USING TNS FOR OUT-OF-EQUILIBRIUM DYNAMICS

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PRB 97, 094206 (2018)

MCQST

MPQ Max Planck Institut of Quantum Optics (Garching)

KITP 17.4.2019

In this talk...

Tensor Network States: MPS techniques for dynamics

Some applications to out-of-equilibrium problems

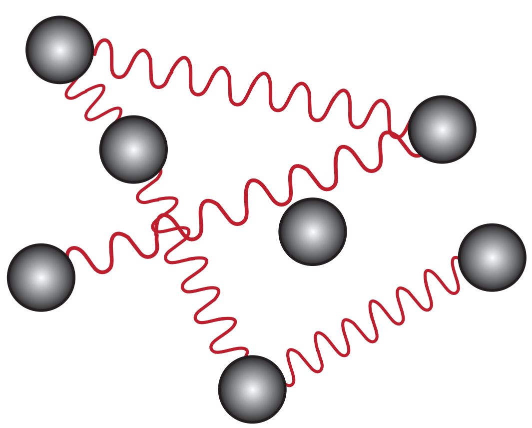
WHAT ARE TNS?

- TNS = Tensor Network States

Context: quantum many body systems

$\{|i\rangle\}_{i=0}^{d-1}$

N



interacting with each other

Goal: describe interesting states

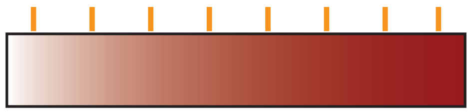
ground, thermal states

WHAT ARE TNS?

- TNS = Tensor Network States

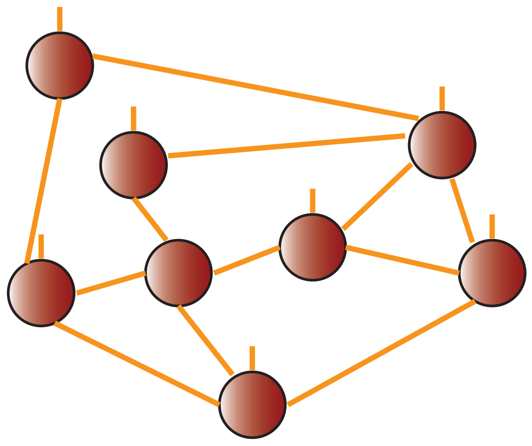
A general state of the N-body Hilbert space has exponentially many coefficients

$$|\Psi\rangle = \sum_{i_j} c_{i_1 \dots i_N} |i_1 \dots i_N\rangle$$



N-legged tensor

d^N



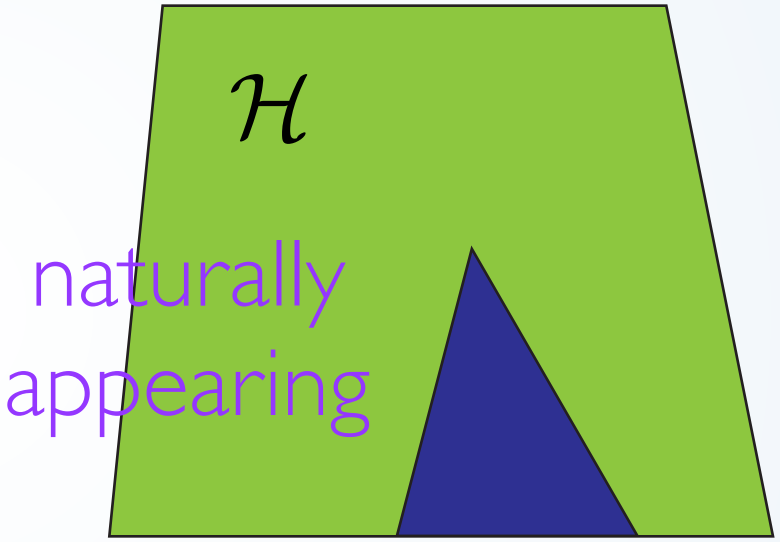
$\mathrm{poly}(N)$

A TNS has only a polynomial number of parameters

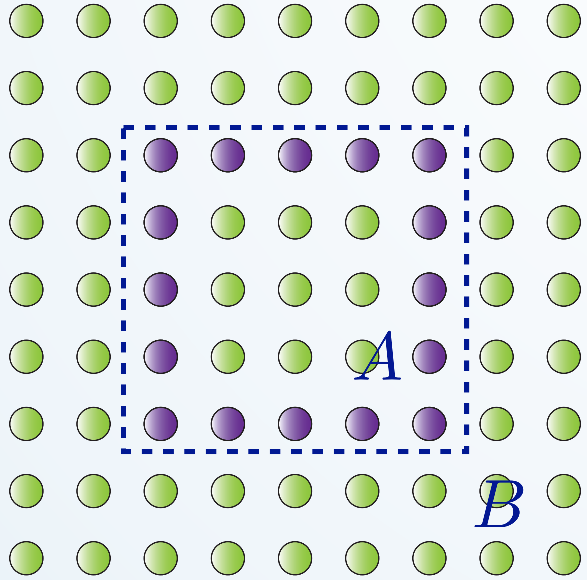
WHY SHOULD TNS BE USEFUL?

States appearing in Nature are peculiar

State at random from Hilbert space is not close to product



area law



Hastings 2007

Calabrese, Cardy 2004; Wolf 2006

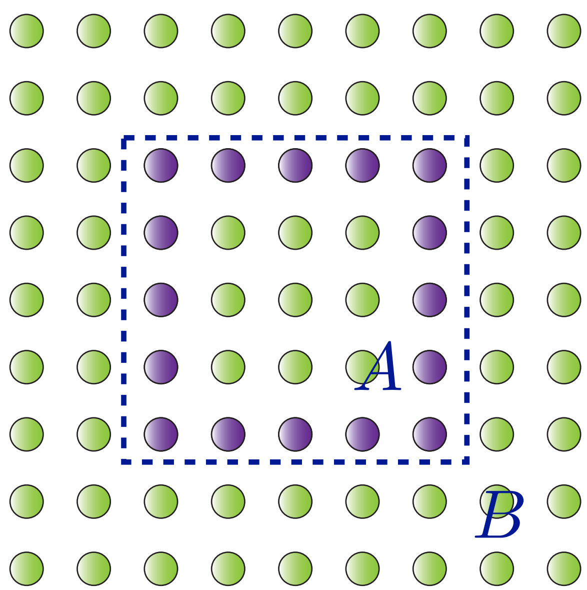
We look for states with *little* entanglement

