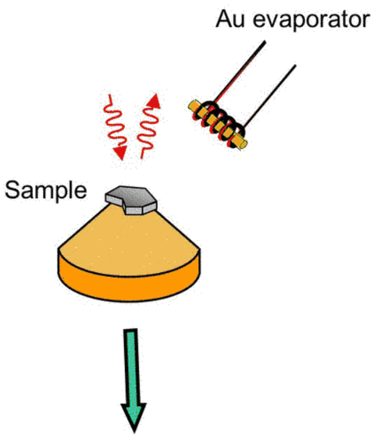
Scale invariant optical response in cuprate superconductors

19-01-05

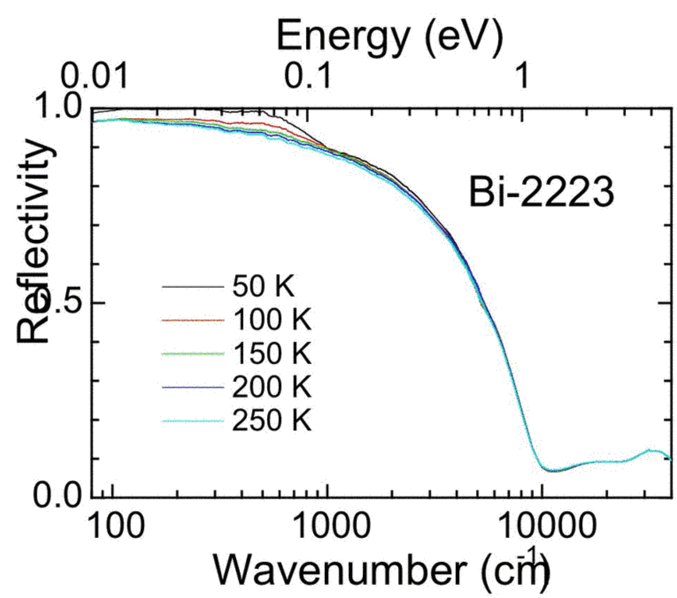
Santa Barbara

D. Van der Marel, H. J. A. Molegraaf, F. Carbone, A. Kuzmenko, E. van Heumen
Université de Genève
J. Zaanen, Z. Nussinov, Universiteit Leiden
P. Kes, M. Li, Universiteit Leiden
H. Eisaki, NIAIST Tsukuba
M. Greven, Stanford University
A. Damascelli, University of British Columbia
Enrico Giannini, Université de Genève
http://optics.unige.ch

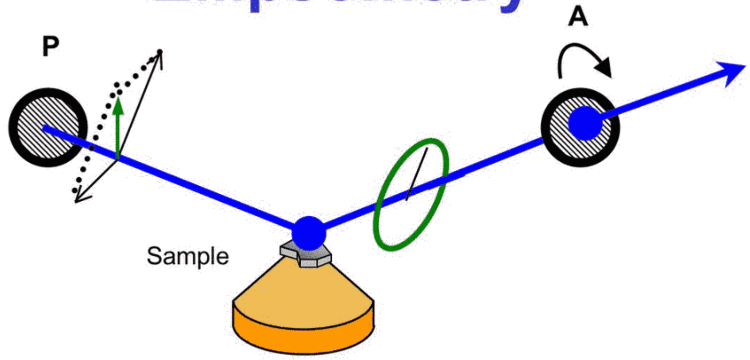
Reflectivity

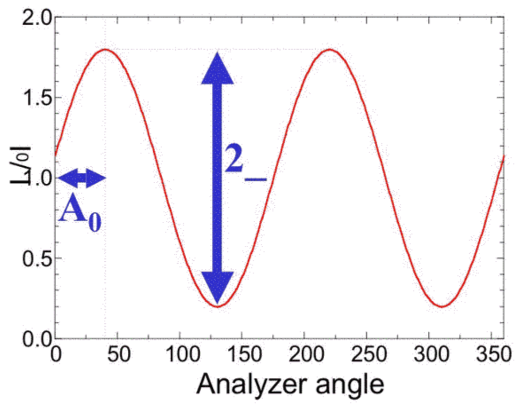


Kramers Kronig gives optical conductivity $\sigma(\omega)$

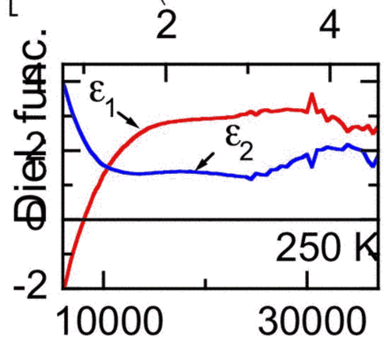


Ellipsometry





$$\varepsilon_1 + \frac{4\pi i}{\omega}\sigma_1 = \sin^2\theta\left[1+\tan^2\theta\left(\frac{\cos P - i\sqrt{1-\gamma^2}\sin P - \cos(2A_0 - P)}{\cos P + i\sqrt{1-\gamma^2}\sin P - \cos(2A_0 + P)}\right)^2\right]$$

