UNIVERSITÄT DES SAARLANDES

UNIVERSITY OF WATERLOO

IQC Institute for Quantum Computing

Making optimal control work for superconducting qubits

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QUAINT

SEVENTH FRAMEWORK PROGRAMME

QUEST

IARPA
BE THE FUTURE

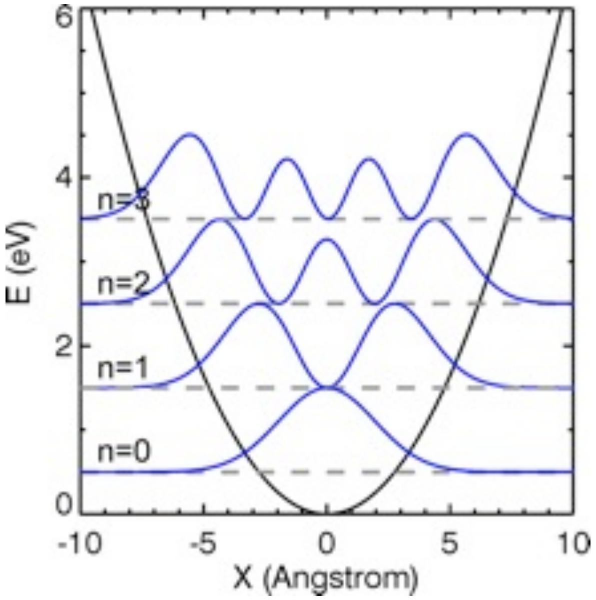
Topics

- Superconducting qubits for control theorists
- Control tools
- Control tasks
- Application: Controlled-Z gates
- Closing the loop - control and tuneup

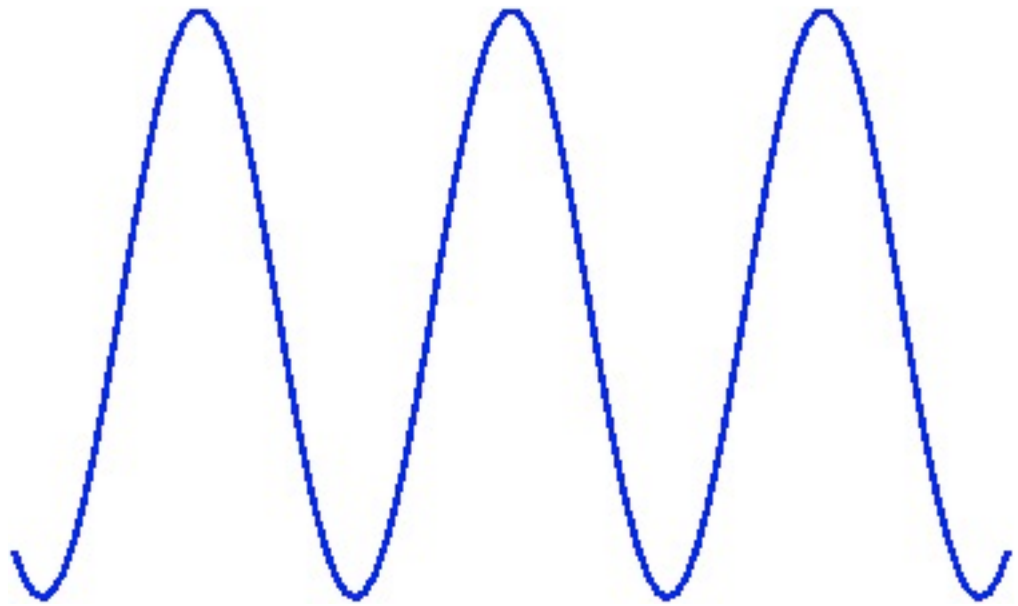
Superconducting qubits for control theorists

Superconducting artificial atoms

Linear LC-oscillator



Josephson junction:
nonlinear inductor



J. Clarke, FKW, Nature 2008

Superconducting artificial atoms

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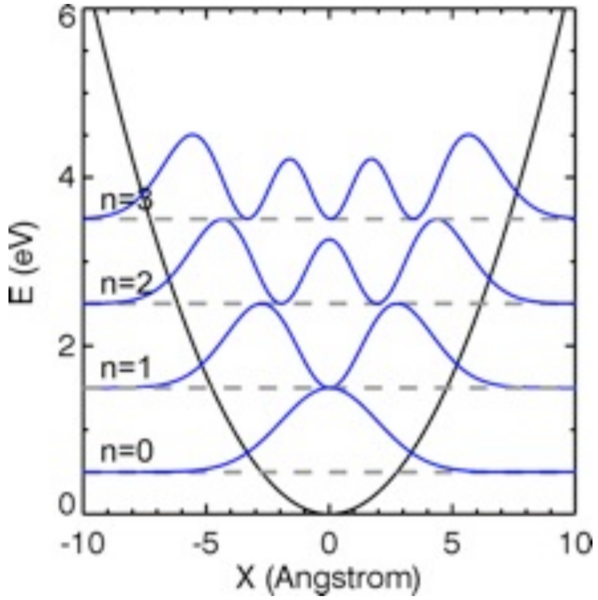

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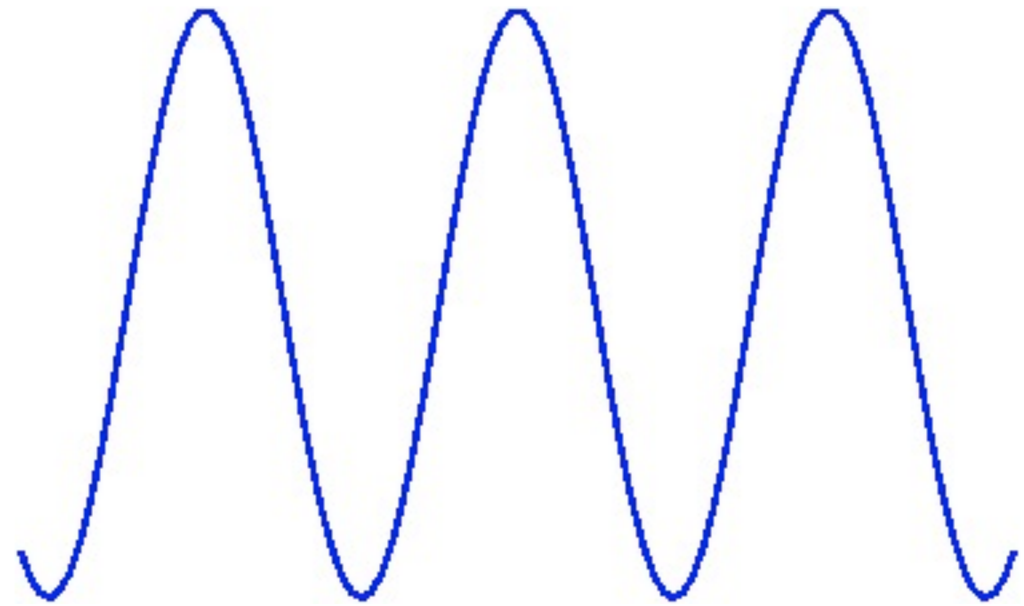
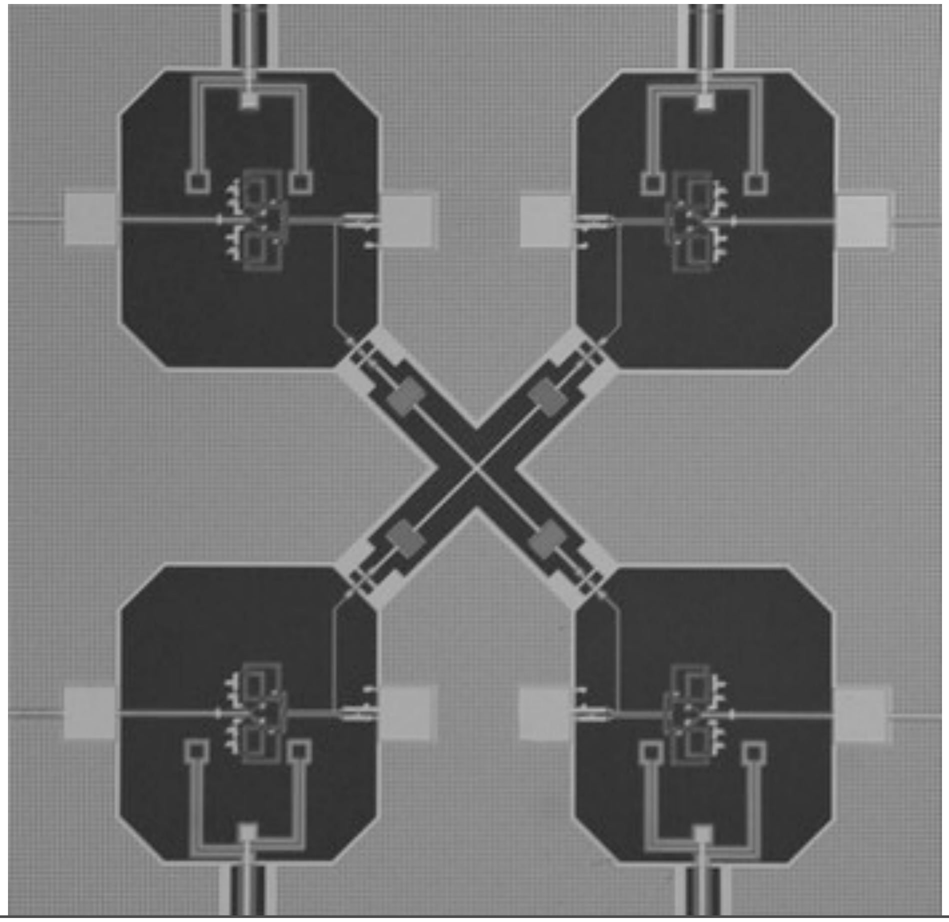
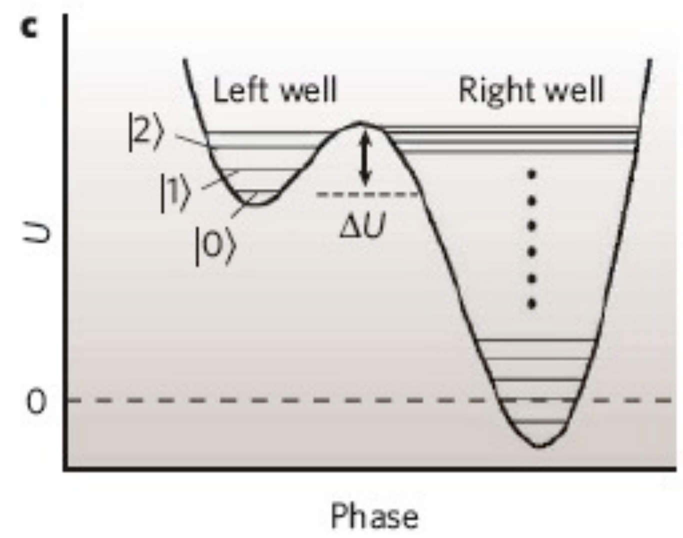


Phase qubit, e.g.

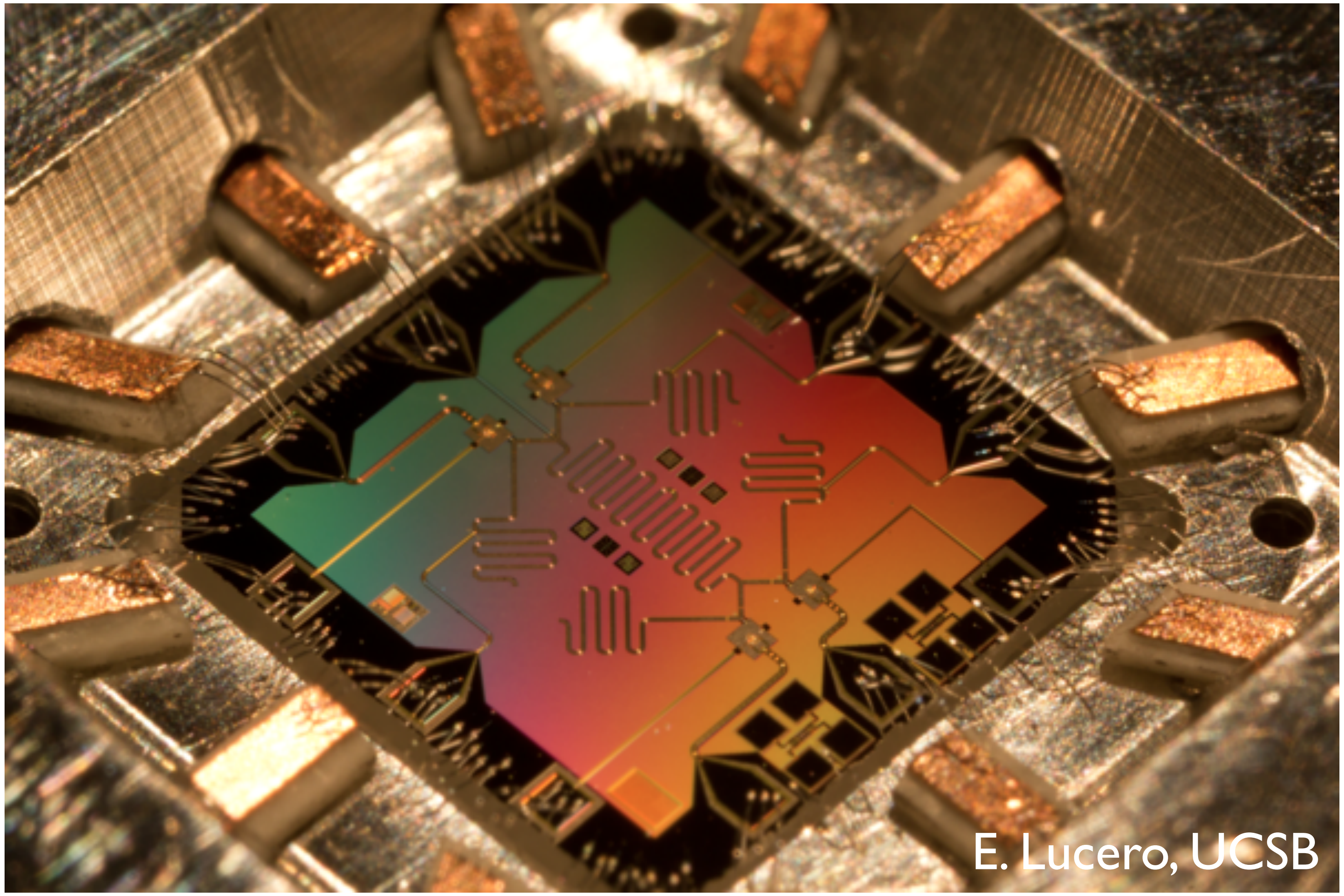




Anharmonic energy levels

J. Clarke, FKW, Nature 2008

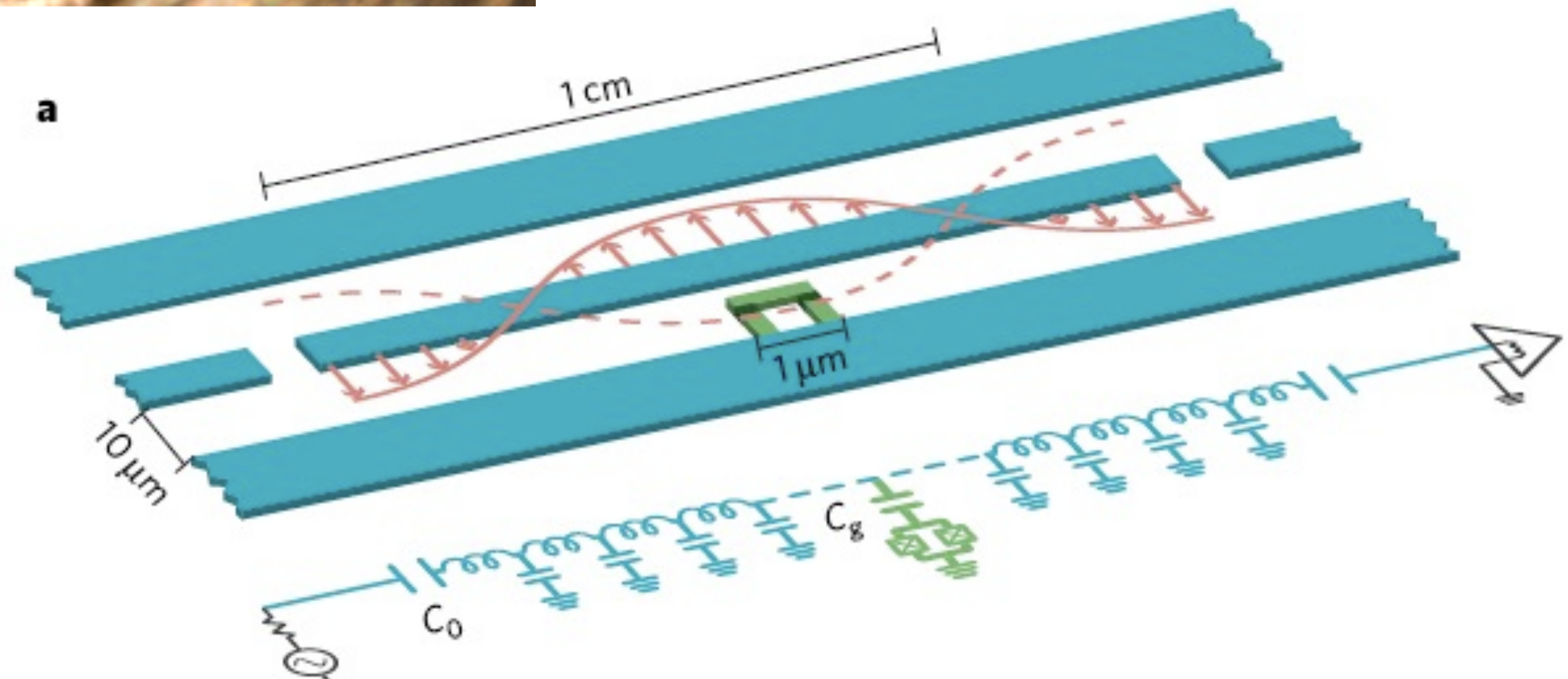
Circuit QED



Key element:
Superconducting resonator at microwave frequencies

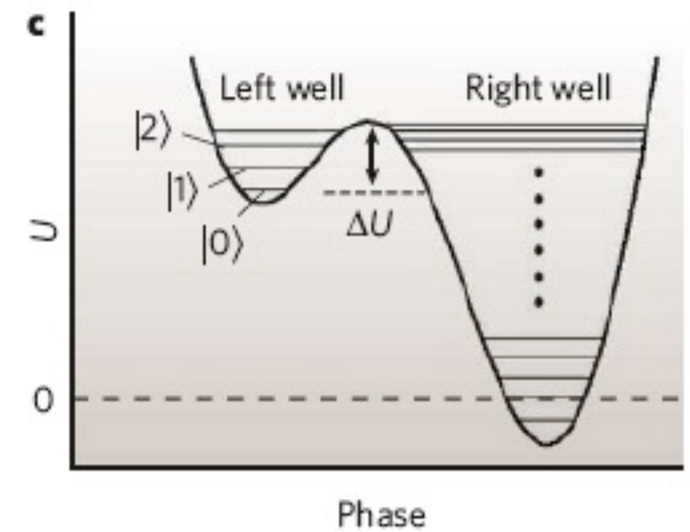
Bus, memory ...

