Quantum metrology of open dynamical systems: Precision limits through environment control

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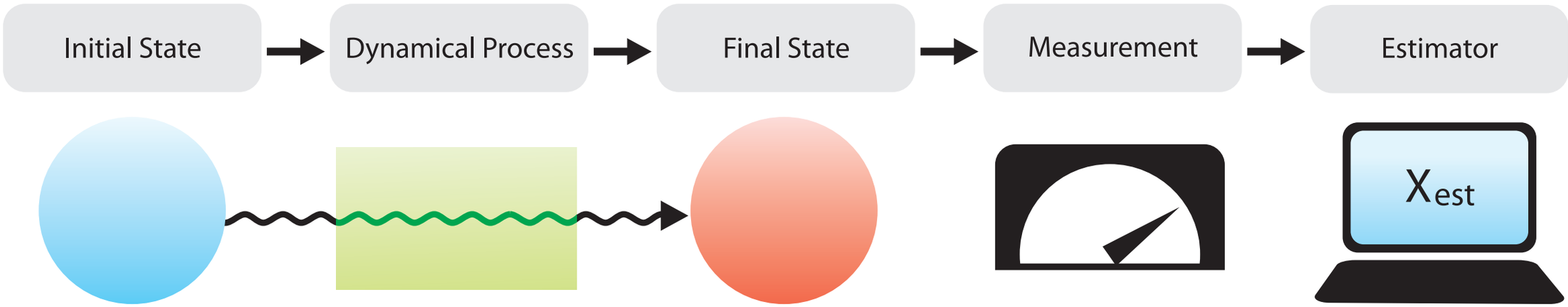
Based on work with Bruno M. Escher, Camille L. Latune, Marcio M.Taddei, Nicim Zagury, and Ruynet L. de Matos Filho

Parameter estimation in classical and quantum physics



1. Prepare probe in suitable initial state
2. Send probe through process to be investigated
3. Choose suitable measurement
4. Associate each experimental result with estimation

Data analysis: Generate estimate $X_{est} = X_{est}(\xi_1, \ldots, \xi_N)$

$$\delta X \equiv \sqrt{\langle [X_{\text{est}}(j) - X]^2 \rangle_j \Big|_{X = X_{\text{true}}}} \rightarrow \text{Merit quantifier}$$

$$\langle X_{\text{est}} \rangle = X \rightarrow \text{Unbiased estimator}$$

Cramér, Rao, and Fisher

H. Cramér

C. R. Rao

R. A. Fisher

Cramér-Rao bound for unbiased estimators:

Fisher information

$$\delta X \geq 1/\sqrt{\nu F(X_{\text{true}})}, \quad F(X) \equiv \int d\xi \; p(\xi \mid X) \left(\frac{d \ln\left[p(\xi \mid X) \right]}{dX} \right)^2$$

$\nu \rightarrow$ Number of repetitions of the experiment

$p(\xi \mid X) \rightarrow$ probability density of getting an experimental result ξ

Fisher's theorem: Inequality can be saturated (i.e., it is possible to make it an equality) when $\nu \rightarrow \infty$, by choosing an appropriate estimator X_{est}.

Quantum Fisher Information

$$F\left(X;\{\hat{E}_\xi\}\right) \equiv \int d\xi\, p(\xi \mid X)\left(\frac{d\,\ln\left[p(\xi \mid X)\right]}{dX}\right)^2$$

$$p(\xi \mid X) = \mathrm{Tr}\left[\hat{\rho}(X)\hat{E}_\xi\right]$$

$$\int d\xi \hat{E}_\xi = \hat{1}$$

POVM

This corresponds to a given quantum measurement. Ultimate lower bound for $\langle(\Delta X_{\mathrm{est}})^2\rangle$: optimize over all quantum measurements so that

$$\mathcal{F}_Q(X) = \max_{\{E_\xi\}} F\left(X;\{E_\xi\}\right)$$

Quantum Fisher Information

$$\delta X \equiv \sqrt{\langle(\Delta X_{\mathrm{est}})^2\rangle} \geq 1/\sqrt{\nu \mathcal{F}_Q(X)}$$

Ultimate precision limit

Asymptotically attainable when $\nu \to \infty$ (Braunstein and Caves 1994)

Bures' Fidelity: $\Phi\left(\hat{\rho}_1,\hat{\rho}_2\right) \equiv \left(\mathrm{Tr}\sqrt{\hat{\rho}_1^{1/2}\hat{\rho}_2\hat{\rho}_1^{1/2}}\right)^2$

$$\Rightarrow \Phi\left[\hat{\rho}(X_{\mathrm{true}}),\hat{\rho}(X)\right] = 1-(\delta X/2)^2\mathcal{F}_Q\left[\hat{\rho}(X_{\mathrm{true}})\right]+O\left[(\delta X)^4\right]$$

Related to distance between states!

Quantum Fisher information for pure states

Initial state of the probe: $|\psi(0)\rangle$

Final X-dependent state: $|\psi(X)\rangle = \hat{U}(X)|\psi(0)\rangle$, $\hat{U}(X)$ unitary operator.

Then (Helstrom 1976):

$$\mathcal{F}_Q(X) = 4\langle(\Delta\hat{H})^2\rangle_0\,, \quad \langle(\Delta\hat{H})^2\rangle_0 \equiv \langle\psi(0)|\left[\hat{H}(X) - \langle\hat{H}(X)\rangle_0\right]^2|\psi(0)\rangle$$