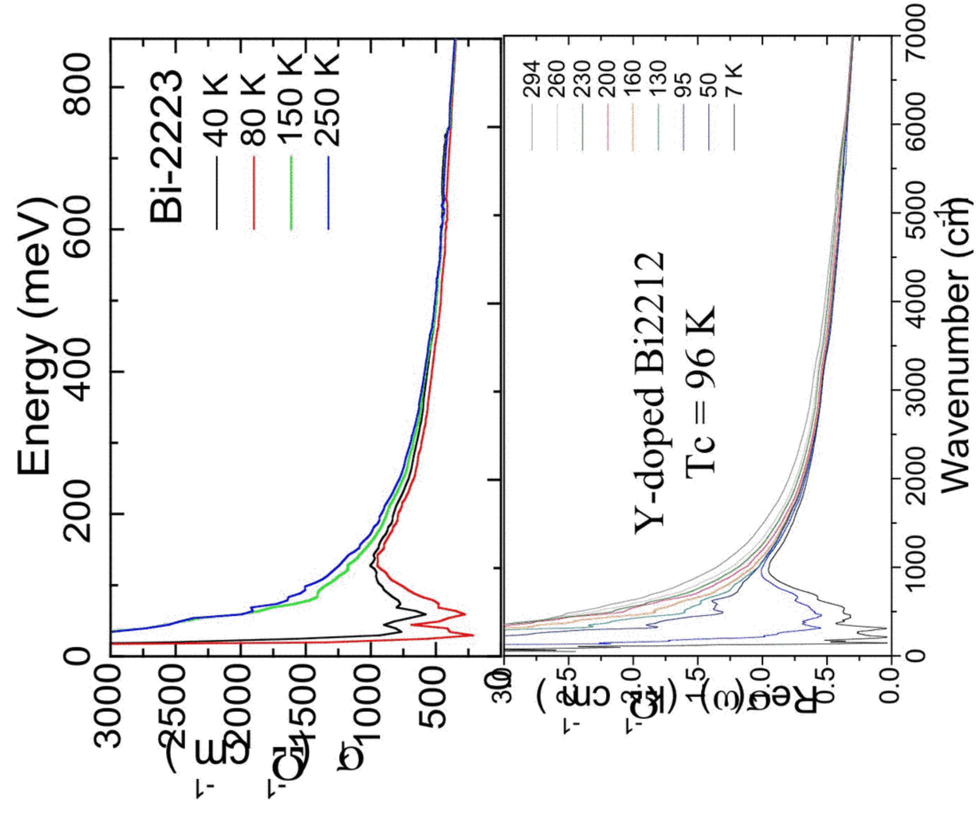


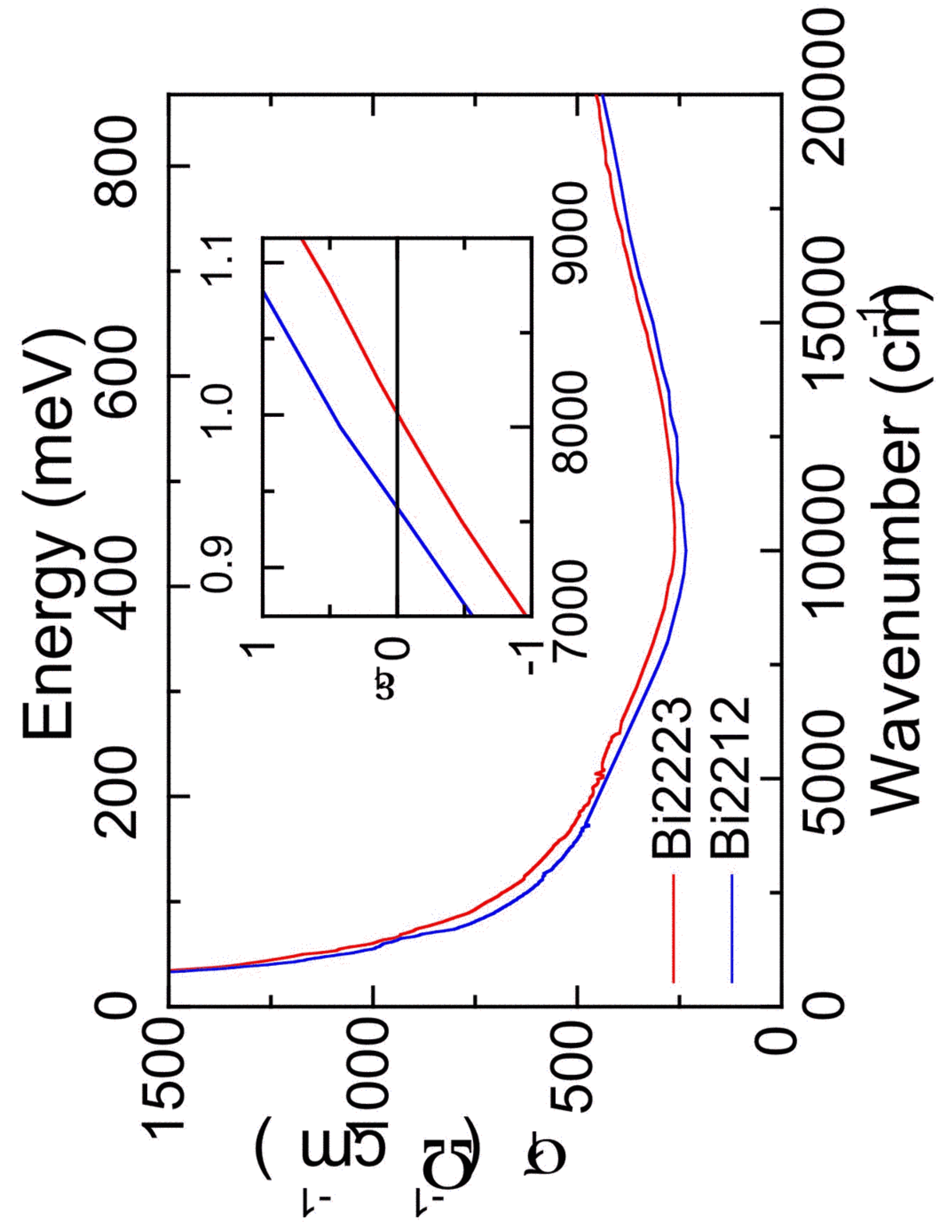
Optical Conductivity $\sigma = \mathbf{j} / \mathbf{E}$