TNS = entanglement based ansatz

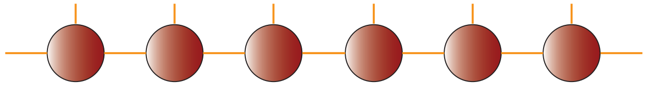
TNS = entanglement based ansatz

successful GS, thermal…

area law

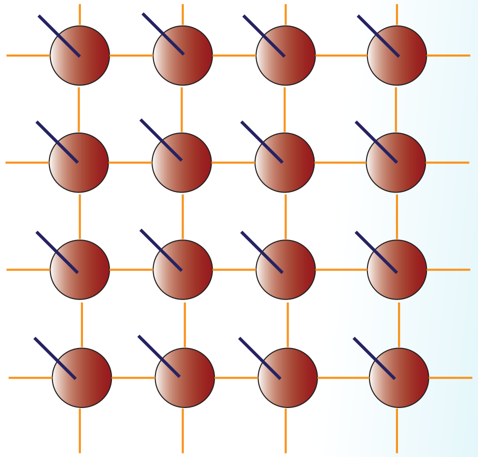


MPS



Schollwöck Ann.Phys.2011

PEPS

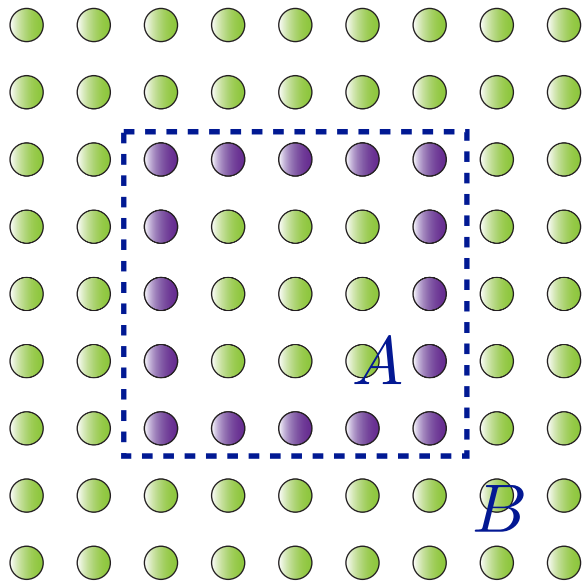


Verstraete et al. Adv. Phys. 2008

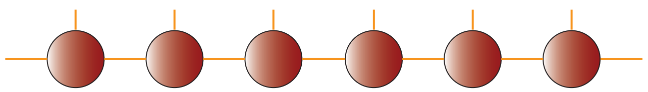
TNS = entanglement based ansatz

algorithms exist to simulate time evolution

area law

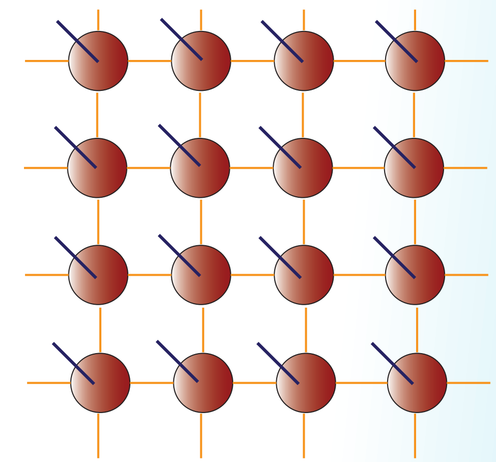


MPS



Schollwöck Ann.Phys.2011

PEPS



Verstraete et al. Adv. Phys. 2008

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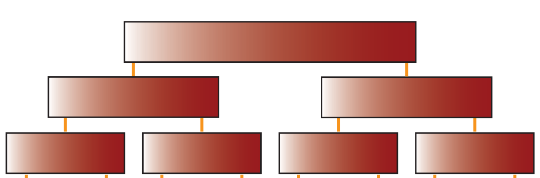
other TNS

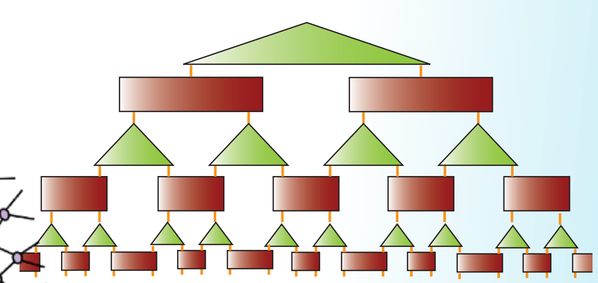
TTN

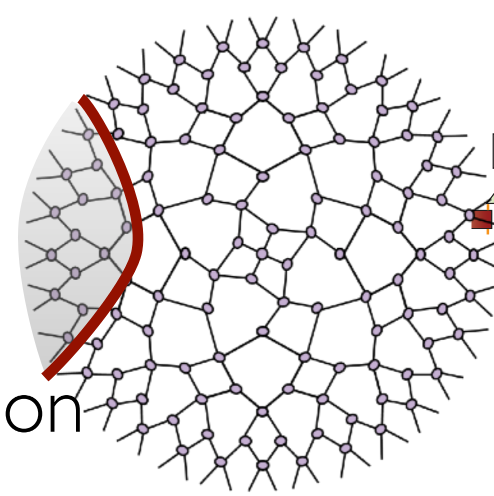


Shi et al PRA 2006

Vidal PRL 2007

MERA





suggested connection to AdS/CFT

Swingle PRD 2012
Molina JHEP 2013
Nozaki et al JHEP 2012
Bao et al PRD 2015

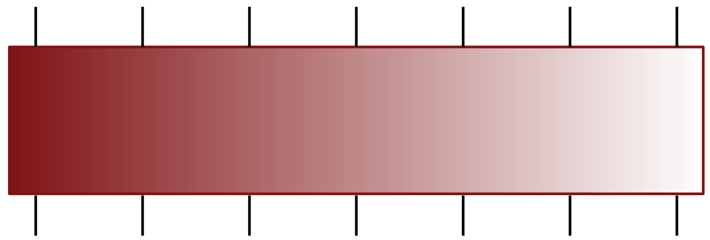
TIME EVOLUTION WITH MPS

basic ideas

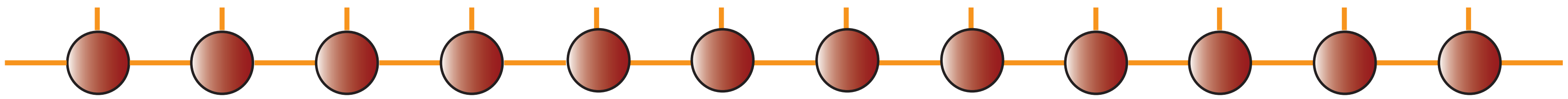
BASIC ALGORITHMS

Simulate time evolution

$U(t)$



initial MPS



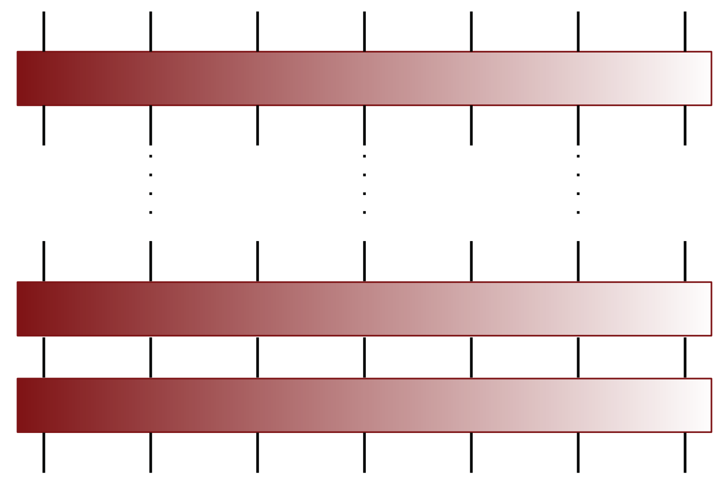
(i)TEBD
t-DMRG

Vidal, PRL 2003, 2007; Daley et al., JStatMech 2004
Verstraete, García-Ripoll, Cirac, PRL 2004

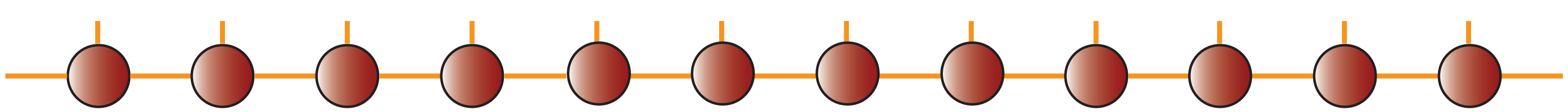
BASIC ALGORITHMS

Simulate time evolution

$$U(t) \rightarrow [U(\delta)]^M$$



initial MPS



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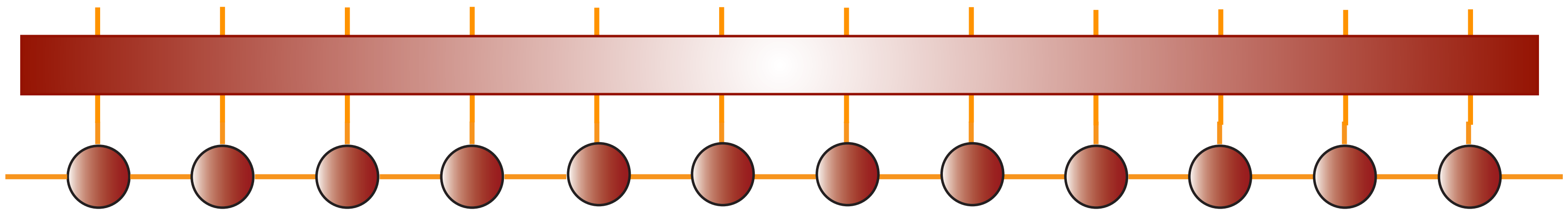