where

$$\hat{H}(X) \equiv i\frac{d\hat{U}^\dagger(X)}{dX}\hat{U}(X)$$

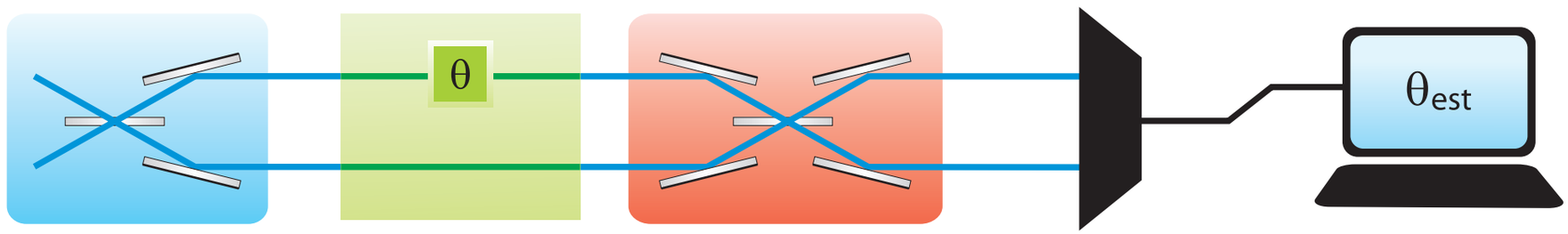
Proper framework to discuss measurements of quantities like elapsed time or harmonic oscillator phase

If $\hat{U}(X) = \exp(i\hat{O}X)$, $\hat{O}$ independent of X, then $\hat{H} = \hat{O}$

$$\delta X \geq 1/2\sqrt{\nu\left\langle\Delta\hat{H}^2\right\rangle}$$

$\Rightarrow$ Should maximize the variance to get better precision!

Example of parameter estimation: Optical interferometry



$\hat{n} = \hat{a}^\dagger a \rightarrow$ Generator of phase displacements

$\Rightarrow \mathcal{F}_Q(\theta) = 4\langle(\Delta\hat{n})^2\rangle_0$ where $\langle(\Delta\hat{n})^2\rangle_0$ is the photon-number variance in the upper arm.

Standard limit: coherent states $\mathcal{F}_Q(\theta) = 4\langle(\Delta\hat{n})^2\rangle_0 = 4\langle\hat{n}\rangle \Rightarrow \delta\theta \geq \dfrac{1}{2\sqrt{\langle n\rangle}}$

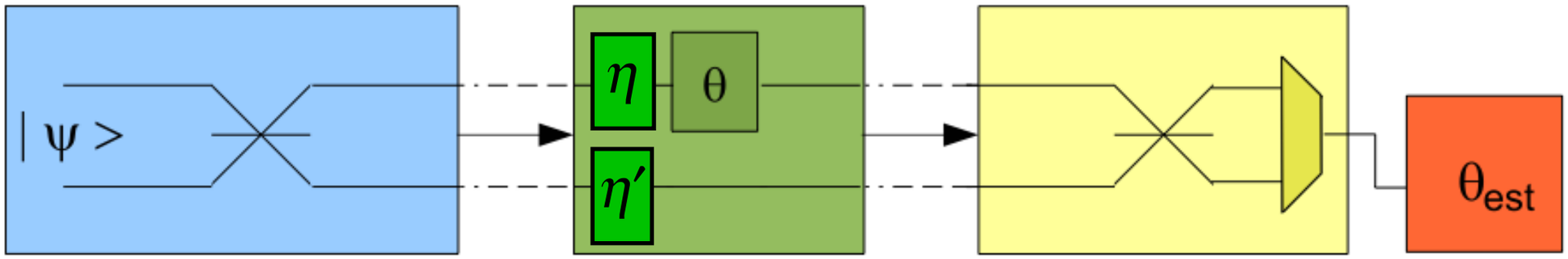
Increasing the precision: maximize variance with NOON states:

$$|\psi(N)\rangle = \left(|N,0\rangle + |0,N\rangle\right)/\sqrt{2} \rightarrow |\psi(N,\theta)\rangle = \left(|N,0\rangle + e^{iN\theta}|0,N\rangle\right)/\sqrt{2}$$

$$\left\langle(\Delta\hat{n})^2\right\rangle_0 = \frac{N^2}{4} \Rightarrow \delta\theta \geq \frac{1}{N}$$

Precision is better, for the same amount of resources.

Parameter estimation with decoherence



Loss of a single photon transforms NOON state into a separable state!

$$|\psi(N)\rangle = \frac{|N,0\rangle + |0,N\rangle}{\sqrt{2}} \rightarrow |N-1,0\rangle \text{ or } |0,N-1\rangle$$

No simple analytical expression for Fisher information!

For small N, more robust states can be numerically calculated

Experimental test with more robust states (for N=2):

nature photonics

LETTERS

PUBLISHED ONLINE: 4 APRIL 2010 | DOI: 10.1038/NPHOTON.2010.39

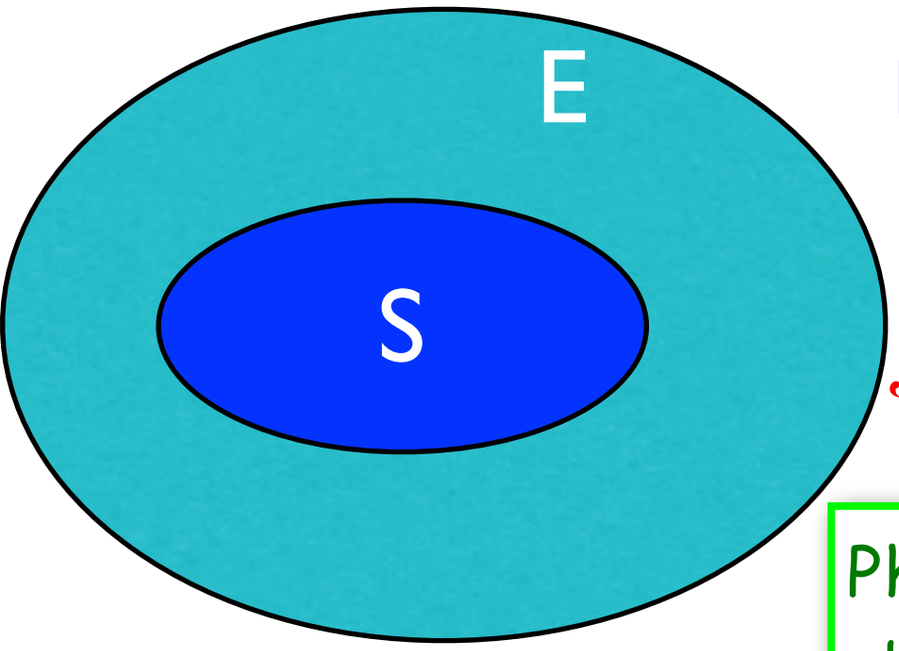
Experimental quantum-enhanced estimation of a lossy phase shift