$$\sigma(\mathbf{q},\omega,T) = \frac{2i\omega}{V}\sum_n e^{(\Omega - E_n)/k_BT}\langle n|\hat{\mathbf{j}}_{-\mathbf{q}}\frac{\hbar(H-E_n)^{-1}}{\omega^2 - \hbar^{-2}(H-E_n)^2 + i0^+}\hat{\mathbf{j}}_{\mathbf{q}}|n\rangle$$

Current operator : $$\hat{\mathbf{j}}_{\mathbf{q}} = \frac{e}{2\hbar}\sum_{p\sigma}\left\{\left.\frac{\partial\varepsilon_k}{\partial\mathbf{k}}\right|_{k=\mathbf{p}+\mathbf{q}/2} - \left.\frac{\partial\varepsilon_k}{\partial\mathbf{k}}\right|_{k=-\mathbf{p}+\mathbf{q}/2}\right\} c^{t}_{\mathbf{p}-\mathbf{q}/2,\sigma}\, c_{\mathbf{p}+\mathbf{q}/2,\sigma}$$

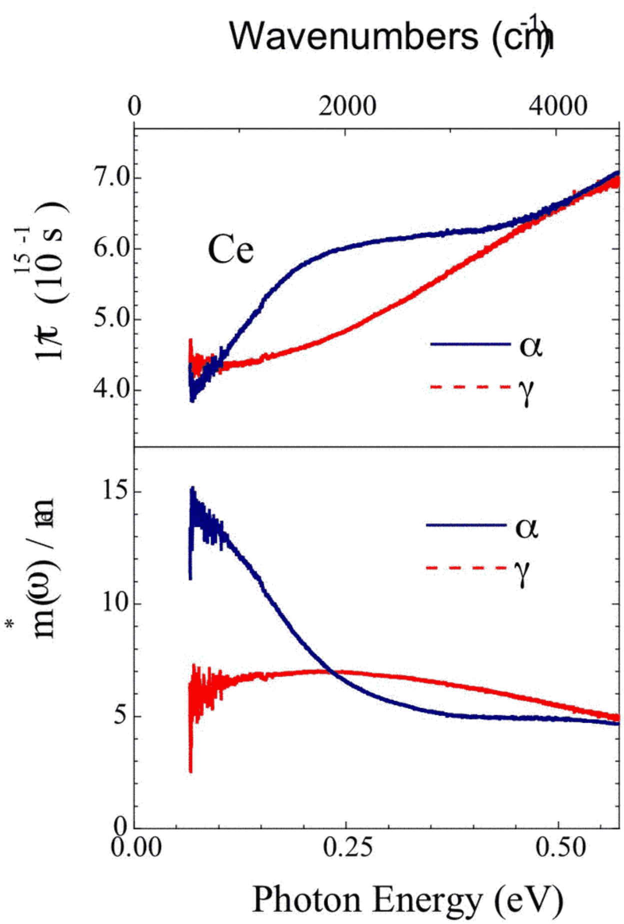
Kubo formula (1957)

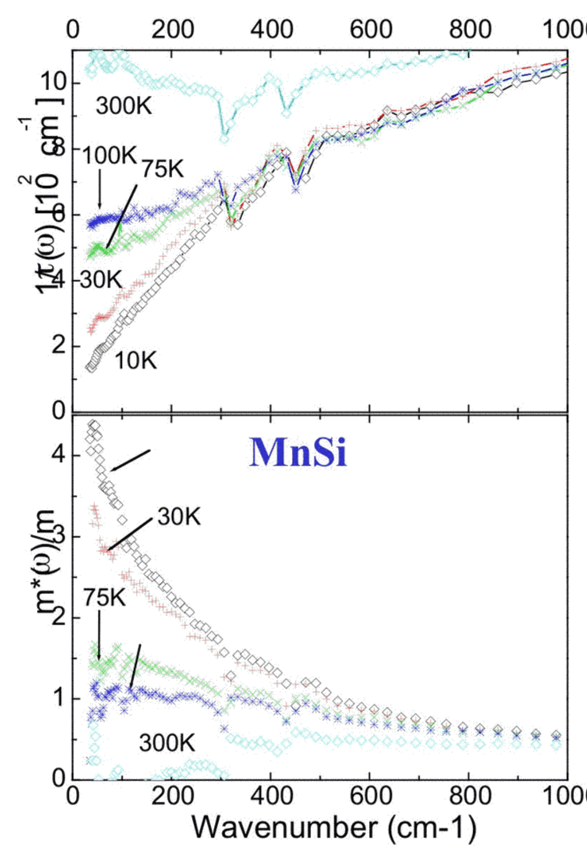

Energy (meV)
Bi-2223
40 K
80 K
150 K
250 K
Y-doped Bi2212
Tc = 96 K
Wavenumber (cm-1)

