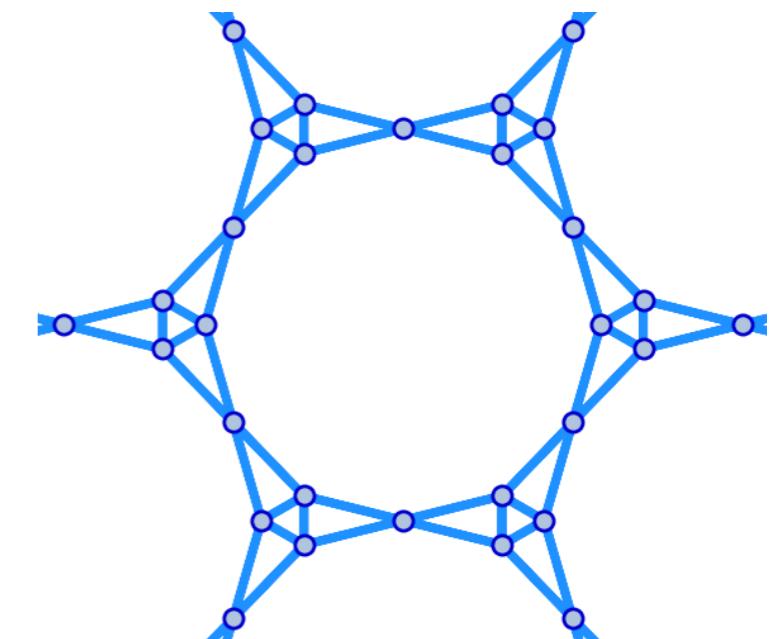
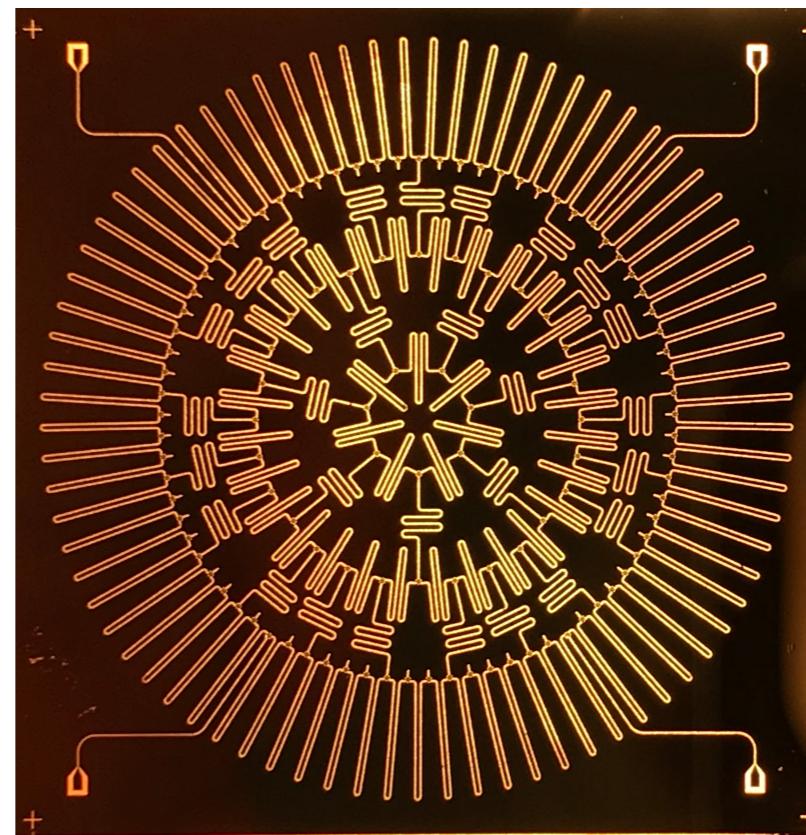
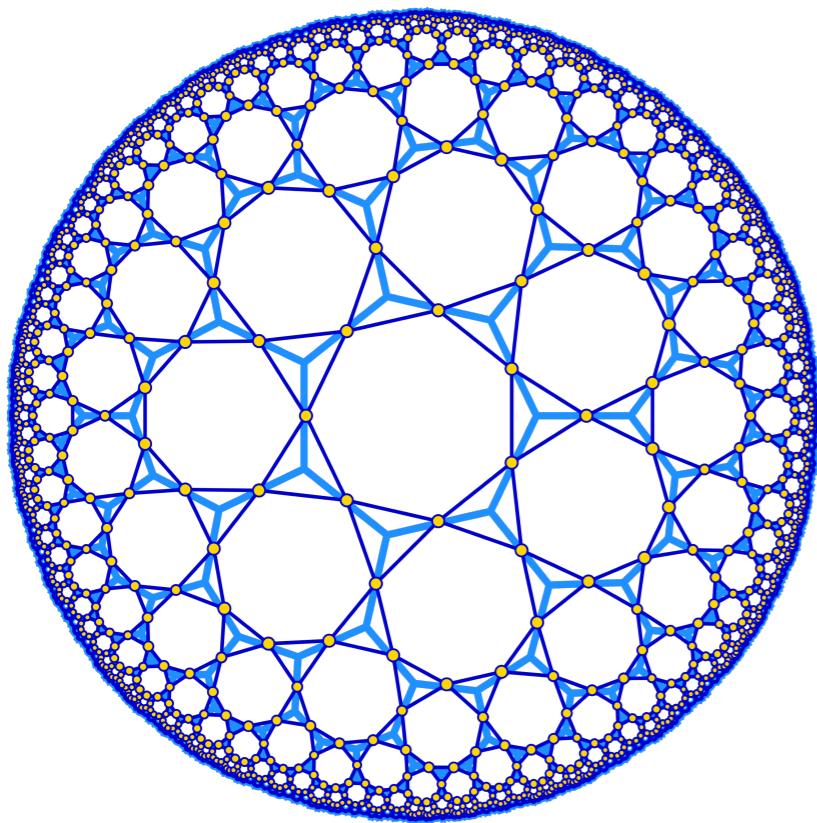


Lattice Simulators in Circuit QED

Alicia Kollár

Department of Physics, University of Maryland

Department of Electrical Engineering, Princeton University



Outline

- Coplanar Waveguide (CPW) Lattices
 - Interacting photons
- Hyperbolic lattices
 - Projection to flat space
 - Deformable resonators
- Flat-band lattices
 - Line graphs
 - Maximal gaps
- Non-linear lattices
 - Limit cycles
 - Chaos

Outline

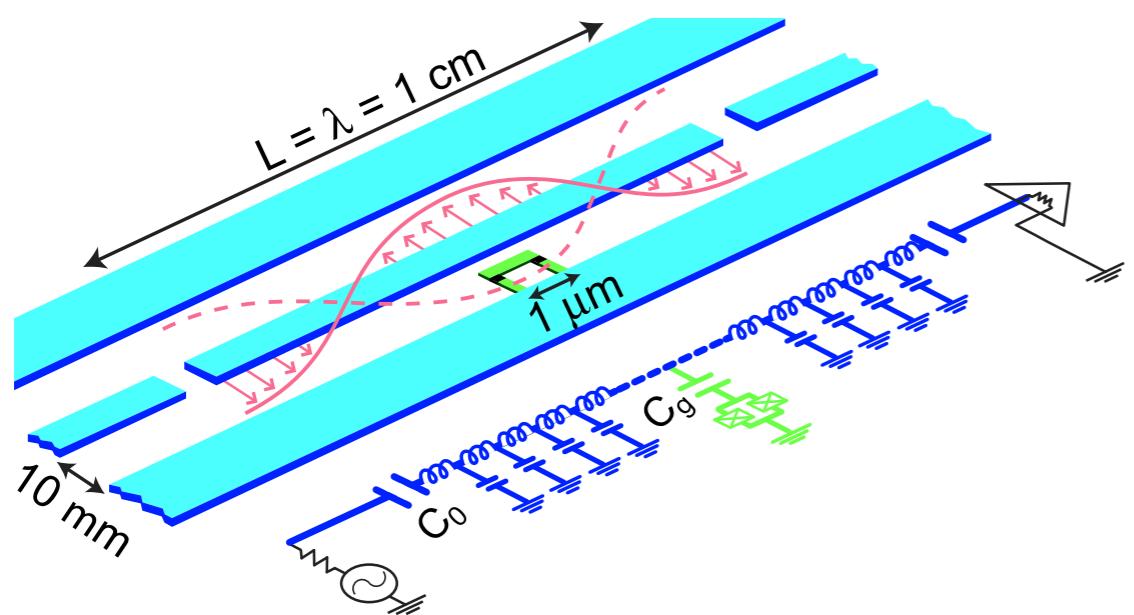
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Microwave Lattice Sites

