Quenching Across Quantum Critical Points: dependence of scaling laws on spatial periodicity

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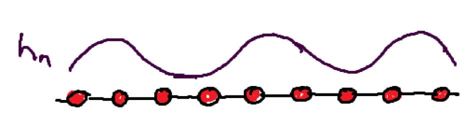
Summary

Initializing a system in a certain ground state and dynamically varying a parameter of its Hamiltonian typically produces defects above the groundstate of the final Hamiltonian. Such quenching at a rate τ^{-1} through a quantum critical point is dominated by dynamic critical behavior and the post-quench defect density obeys the Kibble-Zurek power-law scaling form

$$n_D \sim \tau^{\frac{-d\nu}{z\nu+1}}$$

Here it is found for periodic systems

$$h_n = h\cos(\pi n/q + \phi)$$



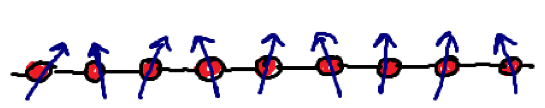
that periodicity alters post-quench dynamics, yielding

$$\boxed{n_D \sim \tau^{\frac{-q}{q+1}}} \quad (d=1, z=1)$$

Models

- XY spin chain with periodically varying magnetic field

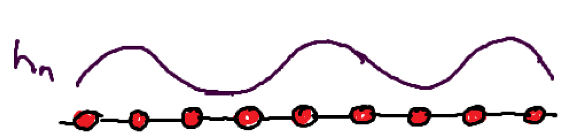
$$\boxed{H = -J\sum_{n=-N}^{N}\left[\sigma_n^x\sigma_{n+1}^x + \sigma_n^y\sigma_{n+1}^y + h_n\sigma_n^z\right]}$$



Phases: Ferromagnetic/Paramagnetic

- Maps to fermionic tight-binding system with varying chemical potential

$$\boxed{H = \sum_n (Jc_n^\dagger c_{n+1} + h.c. + h_n c_n^\dagger c_n) \quad h_n = h\cos(\pi n/q+\phi)}$$



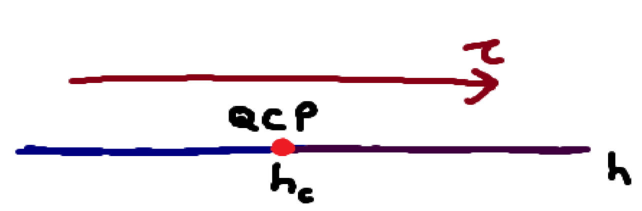
- Also relevant to spinless p-wave superconducting wire

Phases: Topological/Non-topological

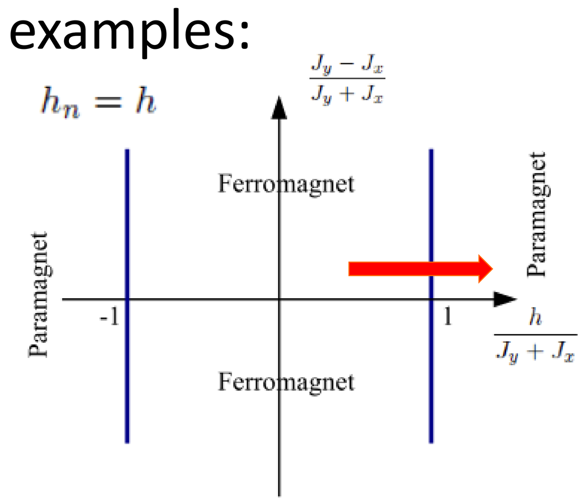
Quench dynamics

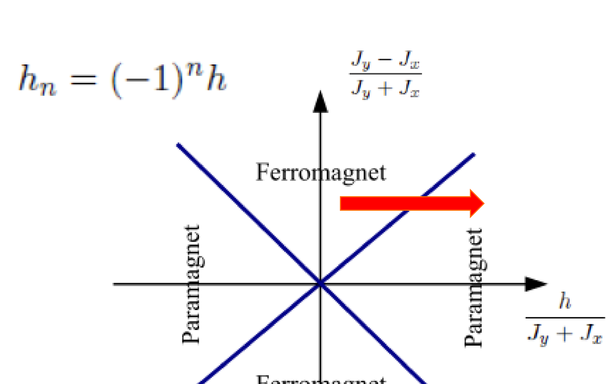
Linear quench in the periodic potential strength

$$\boxed{h = -\frac{t}{\tau}}$$



Some examples:

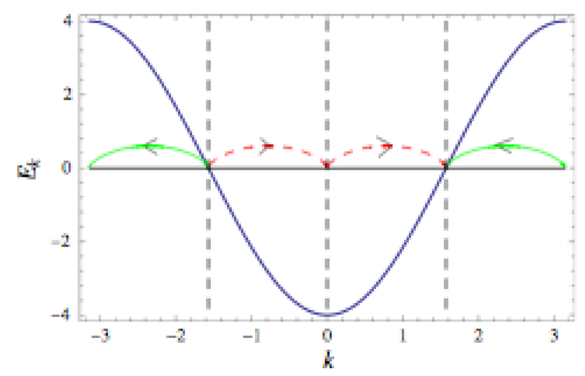




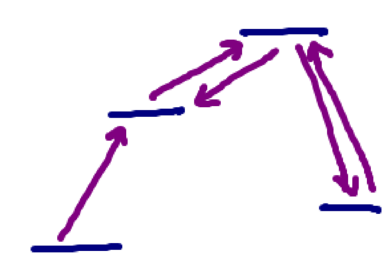
Quenching in periodic systems

- Periodic potential divides and couples Brillouin zone into *q* regions.

$$h_n = \frac{h}{2}\left(e^{i(\pi n/q+\phi)} + e^{-i(\pi n/q+\phi)}\right)$$



- Excitations correspond to probability transfer between low-lying states and occur near QCP of h = 0. Can use pertubation theory to map to effective two-state system with *non-linear* quench.



Effective description:

$$\boxed{H_{eff,\delta k} = \begin{pmatrix} -4\delta k & \Delta^* \\ \Delta & 4\delta k \end{pmatrix}, \quad (J=1); \Delta = h^q f(\phi), \quad f(\phi) \sim \cos(q\phi)/\sin(q\phi), \text{ q-odd/even}}$$

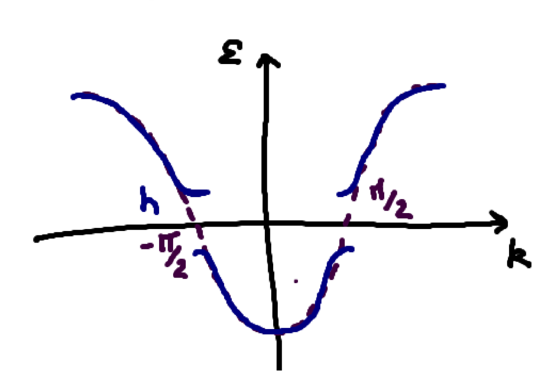
$$\Delta \sim (t/\tau)^q$$

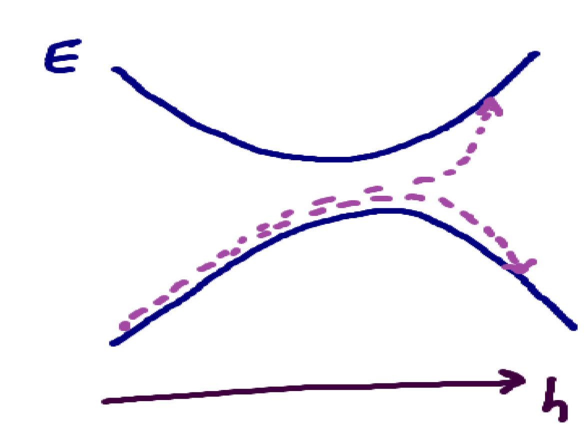
- Close to QCP: $\boxed{\Delta E \sim \delta k; \sim h^q \quad z = 1; \nu = q}$

- Scaling Analysis: $\boxed{p_k(\delta k(\tau^q/f(q\phi))^{1/(q+1)}) \quad N_{exc} \sim (f(q\phi)/\tau^q)^{1/(q+1)}}$

Illustrative Example

Case of $h_n = (-1)^n h$: Coupling of $(k, k+\pi)$ modes
Gap for $h \neq 0$.