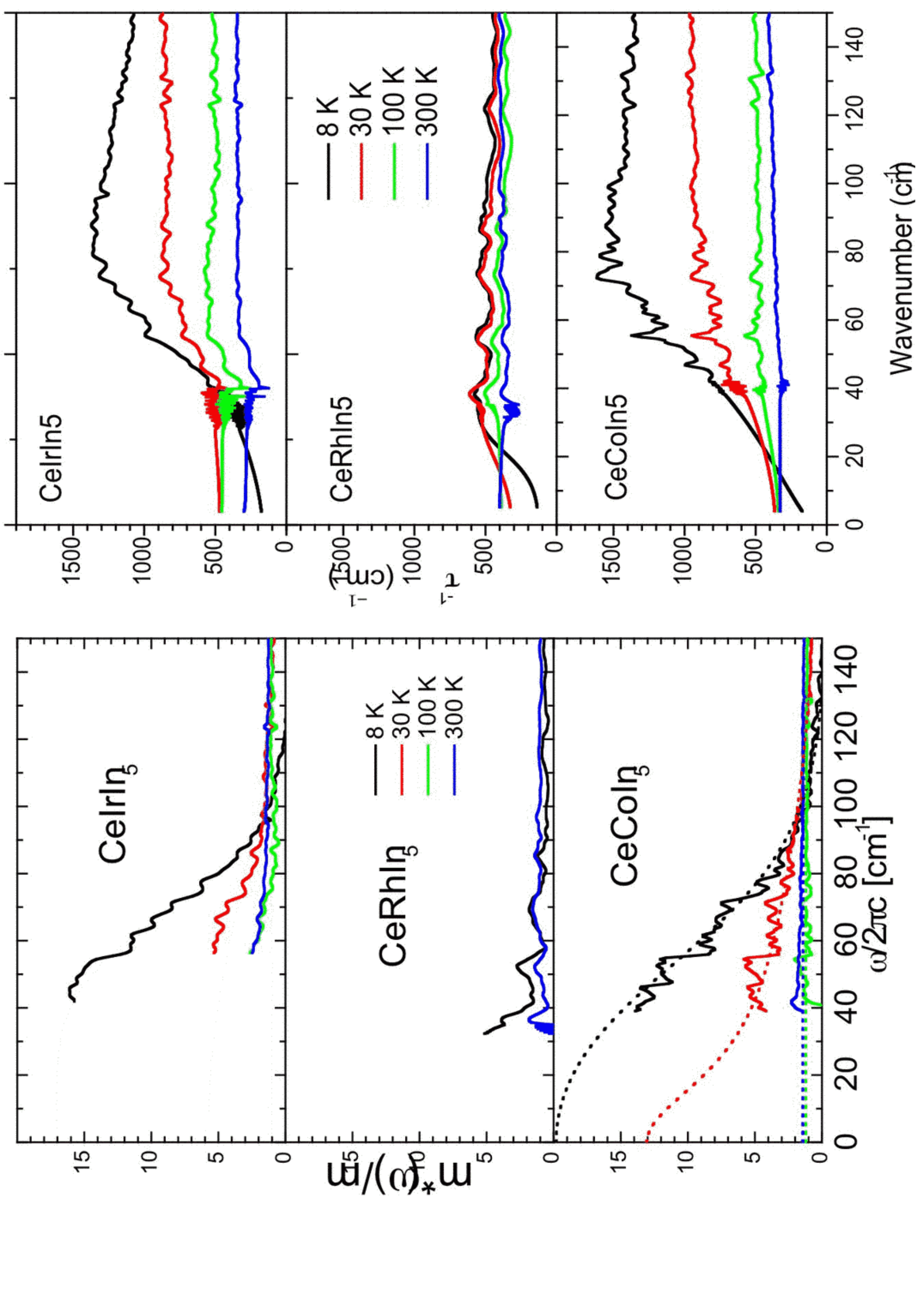

Energy (meV)
Bi2223
Bi2212
Wavenumber (cm-1)

