A Road to local quantum criticality

Karyn Le Hur

Sherbrooke & CIAR

Collaborators: Mei-Rong Li (Sherbrooke)
Philippe Doucet-Beaupré (Sherbrooke)
Walter Hofstetter (MIT and Aachen)

Observability: Heavy-fermion materials?
Noisy mesoscopic qubits!

sketch of the summary

Bose-Fermi Kondo model? Results from ε expansion

Ohmic case and Ising coupling with the bosons:
Hidden Caldeira-Leggett model

Quantum phase transition

Exact NRG for bosons coupled to spins: Computation of spin properties close to the quantum phase transition

Applicability to noisy mesoscopic qubits

Spin coupled to bosons

From Zarand & Demler, PRB 2002

$$H_B^{\mathrm{int}} = \sum_\alpha \Lambda^{\epsilon/2} \gamma_\alpha S^\alpha \phi^\alpha$$

ε measures deviations from ohmic bosons

« Sub-ohmic case »

$$g_\mu = \gamma_\mu{}^2$$

$$\langle T\phi_\alpha(\tau)\phi_\beta(0)\rangle = cst.\ \frac{\delta_{\alpha\beta}}{\tau^{2-\epsilon}}$$

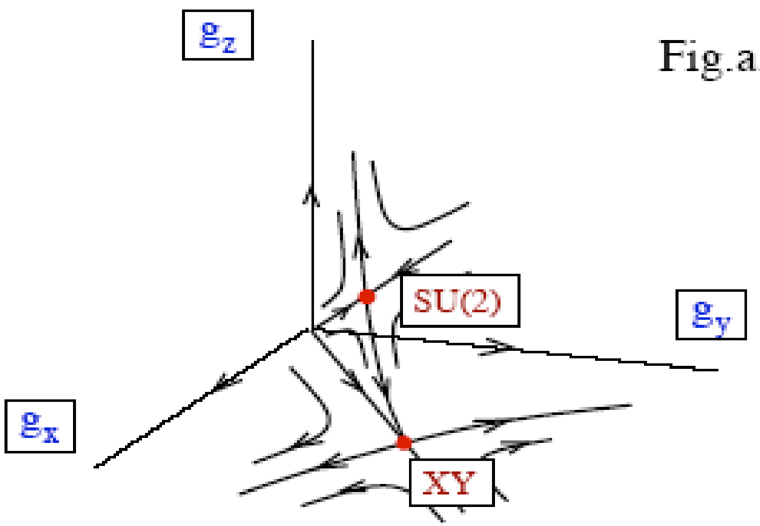


Fig.a.

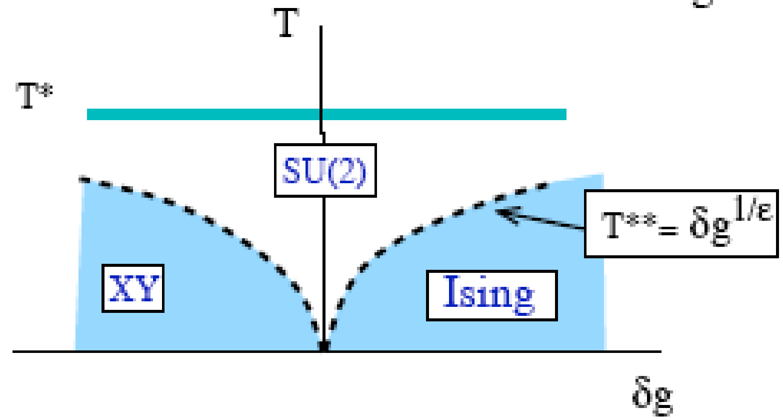


Fig.b.

Large N + NCA: M. Vojta, C. Buragohain, S. Sachdev, PRB 2000

By including the fermions: SU(2) symmetry

ε expansion

$$\chi^{SU(2)}_{\alpha\beta}(\tau) = \langle T S_\alpha(\tau) S_\beta(0)\rangle = \delta_{\alpha\beta}\chi(\tau)$$