(i)TEBD
t-DMRG

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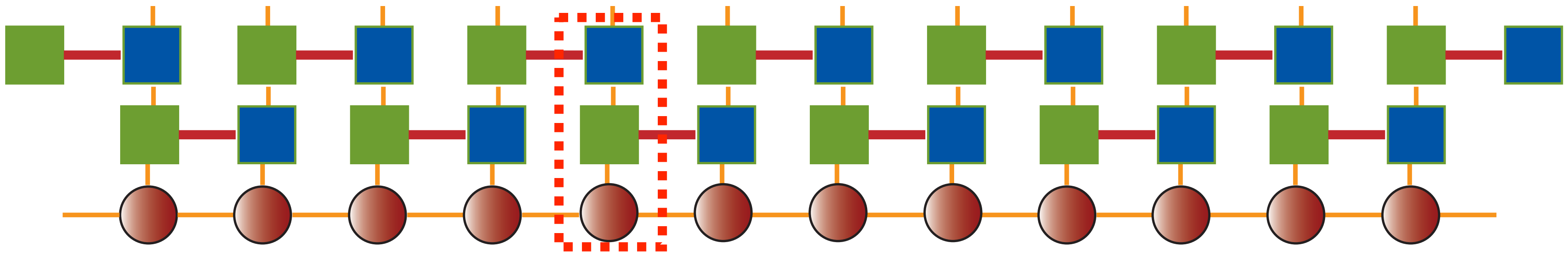
BASIC ALGORITHMS

Simulate time evolution

$$U(t) \to [U(\delta)]^M$$

truncate bond dimension

initial MPS



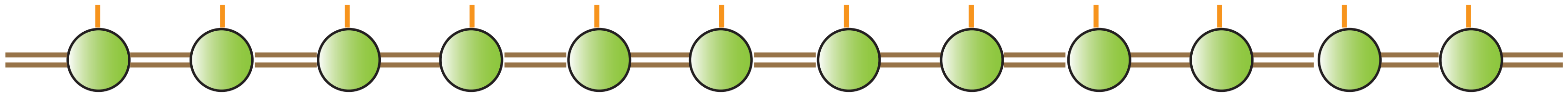
(i)TEBD
t-DMRG

Vidal, PRL 2003, 2007; Daley et al., JStatMech 2004
Verstraete, García-Ripoll, Cirac, PRL 2004

BASIC ALGORITHMS

Simulate time evolution

iterate



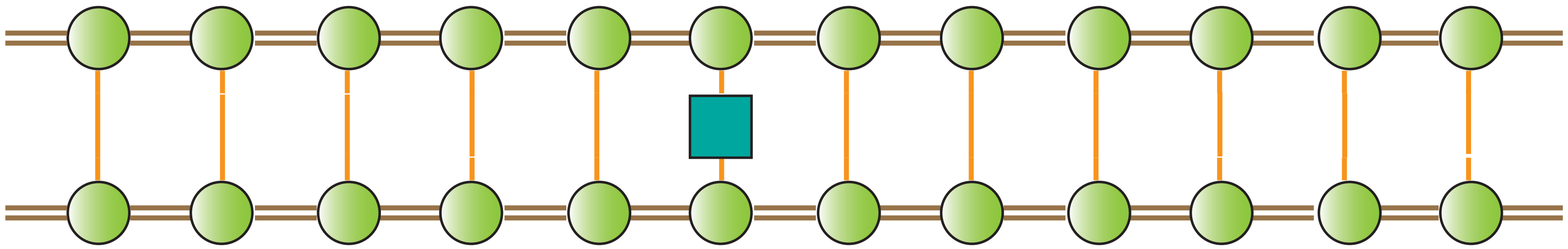
(i)TEBD
t-DMRG

Vidal, PRL 2003, 2007; Daley et al., JStatMech 2004
Verstraete, García-Ripoll, Cirac, PRL 2004

BASIC ALGORITHMS

Simulate time evolution

compute observables



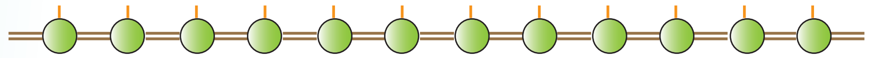
(i)TEBD
t-DMRG

Vidal, PRL 2003, 2007; Daley et al., JStatMech 2004
Verstraete, García-Ripoll, Cirac, PRL 2004

more time evolution with MPS

evolving the (pure state) ansatz

Vidal, PRL 2003, PRL 2007
White, Feiguin, PRL 2004
Daley et al., 2004
Haegeman et al., 2011

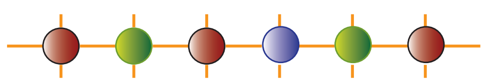


entanglement can grow fast!

Osborne, PRL 2006
Schuch et al., NJP 2008

evolving operators: Heisenberg picture

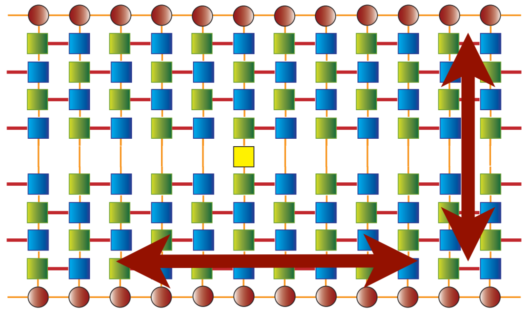
Hartmann et al, PRL 2009



also for mixed states
operator space entanglement

Prosen Pizorn, PRL 2008

observables as TN to contract



different *entanglement* quantities

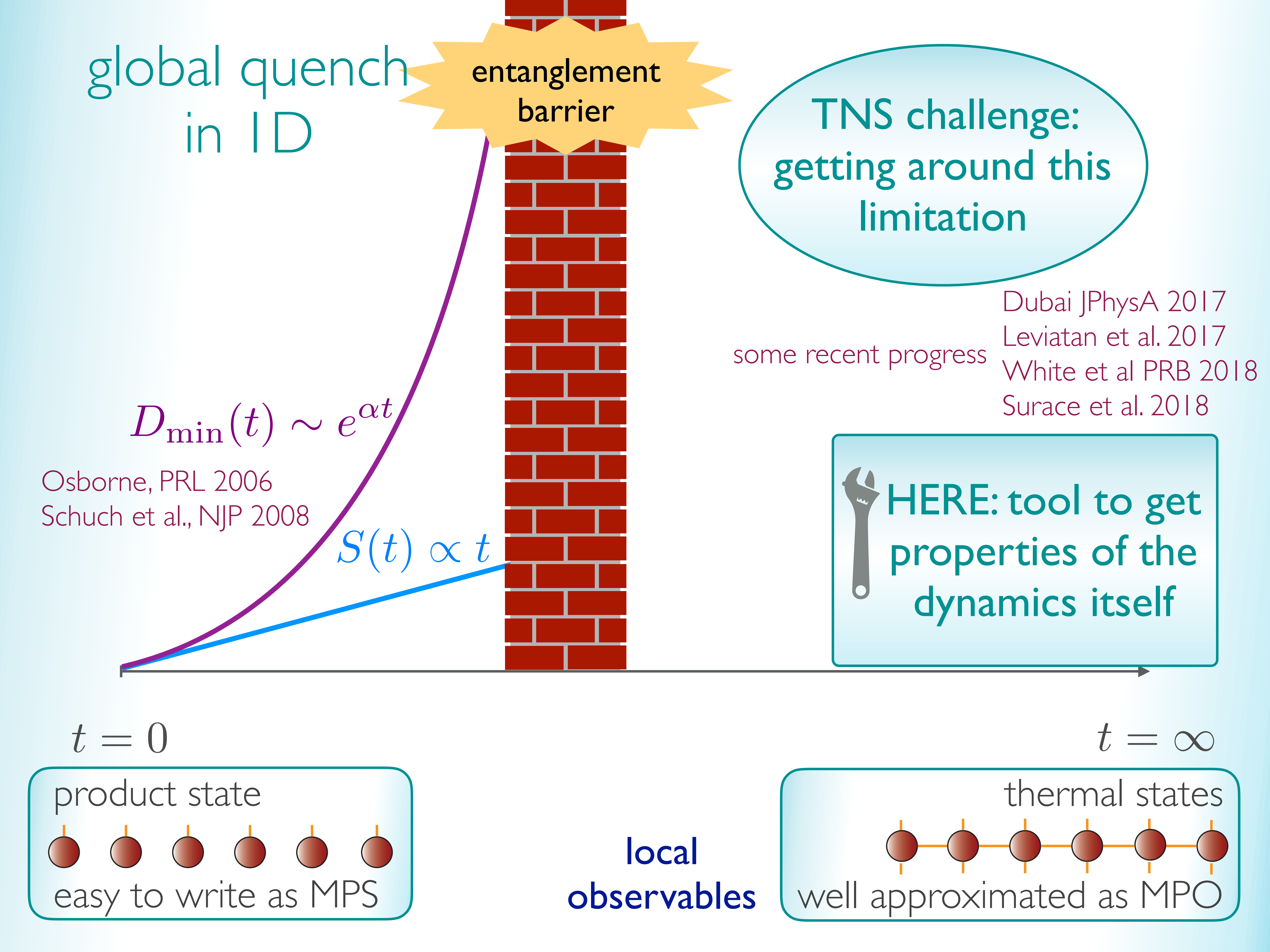
MCB, Hastings, Verstraete, Cirac, PRL 2009
Müller-Hermes et al., NJP 2012
Hastings, Mahajan 2014

Many physical situations can be treated!

