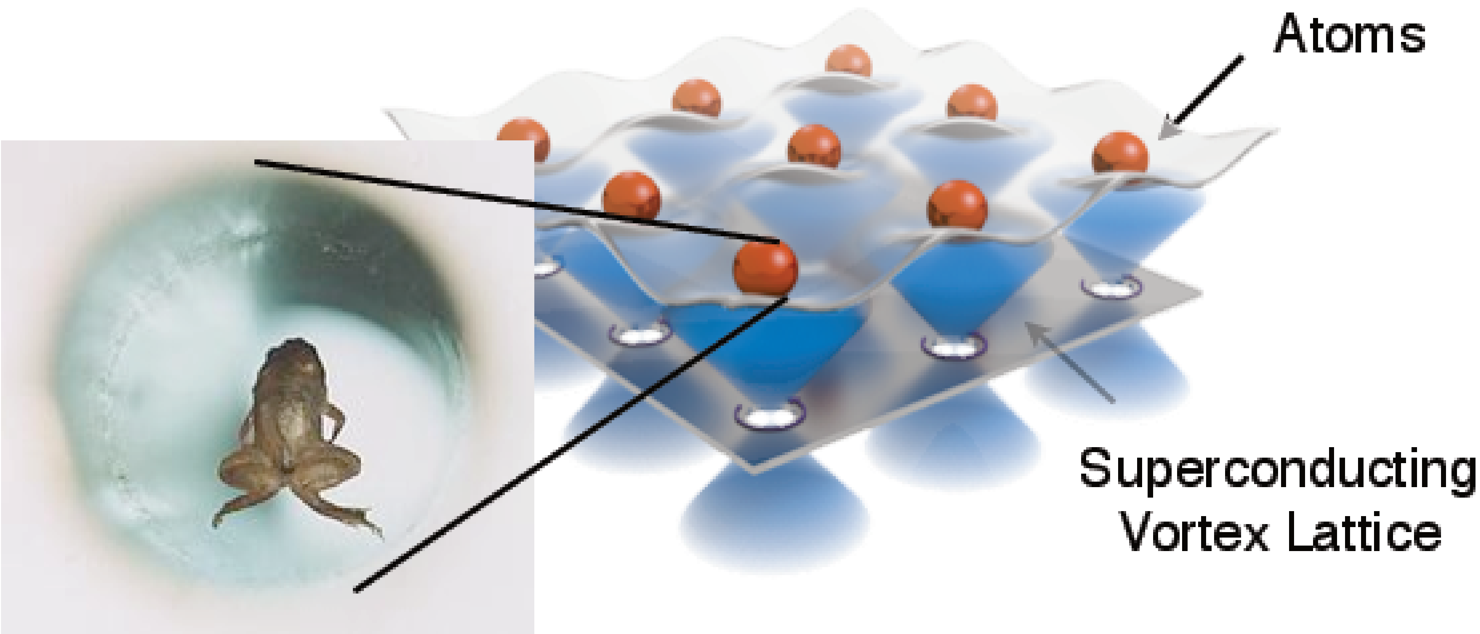
"Is there life after optical lattices?"

Ultracold Atoms (or Molecules) in Superconducting Vortex Lattices



O. Romero-Isart, C. Navau, A. Sanchez, P. Zoller, J. I. Cirac,
arXiv:1302.3504

Thanks to O. Romero-Isart for providing me with some of his slides

UNIVERSITY OF INNSBRUCK

IQOQI
AUSTRIAN ACADEMY OF SCIENCES

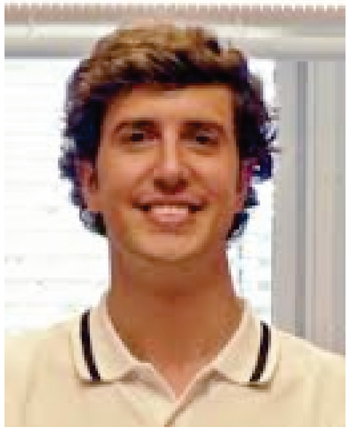
OLE
OPTICAL LATTICE EMULATOR

NAMEQUAM
Nanodesigning of atomic and molecular quantum matter

FWF Der Wissenschaftsfonds.

June & July 2012, and January 2013 at MPQ ...

MPQ



Oriol Romero-Isart

J. Ignacio Cirac

Autonomous University of Barcelona

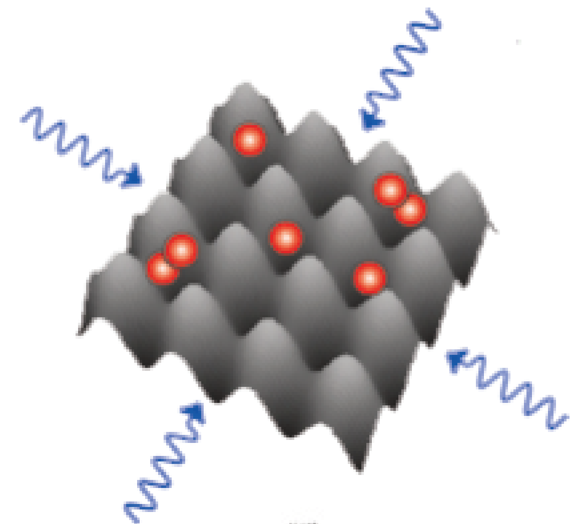
Àlvar Sànchez

Carlos Navau

AMO Hubbard Models with Optical Lattices: Issues?

- **Optical lattice** $V(x) = V_0 \sin^2(kx) \quad k = \frac{2\pi}{\lambda}$

$\lambda/2 \sim$ wavelength of the light



- **Hubbard Models**

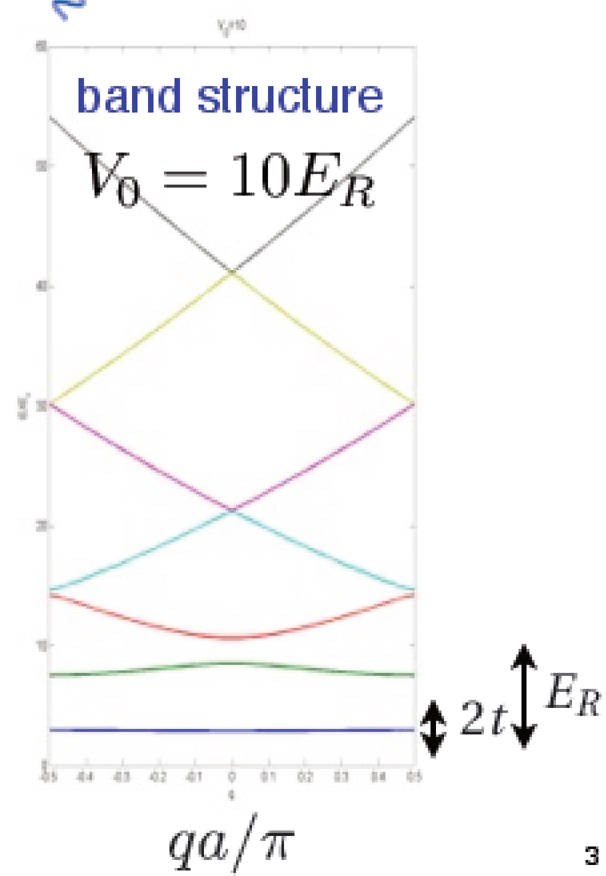
$$H = -t \sum_{i,j,\sigma} (c^{+}_{i\sigma} c_{j\sigma} + c^{+}_{j\sigma} c_{i\sigma}) + U \sum_{i} n_{i\uparrow} n_{i\downarrow}$$

- **Energy scales [& temperature requirements]**

$$T < J \equiv \frac{t^2}{U} < t < E_R = \frac{\hbar^2 k^2}{2 m_a}$$

temperature, exchange, hopping, recoil energy

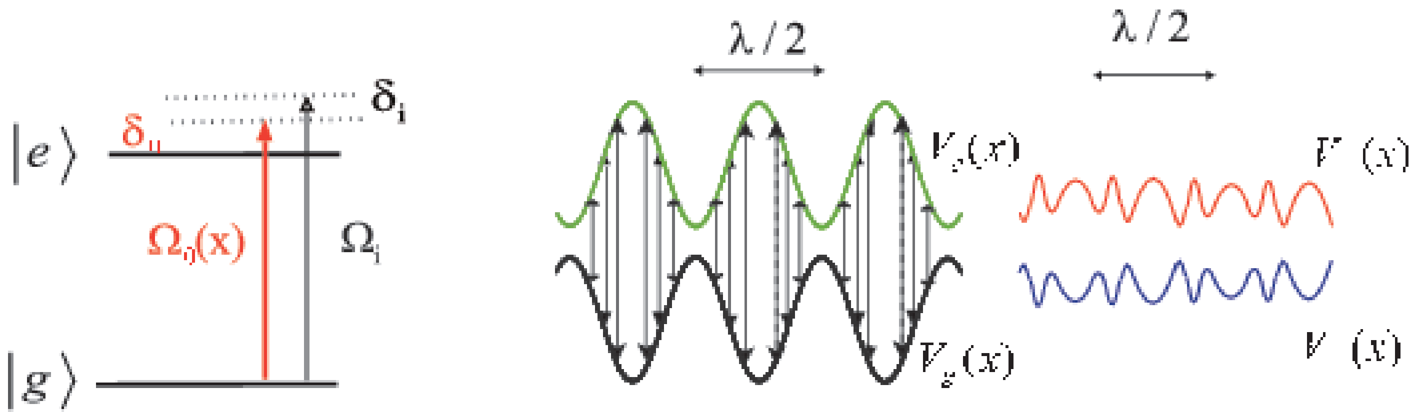
+ decoherence rates (~spontaneous emission)



Q.: Can we scale down the lattice: "sub-wavelength lattices"

Answer 1: Let atoms or molecules “make” a sub-wavelength lattice

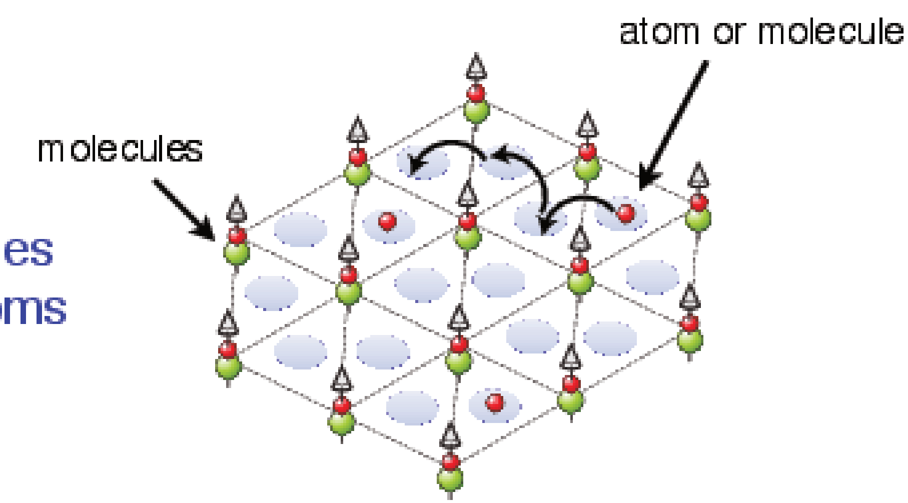
- **Nonlinear Optical (Sub-Wavelength) Lattices**



Alkaline Earth atoms

W Yi, A J Daley, G Pupillo and P Zoller, NJP (2008)

- **Cold Atoms and Molecules in *Self-Assembled Dipolar Lattices***



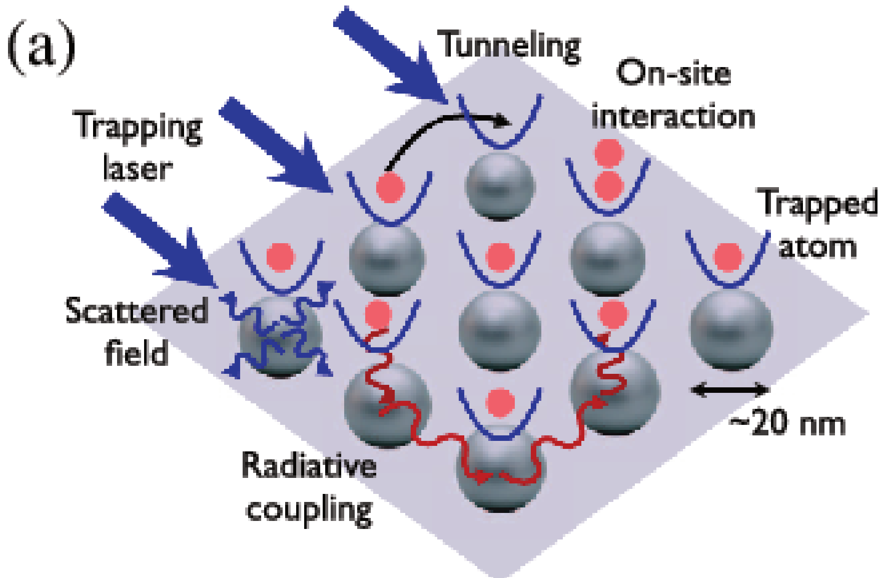
self-assembled dipolar crystal of molecules
as a “floating nano-structure” for cold atoms

