Fault Tolerance in Small Experiments

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Error Correction vs. Fault Tolerance

Error Correction:

- Encodes qubits using extra qubits to protect against errors.
- Works vs. errors during storage or transmission.
- Assumes perfect encoding and decoding.
- Cannot safely perform computations on encoded qubits.
- Reduces error rate from p to $O(p^2)$ or less for storage or transmission errors *only*.
- Has been demonstrated experimentally.

Fault Tolerance:

- Supplements error-correcting code with protocols for error correction and computation.
- Corrects errors in any part of the protocol.
- Handles general weak errors affecting individual qubits or gates.
- Reduces error rate from p to $O(p^2)$ or less.
- Uses extra ancilla qubits compared to error correction.
- Has not yet been demonstrated experimentally.

What Are the Conditions?

How can we evaluate if an experiment has successfully demonstrated a fault-tolerant quantum protocol?

We need to come up with a criterion that has the following properties:

- Distinguishes fault-tolerant circuits from non-fault-tolerant circuits, i.e., accomplishes something in line with the goals of fault tolerance.
- Makes sense in non-asymptotic small systems.
- Is experimentally accessible.
- Does not make assumptions about the error model for which the protocol works. In particular, does not assume non-Markovian or independent errors.
- Is a natural criterion. In particular, it should be robust against small changes in parameters.

This is surprisingly hard to do.

What Constitutes Demonstrating FT?

Consider these possible criteria:

Test scaling with error rate?

But this is not an accessible experimental parameter; and is in any case, not a single parameter but many.

Test scaling with circuit size?

Better -- but in a small experiment, we cannot test arbitrarily large circuits. If we set an error cutoff, it will be arbitrary.

Threshold Theorem: If error rate is below some threshold value, arbitrarily long fault-tolerant quantum computation is possible.

Test if we are below the threshold?

This can be done without actually running a fault-tolerant protocol. Besides, for most types of errors we don't know the threshold and it is not straightforward to measure error rates.

What is a Fault-Tolerant Protocol?

A fault-tolerant protocol is a map:

Ideal circuits C → Fault-tolerant circuits C'

The output bits of C' can be decoded to give an output distribution, which in the absence of error should be the same as the output of C.

We can also consider non-universal protocols, where C is drawn from some family F of possible circuits.

Often the protocol has a gadget for each different circuit element.

Compare the error rates of the circuit outputs:

Error rate if C is implemented in an experiment	Error rate if C' is implemented in an experiment

We want: for all C, the error rate of C' is less than that of C.

Important Notes

Note:

- We need a reasonable family F of circuits, including both small and large circuits. This is to make sure all circuit elements are important at times.
- Only measure error rate for a complete circuit, not individual gates. This is partially for simplicity and partially to account for the possibility (likelihood) of non-Markovian errors.
- We should compare encoded vs. unencoded error rates for experiments in the same system. No other comparison is fair. This also means the number of physical qubits is fixed.

Complications:

- For large circuits, it is hard to know the ideal output distribution to compare to.
- A lot of statistics may be needed to learn error rates.
- An experiment can only test a limited number of circuits from the family.

Four-Qubit Code

There is a [[4,2,2]] code (4 physical qubits, 2 logical qubits, ability to detect one error).

Stabilizer:

X X X X

Z Z Z Z

Codewords:

$$|\overline{00}\rangle = |0000\rangle + |1111\rangle$$

← Cat state

$$|\overline{10}\rangle = |1100\rangle + |0011\rangle$$

$$|\overline{01}\rangle = |1010\rangle + |0101\rangle$$

$$|\overline{11}\rangle = |0110\rangle + |1001\rangle$$

This code cannot correct an arbitrary single-qubit error, but it can detect an arbitrary error.

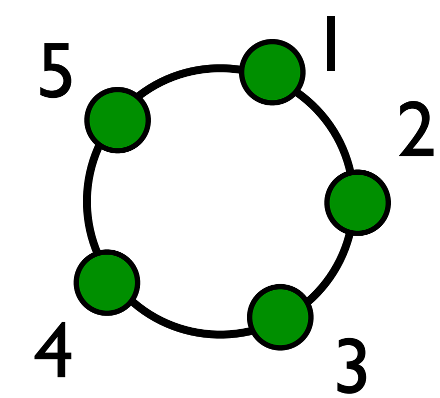
Also note:

$$|\overline{+0}\rangle = (|00\rangle + |11\rangle)(|00\rangle + |11\rangle)$$

Fault Tolerance with 4-5 Qubits

Gadgets = {State preparation, measurement, Pauli, Hadamard w/ SWAP, controlled-Z}

(With only 5 qubits, no FT error correction.)