M. Kacprowicz[1], R. Demkowicz-Dobrzański[1,2]*, W. Wasilewski[2], K. Banaszek[1,2] and I. A. Walmsley[3]

Parameter estimation in open systems: Extended space approach

B. M. Escher, R. L. Matos Filho, and L. D., **Nature Physics** 7, 406 (2011); **Braz. J. Phys.** 41, 229 (2011)

Given initial state and non-unitary evolution, define in S+E



$$|\Phi_{S,E}(x)\rangle = \hat{U}_{S,E}(x)|\psi\rangle_S|0\rangle_E$$ (Purification)

Then

$$\mathscr{F}_Q \equiv \max_{\hat{E}_j^{(S)}\otimes\hat{1}} F\left(\hat{E}_j^{(S)}\otimes\hat{1}\right) \leq \max_{\hat{E}_j^{(S,E)}} F\left(\hat{E}_j^{(S,E)}\right) \equiv \mathscr{C}_Q$$

Physical meaning of this bound: information obtained about parameter when S+E is monitored

Least upper bound: Minimization over all unitary evolutions in S+E - difficult problem

Bound is attainable - there is always a purification such that $\mathscr{C}_Q = \mathscr{F}_Q$

Then, monitoring S+E yields same information as monitoring S

$\Phi_{S,E}(x)\rangle$
$\Phi_{S,E}(x)\rangle$

Minimization procedure

PRL **109,** 190404 (2012) PHYSICAL REVIEW LETTERS week ending 9 NOVEMBER 2012

Quantum Metrological Limits via a Variational Approach

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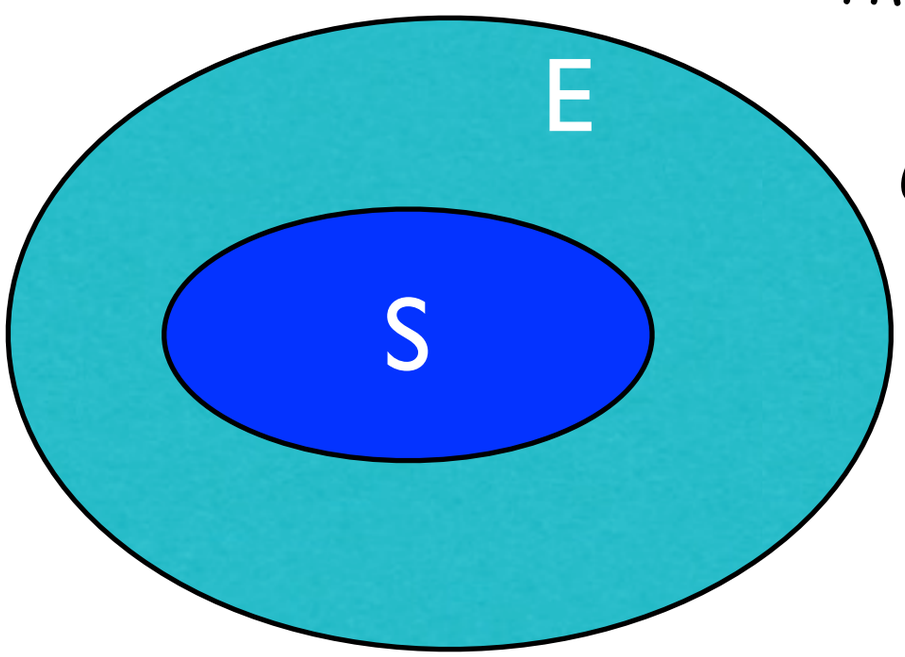
(Received 29 June 2012; published 9 November 2012)



There is always an unitary operator acting only on E that connects two different purifications of ρ_S

Given $|\Phi_{S,E}(x)\rangle = \hat{U}_{S,E}(x)|\psi\rangle_S|0\rangle_E$,

$$i\frac{d|\Phi_{S,E}(x)\rangle}{dx} = \hat{H}_{S,E}(x)|\Phi_{S,E}(x)\rangle,$$

then any other purification can be written as:

$$|\Psi_{S,E}(x)\rangle = u_E(x)|\Phi_{S,E}(x)\rangle$$

Define $\hat{h}_E(x) = i\frac{d\hat{u}_E^\dagger(x)}{dx}\hat{u}_E(x)$

Then $C_Q = 4\langle[\hat{\mathcal{H}}(x) - \langle\hat{\mathcal{H}}(x)\rangle_\Phi]^2\rangle_\Phi, \quad \hat{\mathcal{H}}(x) = \hat{H}_{S,E}(x) - \hat{h}_E(x),$ and

