EIT and Slow Light in Nonlinear Magneto Optic Rotation

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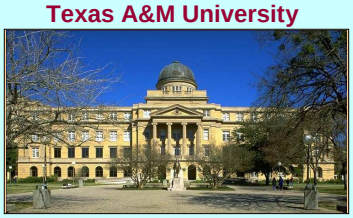


Institute for Quantum Studies

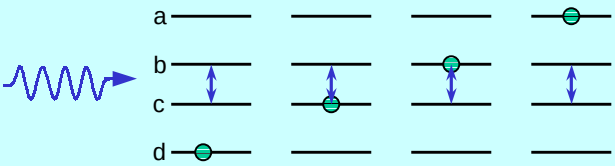
Office of Naval Research

Texas Advanced Research Program

Outline:

- Atomic Coherence
  - ♦ Three-level coherence
  - ♦ Modified susceptibility
    Electromagnetically Induced Transparency
    ➔ Index of refraction:
    Enhanced Index, **Slow light**
- Applications!
  - ♦ **Ultra-sensitive magnetometry**
    ➔ **Non-linear magneto-optic rotation**
    – Sensitivity
    – Limitations (ac Stark, radiation trapping)
    – Vacuum Squeezing
    – Slow Light

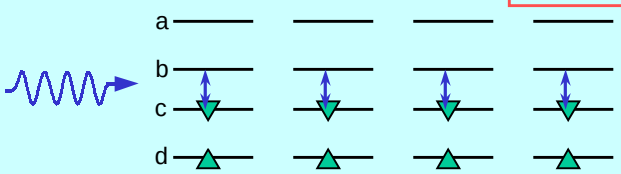
Statistical Mixture of States: **Only two relevant energy levels**



Ordinary Absorption, Refraction, and Amplification

$$\psi = |a\rangle \quad \text{or} \quad \psi = |b\rangle \quad \text{or} \quad \ldots$$

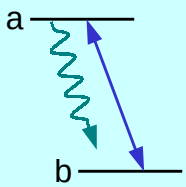
Coherent Superposition of States: **Three or more levels!**

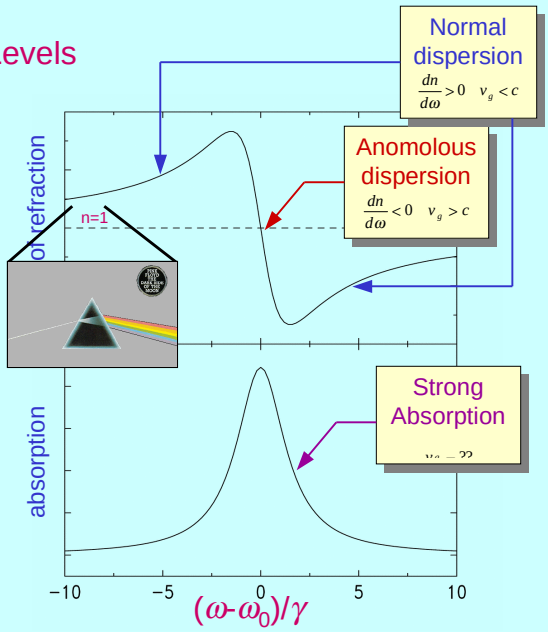


New effects

$$\psi = \alpha|c\rangle + \beta|d\rangle$$

Susceptibility: Two Atomic Energy Levels



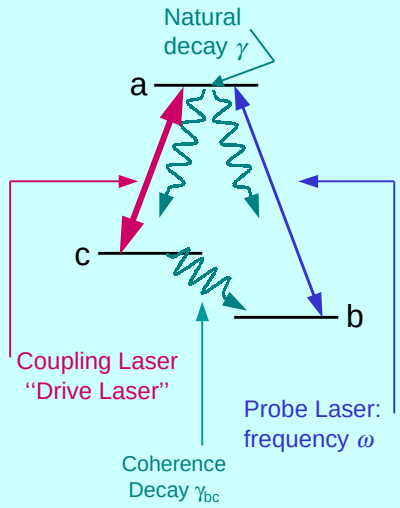


Vg = c/60 in sodium
Tom Mossberg, 1977
unpublished

Superluminal speeds
(e.g., tunnelling)
Chiao et al. (Berkeley)

Atomic Coherence Effects

Three (or more) Atomic Energy Levels

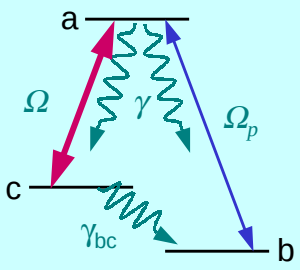


The combined action of the drive and probe lasers produces a quantum superposition of the two lower states:

$$\psi = \alpha|b\rangle + \beta|c\rangle$$

Then, the probe field interacts with this superposition state.

Three Level System



For:

- Low density (single atom response)
- Monochromatic probe
- Weak probe $\Omega > \Omega p$

Calculate susceptibility of homogeneously broadened 3-level system. See for example, Scully and Zubairy, *Quantum Optics*, Cambridge University Press, 1997.

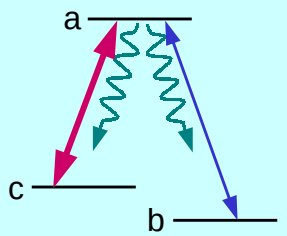
$$\chi = \eta\,\gamma_a \frac{i\gamma_{bc} - (\Delta - \Delta_d)}{[\gamma_{ab} + i\Delta][\gamma_{bc} + i(\Delta - \Delta_d)] + \Omega^2}$$

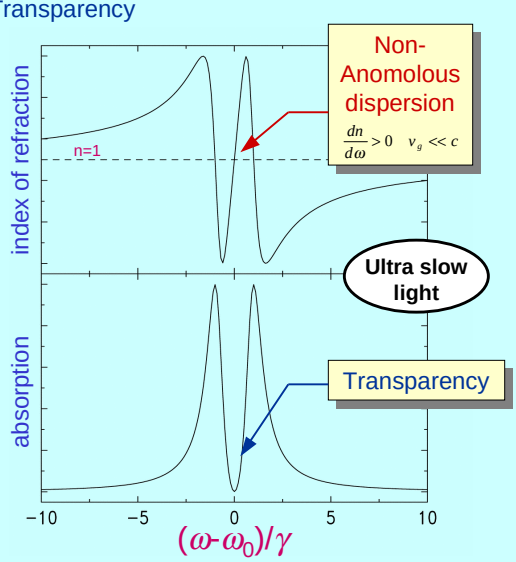
where

$$\begin{aligned} \eta &= 3N\lambda^3/4\pi^2 \\ \Delta_d &= \text{drive detuning} \\ \Delta &= \text{probe detuning} \\ \gamma_{ab}\,,\,\gamma_{bc} &= \text{coherence decay rates} \end{aligned}$$

Three Atomic Energy Levels

Electromagnetically Induced Transparency



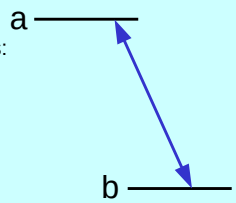


Transmission through 10,000 absorption lengths, Harris et al., 1998.

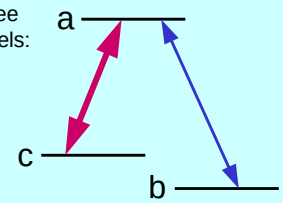
Vg = 1 m/s (c/300,000,000) Ketterly et al., 2001.

Effect of Coherence on Interaction

Two Levels:



Three Levels:

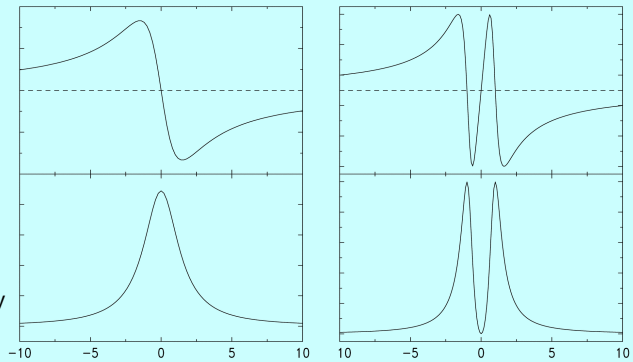


Index of refraction:

•Steep normal dispersion

Absorption:

•Induced transparency

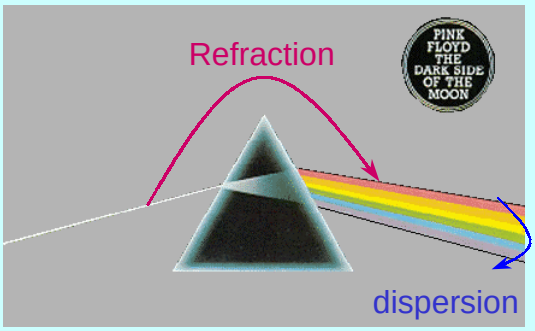


Ultra-Slow Light

Enhanced Index of Refraction

Lasing Without Inversion

Slow Light ?!? What gives?



$v_{red} < c$

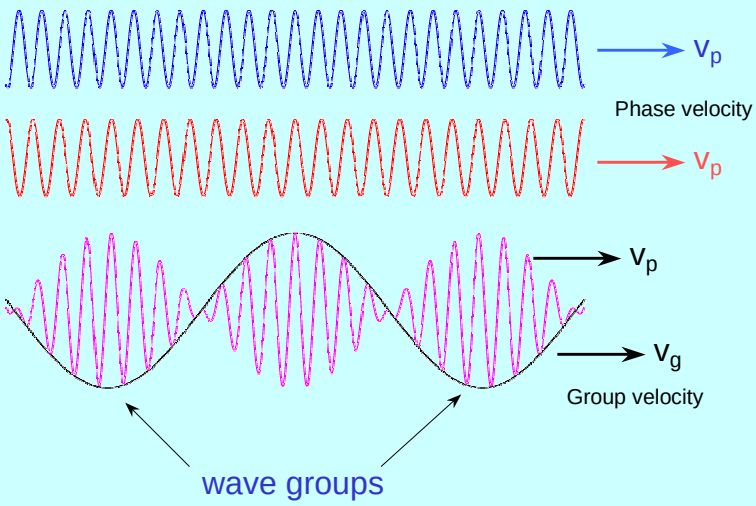
$v_{blue} < v_{red}$

$v = c/n$

Air	1.0003
Water	1.3
Quartz	1.54 – 1.56
Diamond	2.4
Some semiconductors	5 – 6

Phase and Group Velocities

Superposition of travelling waves



Superposition of Travelling Waves

Phase and Group velocity

Consider two travelling waves:

$$\boxed{E_1 = \mathcal{E}_0 \cos(k_1 z - \omega_1 t) \qquad E_2 = \mathcal{E}_0 \cos(k_2 z - \omega_2 t)}$$

$$\boxed{v_{\text{phase}} = \frac{\omega}{k}}$$

Superposition $E = E_1 + E_2$

$$E = 2\mathcal{E}_0 \cos\underbrace{\left(\frac{(k_1 - k_2)z - (\omega_1 - \omega_2)t}{2}\right)}_{\Phi_{\text{envelope}}} \cos\underbrace{\left(\frac{(k_1 + k_2)z - (\omega_1 + \omega_2)t}{2}\right)}_{\Phi_{\text{optical phase}}}$$

Looking at a region of constant phase, we set:

$$\frac{\partial \Phi_{\text{envelope}}}{\partial t} = 0 \quad \rightarrow \quad (k_1 - k_2)\frac{\partial z}{\partial t} - (\omega_1 - \omega_2) = 0$$

So

$$v_{\text{group}} = \frac{\omega_1 - \omega_2}{k_1 - k_2} = \boxed{\frac{\Delta\omega}{\Delta k}}$$

Optical Group Velocity in a Medium

Dispersion

Suppose the frequencies are close together, $\Delta\omega \ll \omega$

And assume a simple dispersion relation: $k_{\text{medium}} = \frac{\omega}{c} n(\omega)$

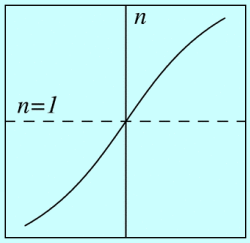
Then:

$$v_{\text{group}} = \frac{1}{\frac{dk}{d\omega}} \quad \longrightarrow \quad \boxed{v_{\text{group}} \approx \frac{c}{n + \omega \frac{dn}{d\omega}}}$$

Since $v_{\text{phase}} = c/n$ we can have:

$v_{\text{phase}_1} \approx v_{\text{phase}_2} \approx c$ and $\boxed{v_{\text{group}} \ll c}$

if $n \approx 1$ and $\boxed{\omega \frac{dn}{d\omega} \gg 1}$



Spatial Dispersion:

The effect of moving atoms.