Qubits arranged in a circle

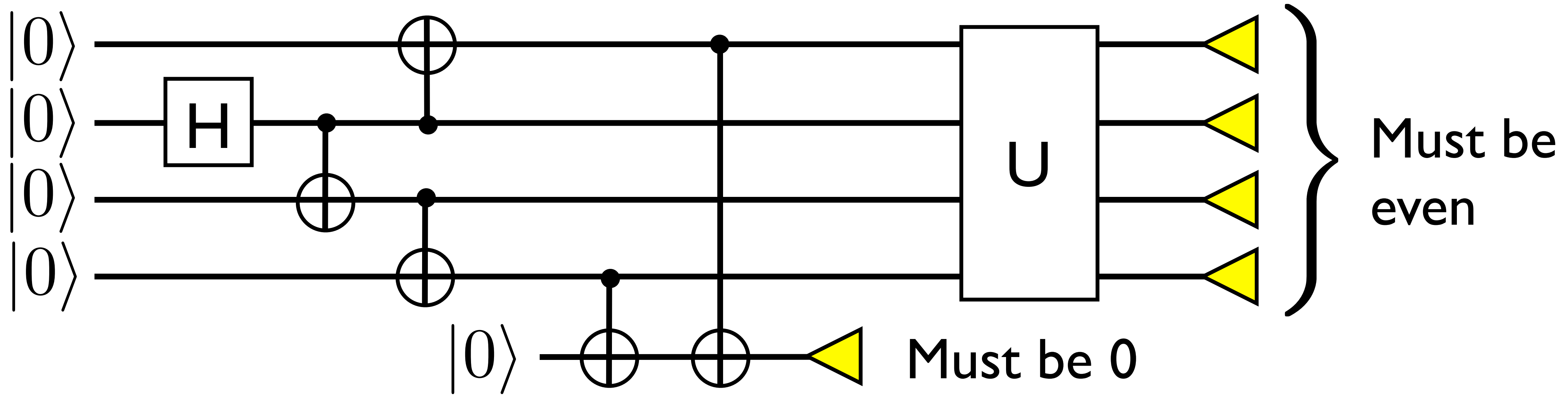
FT measurement gadget for 4-qubit code:

- Measure every qubit, then post-select on even parity.
- First logical qubit = parity of first two qubits.
- Second logical qubit = parity of qubits 1 and 3.

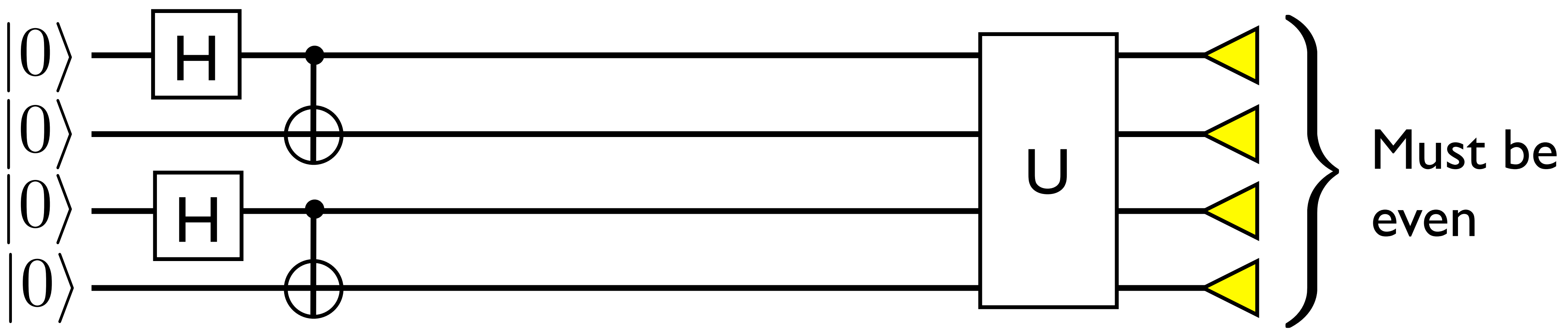
FT state preparation gadgets:

Depends on logical state. For instance, $|\overline{+0}\rangle$ can be made with 2 EPR pairs.
For $|\overline{00}\rangle$, we need CNOTs and then to test to make sure there is no correlated error. The test uses one ancilla qubit.