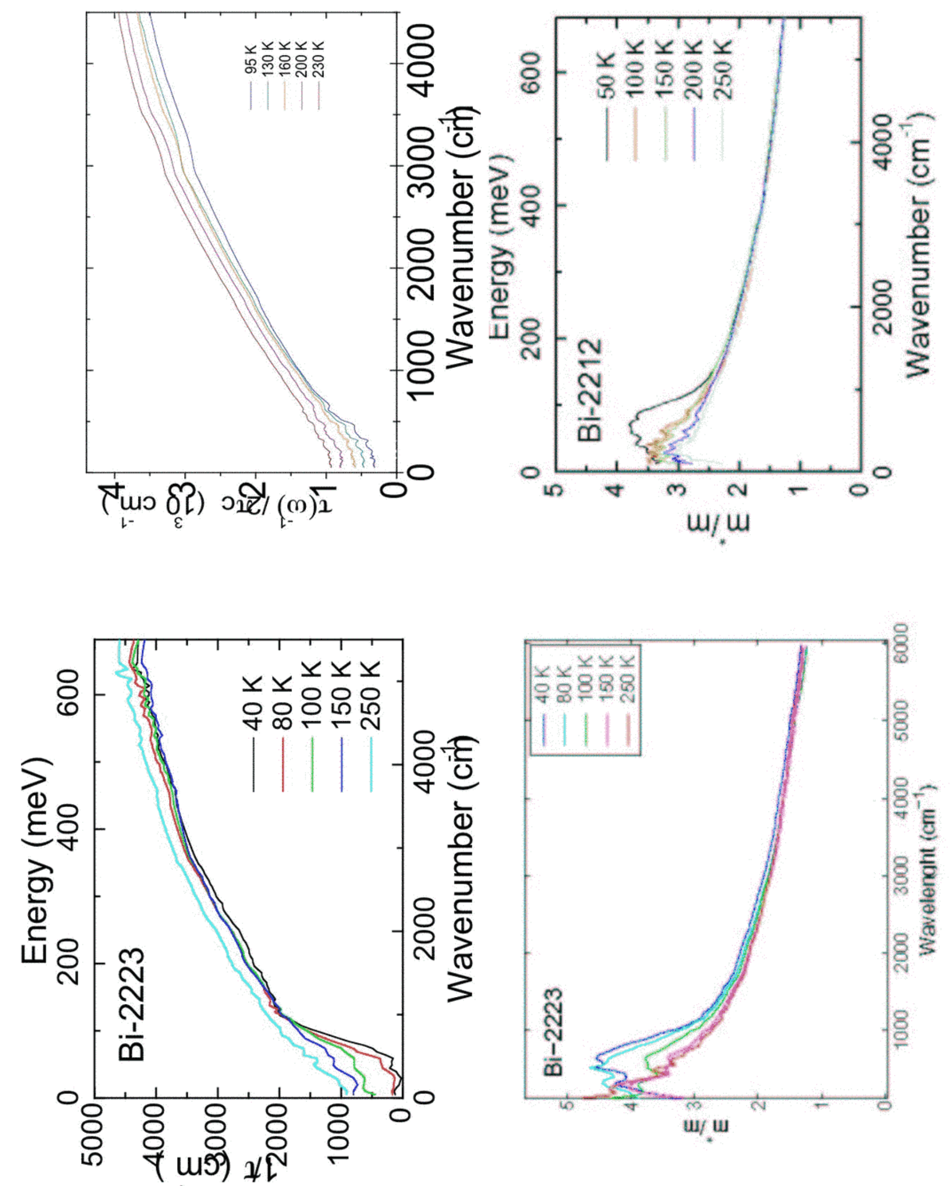
$$\sigma(\omega) = \frac{\omega_p^2 / 4\pi}{\tau^{-1}(\omega) - i\omega m^*(\omega)/m}$$

$$\begin{cases} \tau^{-1}(\omega) \equiv \mathrm{Re}\,\dfrac{\omega_p^2/4\pi}{\sigma(\omega)} \\[2ex] \dfrac{m^*(\omega)}{m} \equiv \mathrm{Im}\,\dfrac{-\omega_p^2/4\pi}{\omega\sigma(\omega)} \end{cases}$$






CeIrIn5
CeRhIn5
CeCoIn5
8 K
30 K
100 K
300 K
Wavenumber (cm-1)
τ-1 (cm-1)
m*(ω)/m
ω/2πc [cm-1]


Bi-2223
Bi-2212
Energy (meV)
Wavenumber (cm-1)
40 K
80 K
100 K
150 K
250 K
50 K
200 K
95 K
130 K
160 K
230 K
m*/m

Marginal Fermi Liquid :

C. M. Varma, et al., PRL 63, 1996 (1989)

$$\tau^{-1}(\omega) = 2\lambda\omega \arctan\left[\frac{\omega}{2\pi T}\right] + \lambda\pi^2 T$$

P. Littlewood, C. M. Varma, JAP 69, 69 (1991)

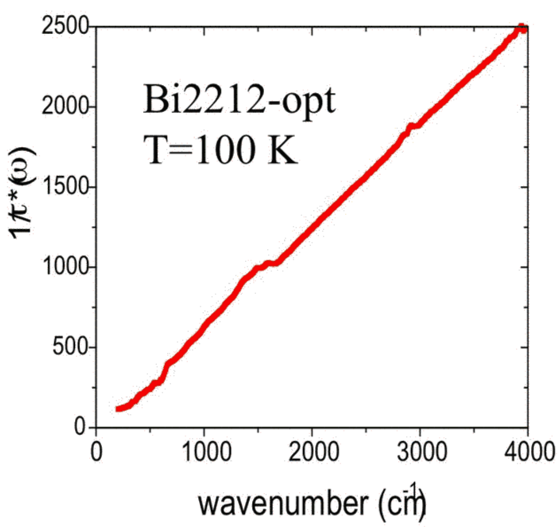
$$\frac{m^*(\omega)}{m} = 1 + \lambda \ln\left[\frac{\pi^2 T^2 + \omega^2/4}{\Omega^2}\right]$$