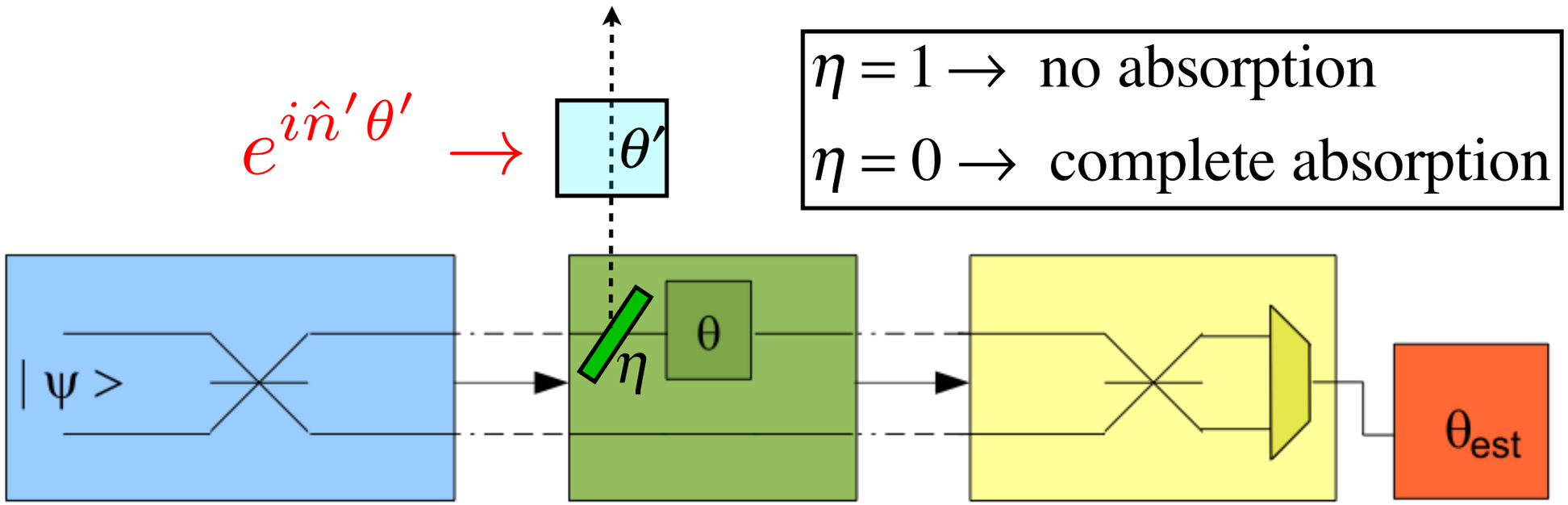
Minimize now C_Q over all Hermitian operators $h_E(x)$ that act on E

Quantum limits for lossy optical interferometry

$e^{i\hat{n}'\theta'} \rightarrow$

$\eta = 1 \rightarrow$ no absorption

$\eta = 0 \rightarrow$ complete absorption



States with well-defined total photon number:

$$|\psi_0\rangle = \sum_{n=0}^{N} \beta_n |n, N-n\rangle$$

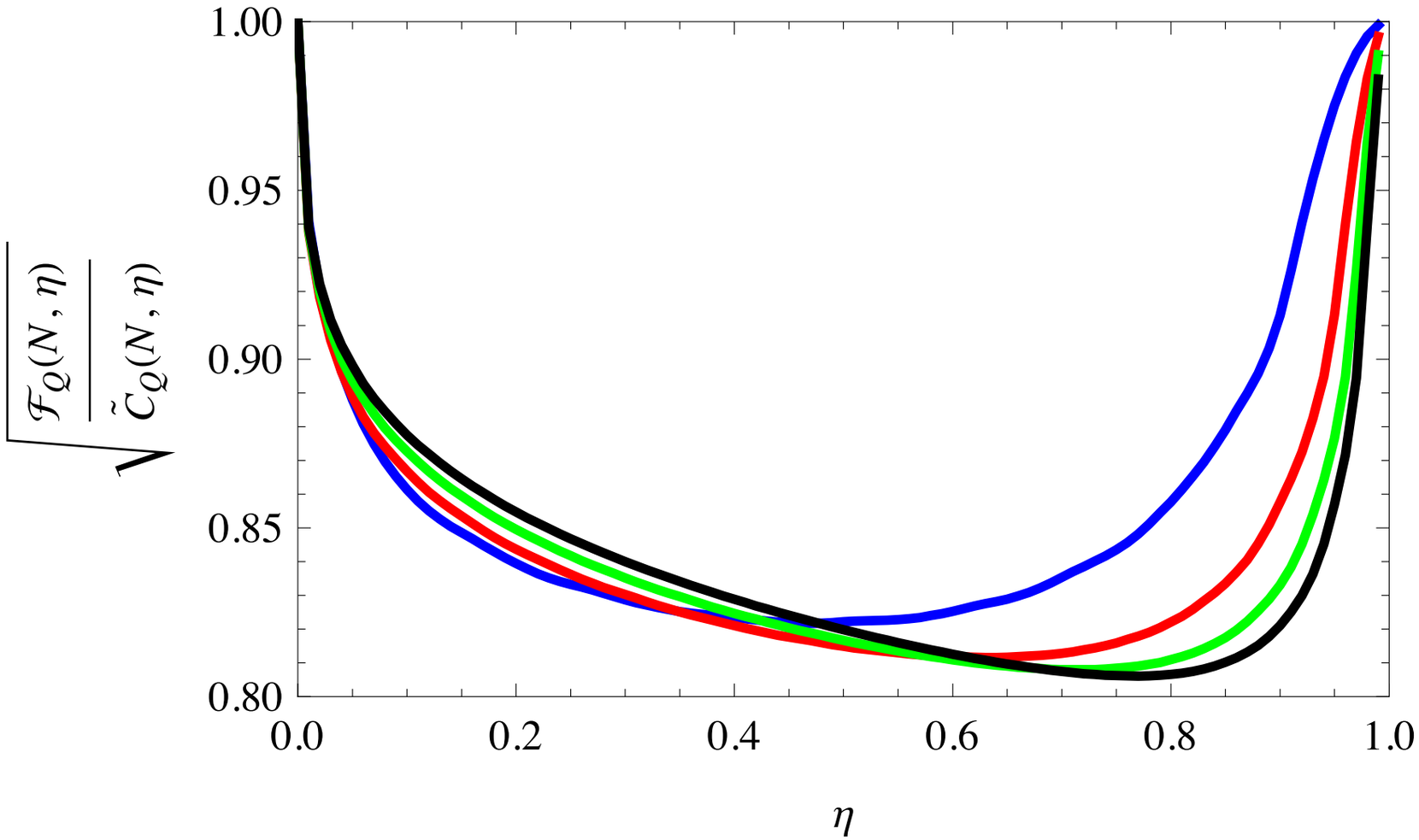
$$2\delta\theta \geq \left[1 + \sqrt{1 + \frac{1-\eta}{\eta} N}\right] / N$$

$$N \ll \frac{\eta}{1-\eta} \Rightarrow \sqrt{\nu}\,\delta\theta \geq 1/N \rightarrow$$ Heisenberg limit

$$N \gg \frac{\eta}{1-\eta} \Rightarrow \delta\theta \geq \frac{\sqrt{1-\eta}}{2\sqrt{\nu\eta N}}$$

For N sufficiently large, $1/\sqrt{N}$ behavior is always reached!