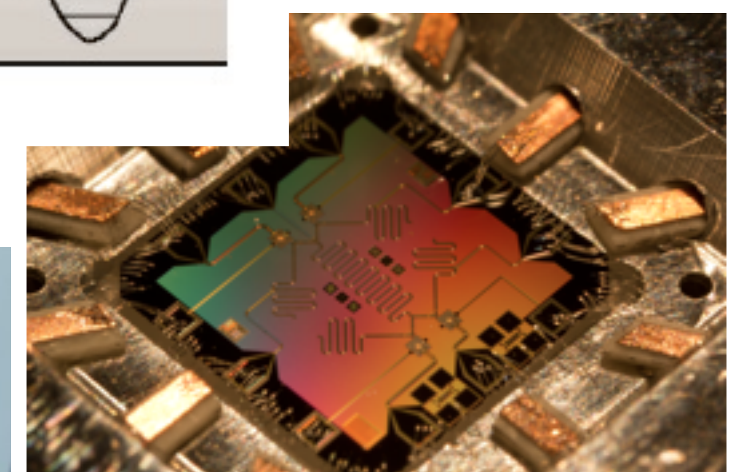
A. Blais et al., PRA 2004
Schoelkopf and Girvin, Nature 2008



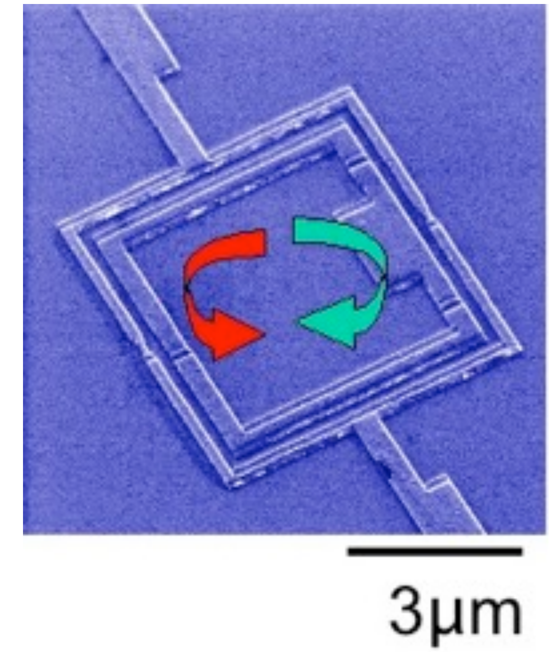
Control implications

- Not a spin - higher levels
- Human made ... parameter uncertainty
- Cryogenic, heavily filtered setup
- Operated at microwave frequencies
- Strong inter-element coupling
- Aiming at unitary gates





Achievements

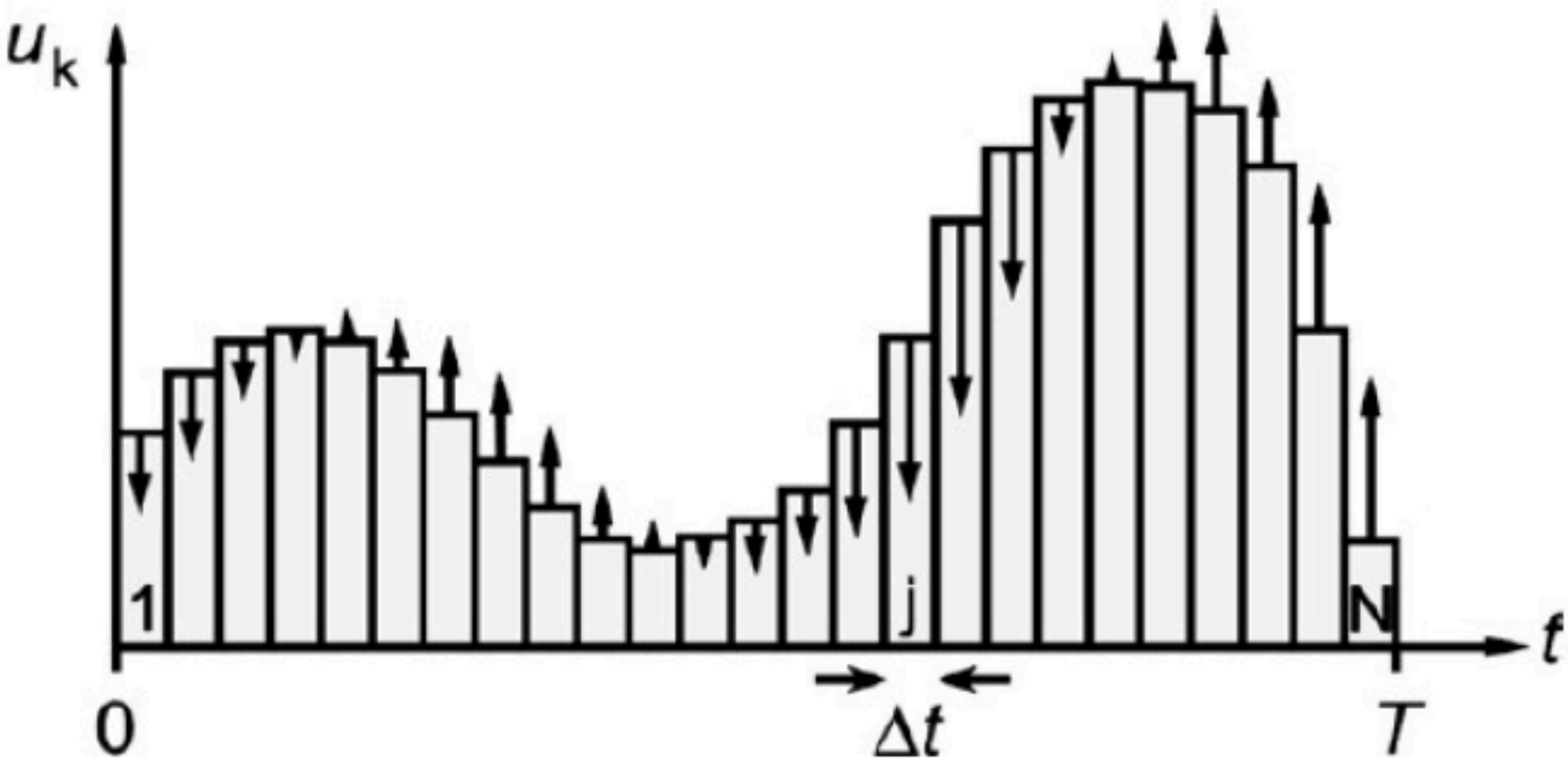


- coherence times $\simeq 50\mu s$
- single qubit gates $\simeq 5ns$
- two- and three qubit gates
- 3-qubit quantum algorithms
- error correction
- Bell states
- Nonclassical resonator states

Control tools

Challenge

Change performance index J in the end in response to u_j



Avoid numerical gradients

Back-propagation: In step j optimize $\hat{U}_{\mathrm{back}} = \hat{U}^\dagger(t_j, T)\hat{U}_{\mathrm{gate}}$

GRAPE

Trotterize performance index

$$\Phi = \left|\mathrm{Tr}(U_{\mathrm{gate}}^{\dagger}U(t_f))\right|^2 = \left|\mathrm{Tr}(U^{\dagger}(t_j, t_N)U_{\mathrm{gate}})^{\dagger}U(t_j, t_1)\right|^2$$

$$= \left|\mathrm{Tr}\left(U_{j+1}^{\dagger}\dots U_N^{\dagger}U_{\mathrm{gate}}\right)^{\dagger}U_j\dots U_1\right|^2$$

Time-pixel propagator

$$U_i = \exp\left(-i\Delta t\left(H_d + \sum u_k(t_i)H_k\right)\right)$$

Analytical gradient - backward loop

$$\frac{\partial\Phi}{\partial u_k(t_j)} = \delta t \mathrm{Re}\left[\left(\mathrm{Tr}U_{j+1}^{\dagger}\dots U_N^{\dagger}U_{\mathrm{gate}}H_kU_j\dots U_1\right)\left(\mathrm{Tr}U_1^{\dagger}\dots U_j^{\dagger}U_{\mathrm{gate}}U_N\dots U_{j+1}\right)\right]$$

N. Khaneja, T. Reiss, C. Kehlet, T. Schulte-Herbruggen, S.J. Glaser, Journal of Magnetic Resonance **172**, 296 (2005).

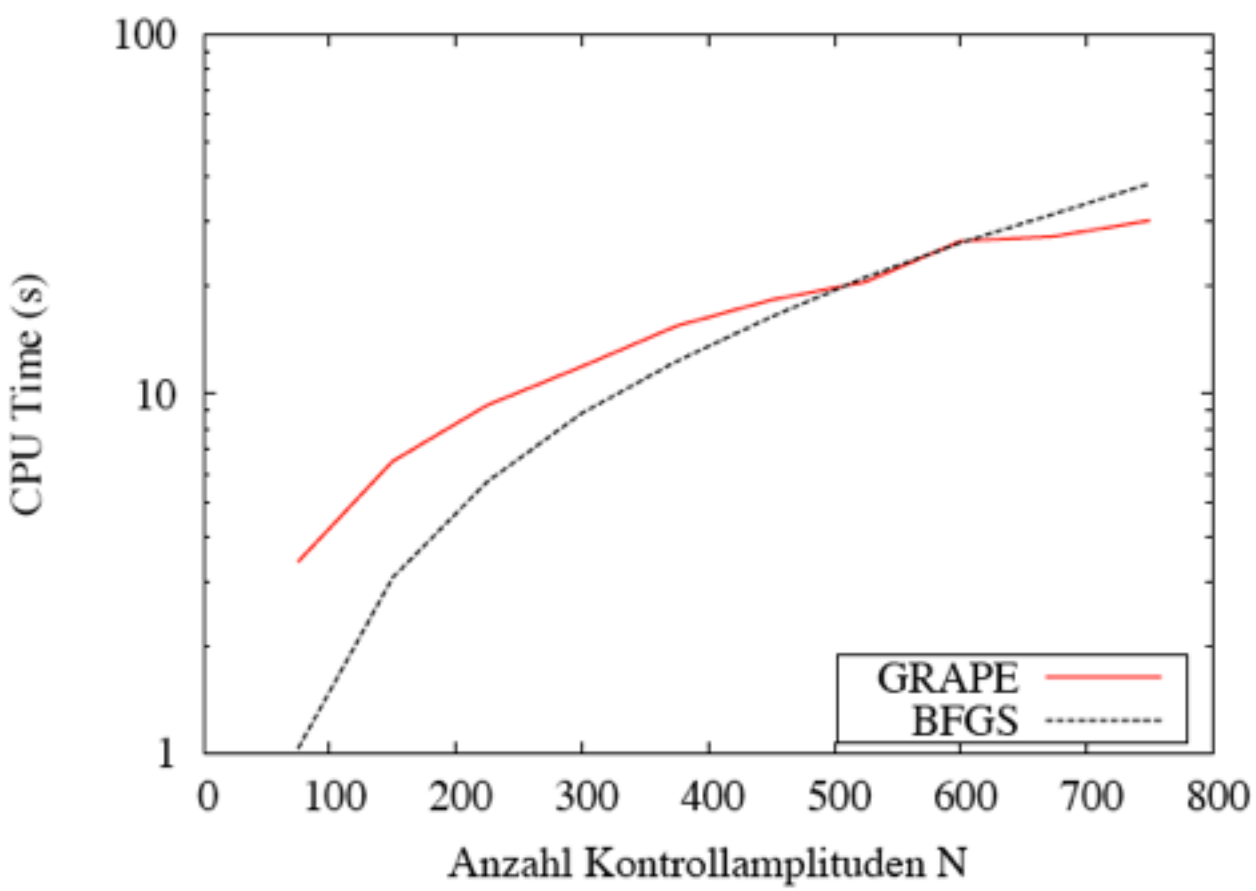
BFGS

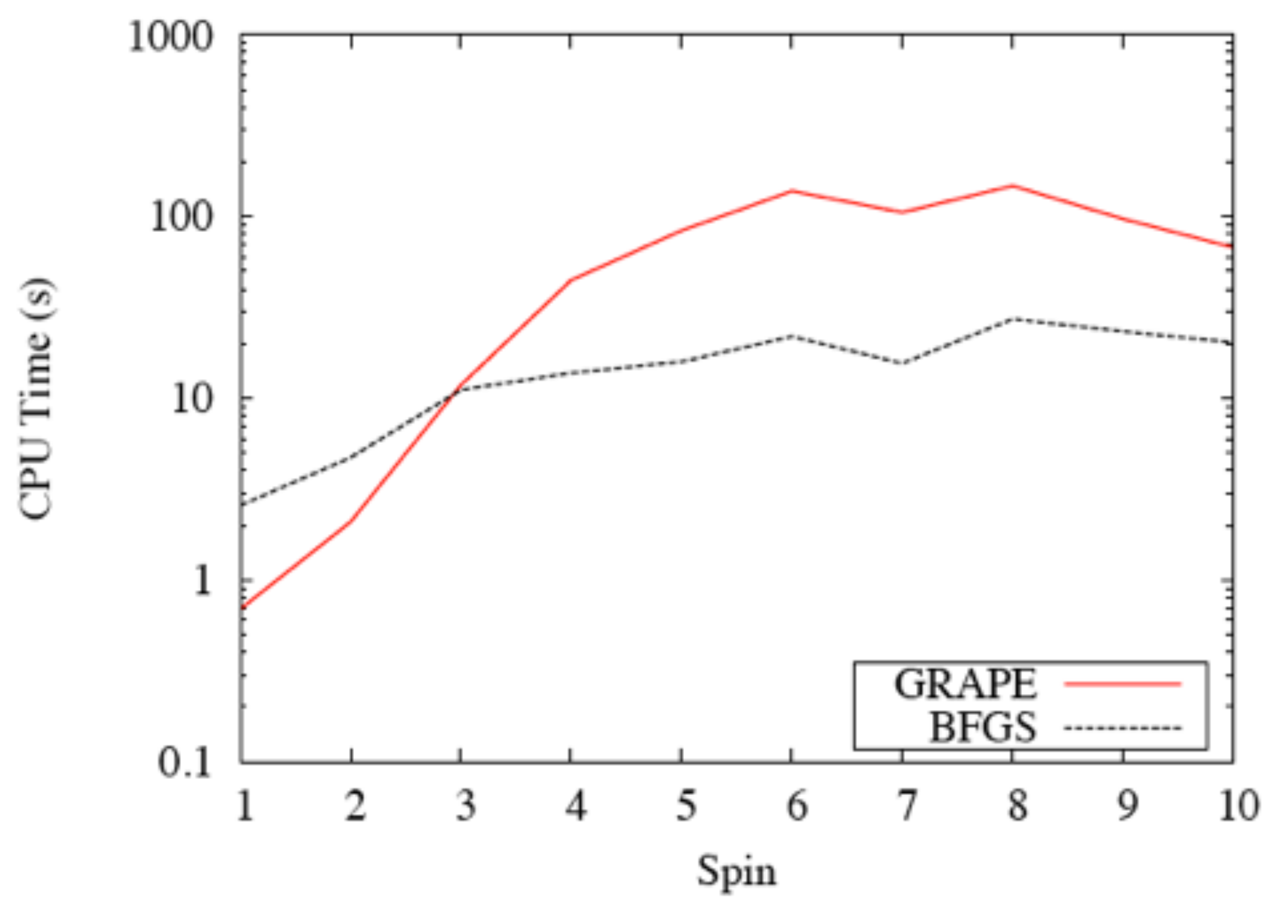
Replace gradient by Quasi-Newton method.

Approximate by Hessian

$$f(x_k + p) \simeq f(x_k) + p^T \nabla f + \frac{1}{2} p^T \left(\nabla^2 f \right) p$$

Minimized by search direction $p_k = -\left(\nabla^2 f_k \right)^{-1} \nabla f_k$





Machnes et al., PRA 2011

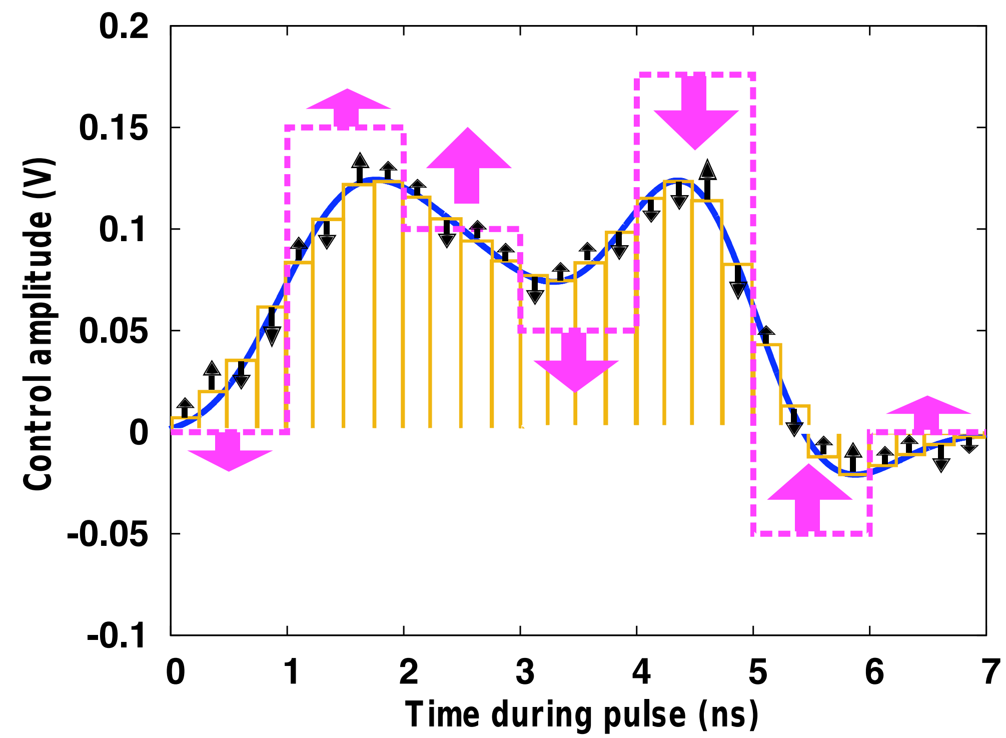
Adapting to experiments

Include linear filters using transfer matrices

Motzoi et al., PRA 2011

Adapting to experiments

Include linear filters using transfer matrices



Motzoi et al., PRA 2011

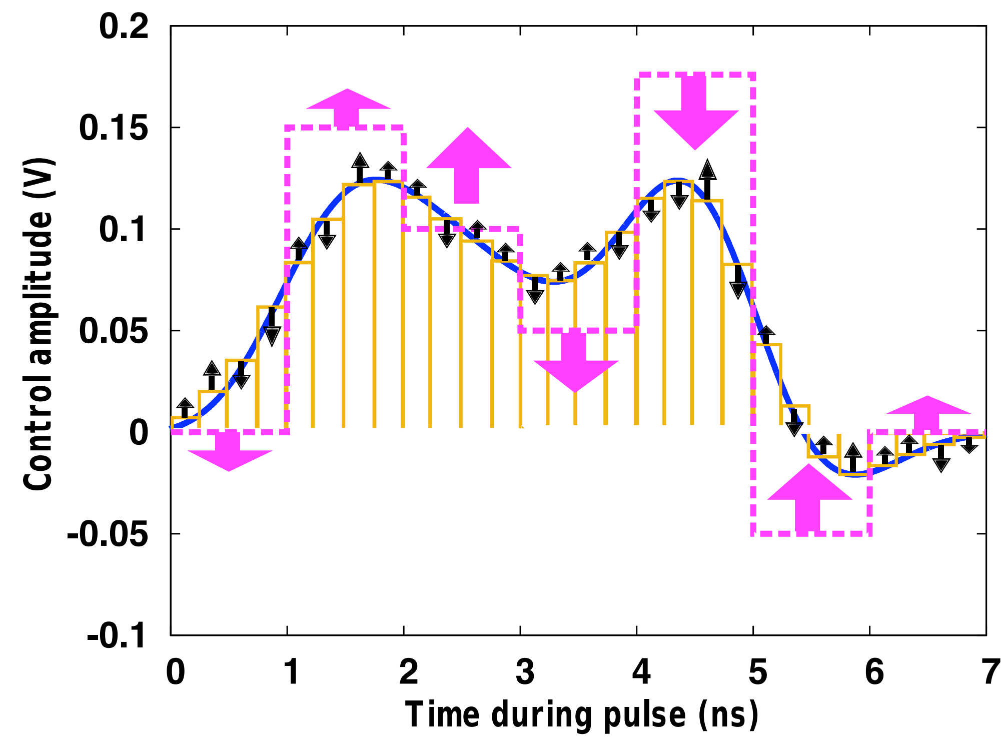
Adapting to experiments

Include linear filters using transfer matrices

For linear systems:

$$o(t) = \int_{-\infty}^{t} dt' L(t-t') i(t')$$

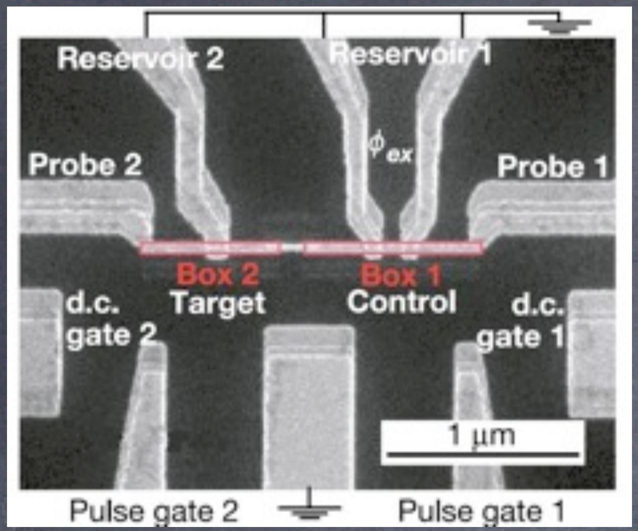
Include filter transfer matrix into GRAPE



Motzoi et al., PRA 2011

Control tasks

Getting started: CNOT



Nakamura group, Nature 2003

Charge basis: $|n_1, n_2\rangle$

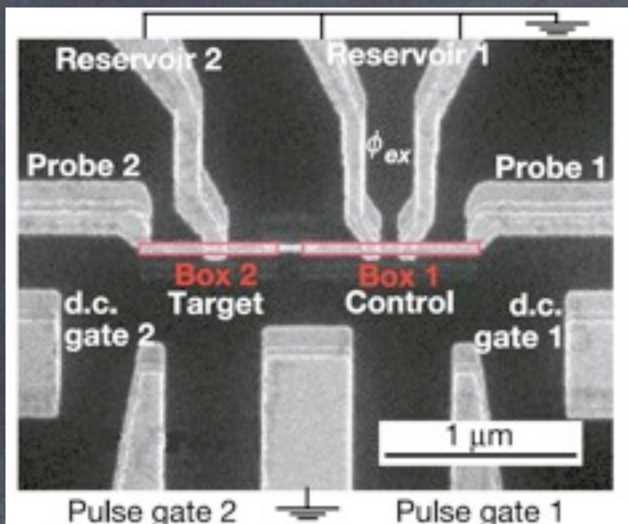
$$\hat{H} = \sum_{n_1,n_2} E_{\mathrm{ch},,n_1,n_2}(V_1,V_2)|n_1,n_2\rangle\langle n_1,n_2| + \frac{E_{J1}}{2}\sum_n (|n\rangle\langle n+1| + \mathrm{h.c.}) \otimes \hat{1} + (1 \leftrightarrow 2)$$

Logical basis: $|\sigma_1, \sigma_2\rangle$

$$\hat{H} = \sum_i E_{ci}\delta n_i(t)\hat{Z}_i + \frac{E_{J,i}}{2}\hat{X}_i + E_{c12}\hat{Z}_1\hat{Z}_2$$

Spörl et al., PRA 2007

Getting started: CNOT



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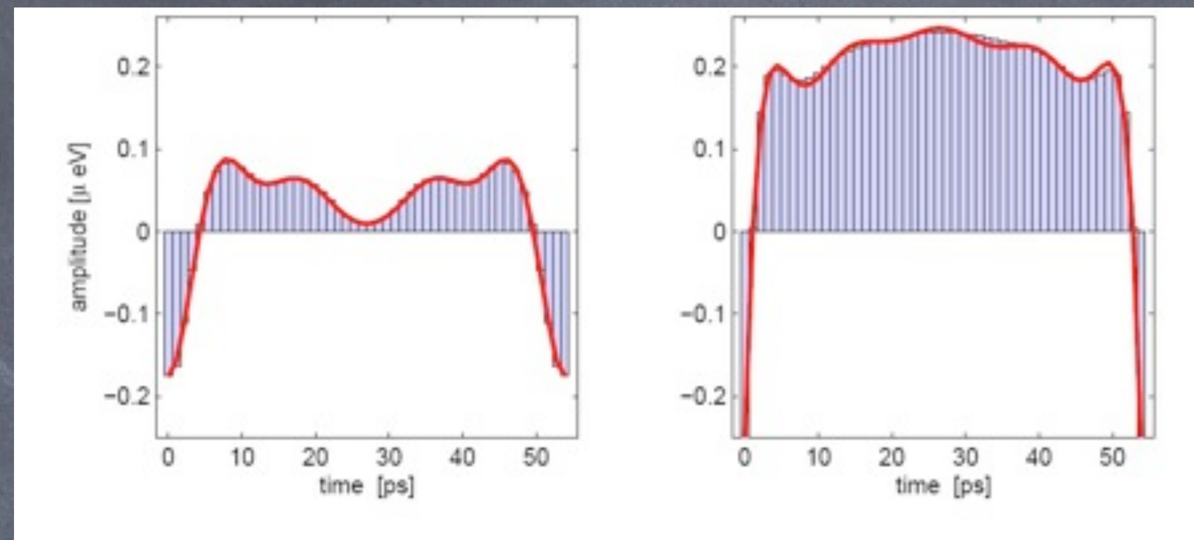

