Design and realization of exotic quantum phases in atomic gases

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Atomic quantum gases

Bose-Einstein condensation

- Gross-Pitaevskii equation
- non-linear dynamics

Quantum degenerate *dilute* atomic gases of fermions and bosons

control and tunability

Rotating condensates

- vortices
- fractional quantum Hall

Molecules

- Feshbach resonances
- BCS-BEC crossover
- dipolar gases

Optical lattices

- quantum information
- Hubbard models
- strong correlations
- exotic phases

Atomic gases in an optical lattice

Preparation

- lattice loading schemes
- controlled single particle manipulations (entanglement)
- decoherence of qubits

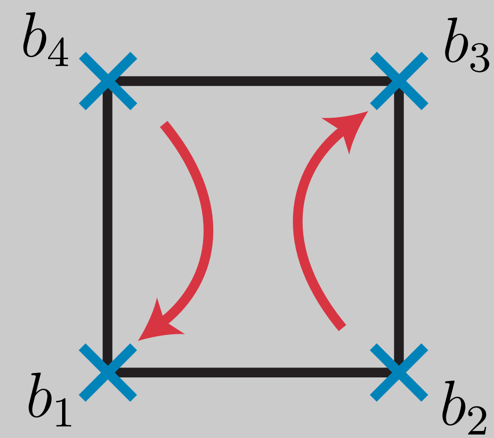
Thermodynamics

- Hubbard models
- design of Hamiltonians
- strongly correlated many-body systems

Measurement

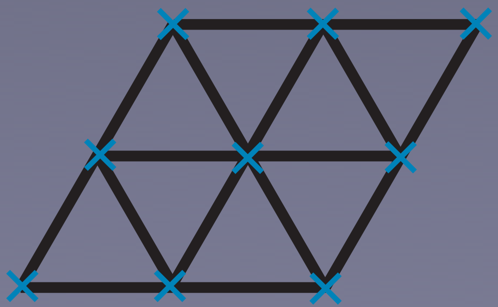
- momentum distribution
- structure factor
- pairing gap
- ...

Ring exchange interaction



Exotic phases?