$$\chi(\tau) = \frac{1}{(T^*\tau)^\epsilon}$$

$\chi(T) \sim 1/T^{1-\varepsilon}$

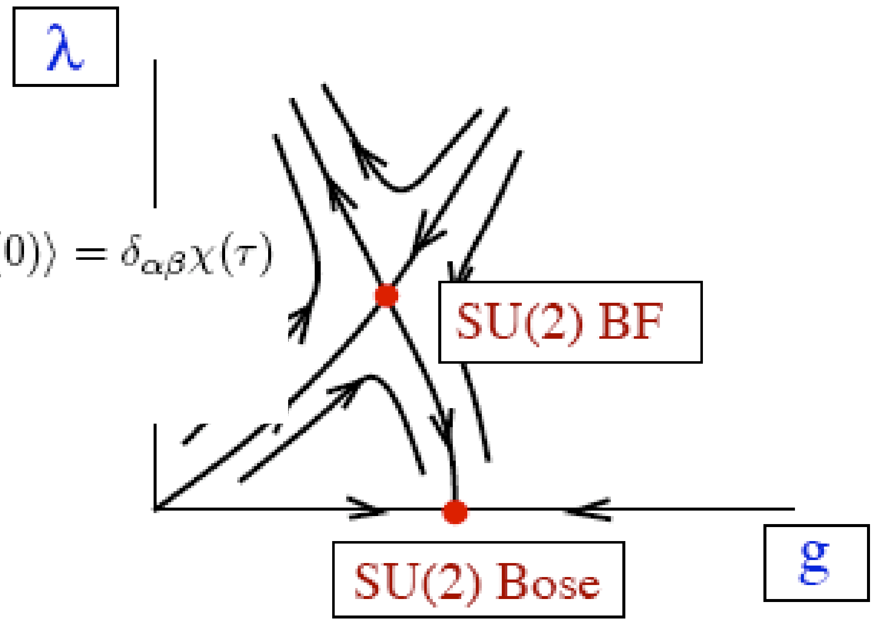
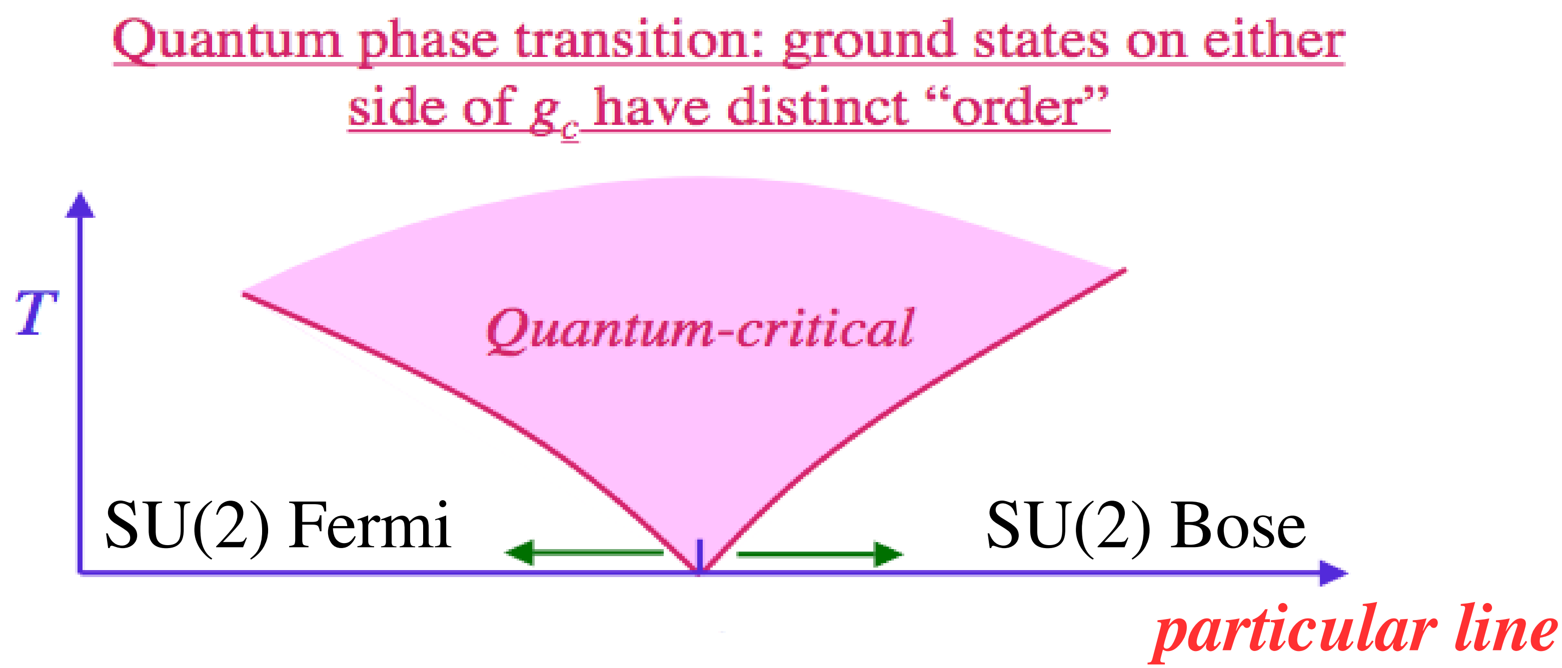


FIG. 2. Renormalization group flows for the SU(2) symmetrical Bose-Fermi Kondo model. "BF" denotes the SU(2) symmetrical Bose-Fermi fixed point. Here the spin is partially screened and both the Fermi and Bose fields couple to it. At the Bose fixed point the fermionic degrees of freedom fully decouple from the spin.

Quantum critical point $g \sim \lambda \sim \varepsilon$

λ denotes the Kondo coupling with electrons

Road to local quantum criticality…



Smith and Q. Si, EuroPhys. Lett 1999
A. Sengupta, PRB 2000

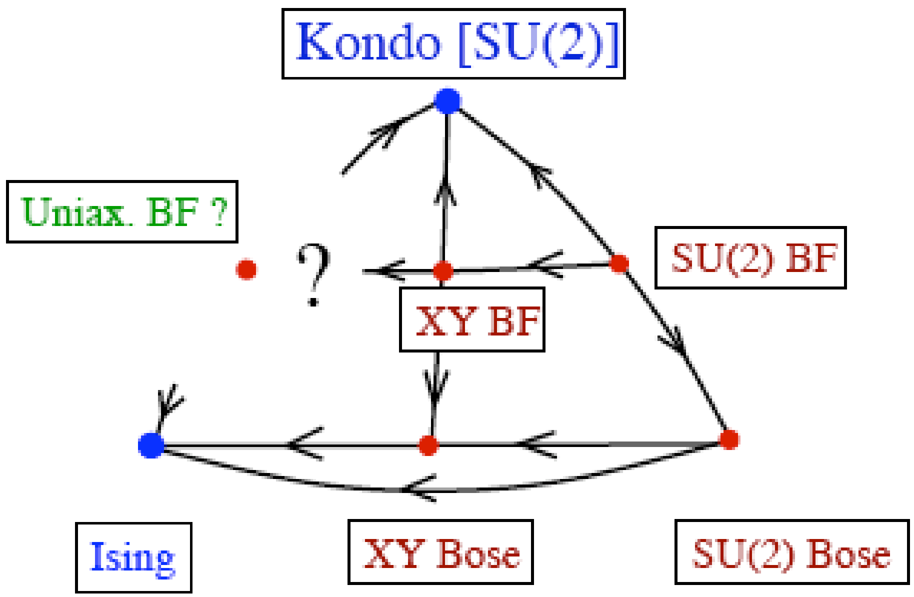
On the Bose-Fermi Kondo model: **Q. Si et al. Nature 413, 804 (2001)**

Exact impurity solver?