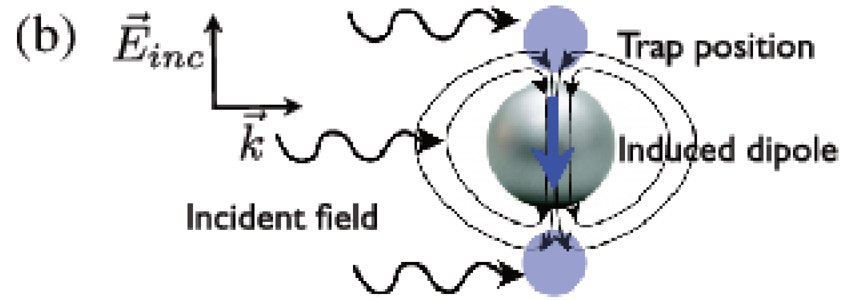
✓ AMO Hubbard + phonons

G. Pupillo, A. Griessner, A. Micheli, M. Ortner, D.-W. Wang, and P. Zoller, PRL 2008

Answer 2: Near field & close to surfaces / nanostructuring

- **Nano-plasmonic Lattices for Cold Atoms**





$$E = E_0 + \frac{\alpha(\omega)}{4\pi\epsilon_0}\frac{3(\hat{r}\cdot E_0)\hat{r} - E_0}{r^3}$$

$$\alpha(\omega) = 4\pi\epsilon_0 a^3 \frac{\varepsilon(\omega)-1}{\varepsilon(\omega)+2}$$

plasmon resonance

Subwavelength confinement of light associated with the near field of plasmonic systems creates a nanoscale optical lattices for ultracold atoms.

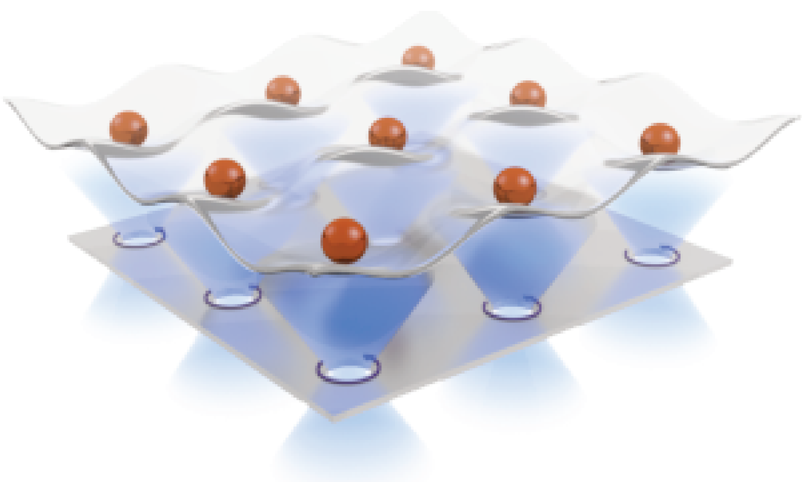
✓ strongly coupled atom-plasmon system
✓ coherent: interaction via plasmonic nanosphere
✓ dissipative: decoherence close to surface

M. Gullans, T. G. Tiecke, D. E. Chang, J. Feist, J. D. Thompson, J. I. Cirac, P. Zoller, and M. D. Lukin, PRL 2012

Experiments see, for example, C. Stehle et al. Nat. Photonics 2011.

This talk ...

- **Nano-scale Superconducting Vortex Lattices for Cold Atoms**



magnetic *nano*-lattice

An array of holes (antidots) with vortices creates a magnetic field to trap atoms

✓ purely magnetic trapping & manipulations
✓ [no laser]

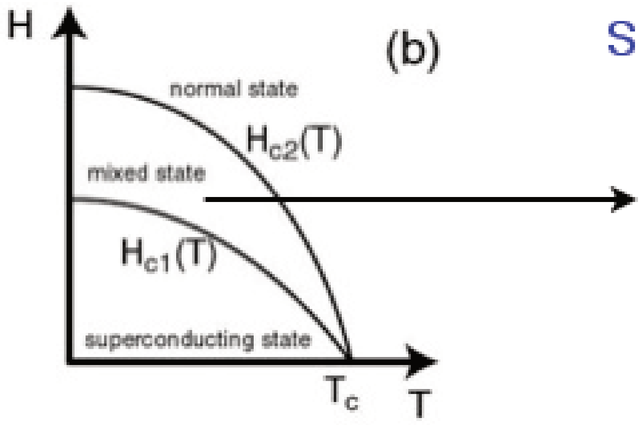
O. Romero-Isart, C. Navau, A. Sanchez, P. Zoller, J. I. Cirac, arXiv:1302.3504

Mesoscopic magnetic traps: Pfau, Hannaford, Spreeuw, ...
Mesoscopic superconducting traps: Zimmermann, Dumke, Haroche, ...

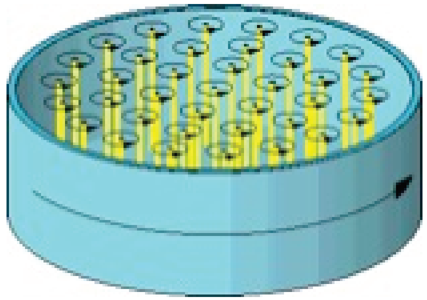
Superconducting vortices in thin films

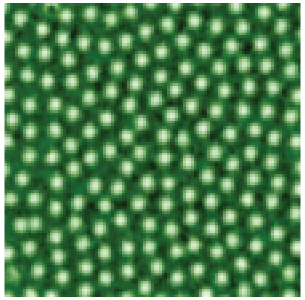
Vortices in type-II superconductors

- **type-II superconductor**

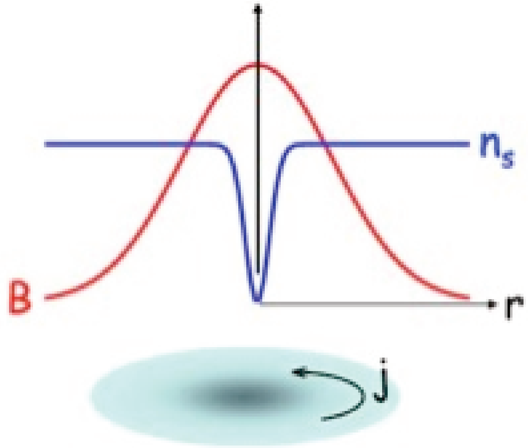


Superconducting vortices





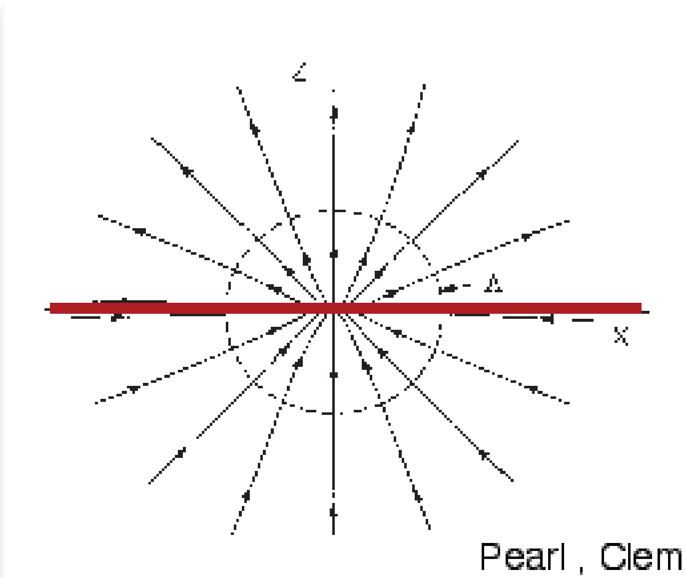
- **superconducting vortex**



- London penetration depth λ
- Correlation length ξ
- For type-II $\kappa = \frac{\lambda}{\xi} > \frac{1}{\sqrt{2}}$
- Quantum of flux $\Phi_0 = \frac{h}{2e} = 2 \times 10^{-15}\ \mathrm{Tm}^2$

Vortices in Thin Films: Pearl Vortex

- Thin film



$$\Lambda \equiv \frac{\lambda^2}{d} \gtrsim \lambda \gtrsim d > \xi$$

thin film
penetration depth

Nanometer regime

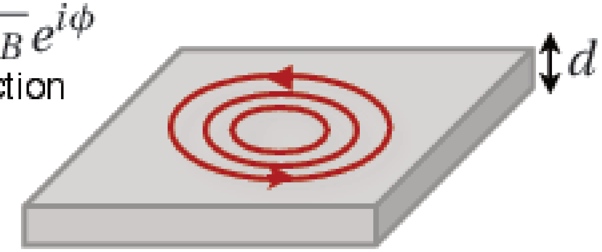
- Pearl vortex

supercurrent $\vec{j} = \frac{1}{2}\psi^*(\frac{\hbar}{i}\nabla + 2e\vec{A})\psi + \text{c.c}$ with vortex $\psi = \sqrt{n_B}\, e^{i\phi}$

wave function

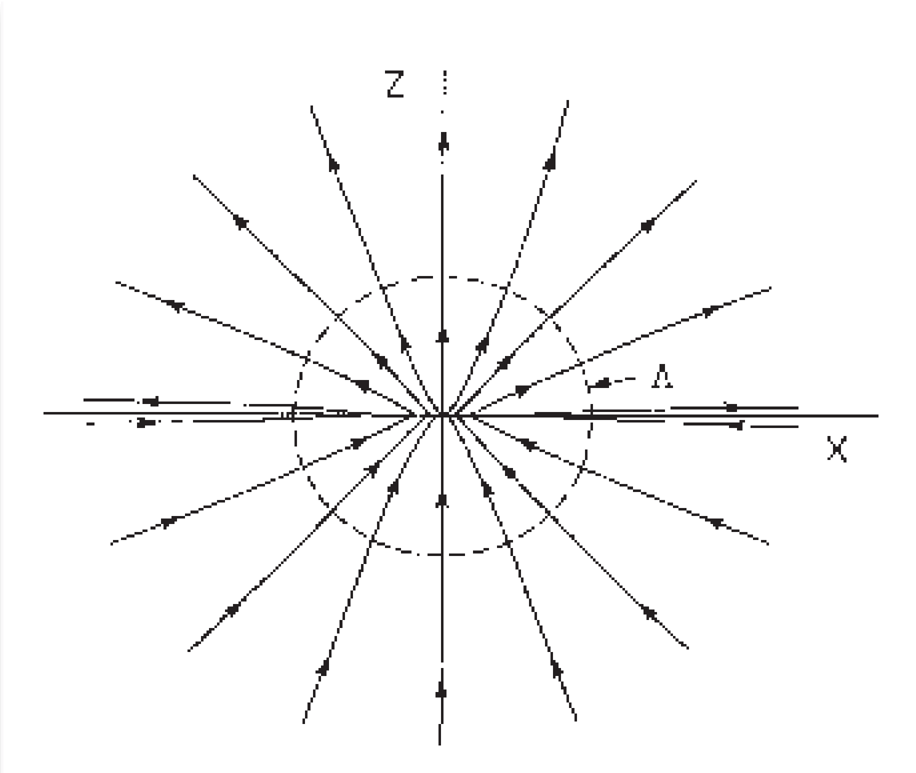
London theory

$$\mu_0^{-1}\nabla \times \vec{B} = \vec{K}\delta(z)$$
$$\equiv \frac{\mu_0}{\Lambda}\left[-\vec{A} + \frac{\phi_0}{2\pi}\frac{1}{\rho}\vec{e}_\phi\right]\delta(z)$$