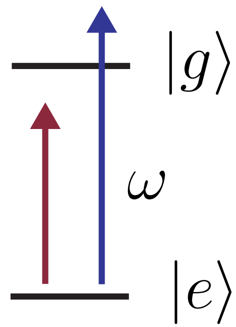
Bose-Hubbard tool box



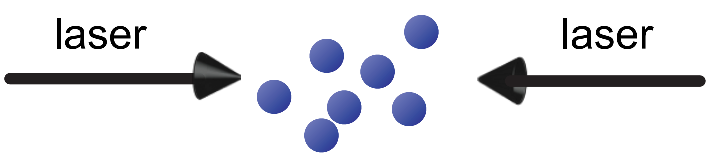
Optical lattices

- AC Stark shift

off-resonant
laser



- standing laser configuration



$$V(\mathbf{x}) = V_0 \sin^2 \mathbf{k}\,\mathbf{x} + \ldots$$

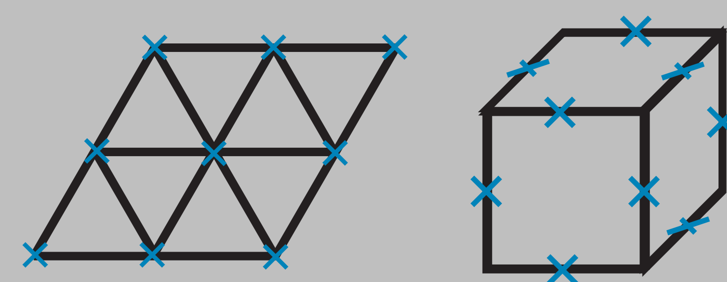
- characteristic energies

$$E_r = \frac{\hbar^2 \mathbf{k}^2}{2m} \sim 10\mathrm{kHz}$$

$$V_0 / E_r \sim 50$$

- high stability of the
  optical lattice

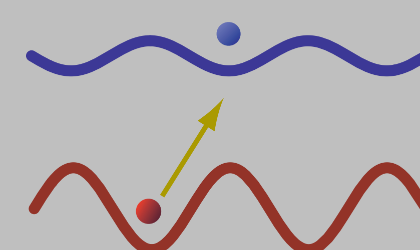
1D, 2D, and 3D Lattice
structures



Internal states

- spin dependent
  optical lattices

- alkaline earth
  atoms

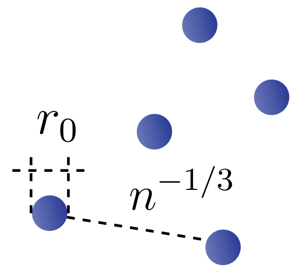


Control of interaction

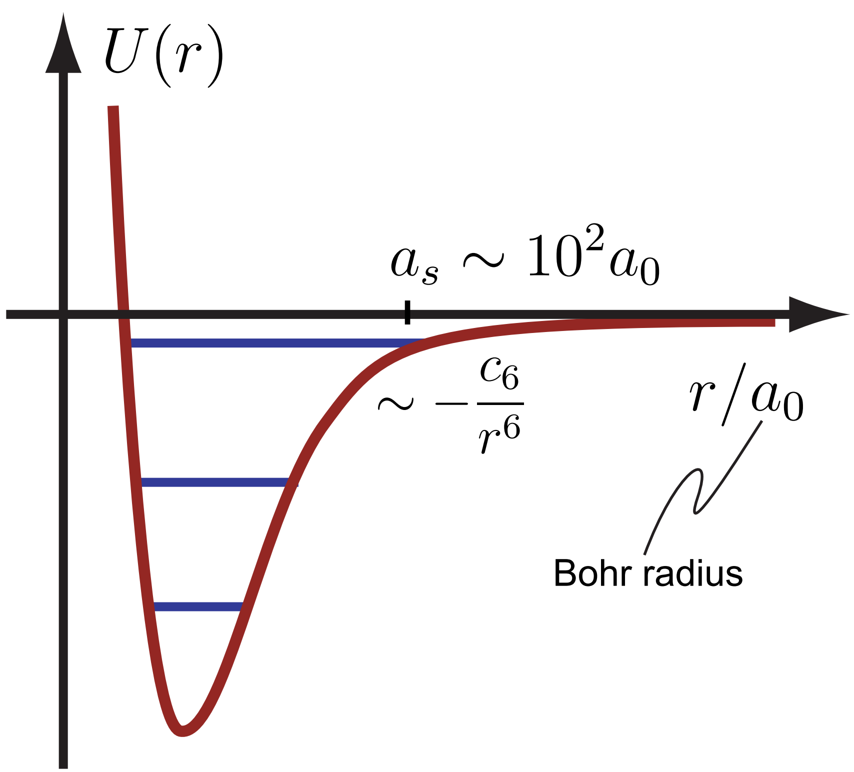
Interaction potential:

- effective range

$$r_0^3 n \ll 1$$



- pseudo-potential approximation



Scattering properties

- scattering amplitude: $$f(k) = -\frac{1}{1/a_s + ik}$$

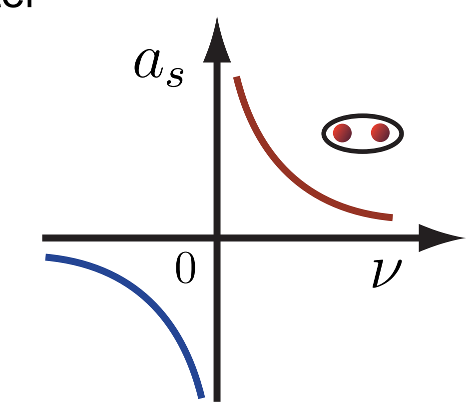
- bound state energy $a_s > 0$: $$E_{\mathrm{M}} = -\frac{\hbar^2}{m a_s^2}$$