Coplanar Waveguide Resonator



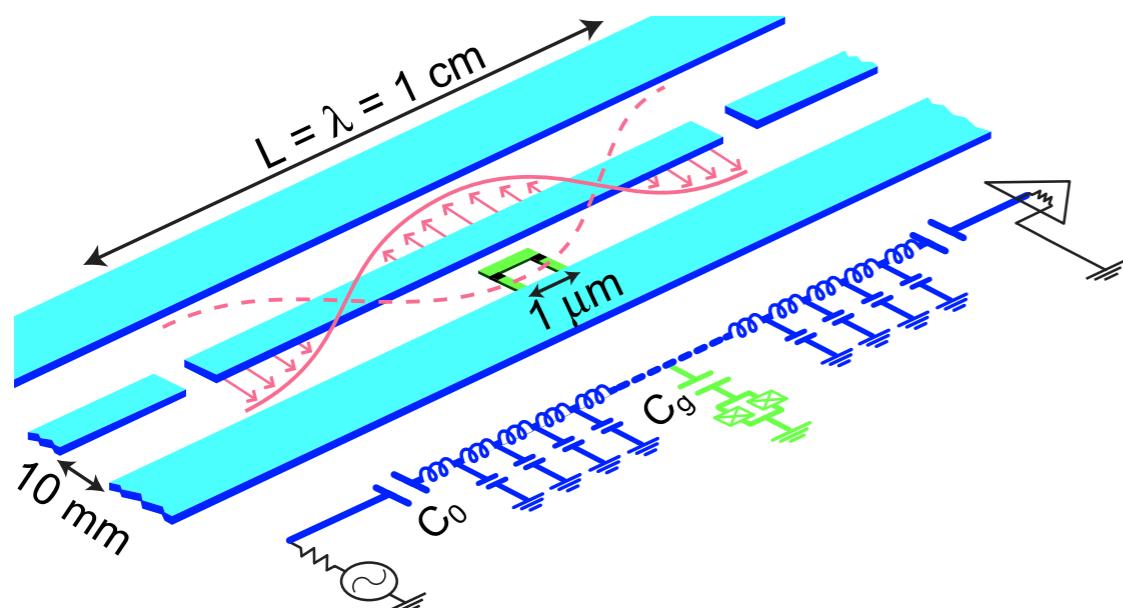
Microwave Lattice Sites

Qubit-Cavity

(Jaynes-Cummings Model)

$$H_{JC} = \omega_c a^\dagger a + \frac{1}{2} \omega_q \sigma_z + g_0 (a^\dagger \sigma^- + a \sigma^+)$$

Coplanar Waveguide Resonator



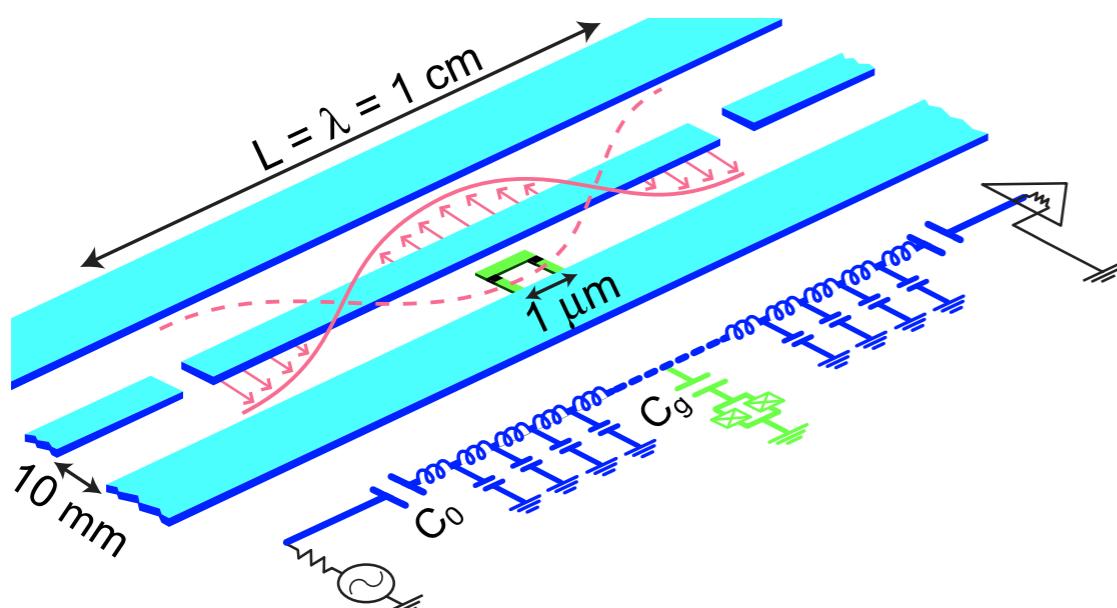
Microwave Lattice Sites

Qubit-Cavity

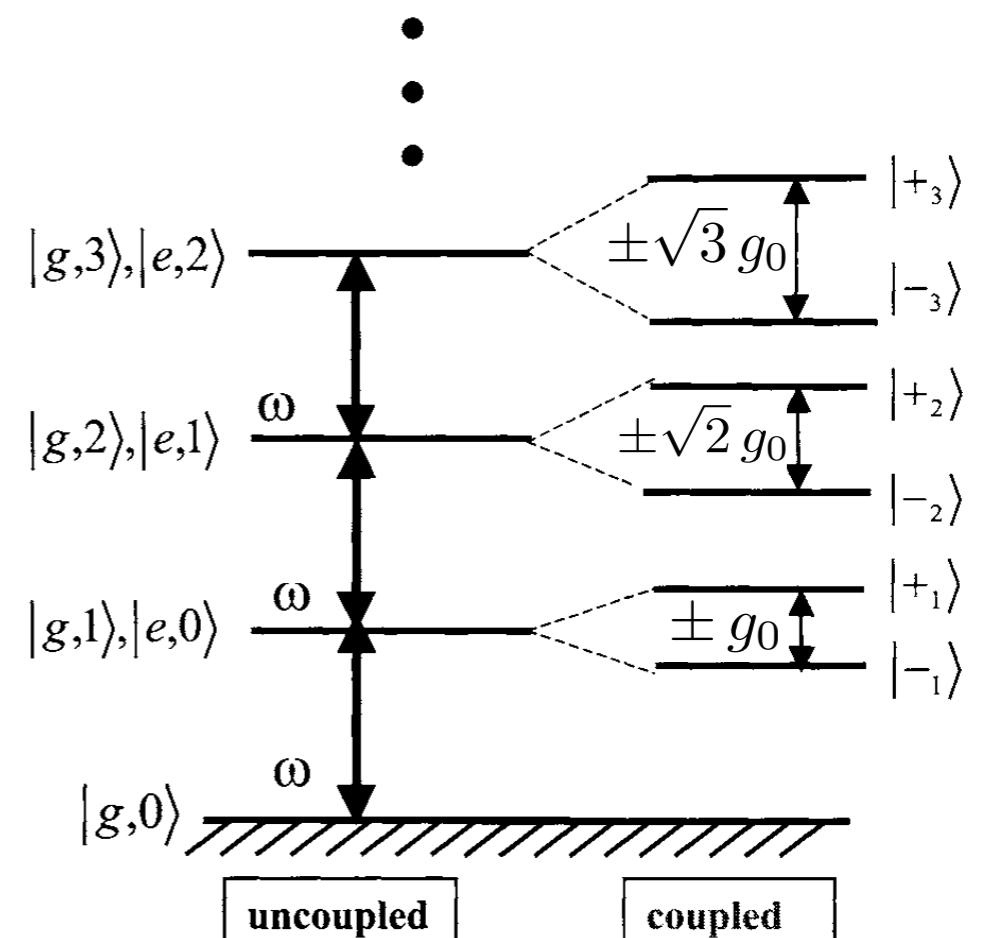
(Jaynes-Cummings Model)

$$H_{JC} = \omega_c a^\dagger a + \frac{1}{2} \omega_q \sigma_z + g_0 (a^\dagger \sigma^- + a \sigma^+)$$