$$\omega >> T: \ \tau^{-1}(\omega) = \lambda\pi\omega + \lambda\pi^2 T$$

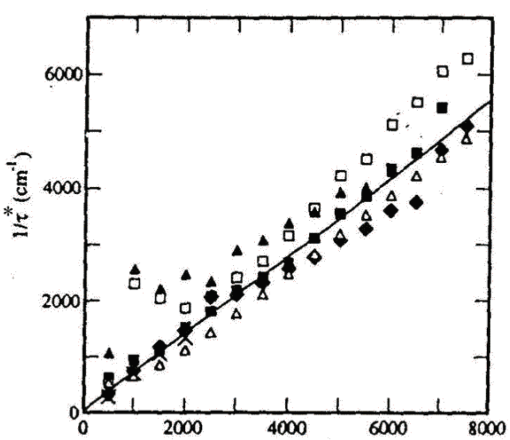
P.W. Anderson,
PRB **55**, 11785 (1997)
Luttinger liquid:

$$\sigma(\omega) \sim \frac{i}{\omega}\left(\frac{\omega}{i\Omega}\right)^{2\alpha} \frac{2\alpha}{\sin \pi\alpha}$$

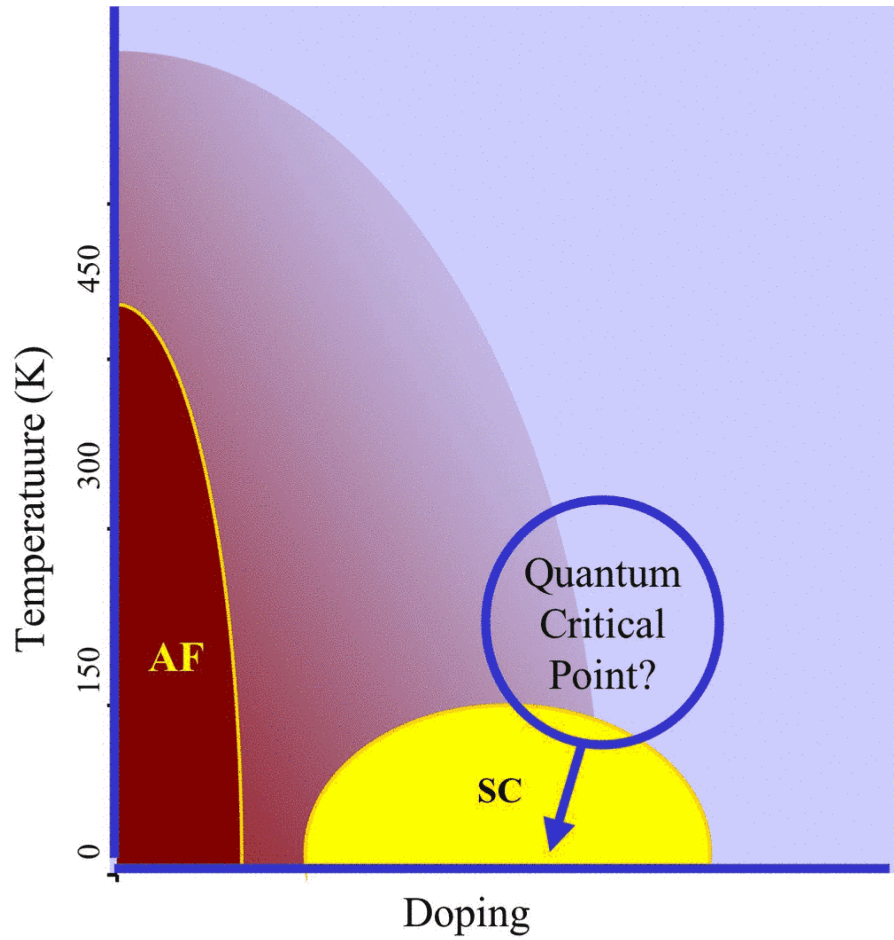
$$\frac{1}{\tau^*(\omega)} \equiv \omega \frac{\mathrm{Re}\,\sigma(\omega)}{\mathrm{Im}\,\sigma(\omega)} = \frac{1}{\tau(\omega)} \frac{m}{m^*(\omega)} = \omega \tan \pi\alpha$$



Baraduc, El Azrak, and Bontemps
J. Superc. 9, 3-6 (1996)



$$\Rightarrow \arctan \frac{\mathrm{Re}\,\sigma(\omega)}{\mathrm{Im}\,\sigma(\omega)} = \text{Phase of } \sigma(\omega) = \text{constant}$$



$T < \omega$:

Quantum critical dynamics ⇒ Time-scale invariance

$$\sigma(p\omega) = \Lambda\, \sigma(\omega)$$
$$\sigma(\omega) = |C|\, e^{i\phi} (-i\,\omega)^{\gamma-2}$$