Usual dispersion relation:

$$\frac{c}{n} = \frac{w}{k} \quad n = n(\omega)$$

Add *spatial* dispersion:

$$c\,k = \omega\, n(\omega, k)$$

Group velocity:

$$\begin{aligned} c\,\mathrm{d}k &= \omega\,\mathrm{d}n + n\,\mathrm{d}\omega \\ &= \omega\left(\frac{\partial n}{\partial \omega}\mathrm{d}\omega + \frac{\partial n}{\partial k}\mathrm{d}k\right) + n\,\mathrm{d}\omega \end{aligned}$$

$$v_g = \frac{\mathrm{d}\omega}{\mathrm{d}k} = \frac{c - \omega\frac{\partial n}{\partial k}}{n + \omega\frac{\partial n}{\partial \omega}}$$

Two ways to reduce v_g:

$$\frac{\partial n}{\partial \omega} \quad \text{or} \quad \frac{\partial n}{\partial k}$$

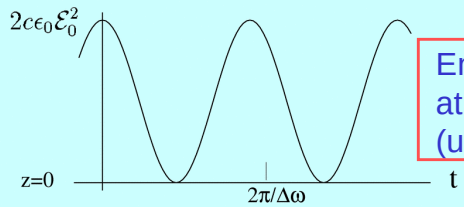
Intensity

$$\vec{S} = \vec{E} \times \vec{H} = \frac{4\mathcal{E}_0^2}{\mu_0 c}\cos^2\underbrace{\left(\frac{\Delta k}{2}z - \frac{\Delta\omega}{2}t\right)}_{\text{slow}}\cos^2\underbrace{\left(\bar{k}z - \bar{\omega}t\right)}_{\text{fast}}\hat{k}$$

$$\underset{\substack{\text{short}\\\text{time}}}{I} = \langle \vec{S}\cdot\hat{k}\rangle = \frac{1}{2}c\epsilon_0(2\mathcal{E}_0)^2\cos^2\underbrace{\left(\frac{\Delta k}{2}z - \frac{\Delta\omega}{2}t\right)}$$

group velocity $v_g = \dfrac{\Delta\omega}{\Delta k}$

modulation frequency $= \Delta\omega$

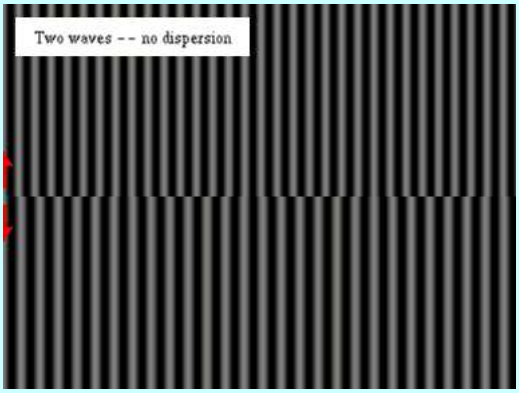


Energy propagates at the group velocity (usually.)

Equal Phase velocities: No dispersion

Short wavelength:

Long wavelength:



Equal Phase velocities: No dispersion

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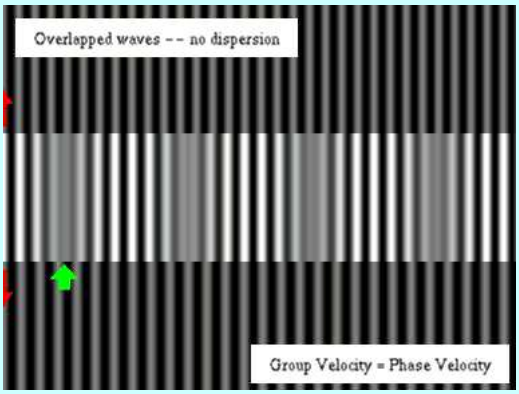
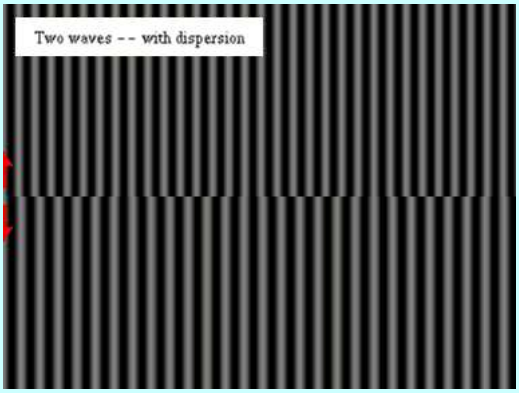


Group Velocity = Phase Velocity

Unequal Phase velocities: "N ormal" dispersion

Short wavelength:

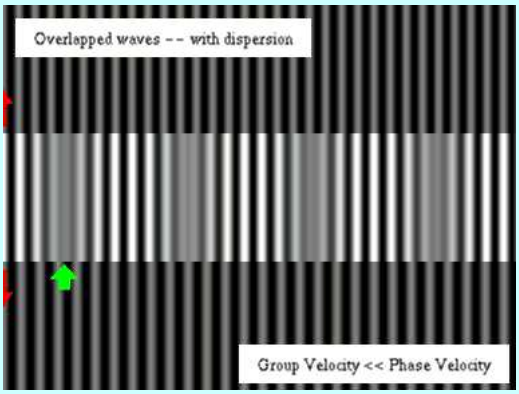
Long wavelength:



Unequal Phase velocities: "N ormal" dispersion

Short wavelength:

Long wavelength:

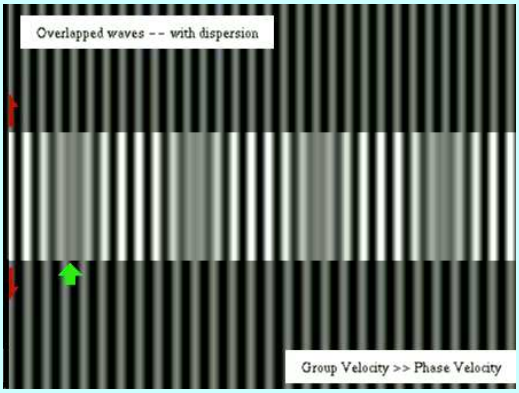


Group Velocity < Phase Velocity

Unequal Phase velocities: "Anomolous" dispersion

Short wavelength:

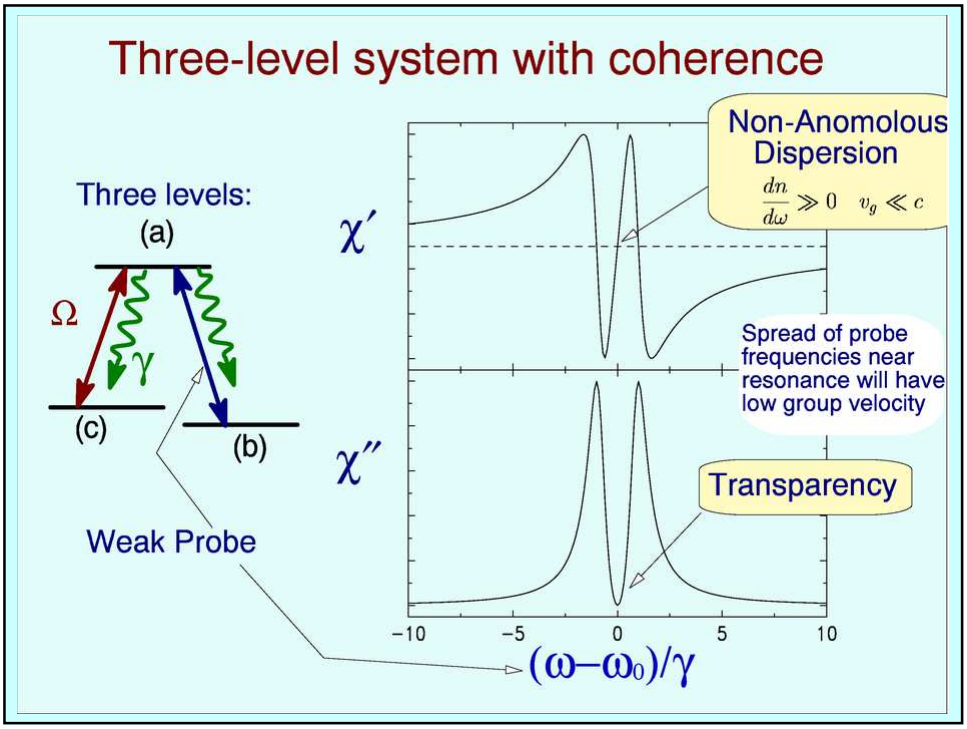
Long wavelength:



Group Velocity > Phase Velocity

Where can we find very steep dispersion?

Answer: Atoms in coherent quantum mechanical superposition states!



EIT Slow light history:

c/13, Xiao et al., PRL **74**, 666 (1995).
c/165, Kasapi, Jain, Yin, and Harris, PRL **74**, 2447, (1995).

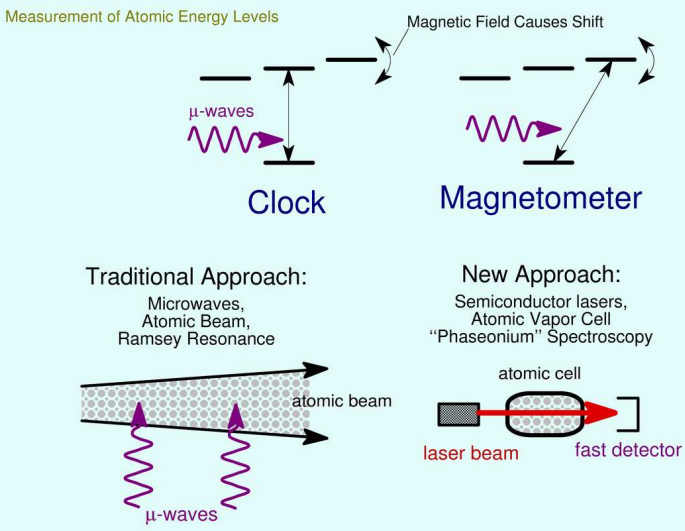
c/3000, Schmidt, Wynands, Hussein, Meschede
PRA **53**, R27, (1996). (Inferred from dn/dω)
~c/10^5 Lukin, Fleischhauer, Zibrov, Robinson, Velichansky, Hollberg, Scully, PRL 79, 2959 (1997).
(Inferred from probe phase shift.)

c/2x10^7 Hau, Harris, Dutton, and Behroozi, Nature **397**, 594 (1999). (BEC)
c/3x10^6 TAMU, PRL **82**, 5229 (1999). (Hot vapor)
c/4x10^7 Budker, Kimball, Rochester, and Yashchuk, PRL **83**, 1757 (1999). (NMOR)
c/6x10^6 Turukhin, Musser, Sudarshanam, Shahriar, and Hemmer, PRL **88**, 023602 (2002). (Solid)

Science Fiction:
''Light of Other Days,'' Bob Shaw (1931-1996), in *Analog Science Fiction*, John W. Campbell, Jr., Ed. August, 1966. (Hugo and Nebula award winner.)