Wall, Carr NJP 2012; Jaksche et al OSMPS

Paeckel et al arXiv:1901.05824

Entanglement growth in non-equilibrium scenarios limits the applicability of MPS

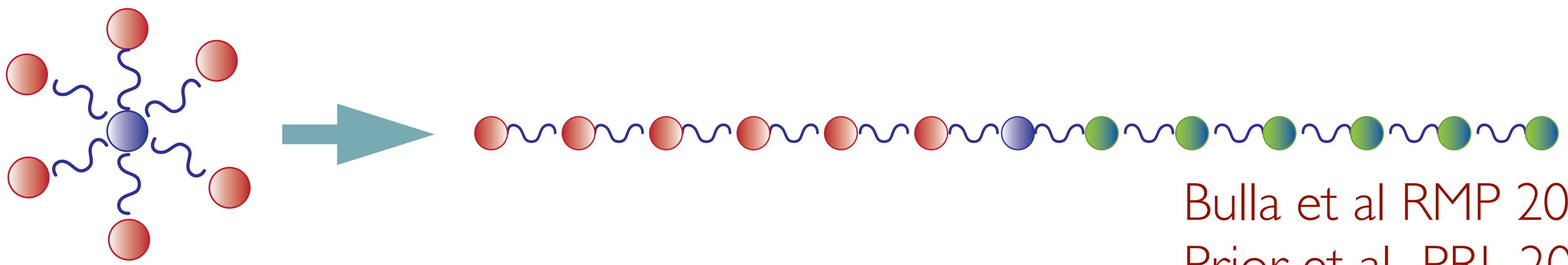


global quench in 1D
entanglement barrier
TNS challenge: getting around this limitation
some recent progress
Dubai JPhysA 2017
Leviatan et al. 2017
White et al PRB 2018
Surace et al. 2018
$D_{\min}(t) \sim e^{\alpha t}$
Osborne, PRL 2006
Schuch et al., NJP 2008
$S(t) \propto t$
HERE: tool to get properties of the dynamics itself
$t = 0$
$t = \infty$
product state
easy to write as MPS
local observables
thermal states
well approximated as MPO

About open systems

open system dynamics

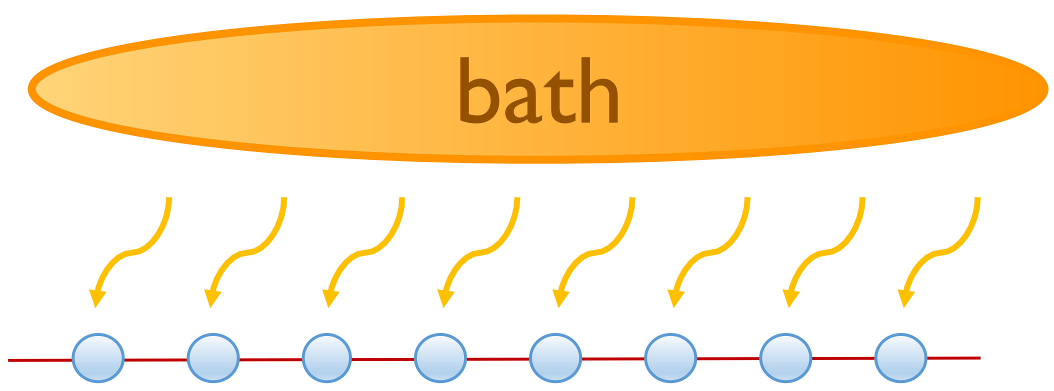
exact description system+bath



e.g. chain mappings

Bulla et al RMP 2008
Prior et al., PRL 2010
de Vega, MCB, PRA 2015

non-equilibrium steady states



quantum trajectories with MPS

Daley, Adv. Phys. 2014

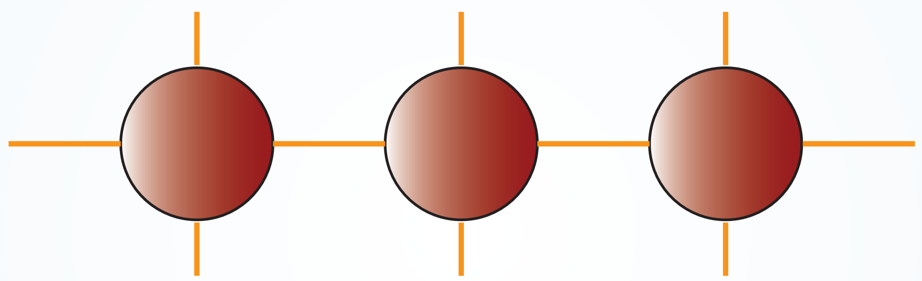
TNS for density operator…

Jaksche et al., QSciTech2019

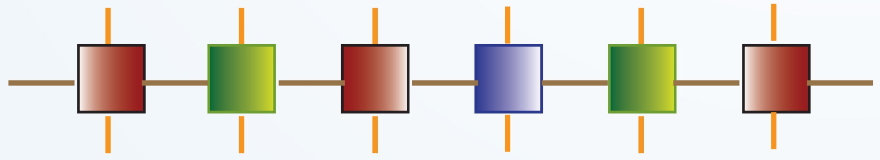
MPO

- MPO = Matrix Product Operator

Same kind of ansatz for operators



$$\hat{M} = \sum_{i_1,j_1\dots i_N,j_N} \mathrm{tr}(M_1^{i_1 j_1} M_2^{i_2 j_2} \dots M_N^{i_N j_N}) |i_1 \dots i_N\rangle\langle j_1 \dots j_N|$$



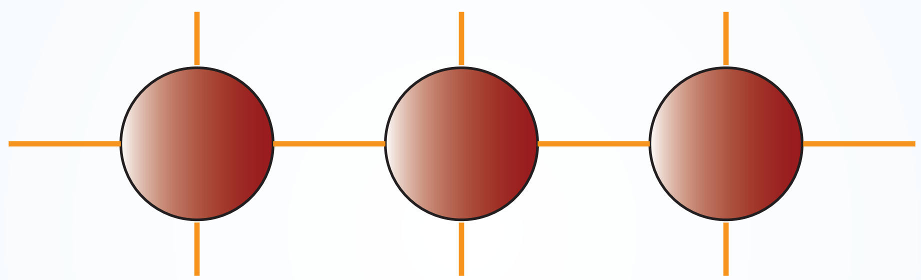
mixed states, H and $U(t)$

efficient exact MPO representation for local, NN, ...

MIXED STATES

- MPDO = Matrix Product Density Operator

Use for density operators



need some properties

$$\rho = \sum_{i_1, j_1 \ldots i_N, j_N} \mathrm{tr}(M_1^{i_1 j_1} M_2^{i_2 j_2} \ldots M_N^{i_N j_N}) |i_1 \ldots i_N\rangle\langle j_1 \ldots j_N|$$

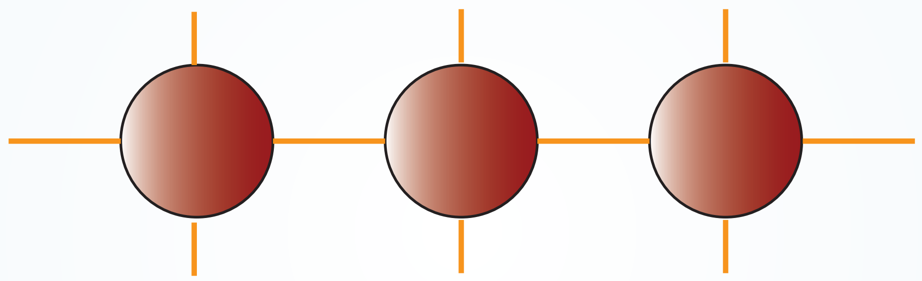
can we impose them *locally*?

