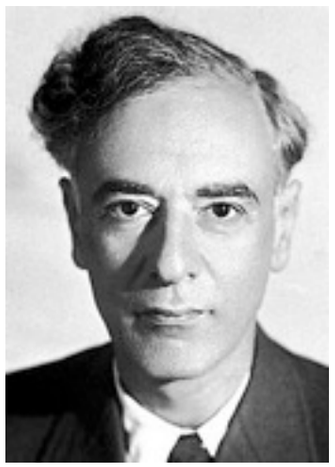
?

The Kibble-Zurek Problem:

Scaling & Universality at Landau & non-Landau transitions

with F. Burnell, A. Erez, S.S. Gubser, V. Khemani, S.L. Sondhi



PRB, 86, 064304 (2012)

Outline

- Kibble-Zurek (KZ) Problem
  - Set up in Landau transition
  - Scaling limit
  - Universal content
- Port to non-Landau transition
- Z_2 gauge theory in (2+1)
- Irrelevant → dangerously irrelevant in KZ
- Analogies in Levin-Wen models

The Kibble-Zurek Problem

- Slow ramps near critical points

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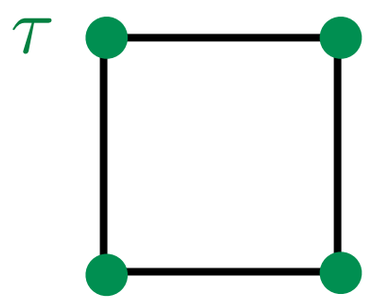
- Critical point



$$-H = J \sum_{ij} \tau_i^z \tau_j^z + \Gamma_M \sum_i \tau_i^x$$

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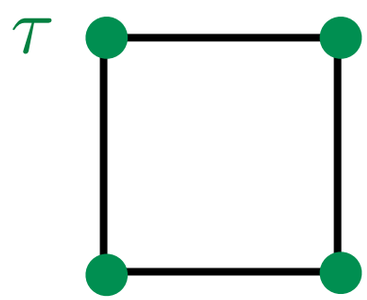

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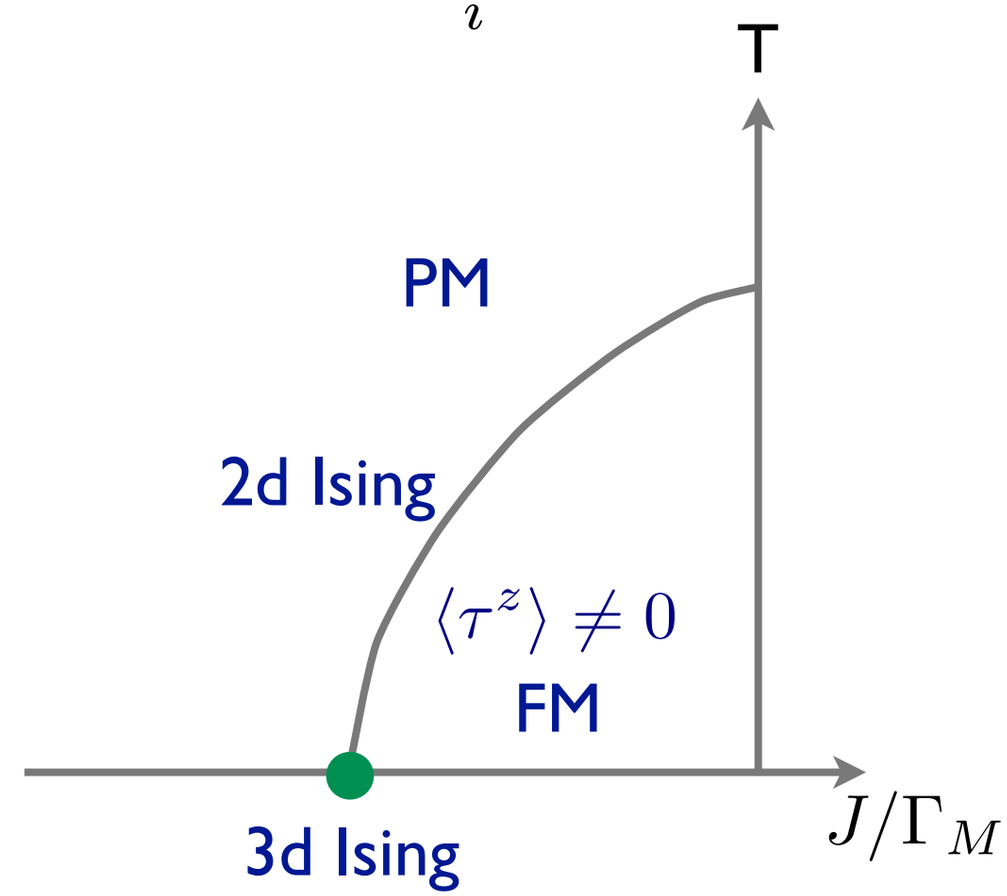

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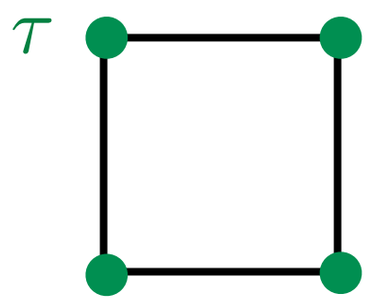
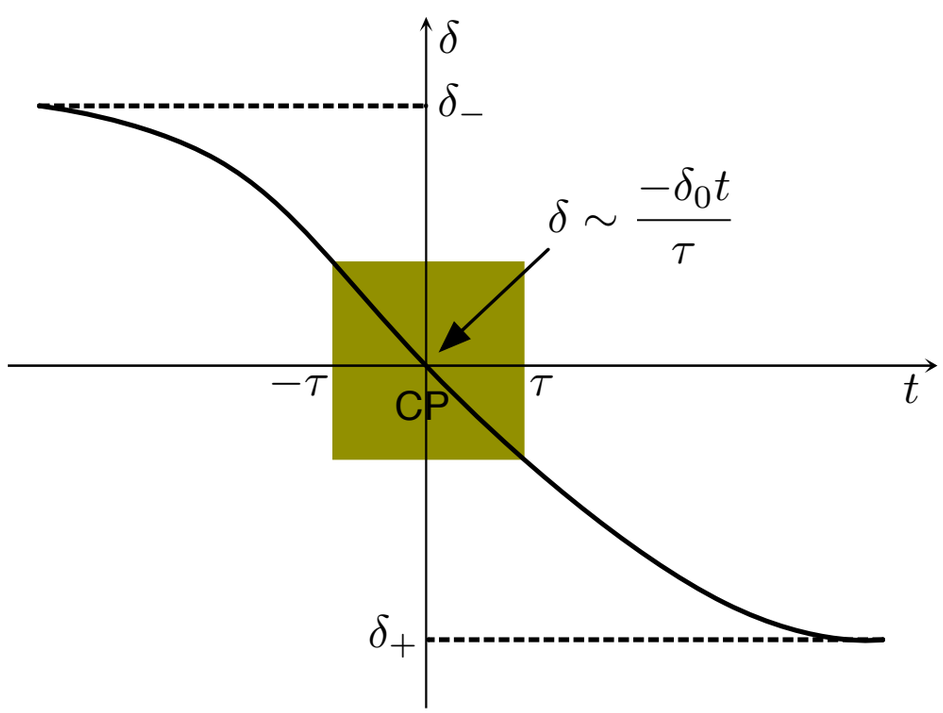
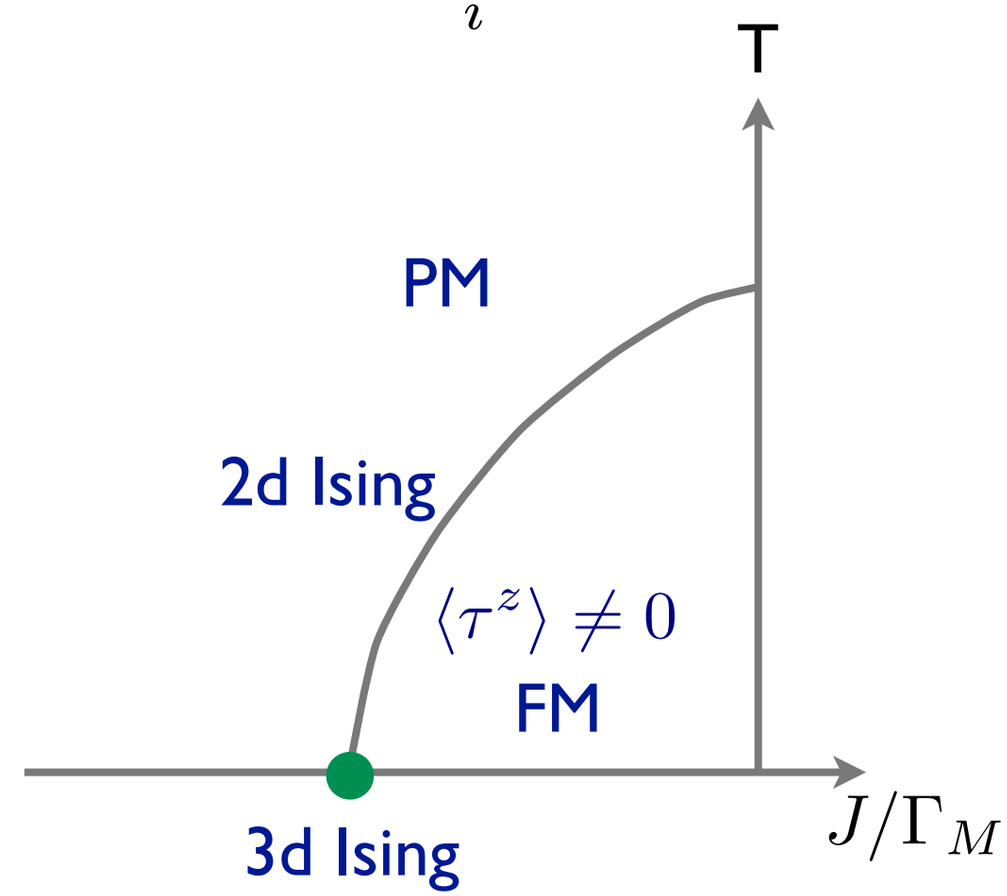


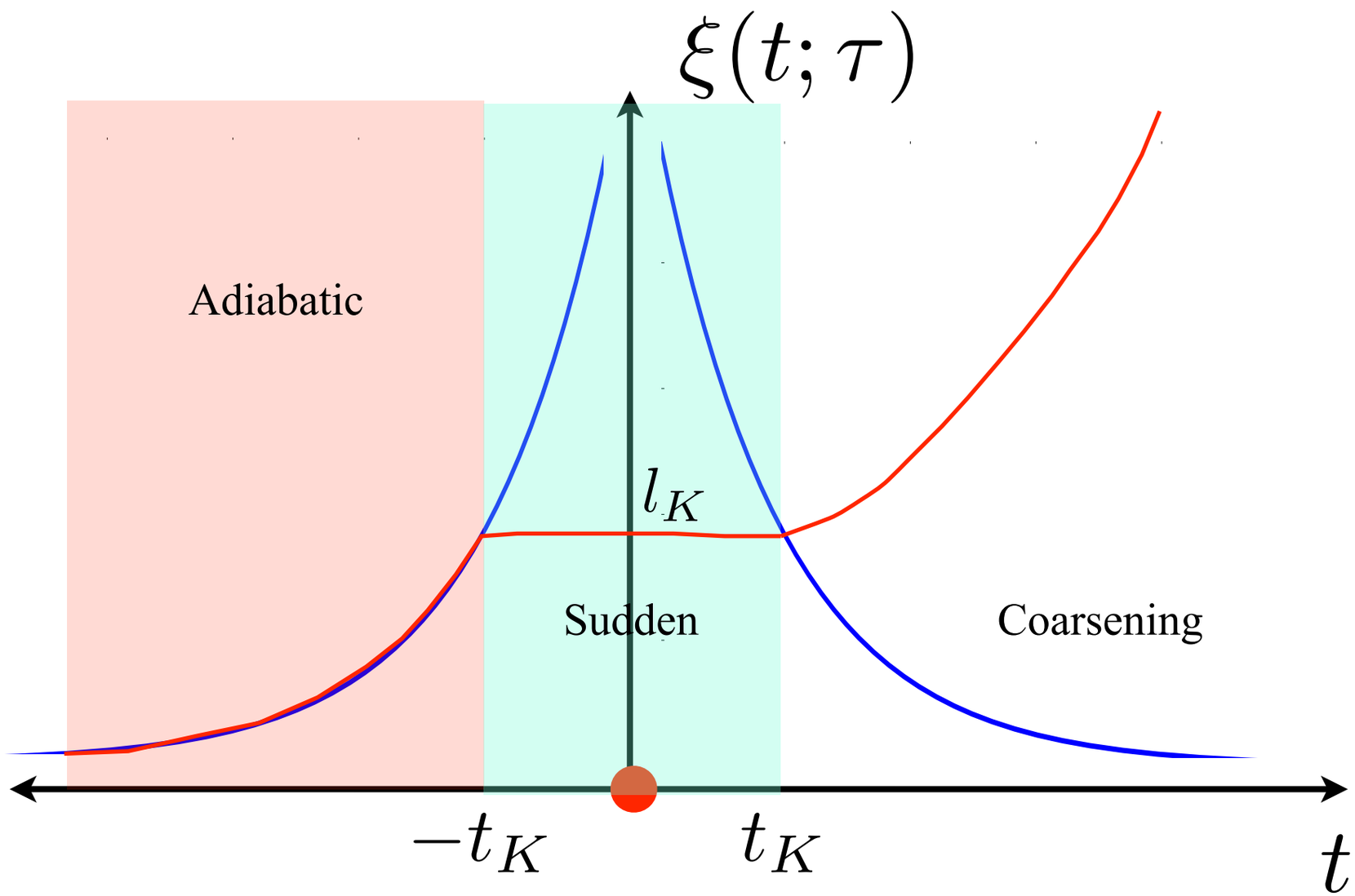
$$-H = J \sum_{ij} \tau_i^z \tau_j^z + \Gamma_M \sum_i \tau_i^x$$

- Ramp $\delta \equiv \Gamma_M / J - (\Gamma_M / J)_c$





Universal scaling of defects



$$t_K = \tau^{\frac{\nu z}{\nu z+1}}$$

$$l_K = t_K^{1/z}$$

τ = Ramp time

Kibble, JPA 9, 1387 (1976)
Zurek, Nature 317, 6037 (1985)

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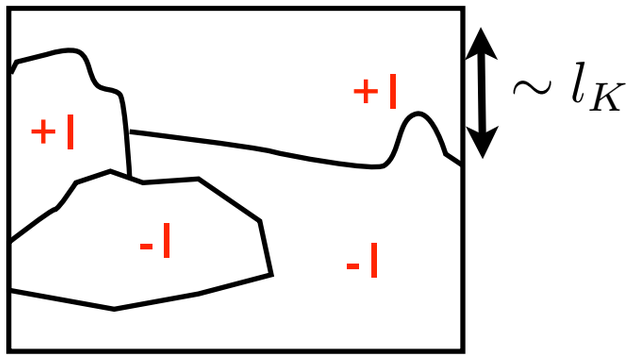

