מכון ויצמן למדע
WEIZMANN INSTITUTE OF SCIENCE

Quantum control of thermodynamic bounds

D. Gelbwaser-Klimovsky, G. Bensky, M. Kolar, N. Erez, R. Alicki & G.K.

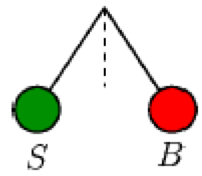
Concept – Progress in quantum technologies is restricted by our ability to minimize environment effects. Instead, *take advantage* of the environment by *non-unitary* manipulations. Yet open-system manipulations must be optimized *within thermodynamic bounds*.

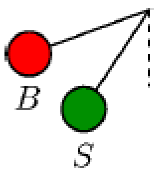
Conceptual difficulty – Thermodynamic bounds are not well understood the system far from equilibrium and are fast enough to break the Markov approximation. These manipulations violate the traditional paradigm of thermodynamics, i.e. *system-bath separability*.

Bounds considered fundamental must be revisited, e.g., the Szilard-Landauer bound on work for information tradeoff, or the Carnot efficiency bound of an engine.

Means: Q. control of steady-state & transient open-system dynamics.

Qubit Evolution



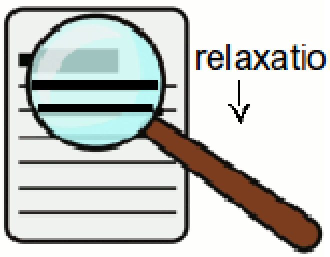


$$H_{SB} = \sum_k \kappa_k \left(\underbrace{(b_k\sigma_+ + b_k^\dagger\sigma_-)}_{RW(\text{Freq. difference})} + \underbrace{(b_k\sigma_- + b_k^\dagger\sigma_+)}_{CR(\text{Freq. sum})} \right)$$

AZE

Long times

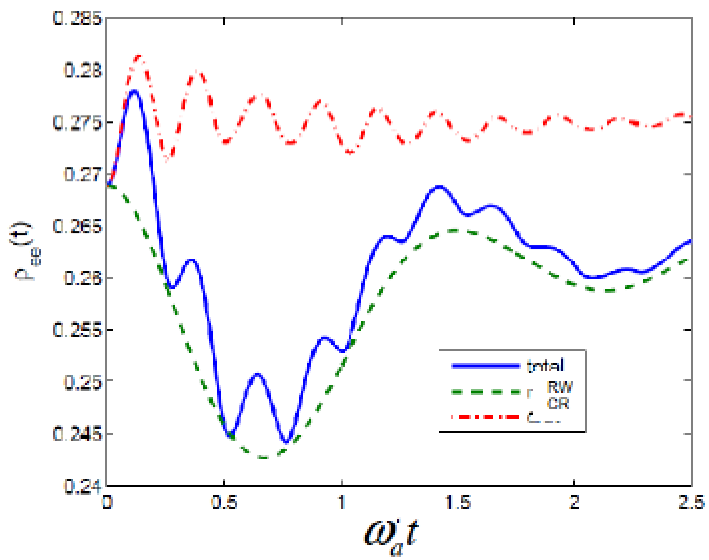
Resolved energy levels



Nature **405** 546 (2000)

$\dot{\rho}_{ee} = -R_e(t)\rho_{ee} + R_g(t)\rho_{gg} < 0$

May yield cooling



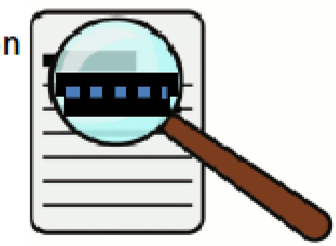
Nature **452**, 724 (2008)

New J. Phys. **11** 123025 (2009)

QZE

Ultrashort times

Unresolved energy levels

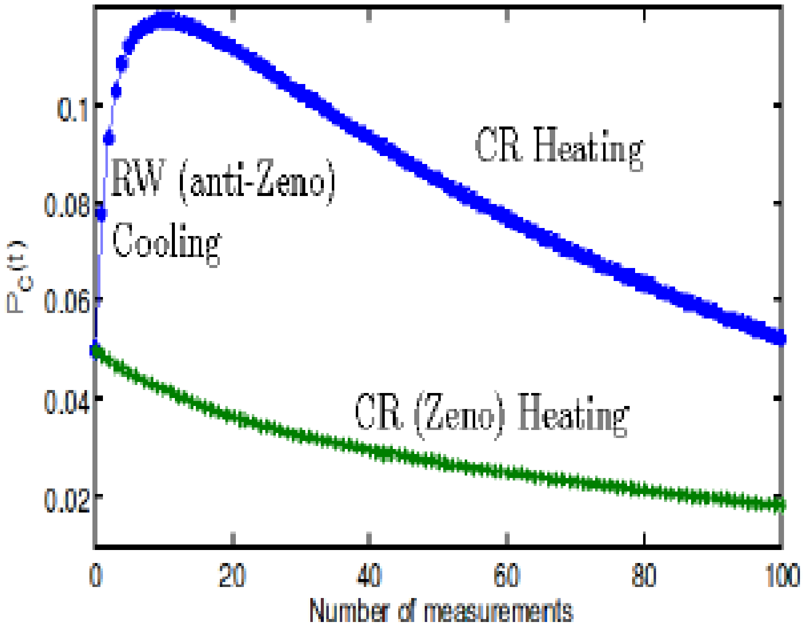


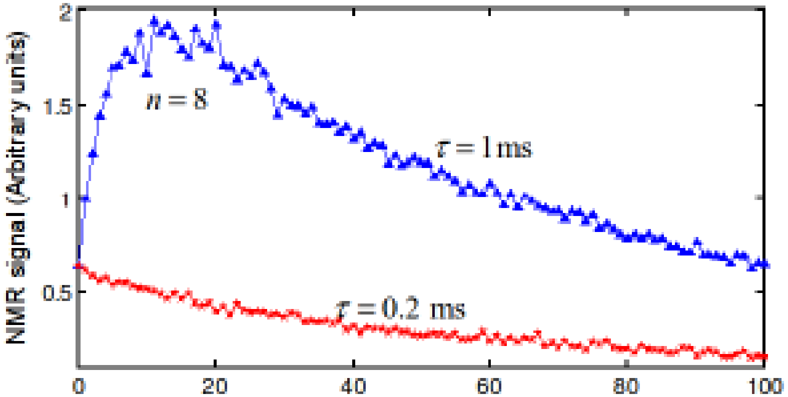
$\dot{\rho}_{ee} \xrightarrow[t\to 0]{} R(t)(\rho_{gg} - \rho_{ee}) > 0$