Fast, palindromic, E_J-limited

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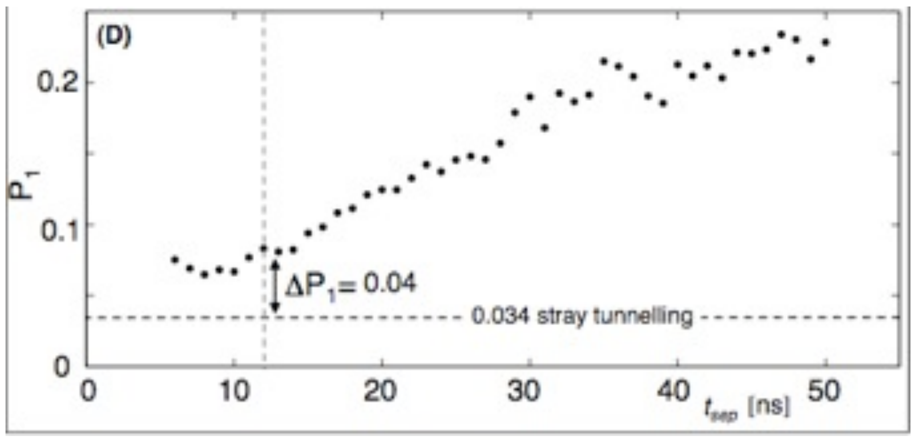
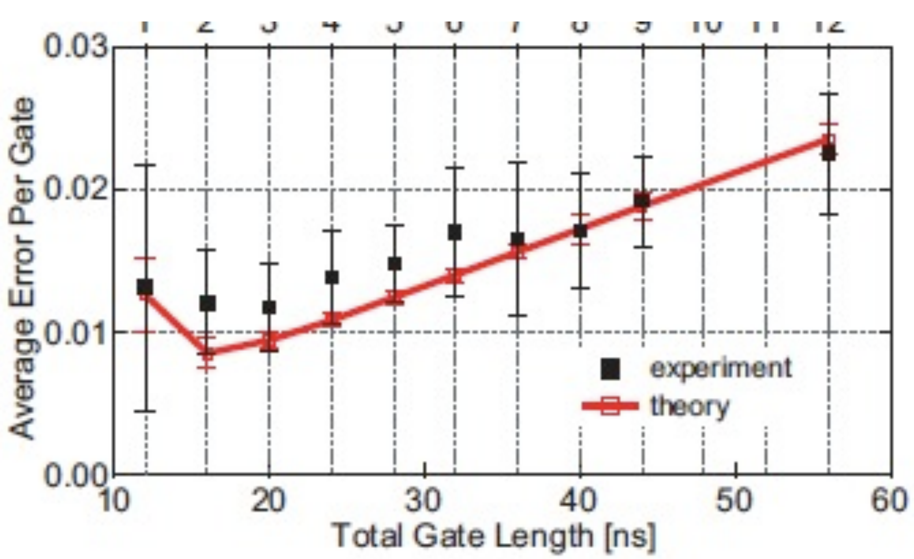
... and then run GRAPE to optimize fidelity J

Spörl et al., PRA 2007

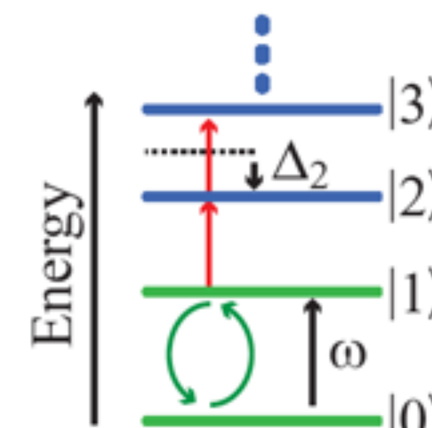
Leakage

Transmon/Phase qubit





Lucero et al., 2008;
Chow et al., 2009



Motzoi et al., PRL 2009

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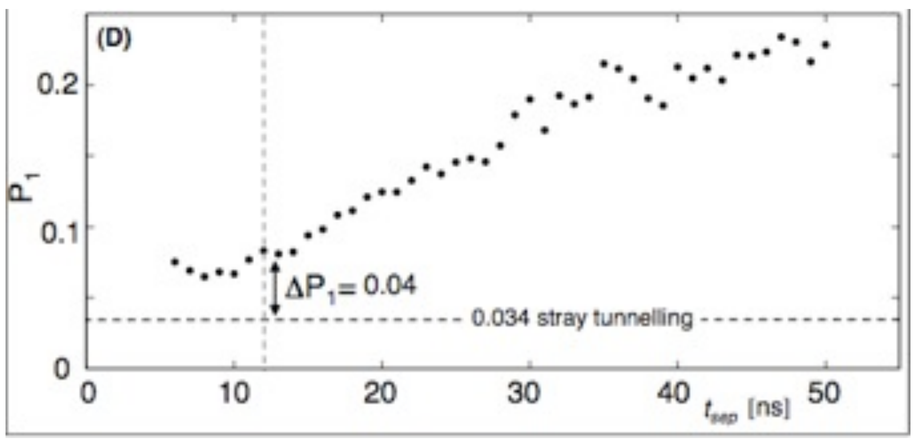

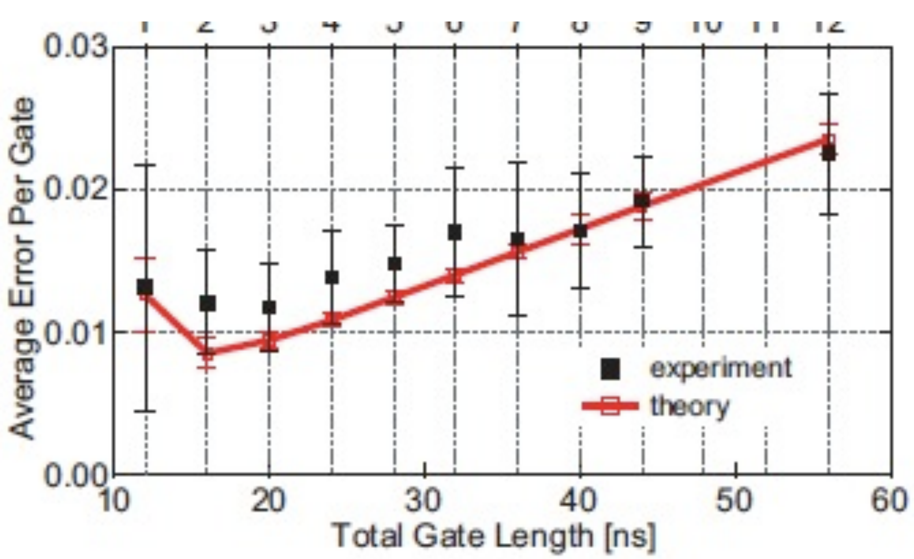

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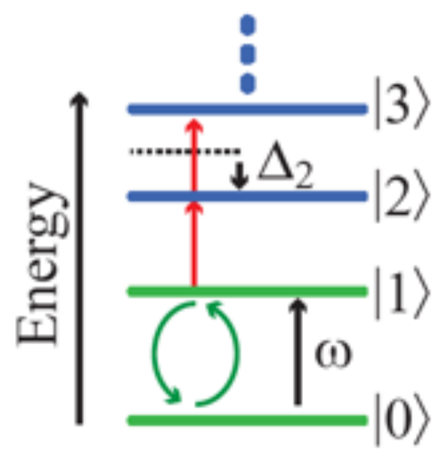

Solution: Envelope shaping
Derivative Removal
by Adiabatic Gate

Motzoi et al., PRL 2009

Leakage

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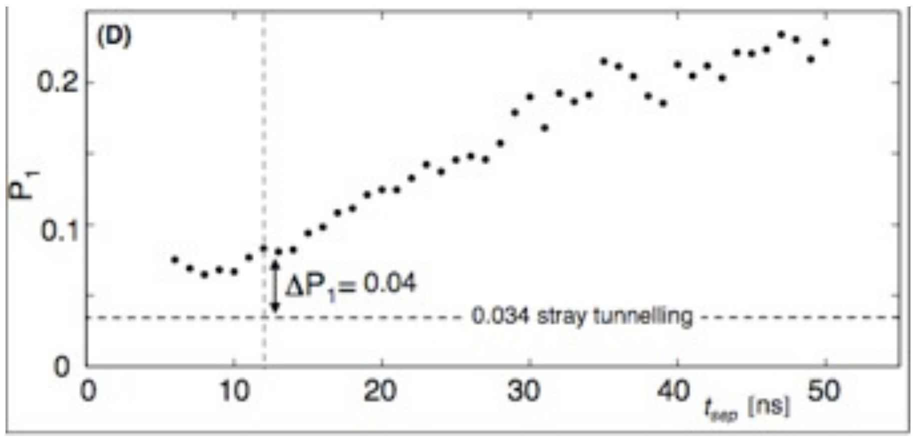

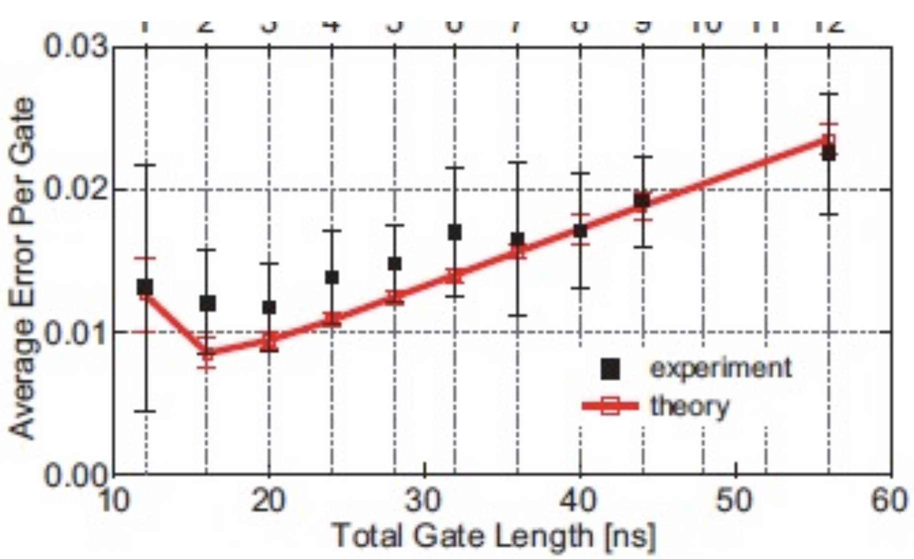

Lucero et al., 2008;
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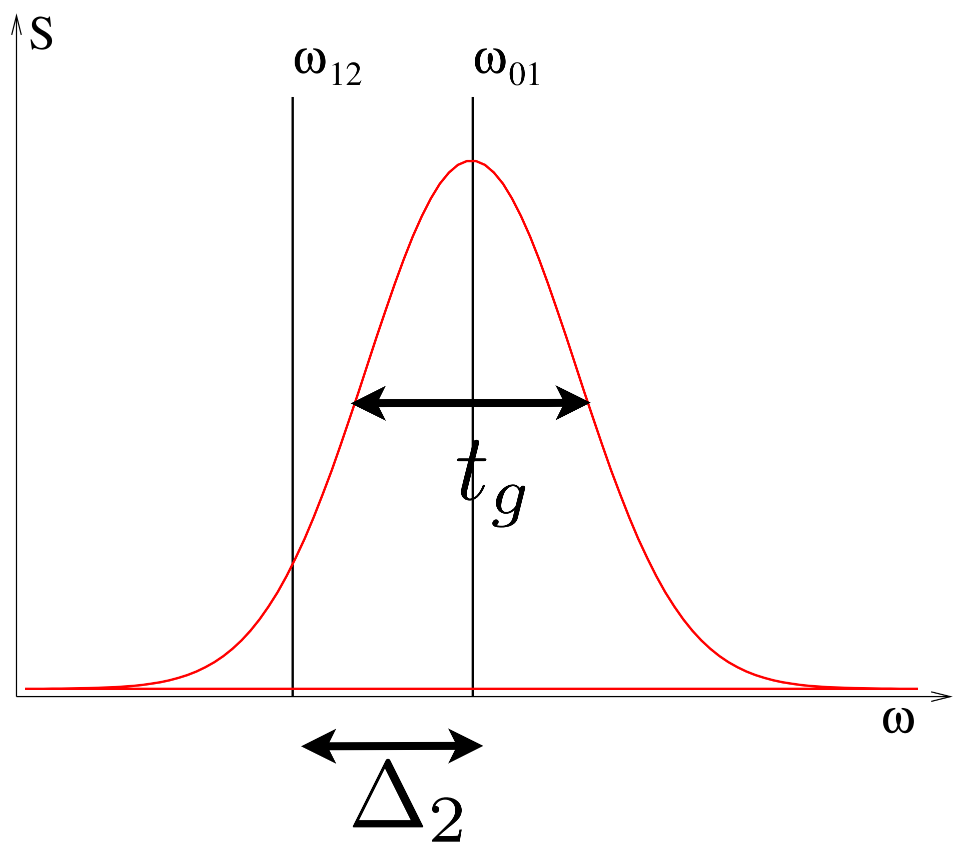
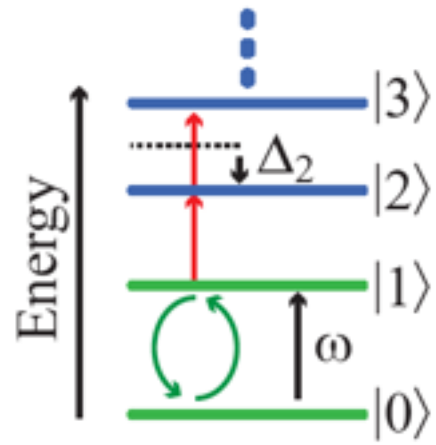
Spectral limitation:
Duration/bandwith uncertainty





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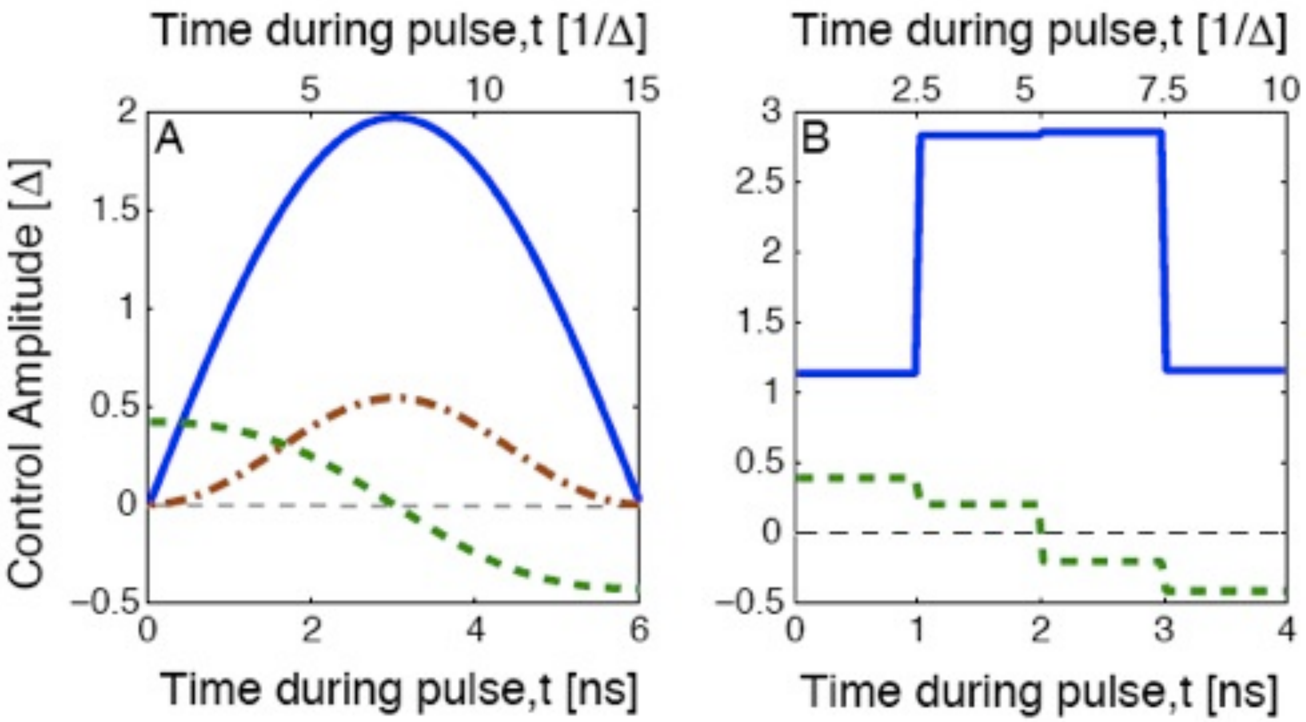
Solution DRAG



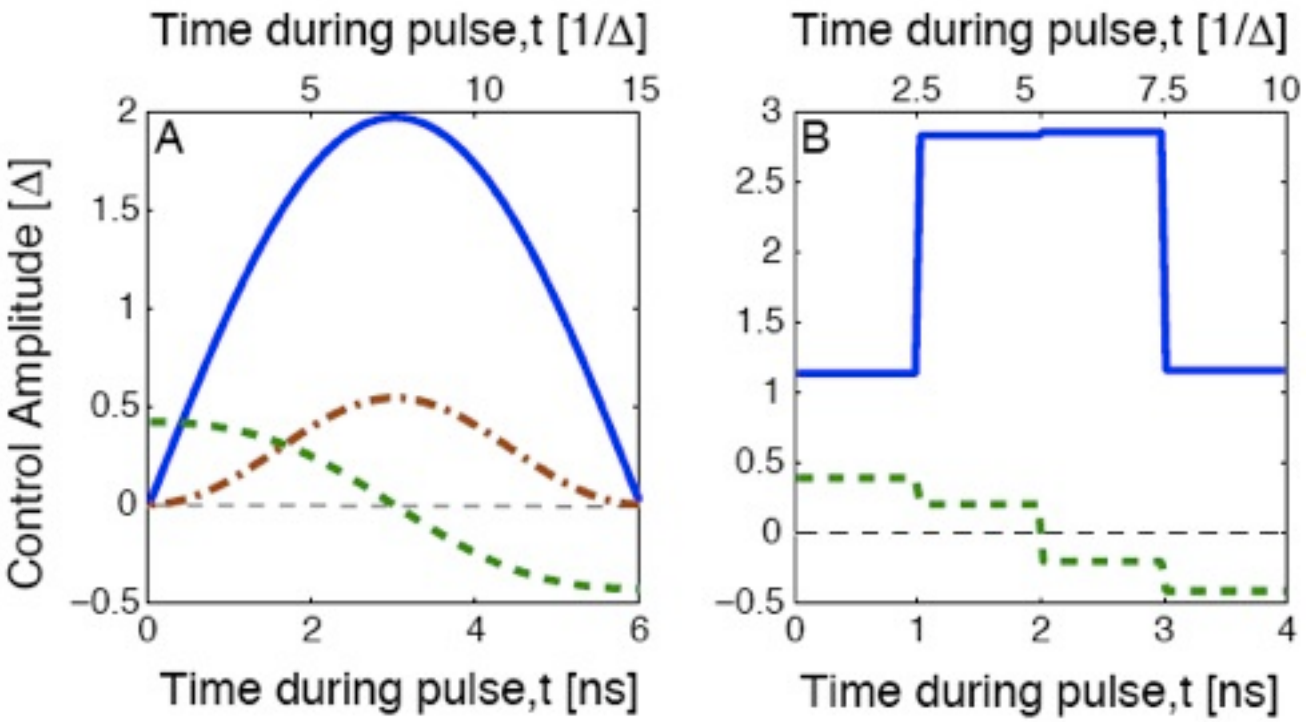
$$u_2 = \frac{\dot{u}_1}{\Delta_2}$$

$$u_1(t)\cos\omega t + u_2(t)\sin\omega t$$

Theorists dream ... experimental reality

Gambetta et al., 2011;
Lucero et al., 2010;
Chow et al., 2010

Solution DRAG



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Theorists dream ... experimental reality

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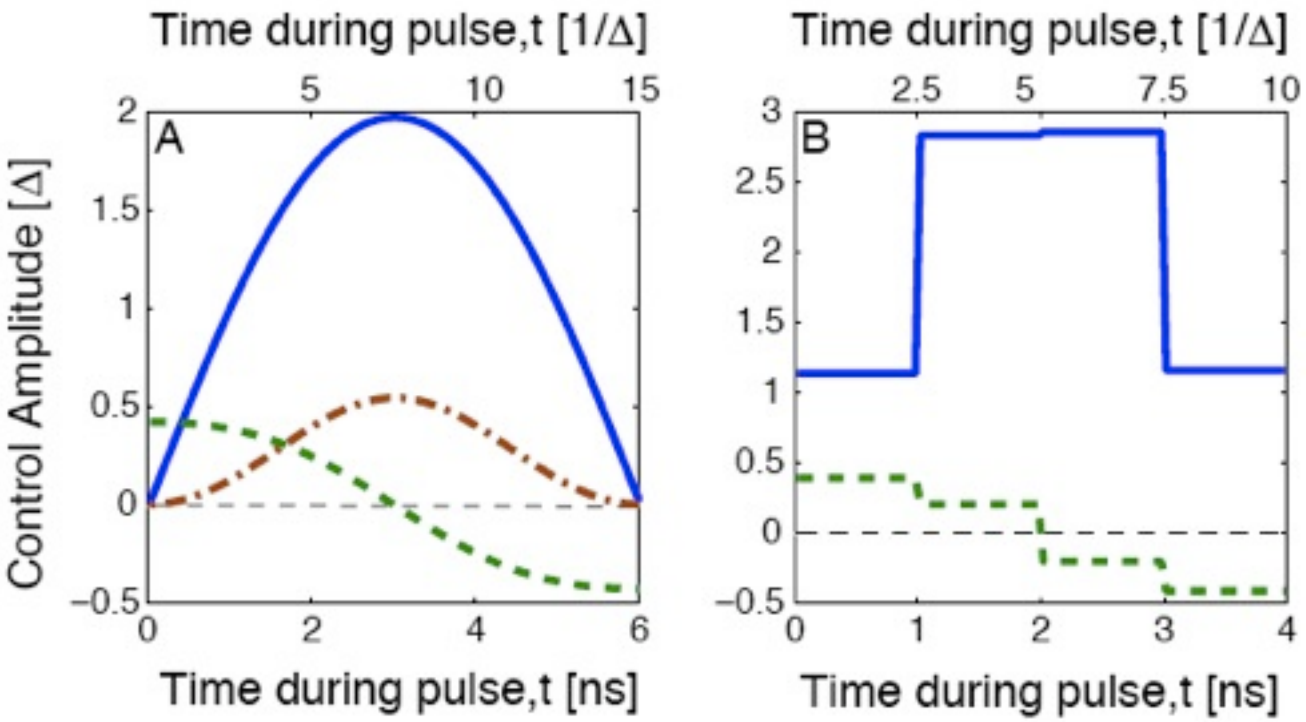
- CAD of analytical scheme
- amenable to long pixels
- third control can be removed - family of DRAG solutions