Take into account different anisotropies

Applicability to noisy qubits

K. Le Hur, PRL 2004



Ohmic case, ε=0

-exact mapping on the Caldeira-Leggett model

-endeavors on NRG applied to bosonic systems coupled to spin

Mei-Rong Li, Karyn Le Hur, and Walter Hofstetter: cond-mat/0501755

The starting point

$$H = hS_z + H_{sf} + H_{sb}, \quad (1)$$

$$H_{sf} = \frac{J_\perp}{2}\left(\Psi^\dagger_\downarrow(0)\Psi_\uparrow(0)S_+ + \text{h.c.}\right)$$
$$+v_f \sum_{\sigma=\uparrow,\downarrow} \int_{-\infty}^{\infty} dx\ \Psi^\dagger_\sigma(x) i\partial_x \Psi_\sigma(x), \quad (2)$$

$$H_{sb} = -\frac{v_b \partial_x \Phi(0)}{\sqrt{2K_b}} S_z + \frac{v_b}{4\pi}\int_{-\infty}^{\infty} dx [\partial_x \Phi(x)]^2, \quad (3)$$

ϕ: bosons with ohmic spectrum

g_z not renormalized when J=0

ψ: fermions with Dirac spectrum

A J_z fermionic coupling will slightly affect the mapping

Caldeira-Leggett mapping

First: Bosonization of the fermionic bath

$$H_{sf} = \frac{\Delta}{2}\left[e^{i\sqrt{2}\varphi(0)}S_{+} + h.c.\right] + \frac{v_f}{4\pi}\int_{-\infty}^{\infty} dx\left[\partial_x\varphi(x)\right]^2$$

$$\Delta = \left(J_{\perp}\omega_c/2\pi v_f\right)$$

Now: Spin coupled to two bosonic baths

(this point was previously noted by Grempel and Q. Si, PRL 2003)

Eventually, for the ohmic case, we can recombine exactly those two environments $\longrightarrow$ Caldeira-Leggett physics

Explore local actions

By integrating over degrees of freedom of « space »:

$$S_{\varphi}^{loc} = \frac{T}{2\pi}\sum_{\omega_n}|\omega_n|\varphi_0(\omega_n)\varphi_0(-\omega_n)$$ **fermionic**

$$S_{\Phi}^{loc} = \frac{T}{2\pi}\sum_{\omega_n}|\omega_n|\Phi_0(\omega_n)\Phi_0(-\omega_n)$$ **bosonic**

$$\Phi_0 = \Phi(x=0) \text{ and } \varphi_0 = \varphi(x=0)$$

Then, unitary transformation:

$$\mathcal{U}_1 = \exp\{-i\Phi_0 S_z/\sqrt{2K_b}\}$$

Finally…

$$\varphi_s = (2\alpha)^{-1/2}[\sqrt{2}\varphi_0 + (2K_b)^{-1/2}\Phi_0] \text{ and } \varphi_a = (2\alpha)^{-1/2}[(2K_b)^{-1/2}\varphi_0 - \sqrt{2}\Phi_0]$$

$$S_{\varphi_s} = \frac{T}{2\pi}\sum_{\omega_n}|\omega_n|\varphi_s(\omega_n)\varphi_s(-\omega_n) - \frac{\Delta}{2}\left[e^{i\sqrt{2\alpha}\varphi_s(0)}S_+ + h.c.\right]$$

Two-level system with dissipation
Leggett et al. Review of Mod. Phys. (1987)

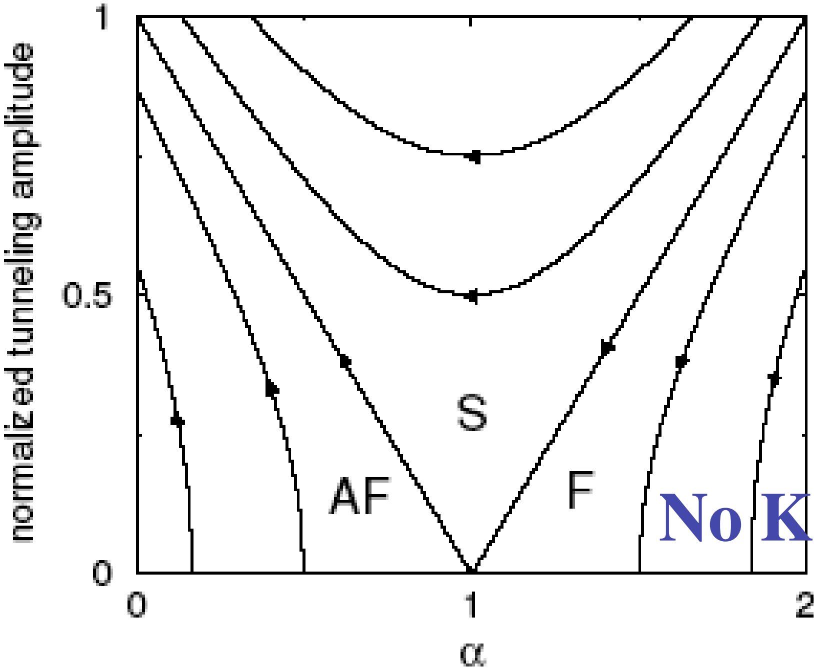
$$\tilde{H} = H_S - \sqrt{2\alpha}v_s\partial_x\varphi_s(0)S_z + \frac{v_s}{4\pi}\int_{-\infty}^{\infty}dx[\partial_x\varphi_s(x)]^2$$