Always yields heating

Measurement-driven control of quantum bits in a spin-bath

G. Alvarez, D. Dasari, L. Frydman & GK *PRL* 104 *040401 (2010)*



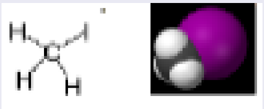


Interaction

$H_{SB} = J_{CH} \sum_k \hat{S}^x \hat{I}_k^x$ (CR+RW)

$P_C(0) = 0.05$, $P_H(0) = 0.2$ } non-equil

Experimental parameters



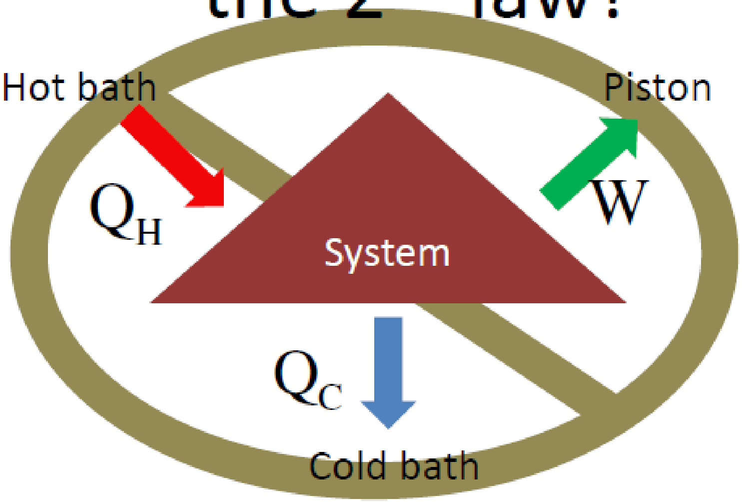
13C-methyl iodide (Iodomethane)

$J_{CH} = 150$Hz; $\frac{\gamma_H \omega_H}{\gamma_C \omega_C} = 2$ (off-resonant)

Induced Dephasings amplify the polarization transfer

No Born: bath changes till $[\rho_{eq}, H_{tot}^{RW}] \approx 0$

Does non-Markovian dynamics contradict the 2nd law?



- Kelvin:

No work can be extracted from a single- bath engine in a cycle. ($\rho_S(t_0) = \rho_S(t_f)$)

What about the second law?

- Single bath engine: a measurement that does not change the system or the bath states
- The initial state is not equilibrium

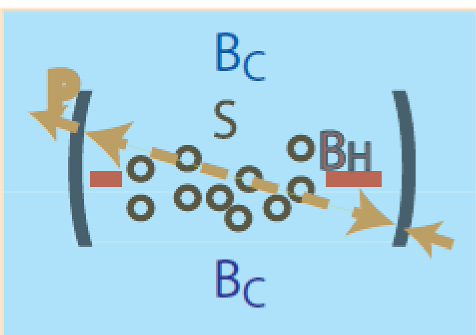
$$\rho_S \otimes \rho_B \neq Z^{-1} e^{-\beta H_{Tot}}$$

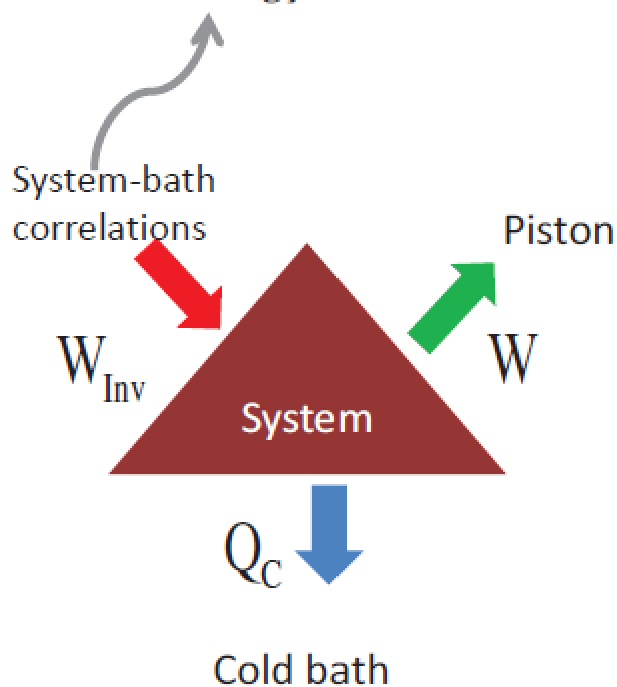
$$Z^{-1} e^{-\beta H_{Tot}} \xrightarrow{\text{Measurement}} \rho_S(0) \otimes \rho_B \xrightarrow{\text{Cyclic\_\_\_Modulation}}$$

$$\rho_S(0) = \rho_S(\tau)$$

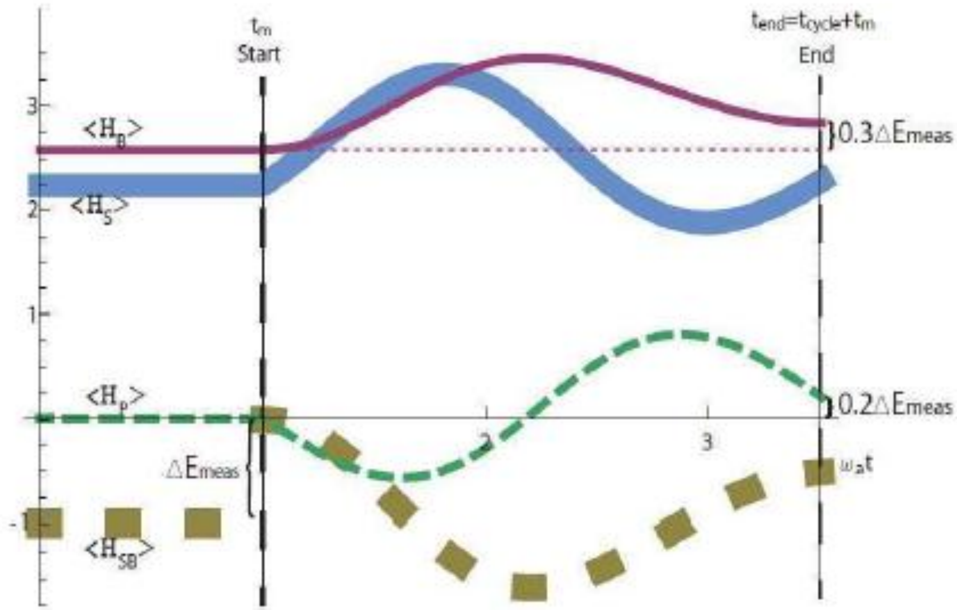
$$\Delta E = \langle \Delta H_B \rangle + \langle \Delta H_P \rangle + \langle \Delta H_{SB} \rangle$$