$\rho = \rho^\dagger$ ✓ $\quad$ $\mathrm{tr}\rho = 1$ ✓ $\quad$ $\rho \geq 0$

MIXED STATES

- MPDO = Matrix Product Density Operator

purification



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$\rho = \rho^\dagger$ ✔

$\mathrm{tr}\rho = 1$ ✔

$\rho \geq 0$

in a way

$$\rho_S = \mathrm{tr}_A |\Psi_{SA}\rangle\langle\Psi_{SA}|$$

Verstraete et al., PRL 2004

Zwolak and Vidal, PRL 2004

MIXED STATES

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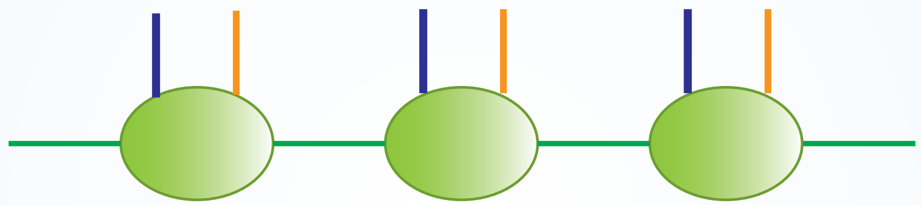

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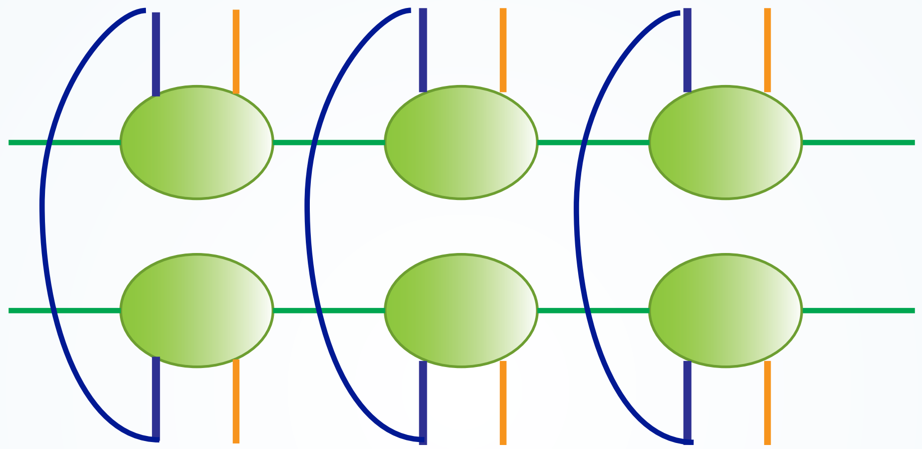

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Verstraete et al., PRL 2004

Zwolak and Vidal, PRL 2004

A tool to get properties of the dynamics itself

finding operators that evolve slowly

can set a long timescale

A DIFFERENT PERSPECTIVE

What are the slowest evolving (local) operators?

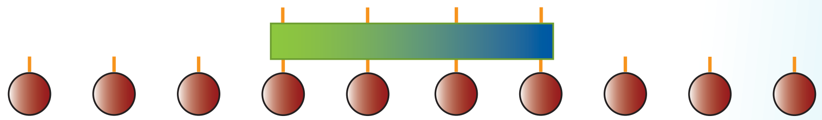
$$\frac{dA(t)}{dt} = i[H, A(t)]$$

numerical study using ED/TNS

Kim, MCB, Cirac, Hastings, Huse, PRE92, 012128 (2015)

(ALMOST) LOCAL CONSERVED OPERATORS

Goal: minimizing $\|[H, A_M]\|$



$$\lambda_M = \min_{A_M} \frac{\|[A_M, H]\|_2^2}{\|A_M\|_2^2}$$

numerically with ED and TNS

$\|A\|_2 = \sqrt{\mathrm{tr}(A^\dagger A)}$ Frobenius

physical meaning

$$\rho \sim I + \epsilon A_M$$

high T state

$$|\langle A_M(t)\rangle - \langle A_M\rangle_\beta| \geq 1 - \frac{1}{2}\lambda_M t^2$$

lower bound thermalization time

$$\tau \geq \frac{1}{\sqrt{\lambda_M}}$$

also slowest evolving at short times

Kim, MCB, Cirac, Hastings, Huse, PRE92, 012128 (2015) can be applied to systematically study different (potentially non-thermalizing) systems

MBL

MANY BODY LOCALIZATION

Anderson localization: single particle states localized due to disorder

environment destroys localization

interactions + disorder = interesting scenario

weak interactions ⇒ MBL phase

Basko, Aleiner, Altshuler, Ann. Phys. 2006
Gornyi, Mirlin, Polyakov, PRL 2005

many-body localization

Altman, Vosk, Ann.Rev.CM 2015
Nandkishore, Huse, Ann.Rev.CM 2015

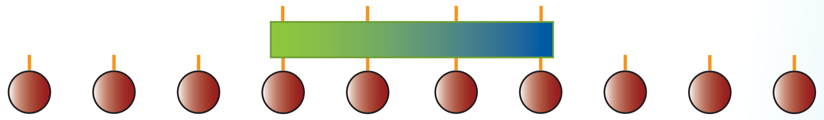
highly excited states localized

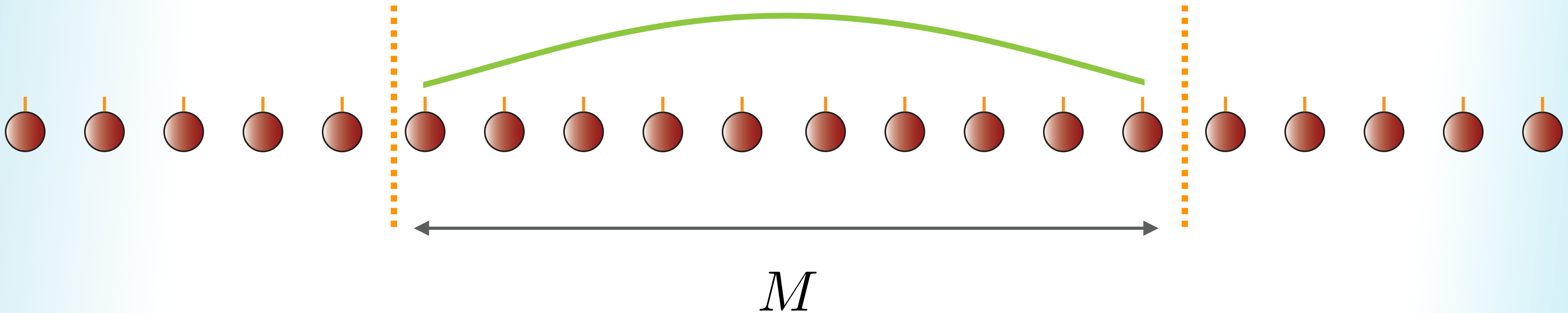
system will not thermalize

TNS *success stories*

Znidaric, Prosen, Prelovsek, PRB 2008
Gogolin, Müller, Eisert, PRL 2011
Bardarson, Pollmann, Moore, PRL 2012
Bauer, Nayak, JStatMech 2013; Chandran et al PRB 2015; Pollmann et al PRB 2016; Khemani et al PRL 2016; Pekker PRB 2017; Wahl et al 2017…

operator acting on M sites





$$\lambda_M = \min_{A_M} \frac{\| [A_M, H] \|_2^2}{\| A_M \|_2^2}$$

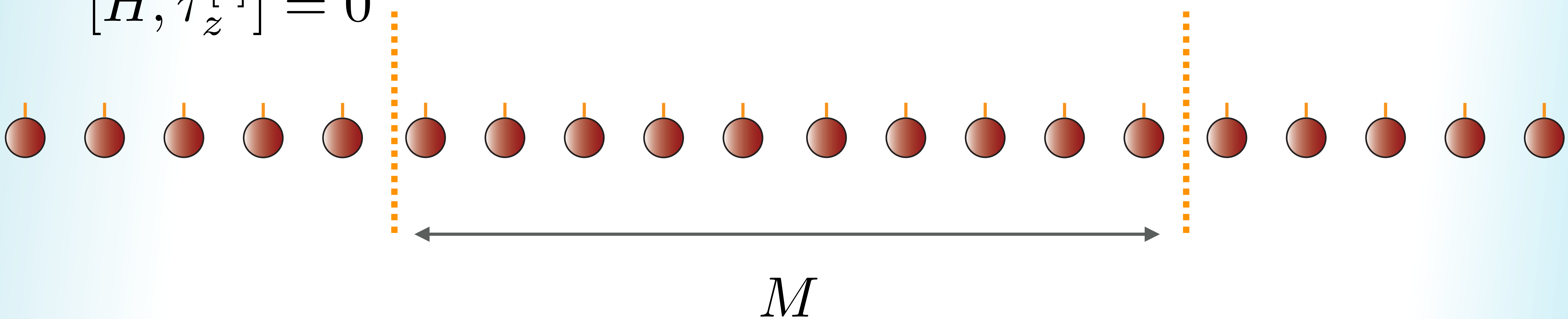
N. Pancotti et al PRB 97, 094206 (2018)

in the localized regime: l-bit model

$$H_{\rm eff} = \sum_{i=0}^{N-1} \epsilon_i \tau_z^{[i]} + \sum_{i,j=0}^{N-1} K_{ij}^{(2)} \tau_z^{[i]} \tau_z^{[j]} + \sum_{i,j,k=0}^{N-1} K_{ijk}^{(3)} \tau_z^{[i]} \tau_z^{[j]} \tau_z^{[k]} + \ldots,$$