- Landau-Zener physics

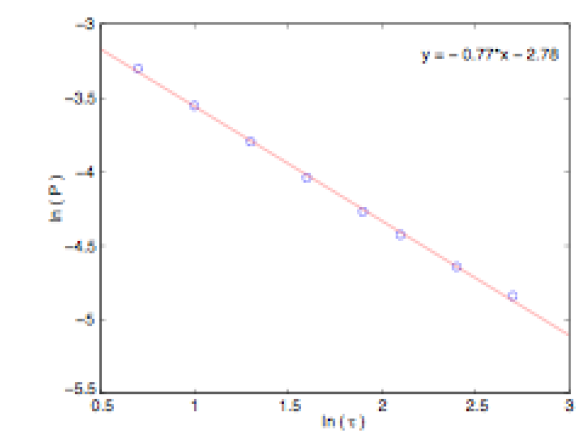
$$\Delta E \sim \delta k (z=1); \sim h(\nu = 1)$$

$$p_k = e^{-2\pi\tau\sin^2 k/J}; N_{exc} \sim 1/\sqrt{\tau}$$

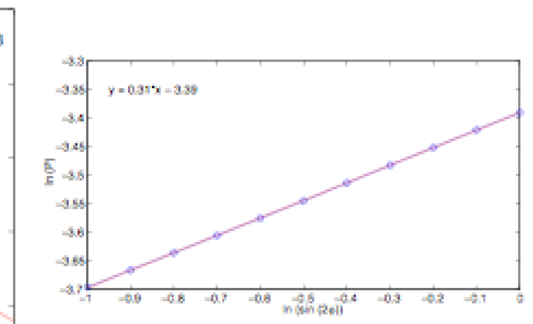
- Scaling ubiquitous to a large class of systems

Numerical Studies

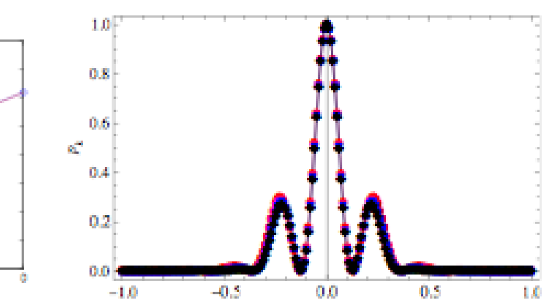
- Scaling of defect density with quench rate and off-set; Scaling and oscillations as a function of momentum



Ln(N_{exc}) vs ln(τ) for q = 3, ϕ = 0



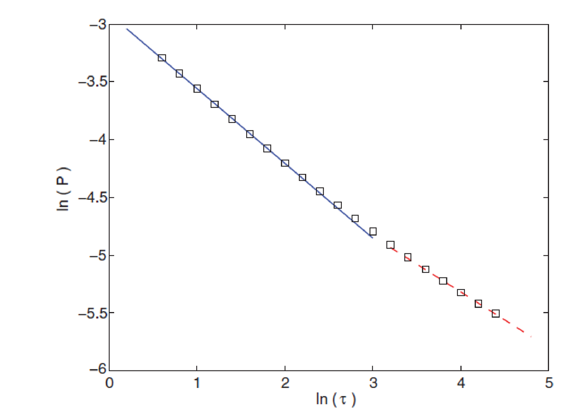
Ln(N_{exc}) vs ln(sin(2ϕ)) for q = 2 and τ = 8



p_k vs $(k+5\pi/6)\tau^{3/4}$ for q = 3, ϕ = 0 and τ = 2 (red), τ = 4 (blue) and τ = 8 (black)

- Cross-over behavior seen in more complicated periodic structures:

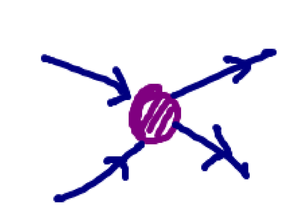
$$h_n = h[a_1\cos(\pi n) + a_2\cos(\pi n/2 + \pi/4)]$$



Interactions

- Jz term maps to interactions in fermionic system

Interactions and linearized modes about $k = \pm\pi/2$ → Luttinger liquid model with Sine-Gordon type form



$$S_q \sim \int\int dt dx h^q \cos(2\sqrt{\pi K}\Phi)$$

Scaling analysis gives

$$\boxed{\nu = q/(2-K) \quad n_D \sim 1/\tau^{q/(q+2-K)}}$$

- Can also be applied to bosons in periodic potentials, modifying results from 1 to q bosons per potential minimum.

Some relevant references

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