Tuning of scattering length

- changing the first “bound state” energy via an external parameter

  - magnetic Feshbach resonance

  - optical Feshbach resonance



Microscopic Hamiltonian

$$H = \int dx\, \psi^+(x) \left(-\frac{\hbar^2}{2m}\Delta + V(x) \right) \psi(x) + \frac{g}{2} \int dx\, \psi^+(x)\psi^+(x)\psi(x)\psi(x)$$

optical lattice

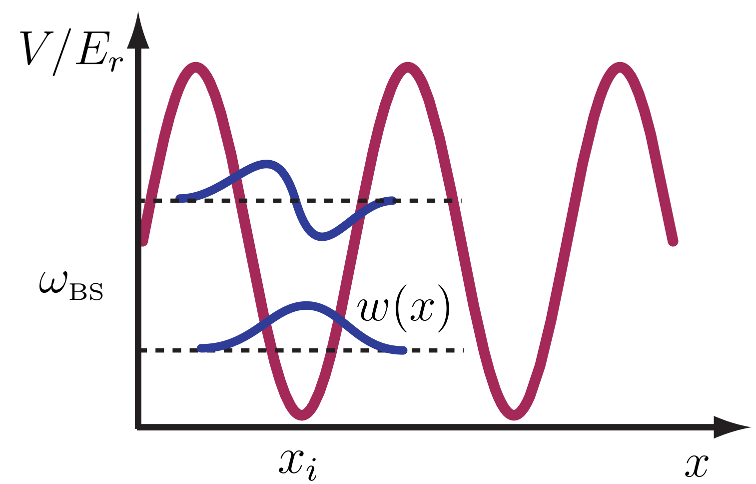
$$g = \frac{4\pi\hbar^2 a_s}{m}$$

: interaction strength

- strong opitcal lattice $V > E_{\mathrm{r}}$
- express the bosonic field operator in terms of Wannier functions
- restriction to lowest Bloch band

(Jaksch et al PRL ‘98)

$$\psi(\mathbf{x}) = \sum_i w(\mathbf{x} - \mathbf{x}_i) b_i$$



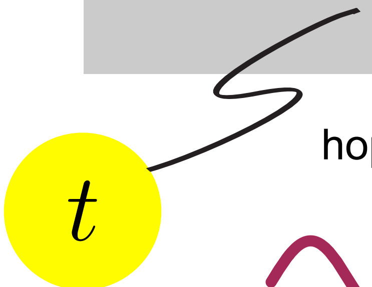
Bose-Hubbard Model

Bose-Hubbard model (Fisher et al PRB '81)

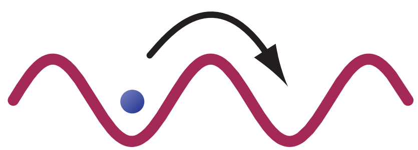
$$H_{\mathrm{BH}} = -J \sum_{\langle i,j \rangle} b_i^+ b_j + U/2 \sum_i b_i^+ b_i^+ b_i b_i$$

