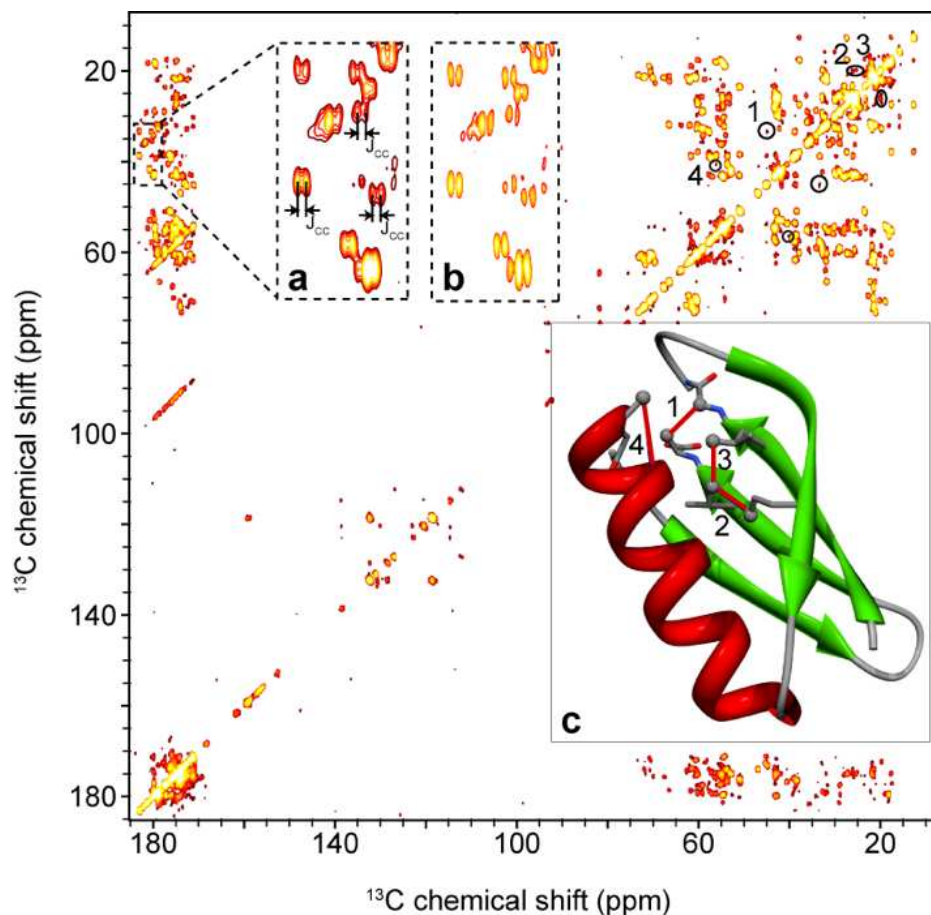
# Conclusions

---



- DQ-CMRR efficient one-bond relayed recoupling sequence for assignment (**without  $^1\text{H}$  decoupling**)
- TSAR recoupling:
  - $^{13}\text{C}$ - $^{13}\text{C}$ ,  $^{15}\text{N}$ - $^{15}\text{N}$  or  $^{15}\text{N}$ - $^{13}\text{C}$  recoupling **assisted by surrounding protons**
  - Very efficient for short, medium and **long distance** transfer
  - Applicable to ***de novo* structure determination**
- Promising techniques for *de novo* atomic structure determination of challenging systems: membrane proteins, fibrils, etc... **that are not accessible by other techniques.**

# $^{13}\text{C}$ - $^{13}\text{C}$ PAR at 65 kHz MAS Frequency on $[\text{U-}^{13}\text{C}, \text{U-}^{15}\text{N}]\text{GB1}$