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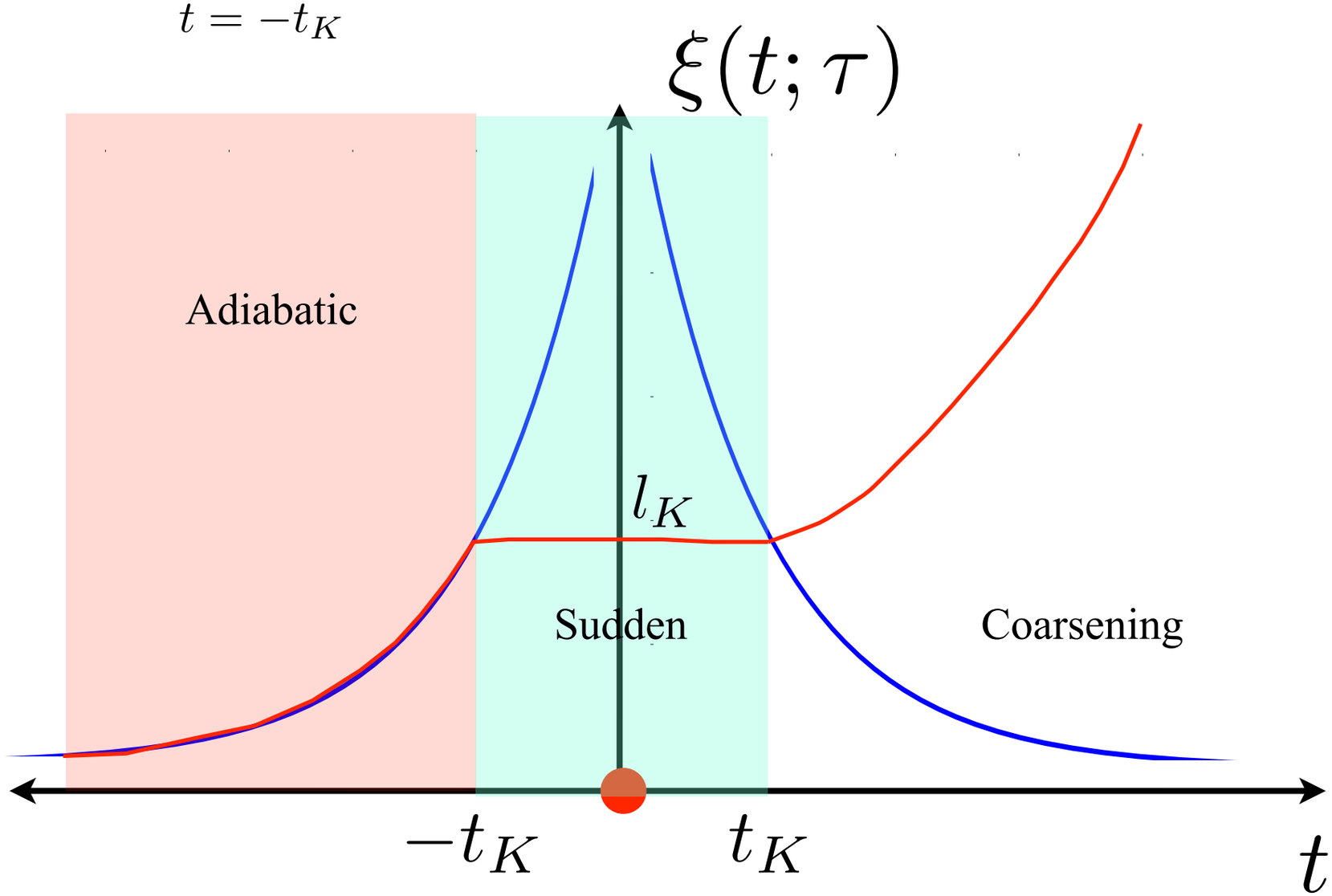

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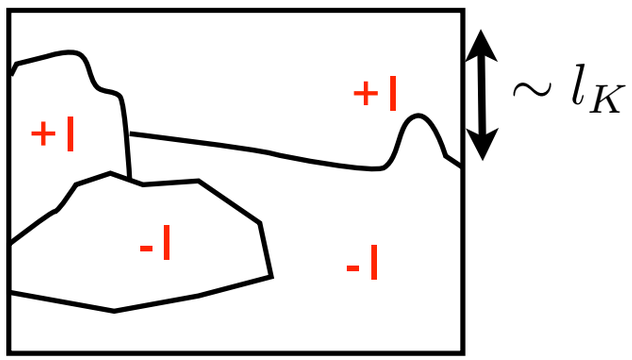

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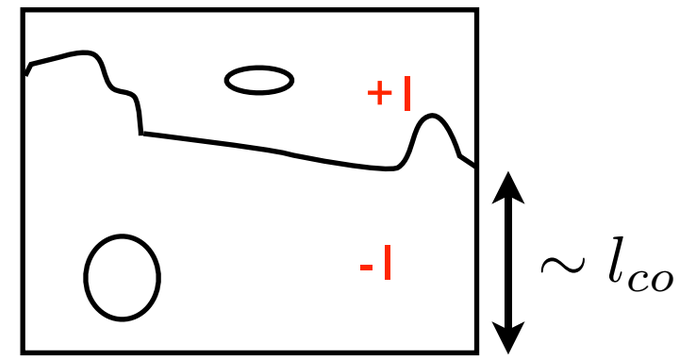
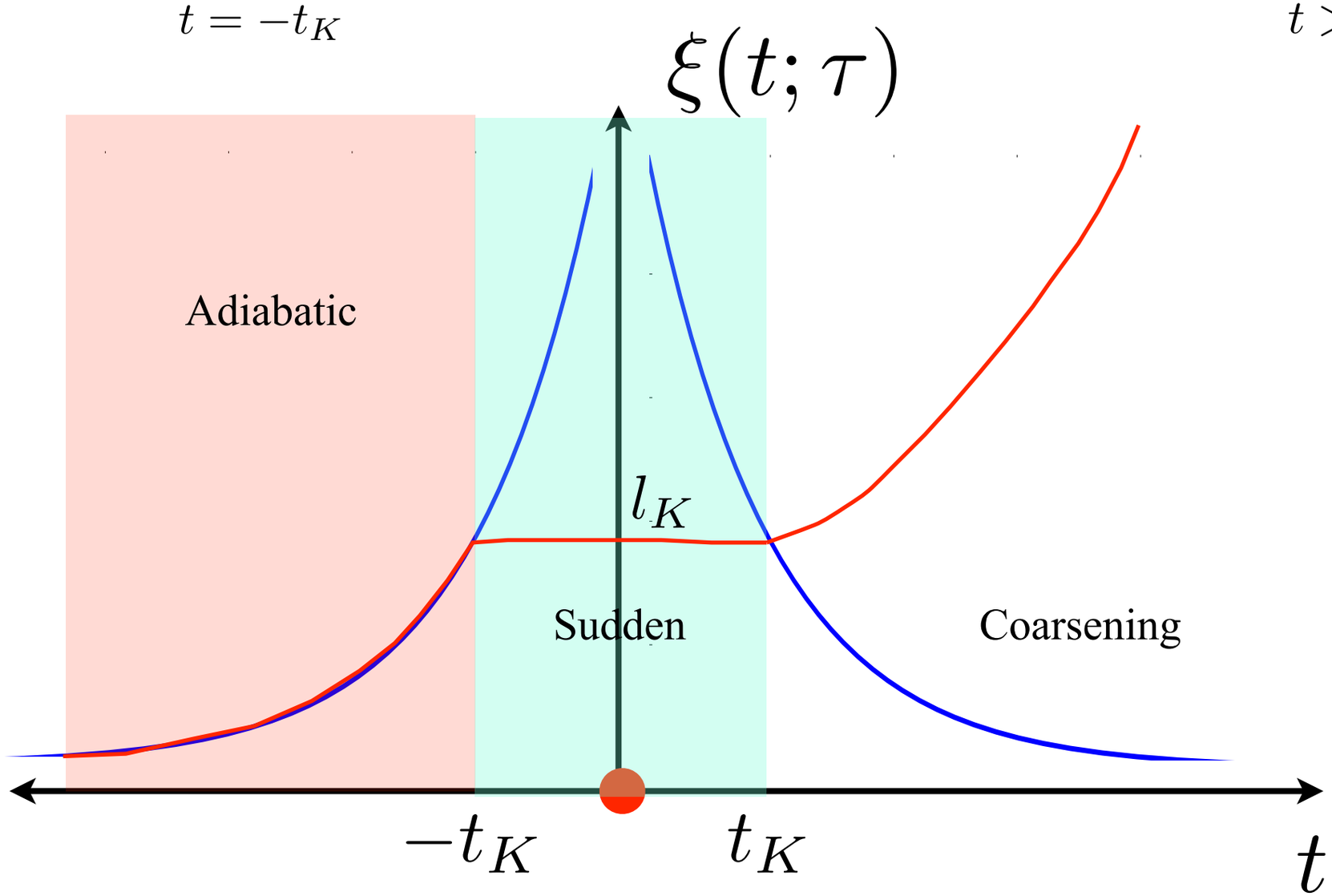


$t \gg t_K$



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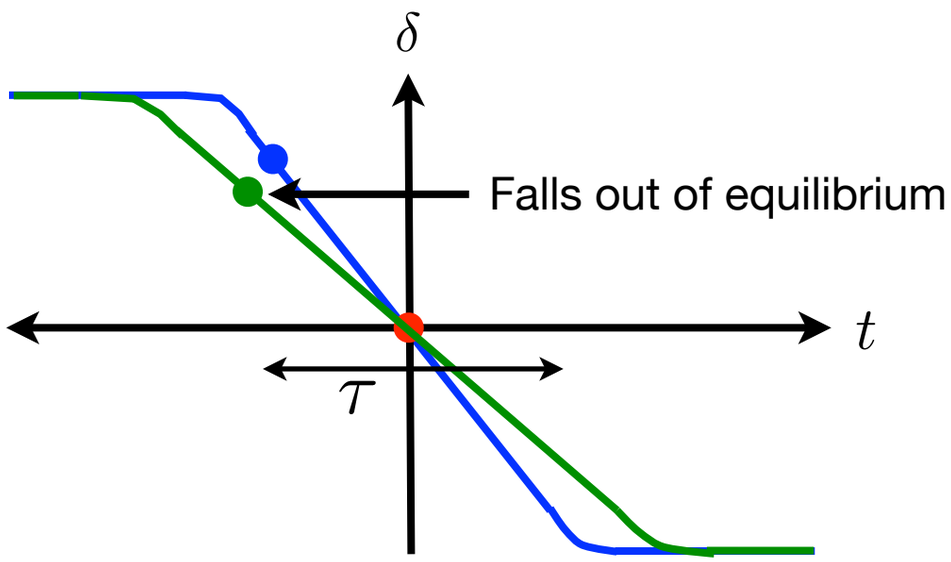
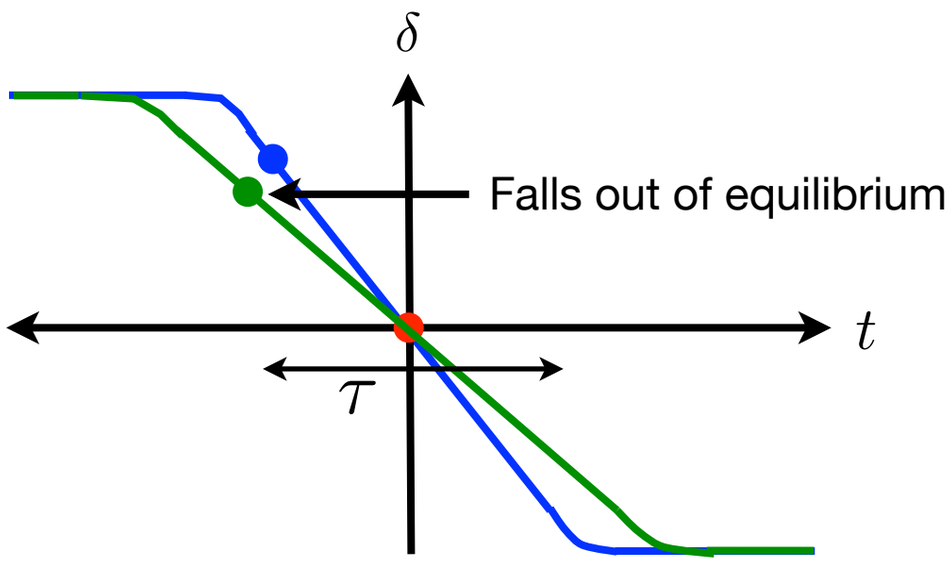


As quench time $\tau \to \infty$

Diverging length and time scale
l_K and t_K

Chandran et al., PRB 86, 064304 (2012)

Universality, scaling



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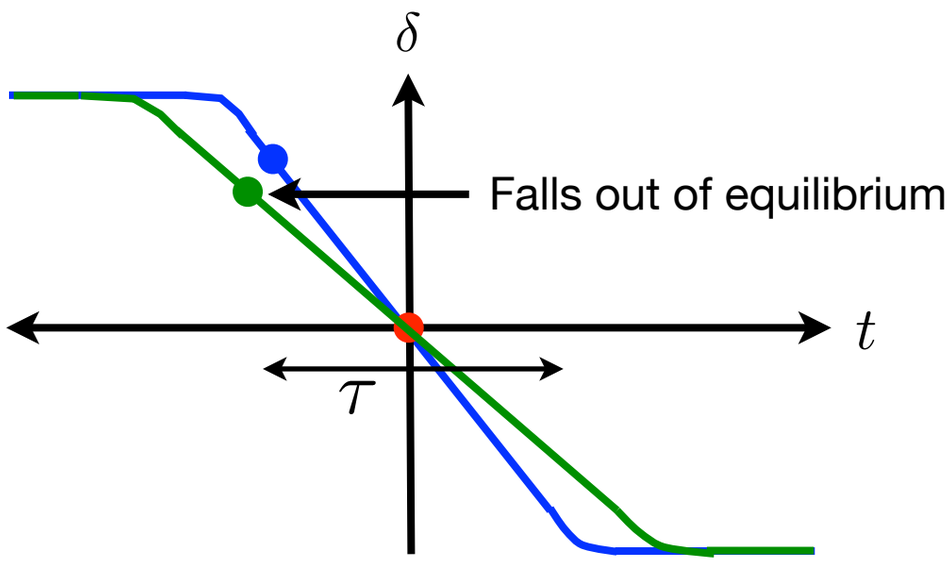

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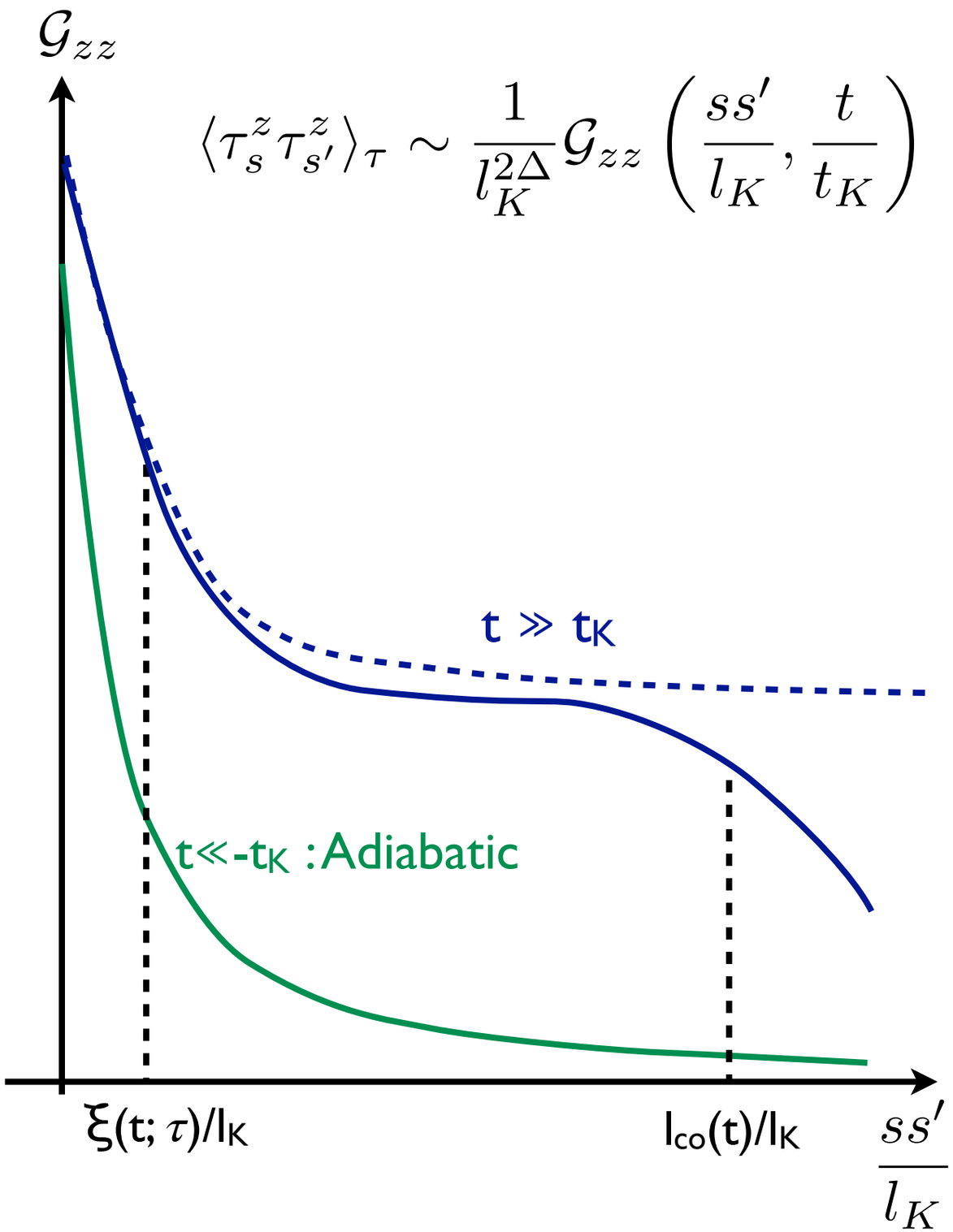
$$\lim_{\substack{\tau \to \infty \\ x/l_K, t/t_K \text{ fixed}}} [\qquad\qquad](x,t;\tau) = \frac{1}{l_K^{\Delta}} f\left(\frac{x}{l_K}, \frac{t}{t_K}\right)$$