> … The most important effect, in the eyes of the average individual, was that light took a long time to pass through a sheet of slow glass. A new piece was always jet black because nothing had yet come through, but one could stand the glass beside, say, a woodland lake until the scene emerged, perhaps a year later. If the glass was then removed and installed in a dismal city flat, the flat would—for that year—appear to overlook the woodland lake. During the year it wouldn' t be merely a very realistic but still picture–the water would ripple in sunlight, silent animals would come to drink, birds would cross the sky, night would follow day, season would follow season. Until one day, a year later, the beauty held in the subatomic pipelines would be exhausted and the familiar gray cityscape would reappear. ...

Applications: Atomic Clocks and Magnetometers



New Applications - TAMU IQS

- Ultra-Sensitive Optical Magnetometry (Novikova/Welch)
- Quantum Information Storage
  (Lukin et al., Hau et al., Zibrov et al.)
- Resonant Four-Wave Mixing (Mikhailov/Welch)
- New IR Detectors (Scully/Boyd)
- New FIR (1-100μ) Lasers (Kocharovsky/Belyanin, Capasso)
- Sub-femtosecond Sub-cycle Laser Pulses (Sokolov, Harris)
- Quantum Nucleonics / g-ray Lasers (Kocharovskaya)
- Quantum Computing (Hemmer/Scully/Zubairy)
- Anthrax Spore Detection (Scully)

Optical Magnetometry

Optical Pumping Magnetomter

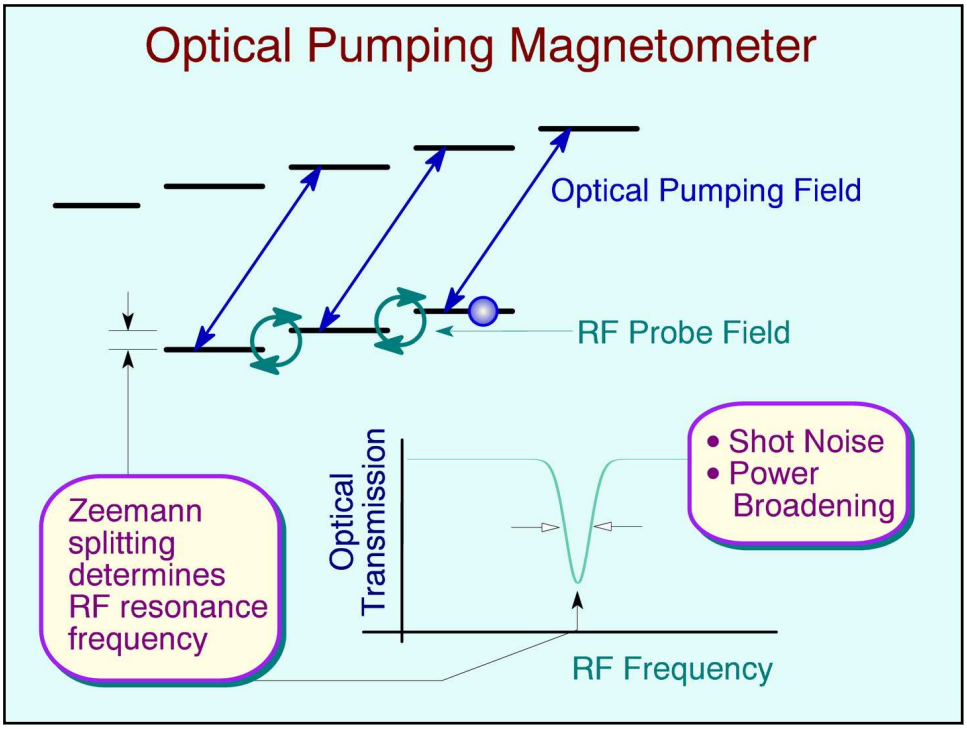
E. B. Alexandrov, V. A. Bonch-bruevich Opt. Eng. **31** 711 (1992).
E. B. Alexandrov et al., Laser Physics **6** 244 (1996).

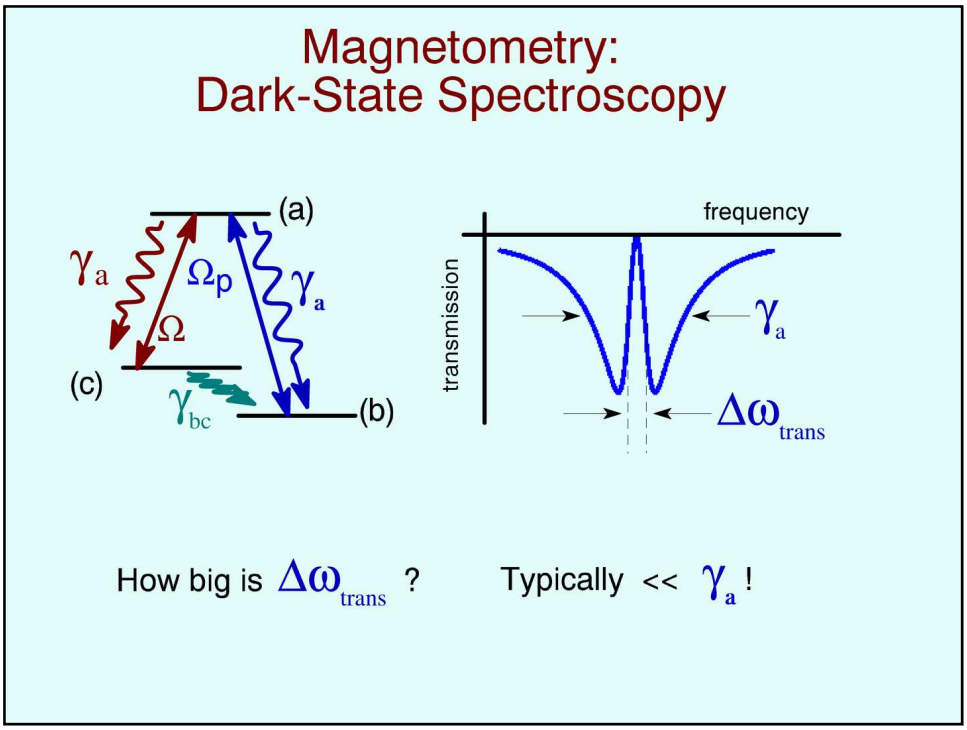
Mean-field Magnetometer

F. Bretenaker et al., PRL **69** 909 (1992).

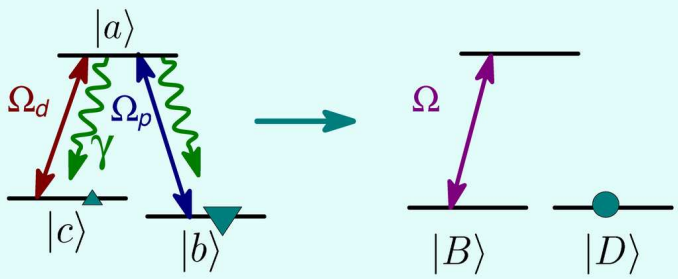
EIT Magnetometers

Proposal:
M. O. Scully and M. Fleischhauer; PRL **69** 1360 (1992); PR**A 49** 1973 (1994).
Λ-EIT:
A. Nagel et al. (Wynands), Europhys. Lett. **44** 31 (1998).
Non-linear Faraday Magnetometer:
D. Budker et al., PRL **81** 5788 (1998), PRL **83** 1767 (1999).
V. A. Sautenkov et al., PRA **62** (2000).


Optical Pumping Magnetometer
Optical Pumping Field
RF Probe Field
Zeemann splitting determines RF resonance frequency
Optical Transmission
• Shot Noise
• Power Broadening
RF Frequency


Magnetometry:
Dark-State Spectroscopy
(a)
(b)
(c)
γ_a
Ω_p
Ω
γ_a
γ_{bc}
frequency
transmission
γ_a
$\Delta\omega_{trans}$
How big is $\Delta\omega_{trans}$?
Typically << γ_a !

EIT as an Optical Pumping Process



$$|D\rangle = \frac{\Omega_d|b\rangle - \Omega_p|c\rangle}{\sqrt{|\Omega_d|^2 + |\Omega_d|^2}} \qquad |B\rangle = \frac{\Omega_d|c\rangle + \Omega_p|b\rangle}{\sqrt{|\Omega_d|^2 + |\Omega_d|^2}} \equiv \Omega$$

Linewidth is the key

Low Density Limit:
(Wynands, Akulshin)

$$\Delta\omega_{\text{trans}} \approx \begin{cases} \dfrac{\Omega^2}{\gamma_a} & \Omega^2 \gg \gamma_{bc}(\gamma_a + \Delta\omega_D) \\ \gamma_{bc} & \text{otherwise} \end{cases}$$

Very High
Density Limit:

$$\Delta\omega_{\text{trans}} \approx \frac{2\Omega^2}{\sqrt{\gamma_{bc}\gamma_a}} \frac{1}{\sqrt{\eta k L}}$$

Good news:

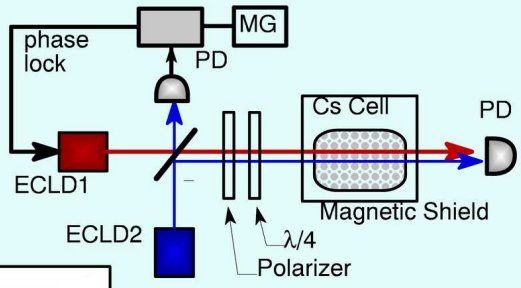
- Linewidth can be made very small.
- Linewidth decreases with increasing density ☞

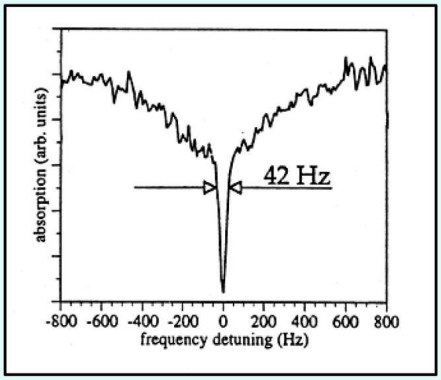
Bad news:

- Proportional to Intensity -- Power broadening
  Need low power ---> Shot Noise

Resolution can be made very good:

Experiment:

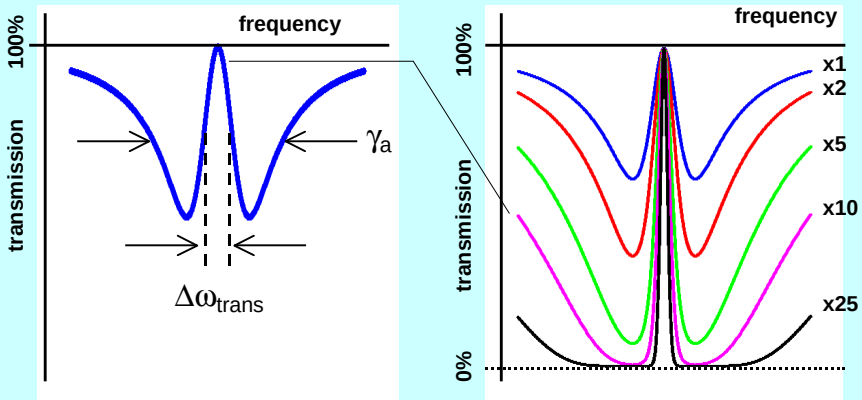




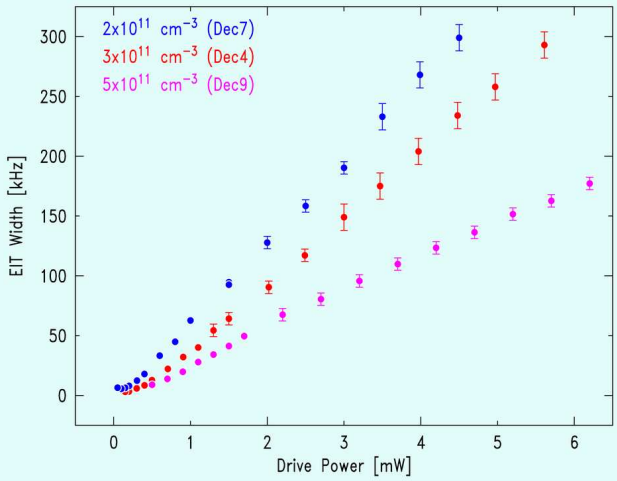
From Robert Wynands
Universität Bonn
Habilitationsschrift 1998