$\tau_z^{[i]}$ exponentially localized

$$[H, \tau_z^{[i]}] = 0$$



$$\lambda_M = \min_{A_M} \frac{\|[A_M, H]\|_2^2}{\|A_M\|_2^2}$$

N. Pancotti et al PRB 97, 094206 (2018)

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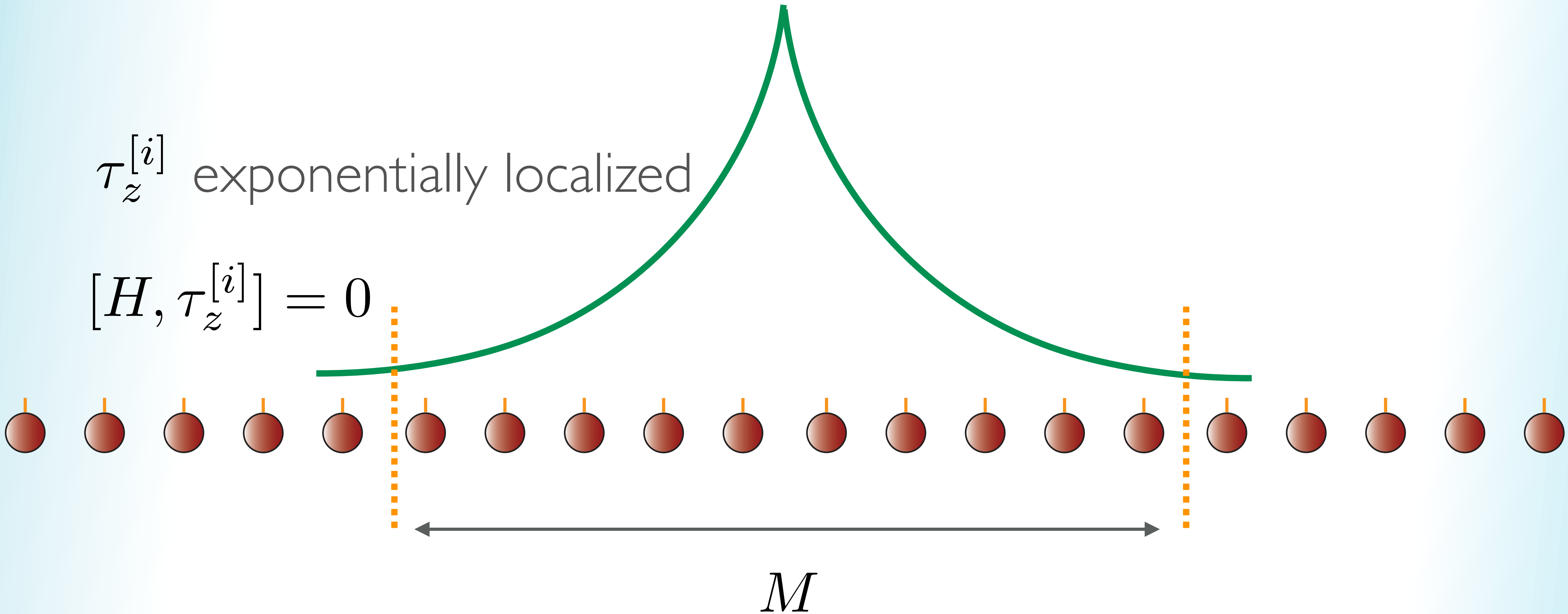


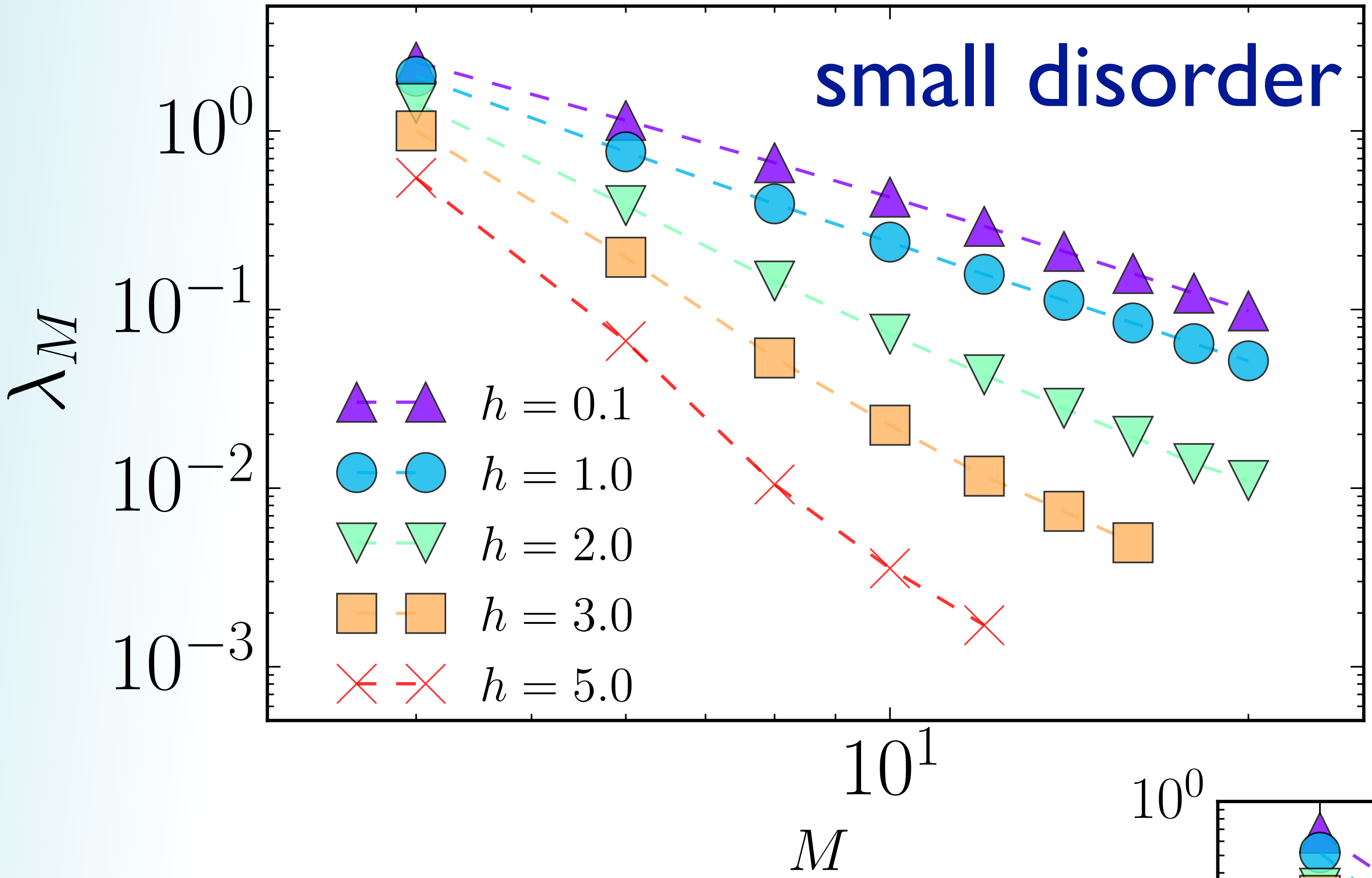
$$\lambda_M = \min_{A_M} \frac{\| [A_M, H] \|_2^2}{\| A_M \|_2^2}$$