Energy resource



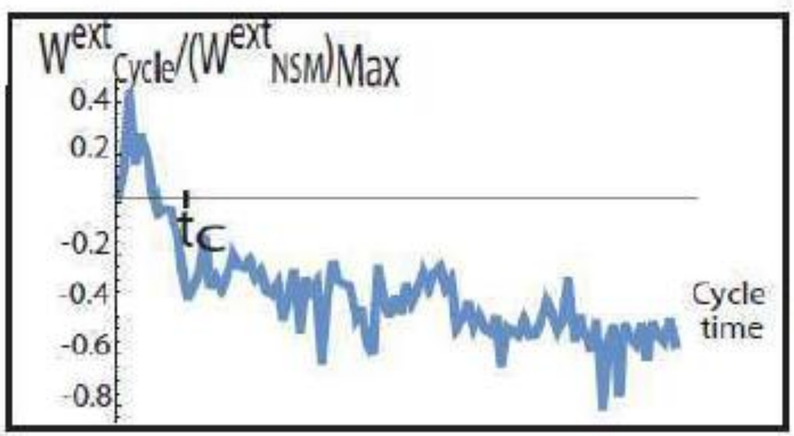


Where does the work come from?



Work extraction only for short cycles

$$W^{ext}_{tot} = -\Delta E_{meas} + W^{ext}_{cycle}$$



Work-Information relation

Szilard-Landauer bound: $(W_{Sel})_{Max} = T\mathcal{H}(\rho_S)$ → Shannon Entropy

- No correlations between system and bath $\rho_S \otimes \rho_B$
- Zero work at zero temperature

By contrast, our bound $(W_{Sel})_{Max} = T\mathcal{H}(\rho_S) + (W_{non-Sel})_{Max}$

- Correlations between the system and the bath are the source: $\rho_{tot}^{eq} \neq \rho_S \otimes \rho_B$
- More work is obtained but higher price is paid for performing the measurement
- Work can be extracted even at zero temperature

$$W_{non-sel} \neq 0 \Rightarrow W_{sel} \neq 0$$

Steady stae under QND control

New J. Phys. **11** 123025 (2009)
New J. Phys. **12** 053033 (2010)

Master Eq. $\dot{\rho}_{ee} = R_G(t)\rho_{gg} - R_e(t)\rho_{ee}$

Solution for n measurements (QND disturbances)

$$\rho_{ee}(n\tau) = e^{-nJ(\tau)}\rho_{ee}(0) + \left(1 - e^{-nJ(\tau)}\right)\chi(\tau)$$

fixed point

$$\chi(\tau) = \frac{\int_0^\tau dt e^{J(t)} R_g(t)}{\int_0^\tau dt e^{J(t)}(R_g(t)+R_e(t))}$$

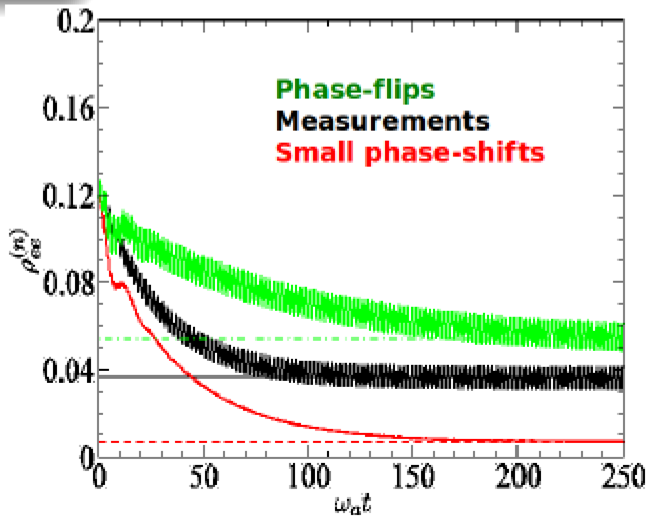
Relax. integral

$$J(t) = \int_0^t dt'(R_g(t')+R_e(t'))$$

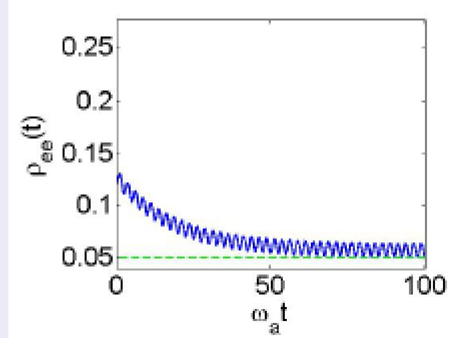
$$\tau \gtrsim \omega_a^{-1} \Rightarrow$$

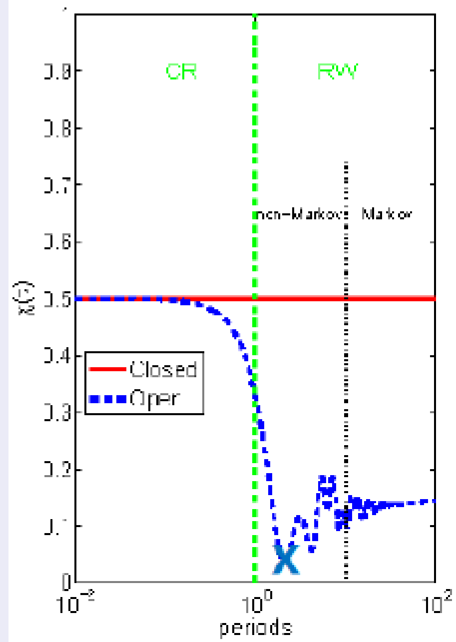
$$\rho_{ee} \approx \chi \ll \rho_{ee}(0) :$$

AZE cooling



Cooling





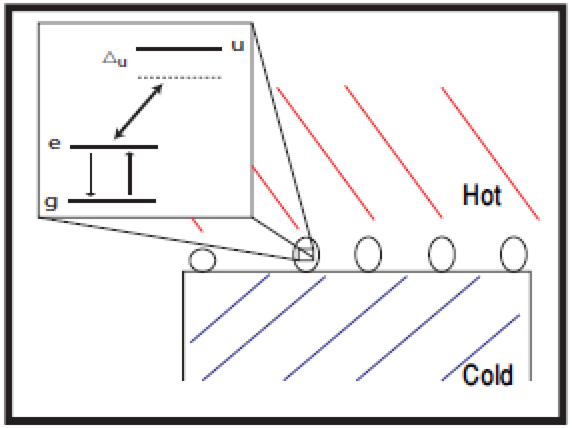
Coarse-graining of non-Markov ME

(A.G. Kofman & G.K., PRL 2001,2004; G. Gordon, N.Erez & G.K. ,J. Phys B(2007))

Start at equilibrium: $\dot{\rho}_{ee}(t) = -\dot{\rho}_{gg}(t) = R_g(t)\rho_{gg} - R_e(t)\rho_{ee}$