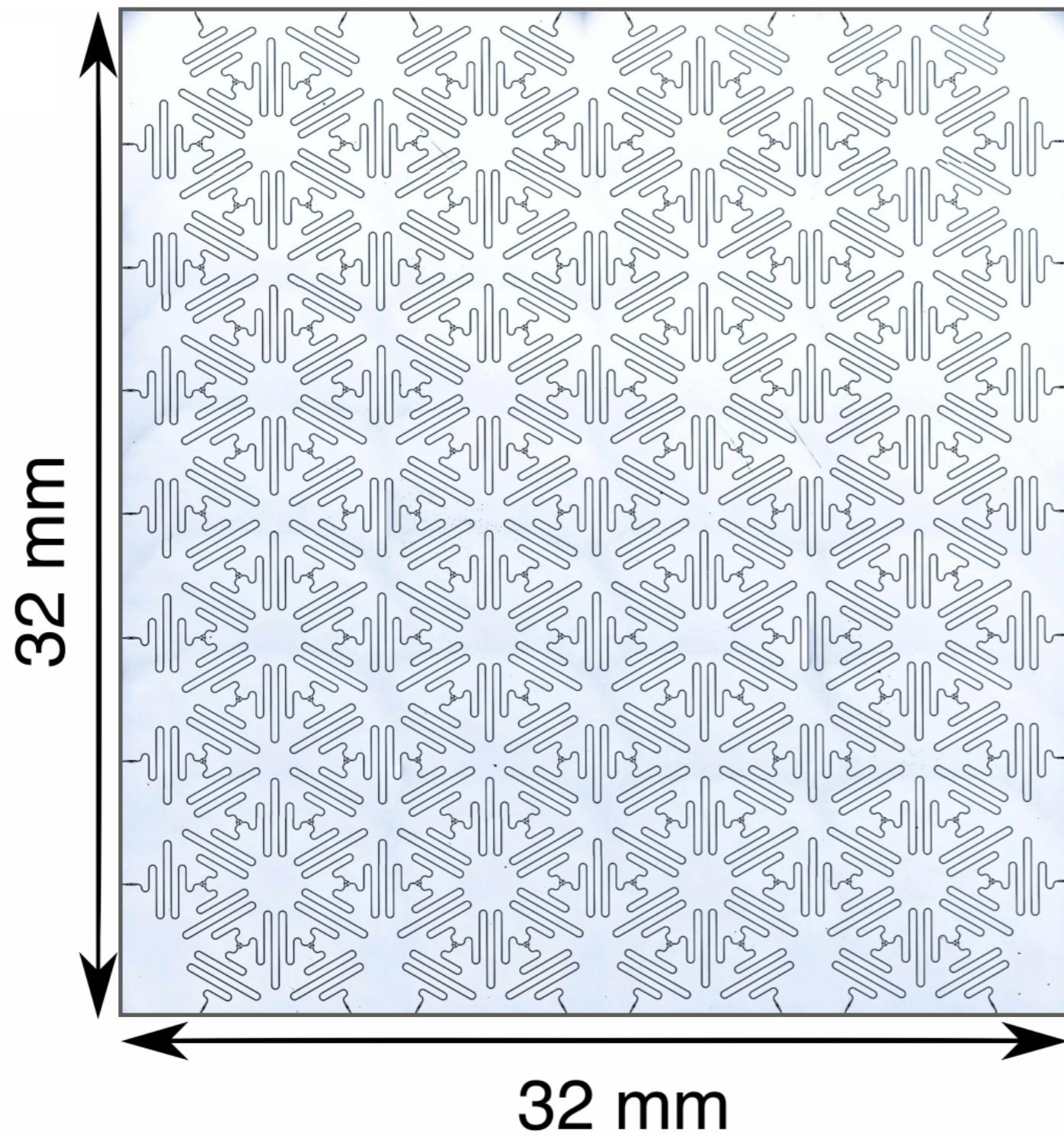
Coplanar Waveguide Resonator



$$|\pm_n\rangle = \frac{1}{\sqrt{2}}(|g, n\rangle \pm |e, n-1\rangle),$$



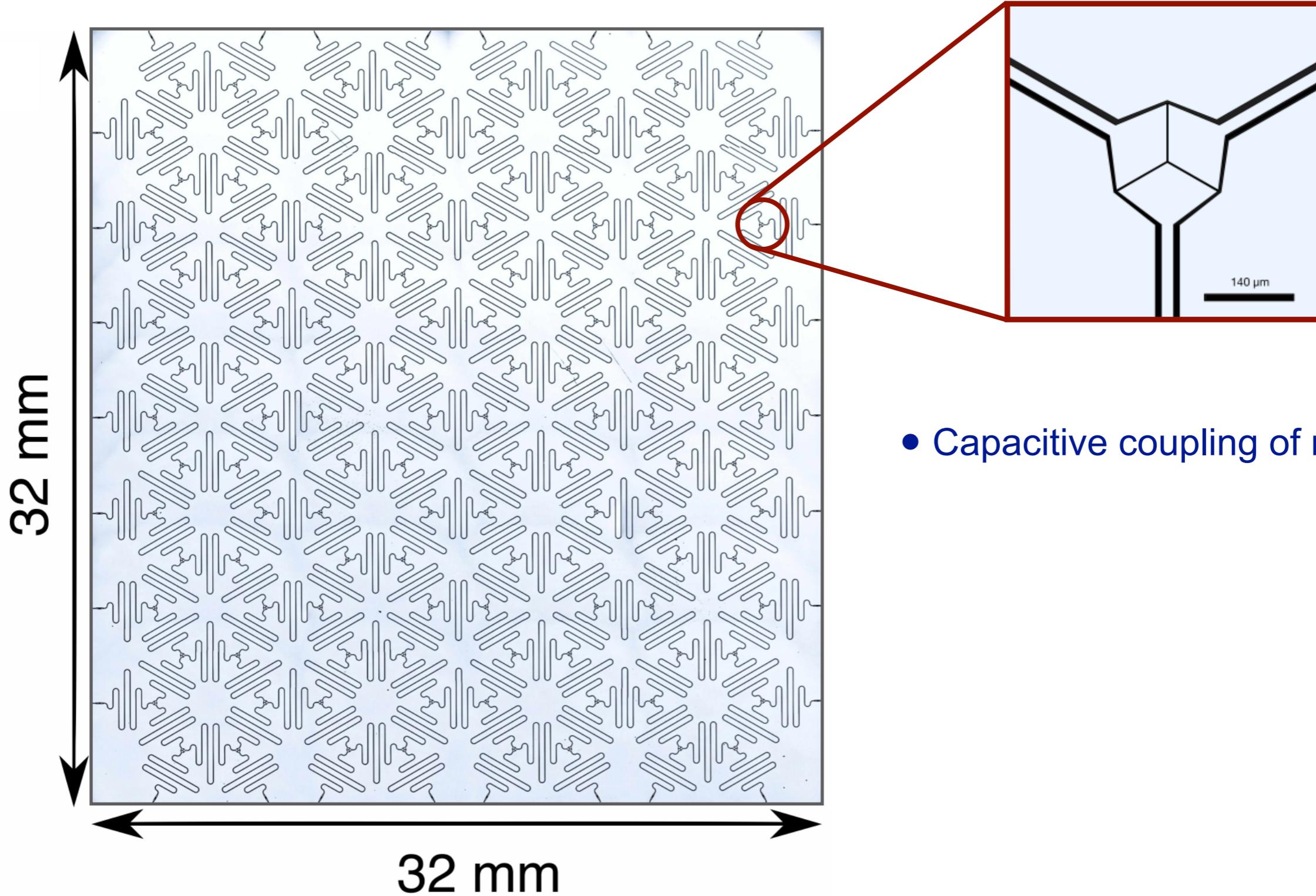
CPW Lattices



Houck *et al.* Nat Phys **8**, (2012)