$$H_S = hS_z + \Delta S_x$$

Keep in mind that here: $\alpha = 1 + (4K_b)^{-1}$

Brief reminder…

SU(2) Kondo



No Kondo effect

S: Strong tunneling

AF: Kondo realm

Bethe-Ansatz region

F: ferromagnetic Kondo

Unscreened spin

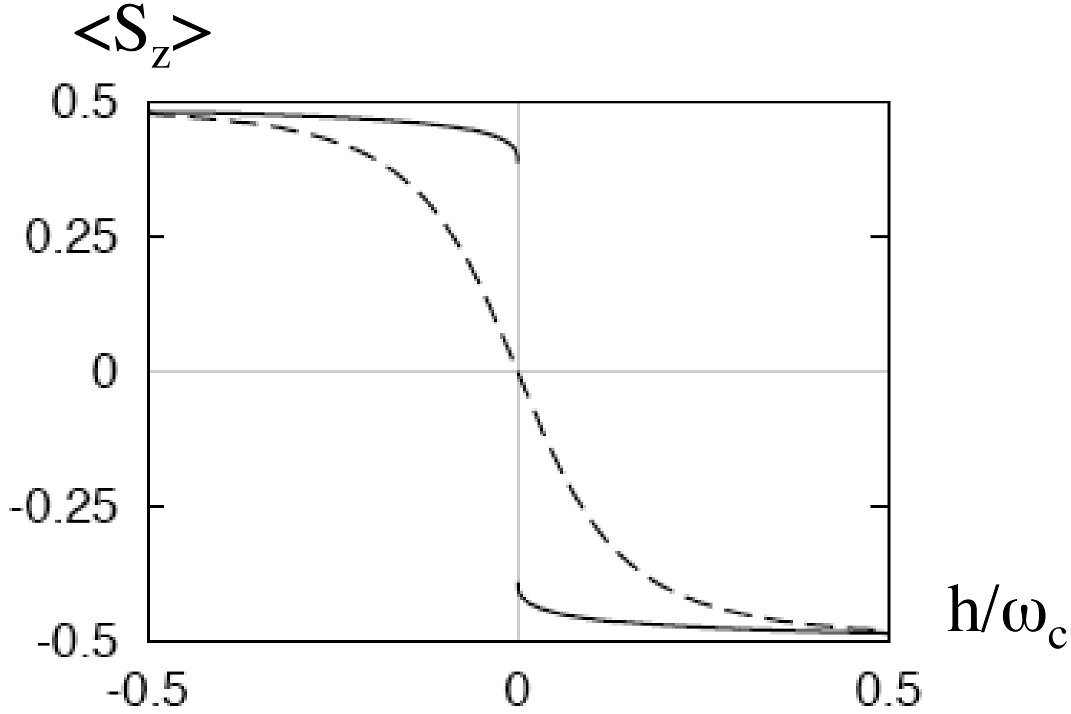
Analogy with the anisotropic Kondo model:

This diagram has been investigated by Anderson, Yuval, Hamman

Bose-Fermi Kondo model: α in the vicinity of the quantum phase trans.:

« How to compute something? »

Kosterlitz-Thouless transition



Kondo realm: exact Bethe-Ansatz calculation
Ferromagnetic region: perturbative expansion in Δ/ω_c

But also necessary to compute quantities versus T close to α_c

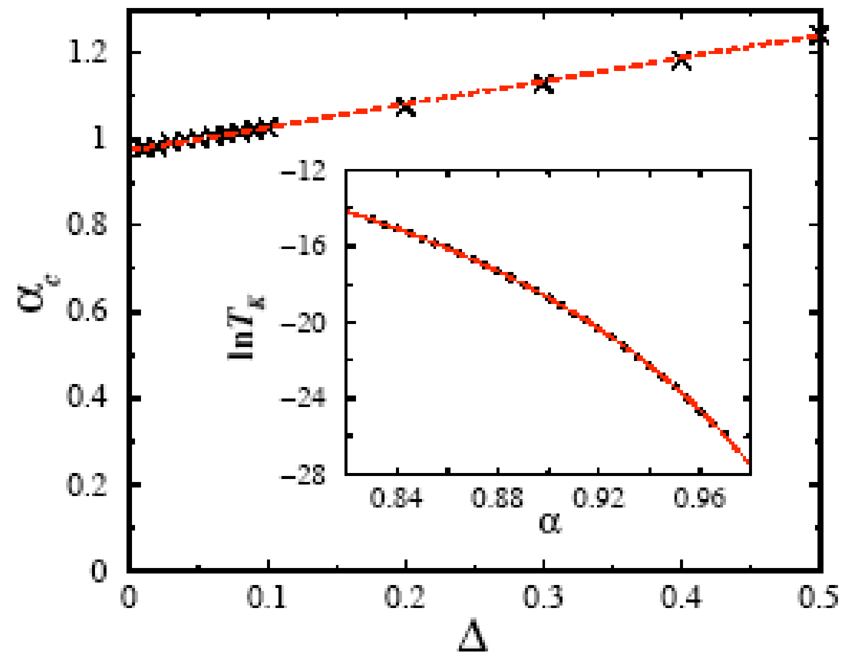
NRG for bosons coupled to spin

Key steps:

-Logarithmic discretization of the energy interval
-Transformation to a spin coupled to a semi-infinite chain
-Recursive parameter N: number of involved sites
-Truncation of the iteration process for a large $N_o \sim 25$
-Truncation of boson states also necessary
(N_{bo}=500, and then N_b= 8)

R. Bulla, N.-H. Tuong, and M. Vojta, PRL (2003)
M.-R. Li, K. Le Hur, and W. Hofstetter, cond-mat/0501755
Explicit computations of $\langle S_z \rangle$ and $\chi_{loc}(T)$
M.T. Glossop and K. Ingersent, cond-mat/0501601