How good is this bound?



Comparison between numerical maximum value of $\mathscr{F}_Q$ and upper bound $\mathscr{C}_Q$ as a function of η, for $N = 10$ (blue), $N = 20$ (red), $N = 30$ (green), and $N = 40$ (black).

Phase diffusion in optical interferometer

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$$\dot{\rho} = \Gamma \mathcal{L}[a^\dagger a]\rho, \quad \mathcal{L}[O]\rho = 2O\rho O^\dagger - O^\dagger O\rho - \rho O^\dagger O$$

$$\Rightarrow \rho(t) = \sum_{m.n} e^{-\beta^2 (n-m)^2} \rho_{n,m}(0)|n\rangle\langle m|, \quad \beta = \Gamma t$$

Possible purification:

$$|\Phi_{S,E}(\phi)\rangle = e^{-i\phi\hat{n}_S} e^{i(2\beta)\hat{n}_S \hat{x}_{\mathrm{E}}} |\psi_S\rangle|0_E\rangle \Rightarrow C_Q = 4\Delta n^2$$ Trivial!

Choose: $\hat{u}_E(\phi;\lambda) = e^{i\phi\lambda\hat{p}_{\mathrm{E}}/(2\beta)} \Rightarrow C_Q = (1-\lambda)^2 4\Delta n^2 + \lambda^2/(2\beta^2)$

$\lambda \longrightarrow$ Variational parameter

Phase diffusion in optical interferometer

$$\delta\phi_{pd} \geq \sqrt{\frac{1}{\nu}\left(\frac{1}{4\Delta n^2} + 2\beta^2\right)}$$

Intrinsic quantum feature

Phase diffusion

Very close to numerical value obtained by Genoni, Olivares, and Paris for Gaussian state - PRL 106, 153603 (2011)

For Gaussian states:

$$\Delta n^2 \leq 2N(N+1)$$

(N is the average photon number)

Then:

$$C_Q^{\rm opt} \leq C_Q^{\rm max} \equiv \left[2\beta^2 + \frac{1}{8N(N+1)}\right]^{-1}$$

Comparison with numerical results

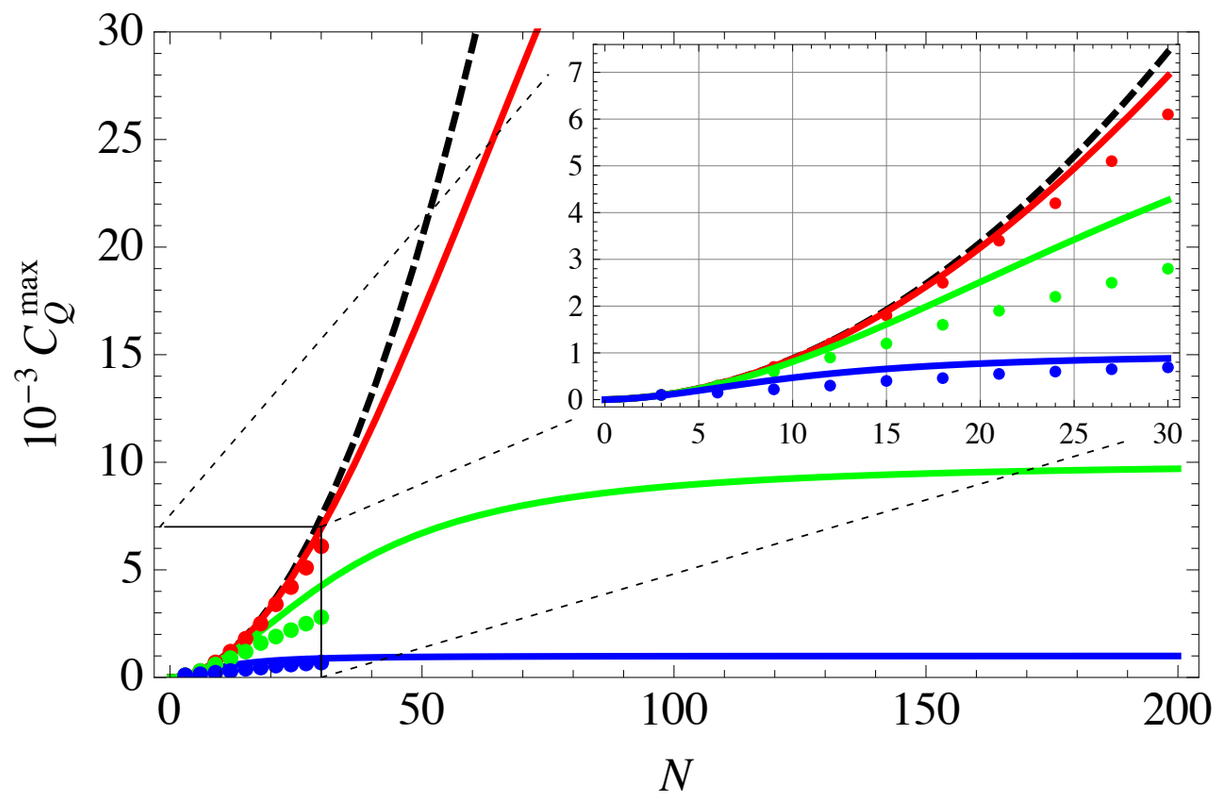

FIG. 1: Comparison between upper bound $C_Q^{\max}$ and the maximum quantum Fisher information $\mathcal{F}_Q^{\max}$ in [14] as a function of the average number of photons N. The dots stand for the values obtained in [14], while the full lines correspond to $C_Q^{\max}$. The inset displays the two quantities up to $N = 30$, which was the range considered in [14]. From top to bottom, $\beta^2 = 0; 5\times10^{-6}; 5\times10^{-5}; 5\times10^{-4}$.