Chandran et al., PRB 86, 064304 (2012)

Observables

$$\langle \tau_s^z \tau_{s'}^z \rangle_\tau \sim \frac{1}{l_K^{2\Delta}} \mathcal{G}_{zz}\left(\frac{ss'}{l_K}, \frac{t}{t_K}\right)$$



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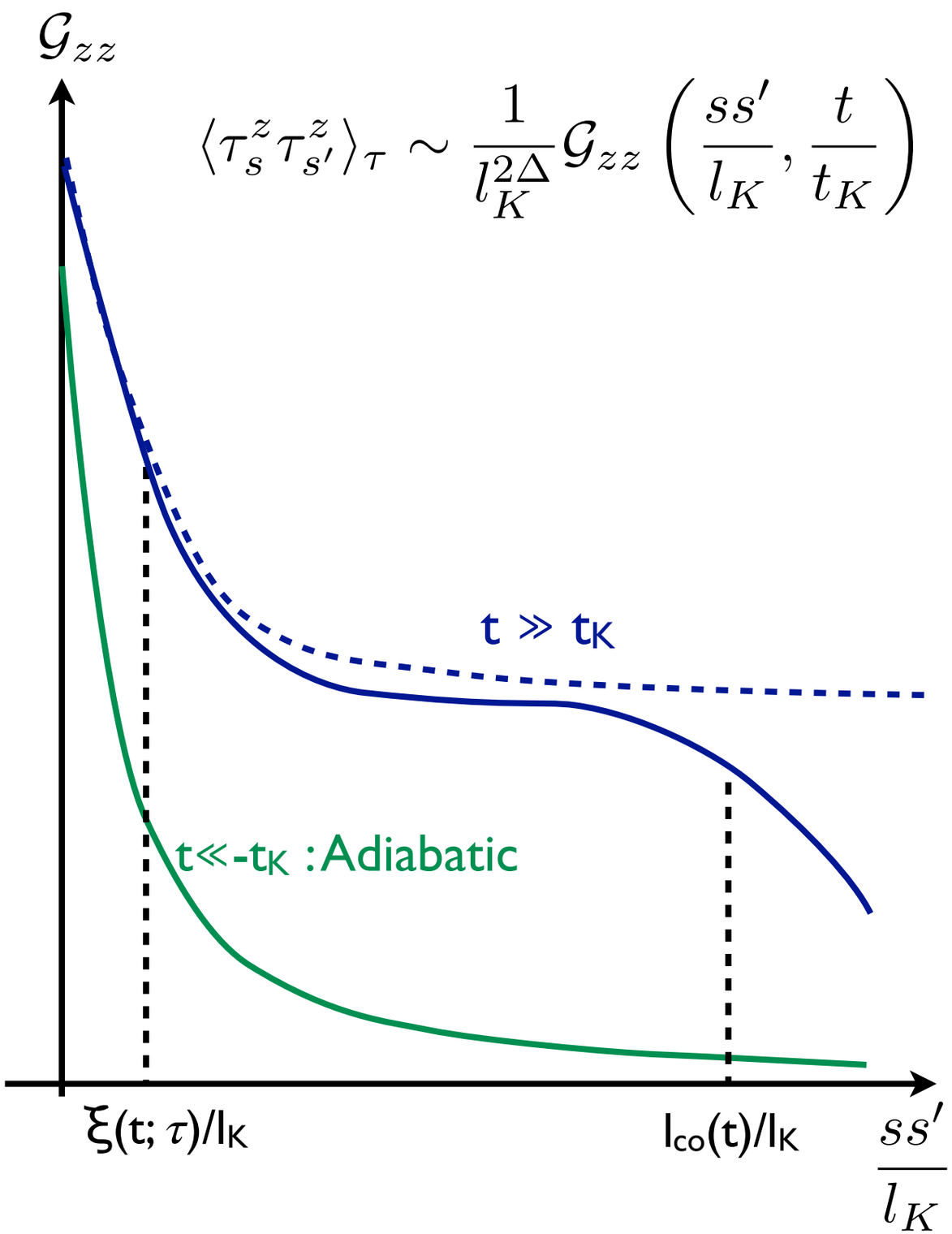

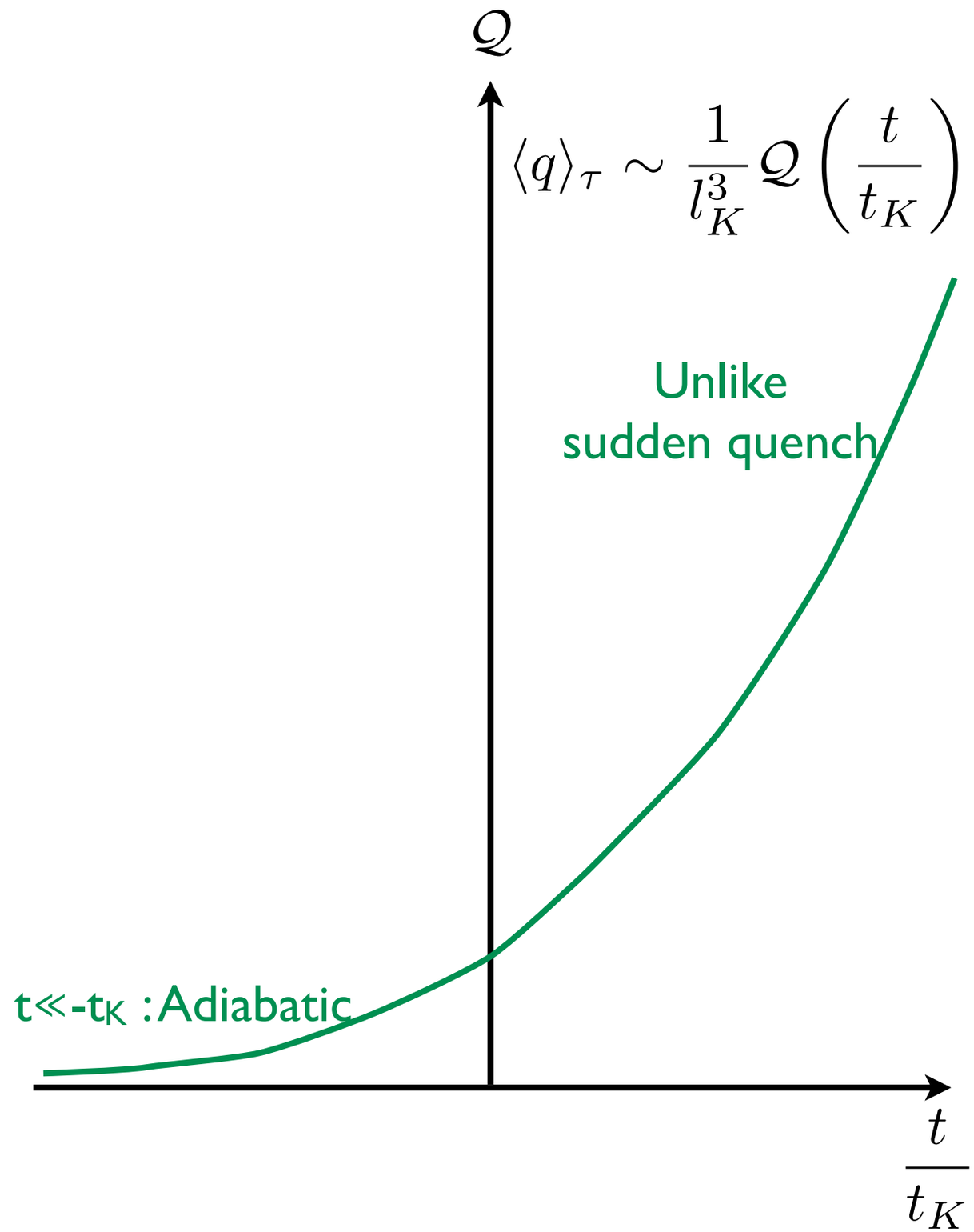

Comments

- Dynamical scaling hypothesis + protocol
  - KZ universality class
  - Content : scaling functions, exponents ..
- Coarsening
  - Finite temperature ordered phase
  - Model C
    - Order parameter relaxes
    - Energy density conserved

Hohenberg & Halperin, RMP 49, 435 (1977)

Landau → Non-Landau

- Wegner lets us cheat!
- Ising gauge theory on the dual lattice

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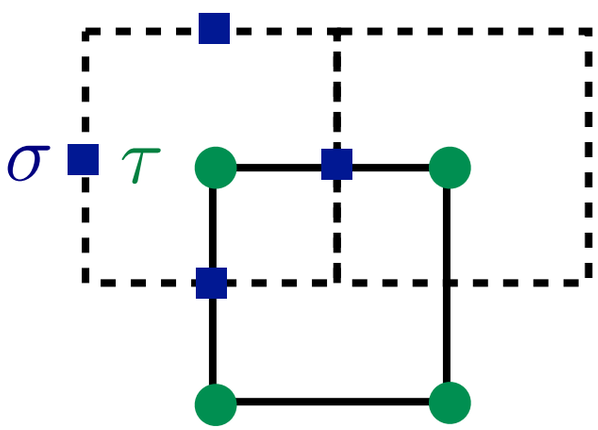

Transverse field FM interaction

$$-H = K \sum_p B_p + \Gamma \sum_l \sigma_l^x$$

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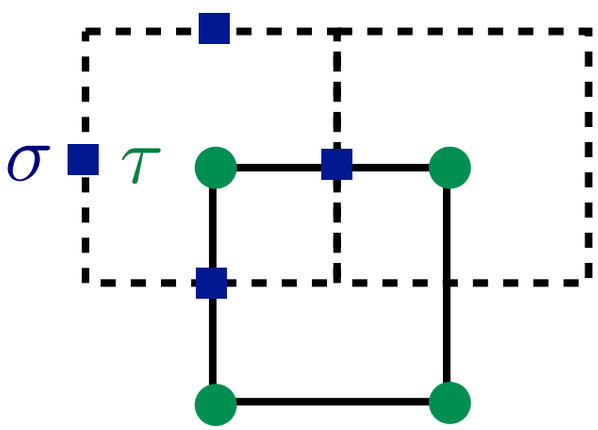

Transverse field FM interaction

$$-H = K \sum_p B_p + \Gamma \sum_l \sigma_l^x$$

$$\sigma_l^x = -1$$

Electric field

$$B_p \equiv \prod_{l \in p} \sigma_l^z$$

$$= -1$$

Vortex

Landau → Non-Landau

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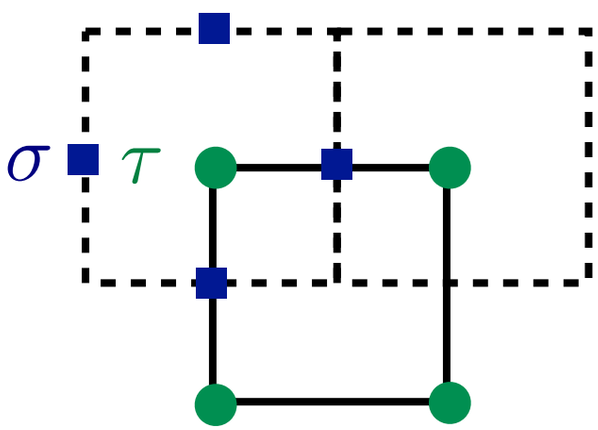

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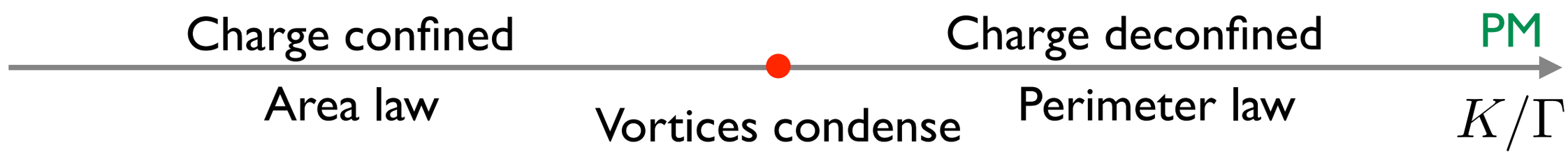
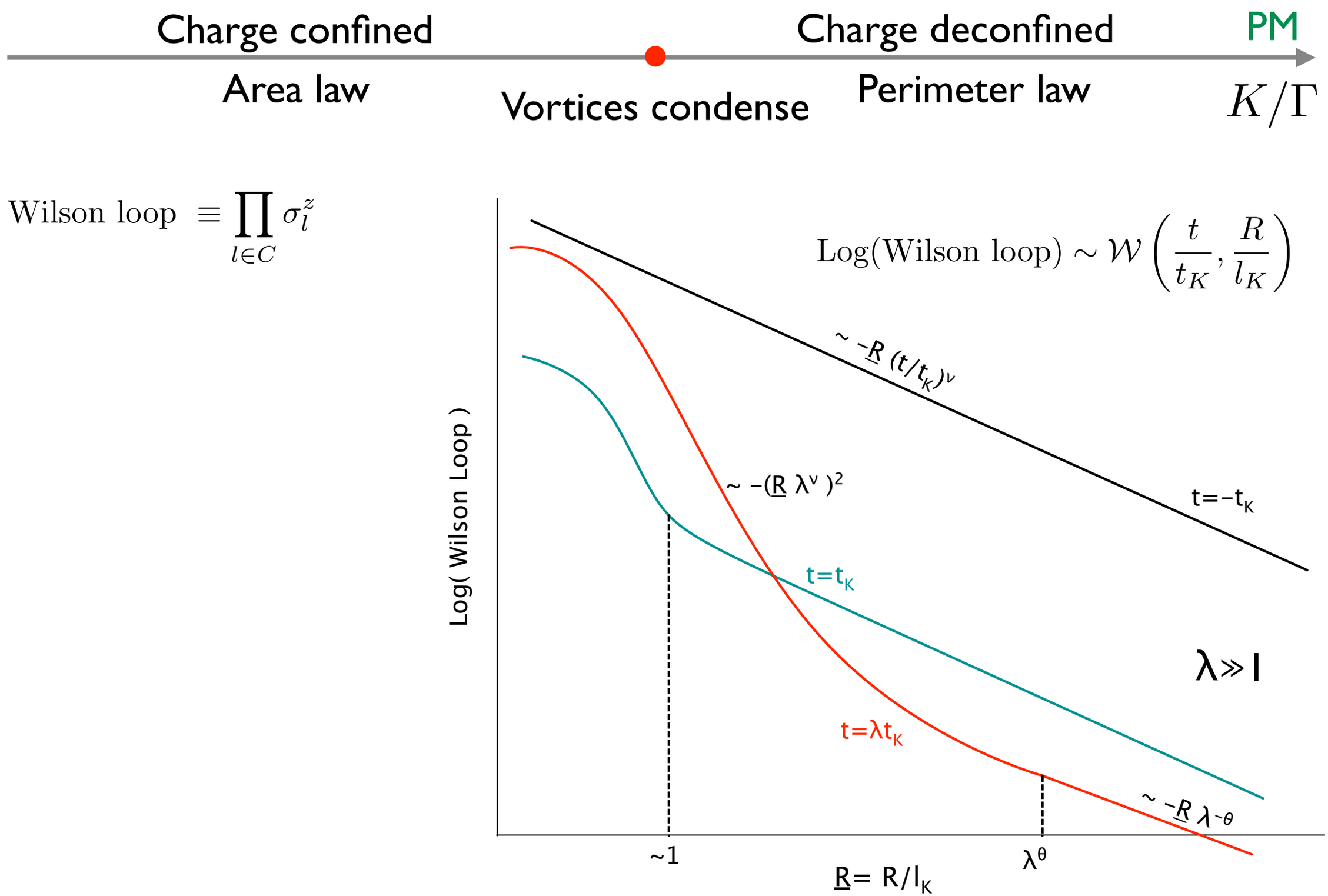
Vortex