sheet current

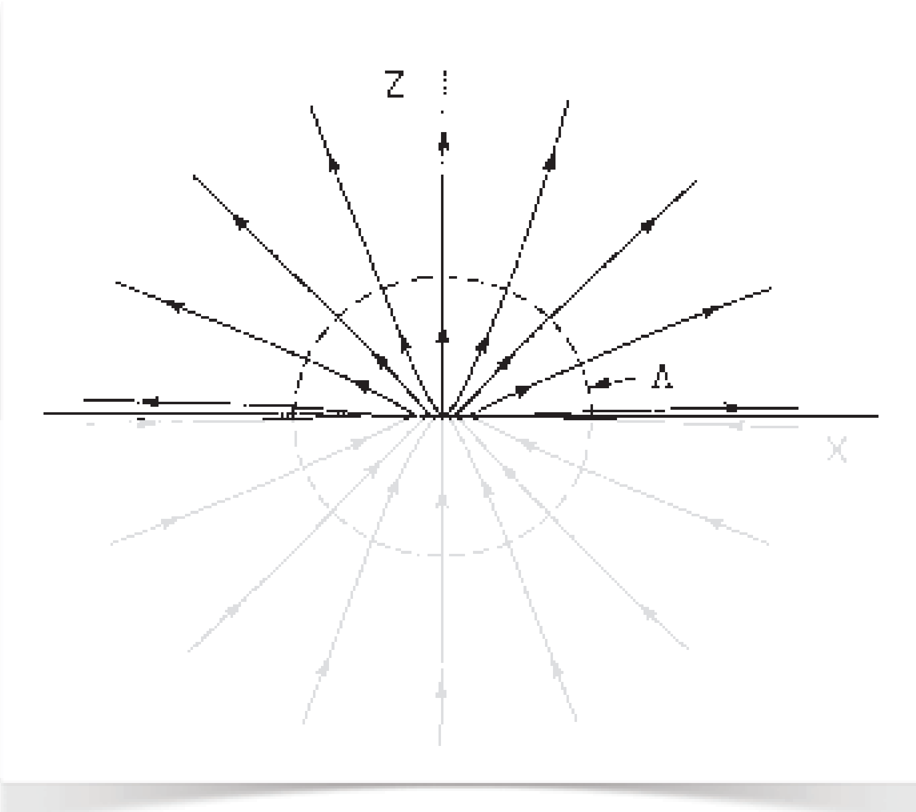
Magnetic Field of a Pearl Vortex



magnetic field:

$$B_\rho(\rho, z > 0) = \int_0^\infty \frac{\Phi_0}{2\pi} \frac{k}{2k\Lambda + 1} J_1(k\rho) e^{-kz},$$
$$B_z(\rho, z > 0) = \int_0^\infty \frac{\Phi_0}{2\pi} \frac{k}{2k\Lambda + 1} J_0(k\rho) e^{-kz}.$$

Magnetic Field of a Pearl Vortex

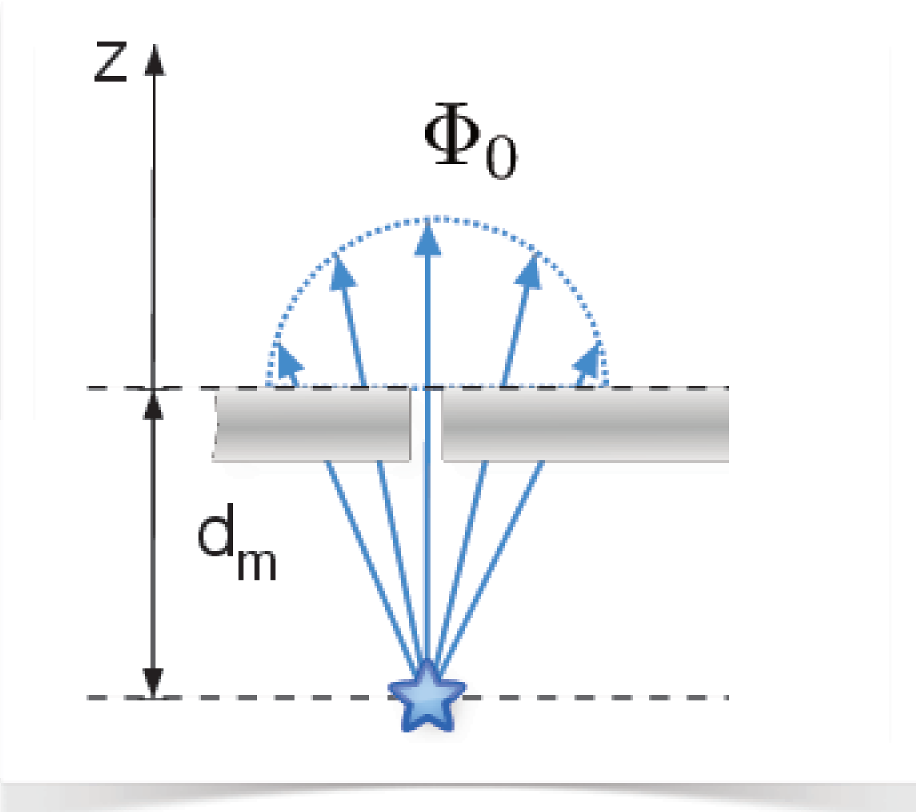


Above thin film, like magnetic charge!

magnetic field:

$$B_\rho(\rho, z>0) = \int_0^\infty \frac{\Phi_0}{2\pi} \frac{k}{2k\Lambda+1} J_1(k\rho) e^{-kz},$$
$$B_z(\rho, z>0) = \int_0^\infty \frac{\Phi_0}{2\pi} \frac{k}{2k\Lambda+1} J_0(k\rho) e^{-kz}.$$

Magnetic Field of a Pearl Vortex



magnetic field:

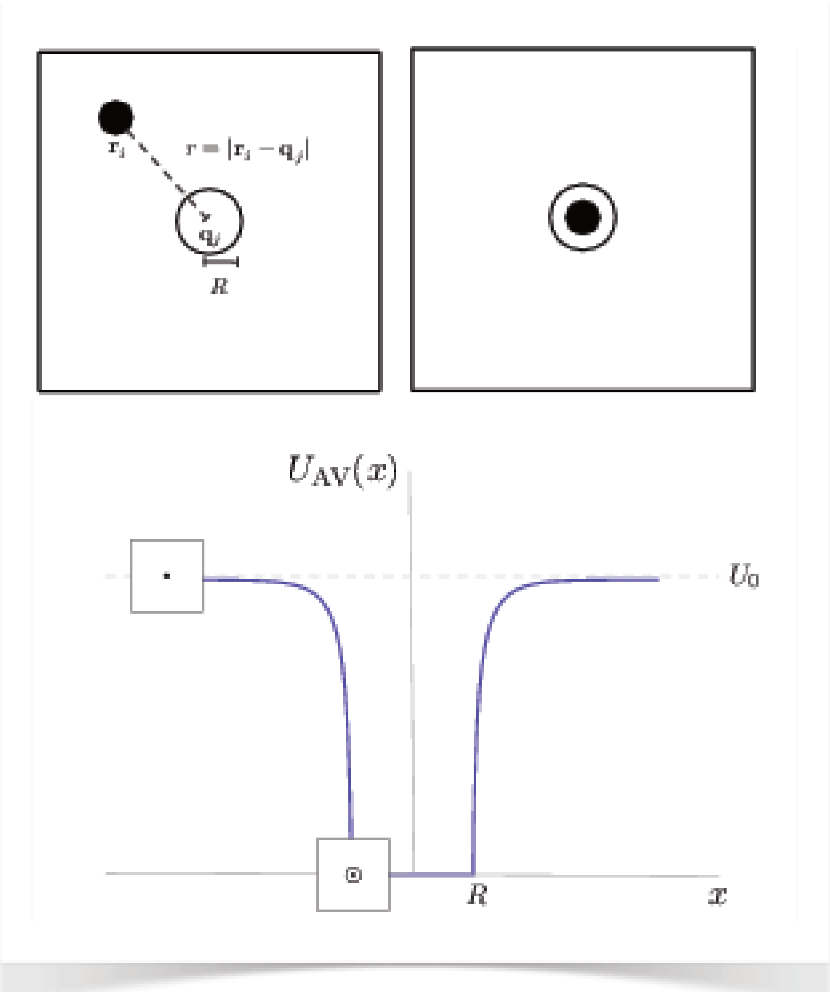
$$B_\rho(\rho, z>0) = \int_0^\infty \frac{\Phi_0}{2\pi} \frac{k}{2k\Lambda+1} J_1(k\rho) e^{-kz},$$
$$B_z(\rho, z>0) = \int_0^\infty \frac{\Phi_0}{2\pi} \frac{k}{2k\Lambda+1} J_0(k\rho) e^{-kz}.$$

- Monopole of strength $2\Phi_0$
- Situated at $z = -d_\mathrm{m}$

$$B_\rho^0(\rho, z>0) = \int_0^\infty \frac{\Phi_0}{2\pi} k J_1(k\rho) e^{-k(z+d_\mathrm{m})} = \frac{\Phi_0}{2\pi} \frac{\rho}{\left[\rho^2 + (z+d_\mathrm{m})^2\right]^{3/2}}$$
$$B_z^0(\rho, z>0) = \int_0^\infty \frac{\Phi_0}{2\pi} k J_0(k\rho) e^{-k(z+d_\mathrm{m})} = \frac{\Phi_0}{2\pi} \frac{z+d_\mathrm{m}}{\left[\rho^2 + (z+d_\mathrm{m})^2\right]^{3/2}}.$$

Manipulation of Vortices: Pinning & Interaction

- Pinning by artificial defect: antidot



✓ Analytics with London approximation

$$U_0 = \frac{\Phi_0^2}{4\pi\mu_0\Lambda}\log\frac{R}{\xi}$$

Nordbourg and Vinokur PRB **62** 12408 (2000)

✓ Vortex-Vortex interaction

$$U_{VV}(r_{ij}) = \frac{\Phi_0^2}{4\Lambda\mu_0}\left[H_0\left(\frac{r_{ij}}{2\Lambda}\right) + Y_0\left(\frac{r_{ij}}{2\Lambda}\right)\right]$$

$$U_{VV}(r_{ij}) \approx \frac{\Phi_0^2}{\mu_0\pi r_{ij}} \quad \text{for} \quad r_{ij} \gg \Lambda$$

like monopoles!

- How near can two pinned vortices be?

$U_{VV}(r_{ij}) \approx U_0$ → $r_{ij} \approx \Lambda$

Brandt PRB 2009; Priour & Ferig PRB 2003

Manipulation of Vortices: Pinning & Interactions

- **Pinning and vortex lattice**

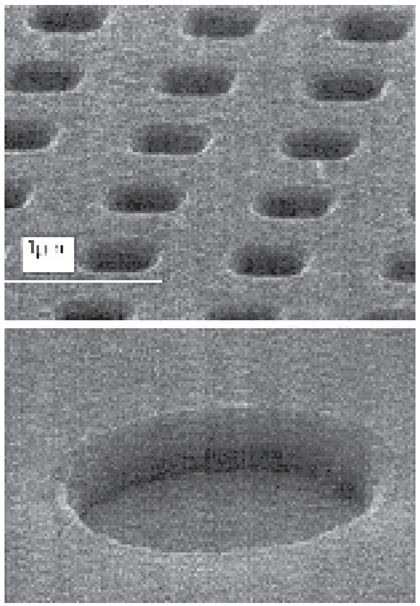
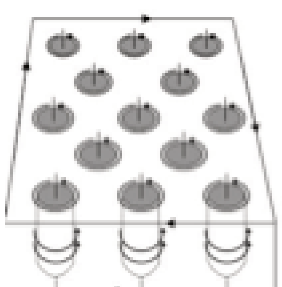


Fig. 1. SEM images of (a) an antidot lattice (d = 1 μm and radius r = 0.2 μm) and (b) an antidot (r = 1 μm) in YBCO films on sapphire.