Magnetic field resolution
$\Delta B \approx 5\times10^{-11}$ G/√Hz

Compensate Power Broadening by Increasing Density



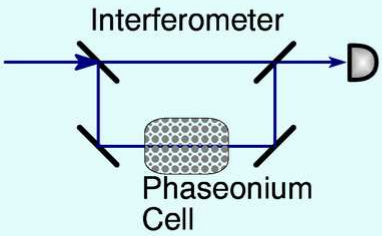
Linewidth Measurements

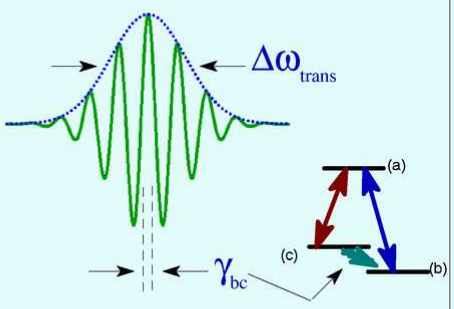


V. A. Sautenkov et al., Laser Physics **9** 889 (1999).

Potentially Can Do Better:

Use ***Dispersive*** Properties of Medium





Scully PRL **67**, 1855 (1991)
Harris et al., PRA **46**, R29 (1992)
Xiao et al., PRL **74**, 666 (1995)
Zibrov et al., PRL **76**, 3935 (1996)
Moseley et al., PRL **74**, 670 (1995)
Schmidt et al., PRA **53**, R27 (1996).

Good News: No power broadening -- Very low shot noise -- High S/N

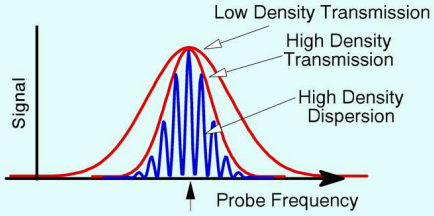
Bad News: Technical Noise (mirror jitter)

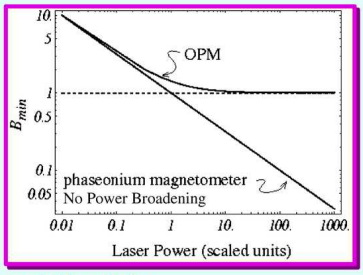
Magnetometry in Dense Coherent Media

Resonant Nonlinear Spectroscopy in High Density Media With Small Absorption

M. O. Scully and M. Fleischhauer; PRL **69** 1360 (1992); PR**A 49** 1973 (1994).

- Transmission Line Narrowing By Increasing Density
- Dispersive Measurement Yields Even Higher Resolution
- Increase Signal-to-Noise by Eliminating Power Broadening





- Goal of Technological and Experimental Simplicity to Create an Extremely Sensitive but Robust Instrument

``Phaseonium´´

Phaseonium = high optical density coherent medium

Avoid collective Effects:

- Density < $1/\lambda^3 \approx 10^{14}$ cm^{-3}

Avoid Radiation Trapping, Collisions:

- Density < ??

Current Research Directions

Novel Spectroscopy and Magnetometry Based on:

- **Coherent Raman Scattering**

  Using resonantly enhanced nonlinearities allows high-resolution measurements of dispersion in a transmission type experiment, effectively eliminating much of the usual technical noise. Elimination of power broadening allows further increase of sensitivity.

- **Self-Locked Parametric Oscillator**

  A new type of Raman parametric oscillator with no optical cavity has been demonstrated. This oscillator generates a stable beat signal at a frequency that is linearly shifted by magnetic field.

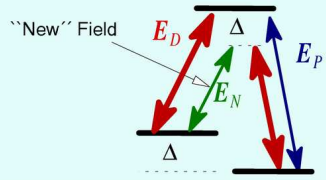
- **Nonlinear Polarization Spectroscopy Based on EIT**

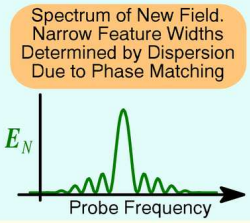
  A new type of polarization spectroscopy based on the non-linear Faraday rotation of polarization in dense media has been developed. Uniquely large rotation angles are observed leading to enhanced sensitivity to magnetic fields.

Coherent Raman Scattering in Dense Media

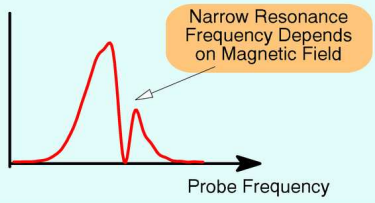
Lukin et al., PRL **79** 2959 (1997).

- Parametrically Generate ``New´´ Field

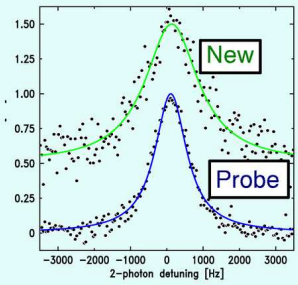




- Heterodyne Signal Displays Narrow Features

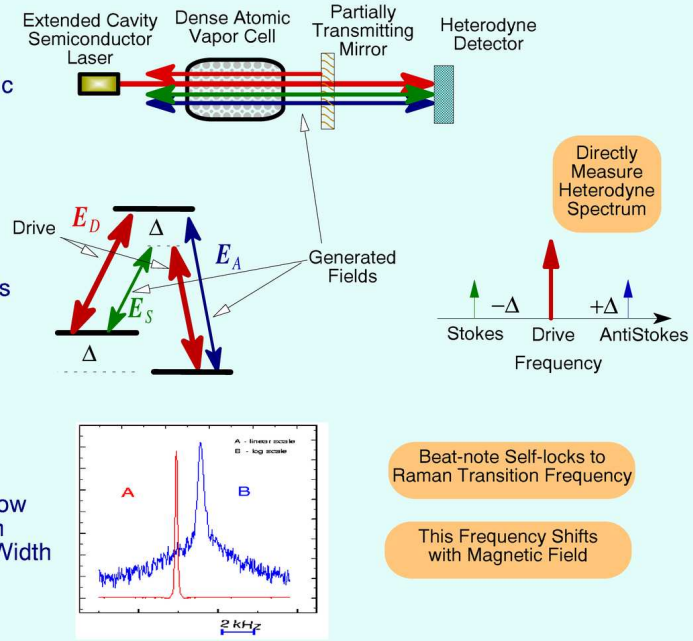


- Mystery: At intermediate density, new field width broader than probe field width



Self-Locked Parametric Oscillator

- Active Parametric Oscillations
- Simple Setup: Single Laser
- Stokes and Antistokes Fields Generated by Medium
- Linewidth Below Two-Photon Homogeneous Width

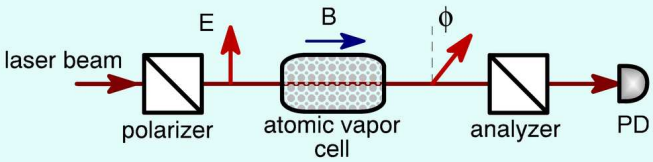


Directly Measure Heterodyne Spectrum

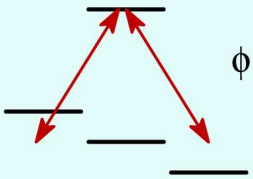
Beat-note Self-locks to Raman Transition Frequency

This Frequency Shifts with Magnetic Field

Ideal System for study of EIT:
Magneto-Optic Polarization Spectroscopy



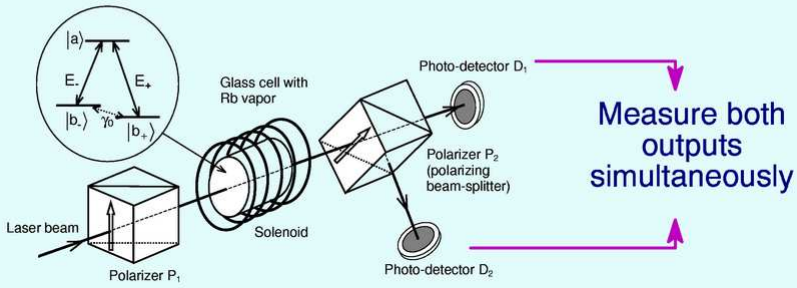
Linear polarized light decomposed as left and right circular polarized beams



$$\phi = (n^+ - n^-)\, kL$$

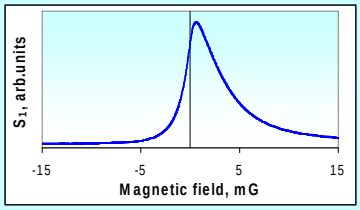
$n^+ - n^-$ changes with magnetic field B

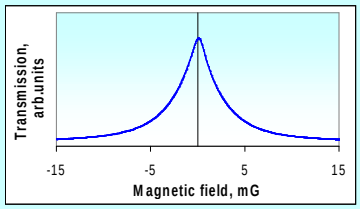
Experimental Setup:



- Obtain transmission and dispersion $\Rightarrow$ χ' and χ''
- Simple and clean experiment.

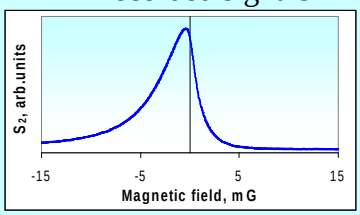
Measurements

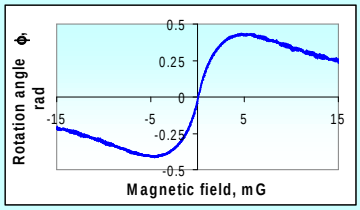




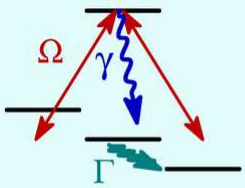
Recorded signals

Transmission $\propto S_1+S_2$





Rotation angle $\phi = \frac{1}{2}\arcsin\left(\frac{S_1 - S_2}{S_1 + S_2}\right)$

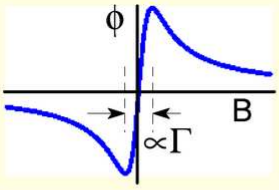


Usual Conditions:

Low Intensity: $\Omega^2 < \Gamma\gamma$
(to avoid power broadening)

Low Density: $N\sigma L < 1$
(to avoid absorption)

$\phi = (n^+ - n^-)\, kL \propto N\sigma L\, (\Omega^2/\gamma)\; B/(B^2+\Gamma^2)$

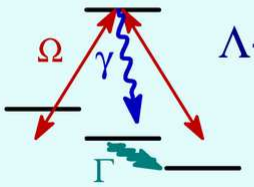


Increase sensitivity by decreasing Γ

Pay the price to keep Ω^2 low
$\Rightarrow$shot noise limited

Berkeley Group: $\Gamma \sim (2\pi)$**1Hz** --> $\Delta B \sim 10^{-12}$ G/$\sqrt{\text{Hz}}$
Budker et al., Phys. Rev. Lett. **81**, 5788 (1998).