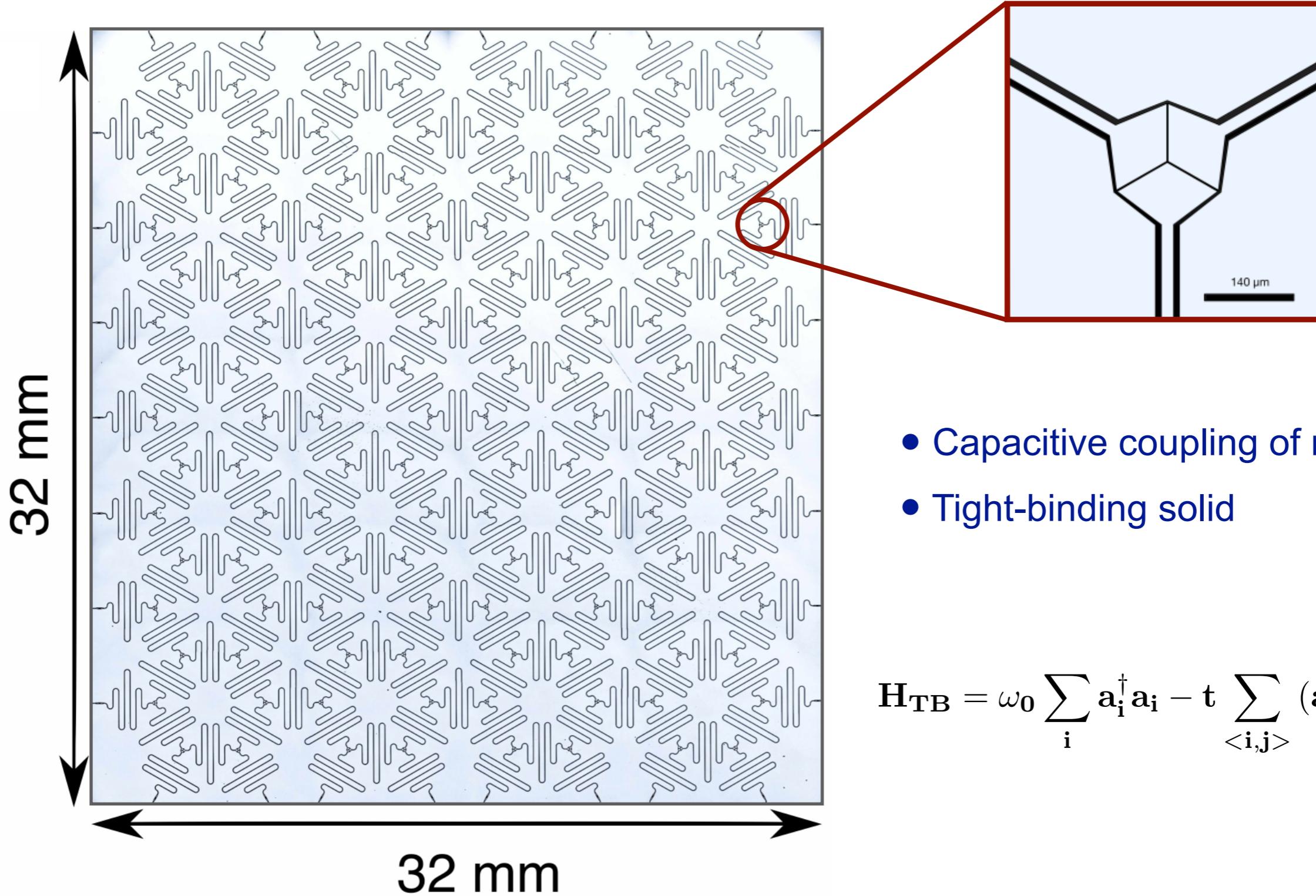
Underwood *et al.* PRA **86**, 023837 (2012)

CPW Lattices



- Capacitive coupling of resonators

CPW Lattices

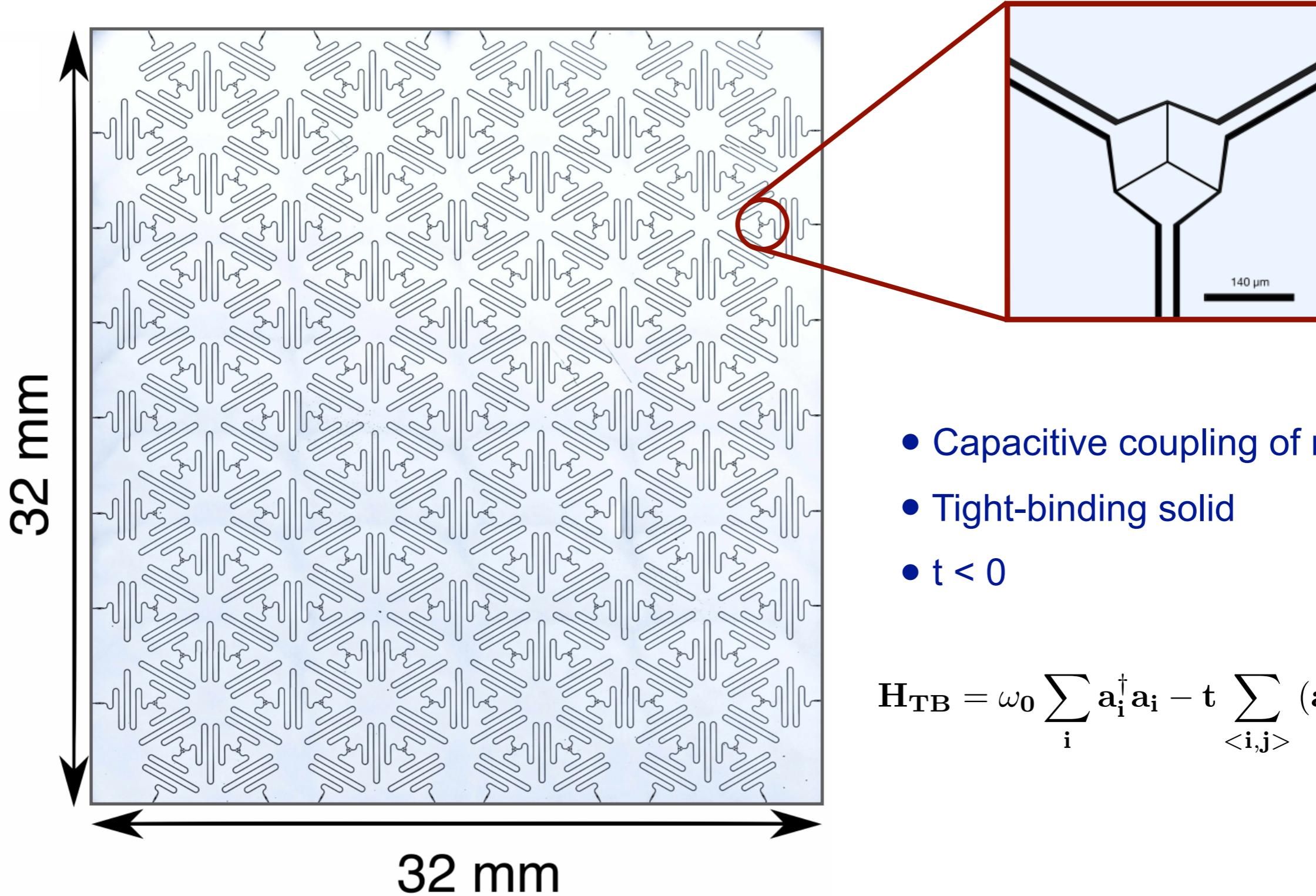


$$H_{TB} = \omega_0 \sum_i a_i^\dagger a_i - t \sum_{\langle i,j \rangle} (a_i^\dagger a_j + a_j^\dagger a_i)$$

Houck *et al.* Nat Phys **8**, (2012)

Underwood *et al.* PRA **86**, 023837 (2012)

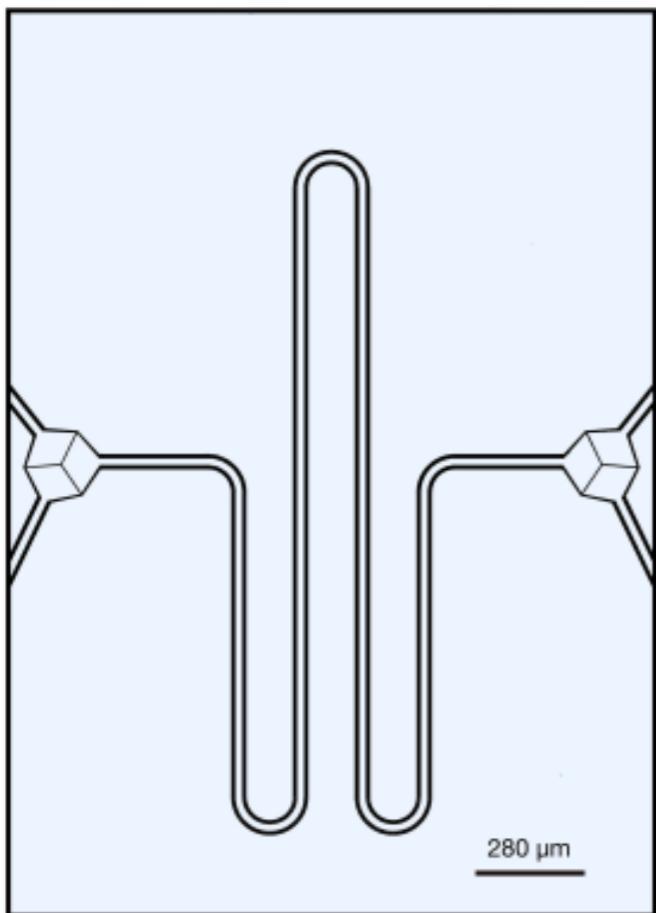
CPW Lattices



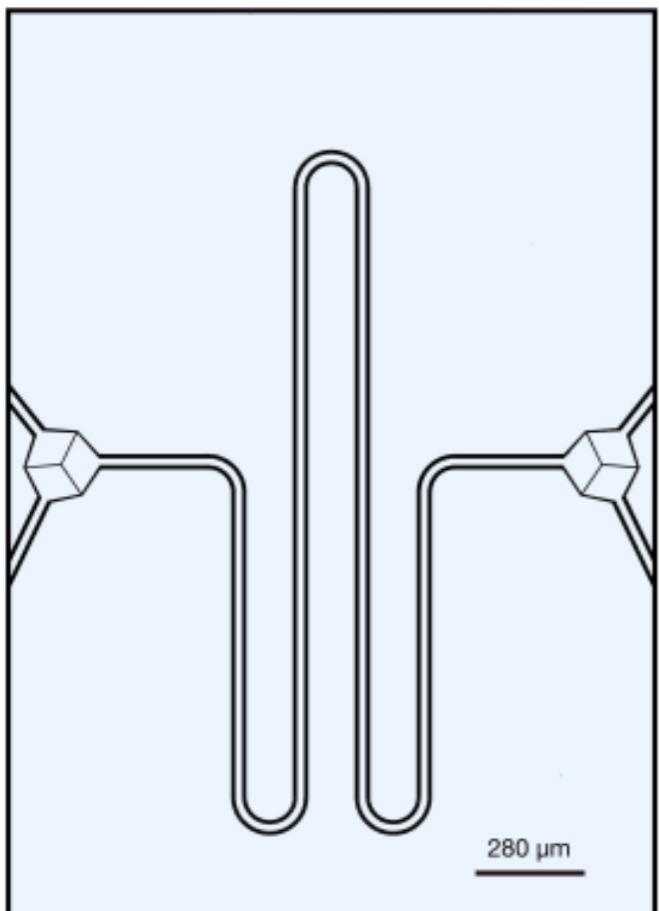
Houck *et al.* Nat Phys **8**, (2012)