Very promissing results

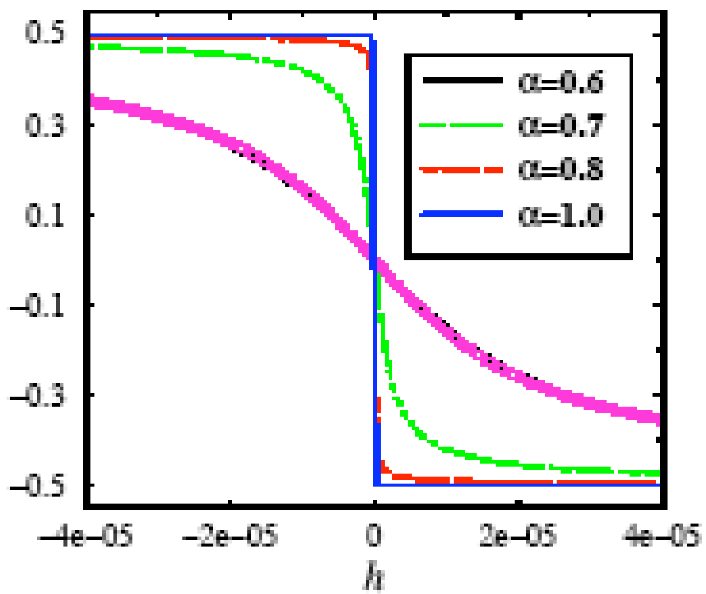


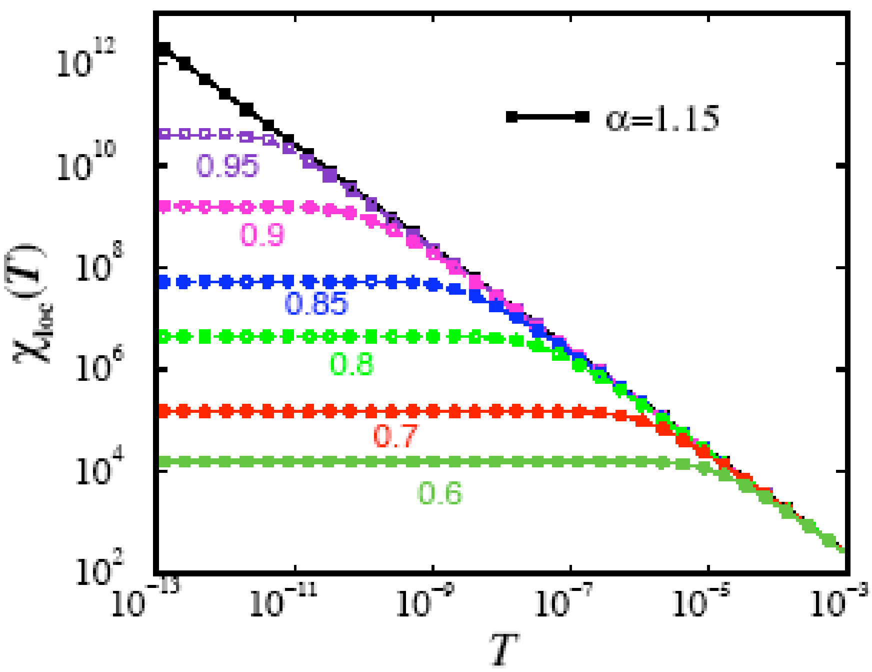
Kosterlitz-Thouless transition: T_K vanishes expt fast close to α_c

We recover the perturbative RG scaling (K. Le Hur, PRL 2004)

$$(K_b)_c^{-1} = J_\perp/\pi v_f = 2\Delta/\omega_c$$

Glimpse on physical quantities





We recover Bethe-Ansatz results for the Kondo phase: Restoration of SU(2) symmetry

χ_{loc}(T) en 1/T at the quantum critical point for ε=0

Deviation from J_z=0

Region around α=1:

Caldeira-leggett model with $\sqrt{2}(\frac{1}{4\pi}J_z - \frac{\sqrt{\alpha}}{2}v_s)\partial_x\varphi_s(0)S_z$

which immediately results in

$$\alpha_c \simeq 1 + \mathcal{O}(\Delta) + \mathcal{O}(J_z)$$

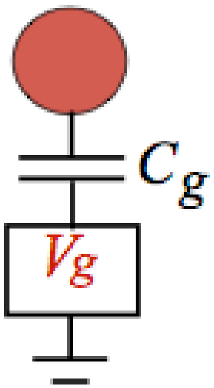
Open question: role of transverse coupling with the bosons?