QUANTUM SPEED LIMIT

THE UNCERTAINTY RELATION BETWEEN ENERGY AND TIME IN NON-RELATIVISTIC QUANTUM MECHANICS

By L. MANDELSTAM * and Ig. TAMM

Lebedev Physical Institute, Academy of Sciences of the USSR

(Received February 22, 1945)

A uncertainty relation between energy and time having a simple physical meaning is rigorously deduced from the principles of quantum mechanics. Some examples of its application are discussed.

1. Along with the uncertainty relation between coordinate q and momentum p one considers in quantum mechanics also the uncertainty relation between energy and time.

The former relation in the form of the inequality

$$\Delta q \cdot \Delta p \geqslant \frac{h}{2}, \qquad (1)$$

An entirely different situation is met with in the case of the relation

$$\boxed{\Delta H \cdot \Delta T \sim h,} \qquad (2)$$

where ΔH is the standard of energy, ΔT — a certain time interval, and the sign $\sim$ denotes that the left-hand side is at least of the order of the right-hand one.

Leonid Mandelshtam

Igor Tamm

Quantum speed limit for physical processes

Quantum speed limit for physical processes

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arXiv:1209.0362

(to be published in PRL)

$$\arccos\sqrt{\Phi_B\left[\hat{\rho}(0),\hat{\rho}(\tau)\right]} \leq \int_0^\tau \sqrt{\mathcal{F}_Q(t)/4}\, dt$$

Bures length of geodesic
Uhlman (1992)

Bures length of actual path followed by state of the system

$\tau \rightarrow$ Minimum time for attaining fidelity $\Phi_B(0,\tau)$ between initial and final states

Attainable bound

Special case: Unitary evolution, time-independent Hamiltonian, orthogonal states

Mandelstam-Tamm

$$\Phi_B\left[\hat{\rho}(0),\hat{\rho}(\tau)\right] = 0, \quad \mathcal{F}_Q(t) = 4\langle(\Delta H)^2\rangle/\hbar^2 \Rightarrow \tau\sqrt{\langle(\Delta H)^2\rangle} \geq h/4$$

Quantum speed limit for physical processes: Purification procedure

$$\mathcal{D} := \arccos\sqrt{\Phi_B\left[\hat{\rho}(0), \hat{\rho}(\tau)\right]} \leq \int_0^\tau \sqrt{\mathcal{F}_Q(t)/4}\, dt$$

$$\Downarrow$$

$$\mathcal{D} \leq \int_0^\tau \sqrt{\mathcal{C}_Q(t)/4}\, dt = \int_0^\tau \sqrt{\langle \Delta \hat{\mathcal{H}}^2_{S,E}(t)\rangle}/\hbar\, dt.$$

$$\hat{\mathcal{H}}_{S,E}(t) := \frac{\hbar}{i}\frac{d\hat{U}^\dagger_{S,E}(t)}{dt}\hat{U}_{S,E}(t)$$

$\hat{U}_{S,E}(t)$: Evolution of the purified state corresponding to $\hat{\rho}_S$

Quantum speed limit for physical processes: amplitude damping channel

Amplitude damping channel:

$$|0\rangle|0\rangle_E \to |0\rangle|0\rangle_E \,,$$

$$|1\rangle|0\rangle_E \to \sqrt{P(t)}|1\rangle|0\rangle_E + \sqrt{1-P(t)}|0\rangle|1\rangle_E$$

Common example: $P(t) = \exp(-\gamma t)$

Unitary evolution corresponding to the map:

$$\hat{U}_{S,E}(t) = \exp[-i\Theta(t)(\hat{\sigma}_+\hat{\sigma}_-^{(E)} + \hat{\sigma}_-\hat{\sigma}_+^{(E)})]$$

$$\Theta(t) = \arccos\sqrt{P(t)}$$

From this and $\mathcal{D} \leq \int_0^\tau \sqrt{\mathcal{C}_Q(t)/4}\, dt = \int_0^\tau \sqrt{\langle \Delta\hat{\mathcal{H}}_{S,E}^2(t)\rangle}/\hbar\, dt.$

one gets:

$$\mathcal{D} \leq \sqrt{\langle\hat{\sigma}_+\hat{\sigma}_-\rangle}\arccos[\exp(-\gamma t/2)]$$

Quantum speed limit for physical processes: amplitude damping channel (2)

$$|0\rangle|0\rangle_E \to |0\rangle|0\rangle_E \,,$$

$$|1\rangle|0\rangle_E \to \sqrt{P(t)}|1\rangle|0\rangle_E + \sqrt{1-P(t)}|0\rangle|1\rangle_E \qquad P(t) = \exp(-\gamma t)$$

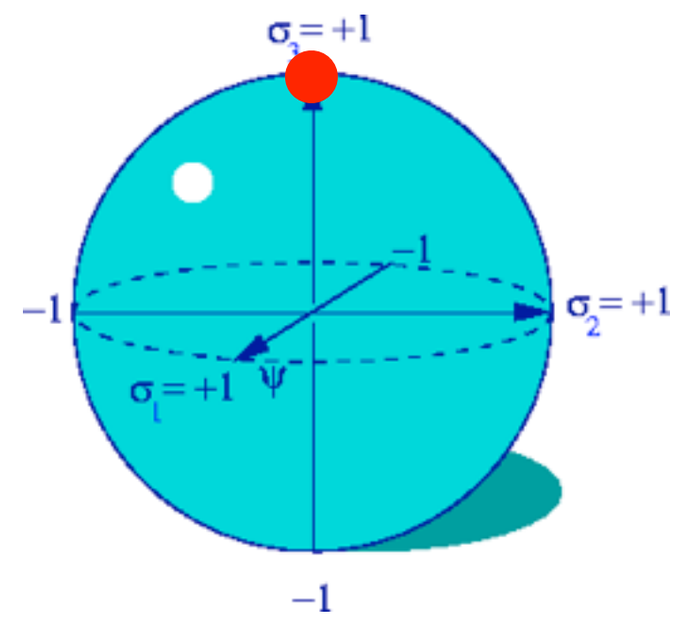
$$\mathcal{D} \leq \sqrt{\langle \hat{\sigma}_+ \hat{\sigma}_- \rangle} \arccos[\exp(-\gamma t/2)] \Rightarrow \gamma\tau \geq 2 \ln \sec(\mathcal{D}/\sqrt{\langle \hat{\sigma}_+ \hat{\sigma}_- \rangle})$$