Gambetta et al., 2011; Lucero et al., 2010; Chow et al., 2010

Solution DRAG



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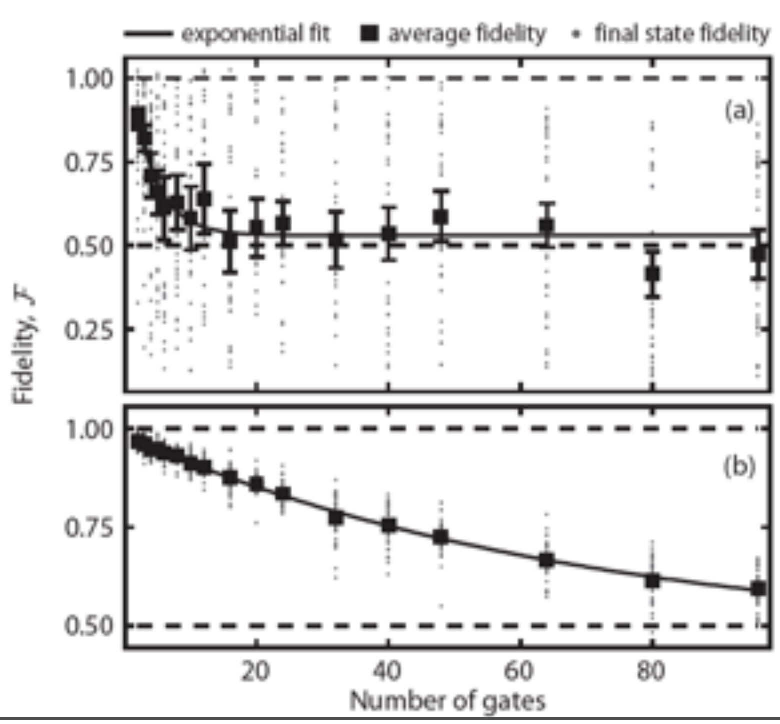
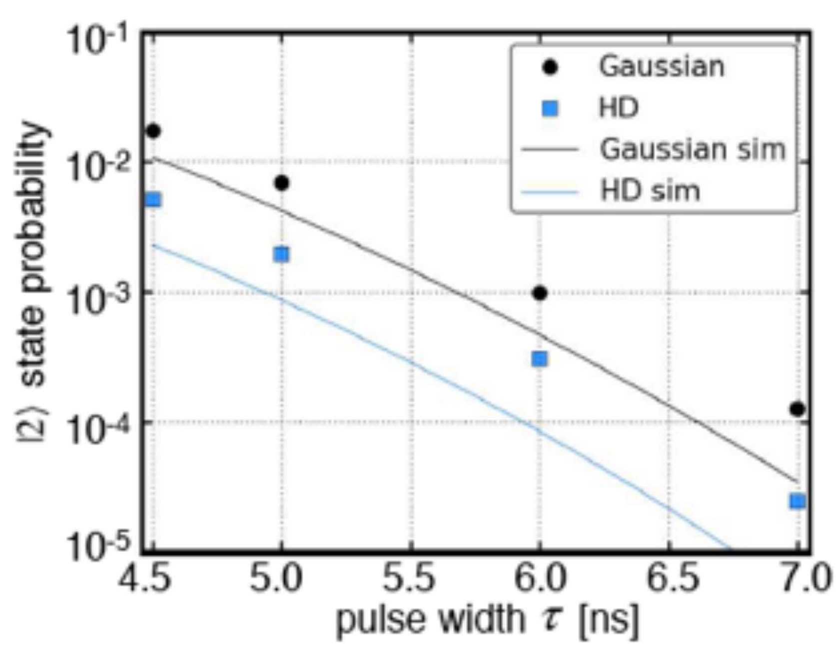




Gambetta et al., 2011;
Lucero et al., 2010;
Chow et al., 2010

Classical: Why derivative?

Excitation profile at *weak drive*: Fourier transform

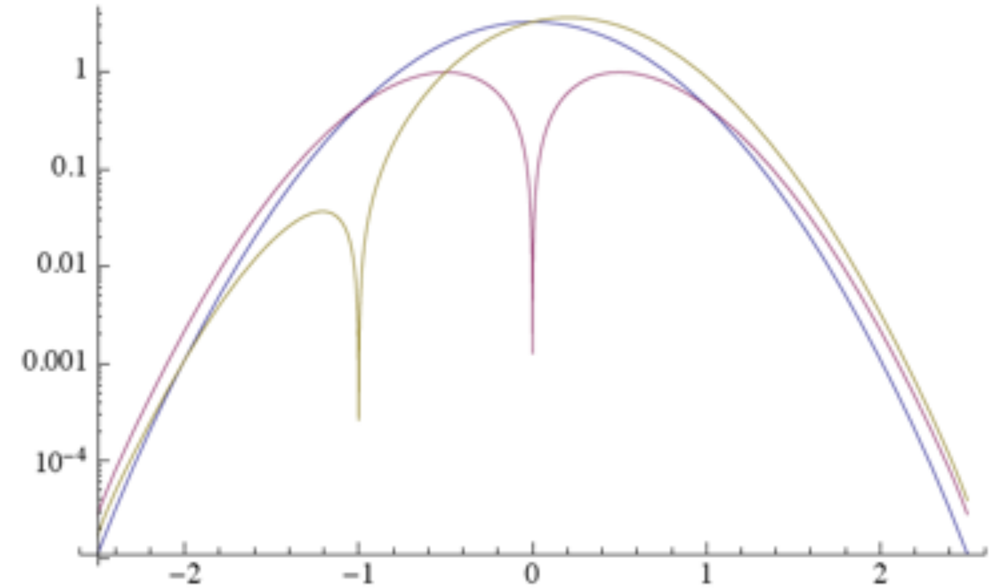
$$S(\delta) = \int_0^T (\Omega(t)e^{-i\phi})e^{-i\delta t}dt$$

Detuning δ

Drive envelope $\Omega(t) = \Omega_1(t) + i\Omega_2(t)$

Simple integration by parts:

$$S(\delta) = e^{-i\phi}\int_0^T (\mathrm{Re}\Omega(t) + i\mathrm{Im}\Omega(t))e^{-i\delta t}dt$$
$$= ie^{-i\phi}\int_0^T \left(\frac{\mathrm{Re}\dot{\Omega}(t)}{\delta} + \mathrm{Im}\Omega(t)\right)e^{-i\delta t}dt$$



Motzoi et al., in preparation

Quantum: The DRAG family

Transformation $H_{\text{eff}} = R^\dagger(t)H(t)R(t) + i\dot{R}^\dagger R$

Generated by: $R(t) = e^{iS(t)}$

Get gate in right frame: $S(0) = S(t_g) = 0$

No leakage $\langle \text{qubit}|H_{\text{eff}}|\text{leak}\rangle = 0$

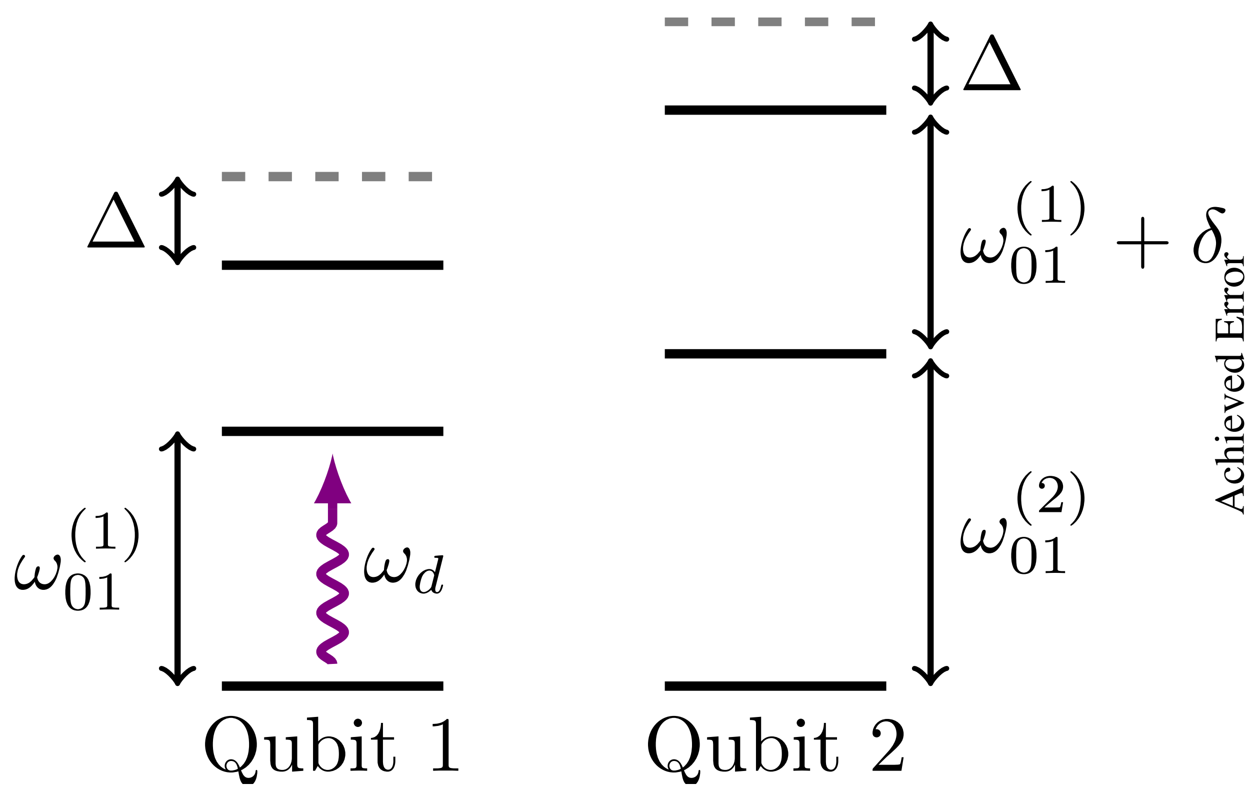
Controllable qubit: $\langle \text{qubit1}|H_{\text{eff}}|\text{qubit2}\rangle$