Underwood *et al.* PRA **86**, 023837 (2012)

Deformable Resonators

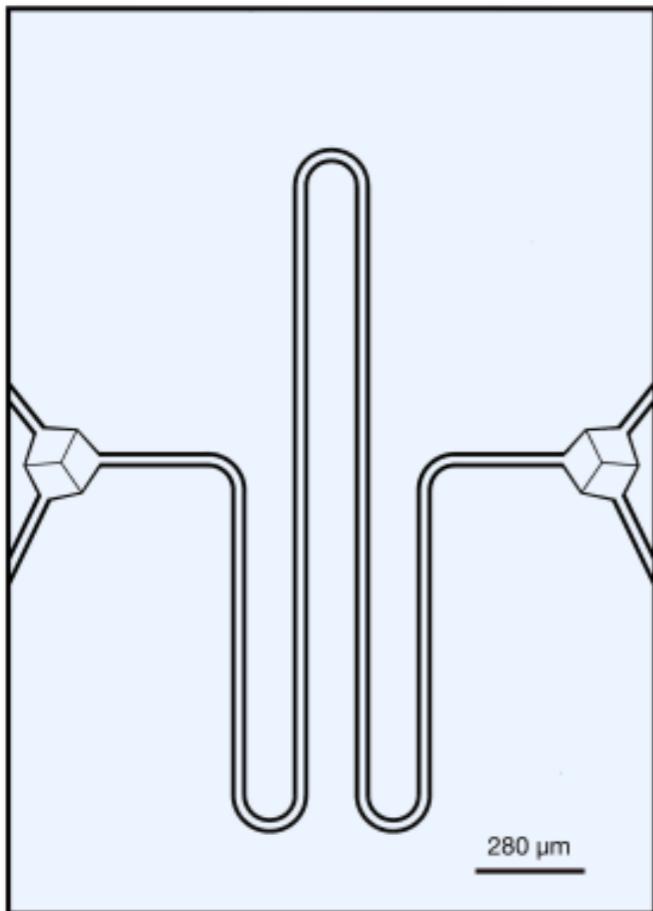


Deformable Resonators



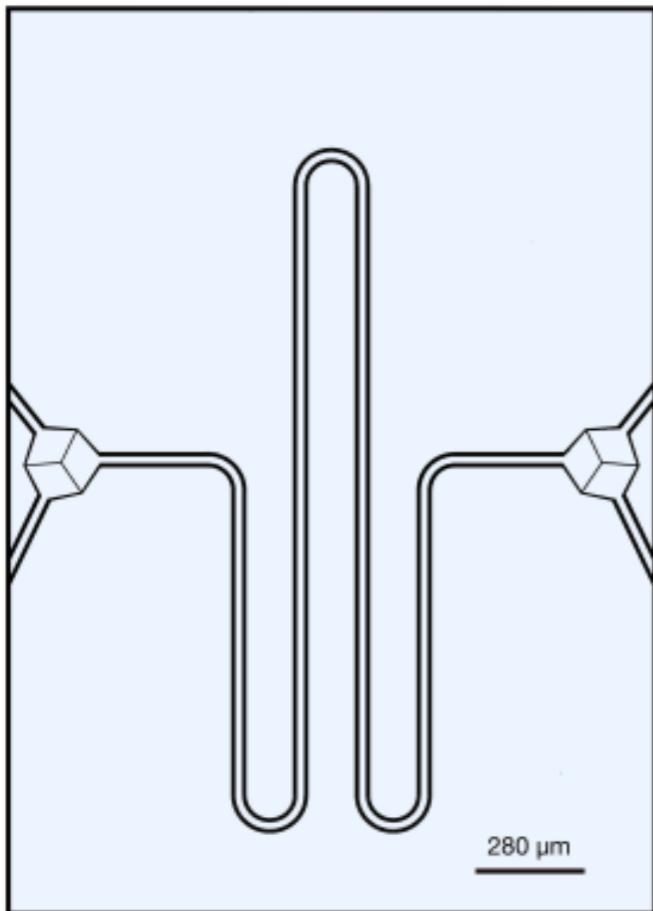
- Frequency depends only on length

Deformable Resonators



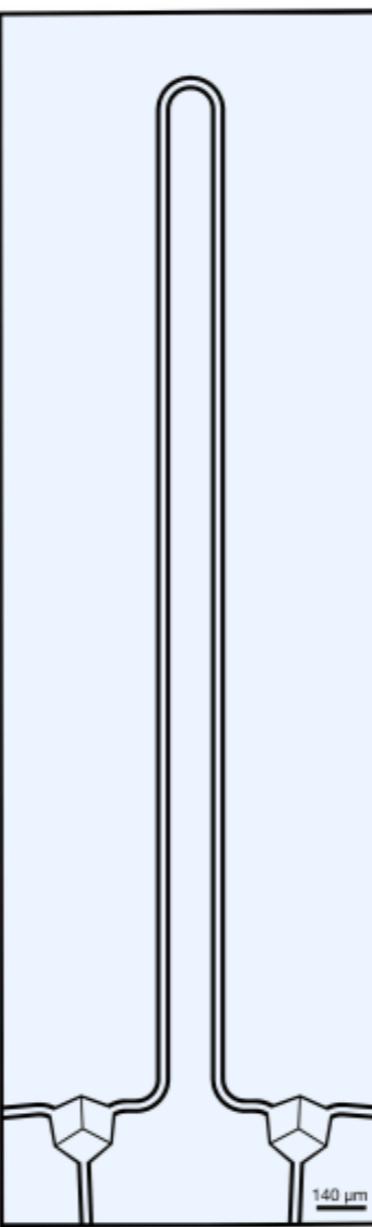
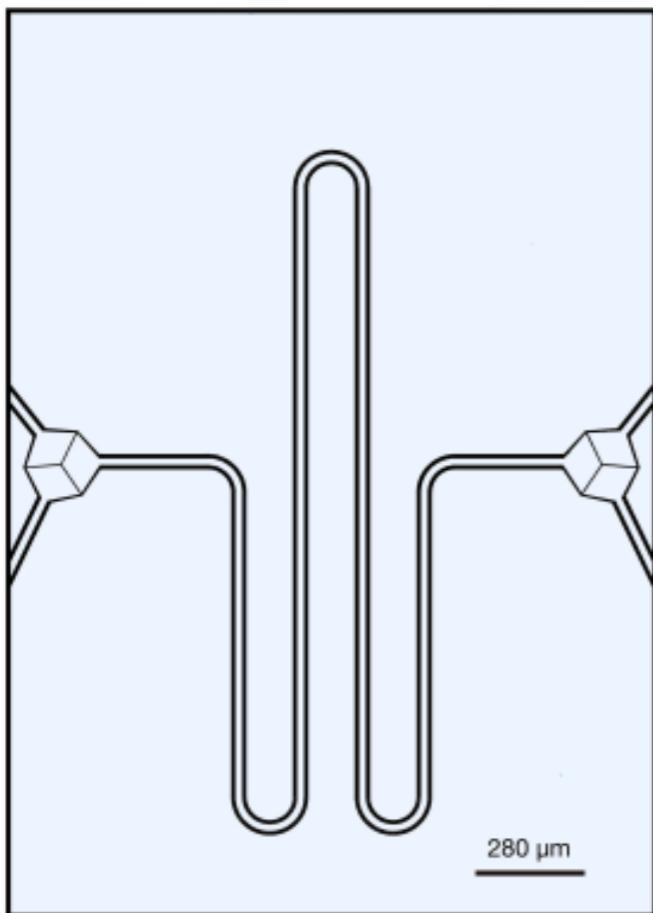
- Frequency depends only on length
- Coupling depends on ends