truncated support → expect exponentially small

see also Chandran et at. PRB 2015

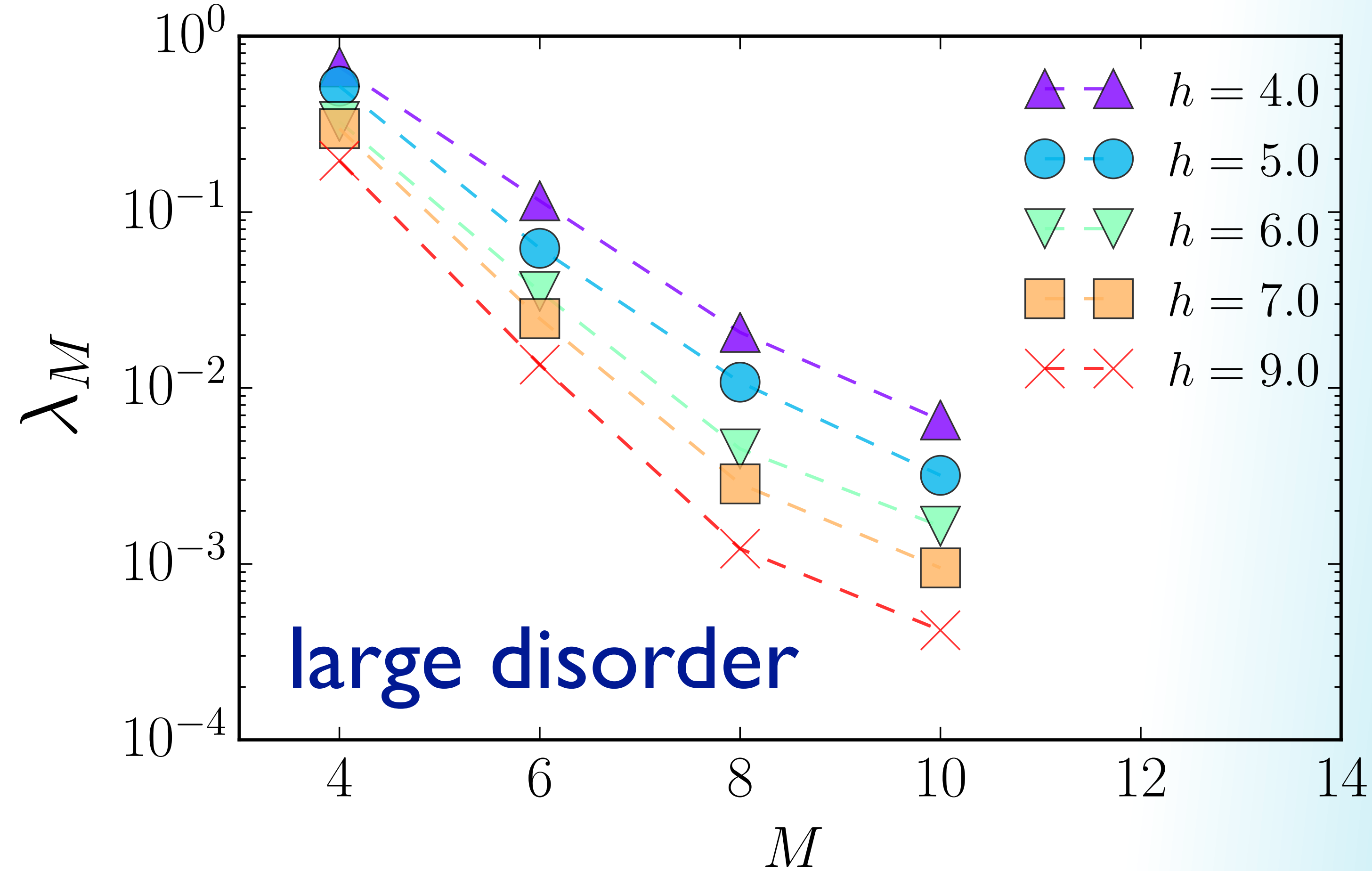
N. Pancotti et al PRB 97, 094206 (2018)

averaging over disorder



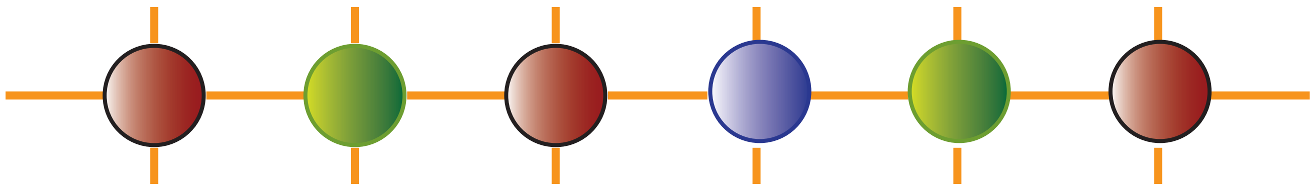
power law

exponential



N. Pancotti et al PRB 97, 094206 (2018)

constructive method



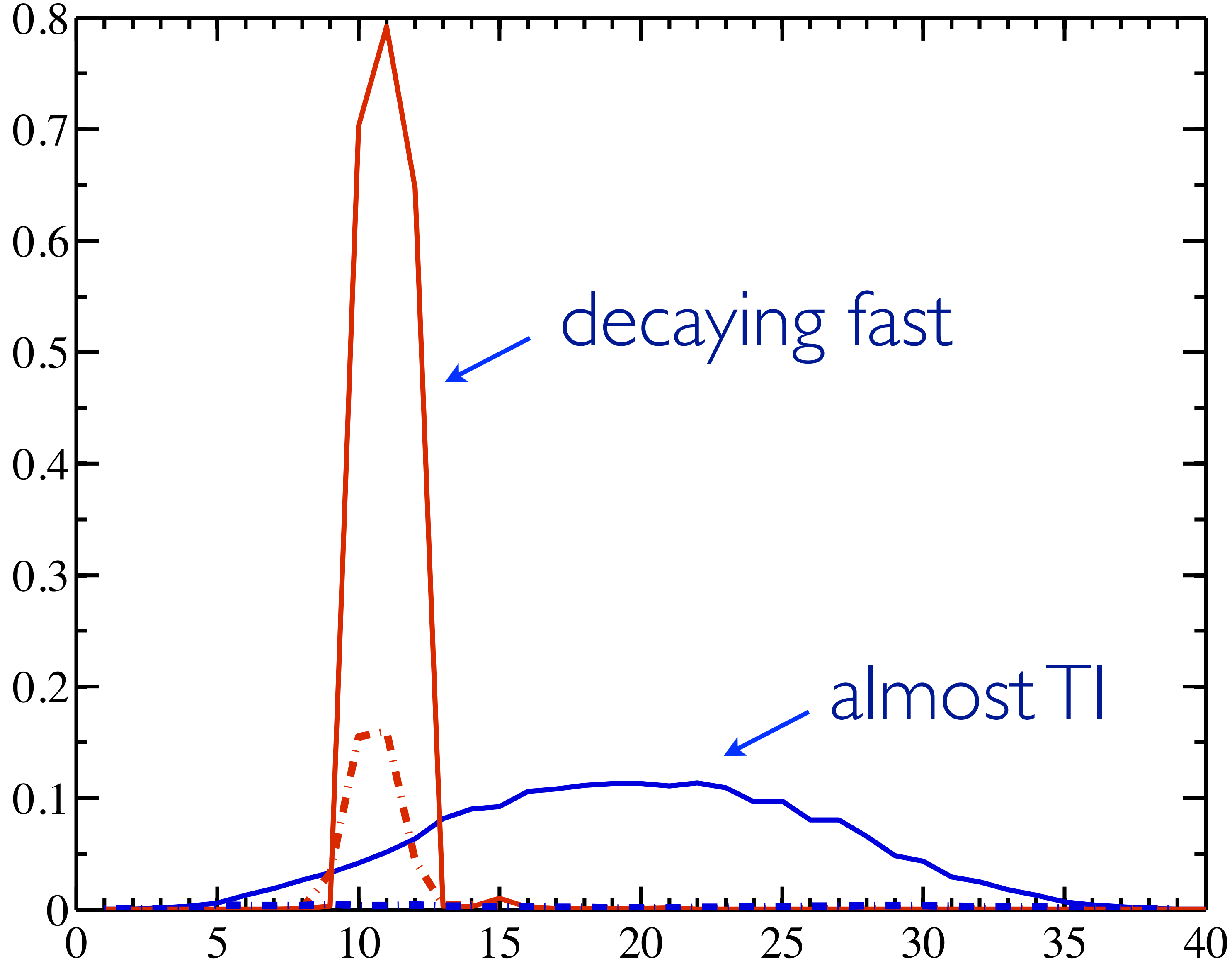
analyze weight of components with different support

$$\sigma_i^{[m]} \otimes \cdots \otimes \sigma_j^{[m+d]}$$

N. Pancotti et al PRB 97, 094206 (2018)

composition of slow operators: how local?