FT Circuits for 4-Qubit Code



U is some transversal gate (e.g., transversal Hadamard = logical Hadamards & switch logical qubits).



No need to test: there are no 2-qubit errors on an EPR pair.

What Circuits Should We Test?

The success criterion says the error rate should be lower for all circuits in our family. It is implausible to test all circuits even for families with a limited number of gates, so we need to do some representative sampling.

We want circuits that test various aspects of the protocol, without making too many assumptions about the error model (e.g., don't assume non-Markovian errors). The subfamily tested should include:

- Small and large circuits
- Random and non-random circuits
- Circuits that emphasize different gadgets

For example:

- Random circuits of a variety of lengths
- Repeat same gate or set of gates over and over again, with both few and many repetitions

Experiments So Far

There have been 3 attempts to experimentally demonstrate FT:

All 3 use the 4-qubit code. The first and third use a subsystem version of the code, with one "protected" and one "unprotected" logical qubit. All restricted to state preparation and measurement.

- Linke et al, 1611.06946 [quant-ph] (U Maryland, ion trap qubits). Compares error rates between protected and unprotected qubits — but they are not comparable. Also introduces artificial errors.
- Vuillot, 1705.08957 [quant-ph] (Aachen, using IBM Quantum Experience, superconducting qubits). Does some non-FT circuits due to limited connectivity.
- Takita et al, 1705.09259 [quant-ph] (IBM group, superconducting qubits). Similar to U Maryland group.

Do Not Do State Tomography

The complete circuit must be done in a fault-tolerant way. Compare the output distribution to the output distribution for an ideal implementation of the circuit.

State tomography is not helpful. The extra gates used in the tomography add extra errors, and the information gained is therefore not reliable.

Randomized benchmarking on the logical state is a little bit more useful, as it gives information about the incremental error rate of adding a logical gate. However, it does not provide information about the error rate for the state preparation and the measurement. All components of the fault-tolerant circuit should have better error rates than the unencoded circuit.

Will the Error Rate Improve?

For a specific error model, e.g., depolarizing channel, it is possible to simulate the protocol and determine if the error rate is likely to improve or not.
(High error rate/physical gate = no, low error rate = yes.)

However, the true error model will not be that simple. Does this same calculation still apply?

We can loosely classify error models into 3 categories:

- Good: Error correction works well; e.g., depolarizing channel
- Bad: Error correction works, but not as well; e.g., 2-qubit correlated errors
- Worse: Error correction does not work; e.g., many-qubit correlated errors

Whether the FT experiment succeeds or not depends on how much of each type of error is present in the system.

Classification of Errors

Depolarizing channel	Good
Asymmetric errors	Good? (maybe depending on protocol)
Coherent errors	Good?
Non-Markovian errors	Good?
Leakage errors	Good or worse (depends on protocol)
2-qubit correlated errors	Bad
Many-qubit correlated errors	Worse
Other errors?	???

How Much of Each Type?

Ideally, an FT experiment could give us information about how much of each of these 3 types of error we have.

Possible protocol:

- Determine physical error rates via randomized benchmarking (or your other favorite method).
- Assume a Pauli channel, and simulate FT circuit to determine expected logical error rate.
- Compare measured logical error rate with expected logical error rate.
  - If logical error rate > physical error rate: Bad or worse errors
  - If logical error rate = expected error rate: Good errors
  - If physical error rate > logical error rate > expected error rate: Some bad errors, quantify by difference? ratio?
  - If logical error rate < expected error rate: Better errors?
- Test multiple circuits using same gadgets.

Summary

- Experiments are reaching the point where they should be able to do experiments demonstrating fault tolerance.
- These experiments can also help to answer important physics questions:
  - How do fault-tolerant protocols behave under coherent or non-Markovian errors? (Good or bad error behavior?)
  - Do the experimental systems have a large component of errors that will be difficult or impossible to handle via fault tolerance?
  - Is there a simple way to measure errors that is good enough to determine if a system supports fault tolerance?
- Most details of error models do not matter for the question of whether we can build a fault-tolerant quantum computer, only how many good/bad/worse errors we have.
- Is there a better way to extract this information from an experiment?