Underconstrained set of equations for S(t) +controls

Gambetta, Motzoi, Merkel, FKW, PRA 2011

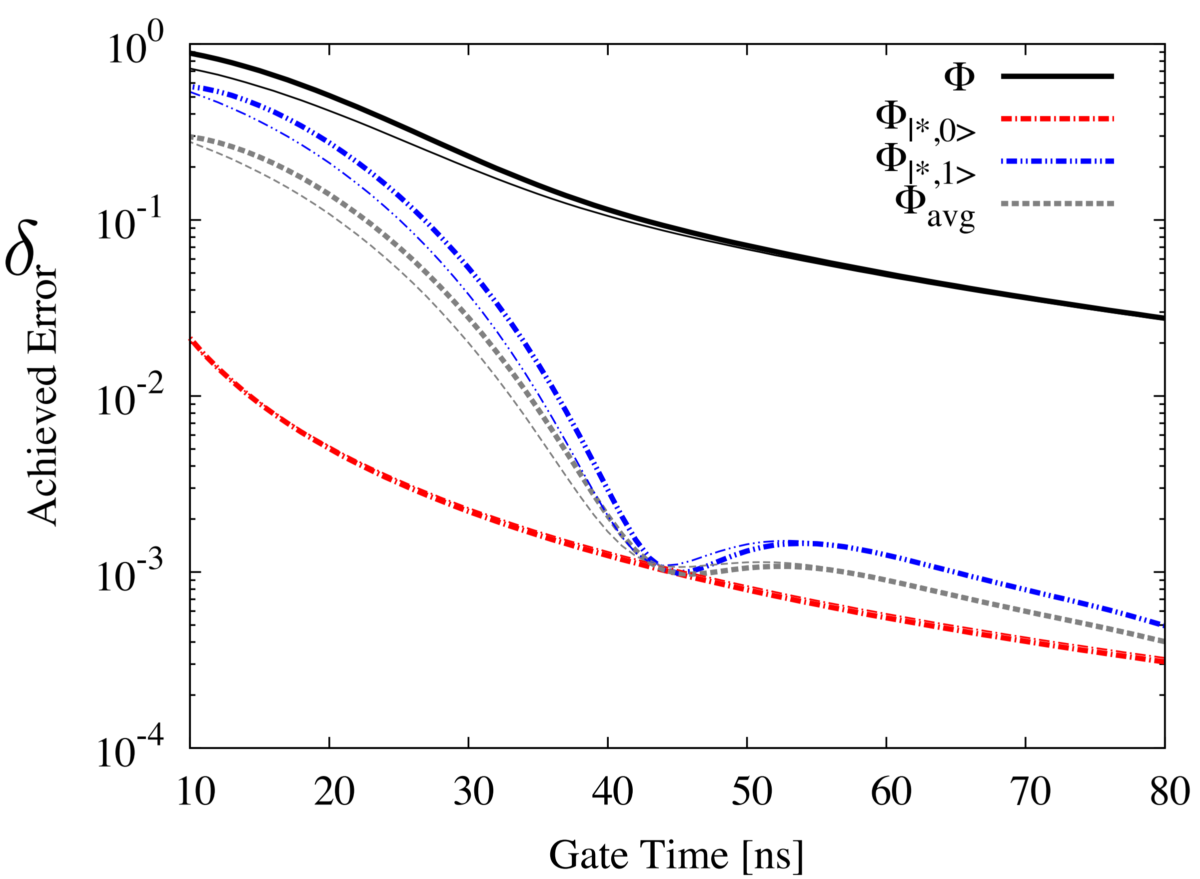
Multi-transition DRAG

Two 3D Transmons



$$\delta = 90\text{MHz} \simeq 0.2\Delta$$

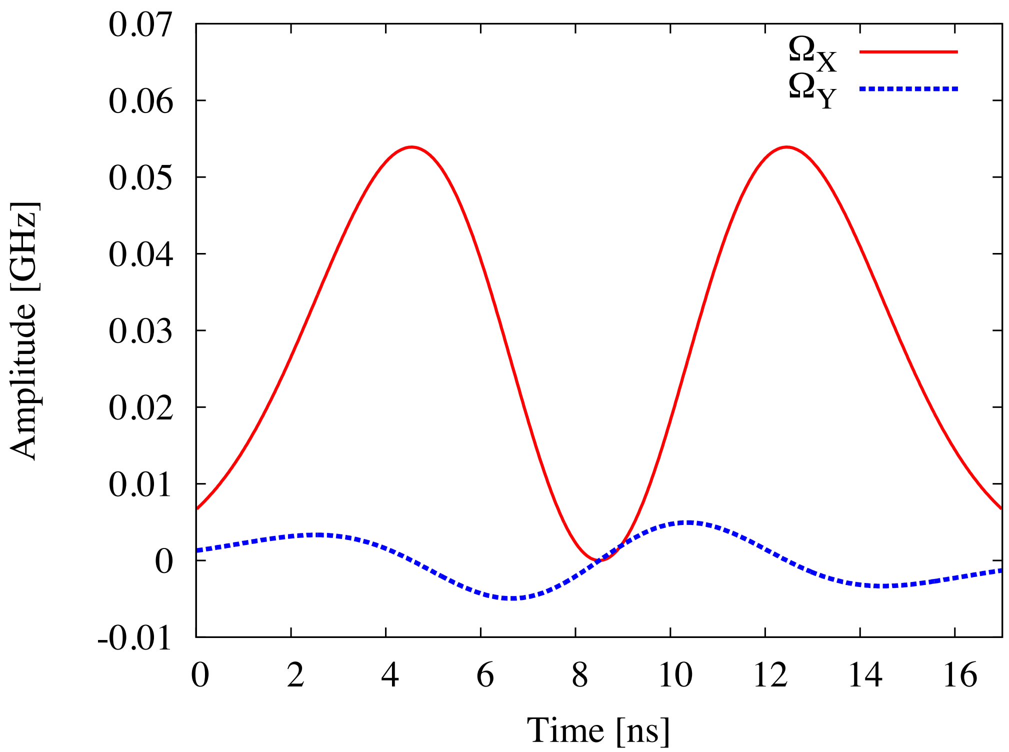
DRAG not effective

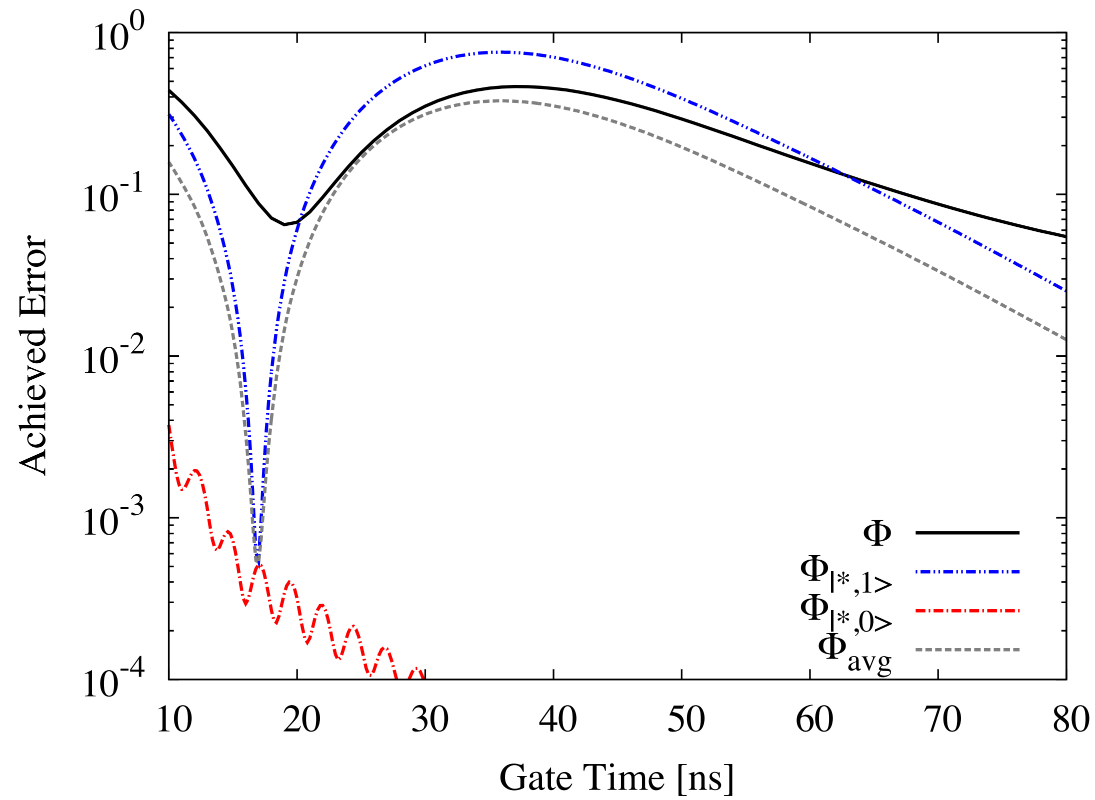


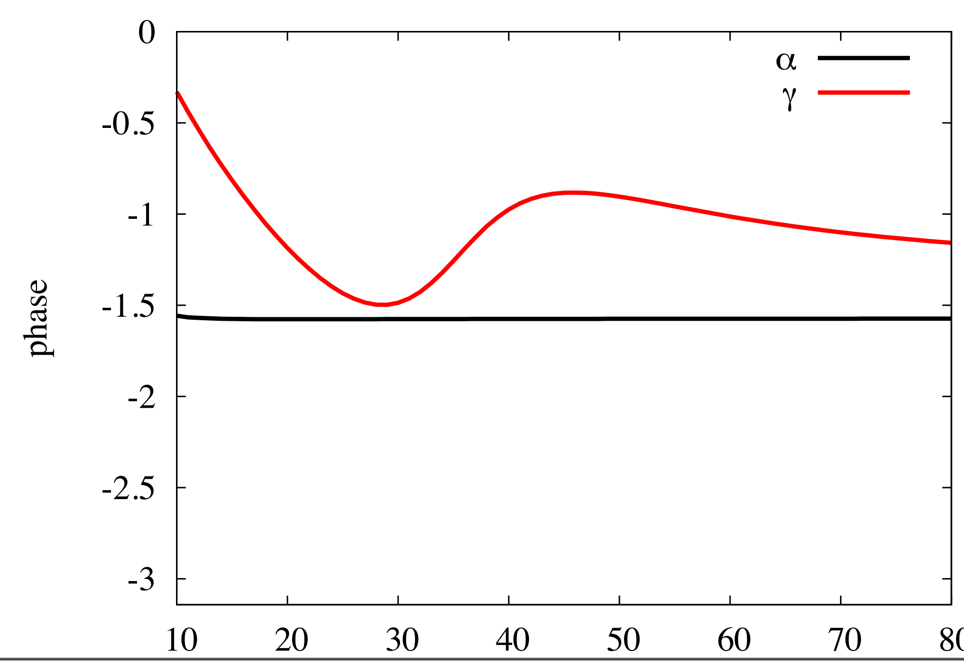
$$u_2 = \frac{\dot{u}_1}{\Delta_2}$$

Wah-wah pulses

Sideband modulation, optimized with Magnus expansion





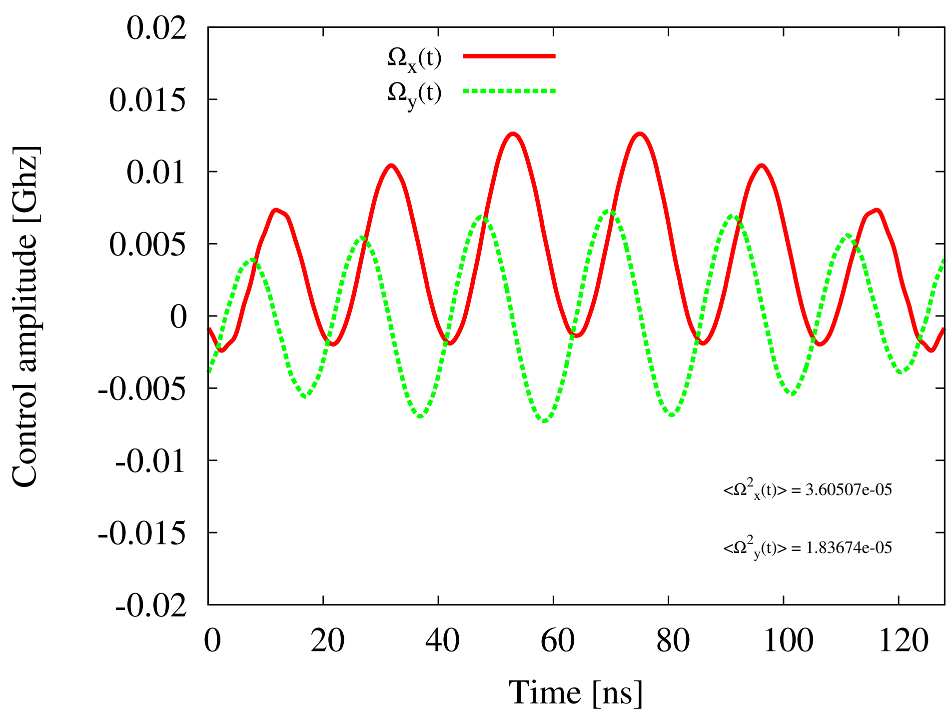


Sideband modulation + DRAG gets

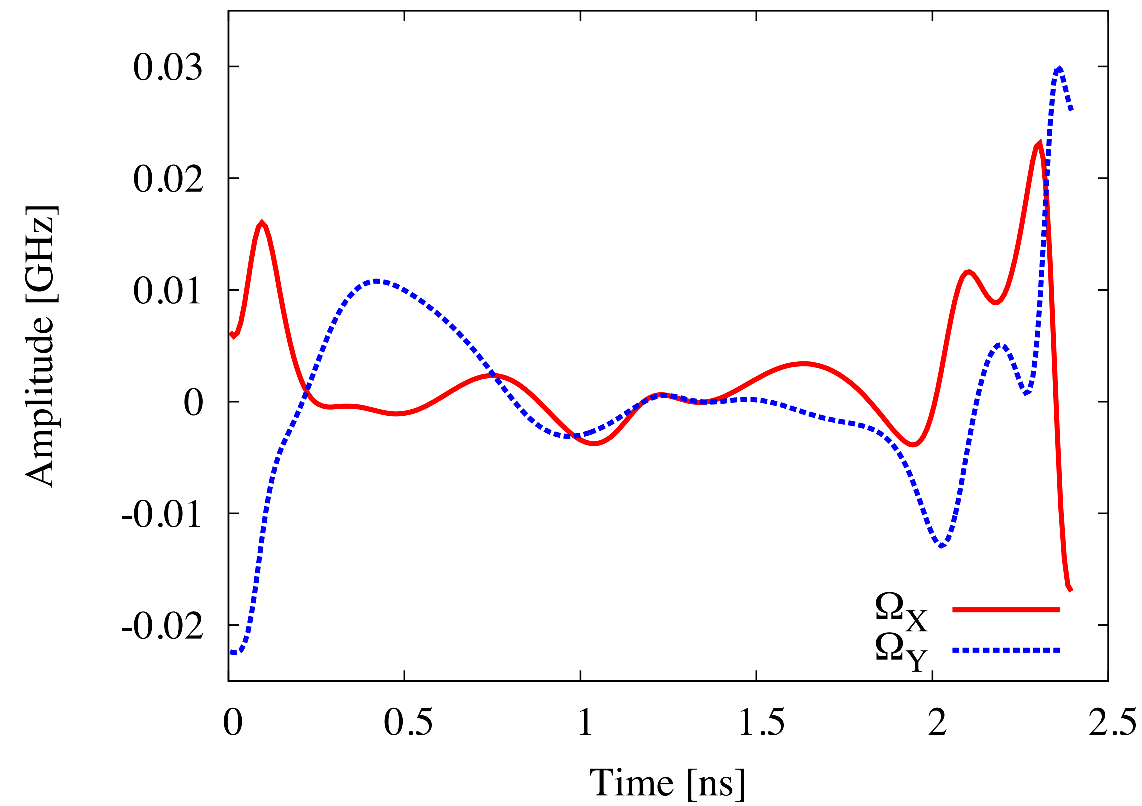
$$\hat{X} \otimes \exp(-i\alpha\hat{Z})$$

Numerical optimization

Long pulse



At the limit



No speed limit other than sufficient # of pixels

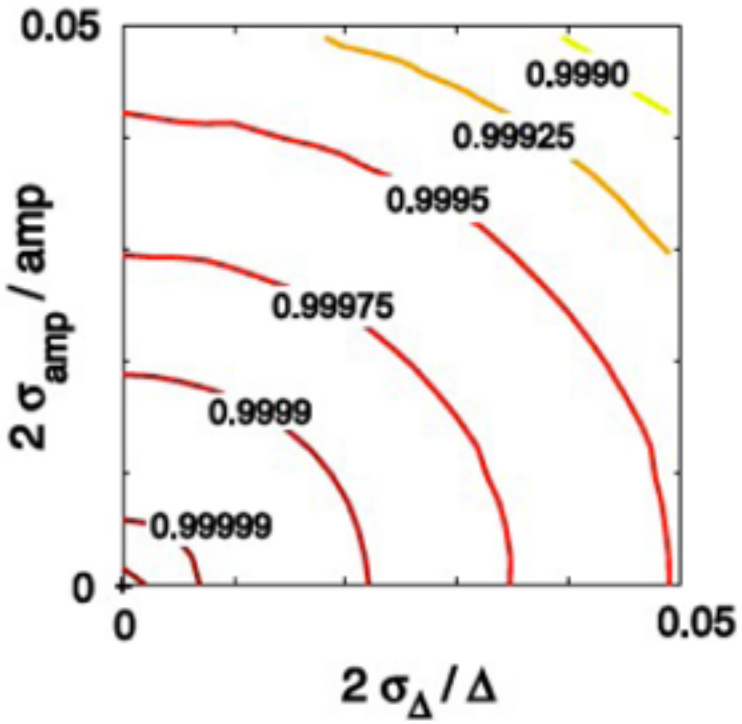
Schutjens, Abu Dagga, Egger, FKW, in preparation

Robustness of optimal pulses

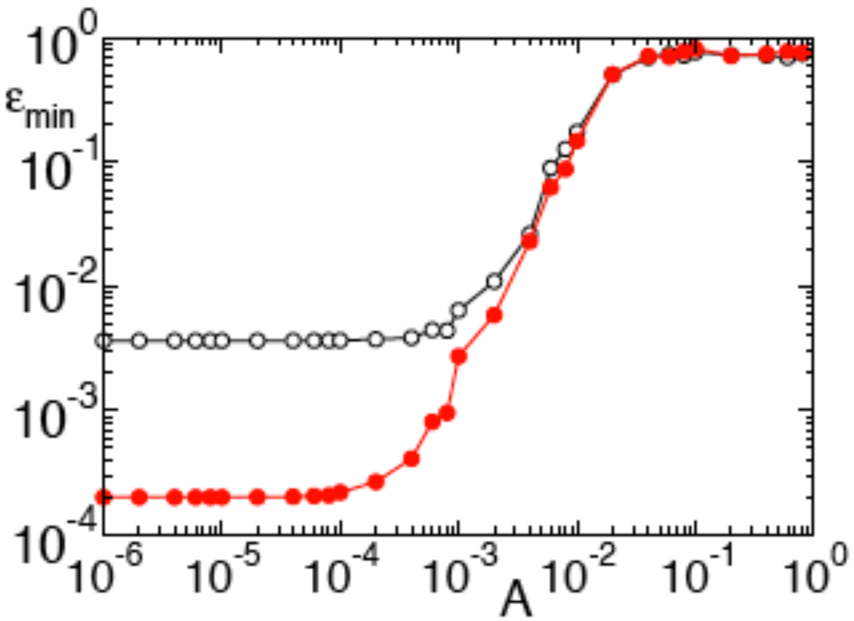
Spörl et al., 2007
Montangero et al., 2007
Khani et al., 2011

Robustness of optimal pulses

Timing jitter



Slow noise



Extremum - flat

Spörl et al., 2007
Montangero et al., 2007
Khani et al., 2011

Robustness of optimal pulses

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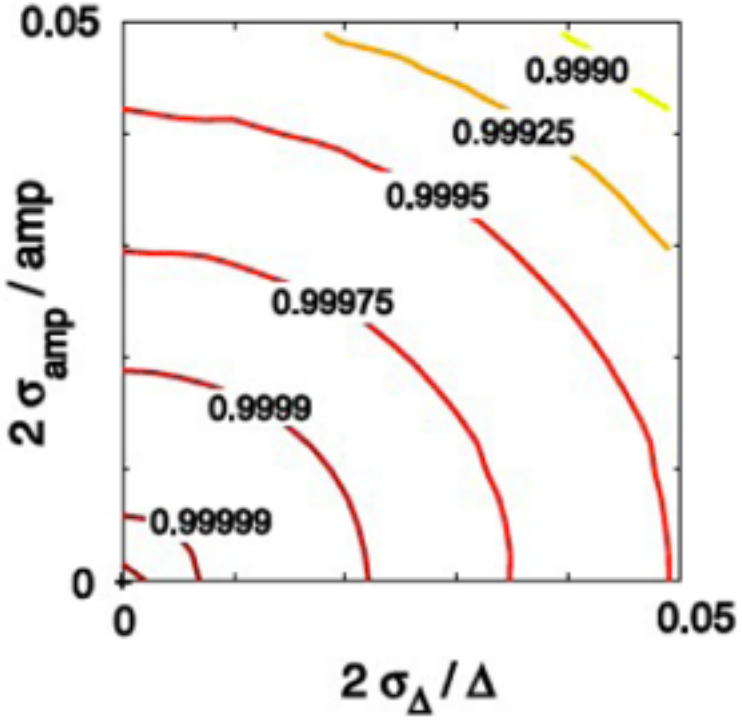

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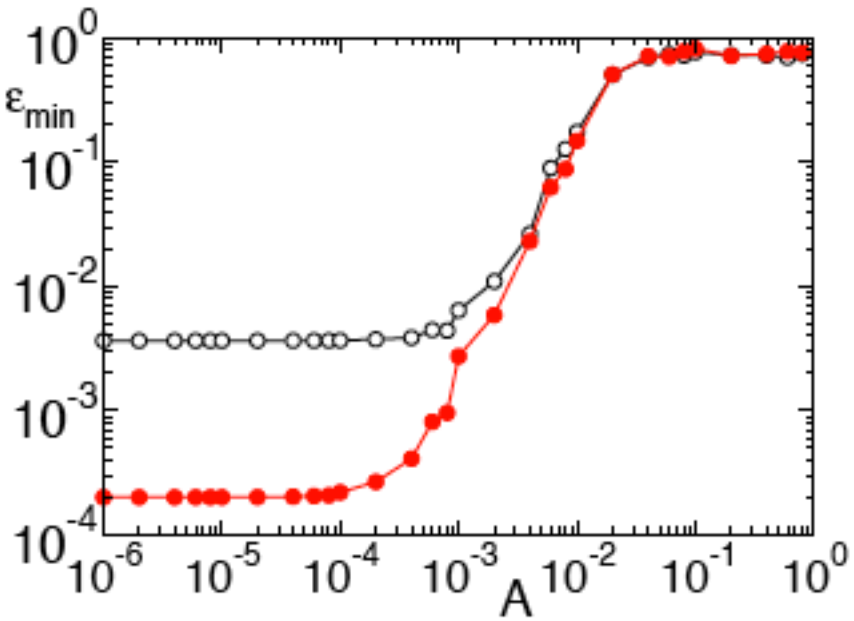

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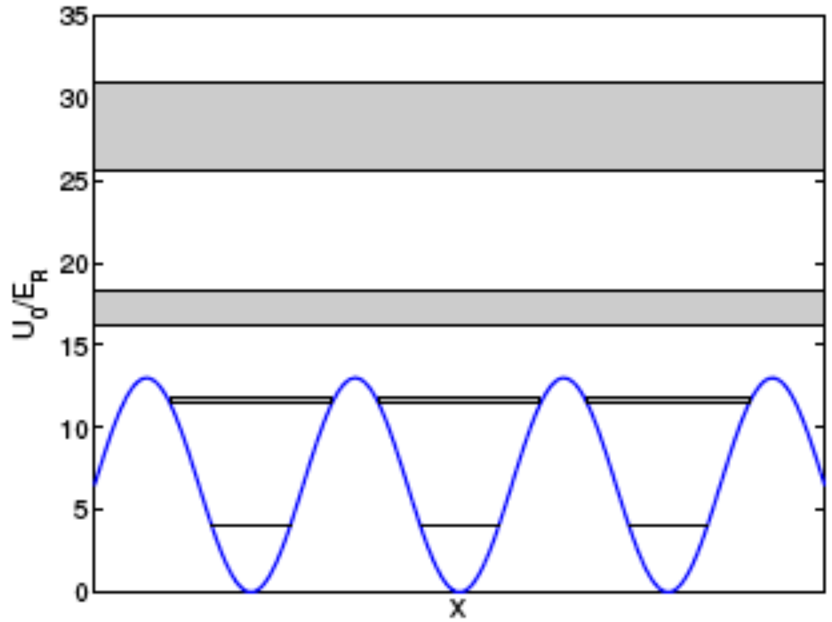
Better: Robust performance index



$$J \to \langle J \rangle$$

e.g. cosine potential

$$H_0 = p^2 + r\sin^2 x = p^2 + \frac{r}{2}(1 - \cos 2x),$$

Spörl et al., 2007
Montangero et al., 2007
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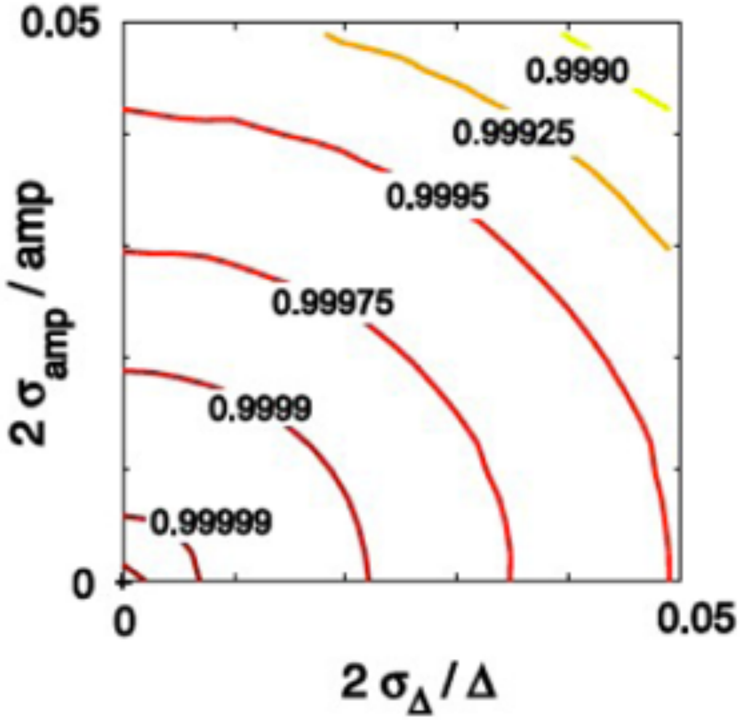
Robustness of optimal pulses

Timing jitter



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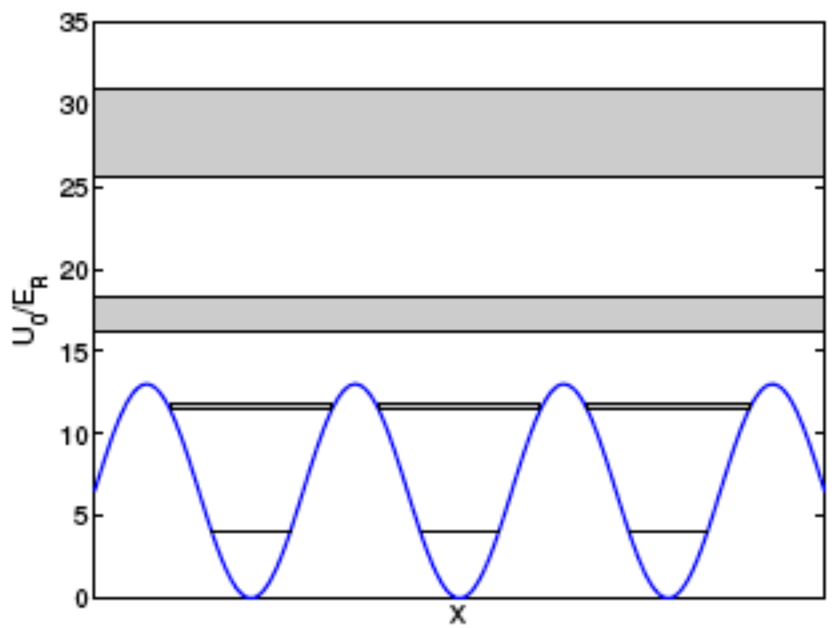

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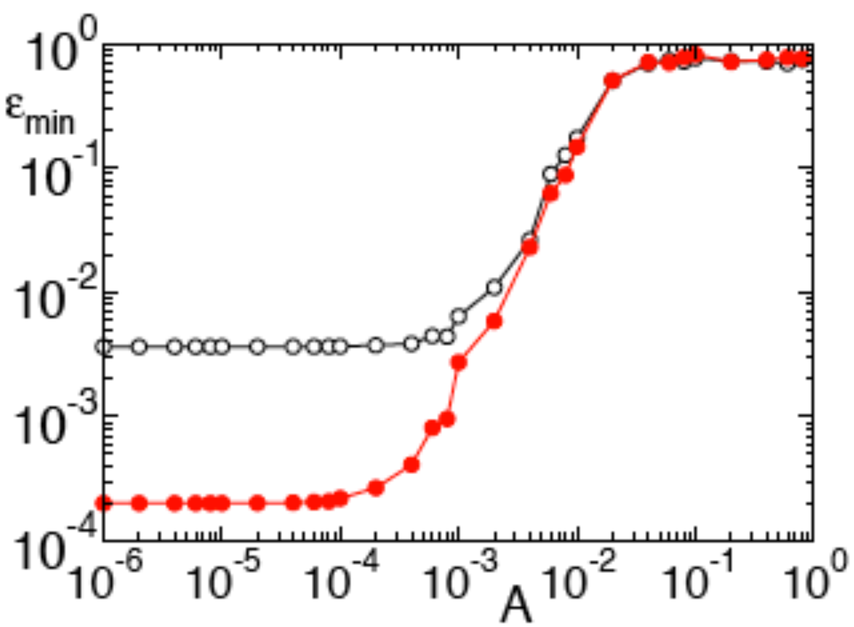

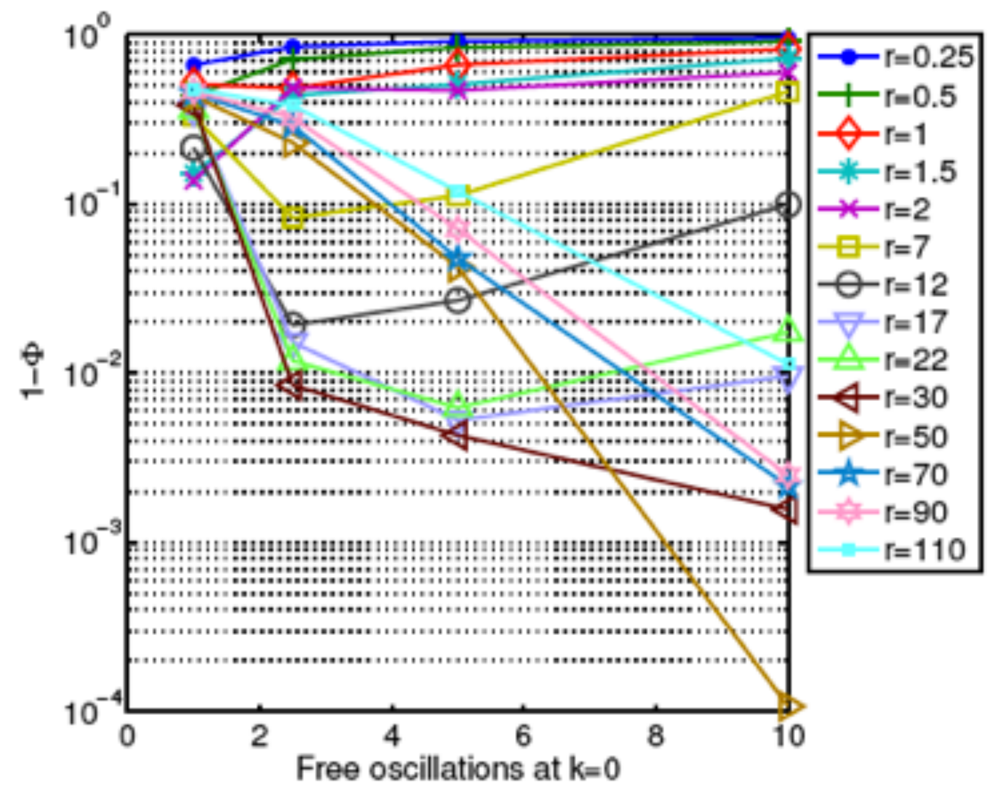

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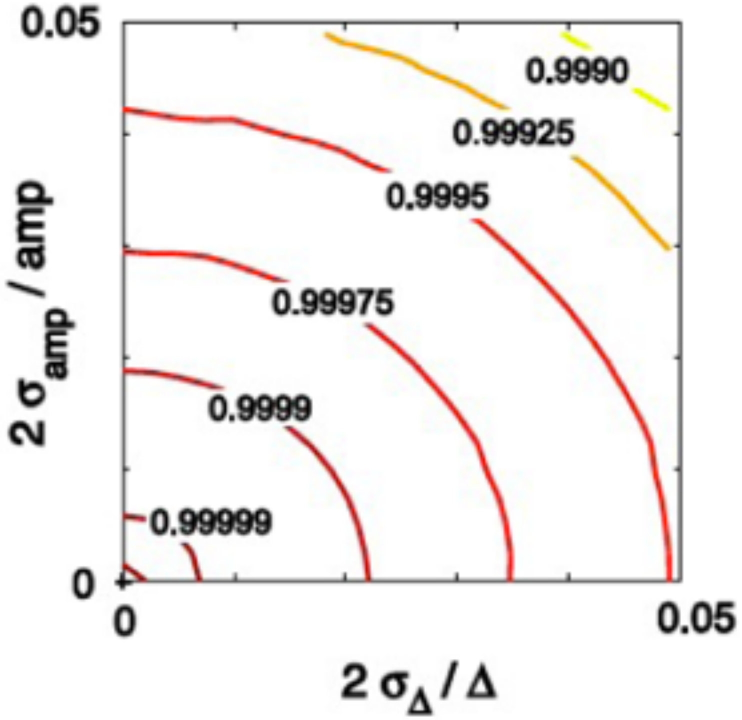
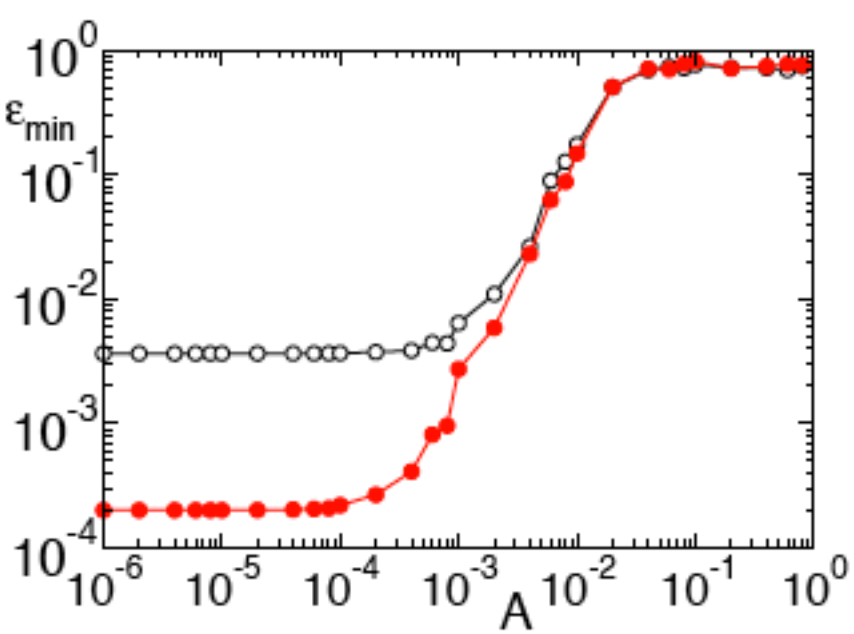
Montangero et al., 2007
Khani et al., 2011