Deformable Resonators



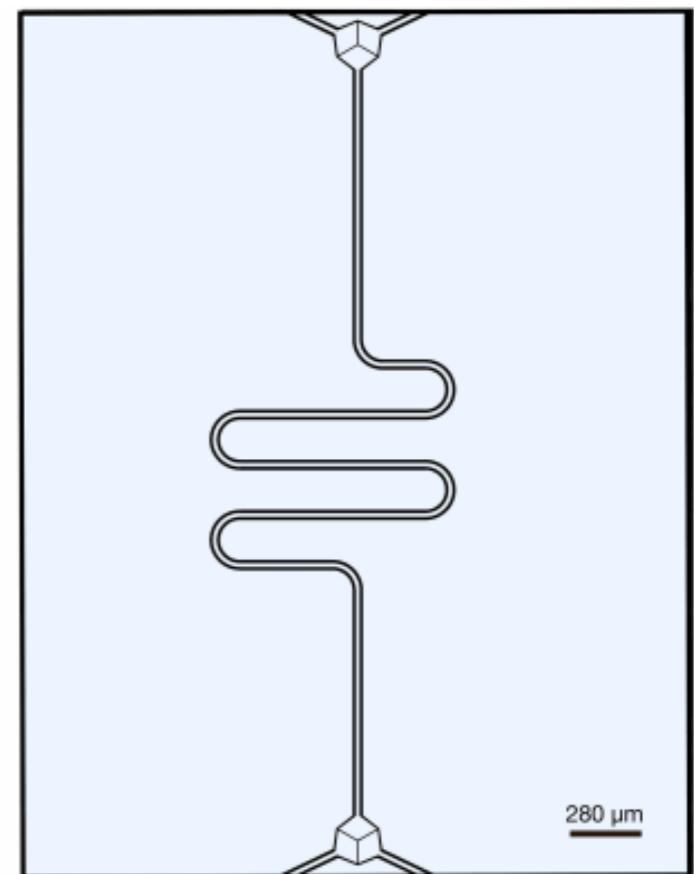
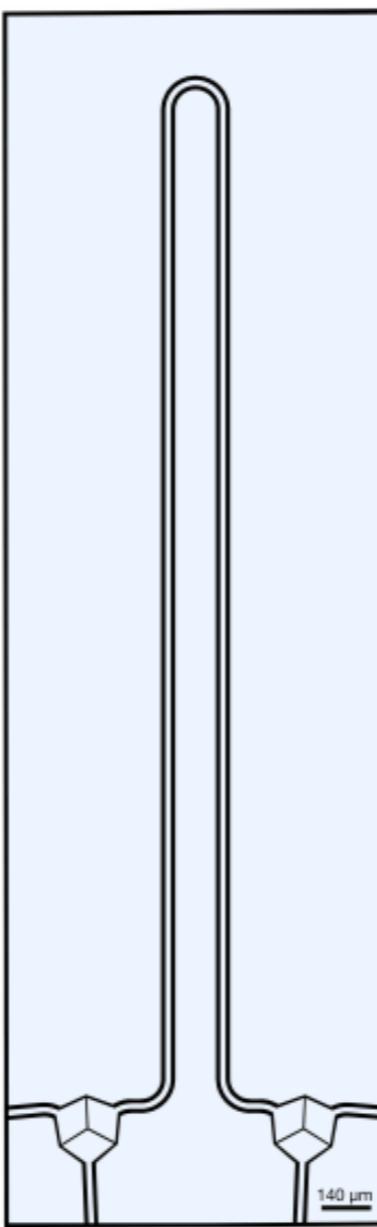
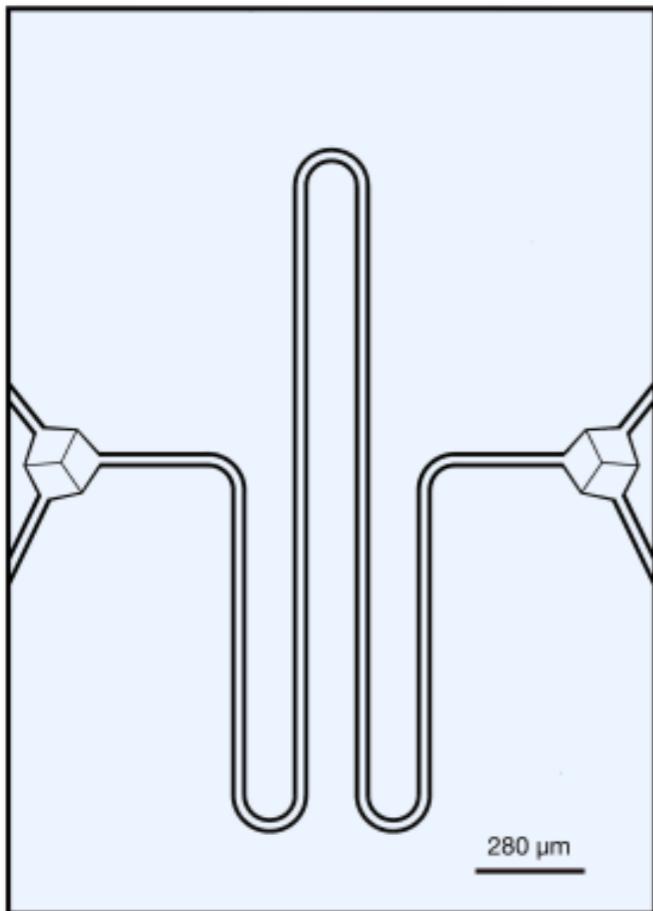
- Frequency depends only on length
- Coupling depends on ends
- “Bendable”

Deformable Resonators



- Frequency depends only on length
- Coupling depends on ends
- “Bendable”