Bound is saturated if $\langle \hat{\sigma}_+ \hat{\sigma}_- \rangle = 0 \;\text{ or }\; 1$

Interpretation:

If initial state is the excited state, then evolution is along a geodesic:

$$|1\rangle\langle 1| \to P(t)|1\rangle\langle 1| + [1-P(t)]|0\rangle\langle 0|$$



Forced harmonic oscillator

$$\hat{H}_S/\hbar\omega = \frac{1}{2}(\hat{P}^2 + \hat{X}^2) - F\zeta(t)\hat{X}, \quad \text{Max } |\zeta(t)| = 1$$

$$\hat{X} = (\hat{a} + \hat{a}^\dagger)/\sqrt{2}$$

$$F = f\sqrt{\hbar/(m\omega)^3}$$

Langevin equation for noisy evolution (interaction picture)

$$d\hat{a}/dt = i\omega F\zeta(t)e^{i\omega t}/\sqrt{2} - \gamma\hat{a}/2 + \hat{f}_\gamma(t)$$

$$\langle\hat{f}_\gamma(t)\rangle = 0 \quad \langle\hat{f}_\gamma(t)\hat{f}_\gamma^\dagger(t')\rangle = \gamma(n_T+1)\delta(t-t') \quad \langle\hat{f}_\gamma^\dagger(t)\hat{f}_\gamma(t')\rangle = \gamma n_T\delta(t-t')$$

$$\hat{a}(t) = \hat{a}(0)e^{-\gamma t/2} + \boxed{iFD(t)/\sqrt{2}} + \int_0^t dt'\,\hat{f}_\gamma(t')e^{\gamma(t'-t)/2}$$

$$D(t) \equiv |D(t)|e^{i\phi_t} = \omega\int_0^t dt'\zeta(t')e^{i\omega t'}e^{-\gamma(t-t')/2}$$

Quantum limit for measurement of a weak classical force coupled to a noisy quantum-mechanical oscillator

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arXiv:1210.3316

Forced harmonic oscillator (2)

$$\hat{a}(t) = \hat{a}(0)e^{-\gamma t/2} + \boxed{iFD(t)/\sqrt{2}} + \int_0^t dt'\, \hat{f}_\gamma(t') e^{\gamma(t'-t)/2}$$

$$D(t) \equiv |D(t)| e^{i\phi_t} = \omega \int_0^t dt' \zeta(t') e^{i\omega t'} e^{-\gamma(t-t')/2}$$

Quadrature $\hat{X}(\phi_t) = [\hat{a}\exp(-i\phi_t) + \hat{a}^\dagger \exp(i\phi_t)]/\sqrt{2}$ does not depend on F

Force displaces state in phase space along generalized momentum quadrature $\hat{P}(\phi_t) = [\hat{a}\exp(-i\phi_t) - \hat{a}^\dagger \exp(i\phi_t)]/i\sqrt{2}$, orthogonal to $\hat{X}(\phi_t)$

Note: For $\zeta(t) = \cos\omega t$, then in the rotating-wave approximation $\phi_t = 0$

Also: $\langle[\Delta\hat{X}(\theta)]^2\rangle_t = \eta\langle[\Delta\hat{X}(\theta)]^2\rangle_0 + (2n_T+1)(1-\eta)/2$

Special solution: Gaussian initial state, generalized momentum measurement

$$\mathcal{F}_P(F) = \int dP \frac{1}{\langle P|\hat{\rho}_t|P\rangle}\left(\frac{\partial\langle P|\hat{\rho}_t|P\rangle}{\partial F}\right)^2 = \frac{|D(t)|^2}{\langle[\Delta\hat{P}(\phi_t)]^2\rangle_t} = \frac{4|D(t)|^2\langle[\Delta\hat{X}(\phi_t)]^2\rangle_0}{\eta + 2(2n_T+1)(1-\eta)\langle[\Delta\hat{X}(\phi_t)]^2\rangle_0}$$

(minimum-uncertainty state)