Applicability of Ohmic case and J_z=0: Noisy mesoscopic qubits

- Large $E_c = e^2/2C$

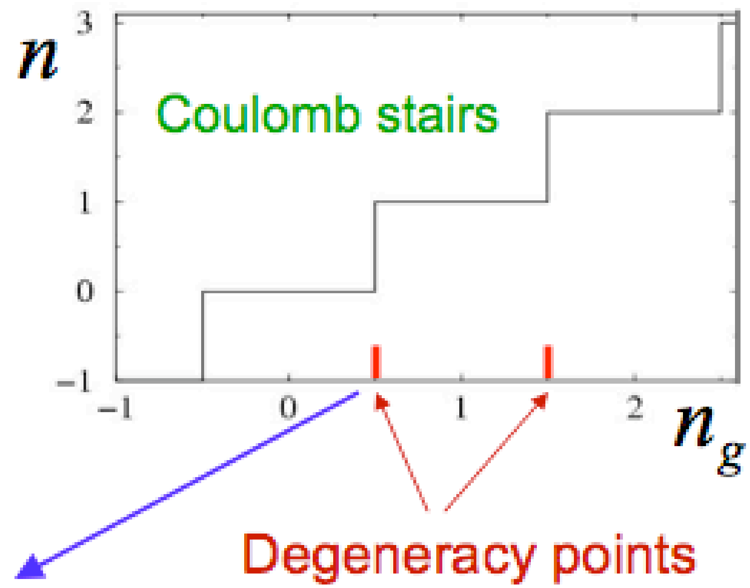
  C : capacitance decreases with size

- Gate voltage controls electron number



$$E_{ch} = E_c(n - n_g)^2$$

$$n_g = C_g V_g / e$$



Two-state system

Artificial spin S =1/2

$\uparrow$ $n = 1$ state

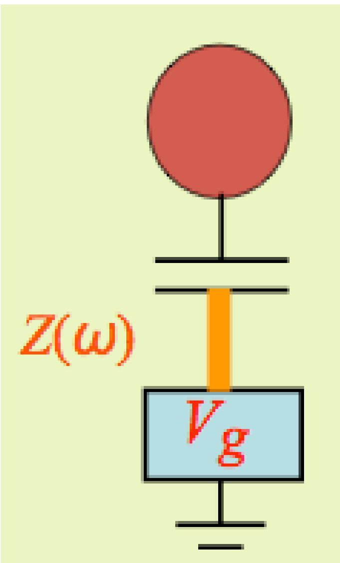
$\downarrow$ $n = 0$ state

$$E_{ch} = h S_z \qquad h = 2E_c(1/2 - n_g)$$

Resonant level

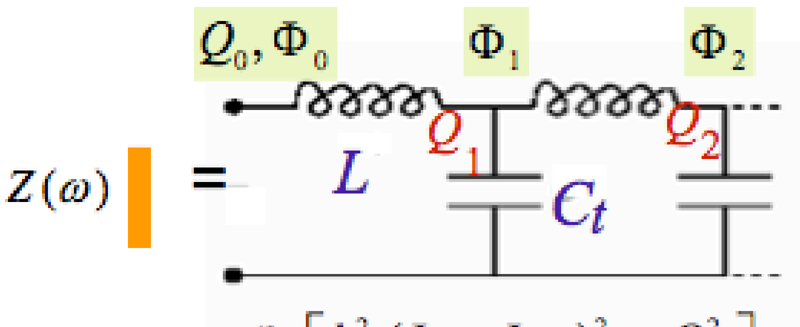
Karyn Le Hur, PRL 92, 196804 2004

Gate voltage fluctuation: quantum noise



Fluctuations in electromagnetic medium …

Model the quantum noise: Transmission lines



$$Z(\omega) \approx R$$

Resistance $R = \sqrt{L / C_t}$

$$H_{noise} = \sum_{i=0}^{\infty} \left[\frac{\hbar^2}{e^2} \frac{(\Phi_i - \Phi_{i-1})^2}{2L} + \frac{Q_i^2}{2C_t} \right]$$

$$\left[\frac{\hbar}{e} \Phi_i, Q_j \right] = i\hbar\delta_{ij}$$

(coupled) quantum harmonic oscillators (bosonic bath)

Voltage fluctuations $\delta V_g = Q_0 / C_g$

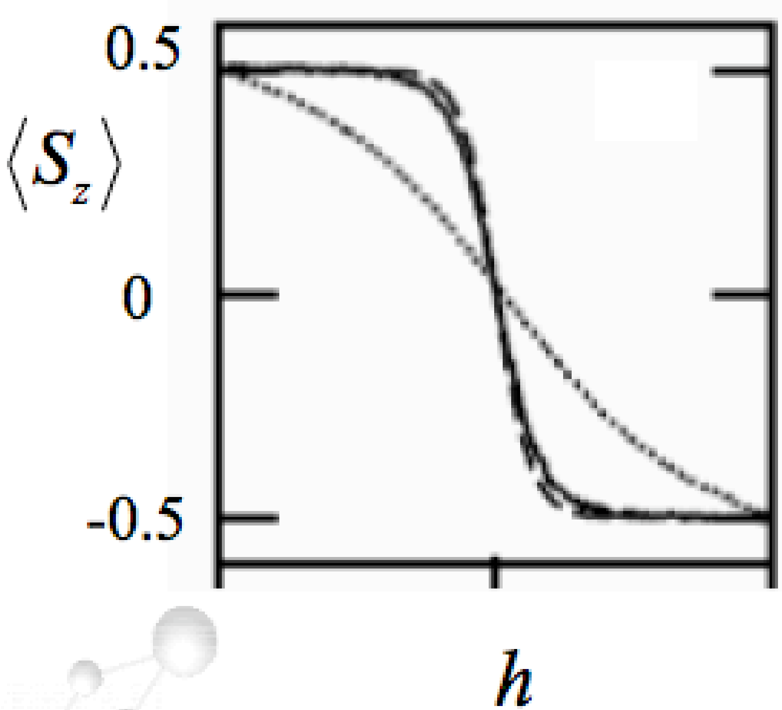
Coupling to bosons comes from eSz. δV_g

Metallic dot + Fermi liquid lead

Dense energy spectrum: $H_t = t\, c^{+}_{lead} c_{dot} S^{-} + h.c.$ Matveev, JETP 72, 892 ('91)

Exotic Kondo effect

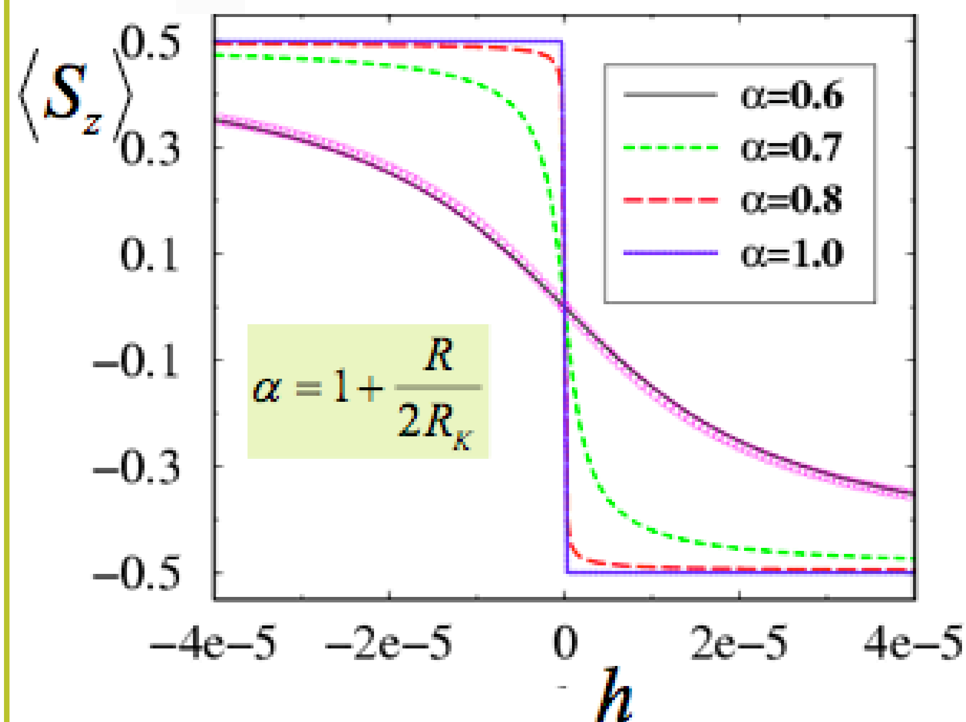
Experimentally measurable



Lehnert et al, PRL 91, 1068011 ('99)