Polarization: $S \equiv (\rho_{ee} - \rho_{gg})/2,$

$$R_{e(g)} t_c \ll 1 : \text{weak coupling, slow change}$$



Polarization at quasi steady-state

$$\overline{S} = \overline{S}^C + \overline{S}^H, \quad \overline{R}_{e(g)} = \overline{R}^C_{e(g)} + \overline{R}^H_{e(g)}$$

$$\dot{\overline{S}}^{C(H)} = -\left[\overline{R_g} + \overline{R_e}\right]^{C(H)} \overline{S} + \left[\frac{\overline{R_g} - \overline{R_e}}{2}\right]^{C(H)}$$

Floquet (harmonic) expansion of steady-state rates

$$\overline{R}_{e(g)}^{C(H)} \equiv 2\pi \sum_m P_m G^{C(H)}[\pm(\omega_0 + m\Delta)]$$

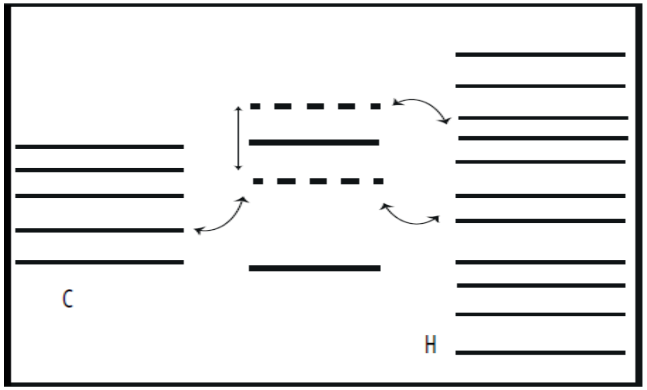
A.G. Kofman and G.K., PRL 2001,2004

$$\text{Bath response } G^{C(H)}(\omega) = |g^{C(H)}(\omega)|^2 \rho^{C(H)}(\omega)(n^{C(H)}(\omega)+1)$$

Probabilities of shifting $G^{C(H)}(\omega)$ by $m\Delta$, $\Delta = \frac{2\pi}{\tau}$, from ω_0

$$P_m = |\varepsilon_m|^2$$

$$\varepsilon_m = \frac{1}{\tau}\int_0^\tau e^{i\int_0^t(\nu(t')-\omega_0)dt'} e^{im\Delta t} dt,$$



Same results for Floquet expansion of Markovian (Lindblad) ME

$$\mathcal{L}^j = \sum_m \mathcal{L}^j, \mathcal{L}^j_m \rho = \frac{P_m}{2}\Big(G^j(\omega_0 + m\Delta)\big([\sigma^-\rho, \sigma^+] + [\sigma^-, \rho\sigma^+]\big) +$$
$$G^j(-\omega_0 - m\Delta)\big([\sigma^+\rho, \sigma^-] + [\sigma^+, \rho\sigma^-]\big)\Big)$$

R. Alicki, D. Gelbwaser-Klimovsky & G.K, arXiv:1205.4552v1 **[quant-ph]**

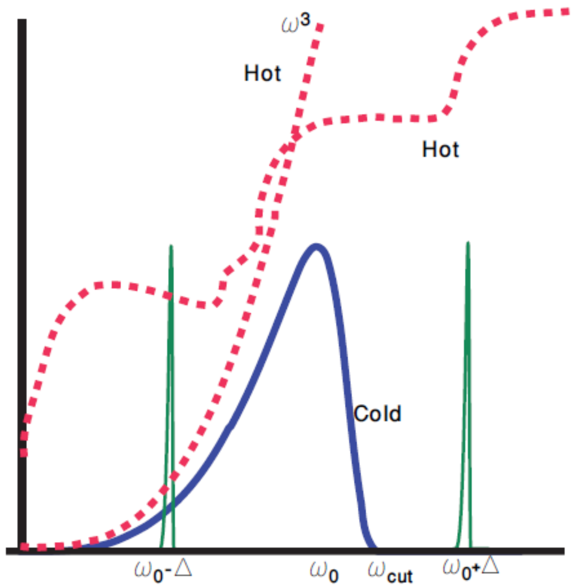
Steady-state Thermodynamic Variables via Floquet

$$J_{C(H)} = \overline{\dot{Q}}_{C(H)} = \sum_m (\omega_0 + m\Delta)\underbrace{\overline{\dot{S}_m^{C(H)}}}_{\propto\, G^{C(H)}(\omega_0 + m\Delta)}$$

Heat flow: $J_C > 0 \text{ refrigerator (QR)}$

1st law: power (work flow): $\mathcal{P} = -(J_C + J_H) > 0 \text{ work (QHE)}$

Spectral separation of C&H baths



qubit π-flips at $\tau = \frac{2\pi}{\Delta}$

cause shifts of $G^{C(H)}(\omega_0) : \omega_0 \to \omega_0 \pm \Delta.$

$P_0 = 0$ and $P_{\pm 1} \approx (2/\pi)^2$

Rising $G^H(\omega)$, localized $G^C(\omega)$
$G^H(\omega_0 + \Delta) \gg G^H(\omega_0 - \Delta), G^C(\omega_0 \pm \Delta)$

Heat pump (QR) condition:

$$n^C(\omega_0 - \Delta) > n^H(\omega_0 + \Delta) \Leftrightarrow \frac{\omega_0 + \Delta}{T_H} > \frac{\omega_0 - \Delta}{T_C}.$$

Minimal model of a universal heat machine (spectral separation of baths)

D. Gelbwaser-Klimovsky, R. Alicki & G.K., PRE **87,** 012140 (2013)

Engine (QHE) regime $(\mathcal{P} > 0)$:

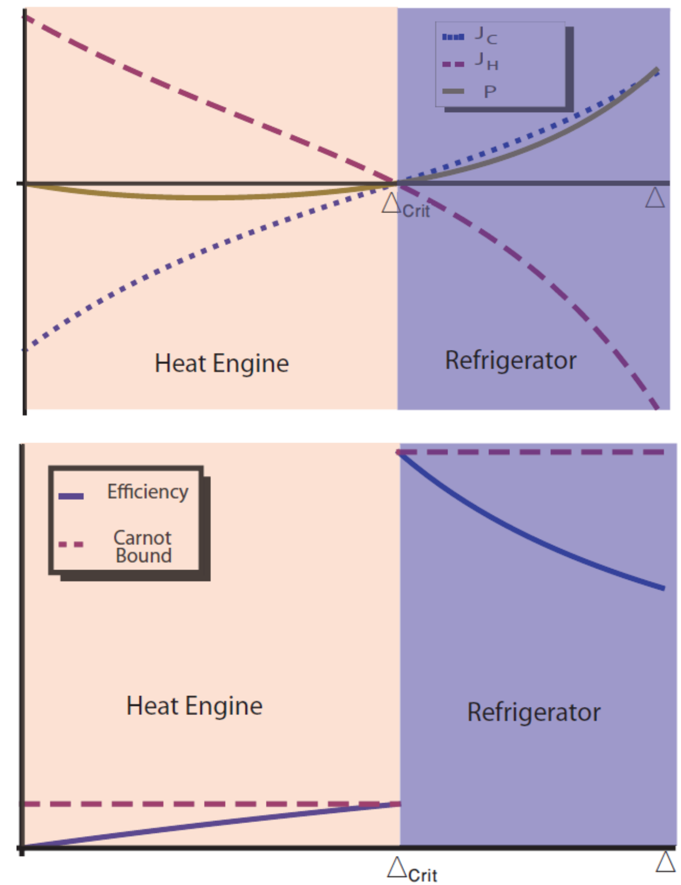
$$\Delta < \Delta_{cr} = \omega_0 \frac{T_H - T_C}{T_H + T_C}$$

$$\eta = \frac{\mathcal{P}}{J_H} = 1 - \frac{T_C}{T_H} \text{ Carnot bound}$$

$\mathbf{Heat\ pump\ (QR)\ regime}$ $(J_C > 0)$:

$$\Delta > \Delta_{cr} = \omega_0 \frac{T_H - T_C}{T_H + T_C}$$

$$COP : \frac{J_C}{\mathcal{P}} = \frac{\omega_0 - \Delta}{2\Delta};$$



How much work can a quantum piston extract from a heat engine?

D. Gelbwaser-Klimovsky, R. Alicki & G.K.

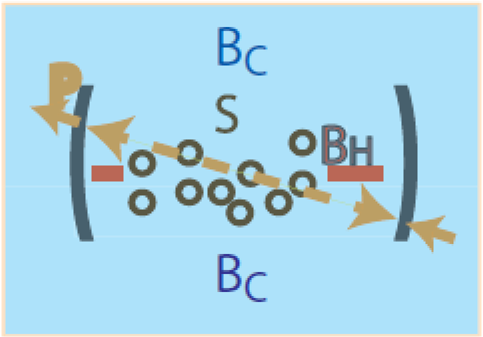
If the piston (p) acts classically (*parametrically*) on S:

$$\Delta E_S = \oint tr(\rho_S H_S)dt = -W + Q;$$

Alicki (1979)

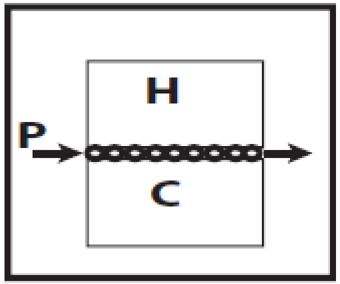
$$W = -\oint tr\{\rho_S \dot{H}_S dt\}; \quad Q = \oint tr\{\dot{\rho_S} H_S dt\}.$$

W=0 if P+S is a quantized (time-indep.) complex?



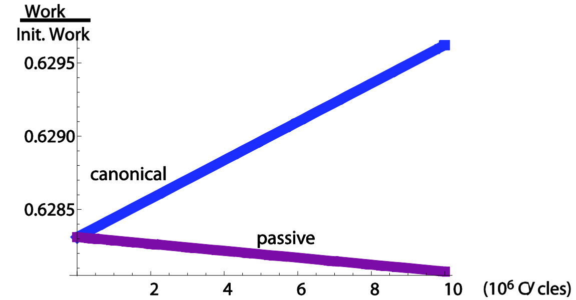
Canonical steady-state balance (1st law):

$$\mathcal{P}_{can} \equiv \frac{d\langle H_P\rangle}{dt} = J_C + J_H,$$



$$\eta_{can} = \frac{\mathcal{P}_{can}}{J_H} = \frac{J_C + J_H}{J_H}$$

Kelvin's 2nd law violated



$$T_C = T_H, \alpha(0) = 1/5$$

Work by passivity definition (Lenard (1978))

$$W_P = \langle H_P(\rho_P) \rangle - \langle H_P(\rho'_P) \rangle_{min}$$

Passive-state energy (unitary minimization)

$$(W_P)_{Max} = \langle H_P(\rho_P) \rangle - \langle H_P(\rho'_P) \rangle_{Gibbs};$$

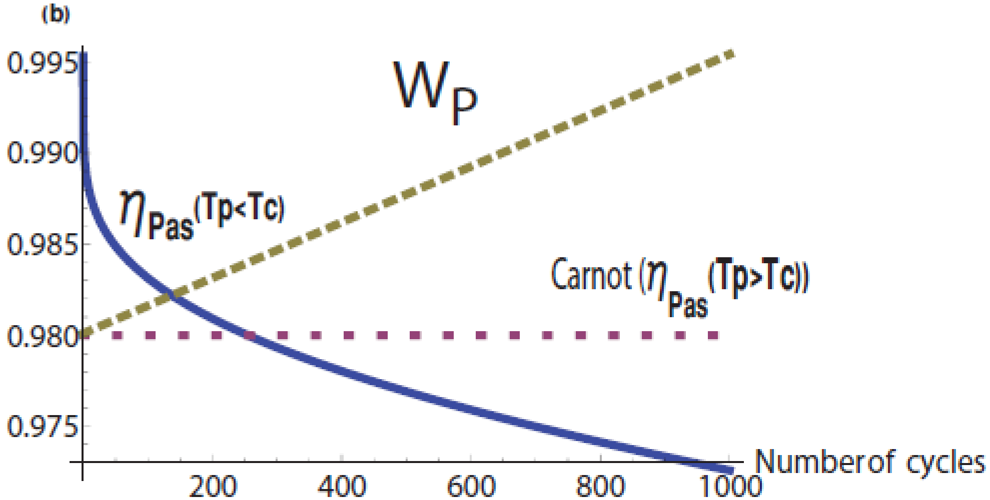
$$(\rho'_P)_{Gibbs} = Z^{-1} e^{-\frac{H_P}{T_P}}$$

Effective temperature

$$(\mathcal{P}_{pas})_{Max} = \frac{d(W_P)_{Max}}{dt} = \frac{d\langle H_P \rangle}{dt} - T_P \dot{\mathcal{S}}_P, \qquad (\eta_{pas})_{Max} = \frac{J_C + J_H - T_P \dot{\mathcal{S}}_P}{J_H}.$$