$$U \sim E_{\mathrm{r}} a_s / \lambda$$

$$J \sim E_{\mathrm{r}} e^{-2\sqrt{V/E_{\mathrm{r}}}}$$



hopping energy

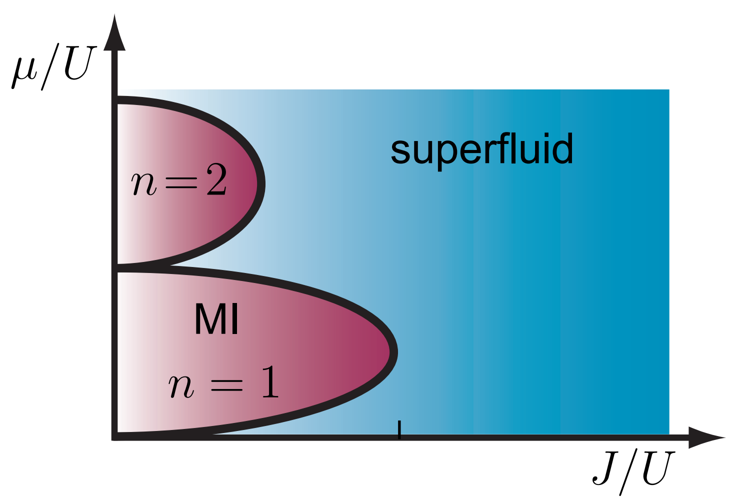


interaction energy

Phase diagram

Mott insulator

- fixed particle number
- incompressible
- excitation gap

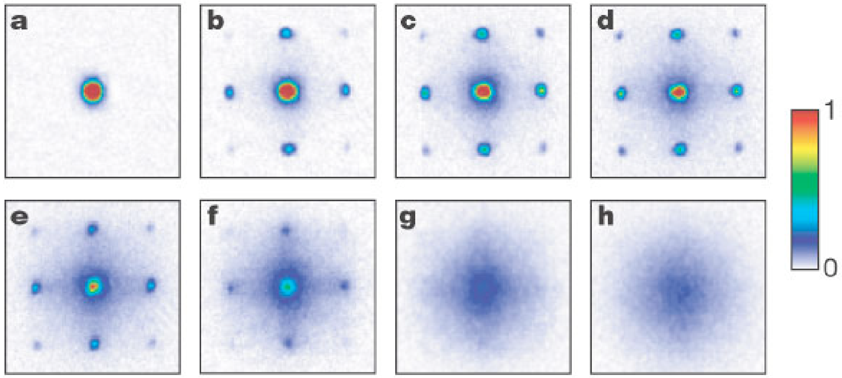


Superfluid

- long-range order
- finite superfluid stiffness
- linear excitation spectrum

Experiments

Long-range order:

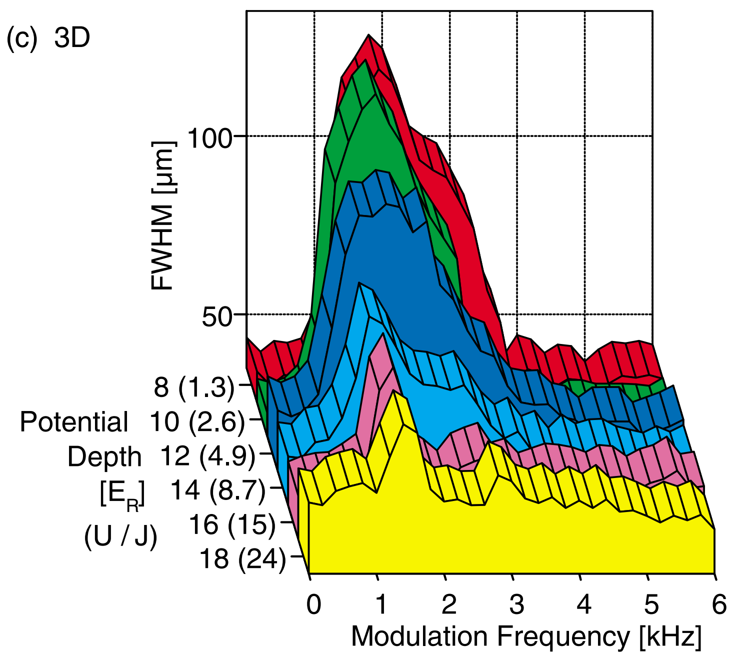


(Greiner et al., 02)

Disappearance of coherence for strong optical lattices (Greiner et al. '02)

$$\frac{V}{E_r} > 13$$

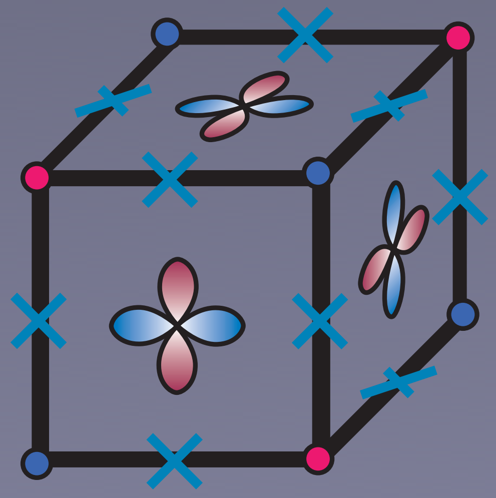
Structure factor



(Esslinger et al., 04)