Robustness of optimal pulses

Timing jitter

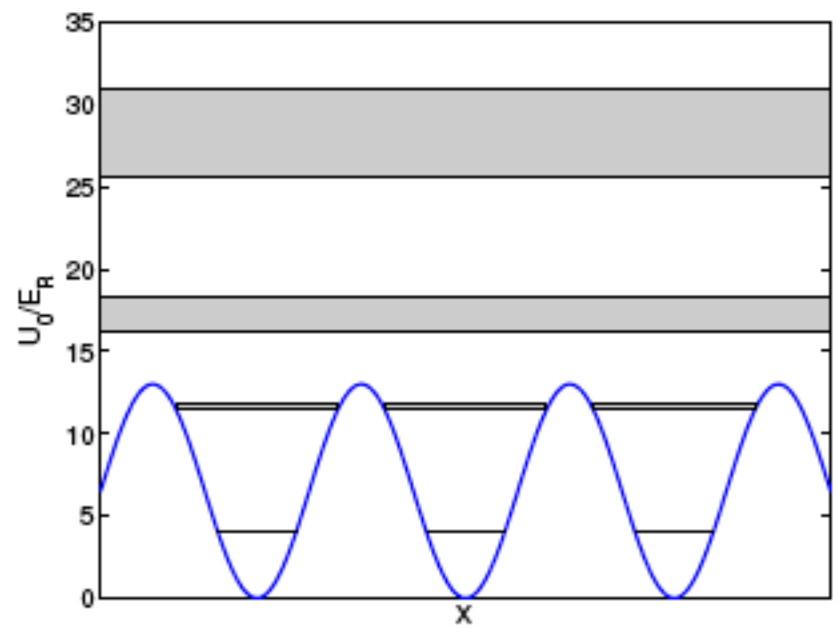


Slow noise



Extremum - flat

Better: Robust performance index



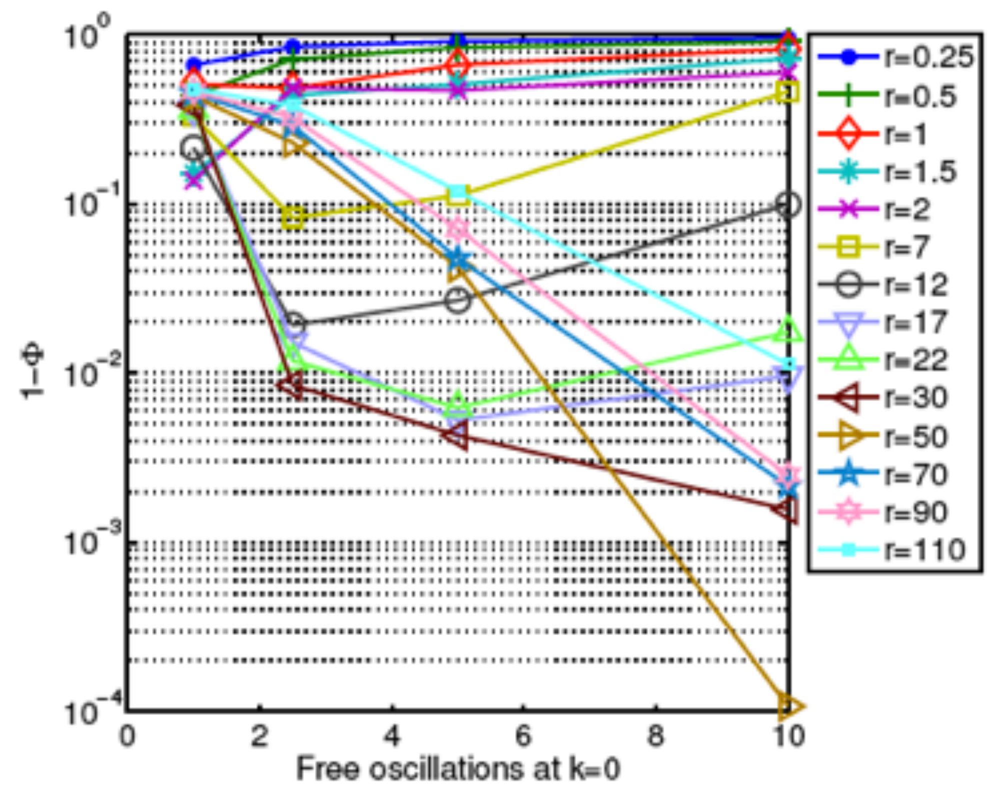
$$J \to \langle J \rangle$$

e.g. cosine potential

$$H_0 = p^2 + r\sin^2 x = p^2 + \frac{r}{2}(1 - \cos 2x),$$

large r: little nonlinearity

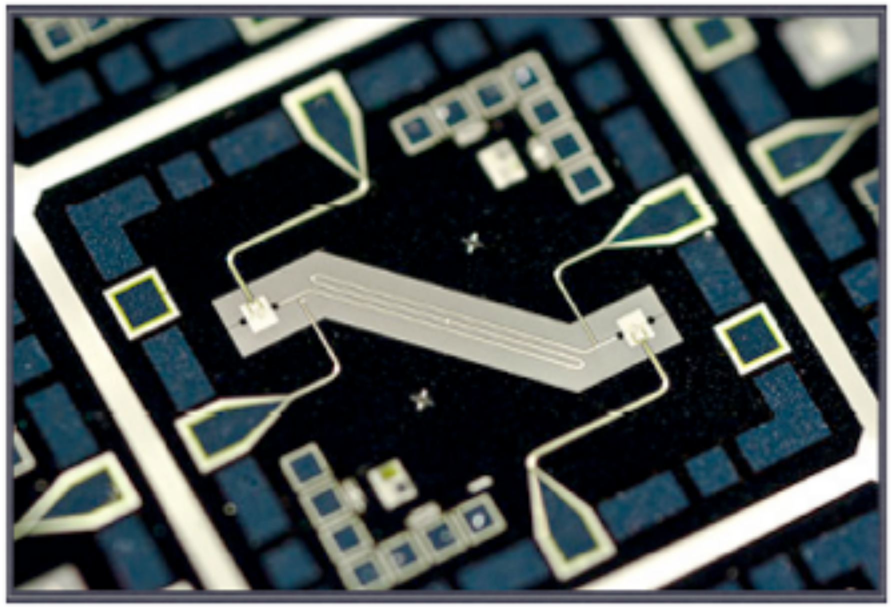
small r: large dispersion

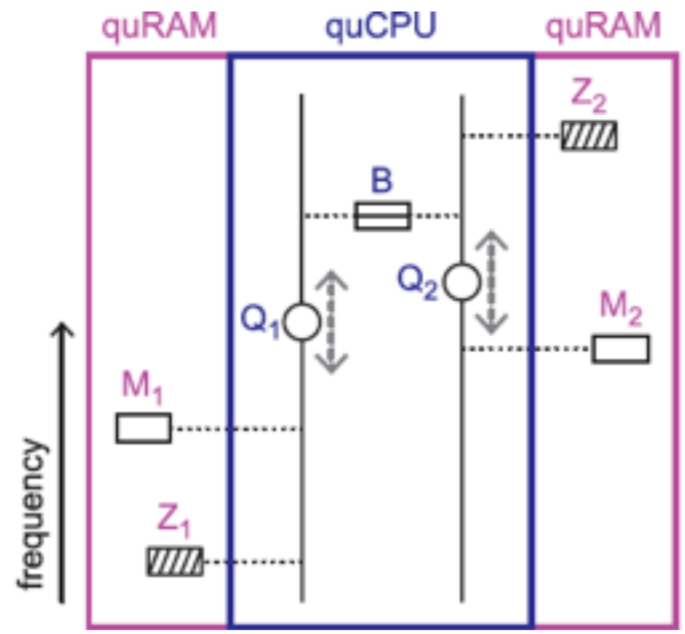


Spörl et al., 2007
Montangero et al., 2007
Khani et al., 2011

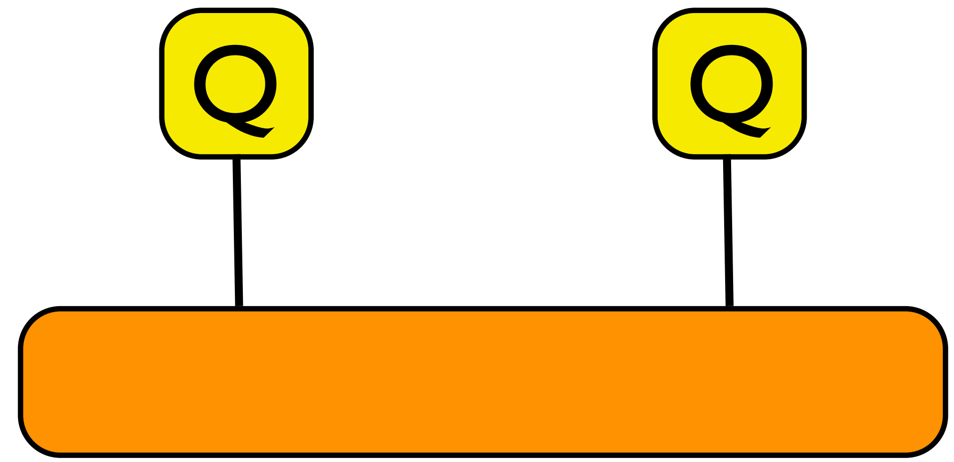
Making a CZ work

RezQU CPHASE





Cavity-assisted CPHASE



$$|0\rangle \rightarrow |0\rangle$$

Qubit Anharmonicity

$$\hat{H}^R = \sum_{k=1}^{2} \delta_k(t)\,\hat{n}_{q_k} + \Delta_k(\delta_k)\,\hat{\Pi}_{2,k} + g_k\left(\hat{\sigma}_k^{+}\hat{a}_b + \hat{\sigma}_k^{-}\hat{a}_b^{\dagger}\right)$$

Qubit Frequency
(Relative to bus)

Qubit-Bus Coupling

Goal: CPHASE between qubits

$$|\mathrm{q}_1, \mathrm{c}, \mathrm{q}_2\rangle$$

$$|\sigma_1, 0, \sigma_2\rangle \mapsto (-1)^{\sigma_1\sigma_2}|\sigma_1, 0, \sigma_2\rangle$$

M. Mariantoni et al., Science 2011

Baseline sequence

Hamiltonian $\hat{H}_{\mathrm{JC}} = g\left(\hat{\sigma}^{+}\hat{a} + \hat{\sigma}^{-}\hat{a}^{\dagger}\right)$

conserves total # of excitations

$t = \pi/g$ iSWAP

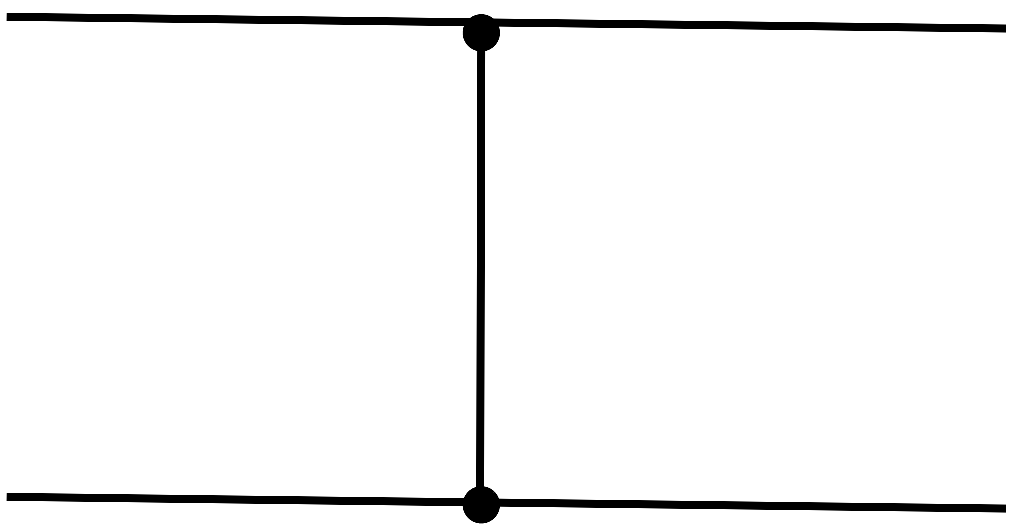
$t = \pi/2g$ -1

- single excitation subspace: iSWAP Qubit 1 into resonator $|1,0,x\rangle \mapsto |0,1,x\rangle$
- two-excitation subspace: conditional phase between res and Qubit 2 $|0,1,1\rangle \mapsto |0,0,2\rangle \mapsto |-0,1,1\rangle$
- iSWAP back into qubit 1

F. Strauch et al., PRL 2002

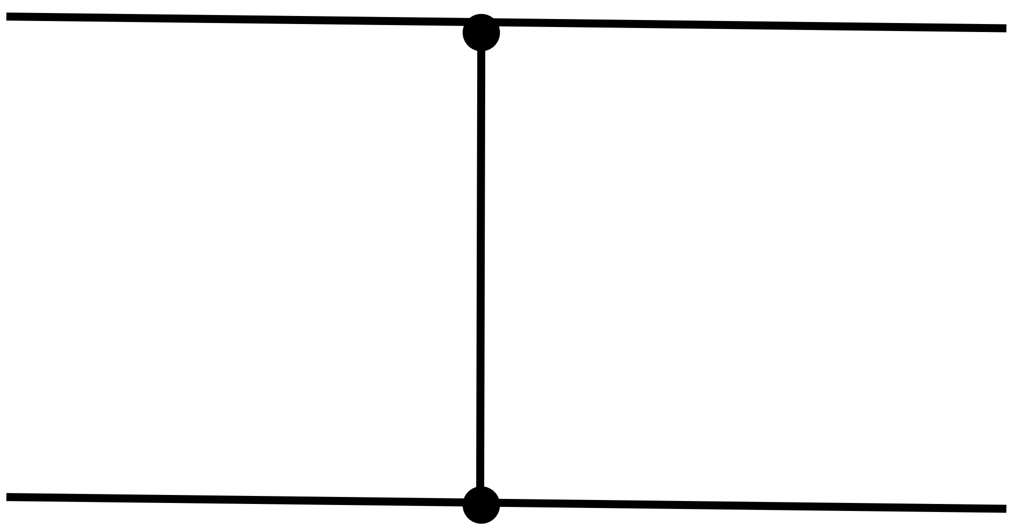
Baseline

CPHASE is symmetric

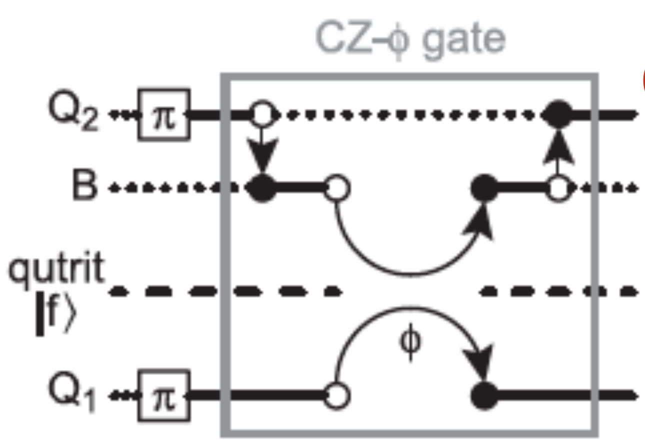


Baseline

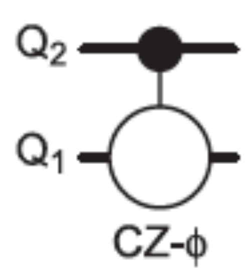
CPHASE is symmetric



Asymmetric sequence:



Ginger

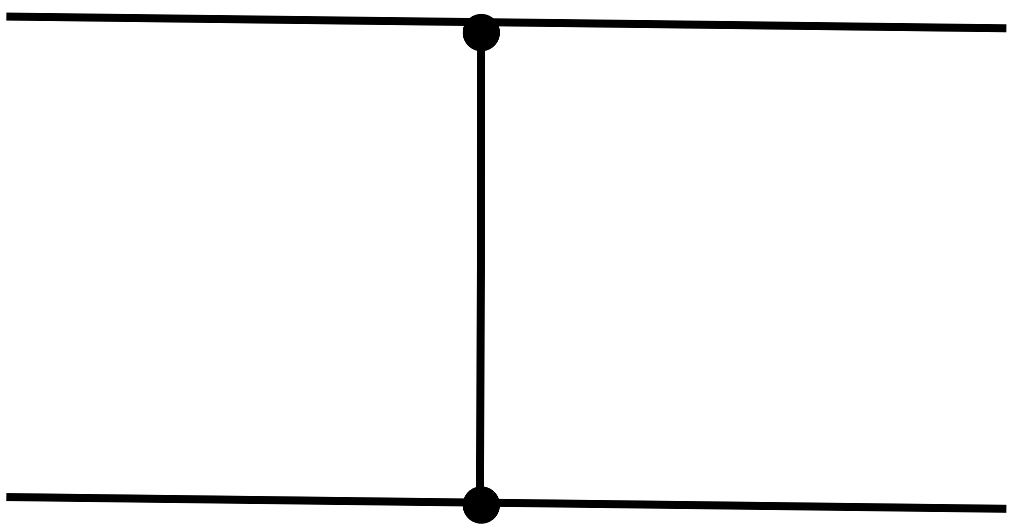


Fred

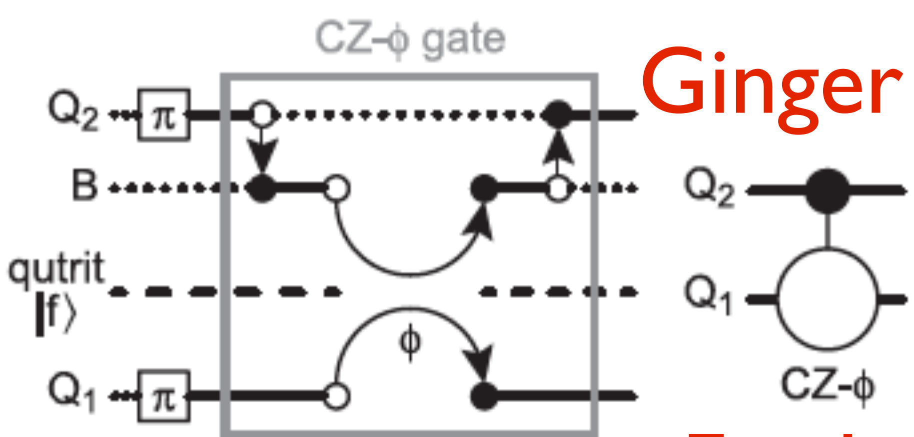
iSWAP - Strauch - iSWAP

Baseline

CPHASE is symmetric



Asymmetric sequence:



Ginger

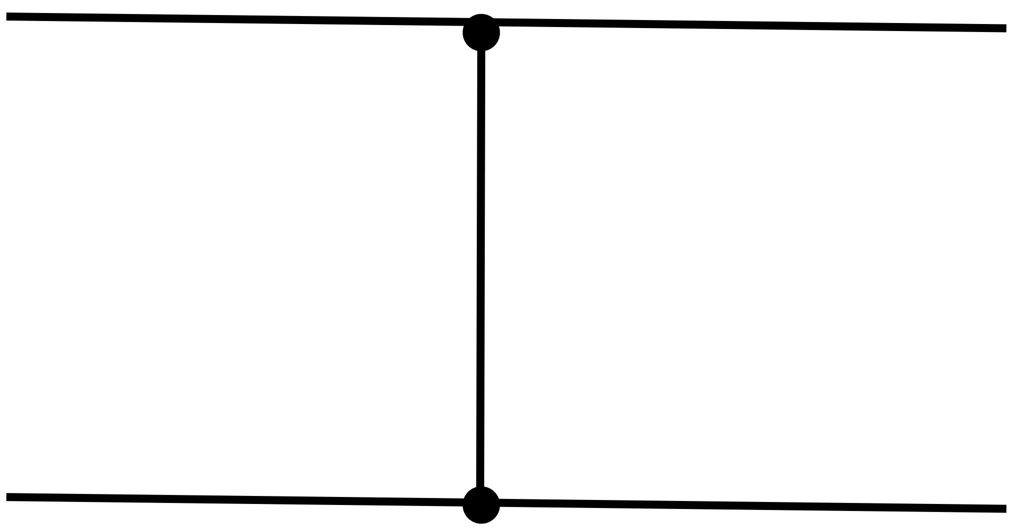
Fred

iSWAP - Strauch - iSWAP

- three sequential steps
- need to correct phases
- third-level errors

Baseline

CPHASE is symmetric



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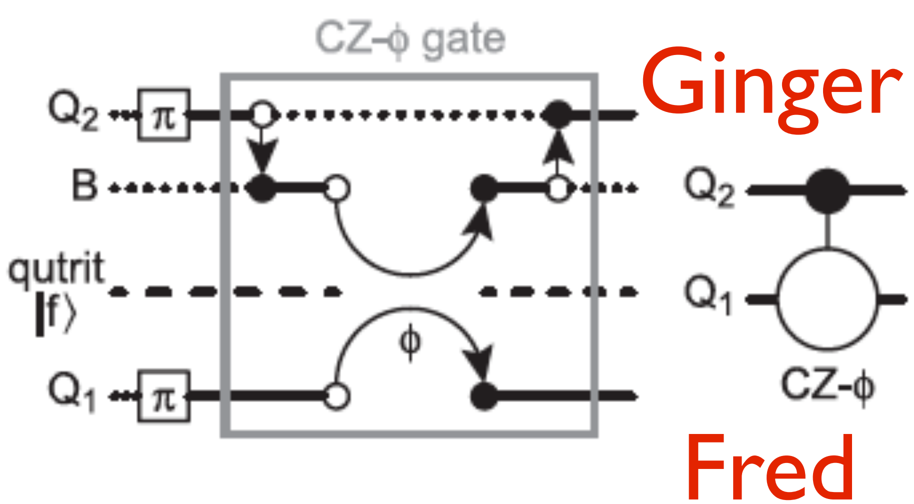
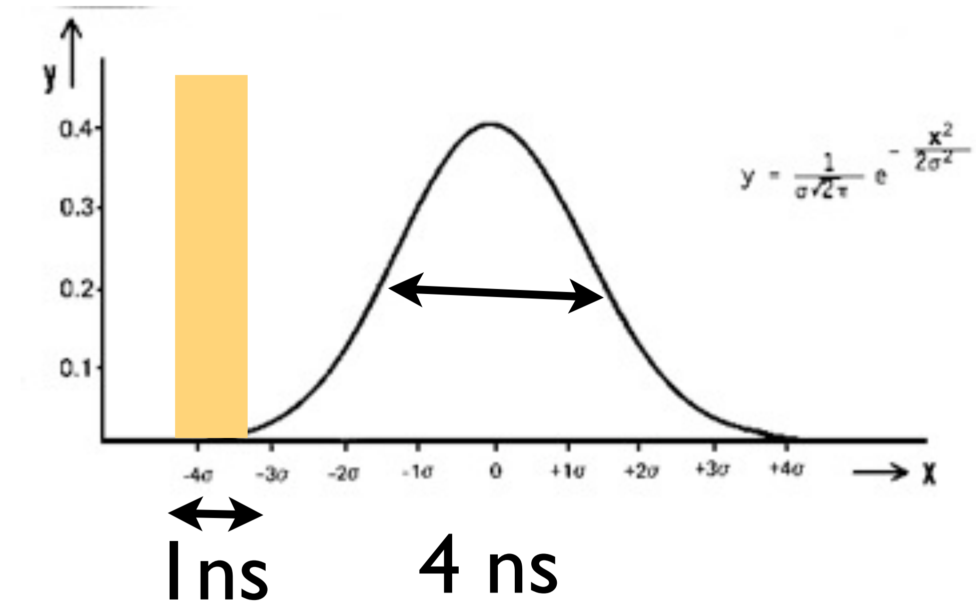


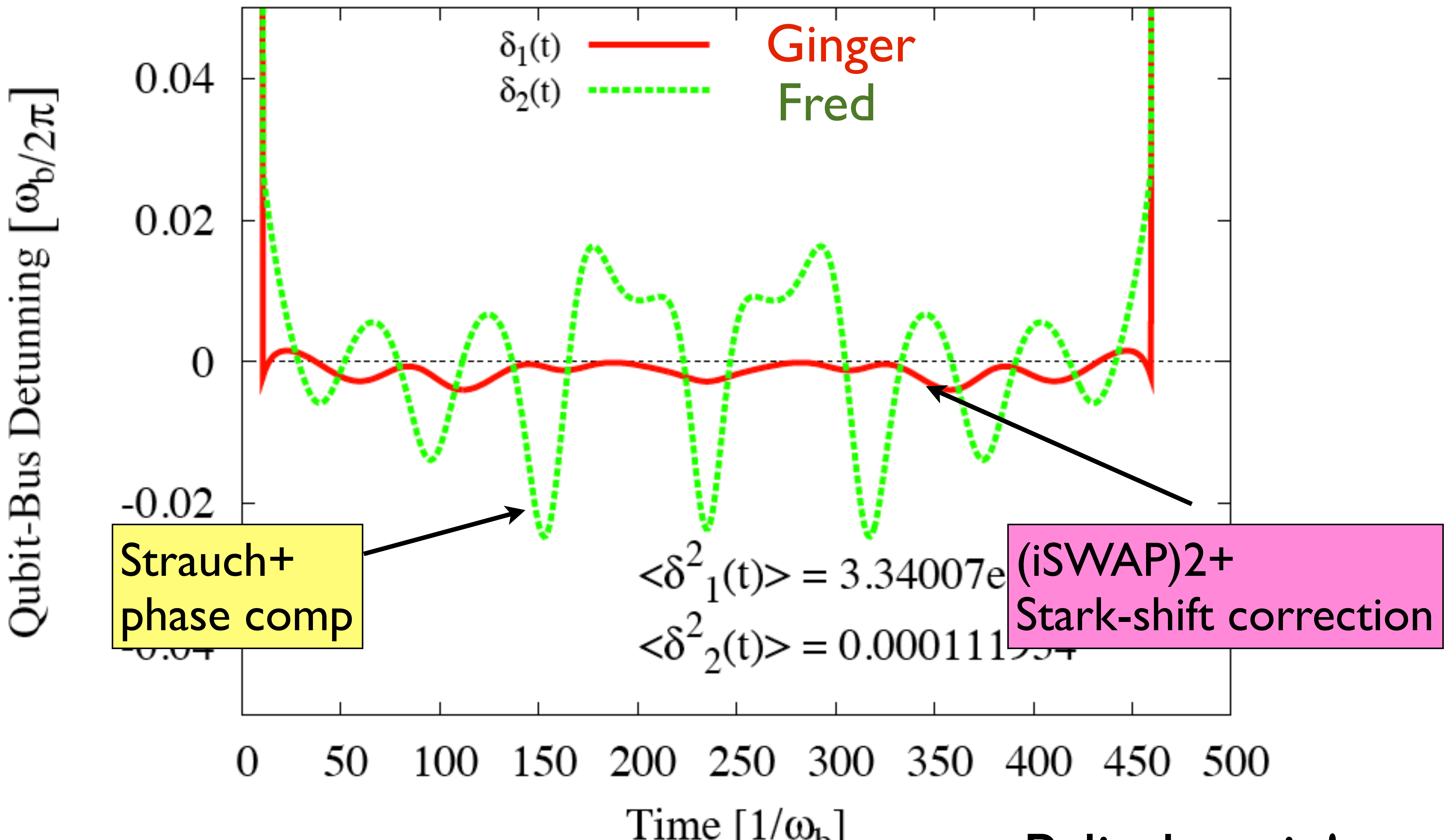
iSWAP - Strauch - iSWAP

Filtering extreme



- three sequential steps
- need to correct phases
- third-level errors

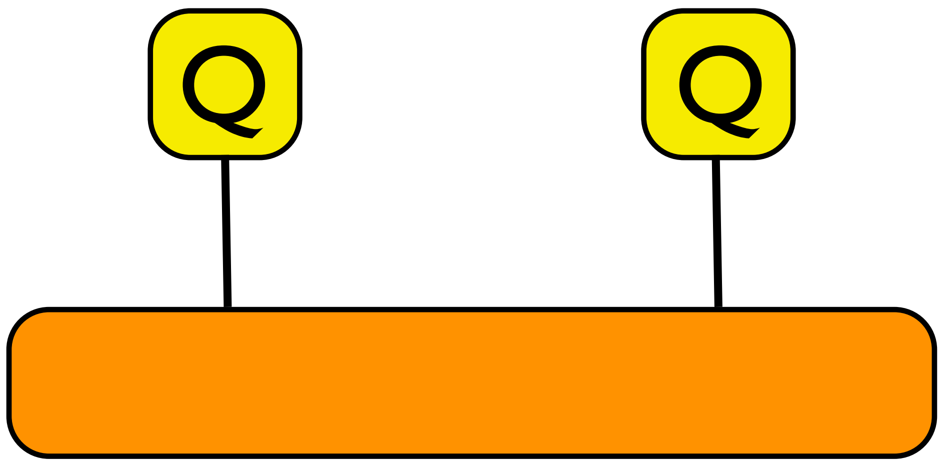
GRAPE pulse, no filter



Palindromic!

A worked example

Cavity-assisted CPHASE



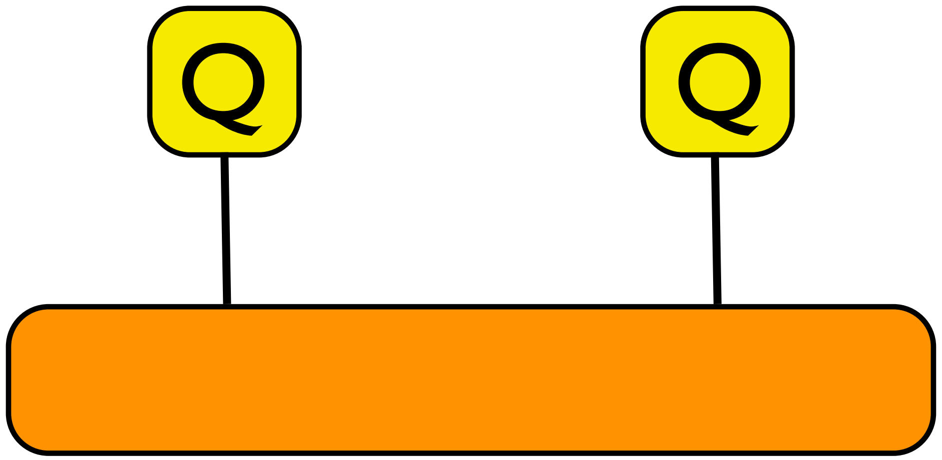
$$|0\rangle \rightarrow |0\rangle$$

- 27 ns gate
- 1 ns pixel length
- 4 ns Gaussian filter
- g= 30 MHz

D.J. Egger and FKW, in preparation

A worked example

Cavity-assisted CPHASE



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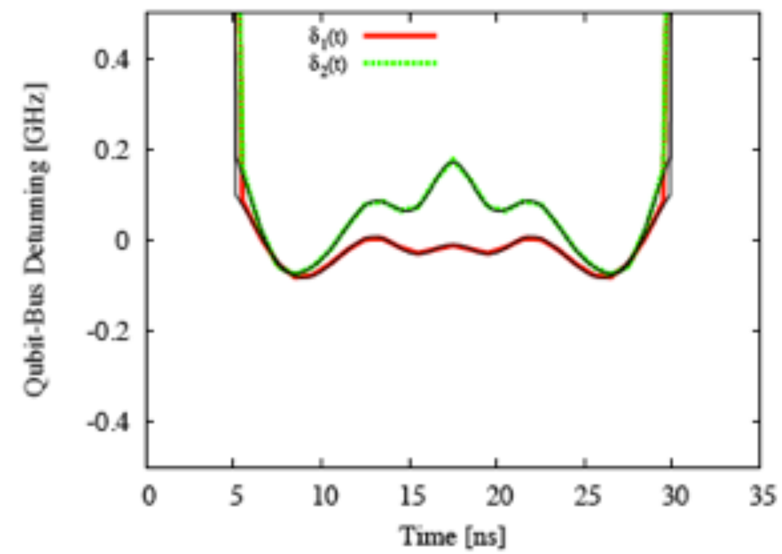


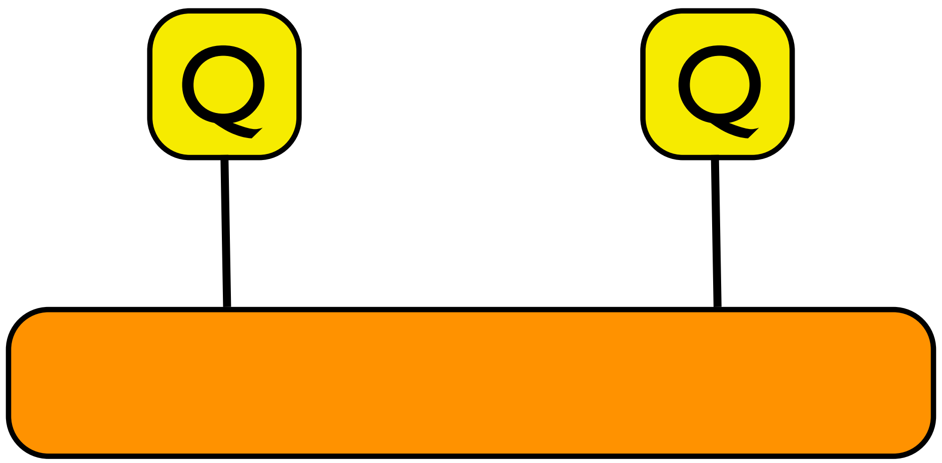
Ideal

- 27 ns gate
- 1 ns pixel length
- 4 ns Gaussian filter
- g= 30 MHz

D.J. Egger and FKW, in preparation

A worked example

Cavity-assisted CPHASE



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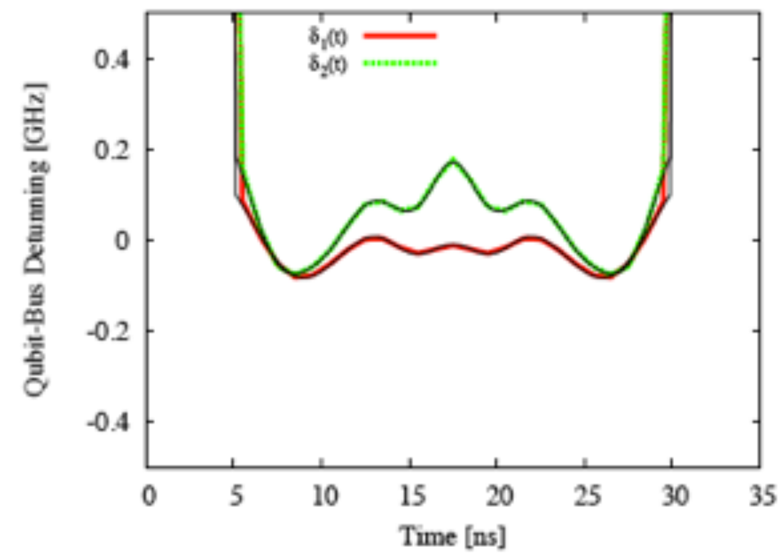
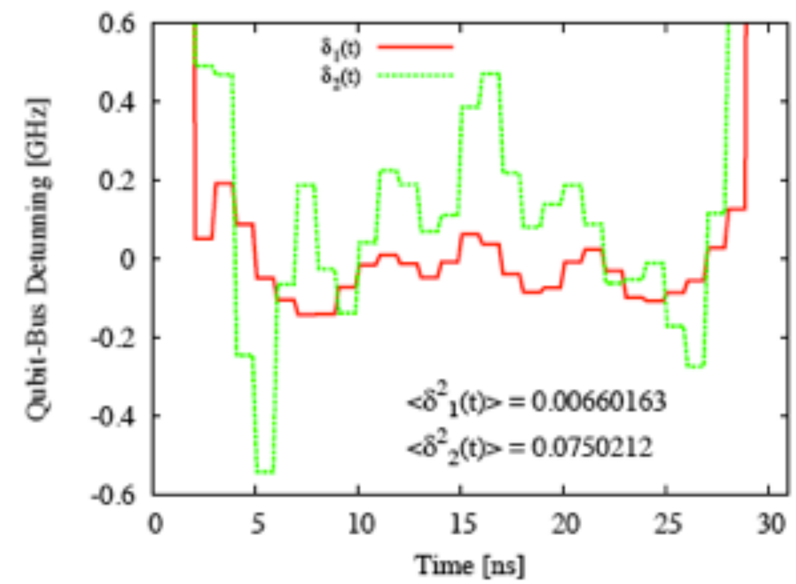


Ideal

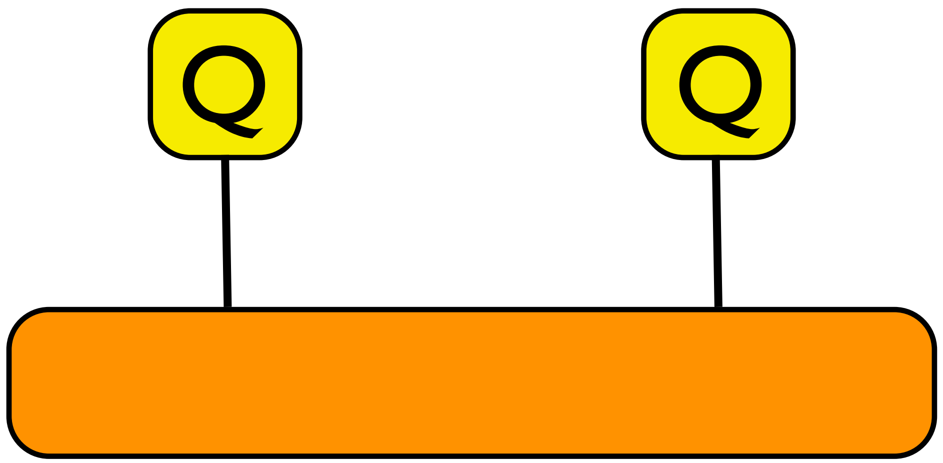


Program

D.J. Egger and FKW, in preparation

A worked example

Cavity-assisted CPHASE



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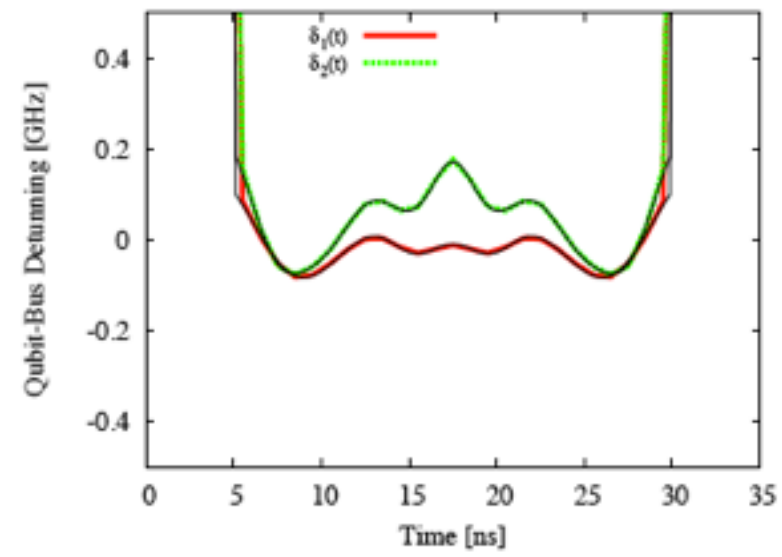
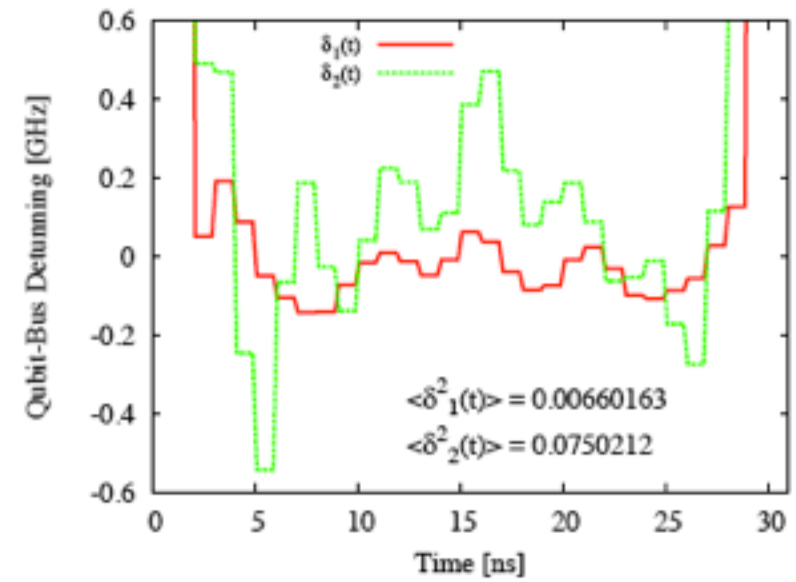
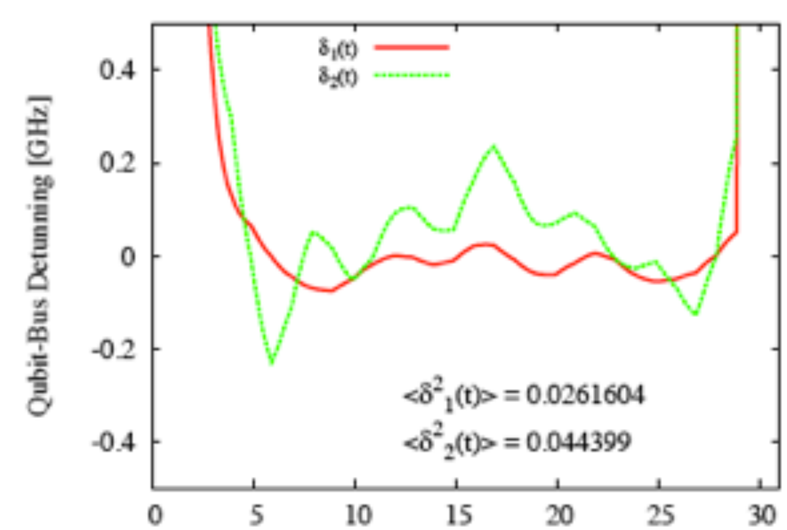
D.J. Egger and FKW, in preparation