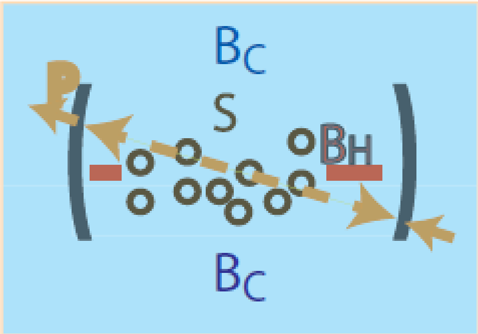
Under Spohn's law :

$$\eta_{pas}(T_P \leq T_C) \leq 1 + \frac{J_H - T_P(\frac{J_H}{T_P} + \frac{J_H}{T_H})}{J_H} = 1 - \frac{T_P}{T_H}$$

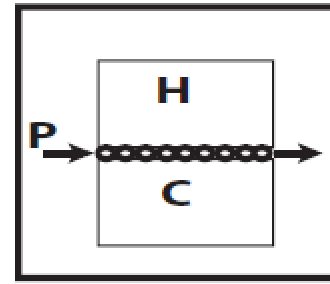


$$T_C/T_H = 1/50, \alpha^2(0) = 0.98$$

Passivity bound can transgress Carnot!
When?



$$H_{S+P} = \frac{1}{2}\omega_0\sigma_Z + \nu a^+ a + \frac{g_p}{2}(a^+ + a)\sigma_Z$$



$$W_P = \nu \int d^2\alpha |\alpha|^2 (\mathbf{P}(\alpha) - \mathbf{P}'(\alpha))$$

Only nonpassive $\frac{\partial P}{\partial |\alpha|} \geq 0$ yields work.

By contrast $\langle H_P(t)\rangle = \nu(D(1 - e^{-\gamma t}) + e^{-\gamma t}\langle \alpha^2(0)\rangle$ energy gain indep. of passivity.

Coherent state

$W_P = \nu\alpha^2(0)e^{-\gamma t}$

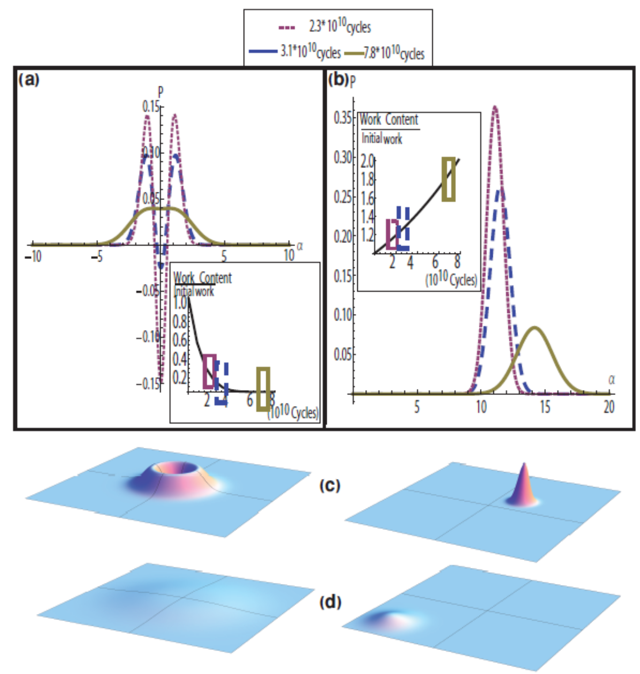
Fock state

$W_P = \nu n(0)$

Thermal state

$W_P < 0$

Carnot Violation: $T_P(t) < T_C$



Coh. state $\frac{1}{T_P(t)} = \frac{1}{\nu} Log(\frac{1+D\gamma t}{D\gamma t})$.

Nonpassive state is negentropy source.

QHM refrigeration: Towards Absolute Zero?

M. Kolar, D. Gelbwaser-Klimovsky, R. Alicki & G.K. PRL **109,** 090601 (2012)

Challenging Nernst's third law (1908)

$$\text{Slow temp. change } c_V \, \frac{\mathrm{d}T_C(t)}{\mathrm{d}t} = J_C = \dot{Q}_C$$

$$\lim_{T_C \to 0} c_V = \frac{d}{dT}\frac{\langle H_B \rangle}{V}|_{T_C} \simeq \frac{d}{dT}\int d\omega\omega\rho(\omega)(n_C(\omega)+1)|_{T_C} \sim T_C^d$$

$$\rho_(\omega) \approx \omega^{d-1}$$

$$J_C \text{ maximized for } \omega_0 - \Delta \approx T_C \ll \omega_{cut} : \lim_{\omega \to 0} |g(\omega)|^2 \propto \omega^\gamma$$

$$J_C(T_C) \propto -T_C^{\gamma+d}.$$

$$T_C^d \text{ scaling of } c_V \text{ is canceled by a similar scaling of the density of modes} \rightarrow$$

$$\mathrm{d}T_C/\mathrm{d}t = -AT_C^\gamma.$$

Cooling rate scaling

$$\mathrm{d}T_C/\mathrm{d}t = -AT_C^\gamma.$$

$\frac{dT_C}{dt}$ scaling only depends on γ scaling power of system-bath coupling $|g(\omega)|^2 \propto \omega^\gamma$.

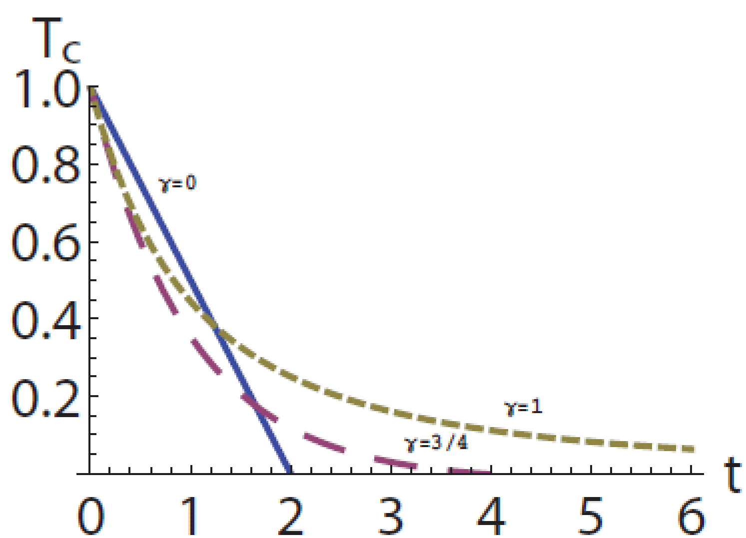
Coupling of a dipolar system to a bath depends on

$$\vec{\sigma} \cdot \nabla \hat{B}(\vec{\mathbf{x}}) \Rightarrow \nabla \hat{B}(\vec{\mathbf{x}}) = \frac{-i}{\sqrt{V}} \sum_{\vec{\mathbf{k}}} \frac{1}{\sqrt{\omega(\vec{\mathbf{k}})}} \left(\nabla \phi_{\vec{\mathbf{k}}}(\vec{\mathbf{x}}) a^\dagger(\vec{\mathbf{k}}) - h.c.\right)$$

i)For *acoustic phonons* $\phi_{\vec{k}} \sim e^{i\vec{k}\cdot\vec{x}}, \omega(\vec{\mathbf{k}}) \simeq v|\vec{\mathbf{k}}|$, $|g(\omega)|^2 \sim \omega$, i.e. $\gamma = 1$.