Physica C **322**, 27 (2000)

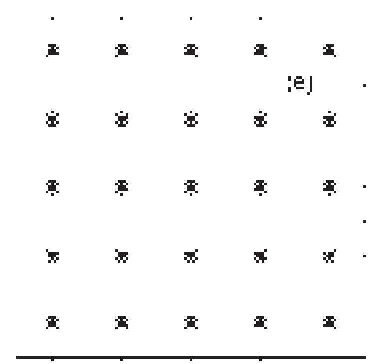
✓ Without pinning: triangular lattice

$$\mathcal{A} \gtrsim \xi^2$$

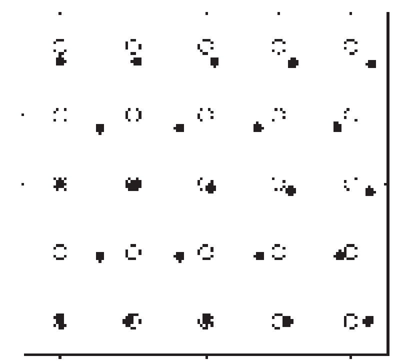


✓ Interplay between vortex-vortex and vortex-antidot interaction (London approximation)

$$\mathcal{A} \gtrsim \Lambda^2$$



$$\mathcal{A} \lesssim \Lambda^2$$



Pogosov, Rakhmanov, and Moshchalkov, PRB **67**, 014532 (2003)

✓ Density of vortices $n = \dfrac{1}{\mathcal{A}} = \dfrac{B_a}{\Phi_0}$

- Can be loaded by field-cooling
- Commensurability, interstitial vortices, …

✓ Vortex lattices in thin films (London approx.)

Brandt PRB **79**, 134526 (2009)

- Elasticity: $U_{\mathrm{pV}}(\mathbf{r}) \approx U_{\mathrm{pV}}(0) + U''_{\mathrm{pV}}(0)\dfrac{r^2}{2}$

$$k_{\mathrm{p}} = U''_{\mathrm{pV}}(0) = \frac{\Phi_0^2}{2\mu_0 \Lambda a^2}$$

Manipulation of Vortices: Vortex Dynamics

- Vortex dynamics

$$\eta \dot{\mathbf{x}}_{\mathrm{v}} + \alpha_{\mathrm{H}} \hat{\mathbf{n}} \times \dot{\mathbf{x}}_{\mathrm{v}} + k_{\mathrm{p}} \mathbf{x}_{\mathrm{v}} = \mathbf{F}_{\mathrm{ext}} + \mathbf{F}_{\mathrm{T}}$$

Pompeo and Silva PRB **78**, 094503 (2008)

✓ Drag viscosity $\eta \approx \frac{\mu_0 H_{c2} \Phi_0}{\rho_{\mathrm{n}}} d$

✓ Pinning force $k_{\mathrm{p}} = U''_{\mathrm{pV}}(0) = \frac{\Phi_0^2}{2\mu_0 \Lambda a^2}$

✓ Depinning frequency $\omega_{\mathrm{d}} \equiv \frac{k_{\mathrm{p}}}{\eta} \sim 2\pi \times 10^{11}\ \mathrm{Hz}$

✓ Inertial vortex mass irrelevant far from Tc, pinned vortices and low frequencies

Golubchik, Polturak and Koren PRB **85**, 060504(R) (2012)

✓ Thermal force

$$\langle F_{\mathrm{T}}^i(t) F_{\mathrm{T}}^j(t') \rangle = \delta_{ij} 2 k_{\mathrm{B}} T \eta \delta(t - t')$$

$$\langle \mathbf{F}_{\mathrm{T}}(t) \rangle = 0$$

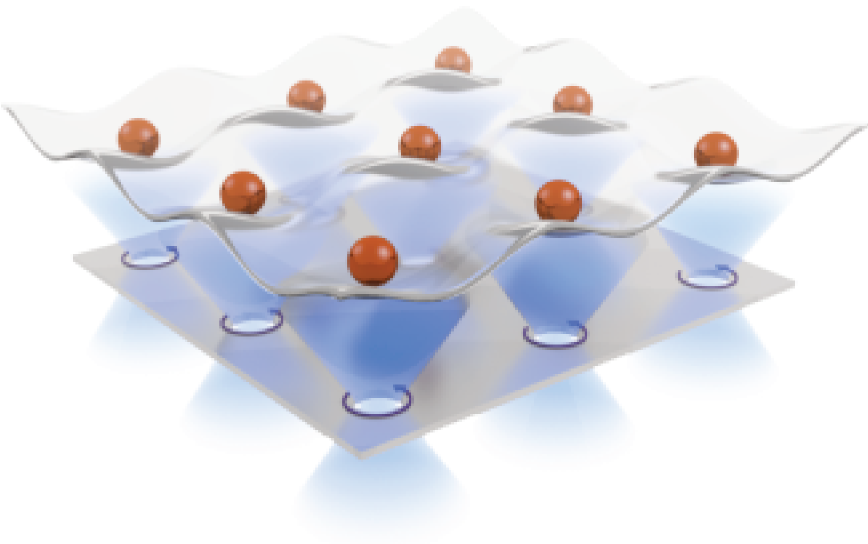
$\alpha_{\mathrm{H}} = 0$, $\mathbf{F}_{\mathrm{ext}} = 0$ →

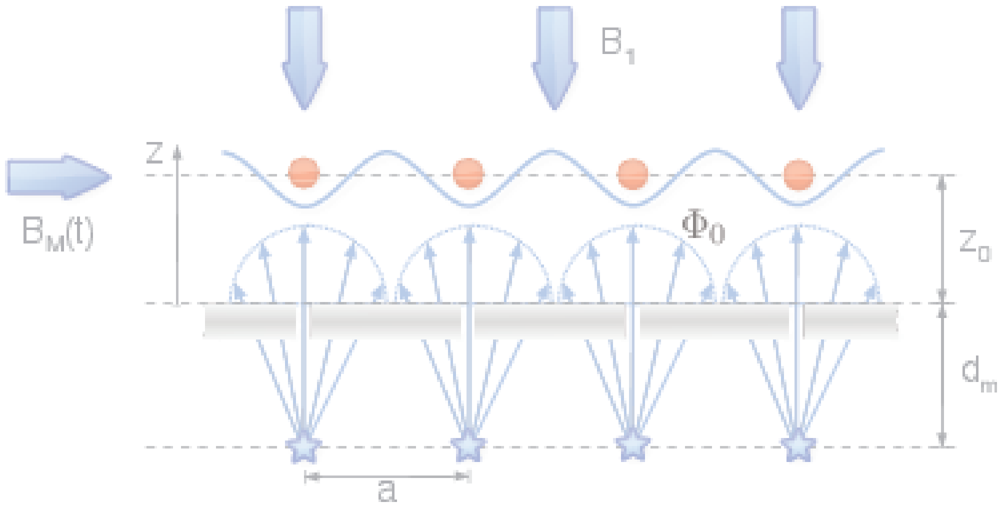
$$\langle x_{\mathrm{v}}(t+\tau) x_{\mathrm{v}}(t) \rangle = e^{-\omega_{\mathrm{d}} \tau} k_{\mathrm{B}} T / (\eta \omega_{\mathrm{d}})$$

Nano-scale magnetic lattice for ultracold atoms

Magnetic nano-lattice for ultracold atoms

- **Superconducting vortex lattice**





✓ Magnetic potential for ultracold atoms

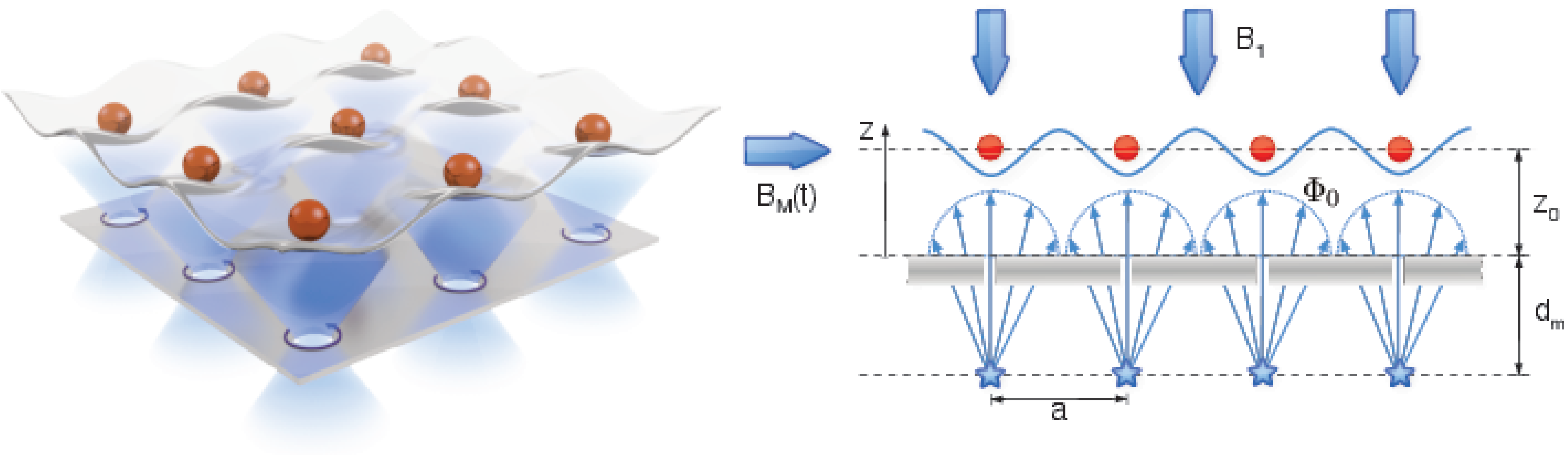
- Atomic magnetic moment parallel to instantaneous direction of the magnetic field

$$V_{\rm lat}(\mathbf{r}, z) = \mu_{m_F} |\mathbf{B}(\mathbf{r}, z)| \qquad \mu_{m_F} \equiv m_F g_F \mu_{\rm B}$$

- Low-field-seeker atomic states can be trapped $g_F m_F > 0$

Magnetic nano-lattice for ultracold atoms

- **Superconducting vortex lattice**

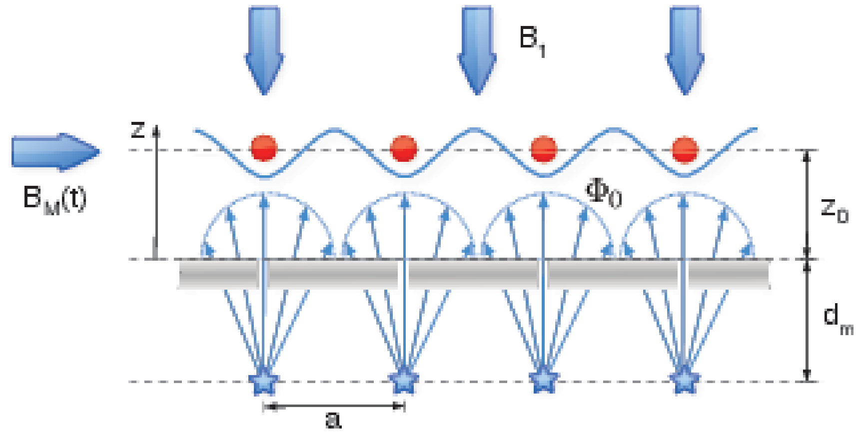


✓ **Total magnetic field to create the lattice:**

- Field created by the array of vortices → $\mathbf{B}_{\mathrm{V}}(\mathbf{r}, z) = \sum_{\mathbf{R}} \mathbf{B}_{\mathbf{R}}(\mathbf{r}, z)$
- Perpendicular bias field to confine atoms along z → $\mathbf{B}_1$
- Parallel time-dependent field to avoid Majorana losses → $\mathbf{B}_{\mathrm{M}}(t)$

Magnetic field generated by array of vortices

- Magnetic field created by array of vortices → $\mathbf{B}_{\mathrm{V}}(\mathbf{r},z) = \sum_{\mathbf{R}} \mathbf{B}_{\mathbf{R}}(\mathbf{r},z)$



✓ Bravais lattice (non-periodic also possible) $\mathbf{R} = n_1\mathbf{a}_1 + n_2\mathbf{a}_2$

✓ Monopole approximation

✓ Solve Poisson equation

$$\nabla^2\phi(\mathbf{r},z) = -\rho(\mathbf{r},z)$$

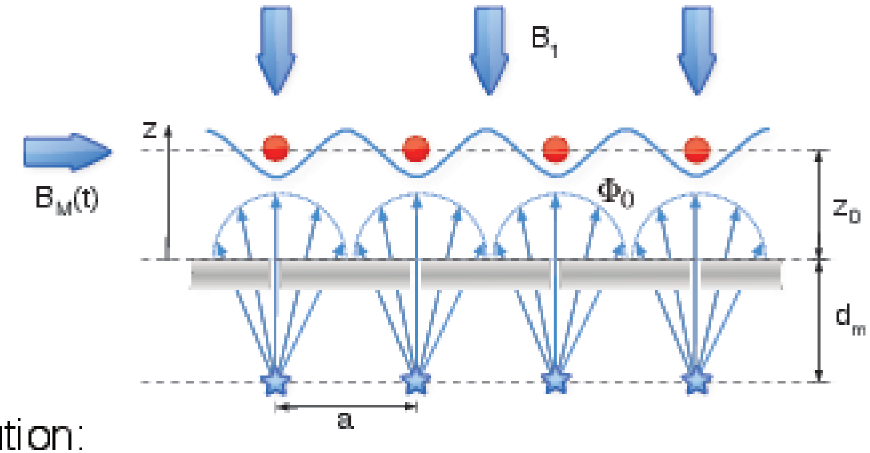
$$\mathbf{B}_{\mathrm{V}}(\mathbf{r},z) = -\nabla\phi(\mathbf{r},z)$$

$$\rho(\mathbf{r},z) = 2\Phi_0\delta\left(z+d_{\mathrm{m}}\right)\sum_{\mathbf{R}}\delta\left(\mathbf{r}-\mathbf{R}\right)$$

✓ Solution: ....