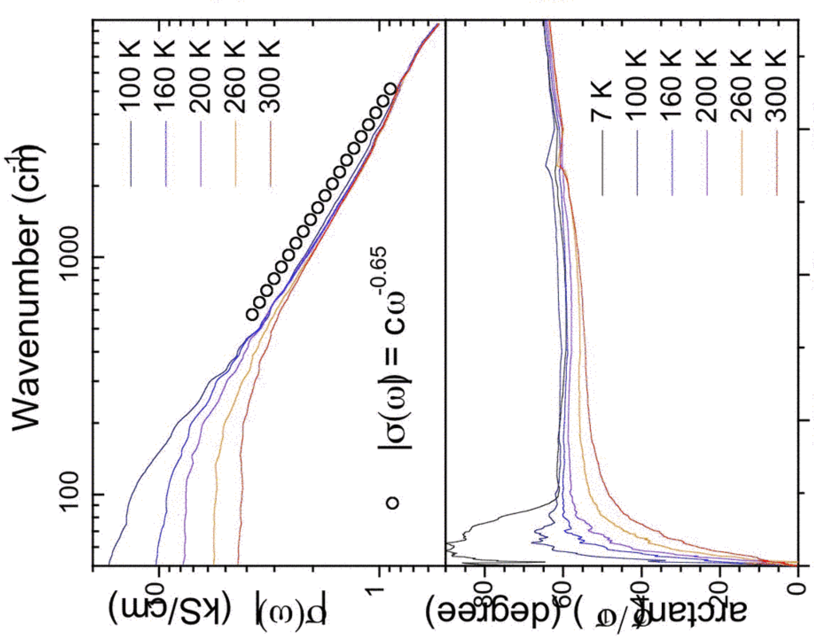
$$\sigma(\omega) = \sigma^*(-\omega)$$

Together: $\phi=0$

$$\sigma(\omega) = |C|\, (-i\,\omega)^{\gamma-2}$$

$$\phi_\sigma = \arctan(\sigma_2/\sigma_1) = \pi - \pi\gamma/2$$

$$d \ln|\sigma| / d \ln\omega = \gamma - 2$$

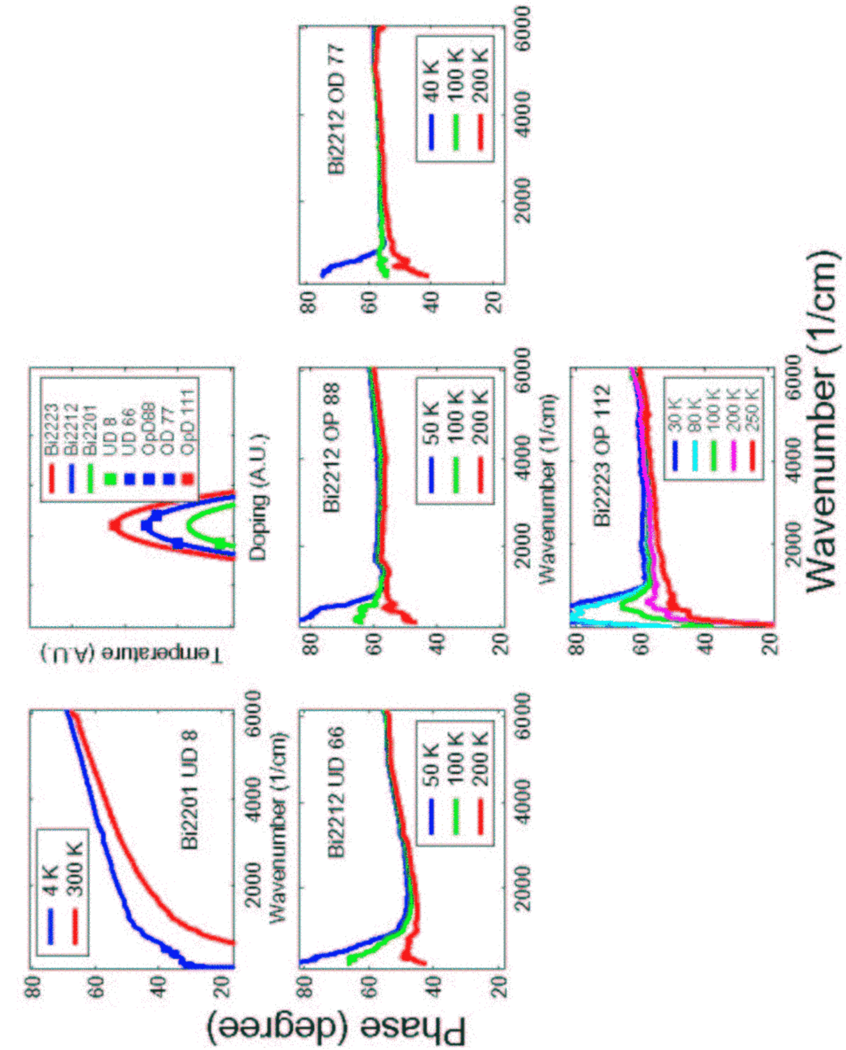


$\gamma-2=-0.65$

$\gamma=1.35$

$\mathrm{Arg}\sigma = 60^{\circ} = \pi(1-\gamma/2)$

$\gamma = 1.33$



ω > Ω: UV-regularization

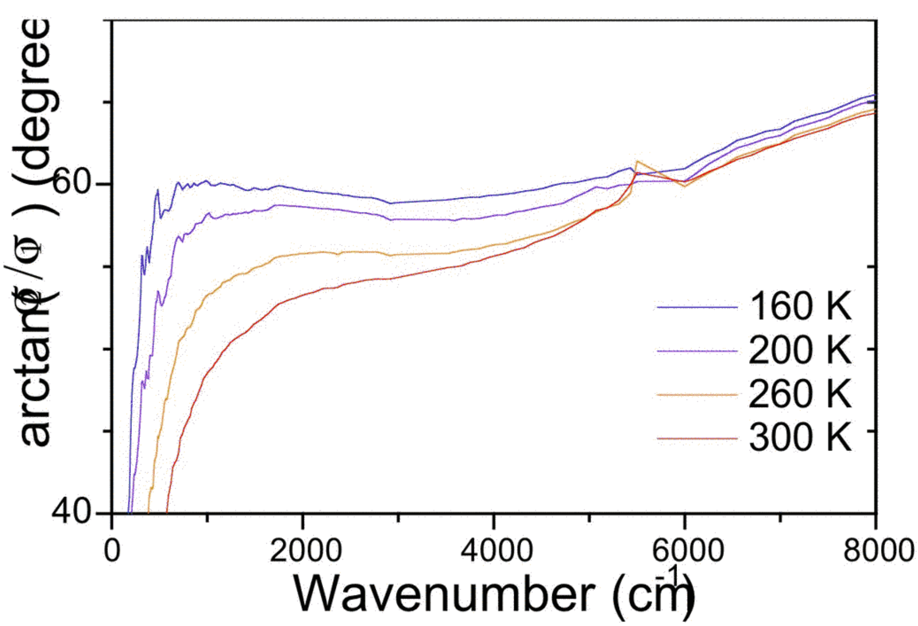
f-sumrule $\int_0^\infty \sigma_1(x)dx = \frac{\pi n e^2}{2m}$

→ High frequency limit of phase → 90 degrees

→ Energy scale of cross-over < bandwidth : Ω~ 1.5 eV

Example:
DvdM, PRB60, R768 (1999)

$$\sigma(\omega,T) = \frac{\omega_p^2/4\pi}{(-i\omega)^{2-\eta}(\Omega - i\omega)^{\eta-1}}$$



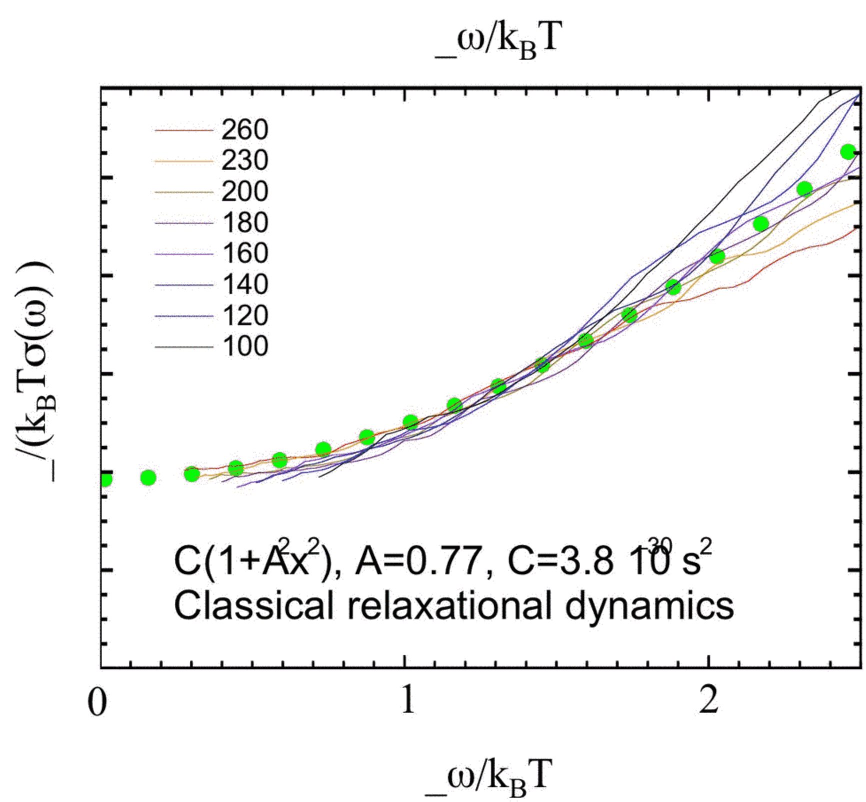
T > ω   T is the only energy scale (Varma)

$$\sigma_1(\omega,T) = T^y f(\omega/T)$$

f - sumrule, $\int T^y f(\omega/T)d\omega = c$, requires $y = -1$