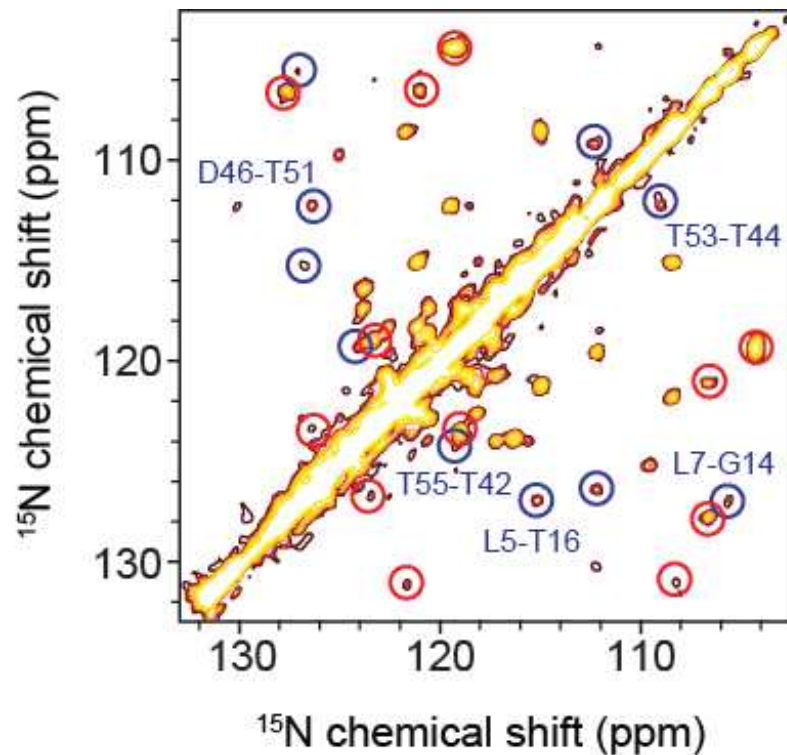


Collaboration with W. Maas et al. at Bruker, MA

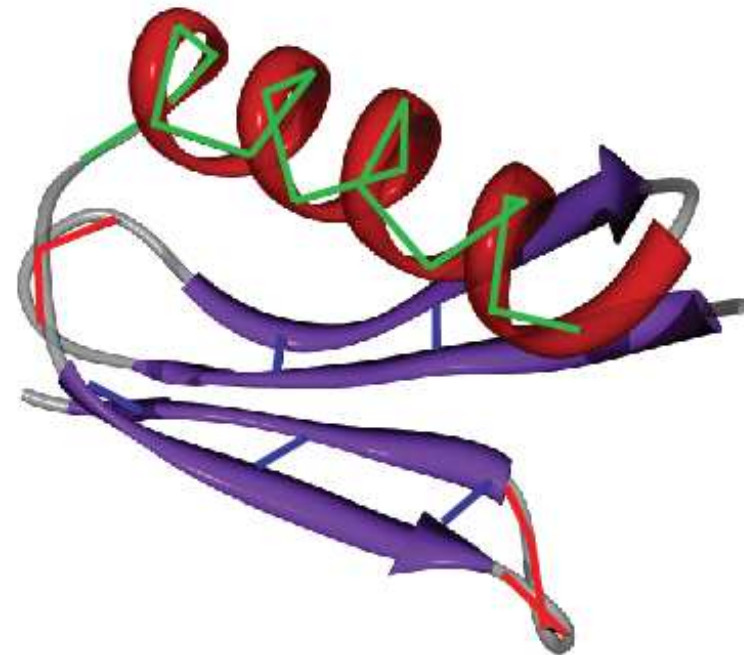
- **Despite being a second order recoupling sequence, PAR is still applicable at 65 kHz MAS !**

# $^{15}\text{N}$ - $^{15}\text{N}$ PAR on $[1,3\text{-}^{13}\text{C},\text{U-}^{15}\text{N}]\text{GB1}$

---



Lewandowski *et al.* (2008) JACS



**18 ms mixing time**

**$\omega_r/2\pi = 20\text{kHz}$ ,  $\omega_{0\text{H}}/2\pi = 900\text{MHz}$**

- direct information about secondary and tertiary structure
- identification of  $\alpha$ -helix and connectivity of strands in  $\beta$ -sheets

# Acknowledgment

---



## MIT, Cambridge (MA):

Jozef Lewandowski

Matt Eddy

Marvin Bayro

Robert Griffin



## CEA-INAC, Grenoble (FR)

Mathilde Giffard

Sabine Hediger

Michel Bardet

Guillaume Gerbaud



## IBCP, Lyon (FR):

Antoine Loquet

Simon Megy

Carole Gardiennet

Anja Böckmann



## CERM, Florence (IT):

Moreno Lelli

Anusarka Bhaumik

Claudio Luckinat

Ivano Bertini **CERM**

## Bruker, Billerica (MA):

Jochem Struppe

Werner Maas