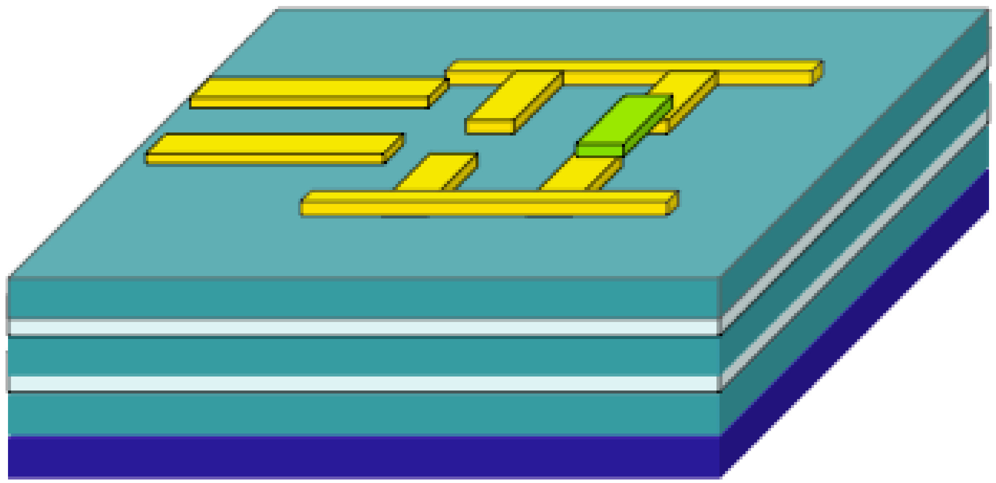
Noise tuned quantum phase transition

Numerical RG result

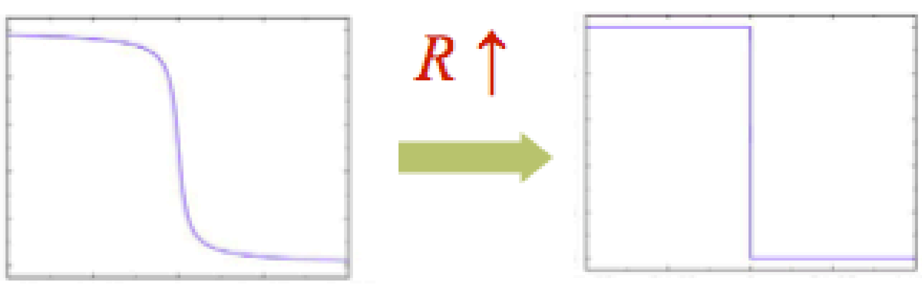


MRL, LeHur & Hofstetter, cond-mat/0501755

What we propose to experimentalists



- First GaAs layer: quantum dot + lead
- Second GaAs layer: *tunable* R through tuning electron density



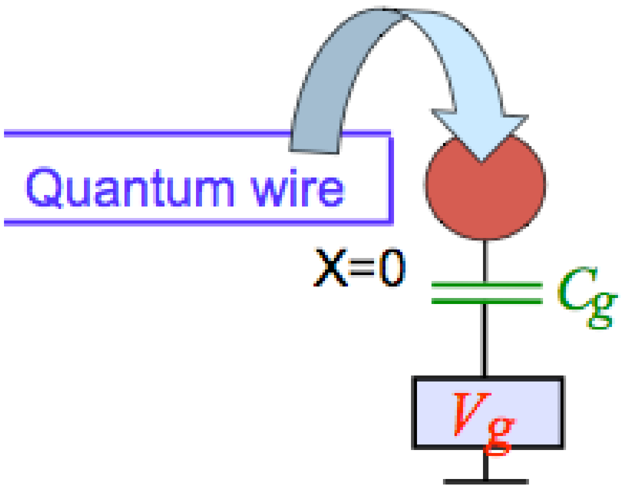
Dissipative Josephson junction (array)

Rimberg *et al.*, PRL78, 2632('91)
Kycia *et al.*, PRL87, 017002('01)

$(K_b)_c^{-1} = J_\perp/\pi v_f = 2\Delta/\omega_c$ **or** **Rc linear with $\Delta.R_K$!!!**

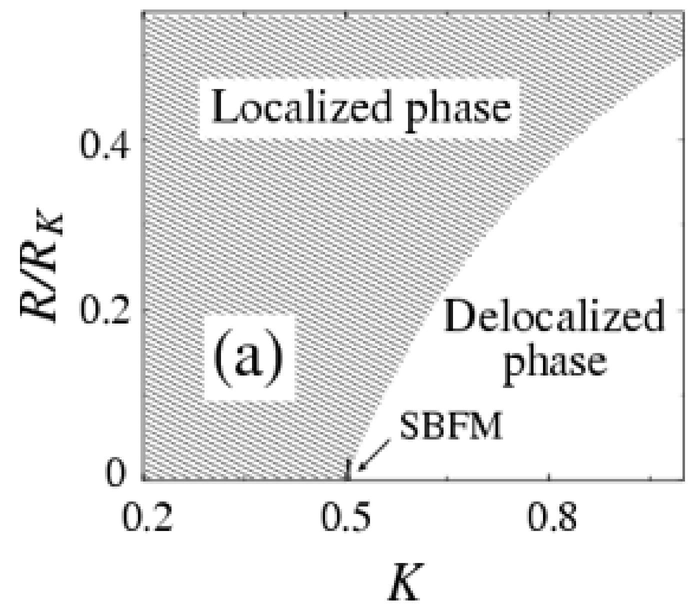
Other fancy proposals…

Unification of 1D Luttinger liquid and electric noise:
K. Le Hur and Meirong Li, cond-mat/0410446