ii) *Magnons (spin-wave) bath* in a ferromagnetic spin lattice (nearest neighbor, $T < T_{cr}$):

Local spin variable a_j *directly coupled* to the qubit by a dipole-dipole (spin-spin) interaction. Absence of dispersive-coupling coefficient $\frac{\vec{\mathbf{k}}}{\sqrt{\omega(\vec{\mathbf{k}})}} \Rightarrow$ Coupling strength to magnons satisfies $|g(\omega)|^2 \sim 1$ ($\gamma = 0$)



Nernst's unattainability principle challenged

Task oriented control

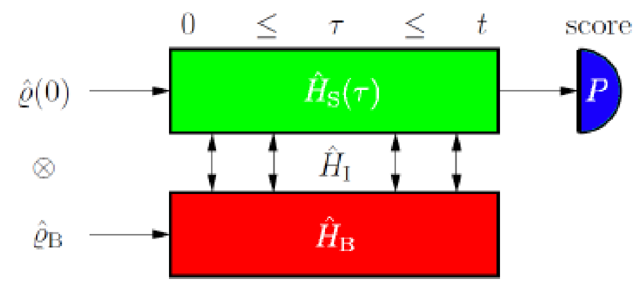
Jens Clausen, Guy Bensky & GK

PRL (2009); PRA (2012)

$P(\rho_f)$ is a measure (score) of how well the task was completed in the presence of a bath/noise. Examples:

Maximize avr. fidelity: $P(\rho_f) = \overline{\langle\psi_0|\,\rho_f\,|\psi_0\rangle}$
Minimize entropy: $P(\rho_f) = \mathrm{Tr}\{\rho_f^2\}$
Maximize entanglement (concurrence): $P(\rho_f) = C(\rho_f)$

$$H = H_0 + H_c(t) + \sum_k S_k \otimes B_k$$



General function, averaged over initial states

$\Delta P(\rho_f) \approx \partial_\rho P \cdot \Delta\rho_f$ (Linear approximation)

$\hat{\Gamma}_{ij} = \partial_\rho P \cdot [\sigma_i, \sigma_j \rho_0]$ — the change in task score after operations σ_i, σ_j

$\Delta P \approx \int_{-\infty}^{\infty} d\omega\, \hat{G}(\omega) \hat{F}_t(\omega)$

Control spectrum $\hat{F}_t(\omega) = t^{-1}\hat{\varepsilon}_t(\omega)\hat{\Gamma}\hat{\varepsilon}_t^\dagger(\omega)$ depends on task

$\hat{G}_{ij}(\omega) = \mathrm{FT}\underbrace{\{\langle B_i(0)B_j(t)\rangle\}}_{\text{bath correlation}}$; $\hat{\varepsilon}_{t,ij}(\omega) = \mathrm{FT}_t\{\varepsilon_{ij}(t)\}$; $S_i(t) = \sum_j \underbrace{\varepsilon_{ij}(t)}_{\text{rotation}}\sigma_j$

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Jens Clausen, Guy Bensky & GK

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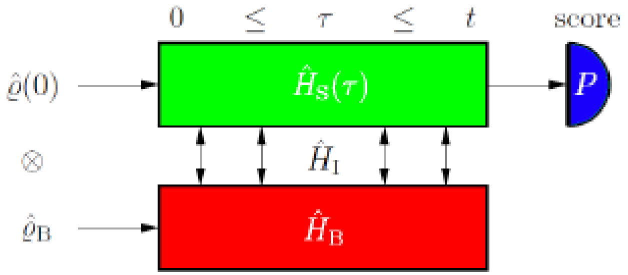
$$H = H_0 + H_c(t) + \sum_k S_k \otimes B_k$$



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$\hat{\Gamma}_{ij} = \partial_\rho P \cdot [\sigma_i, \sigma_j \rho_0]$ — the change in task score after operations σ_i, σ_j

$\Delta P \approx \int_{-\infty}^{\infty} d\omega\, \hat{G}(\omega)\hat{F}_t(\omega)$

Control spectrum $\hat{F}_t(\omega) = t^{-1}\hat{\varepsilon}_t(\omega)\hat{\Gamma}\hat{\varepsilon}_t^\dagger(\omega)$ depends on task

$\hat{G}_{ij}(\omega) = \mathrm{FT}\underbrace{\{\langle B_i(0)B_j(t)\rangle\}}_{\text{bath correlation}}$; $\hat{\varepsilon}_{t,ij}(\omega) = \mathrm{FT}_t\{\varepsilon_{ij}(t)\}$; $S_i(t) = \sum_j \underbrace{\varepsilon_{ij}(t)}_{\text{rotation}}\sigma_j$

Task oriented control

Jens Clausen, Guy Bensky & GK

$P(\rho_f)$ is a measure (score) of how well the task was completed in the presence of a bath/noise. Examples:

Maximize avr. fidelity: $P(\rho_f) = \overline{\langle\psi_0|\,\rho_f\,|\psi_0\rangle}$
Minimize entropy: $P(\rho_f) = \mathrm{Tr}\{\rho_f^2\}$
Maximize entanglement (concurrence): $P(\rho_f) = C(\rho_f)$

$$H = H_0 + H_c(t) + \sum_k S_k \otimes B_k$$

In the interaction picture:

$$H = \sum_k S_k(t) \otimes B_k(t)$$

General function, averaged over initial states

$\Delta P(\rho_f) \approx \partial_\rho P \cdot \Delta\rho_f$ (Linear approximation)

$\hat{\Gamma}_{ij} = \partial_\rho P \cdot [\sigma_i, \sigma_j \rho_0]$ — the change in task score after operations σ_i, σ_j

$\Delta P \approx \int_{-\infty}^{\infty} d\omega\, \hat{G}(\omega)\hat{F}_t(\omega)$

Control spectrum $\hat{F}_t(\omega) = t^{-1}\hat{\varepsilon}_t(\omega)\hat{\Gamma}\hat{\varepsilon}_t^\dagger(\omega)$ depends on task

$\hat{G}_{ij}(\omega) = \mathrm{FT}\underbrace{\{\langle B_i(0)B_j(t)\rangle\}}_{\text{bath correlation}}$; $\hat{\varepsilon}_{t,ij}(\omega) = \mathrm{FT}_t\{\varepsilon_{ij}(t)\}$; $S_i(t) = \sum_j \underbrace{\varepsilon_{ij}(t)}_{\text{rotation}}\sigma_j$