- Domain wall $\Longleftrightarrow$ Electric field
- $\tau_i^z\tau_j^z \Longleftrightarrow$ String order parameter $\prod_{l\in C}\sigma_l^x$

Wilson loop


Charge confined
Charge deconfined
PM
Area law
Vortices condense
Perimeter law
K/Γ

Wilson loop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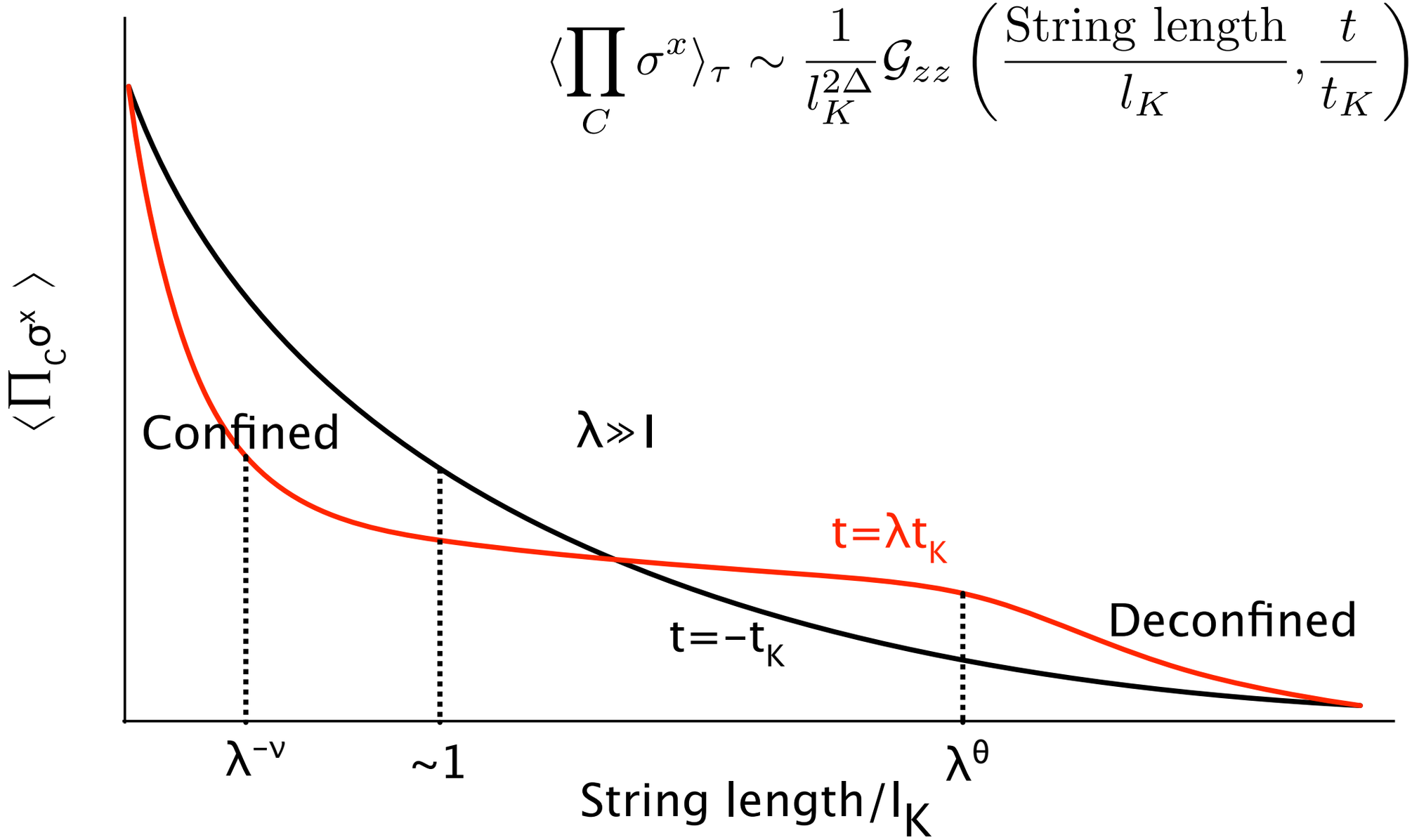
Wilson loop $\equiv \prod_{l \in C} \sigma_l^z$
$\text{Log(Wilson loop)} \sim \mathcal{W}\left(\frac{t}{t_K}, \frac{R}{l_K}\right)$
Log(Wilson Loop)
$\sim -\underline{R}\,(t/t_K)^\nu$
$\sim -(\underline{R}\,\lambda^\nu)^2$
$t=-t_K$
$t=t_K$
$\lambda \gg 1$
$t=\lambda t_K$
$\sim -\underline{R}\,\lambda^{-\theta}$
~1
λ^θ
$\underline{R} = R/l_K$

Electric field coarsening

Creates a pair of vortices at ends of C

$$\langle \prod_C \sigma^x \rangle_\tau \sim \frac{1}{l_K^{2\Delta}} \mathcal{G}_{zz} \left(\frac{\text{String length}}{l_K}, \frac{t}{t_K} \right)$$



Topological degeneracy?

Confined/Trivial

K/Γ

Topological degeneracy

Charge-flux statistics

-Banksey

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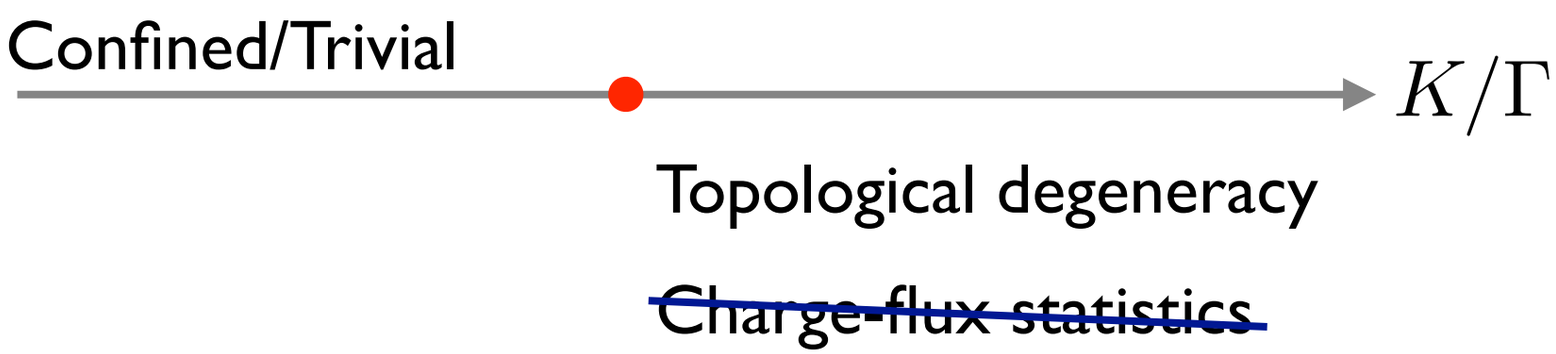
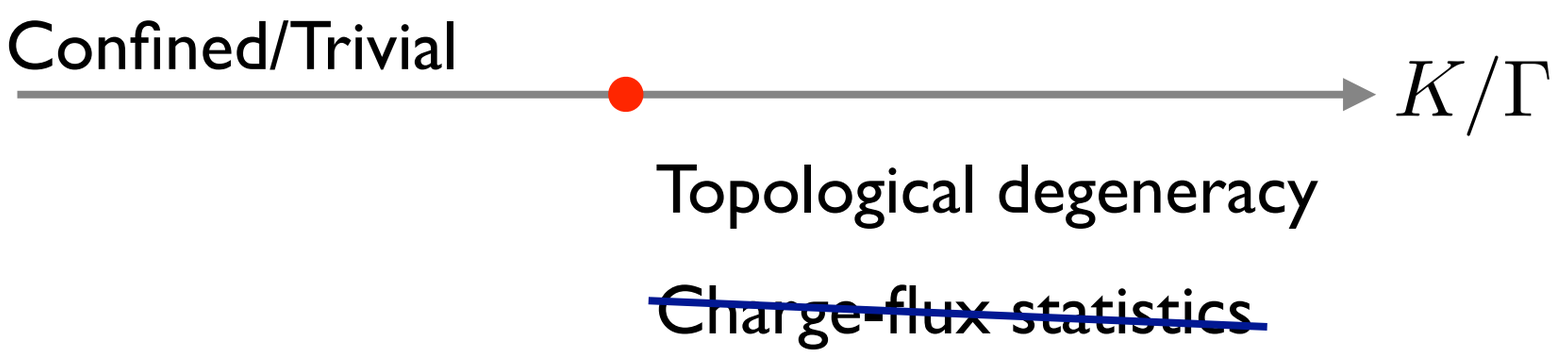


-Banksey

- Each topological sector ➛different TFIM
- Deconfined phase : exp(-Δ/L) energy splitting
- $L \gg l_K, l_{co}$ ..

Topological degeneracy?



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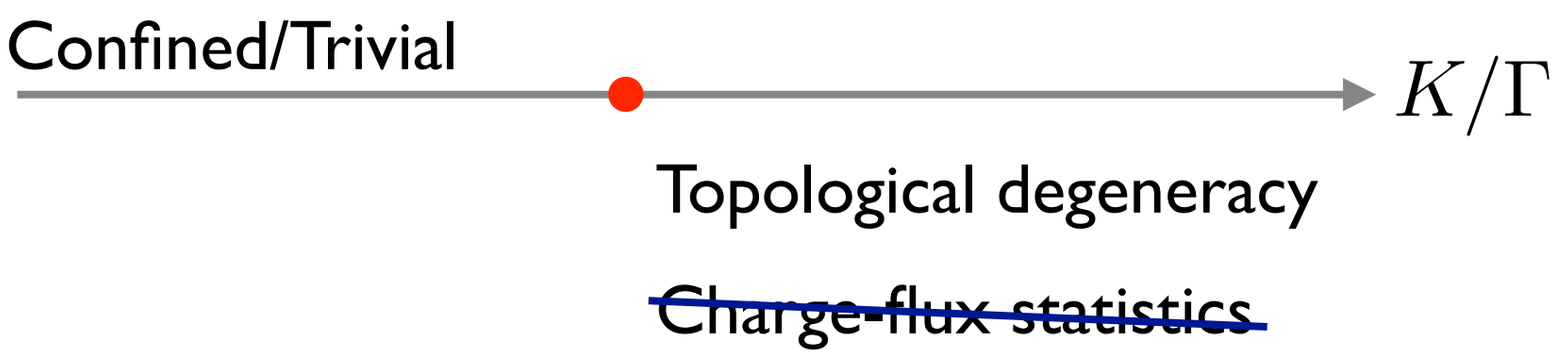
$$q(t;\tau) \sim \frac{L^2}{l_K^3}\left(\frac{t}{t_K}\right)^{\alpha} + \frac{L}{l_K}\left(\frac{t}{t_K}\right)^{\nu} + \ldots$$

Topological degeneracy?



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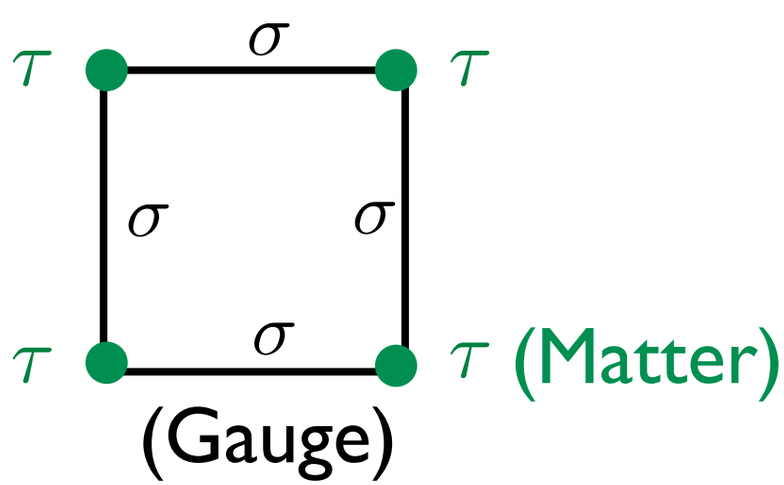
- Window for O(L) term to beat O(L^2)

Z_2 lattice gauge theory



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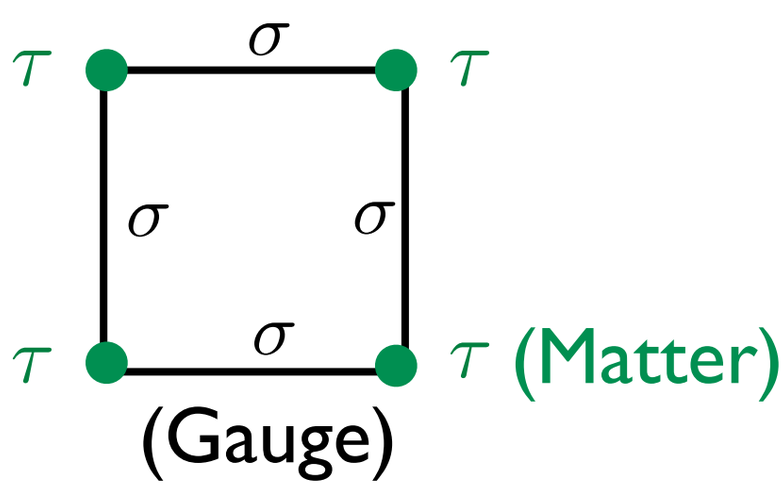

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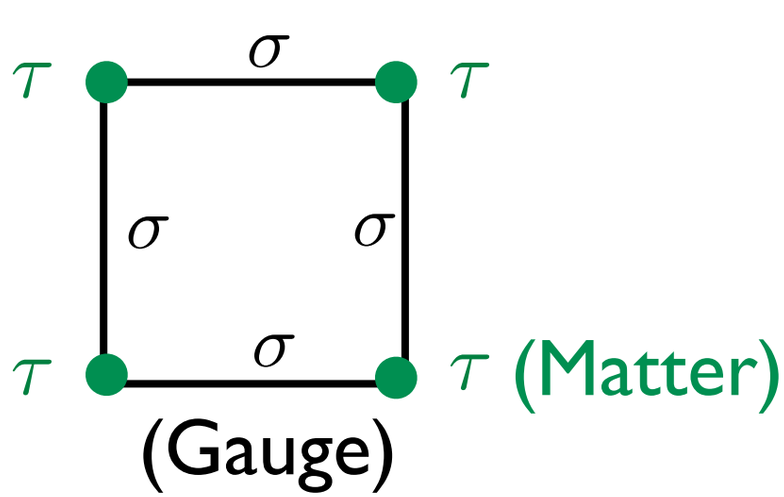