Appearance of well defined two particle excitations

Ring exchange interaction

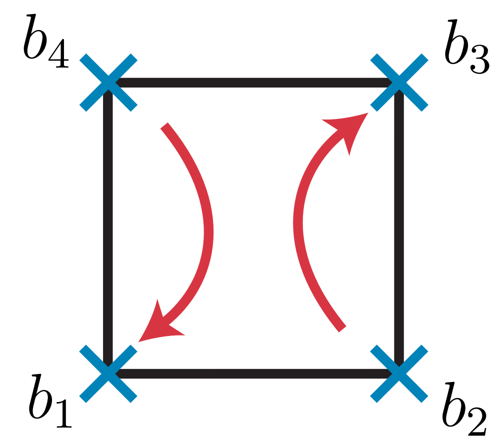


Ring exchange

Ring exchange

- bosons on a lattice

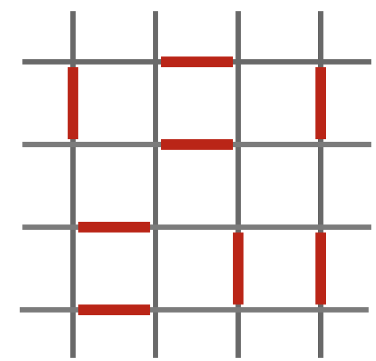
$$H_{\mathrm{R-E}} = K\left[b_1^+ b_2 b_3^+ b_4 + b_1 b_2^+ b_3 b_4^+\right]$$



Applications:

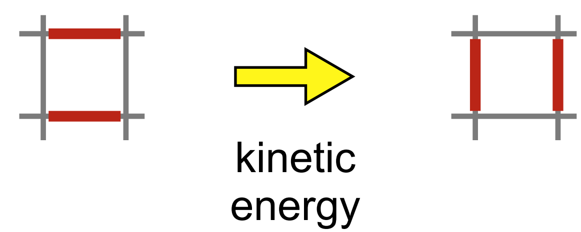
Dimer models

- spin liquids, VBS - phases
- topological protected quantum memory



2D spin systems

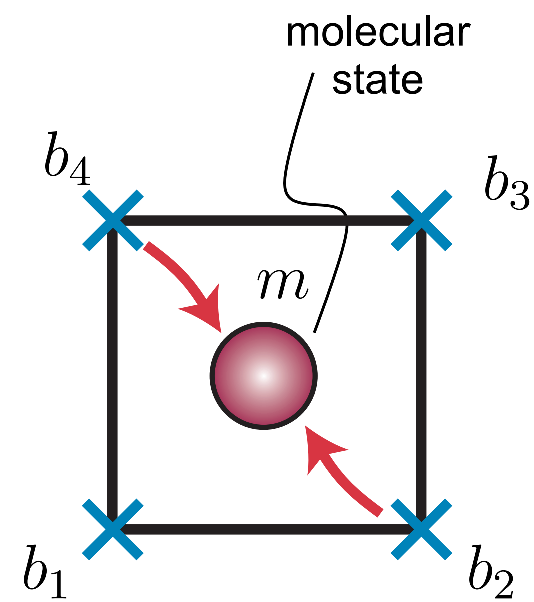
- Neel order versus VBS
- deconfined quantum critical points



Lattice gauge theories

- U(1) lattice gauge fields
- a model QED

Ring exchange



Toy model:

- bosons on a lattice
- resonant coupling to a molecular state via a Raman transition
  - molecule is trapped by a different optical lattice

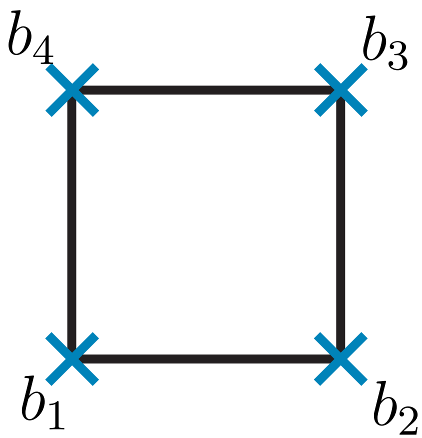
Effective coupling Hamilton

detuning

coupling
(Rabi frequency)