$$\Rightarrow \frac{1}{T\sigma_1(\omega)} = g(\omega/T) = C\left(1 + A^2\left(\frac{\omega}{T}\right)^2 + \ldots\right)$$

Note: MIT in 2D has y=0 (Fisher, Grinstein, Girvin, PRL 1990)



Ioffe-Millis cold spot model

$$\sigma(\omega,T)=\frac{C}{\sqrt{T^2-i\omega T_0}}$$

$$\Rightarrow \frac{1}{T\sigma_1(\omega,T)}=f\left(\omega/T^2\right)$$

Summary

Optical scattering rate of cuprates is slighty sublinear

Optical conductivity below ~5000 cm-1 follows a power law

Low temperature behaviour is masked by the pseudo-gap in the optical conductivity

2D MIT scaling behaviour of optical conductivity not observed

HTSC near optimal doping:

*1)***Region 1** $(\hbar\omega < 1.5k_BT)$: $\tau_R = A\hbar/k_BT$, $A = 0.77$

2) **Region 2:**

a: $\sigma(\omega)$ is proportional to $(i\omega)^{\gamma-2}$

b: Phase of $\sigma(\omega)$ is $\pi(1-\gamma/2)$, independent of frequency

c: $\gamma = 4/3 \pm 0.02$

Details: **Nature 425,** 27 **(2003)**