Magnetic field generated by array of vortices

- Magnetic field created by array of vortices → $\mathbf{B}_{\mathrm{V}}(\mathbf{r}, z) = \sum_{\mathbf{R}} \mathbf{B}_{\mathbf{R}}(\mathbf{r}, z)$



✓ Solution:

$$B_{\mathrm{V}}^{x(y)}(\mathbf{r}, z) = B_0 \sum_{\mathbf{K}\neq\mathbf{0}} \frac{K_{x(y)}}{|\mathbf{K}|} \sin(\mathbf{K}\cdot\mathbf{r}) e^{-\Delta_z|\mathbf{K}|/k},$$

$$B_{\mathrm{V}}^{z}(\mathbf{r}, z) = B_0 + B_0 \sum_{\mathbf{K}\neq\mathbf{0}} \cos(\mathbf{K}\cdot\mathbf{r}) e^{-\Delta_z|\mathbf{K}|/k}.$$

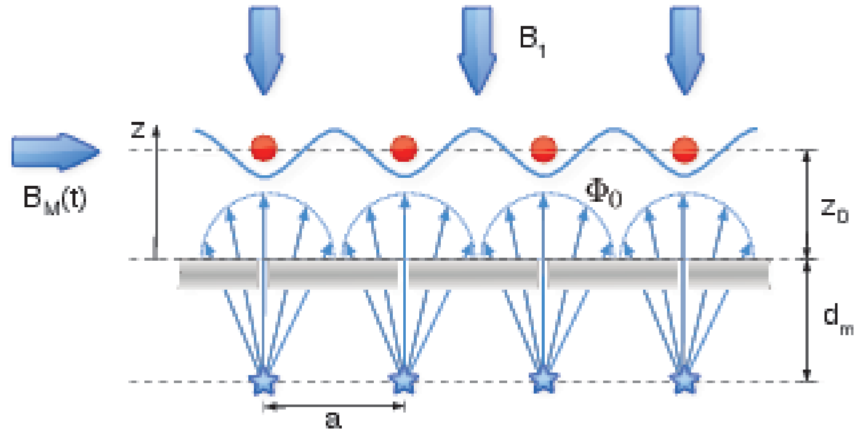
✓ Important parameters: $\Delta_z \equiv 2k(z + d_{\mathrm{m}}) > 2kd_{\mathrm{m}} \equiv \Delta_{\min}$ $B_0 \equiv \frac{\Phi_0}{a^2}$ $k \equiv \frac{\pi}{a}$

✓ x-y components are zero on top of the vortices $B_{\mathrm{V}}^{x(y)}(\mathbf{r} = \mathbf{R}, z) = 0$

✓ In the long-distance limit (infinite plane) $|\mathbf{B}_{\mathrm{V}}(\mathbf{r}, z \to \infty)| = B_0$

Perpendicular bias field

- perpendicular bias field to confine atoms along z → $\mathbf{B}_1$



$$\mathbf{B}_1 = B_1(0,0,-1)$$

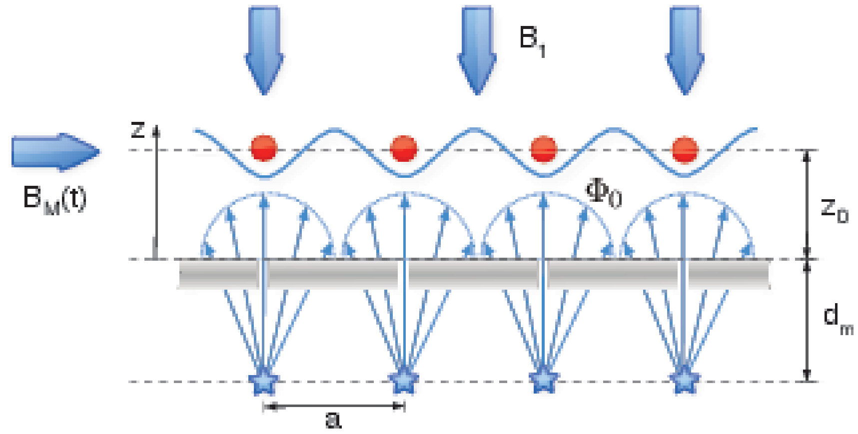
✓ Should not create vortices: $B_1 < B^\star + \min_{\mathbf{r}} B_{\mathrm{V}}^z(\mathbf{r},0) \qquad B^\star \approx \frac{\Phi_0}{4\pi\Lambda^2}$

✓ Homogeneous provided thin film

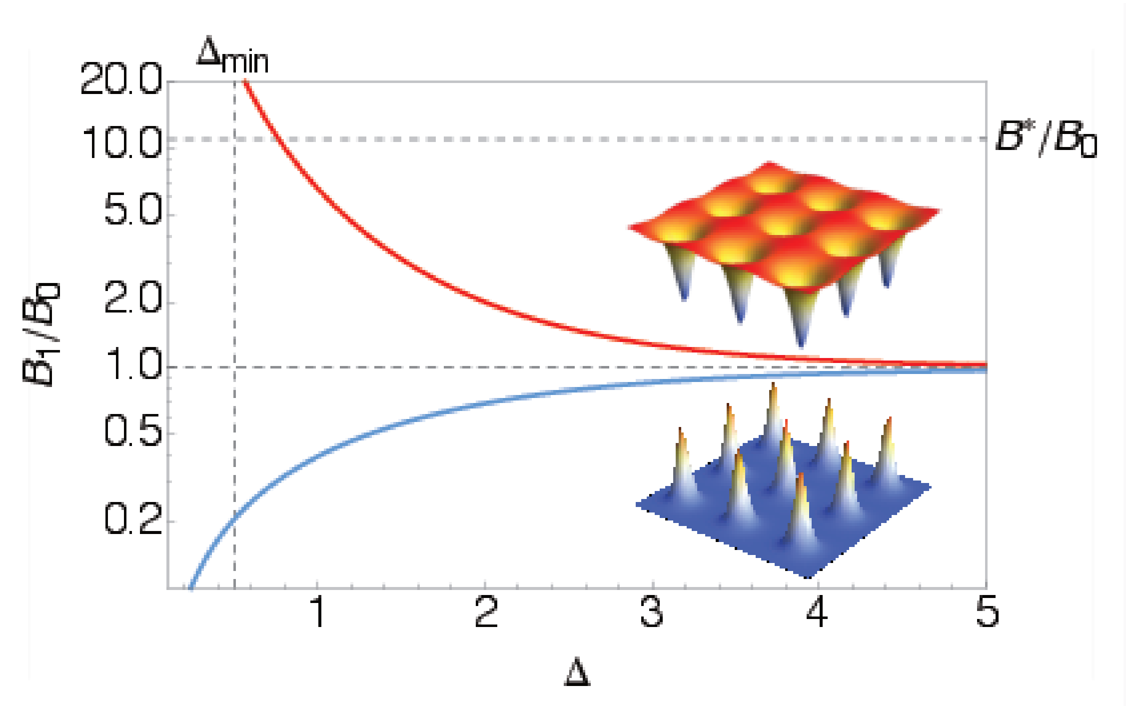
✓ Trapping at z_0 depending on B_1

Perpendicular bias field

- perpendicular bias field to confine atoms along z → $\mathbf{B}_1$



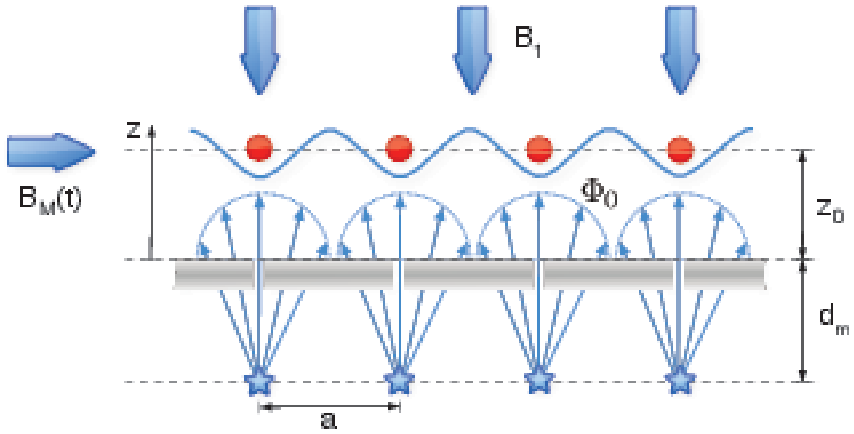
$$\mathbf{B}_1 = B_1(0,0,-1)$$



$$\Delta = \frac{2\pi}{a}(z_0 + d_m)$$

Perpendicular bias field

- perpendicular bias field to confine atoms along z ⟶ $\mathbf{B}_1$



$\mathbf{B}_1 = B_1(0,0,-1)$

✓ Should not create vortices: $B_1 < B^\star + \min_{\mathbf{r}} B_V^z(\mathbf{r},0)$ $\quad B^\star \approx \frac{\Phi_0}{4\pi\Lambda^2}$

✓ Homogeneous provided thin film

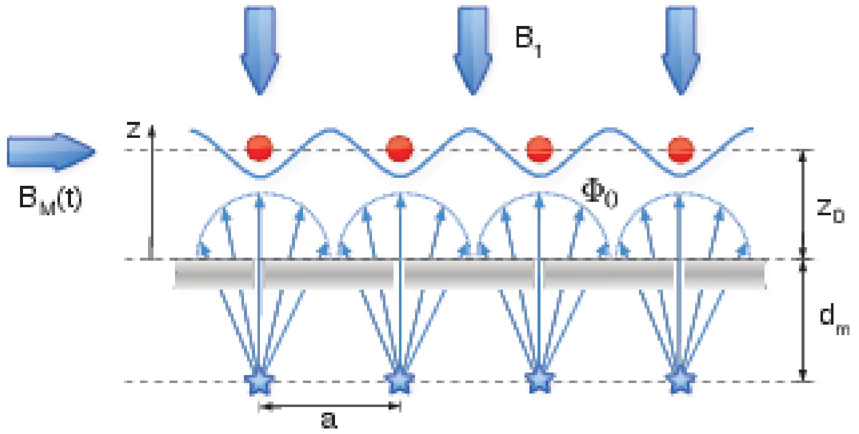
✓ Trapping at z_0 depending on B_1

✓ Total field has 3D local minima $\mathbf{B}_{\text{lat}}(\mathbf{r},z) = \mathbf{B}_V(\mathbf{r},z) + \mathbf{B}_1$

✓ Zero-field minima! (Majorana losses)

Parallel time-dependent field

- **Parallel time-dependent field to avoid Majorana loss**



✓ Use TOP traps (like in standard magnetic traps)

$$\mathbf{B}_{\rm total}(\mathbf{r},z,t) = \mathbf{B}_{\rm latt}(\mathbf{r},z) + B_{\rm M}(\cos\omega_{\rm M}t, \sin\omega_{\rm M}t, 0)$$