New Twist: Use Dense Coherent Media



Λ-EIT $\longrightarrow$

Unusual Conditions:

High Intensity: $\Omega^2 > \Gamma\gamma$
(avoid shot noise)

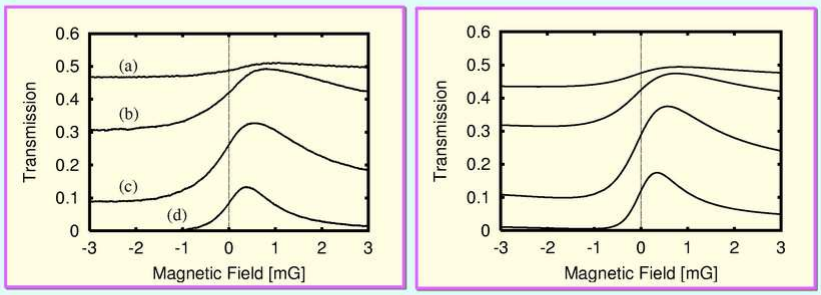
High Density: $N\sigma L > 1$
(increase rotation angle)

For $N\sigma L\, \Gamma\gamma/\Omega^2 \sim 1$ only have approximately 50% absorption.

Interplay:
- Increase Intensity to give increased transmission
- Increase Density to maintain narrow line.

Transmission Curves: Compare with Calculation

``Low´´ Optical Density

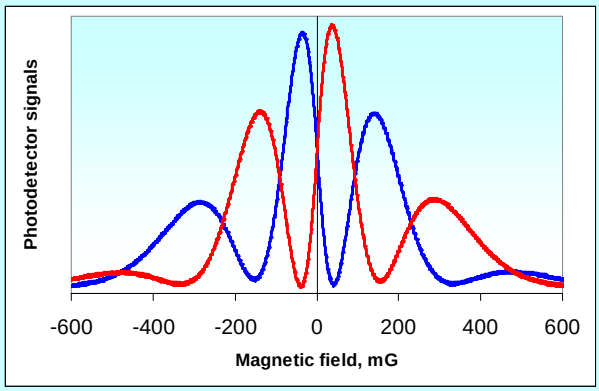


(a) $N=1.1\times10^{10}$ cm^{-3}
(b) $N=3.5\times10^{10}$ cm^{-3}
(c) $N=1.1\times10^{11}$ cm^{-3}
(d) $N=2.5\times10^{11}$ cm^{-3}

Sautenkov et al., PRA **62**, 023810 (2000).

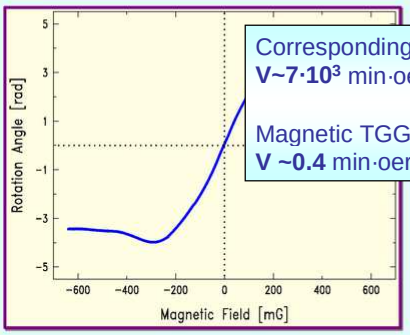
High Optical Density:

Large rotation angle



Scaling to high density and laser power gives multiple oscillations as polarization rotation passes 2π

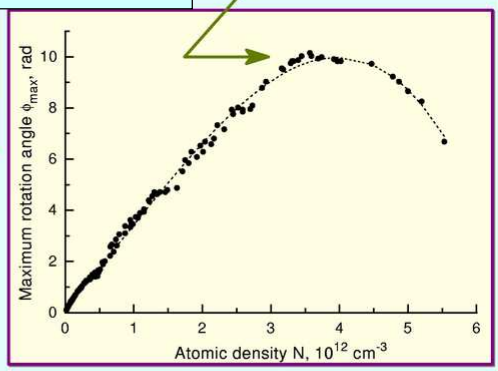
Giant Polarization Rotation



Corresponding Verde constant:
V~7·10³ min·oersted^{-1}·cm^{-1}

Magnetic TGG crystal:
V ~0.4 min·oersted^{-1}·cm^{-1}

10.3 rad



Near Earth-Field
Optical Magnetometry!

Novikova et al.,
Opt. Lett. **26**, 1016 (2001).

Application to Magnetometry

Sensitivity:
Minimum Detectable Field

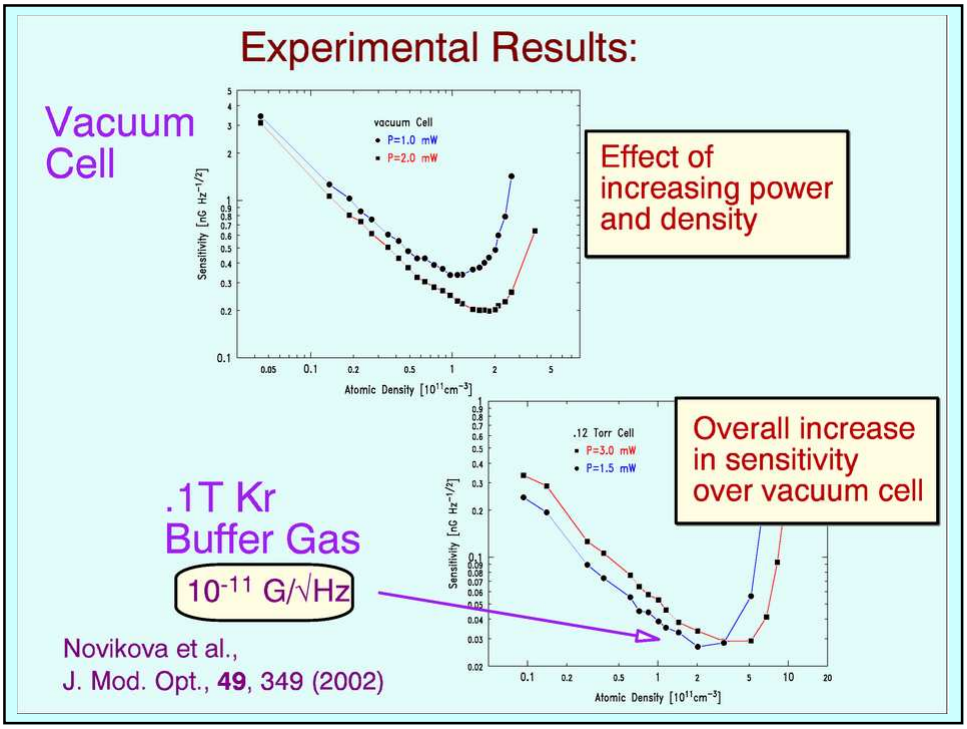
$$S \sim \frac{1}{\frac{d\phi}{dB}} \Delta\phi_{\min}$$

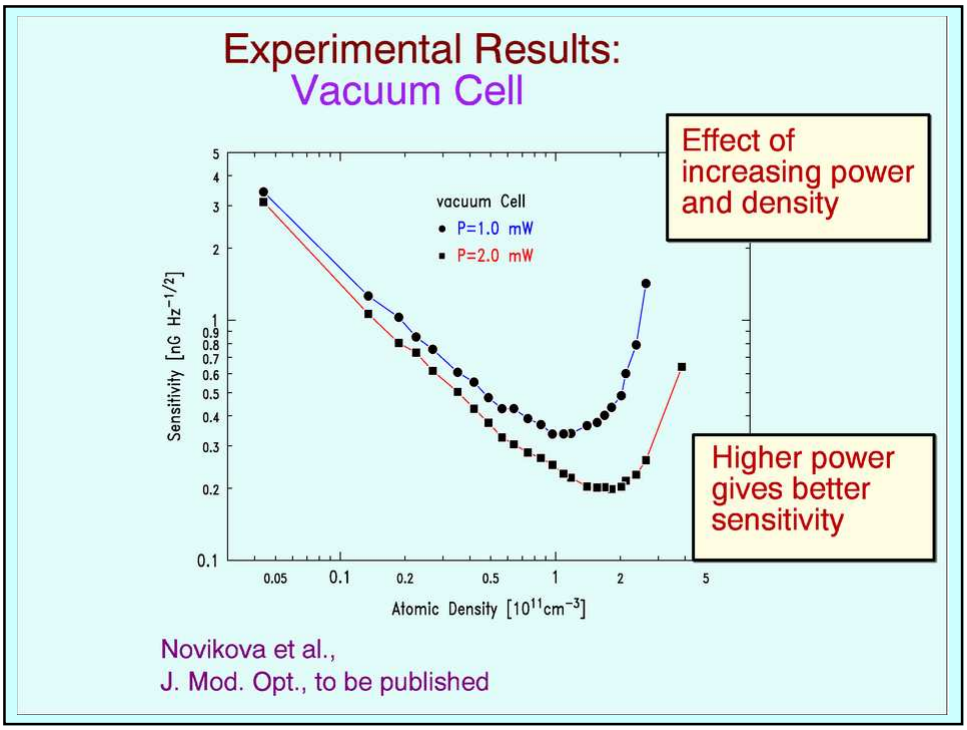
Budker et al.,
PRL **81** 5788 (1998).

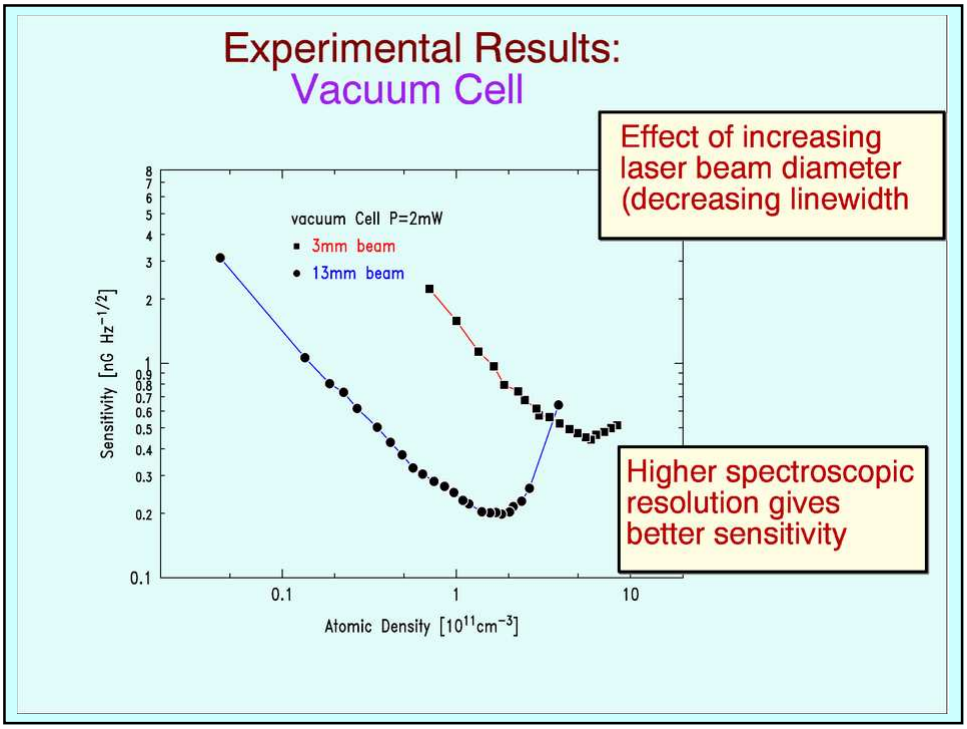
$$\Delta\phi_{\min} = \frac{1}{2}\sqrt{\frac{\hbar\nu}{P(L)\,t}} \sim \frac{1}{\sqrt{N_{\text{photon}}}}$$