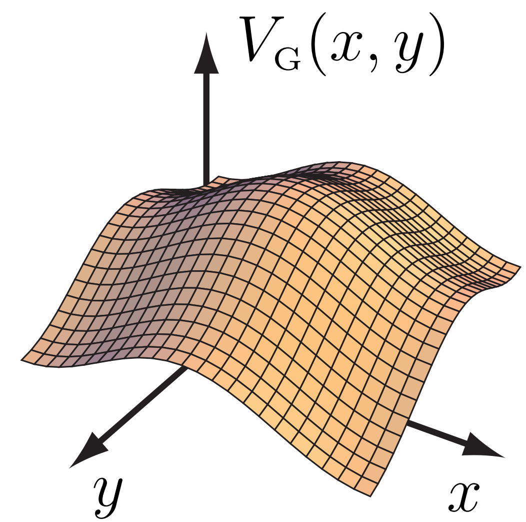
$$H = \nu\, m^{+} m + g \sum_{i \neq j} c_{ij} \left[m^{+}\, b_i b_j + m\, b_i^{+} b_j^{+} \right]$$

Ring exchange

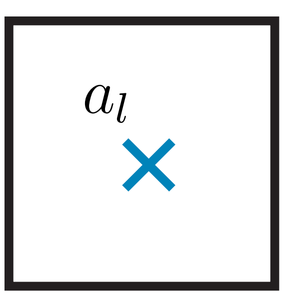


First internal state

- Bosonic atoms in the corners of the square
- Bose-Hubbard model



Raman transition

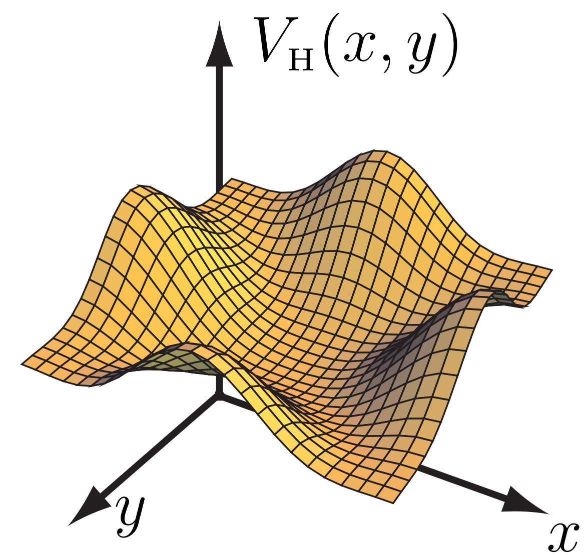


Second internal state

- Trapped in the center of the square
- quenched hopping
- angular momentum

$$l = 0, \pm 1, 2$$

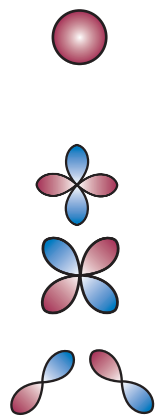
- interaction allow for a molecular state



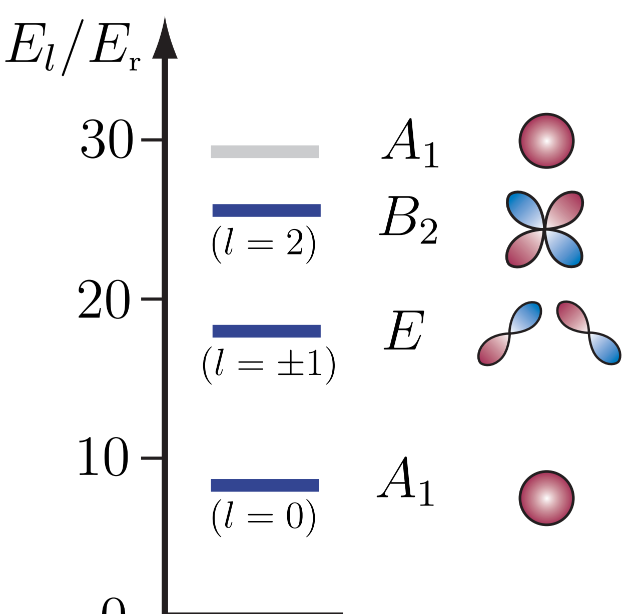
Ring exchange

Symmetries

- Hamilton is invariant under operations of the C_{4v}
- symmetries of single particle states a_l