✓ Using $B_{\rm M} \gg \max|\mathbf{B}_{\rm lat}(\mathbf{r},z_0)|$

$$\omega_{\rm t} \ll \omega_{\rm M} \ll \omega_{\rm L} \equiv \mu_{m_F} B_{\rm M}/\hbar$$

✓ Time-averaged field experienced by atoms is $\langle|\mathbf{B}_{\rm total}(\mathbf{r},z,t)|\rangle \approx B_{\rm M} + \dfrac{|B_{\rm latt}(\mathbf{r},z)|^2}{2B_{\rm M}}$

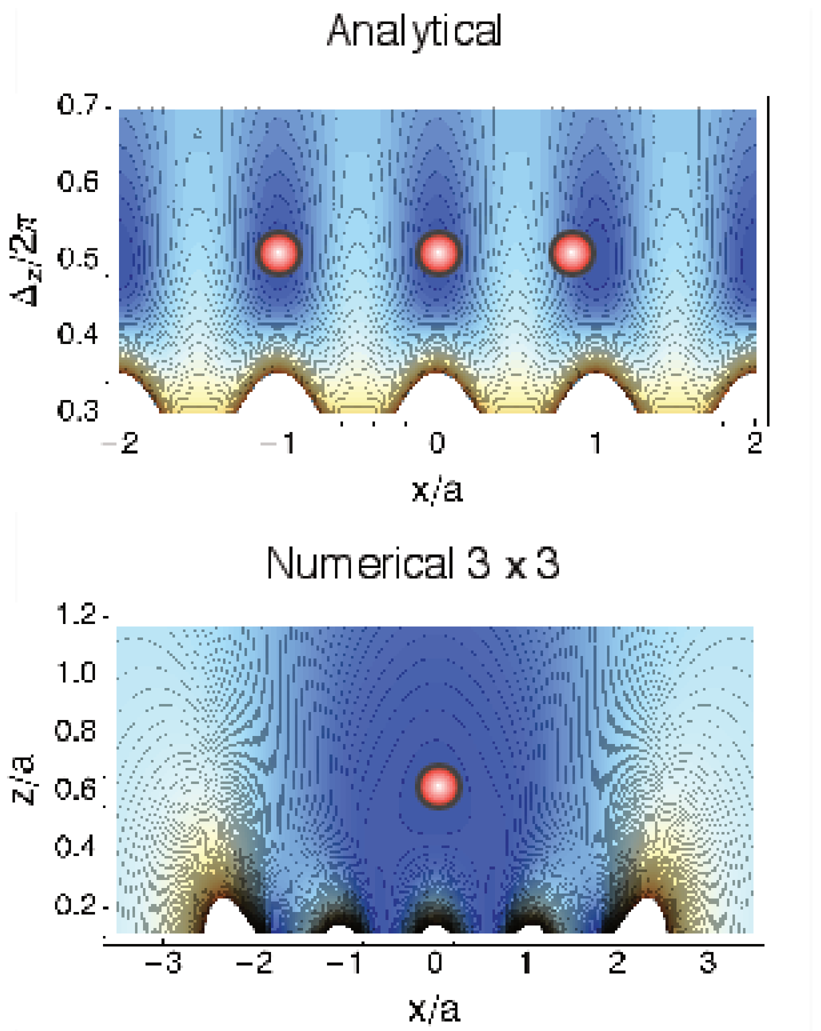
✓ Time-averaged potential

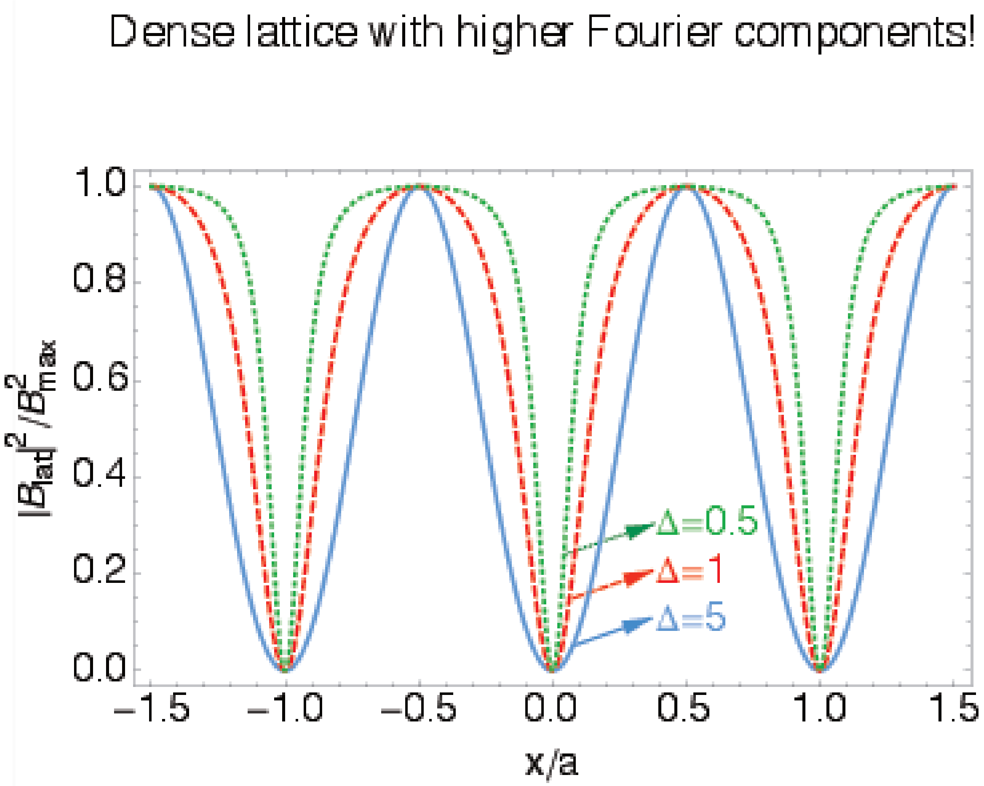
$$V_{\rm lat}(x,y,z) \approx \mu_{m_F} B_{\rm M} + \frac{\mu_{m_F}}{2B_{\rm M}}|\mathbf{B}_{\rm lat}(x,y,z)|^2.$$

Magnetic lattice potential

- **Atomic lattice potential**

$$V_{\mathrm{lat}}(x,y,z) \approx \mu_{m_F} B_{\mathrm{M}} + \frac{\mu_{m_F}}{2B_{\mathrm{M}}} |\mathbf{B}_{\mathrm{lat}}(x,y,z)|^2.$$





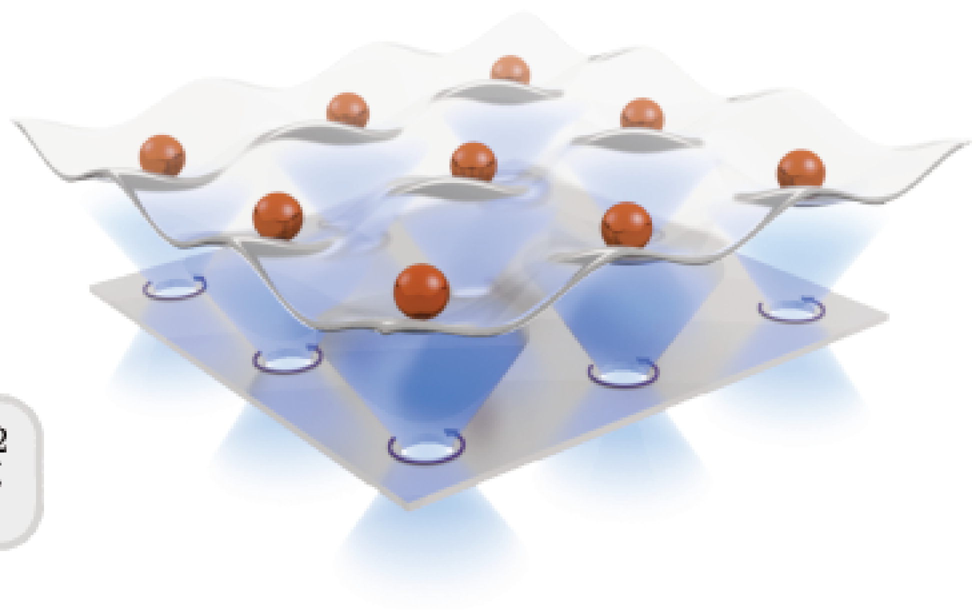
$$\Delta = \frac{2\pi}{a}(z_0 + d_{\mathrm{m}})$$

Quantum simulation of Hubbard models

- **Hubbard Hamiltonians**

$$V_{\mathrm{lat}}(\mathbf{r}, z) \approx V_0[\sin^2(kx) + \sin^2(ky)]$$

$$\hat{H} = -t\sum_{\langle ij\rangle}\left(\hat{a}_i^\dagger \hat{a}_j + \hat{a}_j^\dagger \hat{a}_i\right) + U\sum_i \hat{n}_i^2$$



✓ Formulas from Optical Lattices

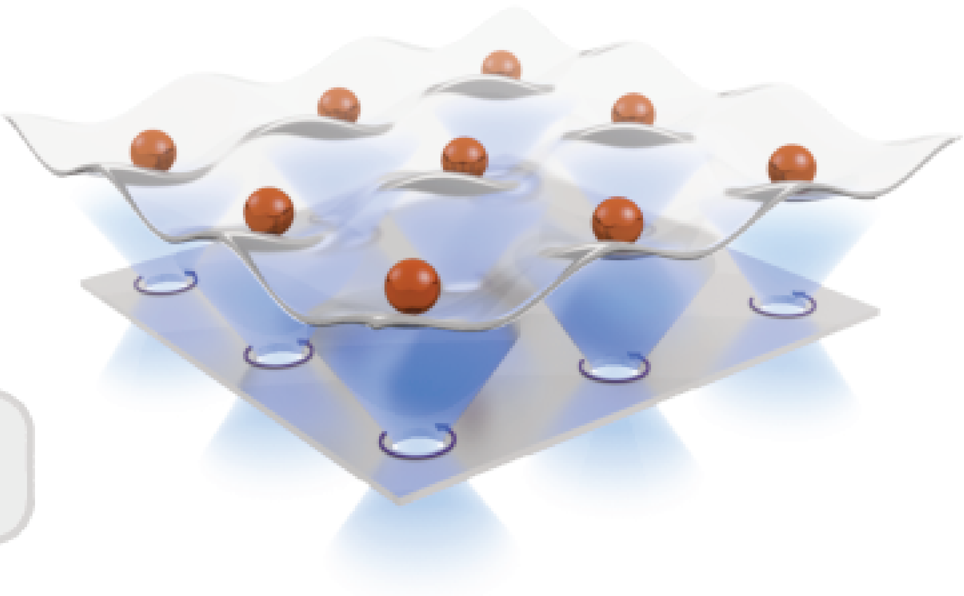
- Recoil energy $E_{\mathrm{R}} = \dfrac{\hbar^2 k^2}{2m_{\mathrm{a}}}$
- Trapping frequency $\omega_{\mathrm{t}} \approx \dfrac{\sqrt{4E_{\mathrm{R}}V_0}}{\hbar}$
- Tunneling rate $t \approx \dfrac{2E_{\mathrm{R}}}{\sqrt{\pi}}\left(\dfrac{V_0}{E_{\mathrm{R}}}\right)^{3/4}\exp\left[-2\left(\dfrac{V_0}{E_{\mathrm{R}}}\right)^{1/2}\right]$
- On-site repulsion $U \approx \sqrt{\dfrac{8}{\pi}}E_{\mathrm{R}}a_{\mathrm{s}}k\left(\dfrac{V_0}{E_{\mathrm{R}}}\right)^{3/4}$

Quantum simulation of Hubbard models

- **Hubbard Hamiltonians**

$$V_{\text{lat}}(\mathbf{r}, z) \approx V_0[\sin^2(kx) + \sin^2(ky)]$$

$$\hat{H} = -t \sum_{\langle ij \rangle} \left(\hat{a}_i^\dagger \hat{a}_j + \hat{a}_j^\dagger \hat{a}_i \right) + U \sum_i \hat{n}_i^2$$