Noisy forced oscillator: Purification procedure

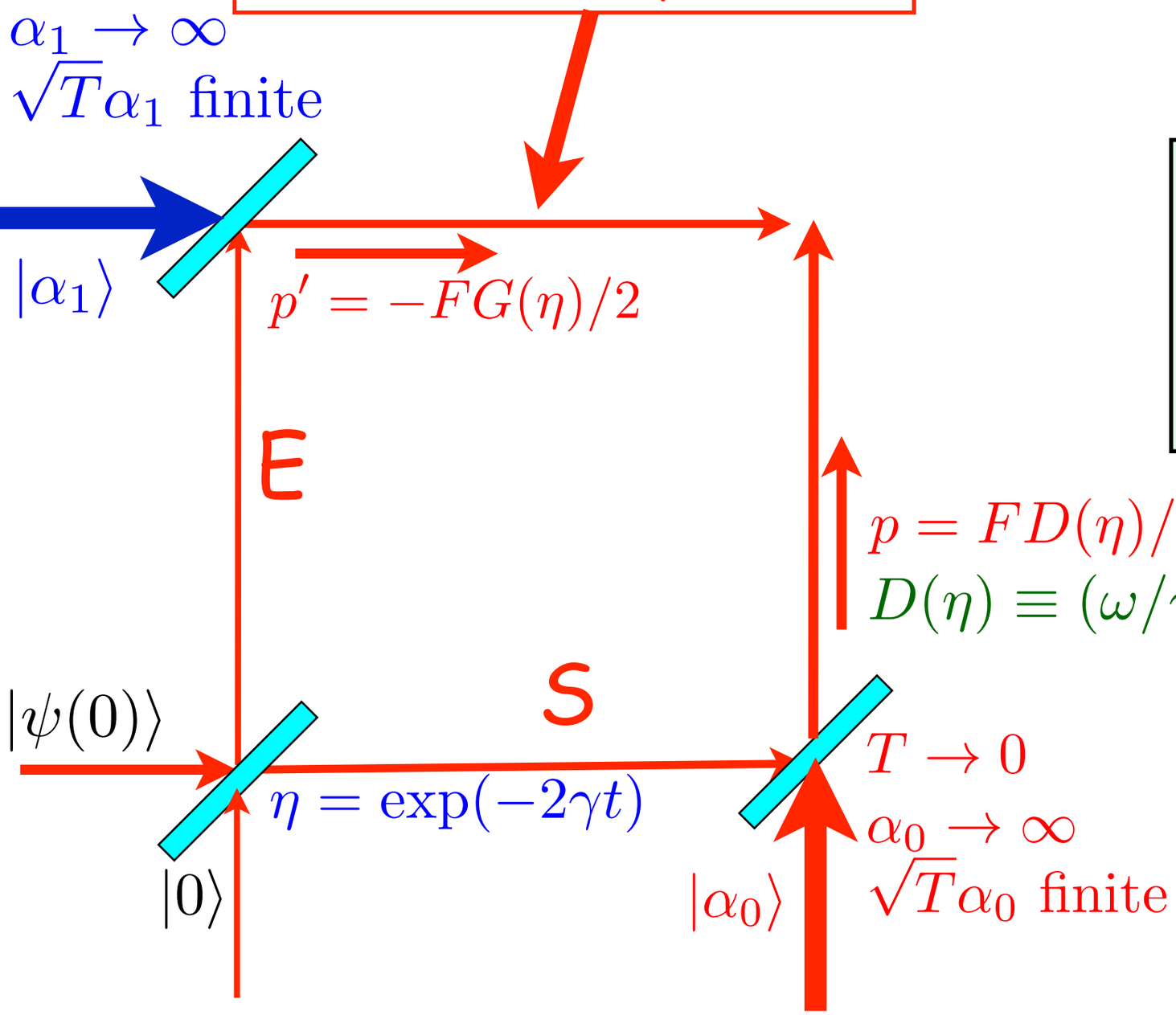
$(T = 0,\ RWA)$

Unitary transformation on environment: does not change the reduced description

Oscillator = Field mode (S)

Environment = Another field mode (E)

Purification!



Calculate quantum Fisher information for SE, choose G to minimize it $\Longrightarrow$ upper bound for quantum Fisher information of S

$$\Downarrow$$

$$\mathscr{F}_{Q\,SE} = \left[D(\eta)\right]^2 \frac{\left\langle \Delta\hat{X}^2 \right\rangle_0}{\eta + 2(1-\eta)\left\langle \Delta\hat{X}^2 \right\rangle_0}$$

Minimum-uncertainty Gaussian state and momentum measurement: best choice!

Coincides with result for damped oscillator with incoming Gaussian state and momentum measurement!

Exact quantum limit

$$\eta = \exp(-2\gamma t) \qquad \boxed{\mathcal{F}_Q = [D(\eta)]^2 \frac{\langle \Delta \hat{X}^2 \rangle_0}{\eta + 2(1-\eta)\langle \Delta \hat{X}^2 \rangle_0}}$$

$$\langle \Delta \hat{X}^2 \rangle_0^{\rm opt} = E + \sqrt{E^2 - 1/4}$$

$$\boxed{\delta F \geq \frac{\gamma}{\omega(1-\sqrt{\eta})\sqrt{\nu}} \left[2(1-\eta) + \frac{\eta}{(E + \sqrt{E^2 - 1/2})} \right]^{1/2}}$$

Minimization of bound implies maximization of variance of position: for fixed average energy E, squeezed state!

Depending on which term dominates, one gets standard or Heisenberg limit

Thermal reservoir:

$$\boxed{2(1-\eta) \to 2(1-\eta)(2\bar{n}_T + 1)}$$

Bound saturates as time grows...

$$\delta F \geq \frac{\gamma}{\omega\sqrt{\nu}} \sqrt{2(1 + 2\bar{n}_T)}$$

It does not pay to wait for a long time...

Better strategy: Divide to conquer...

Force acts during a time t_{total}. Probe force during time τ, measure the probe system, reset this system and repeat this procedure ν times, with $\nu = t_{\mathrm{total}}/\tau$. Minimize measurement uncertainty with respect to τ

$$\tau_{\mathrm{opt}} \approx \left[\langle(\Delta\hat{X})^2\rangle_0(2n_T+1)/12\right]^{-1/3}$$

Diffusive limit: $\gamma \to 0, \quad n_T \to \infty, \quad \text{with} \quad \gamma n_T = \mathcal{D}$

[Maiwald, R. et al. Stylus ion trap for enhanced access and sensing. Nature Phys. 5, 551-554 (2009)]

$$\delta f \geq \sqrt{\frac{4m\hbar\omega\mathcal{D}}{t_{\mathrm{tot}}}}\sqrt{1+\frac{1}{4\mathcal{D}\langle(\Delta\hat{X})^2\rangle_0 t_{\mathrm{tot}}}}$$

Correction to heuristic calculation

Summary

- General framework for estimation of parameters in noisy systems, based on expression of quantum Fisher information for purified evolution (extended space), and on "control" of environment, so as to minimize the quantum Fisher information of S+E .

- Allows analytical calculation of very good bounds on the limits of estimation.

- Bounds obtained for optical interferometry, atomic spectroscopy, minimum evolution time of open systems, and force estimation.