K<1 **Luttinger parameter in the lead**

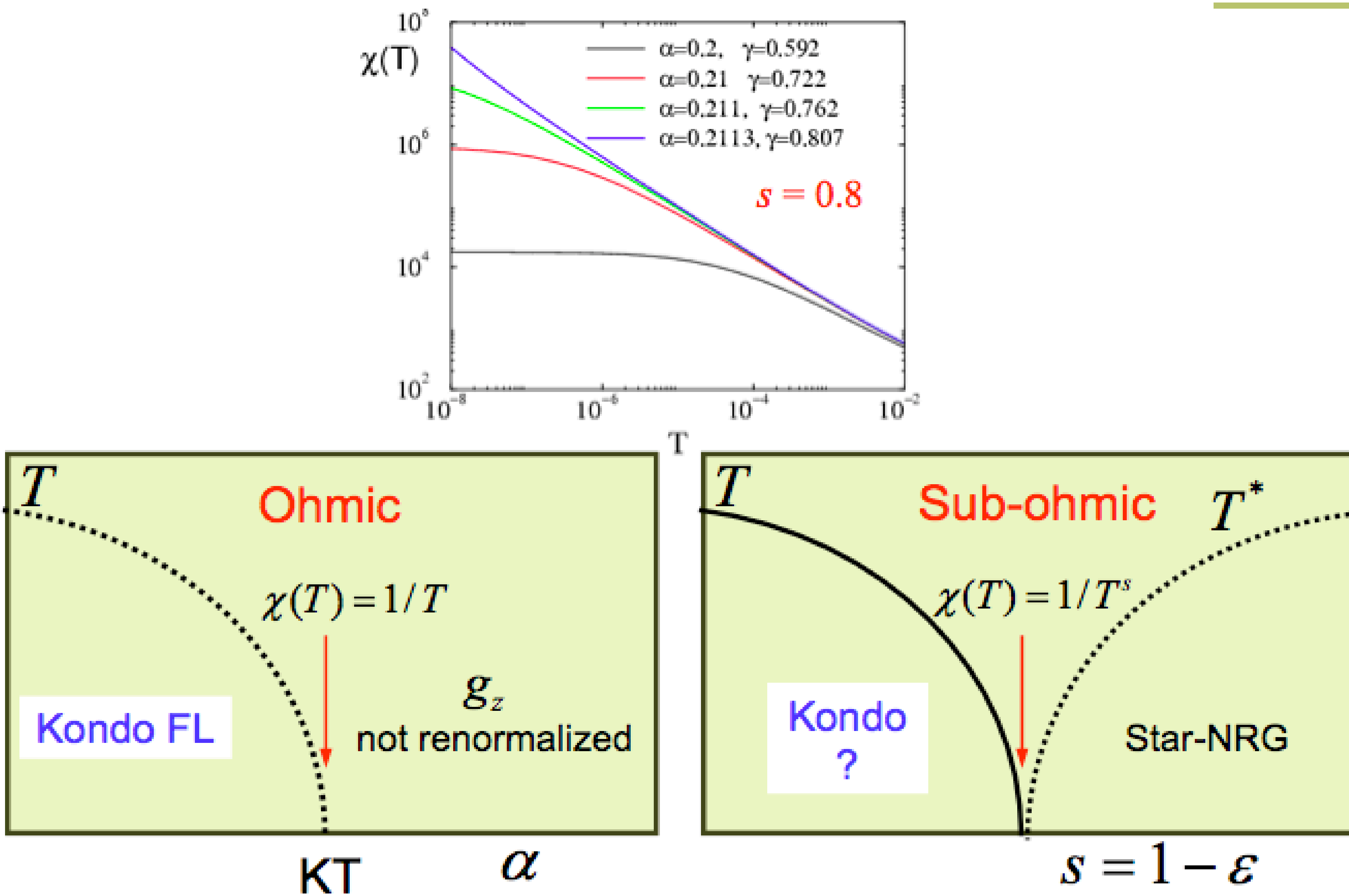
Again exact mapping on the Caldeira-Leggett model



Implication for Transport?

$$\boxed{\frac{R_c}{R_K} = 1 - \frac{1}{2K}}$$

The sub-ohmic situation



Vojta *et al.*, cond-mat/0410132

Summary

For the ohmic situation and Ising coupling with the bosons:

Hidden-Caldeira Leggett model: « single boson bath »

Kosterlitz-Thouless quantum phase transition

Observable in a noisy mesoscopic qubit

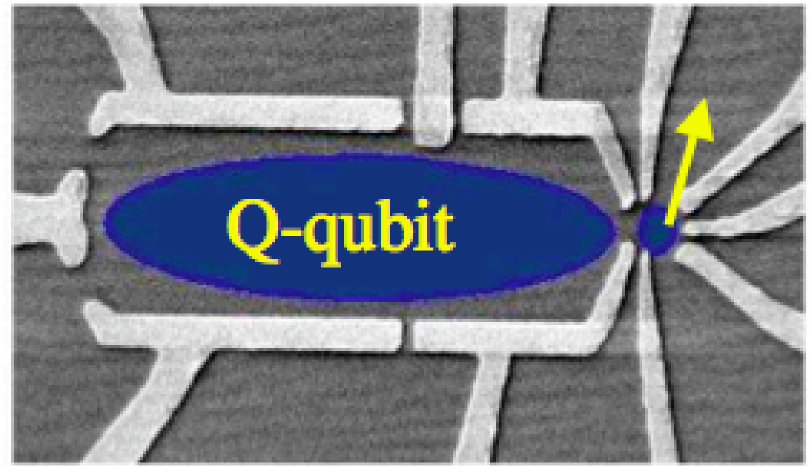
Sub-ohmic case:

A way to perform an ε expansion from the ohmic case?

Irrefutable applicability? Q. Si et al.

Other probes for local « QC »

-SU(4) Kondo liquids



First opportunity to entangle two qubits of different natures: charge & spin qubits

Courtesy of D. Goldhaber-Gordon
& C. Marcus

Another proposal by
Halperin et al. (2003)

(preliminary data from Ron Potock

This set-up also provides the possibility to probe 2-channel KM