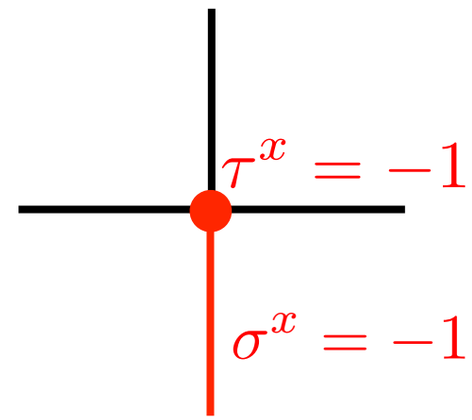

Electric

$$\boxed{\prod \sigma^z} = -1$$

Vortex

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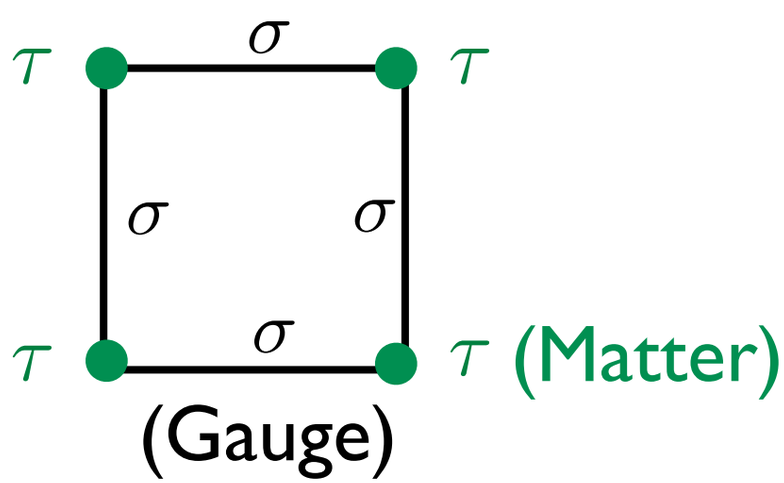

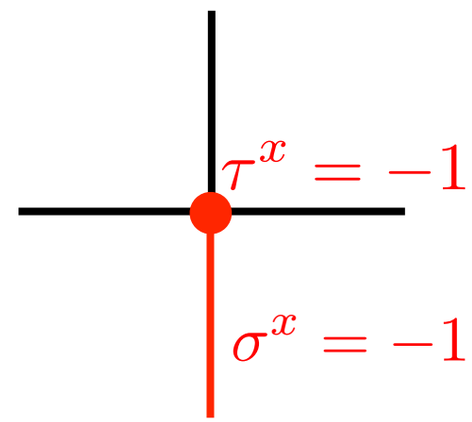

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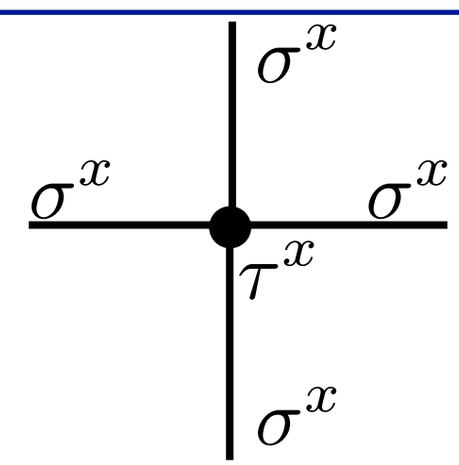
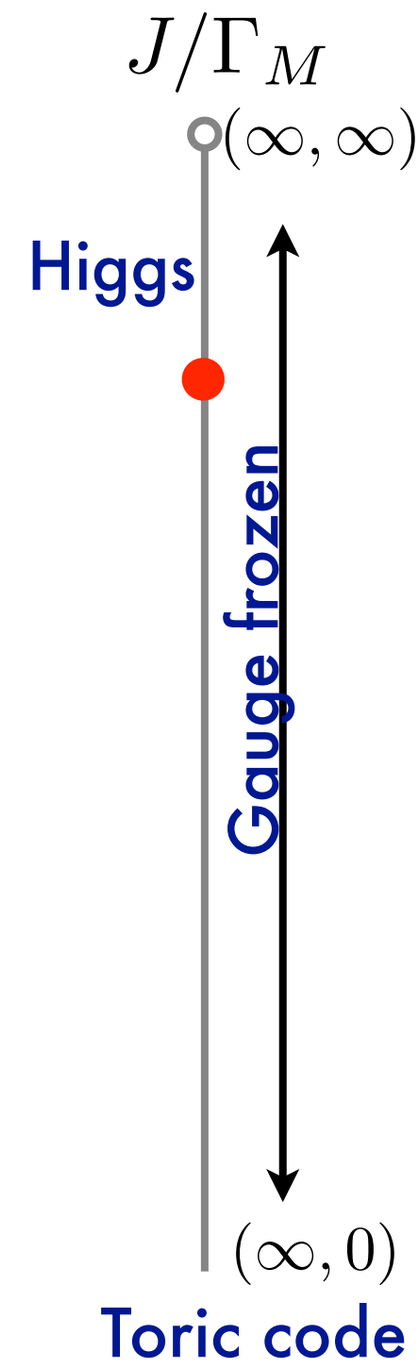
"∇.E = 0"



$| \text{phys} \rangle = | \text{phys} \rangle$

Phase diagram (T=0)



Fradkin & Shenker, PRD 19, 3682 (1979)

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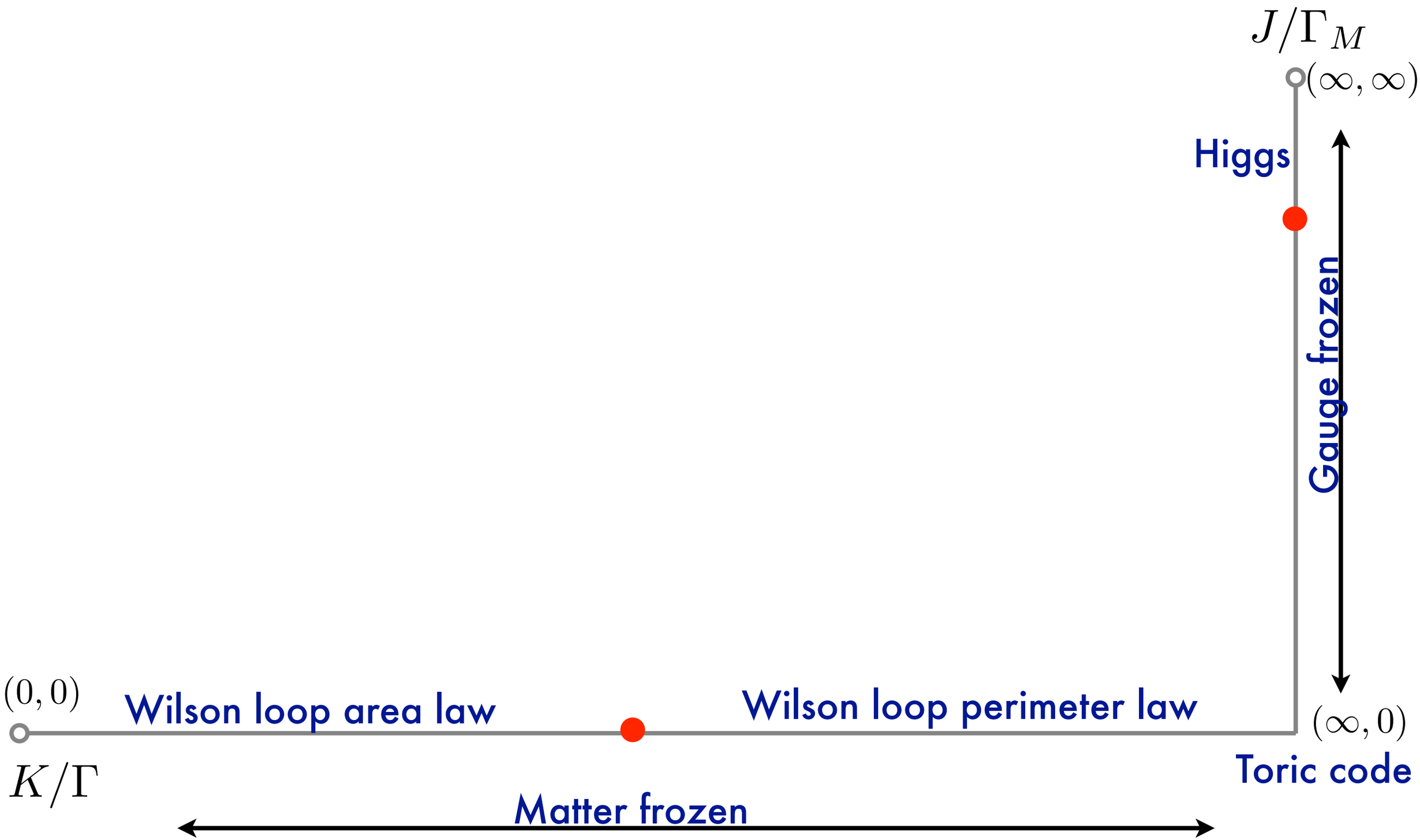

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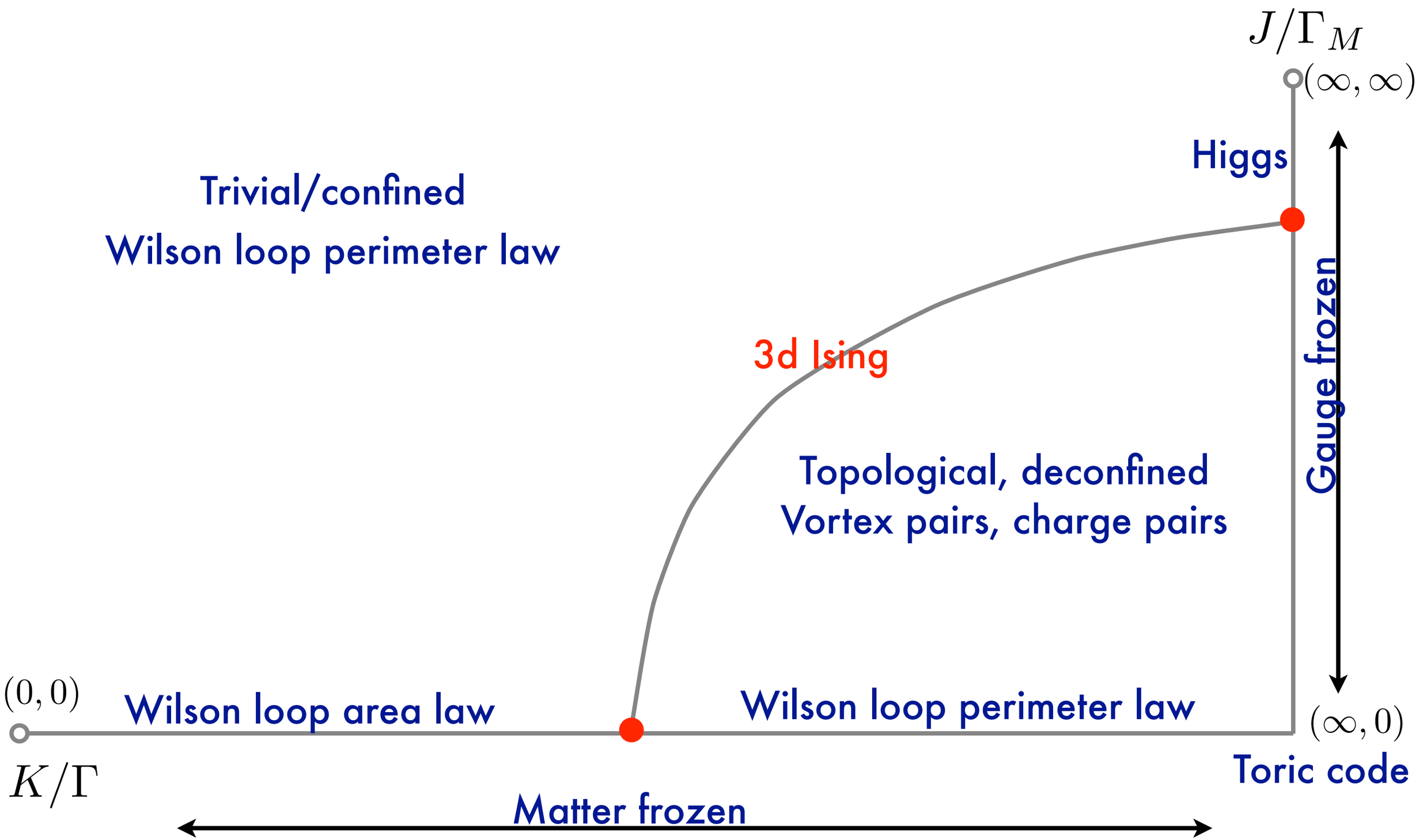

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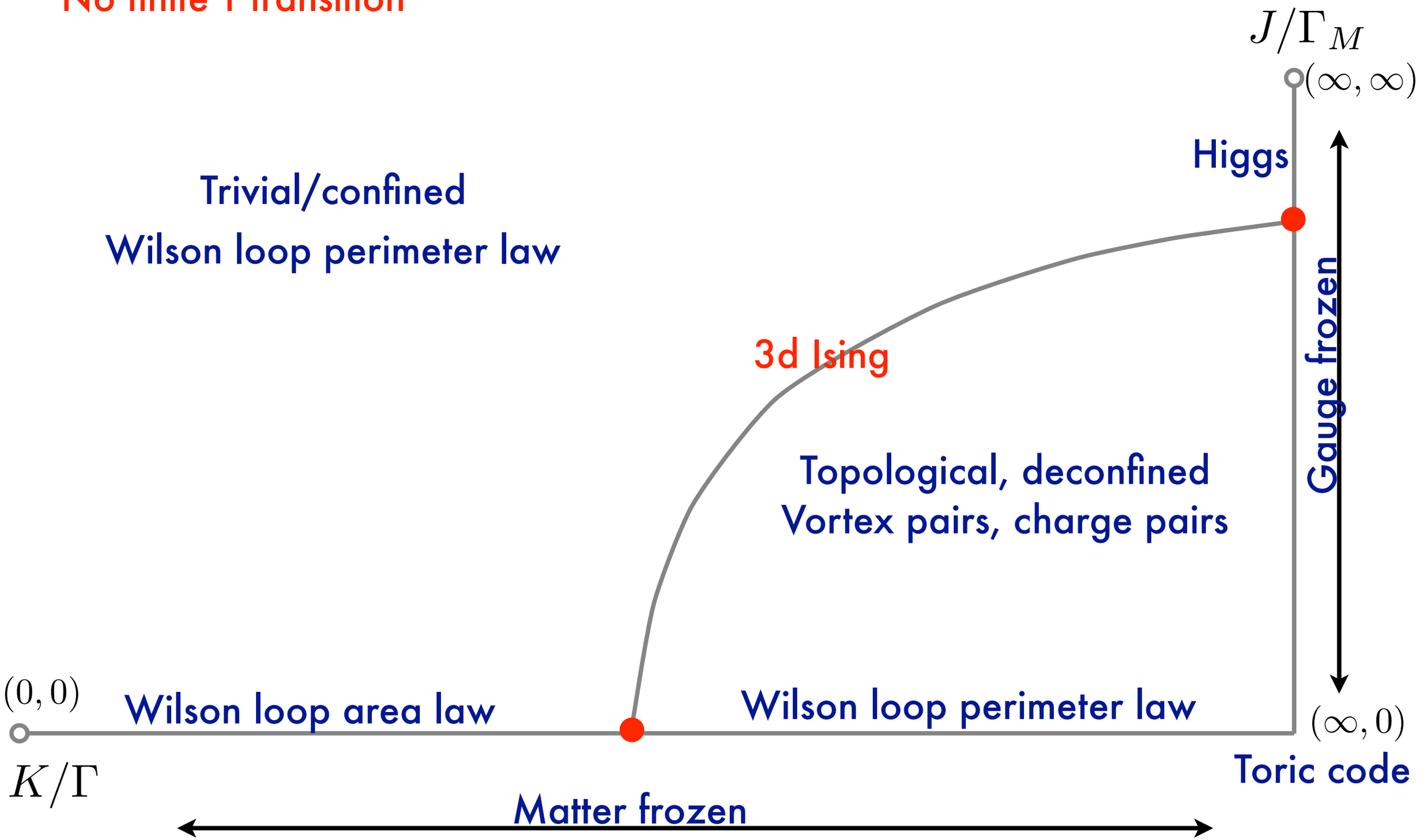
Phase diagram (T=0)

No finite T transition



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Moving inwards

- Coupling to matter RG irrelevant to T=0 transition

Hastings, Wen PRB 72, 045141 (2005)

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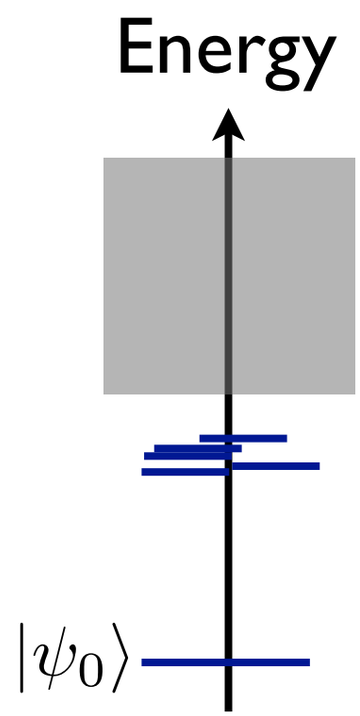