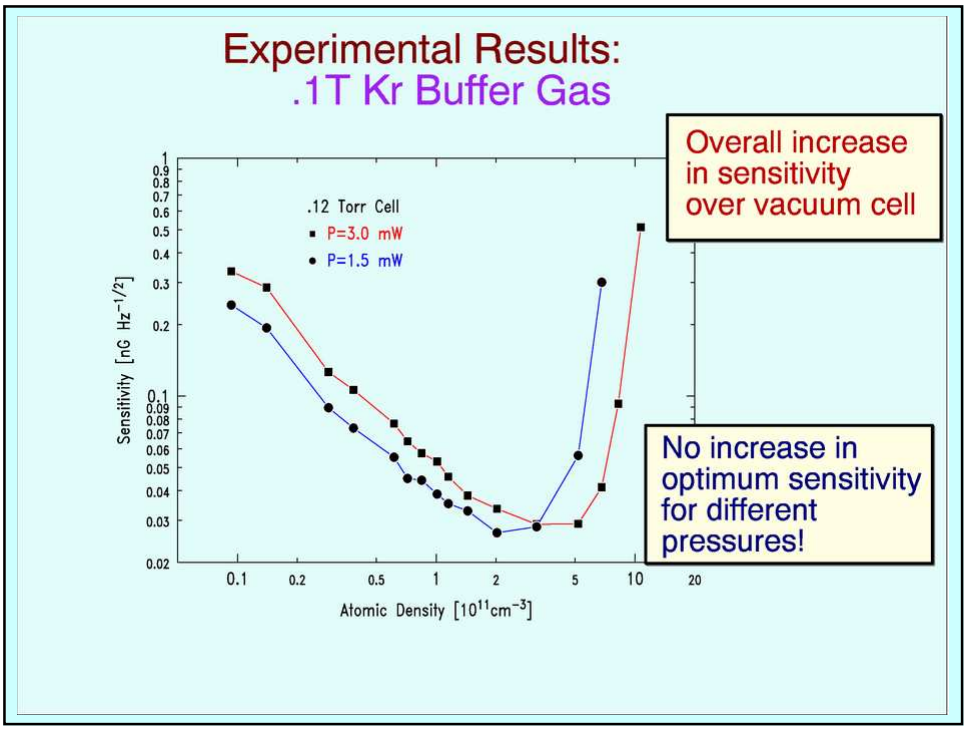
Issues Studied:

- Effect of increasing power and density
- Effect of increasing laser beam diameter (decreasing linewidth)
  Novikova et al., J. Mod. Opt., **49**, 349 (2002)
- Effect of buffer gas for decreasing linewidth
  (Manuscript in progress)


Experimental Results:
Vacuum Cell
vacuum Cell
P=1.0 mW
P=2.0 mW
Sensitivity [nG Hz$^{-1/2}$]
Atomic Density [10^{11}cm^{-3}]
Effect of increasing power and density
.12 Torr Cell
P=3.0 mW
P=1.5 mW
Overall increase in sensitivity over vacuum cell
.1T Kr Buffer Gas
10^{-11} G/√Hz
Novikova et al.,
J. Mod. Opt., 49, 349 (2002)


Experimental Results:
Vacuum Cell
vacuum Cell
P=1.0 mW
P=2.0 mW
Sensitivity [nG Hz$^{-1/2}$]
Atomic Density [10^{11}cm^{-3}]
Effect of increasing power and density
Higher power gives better sensitivity
Novikova et al.,
J. Mod. Opt., to be published


Experimental Results:
Vacuum Cell
Effect of increasing laser beam diameter (decreasing linewidth
vacuum Cell P=2mW
3mm beam
13mm beam
Sensitivity [nG Hz$^{-1/2}$]
Atomic Density [10^{11}cm^{-3}]
Higher spectroscopic resolution gives better sensitivity


Experimental Results:
.1T Kr Buffer Gas
Overall increase in sensitivity over vacuum cell
.12 Torr Cell
P=3.0 mW
P=1.5 mW
Sensitivity [nG Hz$^{-1/2}$]
Atomic Density [10^{11}cm^{-3}]
No increase in optimum sensitivity for different pressures!

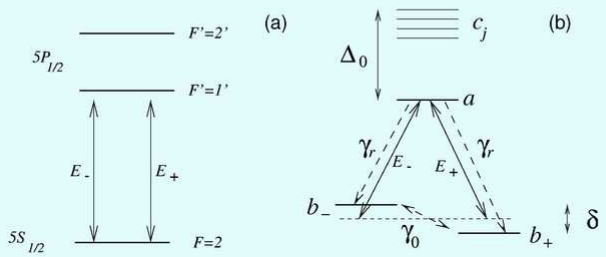
Limitations for Magnetometry

Issues Studied:

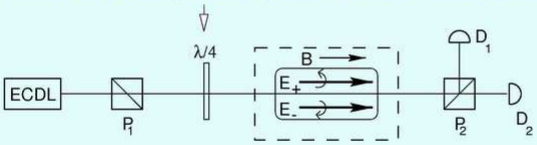
- AC-Stark shifts: line broadening for Faraday magnetometer
  M. Fleischhauer, A. B. Matsko, and M. O. Scully, PR**A 62** (2000)
  Novikova et al., Opt. Lett. **25**, 1851 (2000).
- Compensation of AC-Stark shifts: effect on Sensitivity
  Novikova et al., PRA **63** 063802 (2001).
- Effect radiation trapping in optically dense EIT system
  Matsko et al., J. Mod. Opt., to be published.
  Novikova et al., PRL 87, 133601 (2001)

AC Stark Effect

M. Fleischhauer, A. B. Matsko, and M. O. Scully, PR**A 62** (2000)

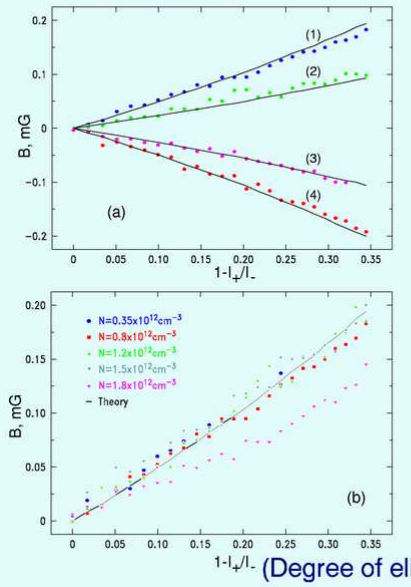


Induce E+/E– asymmetry with quarter-wave plate



Measure field needed to compensate for ac-Stark shift

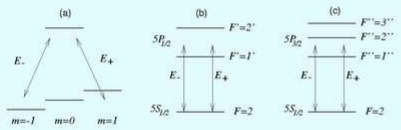
Results



From Experiment:	$\Delta_0 \approx 5\,\mathrm{GHz}$
Optimal laser power:	$P_{\mathrm{in}} \approx 2\,\mathrm{mW}$
Optimal sensitivity:	$\Delta B_{\mathrm{min}} \approx 6 \times 10^{-12}\,\mathrm{G}/\sqrt{\mathrm{Hz}}$

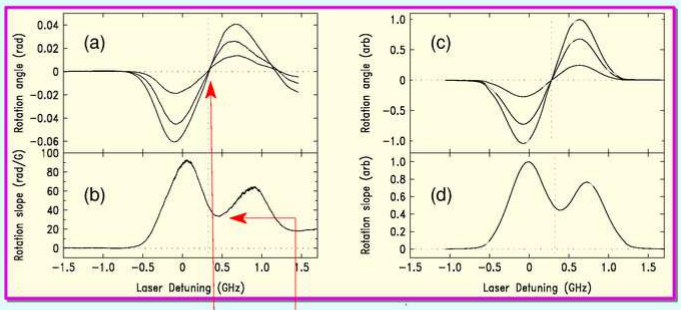
Novikova et al.,
Opt. Lett. **25**, 1851 (2000).

Compensation of AC-Stark Shifts



Compensate for AC-Stark shift by detuning laser into hyperfine manifold

D1 line:

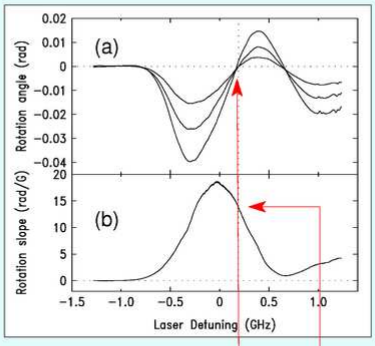


Zero rotation for any ellipticity

Reduced sensitivity

Novikova et al., PRA **63** 063802 (2001).

What about D2 line?



Zero rotation for any ellipticity

Reduced sensitivity

Good News!

What About Radiation Trapping?

``A particularly important effect which must be considered in all optical pumping experiments is the trapping of the pumping light by multiple scattering within the vapor... Detailed studies of the effect of trapping on the ground state have not been made...''

W. Happer, Rev. Mod. Phys. **44**, 169 (1972).

Theory:

D. Peterson and L. W. Anderson, PRA **43**, 4883 (1991).

- Total destruction of optical pumping in vapor cell for N ~ 10^{13} cm^{-3}

Experiment:

Ankerhold et al., PRA **48**, R4031 (1993).

- Radiation transport (spatial and temporal effects) in sodium vapor with argon buffer gas.

EIT: Very little prior work !

What About Radiation Trapping and Coherence Effects ?

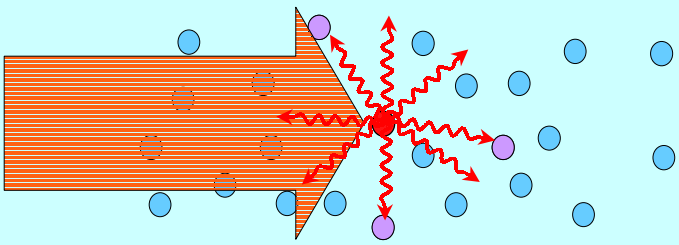
Theory:

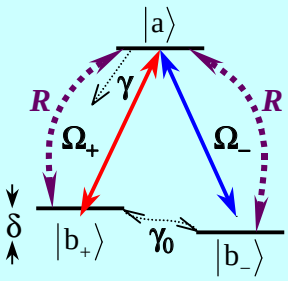
M. Fleischhauer, Europhys. Lett. 45, 659 (1999).

Experiment:

?

Radiation trapping: model





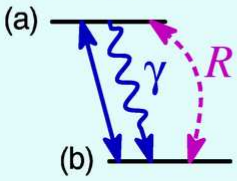
$$\frac{I_{out}}{I_{in}} = 1 - \frac{\kappa L}{|\Omega(0)|^2}\underbrace{(\gamma_0 + R)}$$

$$\left.\frac{d\phi}{dB}\right|_{B\to 0} = -\frac{2\mu_B}{\hbar\underbrace{(\gamma_0 + R)}}\ln\left(\frac{I_{out}}{I_{in}}\right)$$

How to Treat Radiation Trapping?