Ideal

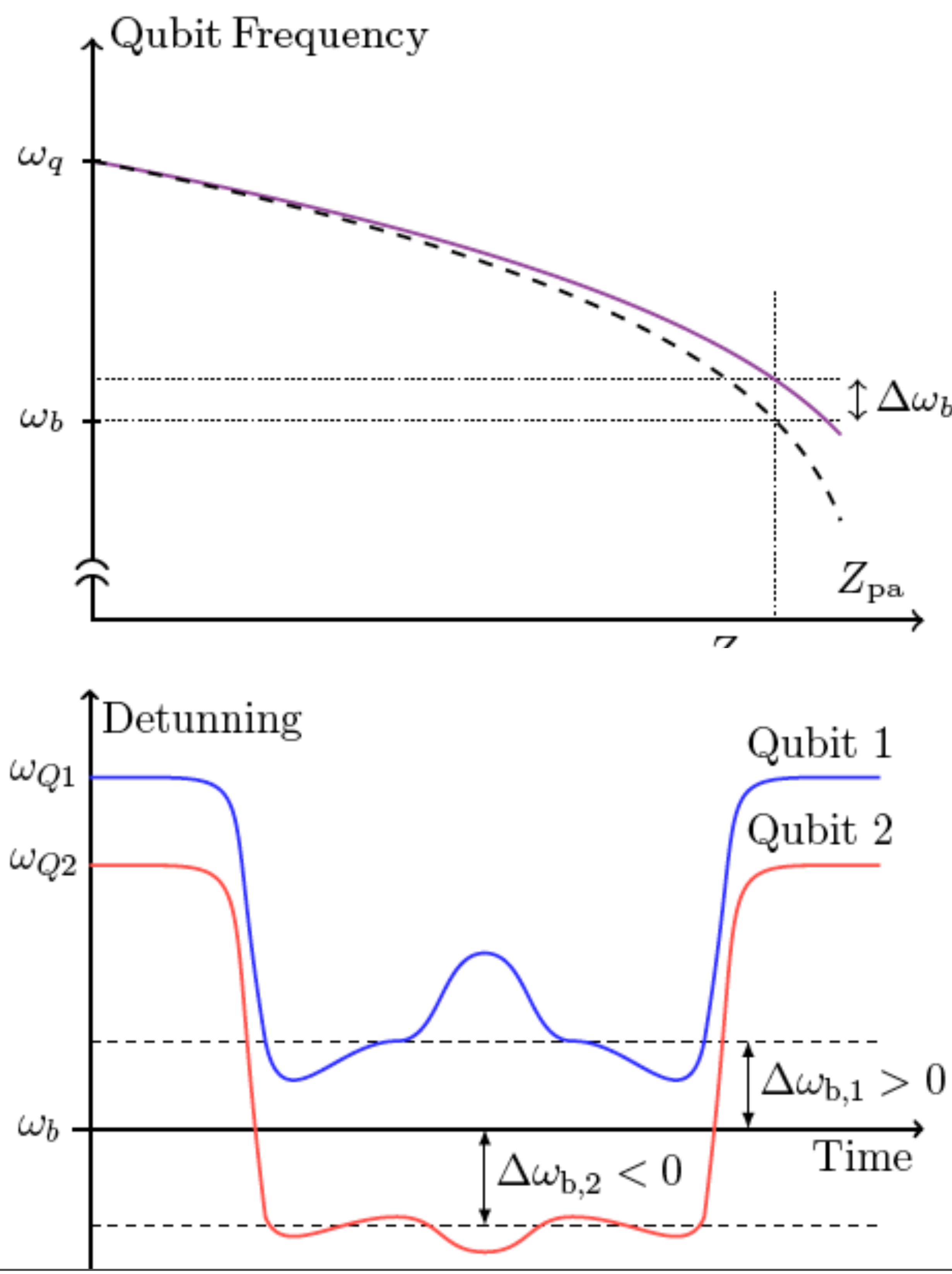


Program



Post-4ns filter

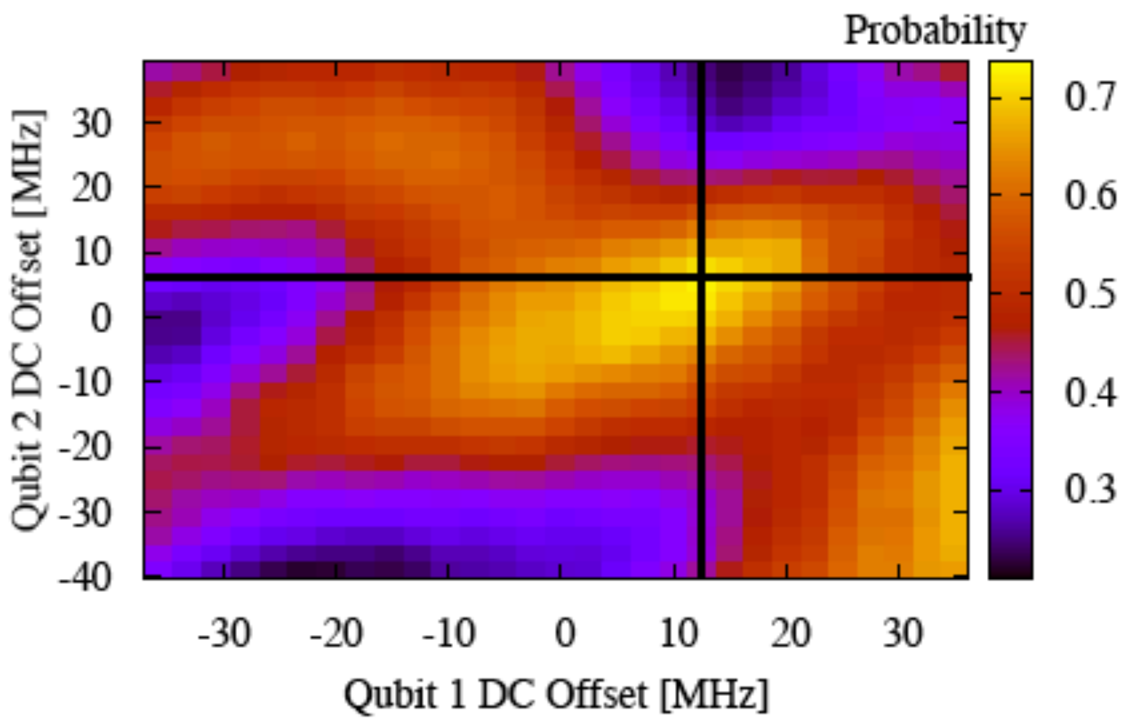
The challenge: Tuneup



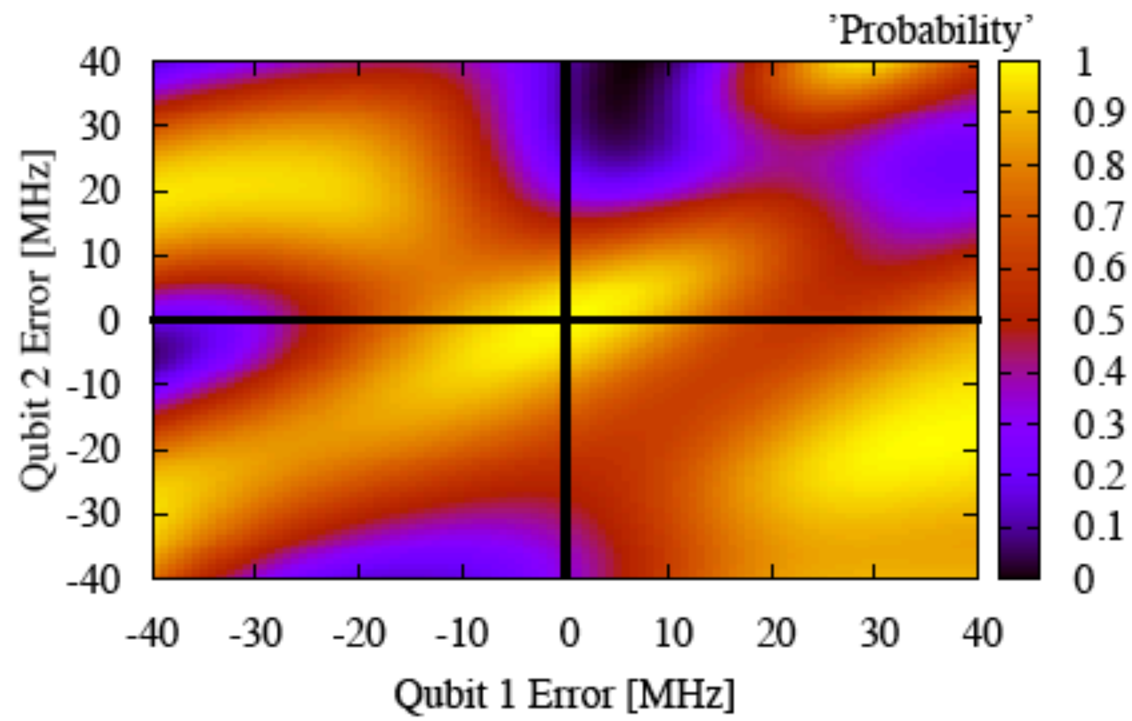
- Translation of control voltage into qubit frequency not known precisely
- Enters everywhere: High sensitivity

Pulse debugging

Phase / leakage level error landscape



Experiment



Theory

- Map out error landscape and minimize by hand
- seems to work

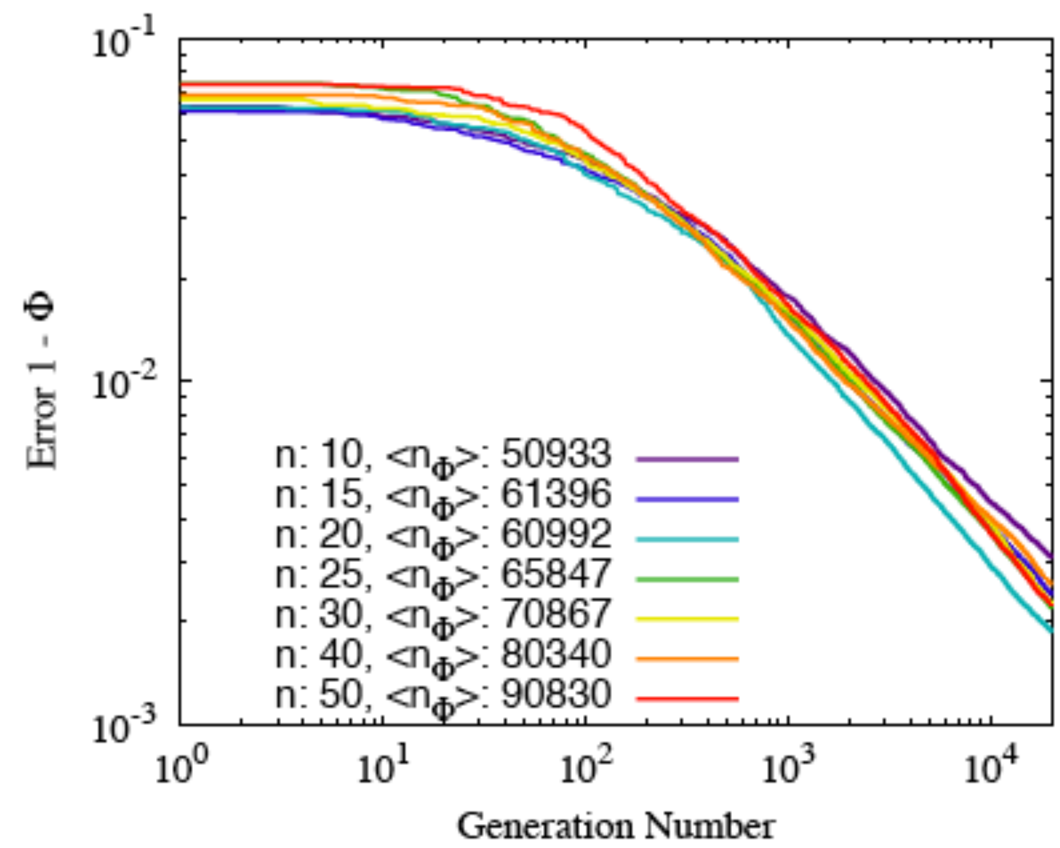
Lessons learned and potential solutions

- GRAPE pulses can work in superconducting qubits, but don‘t right now
- Precise system characterization is crucial, robust GRAPE does not suffice (yet)
- Nonlinear transfer functions limiting factor

So we work on these issues

Self-learning

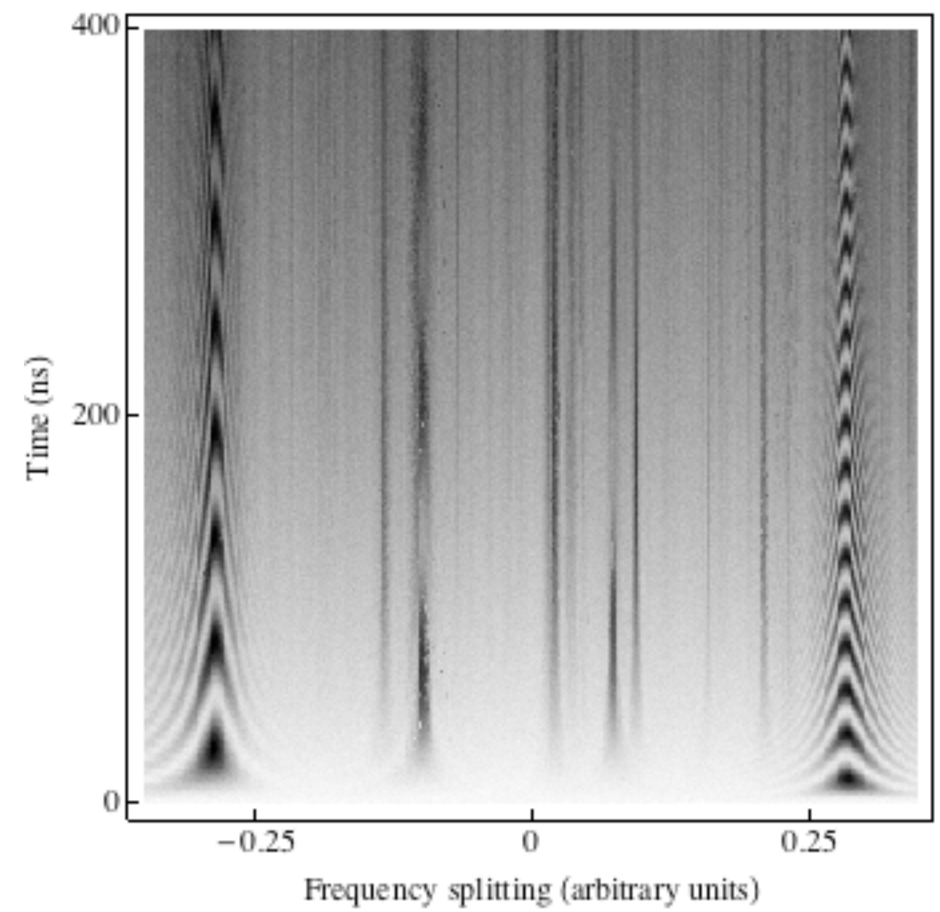
- Include the unmeasurable?
- Way out: Genetic algorithms
- Challenge: Easy measurement of performance (possible in specific cases)
- Note: These are GAs for unitaries, seeded by pretty good guesses

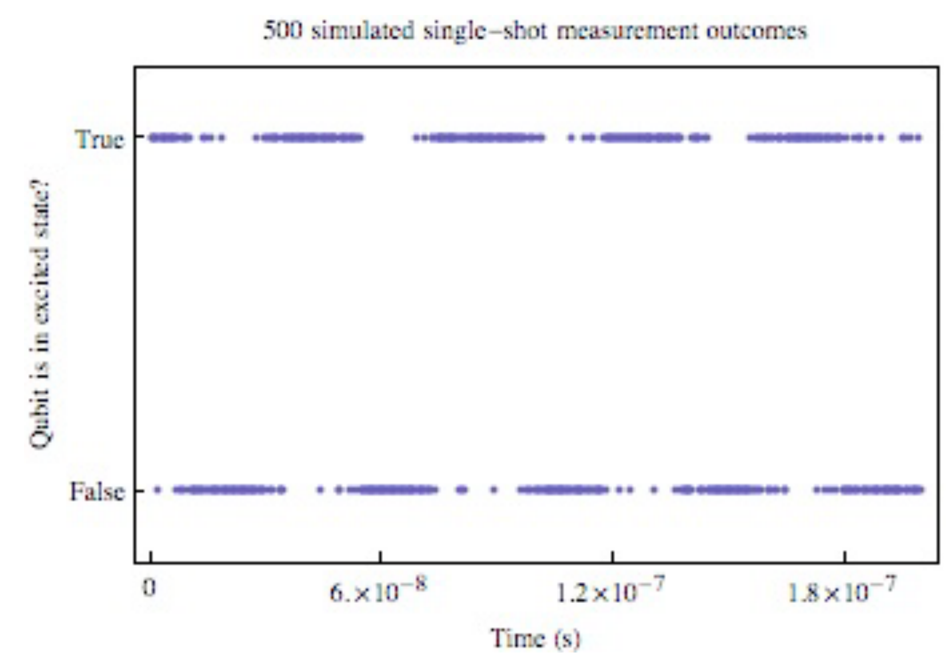


fs-chemistry:
Rabitz, Gerber ...

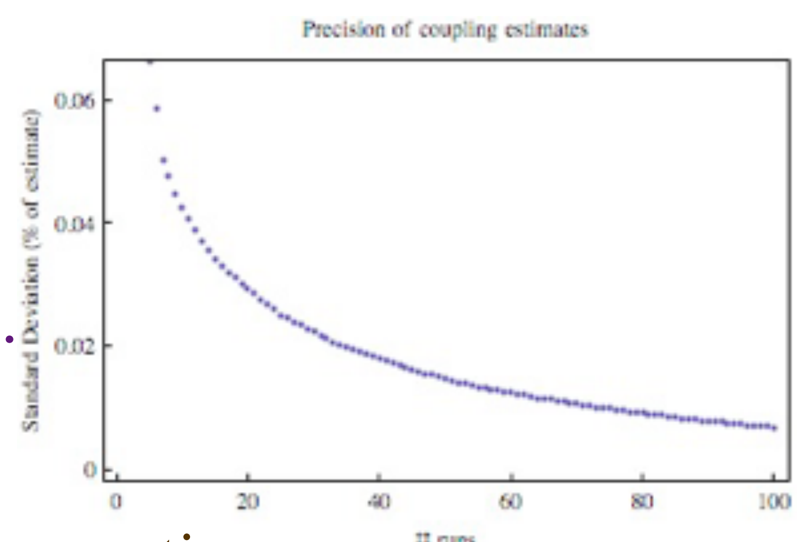
Hamiltonian learning

- Example: Find resonators and couplings
- Input: SWAP-spectroscopy = nonlocal FID
- Previous: 10000 shots / point
- speedup by nonadaptive Bayesian update





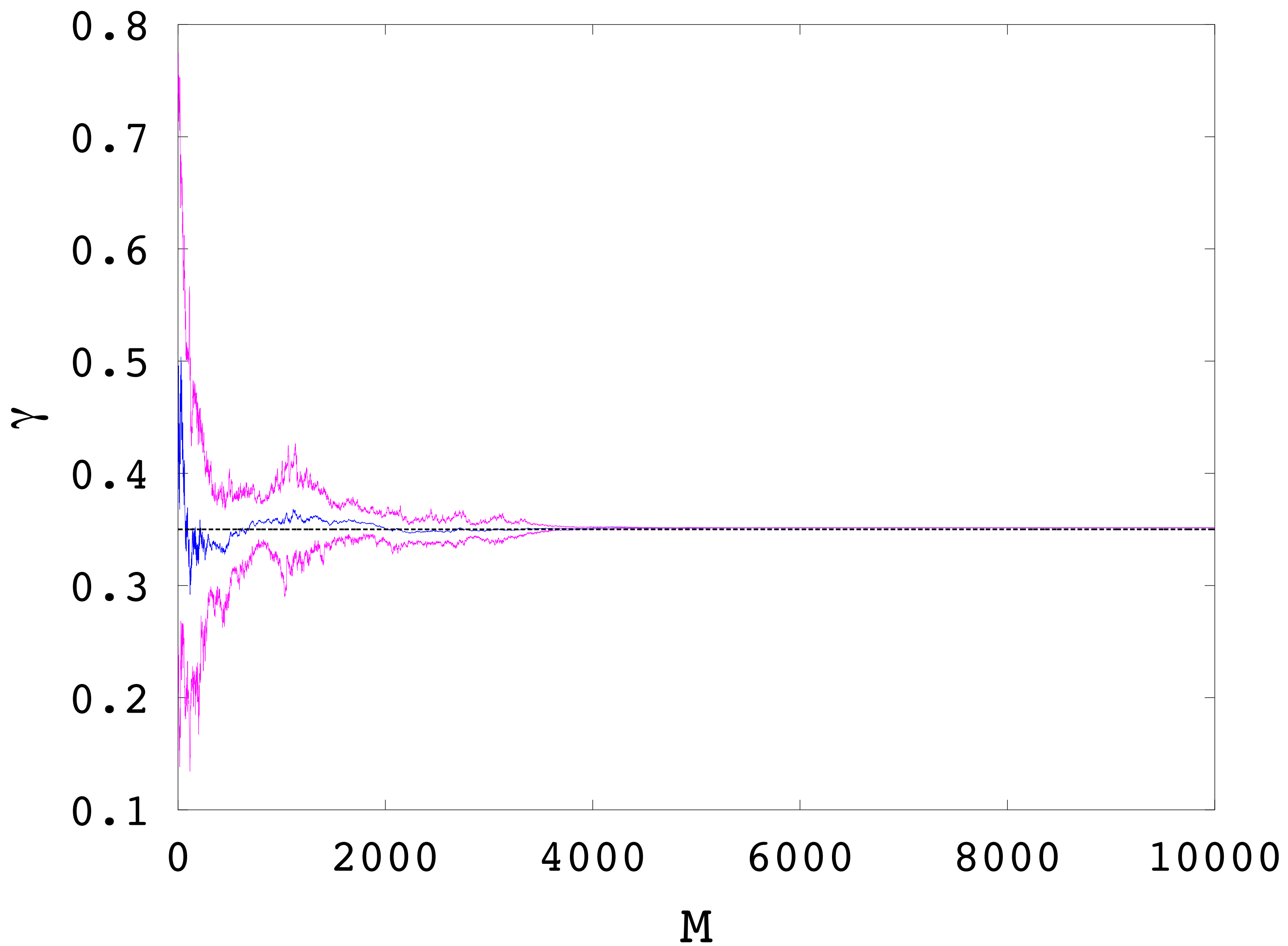
Based on Schirmer, Oi, Langbein, Wiseman, Ferrie ..



Y.R. Sanders and FKW, in preparation

Bayesian tomography

- Measure chi-Matrix efficiently
- diligent use of priors
- adaptive measurement



M. Stenberg and FKW, in preparation

Conclusions

- Superconducting qubits and optimal control mutually beneficial
- Solved higher-level leakage issue
- Ultrafast gates in frequency-crowded 3D-transmons
- challenge in application: closing the control-characterization loop