Confined, J=0

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Hastings, Wen PRB 72, 045141 (2005)

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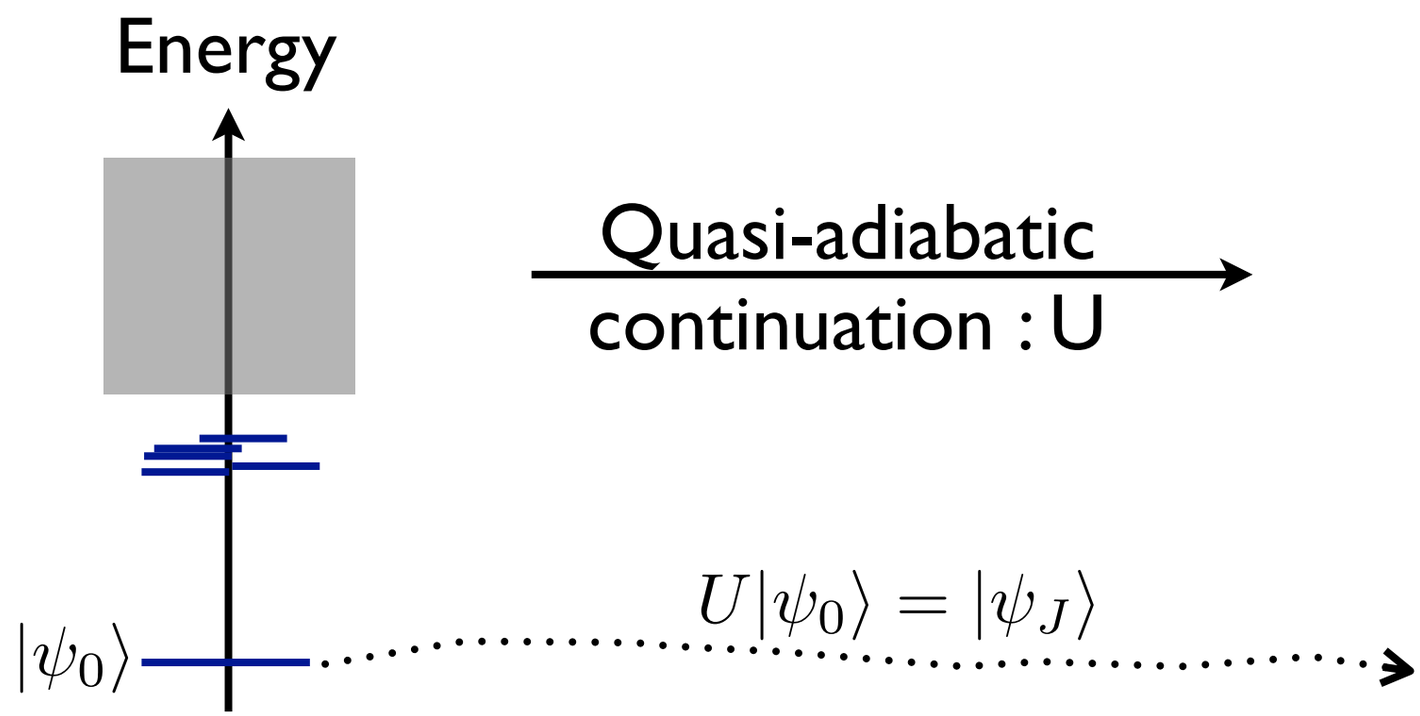

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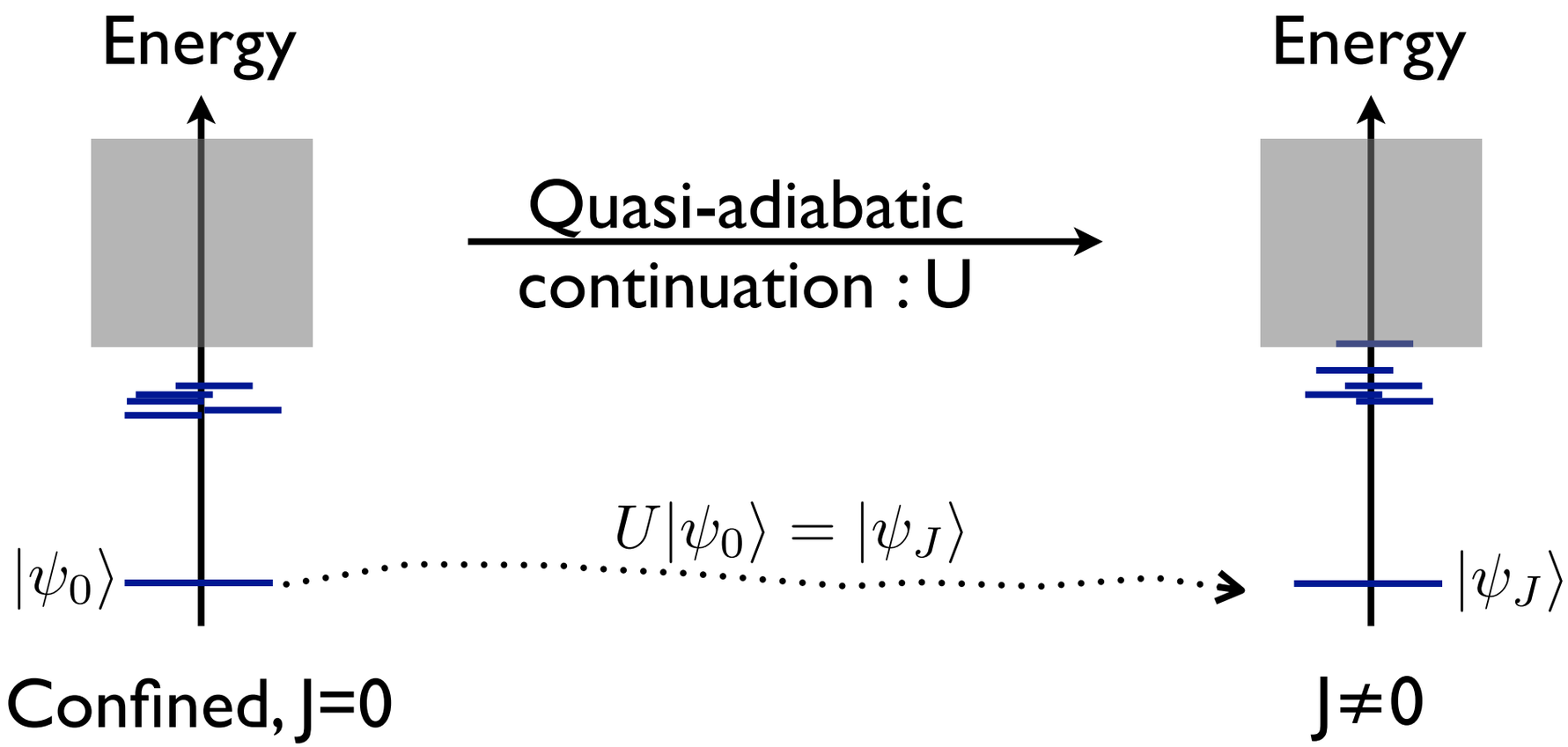
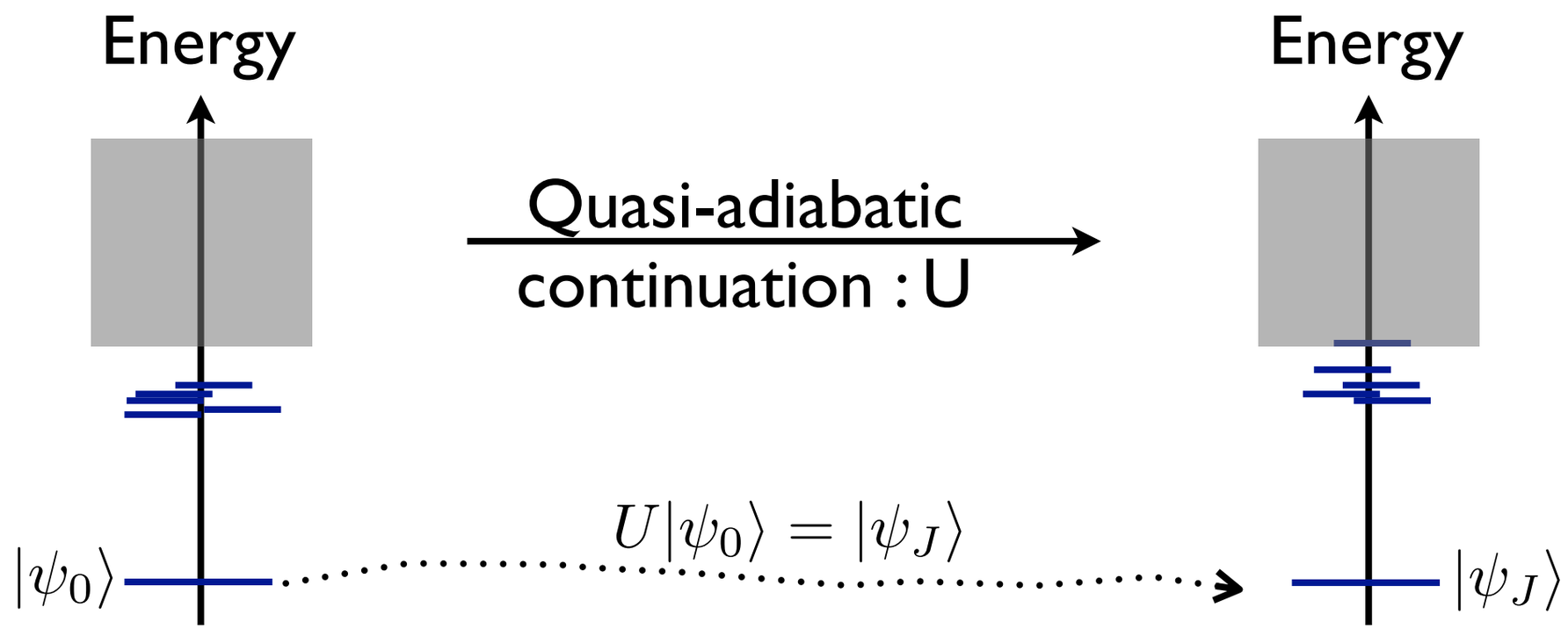


$\langle\psi_0|\tau_s^x|\psi_0\rangle = 1$

Hastings, Wen PRB 72, 045141 (2005)

Moving inwards

- Coupling to matter RG irrelevant to T=0 transition



Confined, J=0

$\langle\psi_0|\tau_s^x|\psi_0\rangle = 1$

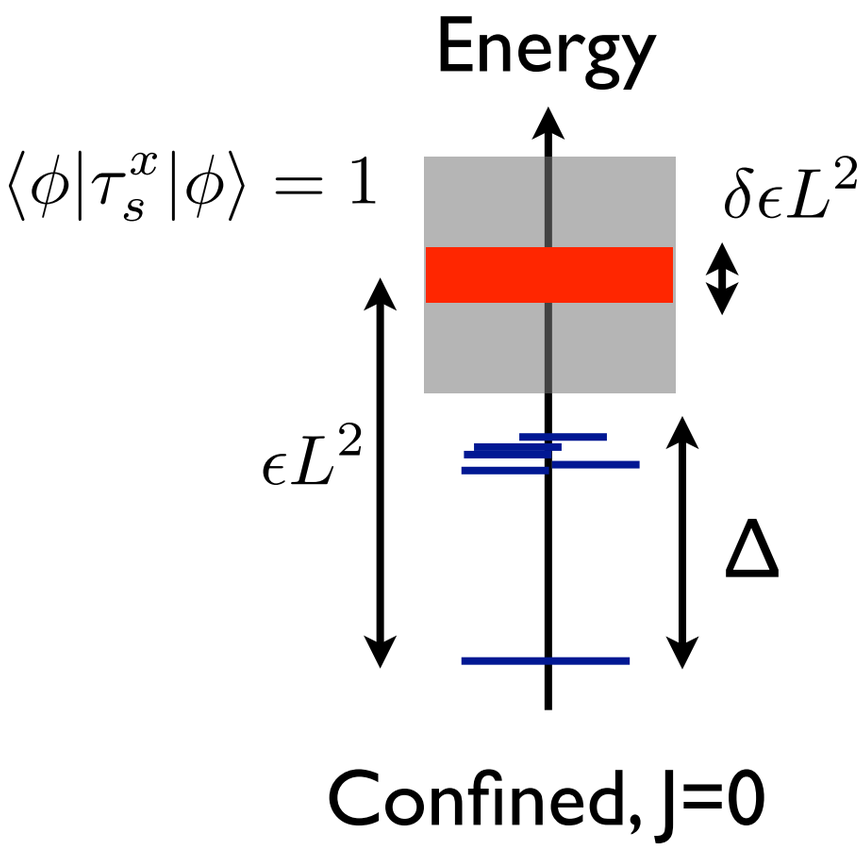
J≠0

$\langle\psi_0|U^\dagger U\tau_s^x U^\dagger U|\psi_0\rangle = 1$

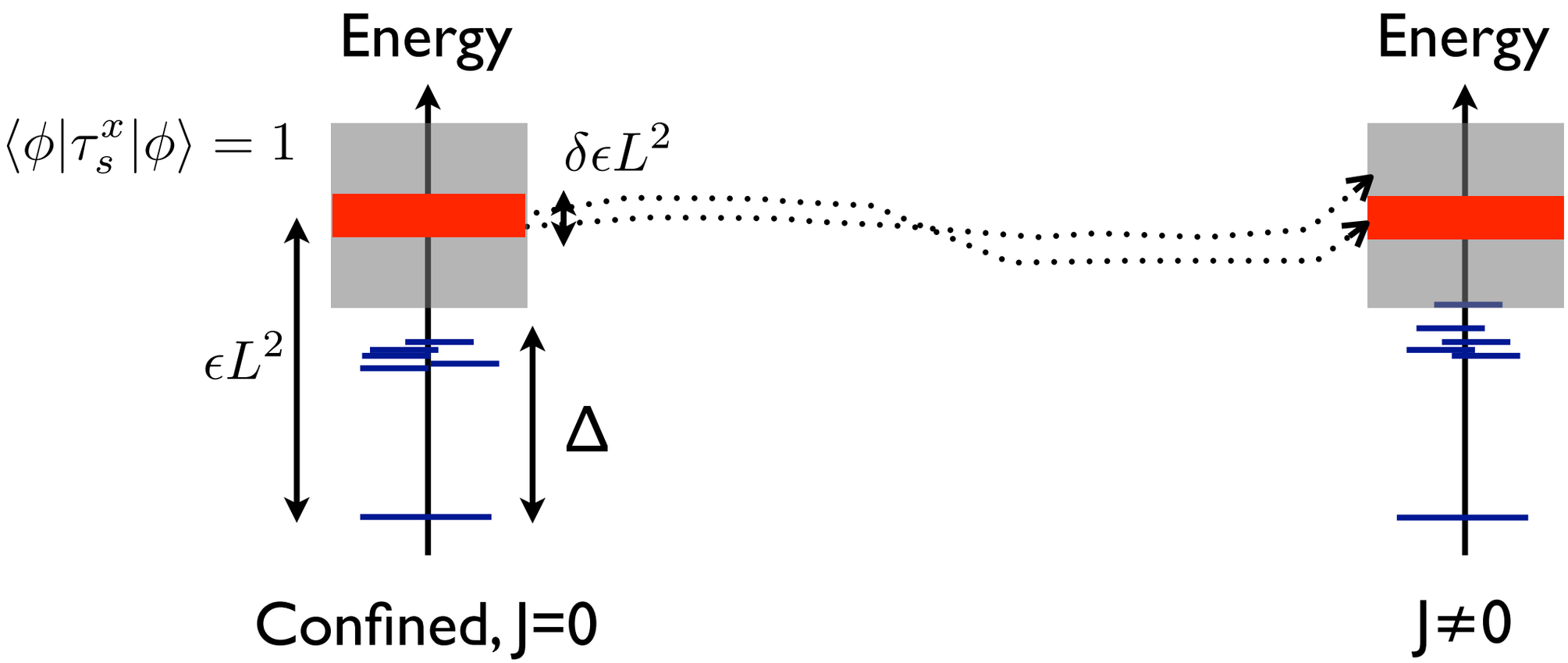
- Defines local dressed operators : “charge”,“vortex” ..