	E	C_2	$2C_4$	$2\sigma_v$	$2\sigma_d$		
A_1 (l = 0)	1	1	1	1	1	z	$b_1b_3 + b_2b_4$ $b_1b_2 + b_2b_3 + b_3b_4 + b_4b_1$
A_2	1	1	1	-1	-1	l_z	
B_1	1	1	-1	1	-1	$x^2 - y^2$	$b_1b_2 - b_2b_3 + b_3b_4 - b_4b_1$
B_2 (l = 2)	1	1	-1	-1	1	xy	m, $b_1b_3 - b_2b_4$
E (l = 1)	2	-2	0	0	0	(x, y)	$(b_1b_2 - b_3b_4,\ b_2b_3 - b_4b_1)$



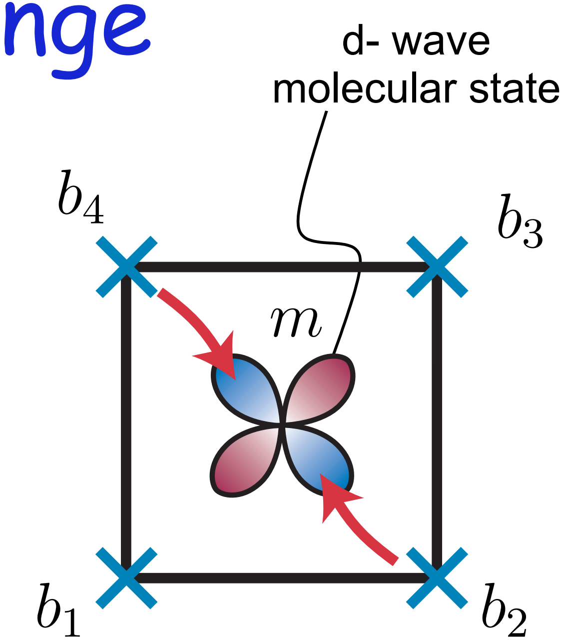
Energy levels

- design of optical lattice
- tune with the Raman transtition close to a *s*-wave molecule in the *d*-wave vibrational state
- *d*-wave symmetry for molecular state

$$m^+ = c a_2^+ a_0^+ + d\left[a_1^+ a_1^+ + a_{-1}^+ a_{-1}^+\right] \ldots$$

- integrate out single-particle states a_l

Ring exchange



Toy model:

- bosons on a lattice
- resonant coupling to a molecular state via a Raman transition
    - molecule is trapped by a different optical lattice

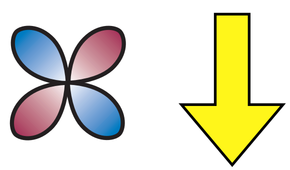
Effective coupling Hamilton

detuning

coupling
(Rabi frequency)

symmetry of the molecule

$$H = \nu\, m^{+} m + g \sum_{i \neq j} c_{ij} \left[m^{+}\, b_i b_j + m\, b_i^{+} b_j^{+} \right]$$



d-wave symmetry

$$m^{+} \left[b_1 b_3 - b_2 b_4 \right] + c.c.$$

Ring exchange

Effective low energy Hamiltonian

$$H = \nu\, m^{+}m + g m^{+}[b_1 b_3 - b_2 b_4] + g m \left[b_1^{+} b_3^{+} - b_2^{+} b_4^{+}\right]$$

Relation to Ring exchange

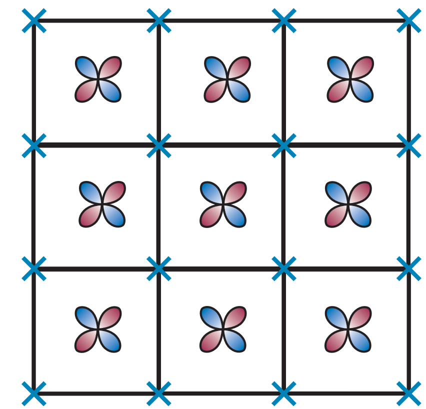
- integrating out the molecule

$$H = K\left[b_1^{+} b_2 b_3^{+} b_4 + b_1 b_2^{+} b_3 b_4^{+} - n_1 n_3 - n_2 n_4\right]$$