✓ Analogous in Superconducting Vortex Lattice

$$V_0 \equiv \frac{8B_0^2}{B_{\text{M}}} \mu_{m_F} \exp[-2\Delta]$$

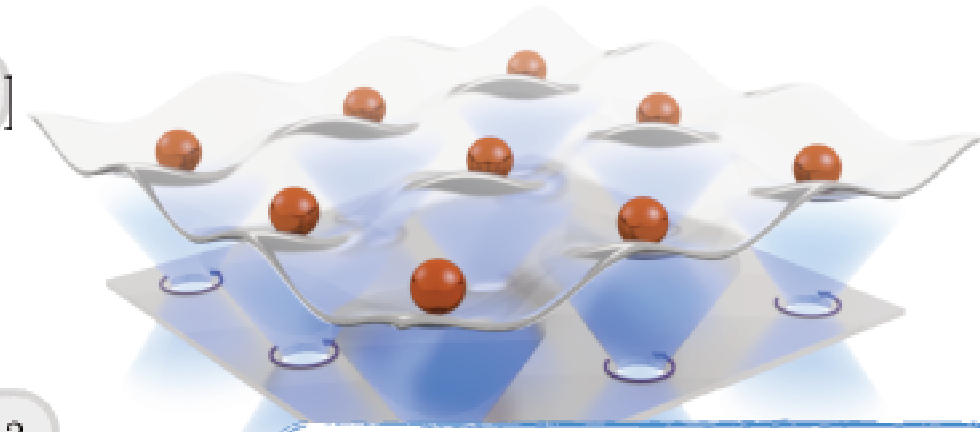
- Twice the inter-vortex distance $2a$ ⟷ Optical wavelength
- The strength of the perpendicular field B_1 ⟷ Laser intensity

Quantum simulation of Hubbard models

- **Hubbard Hamiltonians**

$$V_{\rm lat}(\mathbf{r}, z) \approx V_0[\sin^2(kx) + \sin^2(ky)]$$

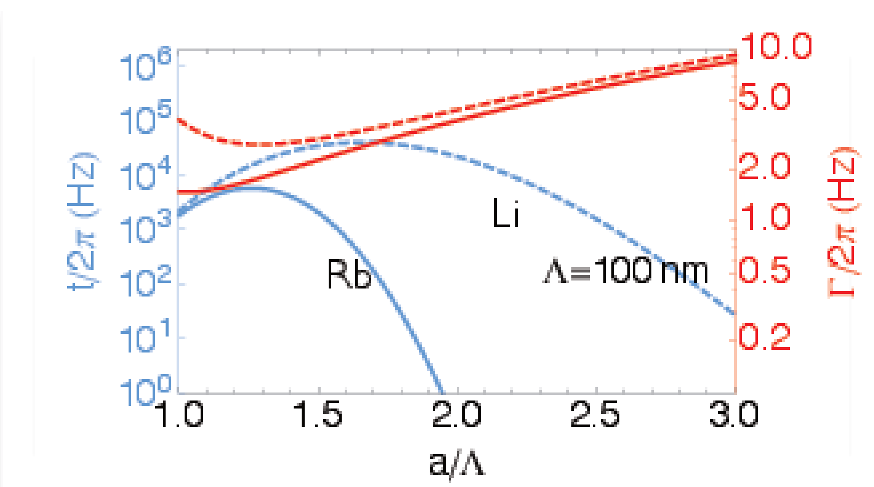
$$\hat{H} = -t \sum_{\langle ij \rangle} \left(\hat{a}_i^\dagger \hat{a}_j + \hat{a}_j^\dagger \hat{a}_i \right) + U \sum_i \hat{n}_i^2$$

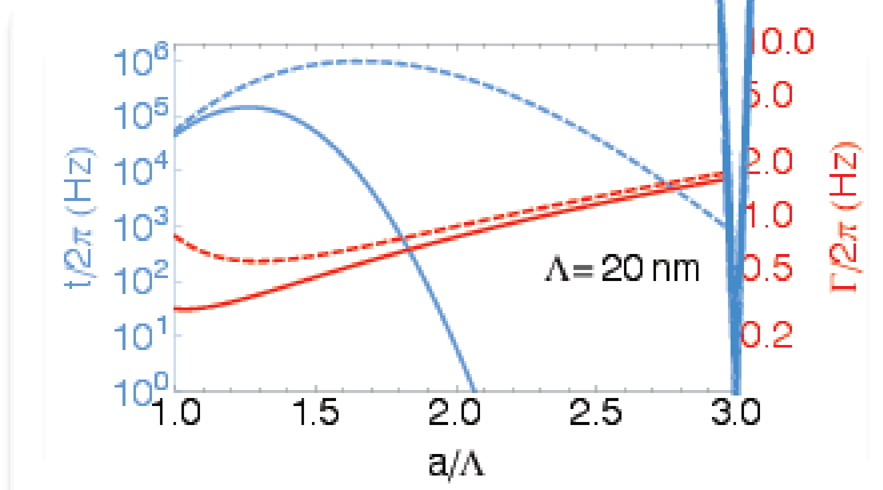


- In thin films of MgB_2 $\quad \lambda \sim 40\ \mathrm{nm}$

APL **102**, 072603 (2013) $\quad \xi \sim 4 - 6\ \mathrm{nm}$

- Analogous in Superconducting Vortex Lattice with potentially larger energy scales





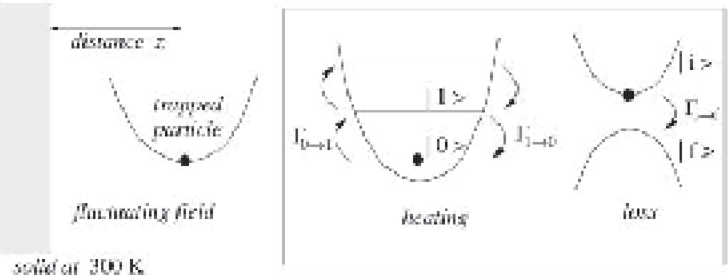
Decoherence of atoms close to [superconducting] surfaces

Magnetic field fluctuations

Loss and heating of particles in small and noisy traps

C. Henkel, S. Pötting, M. Wilkens

Appl. Phys. B 69, 379–387 (1999)



- Heating

$$V(\boldsymbol{r},t) = -\boldsymbol{x}\cdot\boldsymbol{F}(\boldsymbol{r},t) = -a\left(b+b^{\dagger}\right)\boldsymbol{n}\cdot\boldsymbol{F}(\boldsymbol{r},t)$$

$$\Gamma_{0\to1}(\boldsymbol{r}) = \gamma_- = \frac{a^2}{\hbar^2}\sum_{ij} n_i n_j S_F^{ij}(\boldsymbol{r};-\Omega)$$

$$S_F^{ij}(\boldsymbol{r};\omega) = \int_{-\infty}^{+\infty} \mathrm{d}\tau \left\langle F_i(\boldsymbol{r},t+\tau)F_j(\boldsymbol{r},t)\right\rangle \mathrm{e}^{\mathrm{i}\omega\tau}$$

- Spin flips

$$V_Z(\boldsymbol{r},t) = -\boldsymbol{\mu}\cdot\boldsymbol{B}(\boldsymbol{r},t)$$

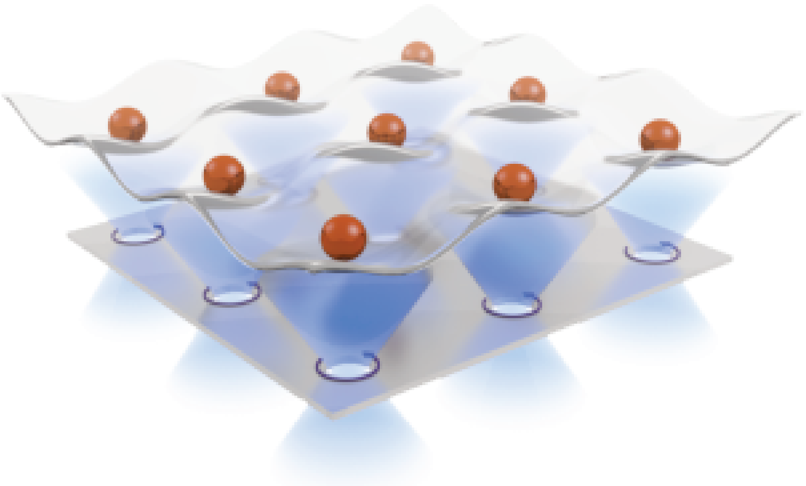
$$\Gamma_{i\to f}(\boldsymbol{r}) = \sum_{\alpha\beta}\frac{\langle i|\mu_\alpha|f\rangle\langle f|\mu_\beta|i\rangle}{\hbar^2} S_B^{\alpha\beta}(\boldsymbol{r};-\omega_{fi})$$

Magnetic field fluctuations due to thermal jiggling of vortices

✔ Magnetic field fluctuations experienced by the atom due to vortex position fluctuations

$$\mathbf{B}_{\mathrm{latt}}(\mathbf{r},z,t)=\sum_{\mathbf{R}}\mathbf{B}_{\mathbf{R}}(\mathbf{r},z,t)$$

$$\mathbf{B}_{\mathbf{R}}(\mathbf{r},z,t)=\frac{\Phi_0}{2\pi}\frac{(\mathbf{r}-\mathbf{R}-\mathbf{r}_{\mathbf{R}}(t),z)}{\left[|\mathbf{r}-\mathbf{R}-\mathbf{r}_{\mathbf{R}}(t)|^2+z^2\right]^{3/2}}$$



✔ Using vortex dynamics $\langle r^i_{\mathbf{R}}(t)r^j_{\mathbf{R}'}(t+\tau)\rangle=\frac{k_{\mathrm{B}}T}{\eta\omega_{\mathrm{d}}}e^{-\omega_{\mathrm{d}}\tau}\delta_{ij}\delta_{\mathbf{R}\mathbf{R}'}$

✔ Estimate heating and spin-flip rates

$$\Gamma_{0\to1}\sim(2\pi)^5\frac{\mu_{m_F}^2}{\hbar^2}\frac{k_{\mathrm{B}}T}{k_{\mathrm{p}}}\frac{\omega_{\mathrm{d}}}{\omega_{\mathrm{t}}^2+\omega_{\mathrm{d}}^2}\frac{x_0^2B_0^2}{a^4\Delta^6}$$

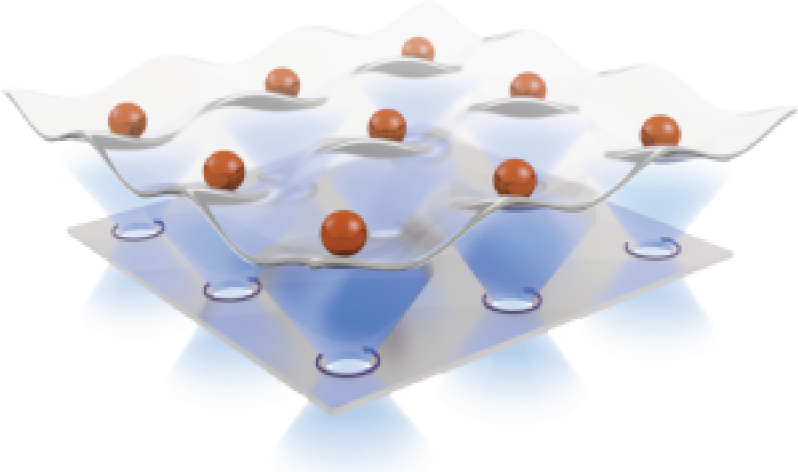
$$\Gamma_{i\to f}\sim3\pi^3\frac{\mu_{m_F}^2}{\hbar^2}\frac{k_{\mathrm{B}}T}{k_{\mathrm{p}}}\frac{\omega_{\mathrm{d}}}{\omega_{\mathrm{L}}^2+\omega_{\mathrm{d}}^2}\frac{B_0^2}{a^2\Delta^4}$$

Magnetic field fluctuations due to thermal jiggling of vortices

✓ Magnetic field fluctuations experienced by the atom due to vortex position fluctuations