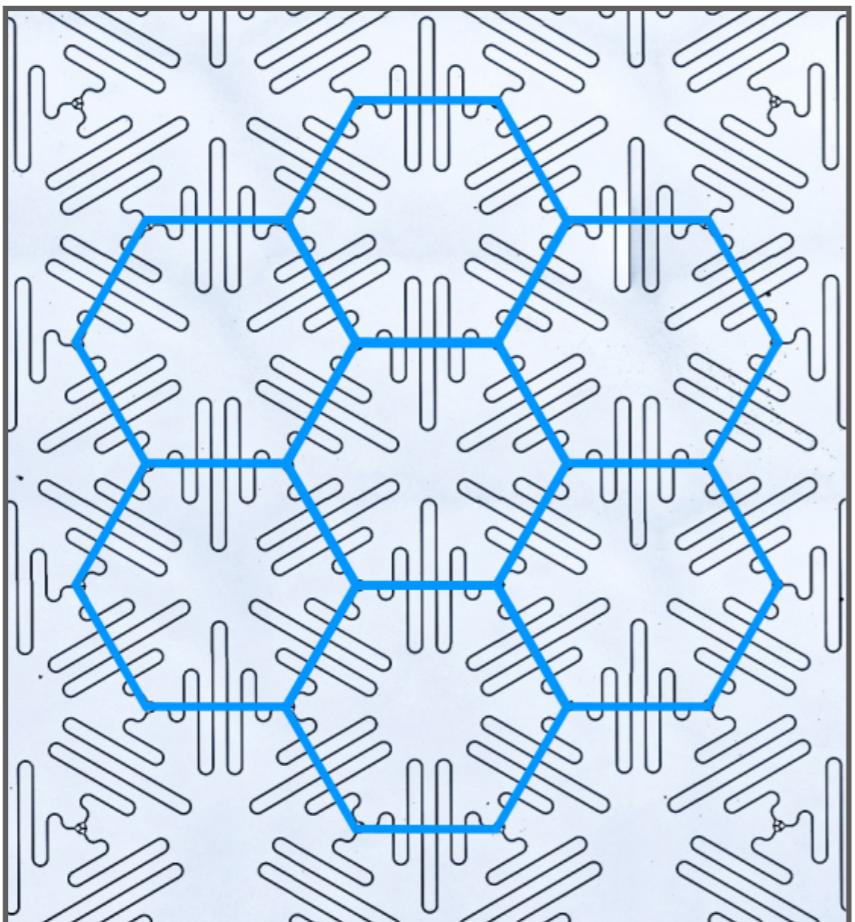
Deformable Resonators



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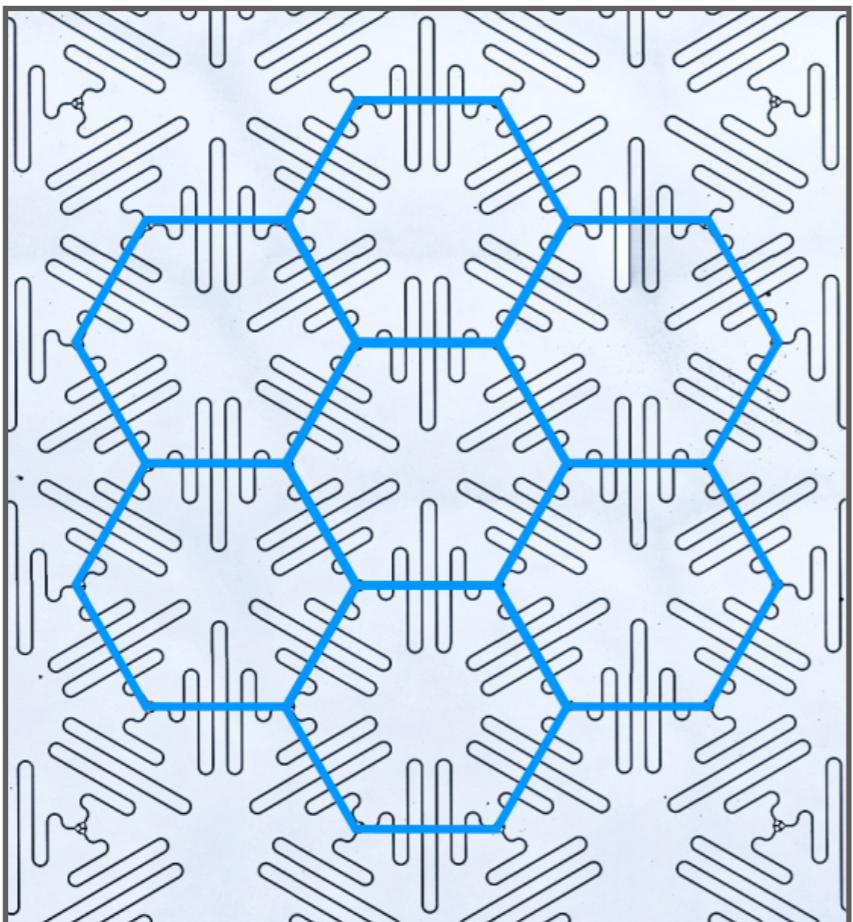
Layout and Effective Lattices