- perturbation theory

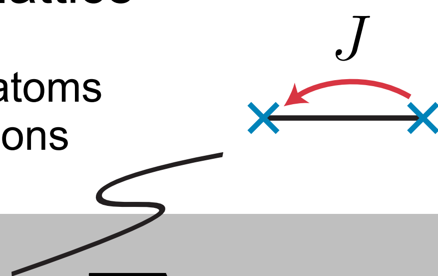
$$K = \frac{g^2}{\nu}$$

Ring exchange



Hamiltonian on a lattice

- add hopping for the atoms
- half-filling for the bosons

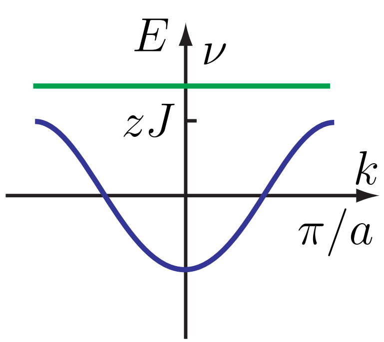


$$H = -J \sum_{\langle ij \rangle} b_i^+ b_j + \nu \sum_i m_i^+ m_i + g \sum_{\Box} m_{\Box}^+ [b_1 b_3 - b_2 b_4] + m_{\Box} [b_1^+ b_3^+ - b_2^+ b_4^+]$$

Superfluid

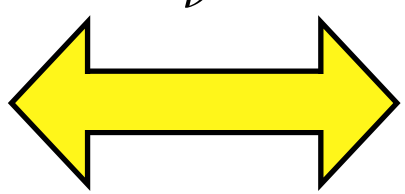
$J \gg K$

- superfluid of bosonic atoms
- long-ranger order



decreasing
detuning

ν

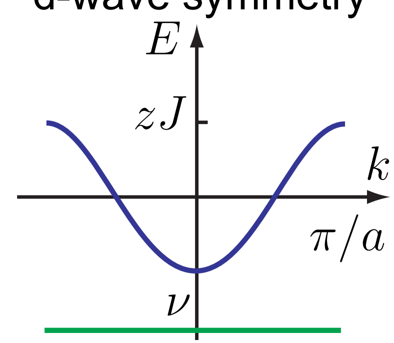


- intermediate regime
- quantum phase transition?
- exotic phases?

Molecules

$J \ll K$

- formation of molecules
- non-trivial structure due to d-wave symmetry



Lattice gauge theory

2D lattice gauge theory

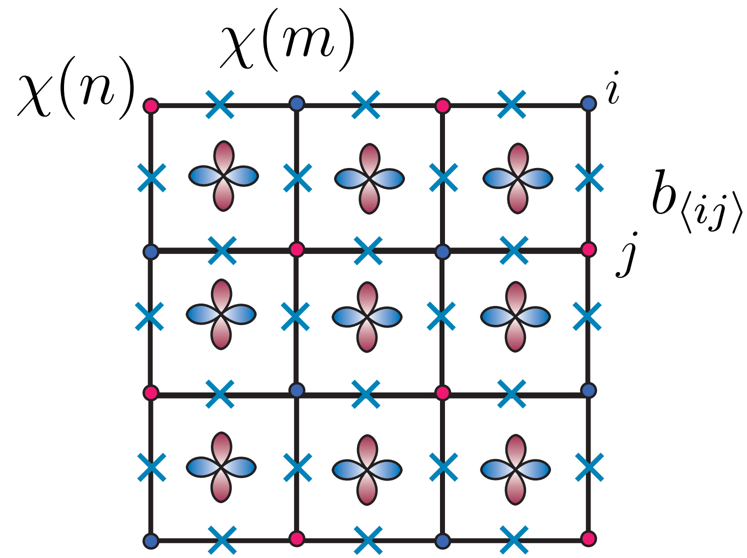
- atoms on links with ring exchange
  and quenched hopping

- gauge transformation

n red corner
m blue corner

$$b_{\langle nm \rangle} \longrightarrow b_{\langle nm \rangle} e^{i[\chi(n) - \chi(m)]}$$

- represents a 2D dimer model



3D lattice gauge theory

- adding an additional dimension
- atoms on the links of the lattice
- moleculs in the center of the faces

- pure U(1) lattice gauge theory exhibits
  a phase transition from the Coulomb
  phase to a confining phase

- presence of a Coulomb phase
  in the present model?
  (M. Hermele et al, PRB 2004)