Hastings, Wen PRB 72, 045141 (2005)

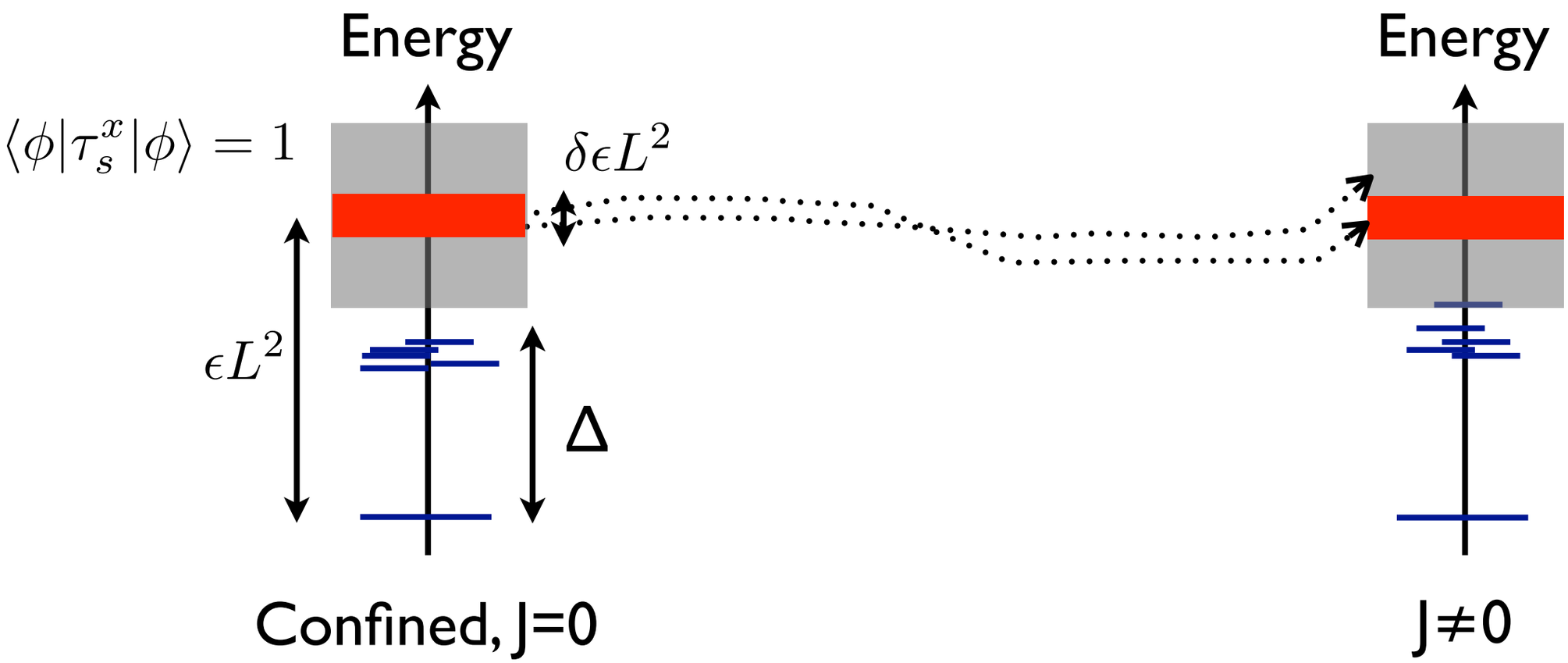
Excited states?



Excited states?



Excited states?

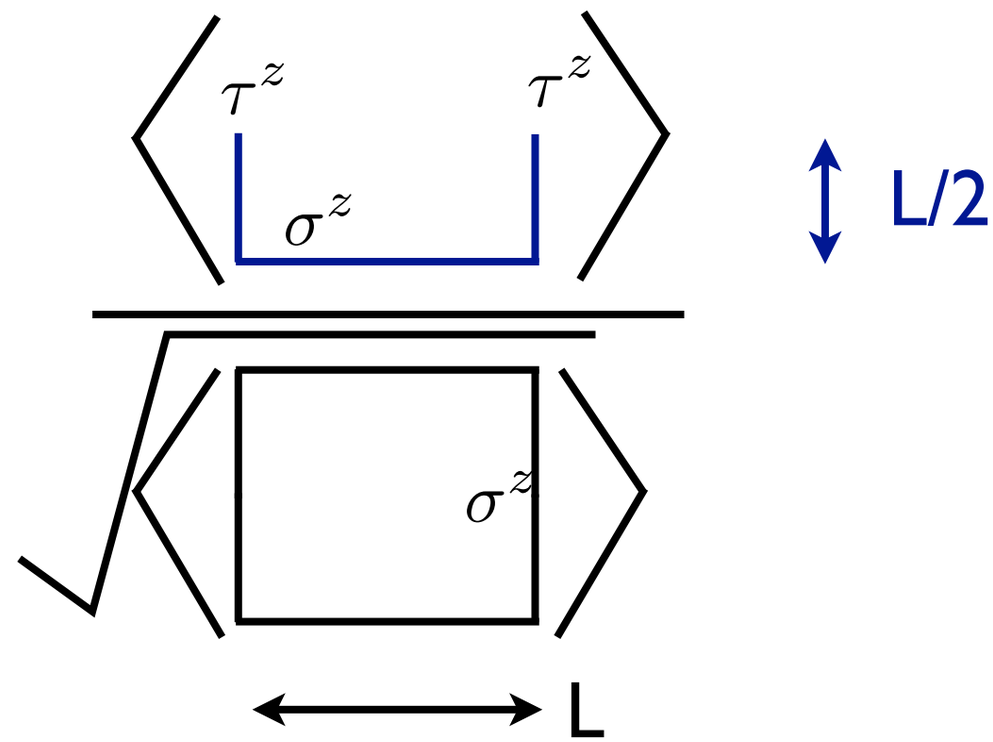


- Freedom in U ⇒ different local "charge"
- No finite T transition
  - "Charge" density ~ exp(-Δ/ε)

Dangerously irrelevant

- Coupling to charge irrelevant for T=0
- T≠0 : destroys confined phase
- Ramp through transition
  - Charge on scale $\ell \sim \exp(\Delta\, l_K / (t/t_K)^{\alpha})$
  - Coarsening stops when $l_{co} \sim \ell$. Thermalization?
  - ℓ / l_K infinite. Outside scaling limit
  - KZ story identical!

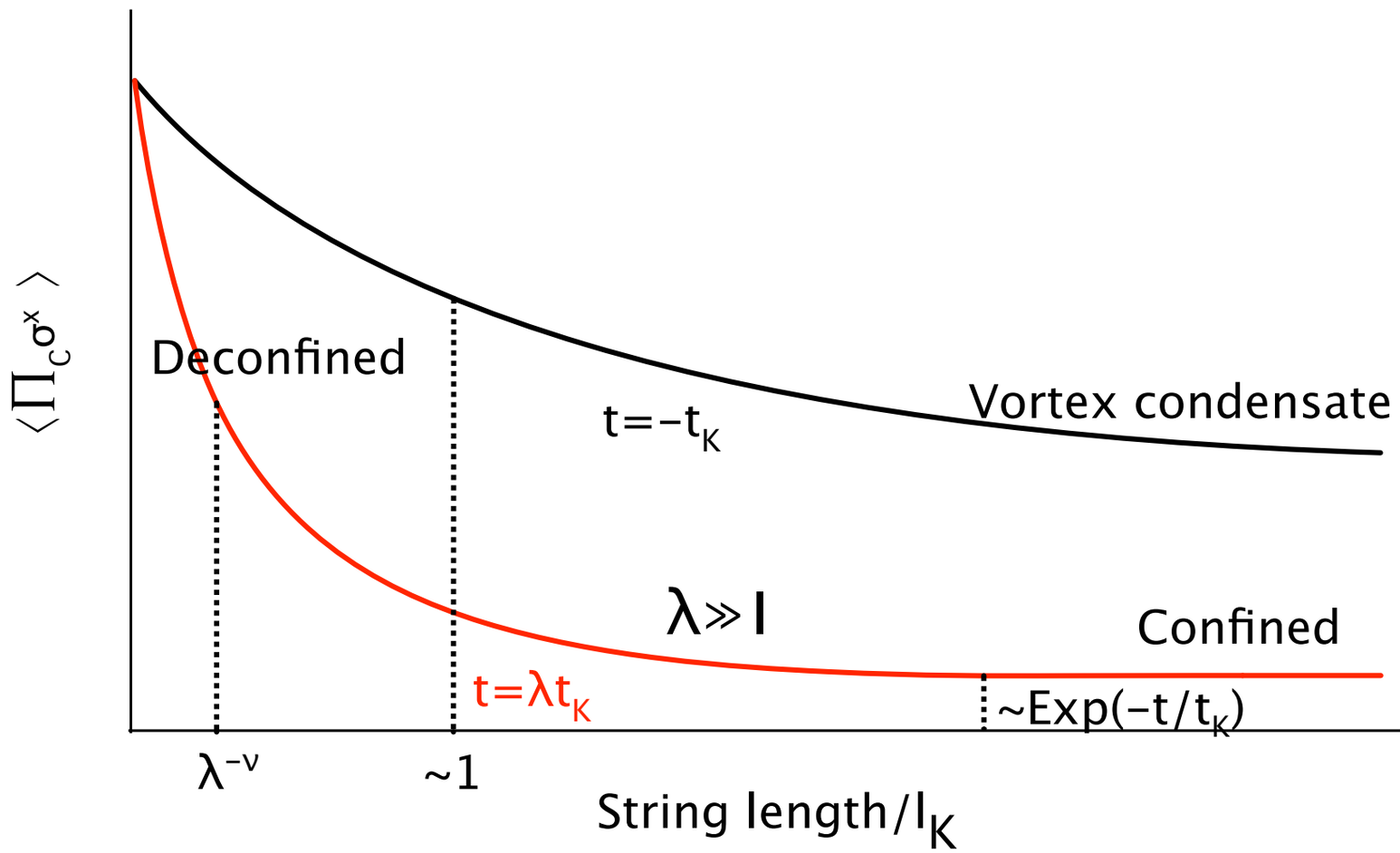
Frendenhagen-Marcu (FM)

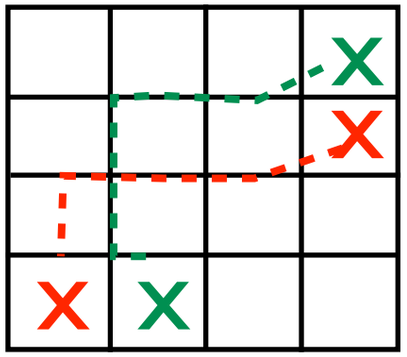
- Observables Hamiltonian dependent
- FM order parameter - Hamiltonian independent
- 

- $L \to \infty$
  - Deconfined : 0
  - Confined : Non-zero

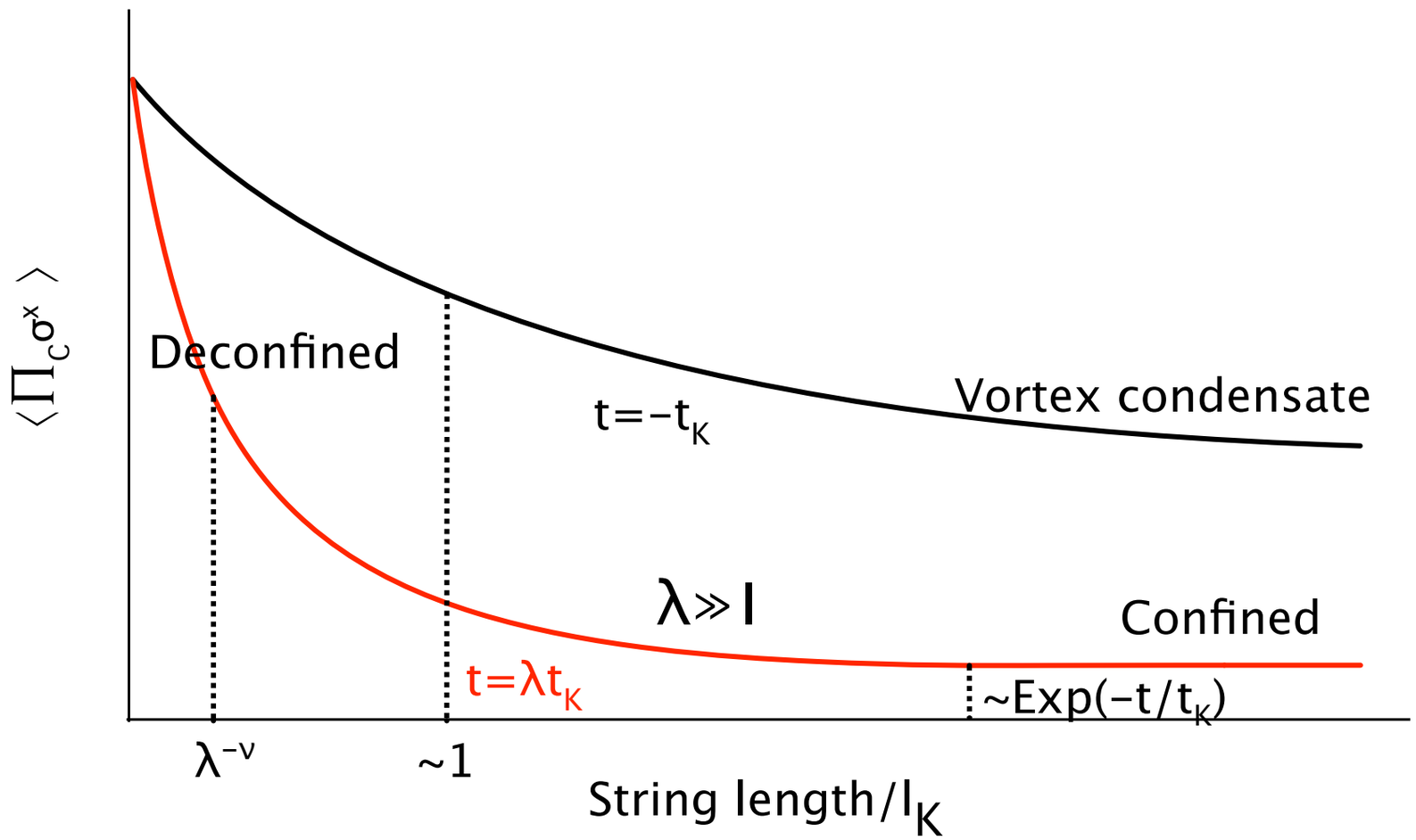
Gregor et al, NJP, 13, 025009 (2011)