landscape of terms with fixed range



sensitive to rare regions

and much more information in the statistics!

single realization $M = 40$

N. Pancotti et al PRB 97, 094206 (2018)

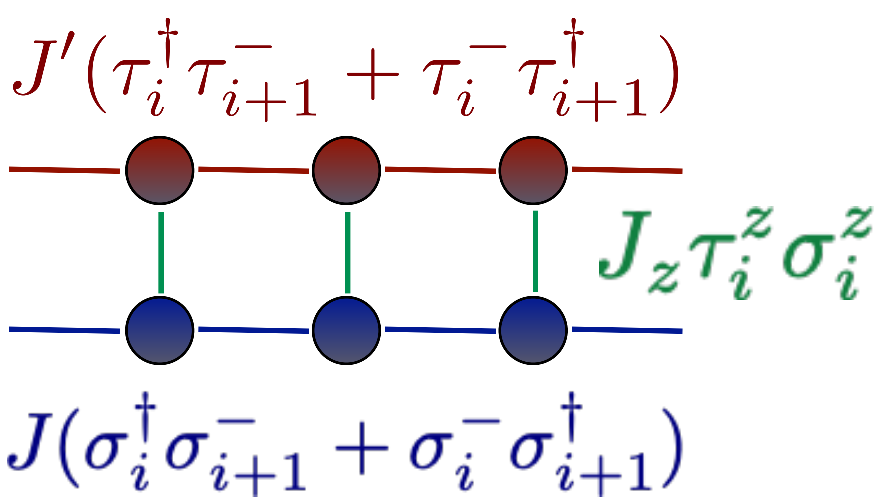
systems without disorder can exhibit slow dynamics

open question: absence of thermalisation in translationally invariant systems?

some models proposed for localization without disorder

De Roeck, Huveneers, Comm. Math. Phys. 2014; PRB 2014
Schiulaz, Müller, AIP Conf. Series 2014

e.g. one slow species could "localize" another one or an ancillary system produces all realisations of disorder



Yao, Laumann, Cirac, Lukin, Moore, PRL 2016

numerical studies inconclusive

Schiulaz, Silva, Müller, PRB 2015
Papic, Stoudenmire, Abanin, Ann. Phys. 2015

in classical systems: glass transition

slowdown of dynamics

can be produced by dynamical constraints

e.g. transitions only allowed if facilitated by neighbouring excitations

$$10 \stackrel{c}{\rightarrow} 11$$

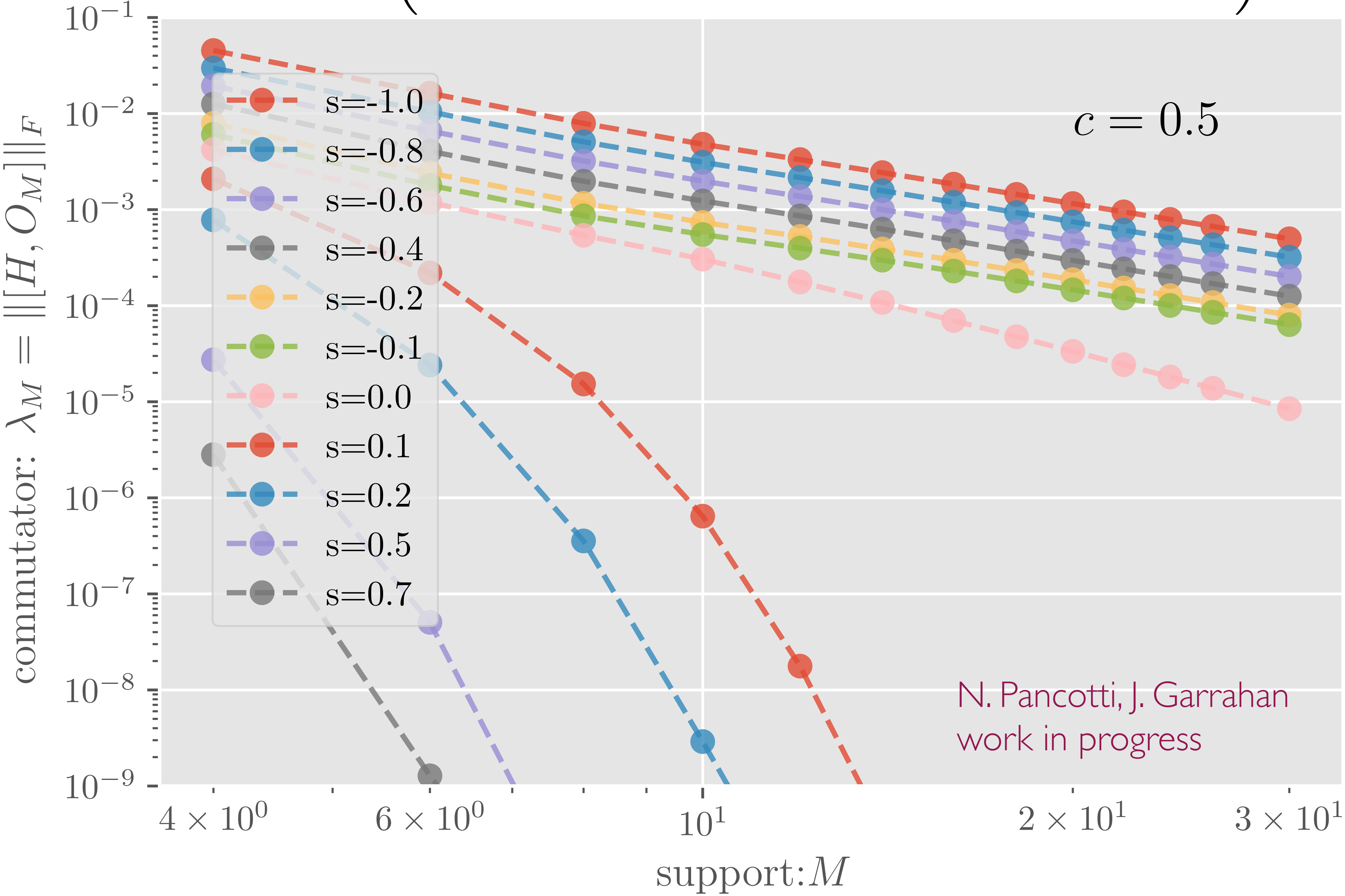
$$11 \stackrel{1-c}{\rightarrow} 10$$

similar constraints in quantum systems?

$$H = -\sum n_i \left(e^{-s} \sqrt{c(1-c)} \sigma^x_{i+1} - (1-2c) n_{i+1} - c \right)$$

van Horssen, Levi, Garrahan, PRB 2015
Lan, van Horssen, Powell Garrahan, PRL 2018

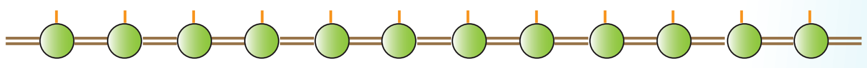
$$H = \sum_{k=0}^{N-1} \left(c + (1-2c) n_k - \exp(-s) \sqrt{c(1-c)} \sigma_k^x \right) n_{k+1}$$



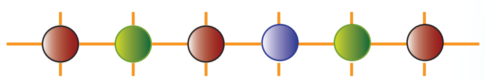
TO CONCLUDE

Various TNS tools can be used for time evolution

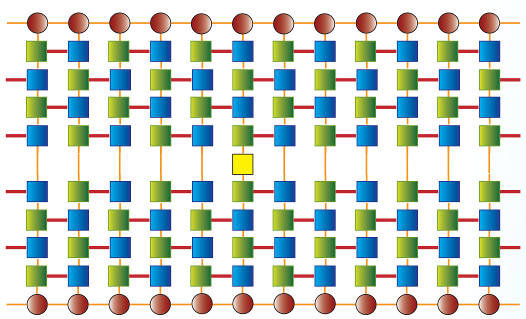
evolving the (pure state) ansatz



evolving operators: Heisenberg picture



observables as TN to contract



global quenches

valid for limited times only

different perspective: slow operators

applicable to different scenarios → e.g. MBL: signatures of localization, and rare regions in the statistics

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ongoing: KCM