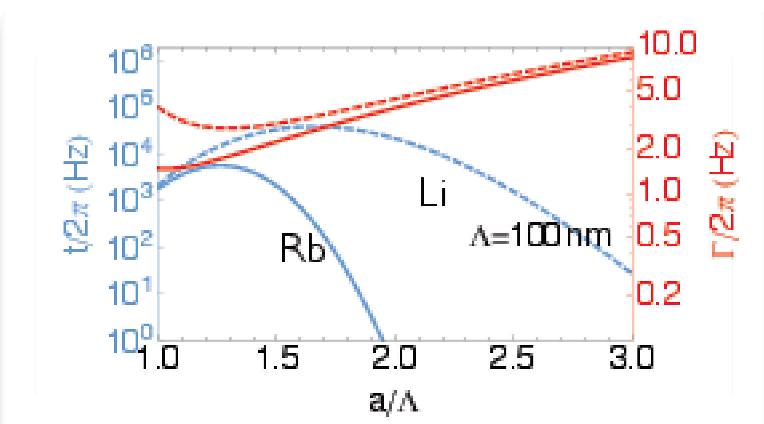
$$\mathbf{B}_{\text{latt}}(\mathbf{r}, z, t) = \sum_{\mathbf{R}} \mathbf{B}_{\mathbf{R}}(\mathbf{r}, z, t)$$

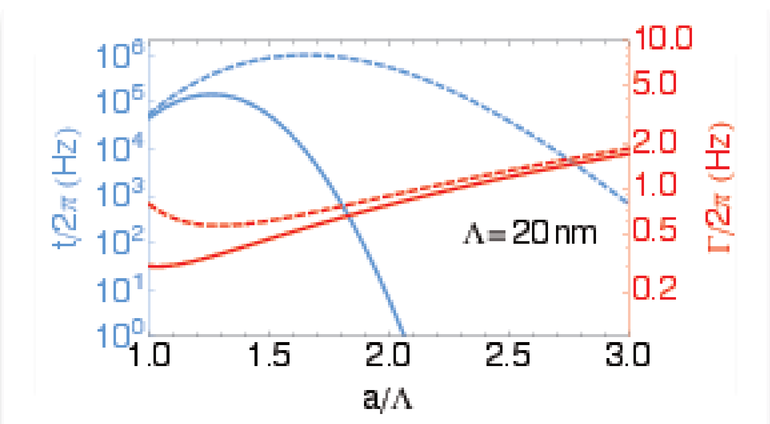
$$\mathbf{B}_{\mathbf{R}}(\mathbf{r}, z, t) = \frac{\Phi_0}{2\pi} \frac{(\mathbf{r} - \mathbf{R} - \mathbf{r}_{\mathbf{R}}(t), z)}{[|\mathbf{r} - \mathbf{R} - \mathbf{r}_{\mathbf{R}}(t)|^2 + z^2]^{3/2}}$$



✓ Using vortex dynamics $\langle r^i_{\mathbf{R}}(t) r^j_{\mathbf{R}'}(t+\tau)\rangle = \frac{k_B T}{\eta \omega_d} e^{-\omega_d \tau} \delta_{ij} \delta_{\mathbf{R}\mathbf{R}'}$

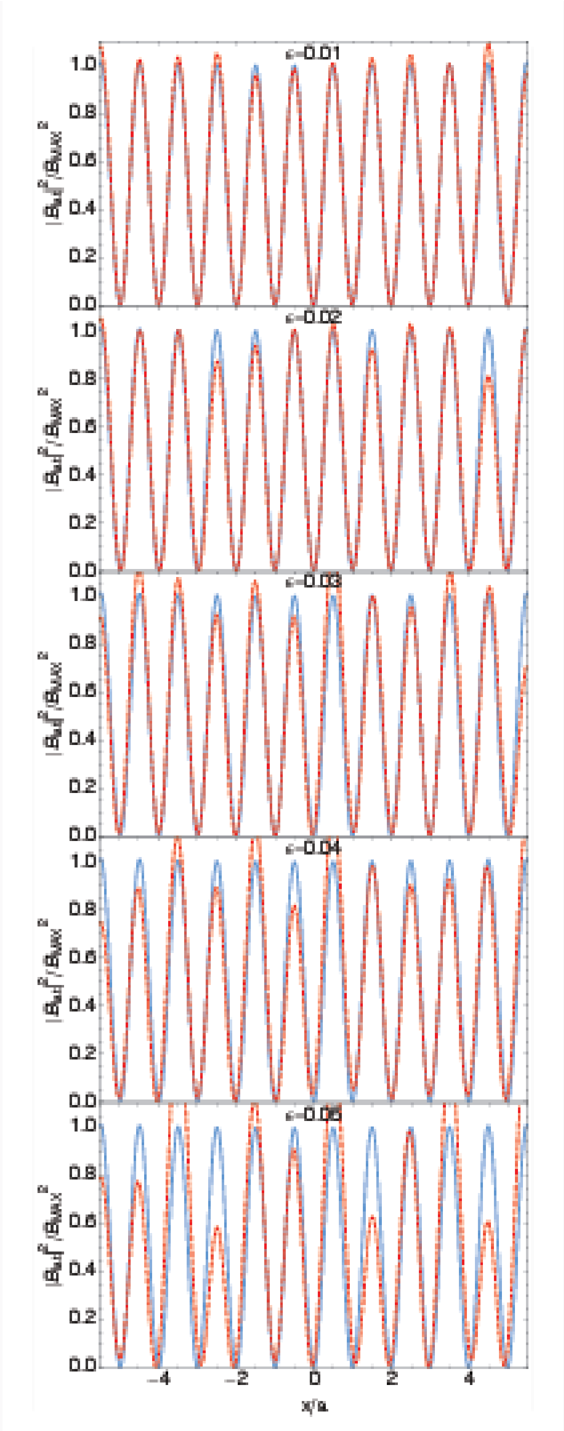
✓ Estimate heating and spin-flip rates





Other imperfections

- **Positioning of vortices**
  - ✓ Low error 1~2 %
  - ✓ Disordered many-body physics
  - ✓ Triangular lattices without pinning
- **Randomness in size and shape of antidots**
  - ✓ Flux in each of them fixed by nature!
- **Time dependent bias fields**
  - ✓ Parallel and $\omega_{\mathrm{M}} \ll \omega_{\mathrm{d}}$
- ...



Towards an all-magnetic toolbox

Magnetic local addressing

- **Approach a magnetic dipole**

✓ Avoid the creation of a vortex

Wei *et al.* PRB **54**, 15429 (1996)

$$a_1 = \Lambda\sqrt{\frac{\mu_0 m_\mathrm{d}}{\Phi_0 \Lambda \ln(\Lambda/\xi)}}$$

$$\Lambda = 10\xi = 100 \text{ nm}$$

$$a_1 \sim 2\Lambda$$

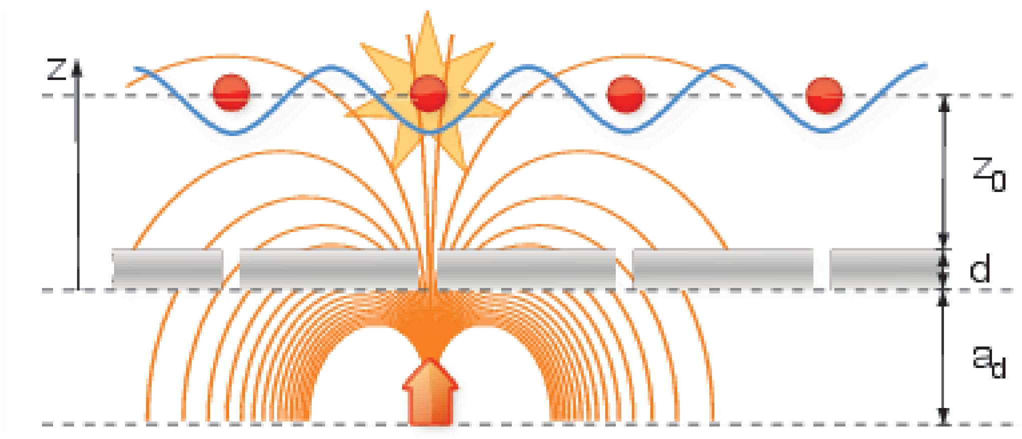
$$m_d \sim 10^8 \mu_\mathrm{B}$$

✓ Coupling to atom (to neighbors reduced by ~ factor 2)

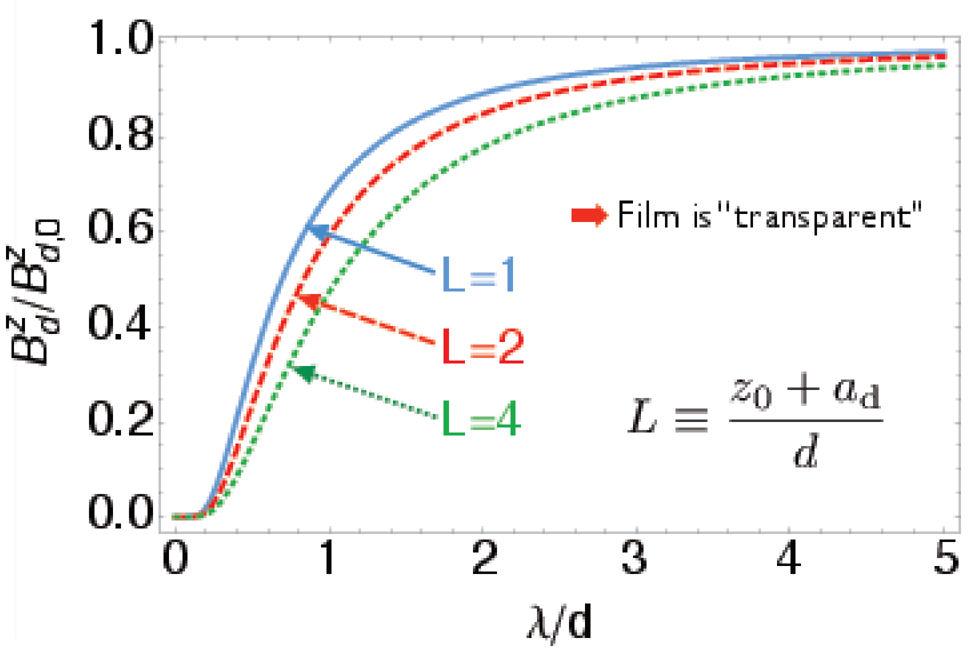
$$\lambda = d = \Lambda = z_0 = \frac{a_\mathrm{d}}{2} = 100 \text{ nm}$$

$$L = 3$$

$$g_\mathrm{d} \sim \frac{\mu_\mathrm{B} B^z_\mathrm{d}}{\hbar} \sim 0.4 \frac{\mu_\mathrm{B} B^z_{\mathrm{d},0}}{\hbar} \sim 2\pi \times 10^8 \text{ Hz}$$



✓ Field above the film compared with the one without the film

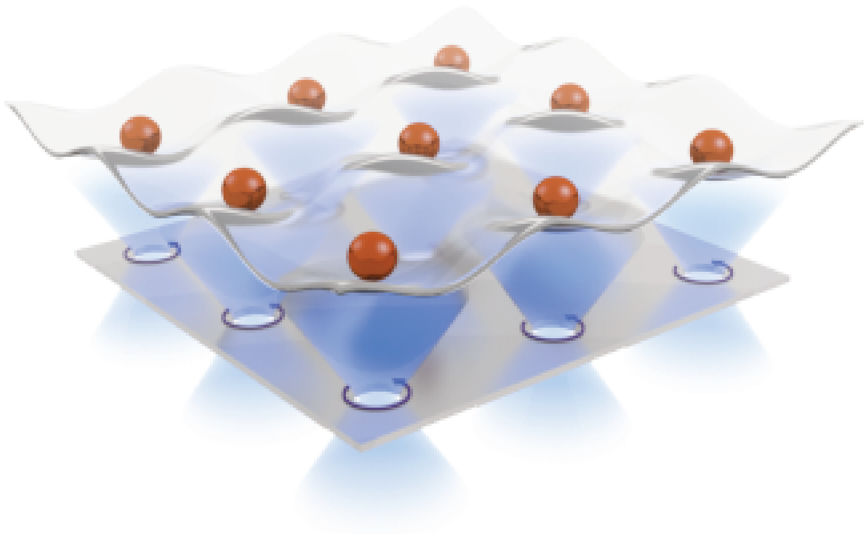


Summary

- Fundamental length scales in superconducting vortex lattices

$$\Lambda \equiv \frac{\lambda^2}{d} \gtrsim \lambda \gtrsim d > \xi$$

Nanometer regime



- Exploit nano-engineering and structuring of vortex matter lattices
- Towards all-magnetic quantum simulation & computation with cold atoms
- Can one use superconductivity to enhance interactions?
- AMO issues: loading, measurement

O. Romero-Isart, C. Navau, A. Sanchez, P. Zoller, and J. I. Cirac arXiv: 1302.3504