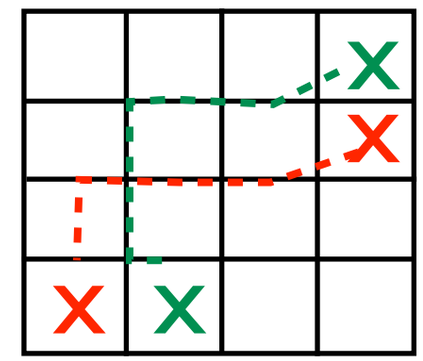
Confined → Deconfined

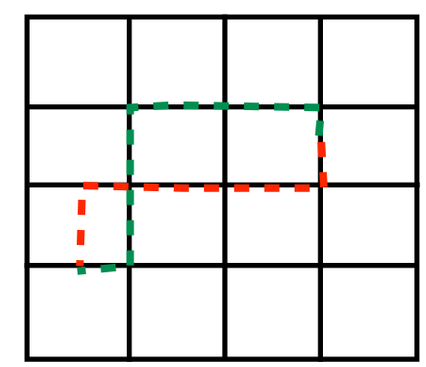




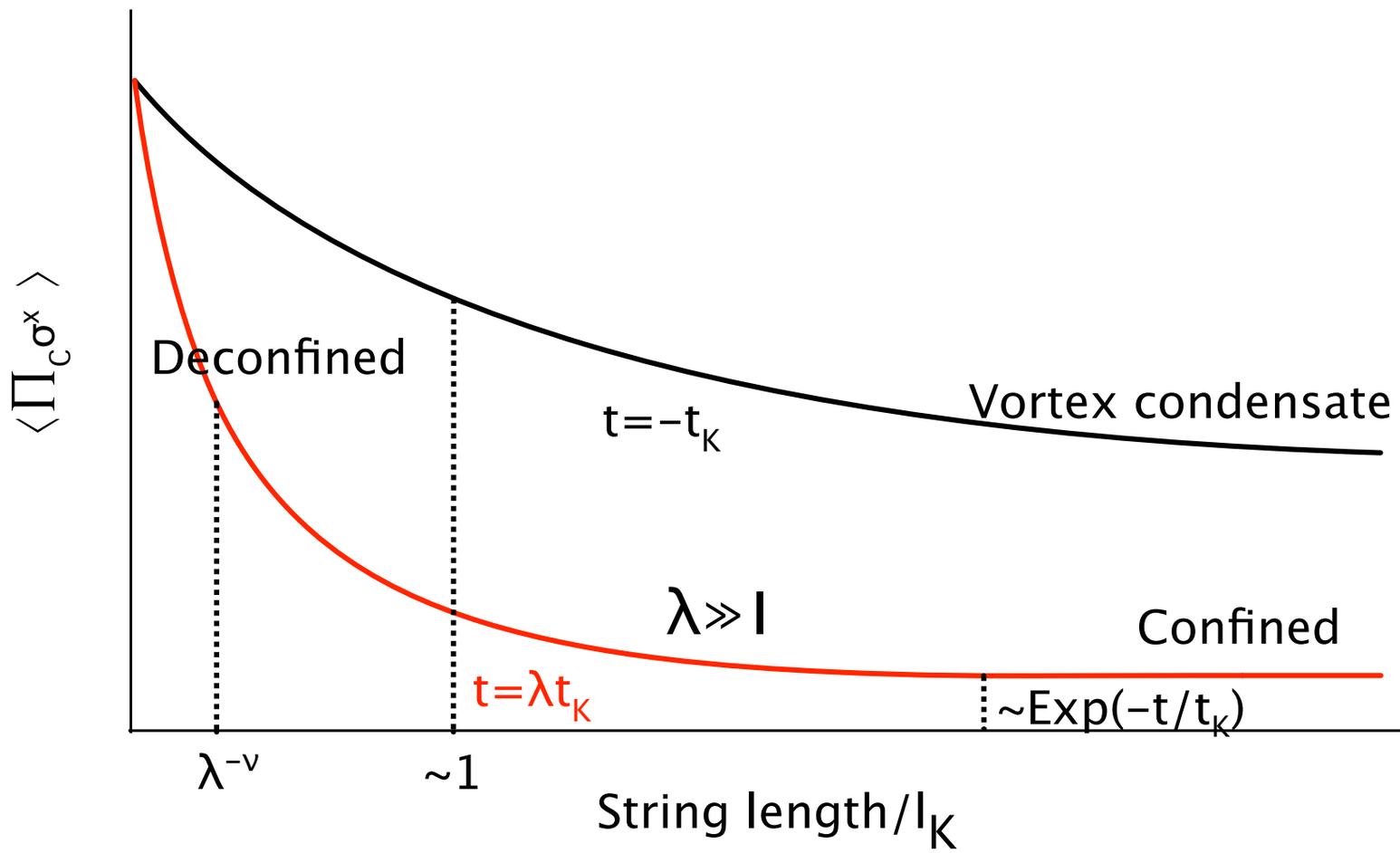
Confined → Deconfined

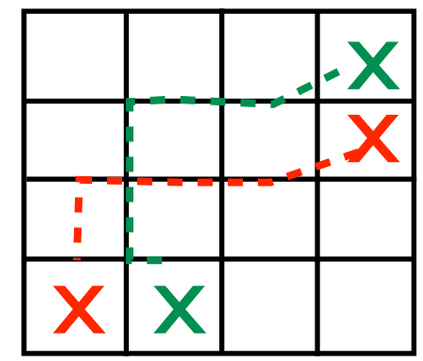


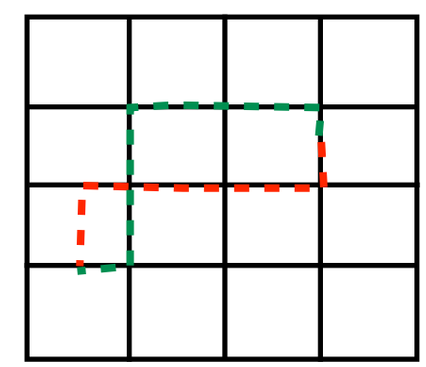




Confined → Deconfined

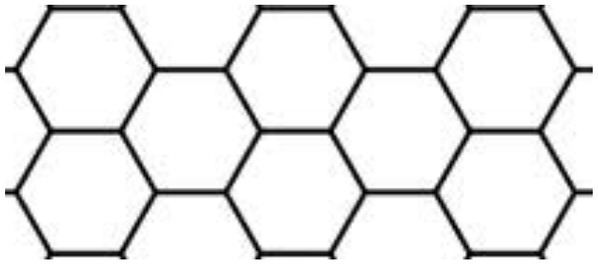




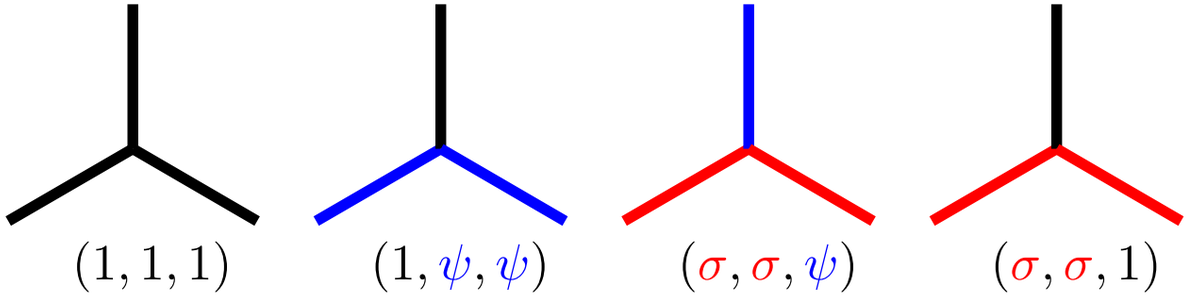


- Vortex pairs of length > l_K at $t = -t_K$
- $t \gg t_K$: annihilate leaving behind contractible strings

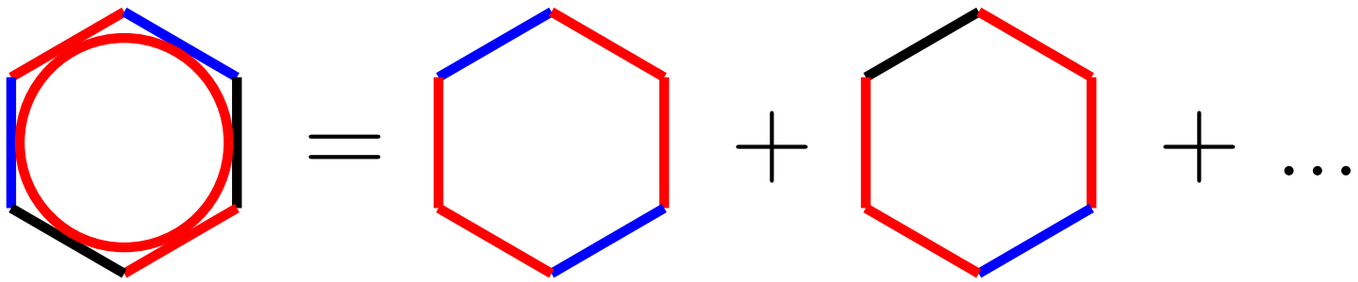
Levin-Wen Models



- 'String net' lattice Hamiltonians
- Hilbert Space : Edge labels 1, ψ, σ
- Allowed vertices ("∇.E = 0")

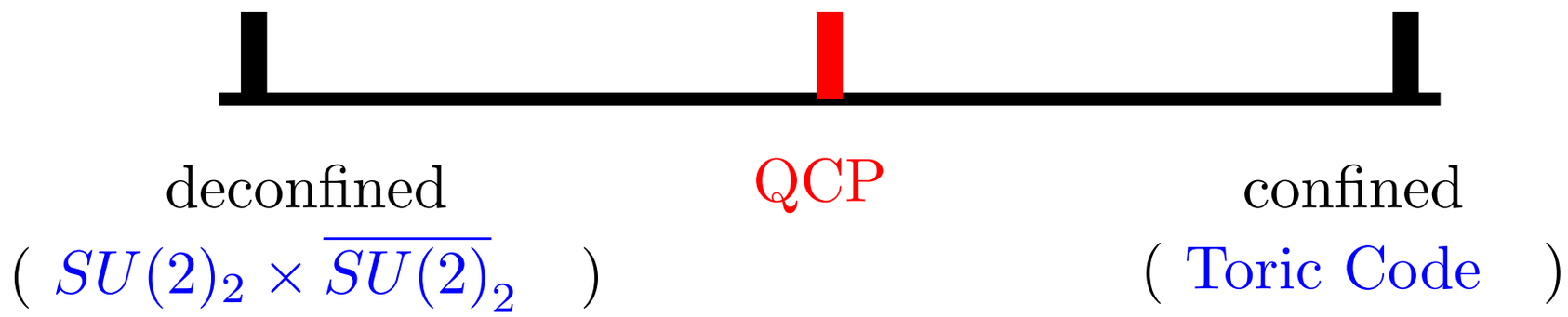


- Allowed label flips (in "B^2" term)

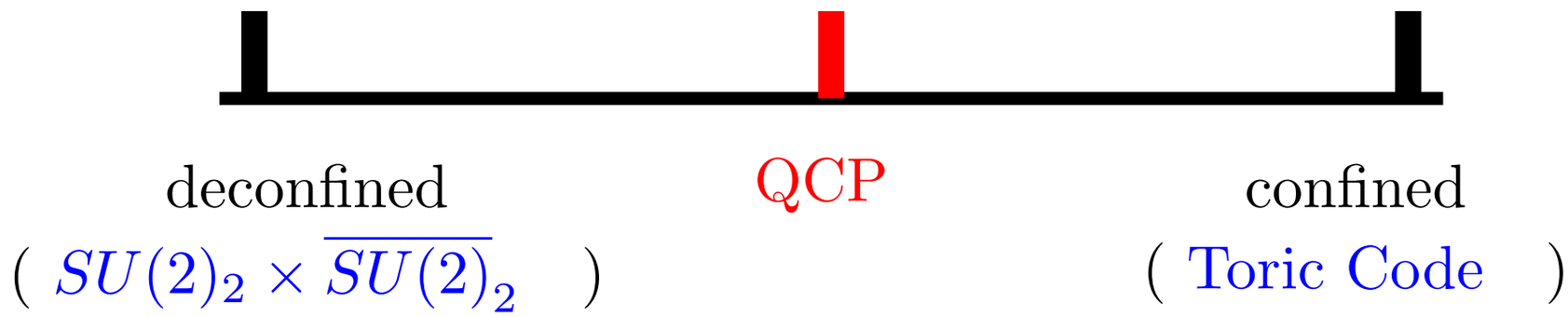


Levin, Wen PRB 71, 045110 (2005)
A. Kitaev, Ann. Phys. 303 (2002)

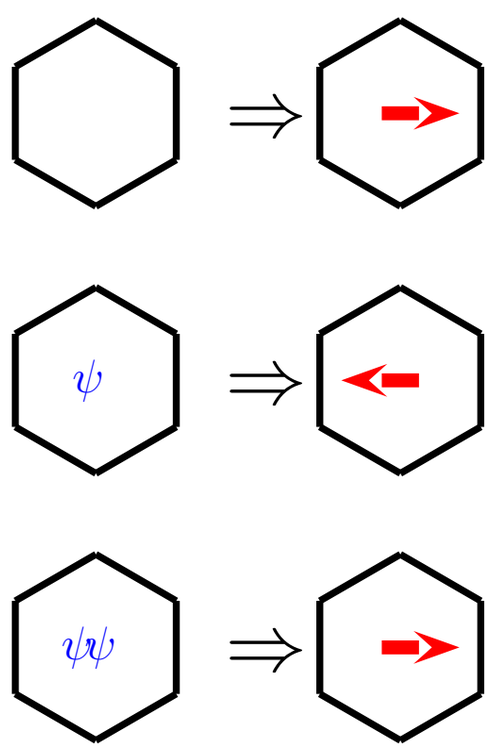
Map to (2+1)-d TFIM



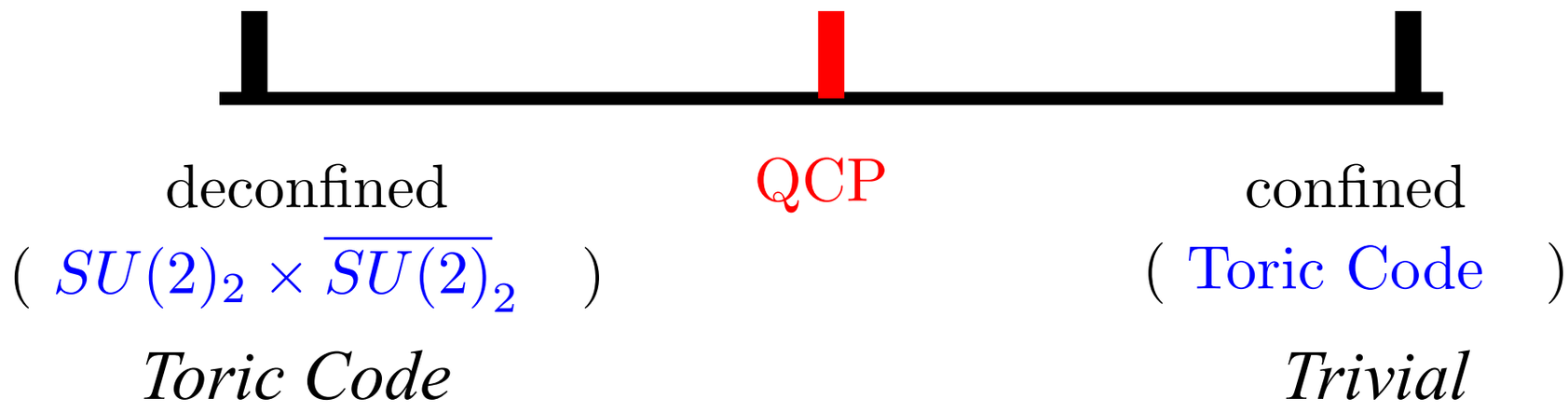
Map to (2+1)-d TFIM



- Condensation of ψ vortex
- Identify



Map to (2+1)-d TFIM



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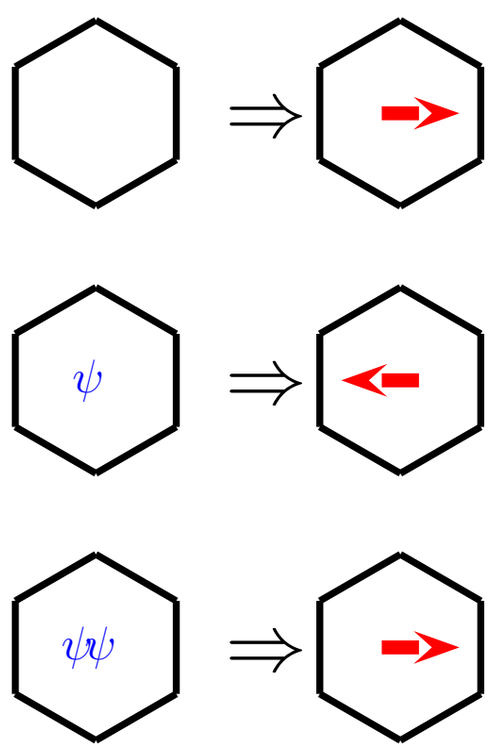

Conclusions

- Kibble-Zurek Problem
  - Set up in Landau transition
  - Scaling limit
  - Universal content
- Scaling theory in non-Landau transition
- Z_2 gauge theory in (2+1)
- Irrelevant → dangerously irrelevant in KZ
- Analogies in Levin-Wen models