Universal paradigms of open-system control

- Minimized bath effect ≡ Quantum Zeno effect (QZE):
    - minimized overlap of $\hat{G}(\omega)$ and $\hat{F}_t(\omega)$ (under constraints)
- Maximized bath effect ≡ Anti-Zeno effect (AZE):
    - maximized overlap of $\hat{G}(\omega)$ and $\hat{F}_t(\omega)$ (under constraints)
- Both are useful:
    - QZE for bath decoupling in QIP / coherence (DD, BOMEC)
    - AZE for bath-assisted processes (transfer, cooling)

Targeted breakthroughs

conceptual breakthroughs

- Use bath engineering as a handle on dynamics, demonstrate its ability to enhance the performance of quantum thermal machines
- Challenge the work-efficiency Carnot limit and the Landauer bound on information "cost", derived within the system-bath separability paradigm and/or Markovian second-law formulations; use bath engineering or system-bath quantum correlations (in spin-ensemble, ultracold-atoms, trapped-ion and optomechanics setups)
- Discover quantum-operations speed limits (for QI storage and retrieval, cooling and engine cycles)
- Revisit the third law, which states that cooling to zero cannot occur at a finite time or rate: examine scaling of bath cooling-rate as $T \to 0$.

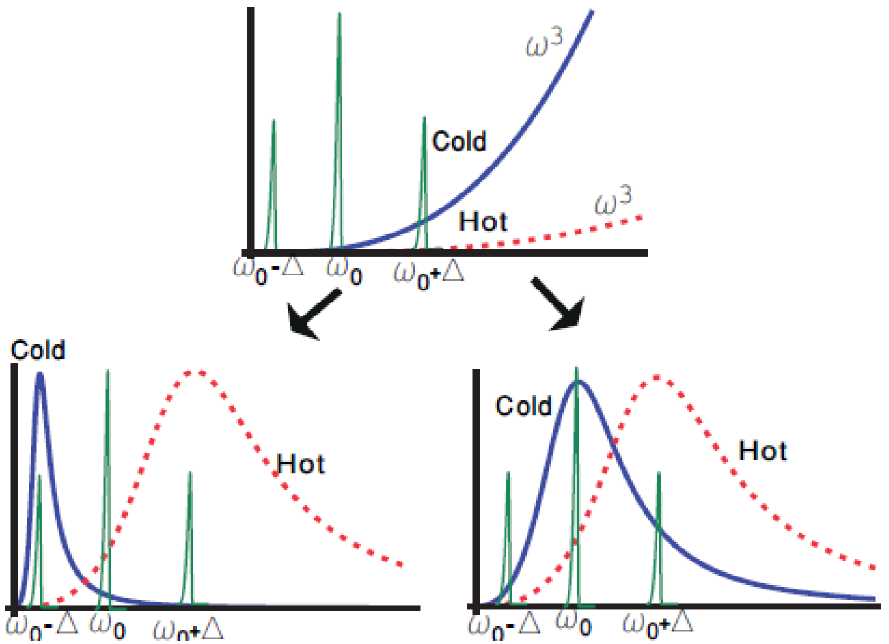
Targeted breakthroughs

Applied breakthroughs

- minimal (single qubit) dual-usage machines
- quantum heat engines (QHE) that may defy the Carnot limit but still adhere to the second law;
- quantum memory that maximies rate and fidelity by speeding up nonunitary operations: aiding resetting the register to zero
- quantum refrigerator (QR) that may surpass the speed limit imposed by the third law

Sinusoidal freq. modulation: Spectral separation by filtering

$$\omega(t) = \omega_0 + \lambda\Delta\sin(\Delta t)$$
$$P_{m=0} \simeq 1 - \frac{\lambda^2}{2}\ ,\ P_{m=\pm 1} \simeq \frac{\lambda^2}{4}$$

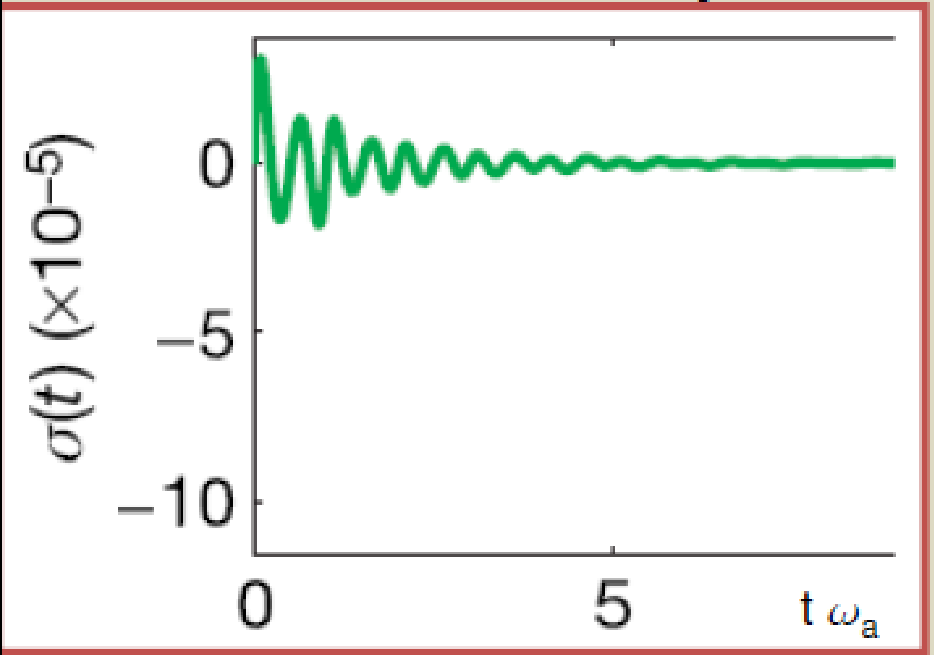


Bath engineering: add filter modes

$$G_f^j(\omega) = \frac{\gamma_f}{\pi}\frac{(\pi G^j(\omega))^2}{(\omega-(\omega_f^j+\Delta_L^j(\omega)))^2+(\pi G^j(\omega))^2},$$

A. Kofman, G. Kurizki, and B. Sherman, Journal of Modern Optics, **41**, 353 (1994).

Non-Markovian "Entropy production"



- It should be positive
- For the Markovian case*:

$$\sigma(t) = -\frac{d}{dt} S(\rho_s(t) \| \rho_{eq})$$

*Spohn, H. Entropy production for quantum dynamical semigroups. J. Math. Phys.19, 1227–1230 (1978).

Erez N, Gordon G, Nest M and Kurizki G Thermodynamic control by frequent Measurements. Nature. 452:724, 2008