Model as Incoherent Pumping:

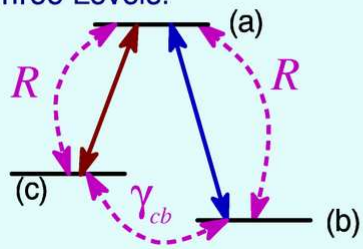
Two Level System:



Assume nearly all population in ground state:

$$\dot{\sigma}_{aa} = R(\sigma_{bb} - \sigma_{aa}) - 2\gamma\sigma_{aa}$$
$$\simeq R - 2\gamma\sigma_{aa} \qquad \sigma_{bb} \approx 1$$

Three Levels:



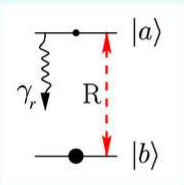
If $\gamma \gg R \sim \gamma_{cb}$

then, effectively:

$$\gamma_{cb} \longrightarrow \gamma_{cb} + R \equiv \gamma_{\text{eff}}$$

Worry: Radiation trapping broadens the resonance

Two level Model:



Reduced density operator

$$\dot{\rho}(t) = -\bar{n}_{th}\gamma_r\left[\hat{\sigma}_-\hat{\sigma}_+\rho(t) - \hat{\sigma}_+\rho(t)\hat{\sigma}_-\right] - (\bar{n}_{th}+1)\gamma_r\left[\hat{\sigma}_+\hat{\sigma}_-\rho(t) - \hat{\sigma}_-\rho(t)\hat{\sigma}_+\right] + H.c.,$$

where

$\bar{n}_{th}$ = thermal photon number in reservoir
γ_r = atomic decay rate
$\hat{\sigma}_- = |b\rangle\langle a|$ and $\hat{\sigma}_+ = |a\rangle\langle b|$

Eq. of motion

$$\dot{\rho}_{aa} = -2\gamma_r(\bar{n}_{th}+1)\rho_{aa} + 2\gamma_r\bar{n}_{th}\rho_{bb}$$

So

$$R = 2\gamma_r\bar{n}_{th}$$

Introduce

γ_R = photon escape rate
r = pumping rate due to atomic decay

Use rate Eq.

$$\dot{\bar{n}}_{th} = -\gamma_R\bar{n}_{th} + r\rho_{aa},$$

Steady state

$$n_{th} = r\rho_{aa}/\gamma_R$$

Introduce $f(N)$ which characterizes the radiation trapping

$$r/\gamma_R = f/(1+f)$$

where $f(N) \geq 0$ $(f(N=0)=0)$. Note: $r/\gamma_R < 1$ characterizes number of spontaneous photons per excited atom. Then

$$\dot{\rho}_{aa} \approx -2\gamma_r \frac{\rho_{aa}}{1+f(N)} \quad \text{and} \quad R = 2\gamma_r \frac{f(N)}{f(N)+1}\rho_{aa}$$

Approximate as weak absorption:

$$d|\Omega|^2/dz \simeq -2\gamma_r^2 \kappa \rho_{aa}$$

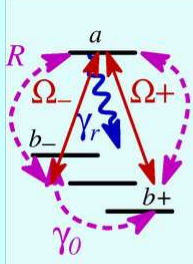
where

$$\Omega = \wp E/\hbar \quad \text{and} \quad \kappa = 3N\lambda^2/8\pi$$

So

$$R = -\frac{1}{\kappa\gamma_r}\frac{f(N)}{1+f(N)}\frac{\mathrm{d}}{\mathrm{d}z}|\Omega|^2$$

Nonlinear Faraday Rotation



Density matrix equations:

$$\begin{aligned}
\dot{\sigma}_{b\pm b\pm} &= -R(\sigma_{b\pm b\pm} - \sigma_{aa}) + \gamma\sigma_{aa} - i(\Omega_\pm \sigma_{ab\pm} - c.c.) \\
\dot{\sigma}_{ab\pm} &= -\Gamma_{ab\pm}\sigma_{ab\pm} - i\Omega_\pm^*(\sigma_{b\pm b\pm} - \sigma_{aa}) - i\Omega_\mp^*\sigma_{b\mp b\pm} \\
\dot{\sigma}_{b-b+} &= -\Gamma_{b-b+}\sigma_{b-b+} - i\Omega_-\sigma_{ab_+} + i\Omega_+^*\sigma_{b-a}
\end{aligned}$$

where

$$\Gamma_{ab\pm} \equiv \gamma + \frac{3}{2}R \pm i\frac{\delta_0}{2} \qquad \Gamma_{b-b+} \equiv \gamma_0 + R + i\delta_0 \quad \text{and} \quad \delta_0 = \frac{2g\mu_B B}{\hbar}$$

Separate phase and intensities

$$\Omega_\pm(z) = |\Omega_\pm(z)|e^{\phi_\pm(z)} \quad \text{and} \quad |\Omega|^2 = |\Omega_+|^2 + |\Omega + -|^2 \quad \phi = \phi_- - \phi_+$$

Find stationary solution of Bloch-equations on resonance to lowest order in γ_0, R, and δ_0. Assume $|\Omega| \gg W_d\sqrt{(\gamma_0+R)/\gamma_r}$

$$\frac{\mathrm{d}}{\mathrm{d}z}|\Omega|^2 = -\kappa(\gamma_0 + R) \quad \text{and} \quad \frac{\mathrm{d}}{\mathrm{d}z}\phi = 2\delta_0\frac{\kappa}{|\Omega|^2}$$

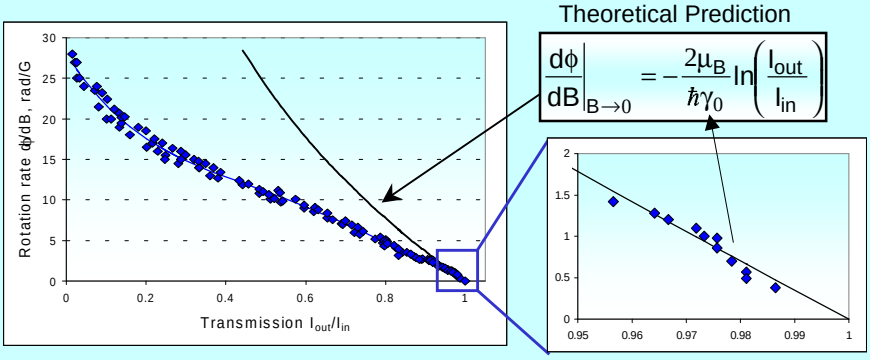
substitute expression for R and integrate

$$\left.\frac{d\phi(z)}{dB}\right|_{B\to 0} = \frac{2\mu_b}{\hbar(\gamma_0+R)}\ln\left|\frac{\Omega(0)}{\Omega(z)}\right|^2$$

Measurement of rotation and transmission gives R

Matsko et al., J. Mod. Opt., to be published.

Radiation trapping



Theoretical Prediction

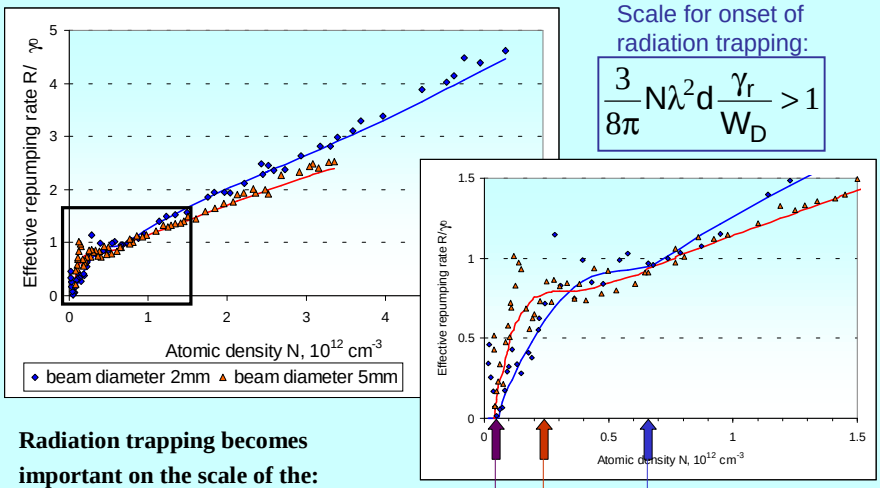
$$\left.\frac{d\phi}{dB}\right|_{B\to 0} = -\frac{2\mu_B}{\hbar\gamma_0}\ln\left(\frac{I_{out}}{I_{in}}\right)$$

Something is causing the **deterioration** of ground-state **coherence** with atomic density. We believe it is the reabsorption of spontaneously emitted photons (**radiation trapping**).

Matsko, Novikova, Scully, Welch, PRL, **87**, 133601 (2001)

Matsko, Novikova, Scully, Welch, JMO**, 49,** 367 (2002).

Experimental confirmation



Scale for onset of radiation trapping:

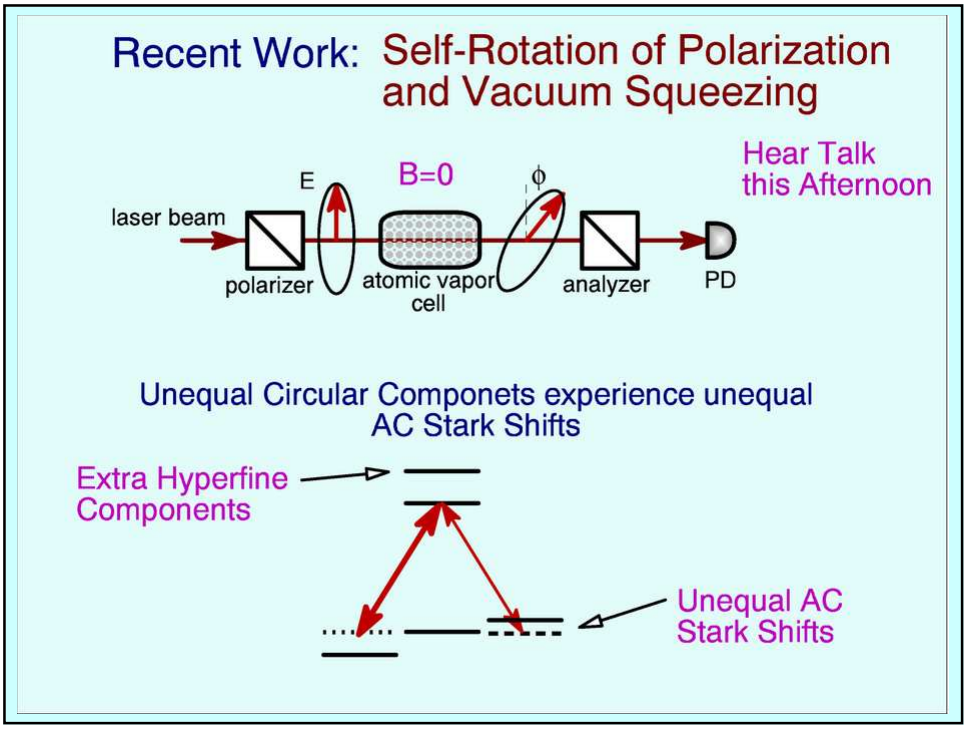
$$\frac{3}{8\pi}N\lambda^2 d\frac{\gamma_r}{W_D} > 1$$

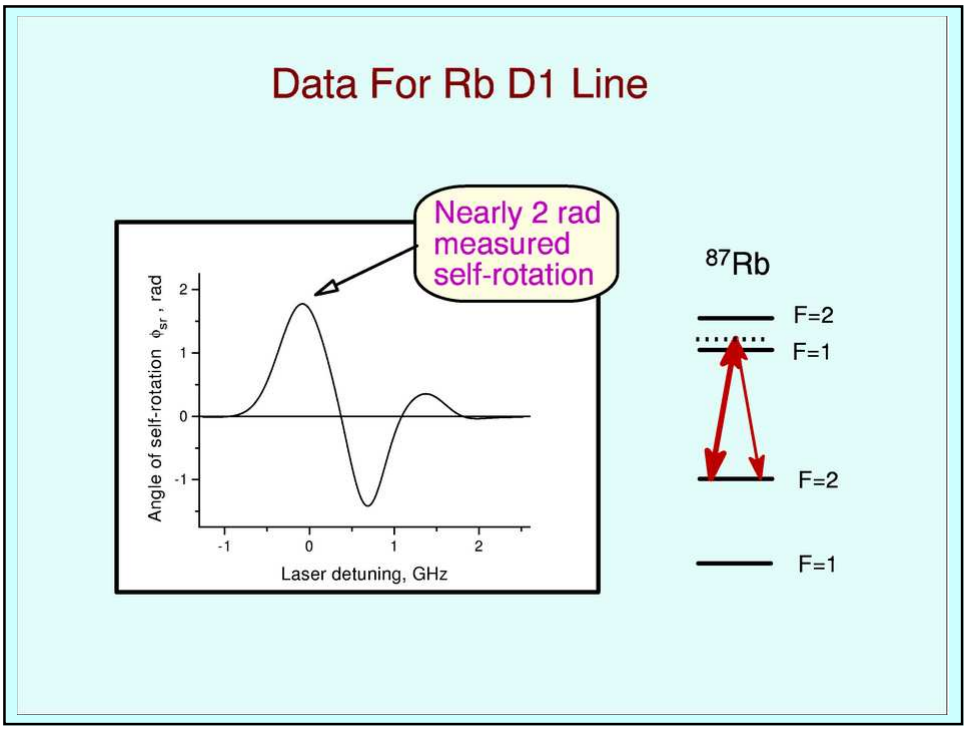
Radiation trapping becomes important on the scale of the:

Cell (diameter 25mm) - N~5·10^{10} cm^{-3}

Laser beam (diameter 2mm) - N~7·10^{11} cm^{-3}

Laser beam (diameter 5mm) - N~3·10^{11} cm^{-3}


Recent Work: Self-Rotation of Polarization and Vacuum Squeezing
Hear Talk this Afternoon
E
B=0
ϕ
laser beam
polarizer
atomic vapor cell
analyzer
PD
Unequal Circular Componets experience unequal AC Stark Shifts
Extra Hyperfine Components
Unequal AC Stark Shifts


Data For Rb D1 Line
Nearly 2 rad measured self-rotation
Angle of self-rotation ϕ_{sr}, rad
Laser detuning, GHz
^{87}Rb
F=2
F=1
F=2
F=1

Vacuum Squeezing

Electromagnetic Field Operator:

$$\begin{aligned}\hat{E}_x &= E_0\left(\hat{a}e^{i(kz-\omega t+\chi)} + \hat{a}^\dagger e^{-i(kz-\omega t+\chi)}\right)\\ &= 2E_0\left(\underbrace{\left(\hat{a}e^{i\chi} + \hat{a}^\dagger e^{-i\chi}\right)}_{\hat{B}_1}\cos(kz-\omega t) + \underbrace{\left(\hat{a}e^{i\chi} - \hat{a}^\dagger e^{-i\chi}\right)}_{\hat{B}_2}\sin(kz-\omega t)\right)\end{aligned}$$

$\hat{B}_1$ and $\hat{B}_2$ are the *quadrature operators* used to characterize the properties of the electromagnetic field.

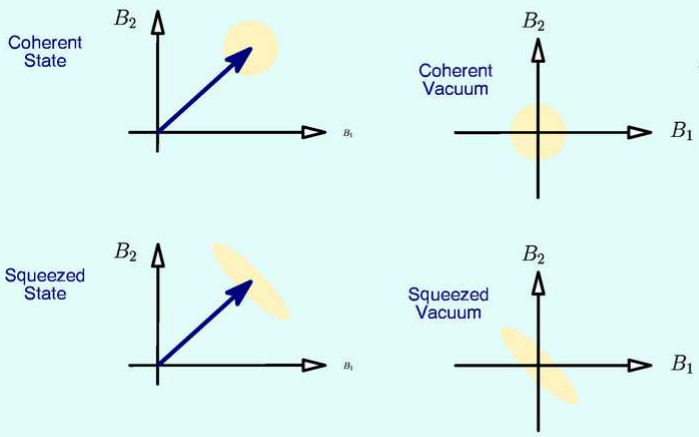
Quadrature Operators:

$$\begin{aligned}\hat{B}_1 &= \hat{a}e^{i\chi} + \hat{a}^\dagger e^{-i\chi}\\ \hat{B}_2 &= \hat{a}e^{i\chi} - \hat{a}^\dagger e^{-i\chi}\end{aligned}$$

Uncertainty relationship

$$\Delta B_1\,\Delta B_2 \geq \frac{1}{4}$$

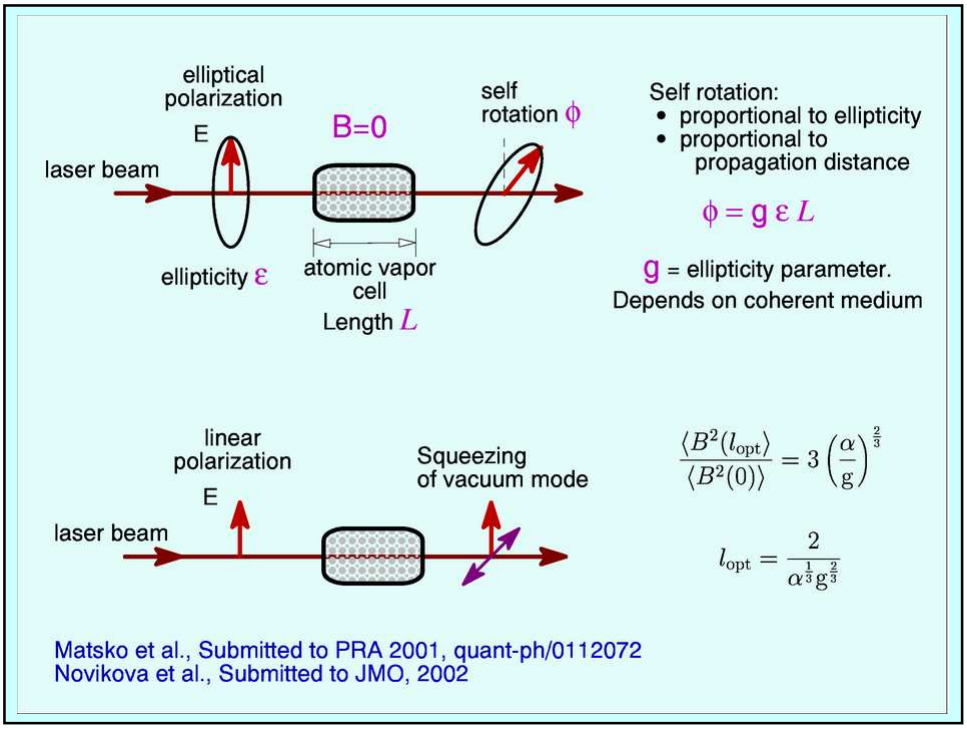
$$\Delta B = \sqrt{\langle \hat{B}^2\rangle - \langle \hat{B}\rangle^2}\,.$$

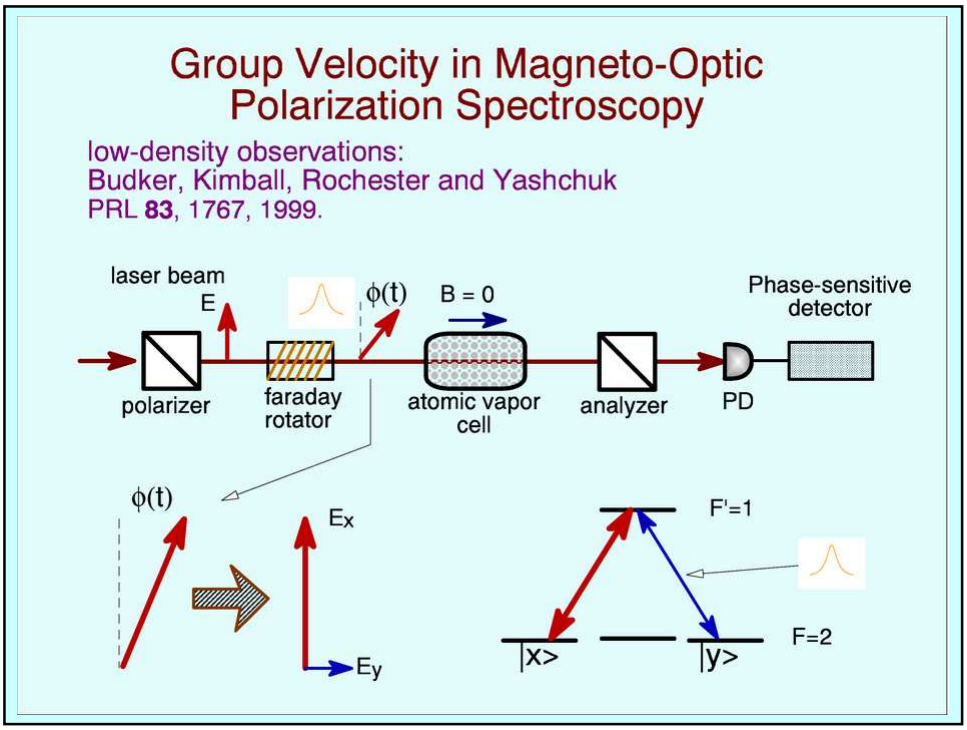


$$\Delta B_1 = \Delta B_2 = \frac{1}{2}$$

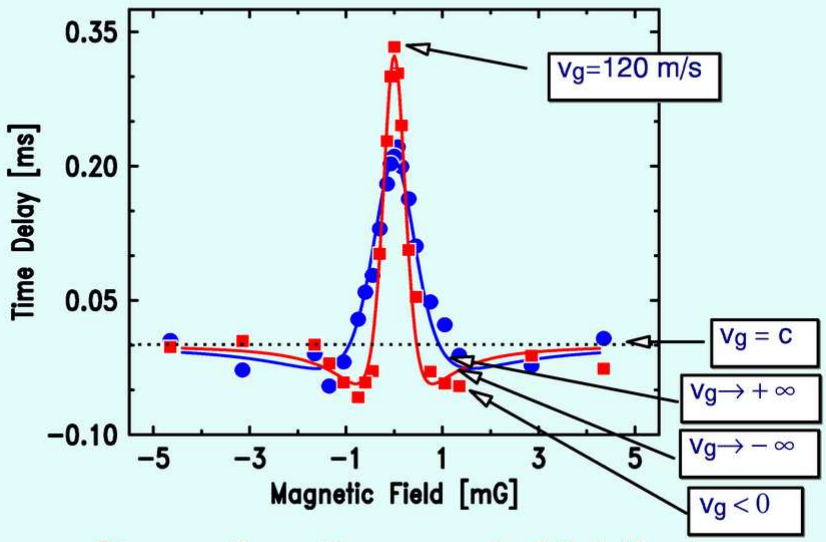
$$\Delta B_1 > \frac{1}{2}$$

$$\Delta B_1 < \frac{1}{2}$$


elliptical polarization
E
laser beam
B=0
self rotation ϕ
ellipticity ε
atomic vapor cell
Length L
Self rotation:
• proportional to ellipticity
• proportional to propagation distance
ϕ = g ε L
g = ellipticity parameter.
Depends on coherent medium
linear polarization
E
laser beam
Squeezing of vacuum mode
$\frac{\langle B^2(l_{\rm opt}\rangle}{\langle B^2(0)\rangle} = 3\left(\frac{\alpha}{g}\right)^{\frac{2}{3}}$
$l_{\rm opt} = \frac{2}{\alpha^{\frac{1}{3}} g^{\frac{2}{3}}}$
Matsko et al., Submitted to PRA 2001, quant-ph/0112072
Novikova et al., Submitted to JMO, 2002


Group Velocity in Magneto-Optic Polarization Spectroscopy
low-density observations:
Budker, Kimball, Rochester and Yashchuk
PRL 83, 1767, 1999.
laser beam
E
ϕ(t)
B = 0
Phase-sensitive detector
polarizer
faraday rotator
atomic vapor cell
analyzer
PD
ϕ(t)
Ex
Ey
F'=1
F=2
|x>
|y>

Measured Group Delay



Slower than the speed of light!

Application to Magnetometry

Status:

Width: Γ=100 mG
Rotation: $d\phi/dB$ = 100 rad/G
Power: P = 3mW

Sensitivity:

$B_{min} = \Delta f_{min}/(d\phi/dB)$
Shot-noise limit:
$d\phi_{min} = (\hbar\omega/Pt)^{1/2}$
$\rightarrow B_{min} <\sim 10^{-10} - 10^{-11}$ G/Hz

Outlook:

Higher Power: P = 100 mW
Higher Density $N = 10^{13}$ cm^{-3}
Buffer Gas, Squeezed Vacuum
$\rightarrow B_{min} <\sim 10^{-11} - 10^{-12}$ G/Hz

Conclusion:

- Slow Light for Fun and Profit?