Resonator Lattice



Layout and Effective Lattices

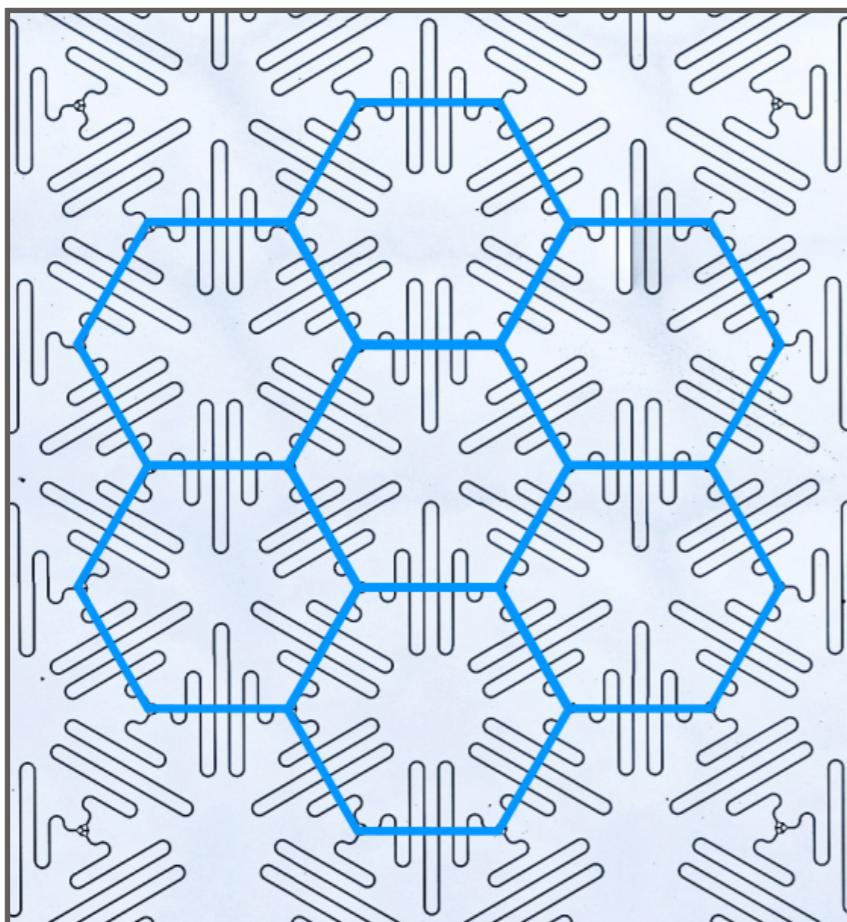
Resonator Lattice



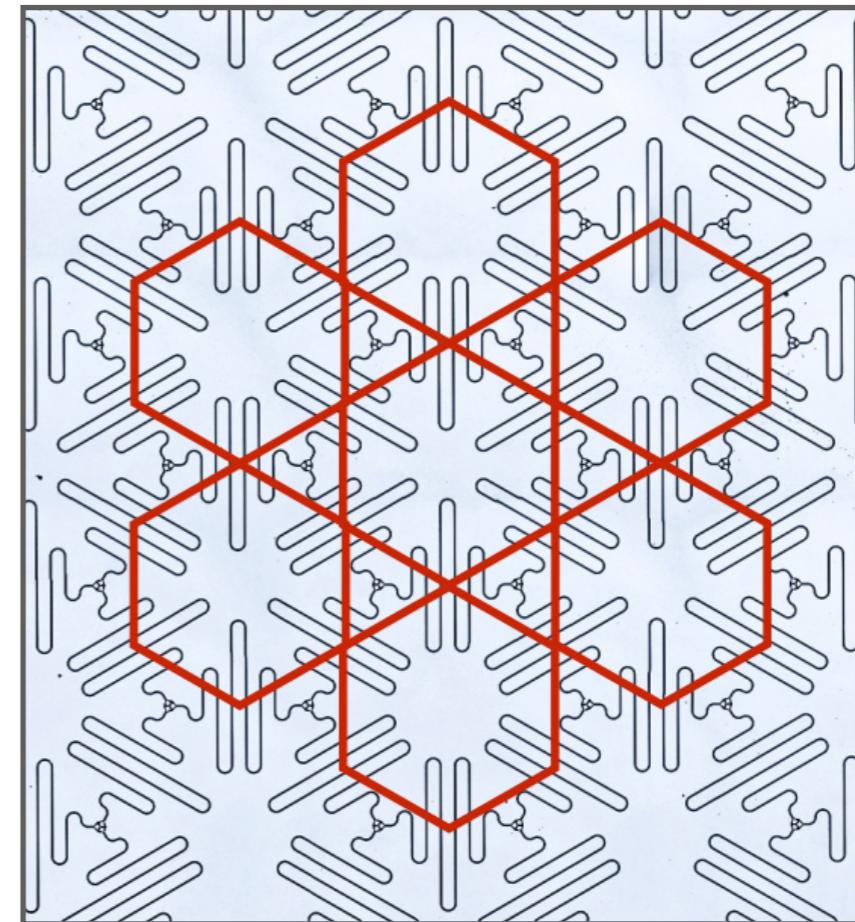
- An *edge* on each resonator

Layout and Effective Lattices

Resonator Lattice



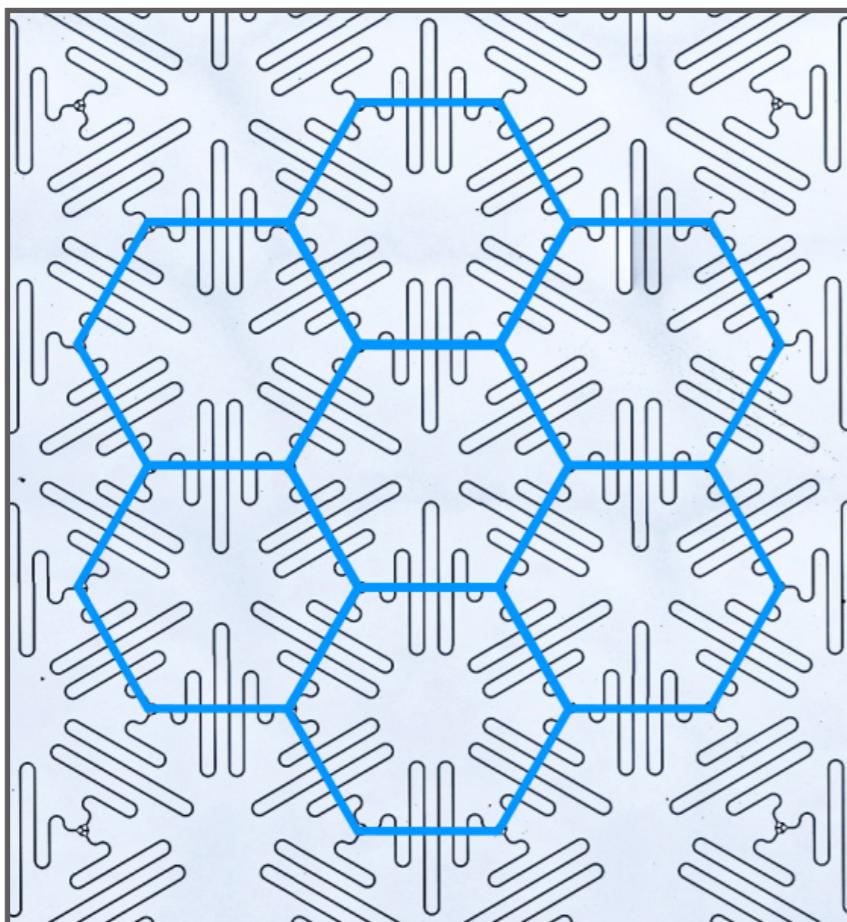
Effective Photonic Lattice



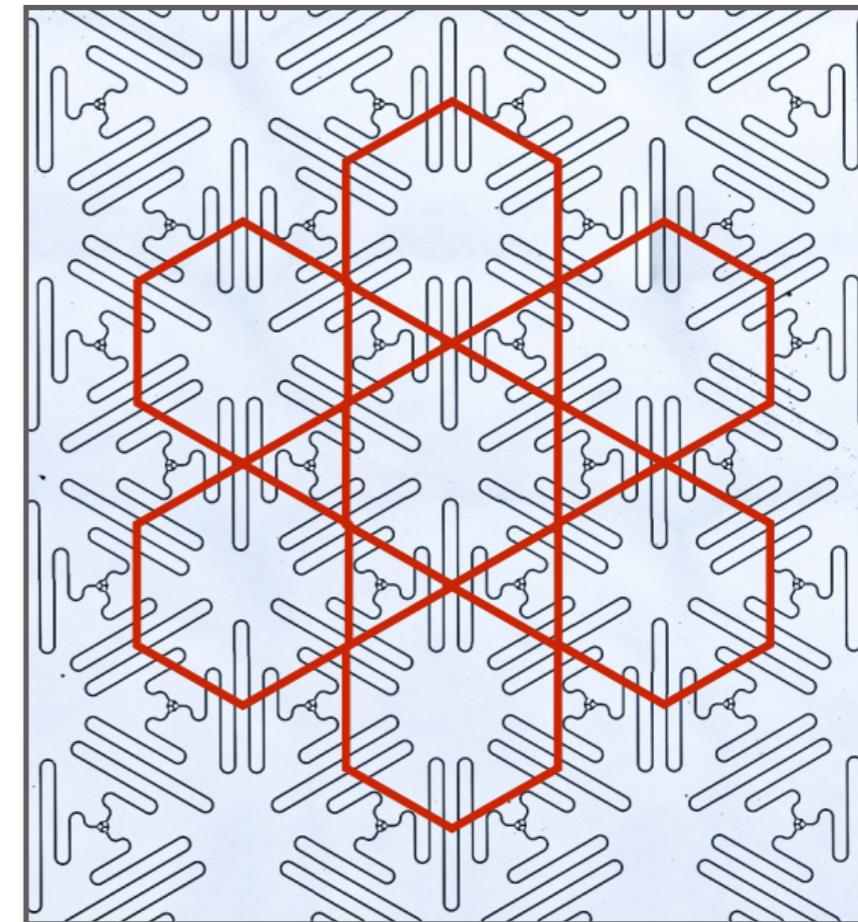
- An *edge* on each resonator

Layout and Effective Lattices

Resonator Lattice



Effective Photonic Lattice

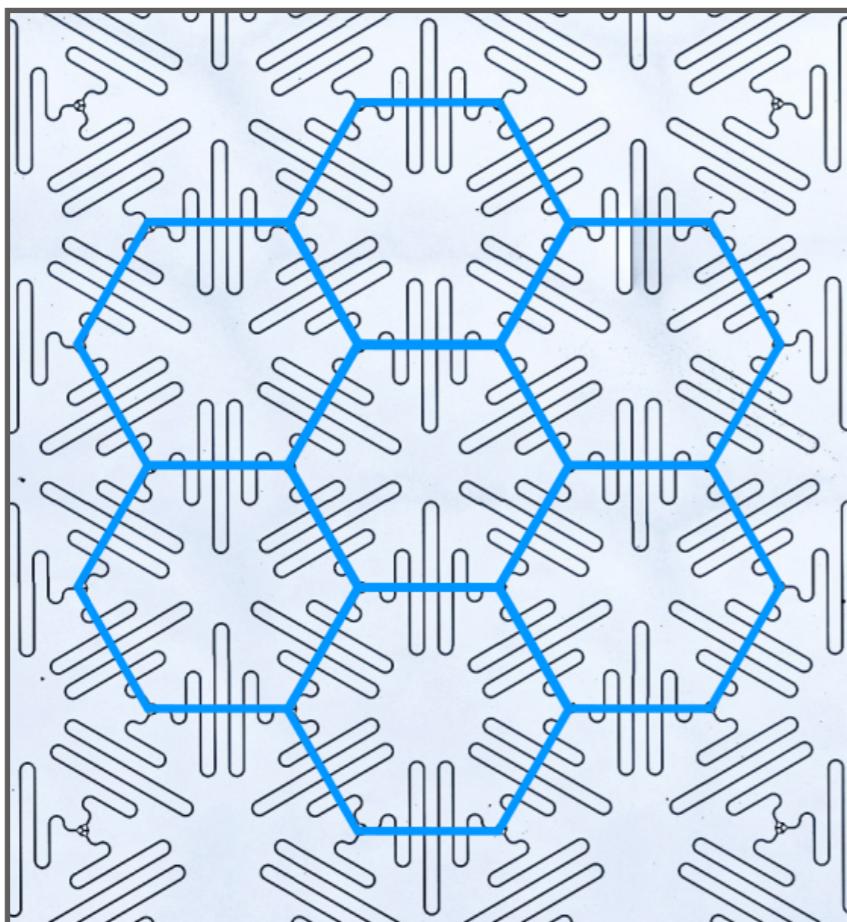


- An *edge* on each resonator

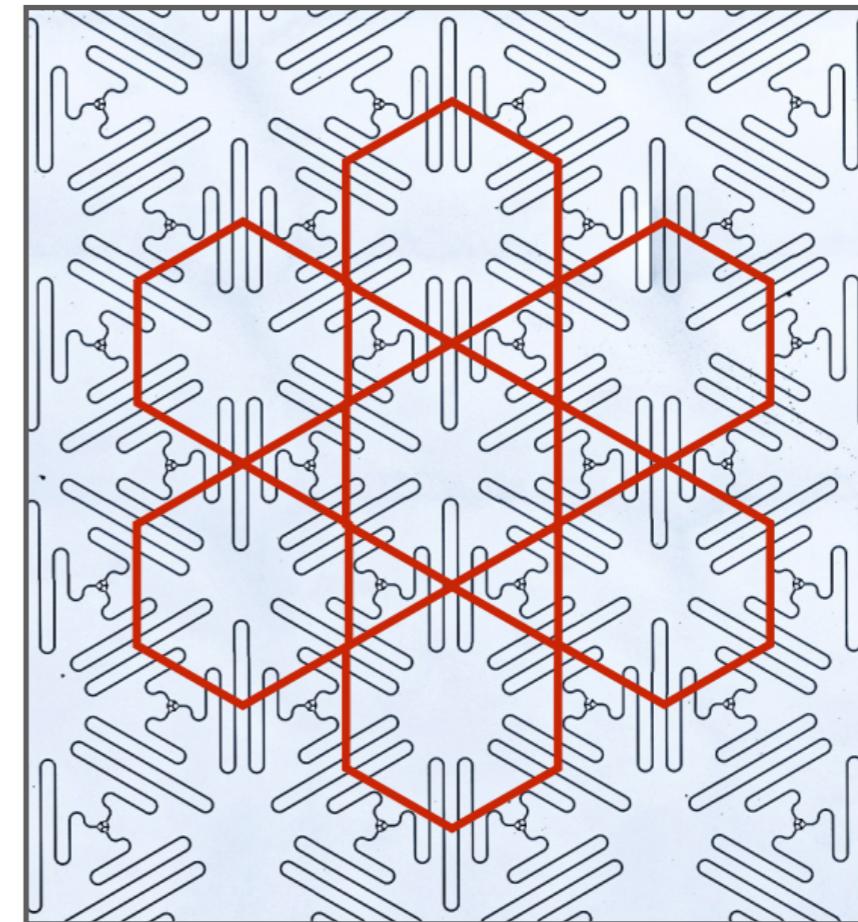
- A *vertex* on each resonator

Layout and Effective Lattices

Resonator Lattice



Effective Photonic Lattice



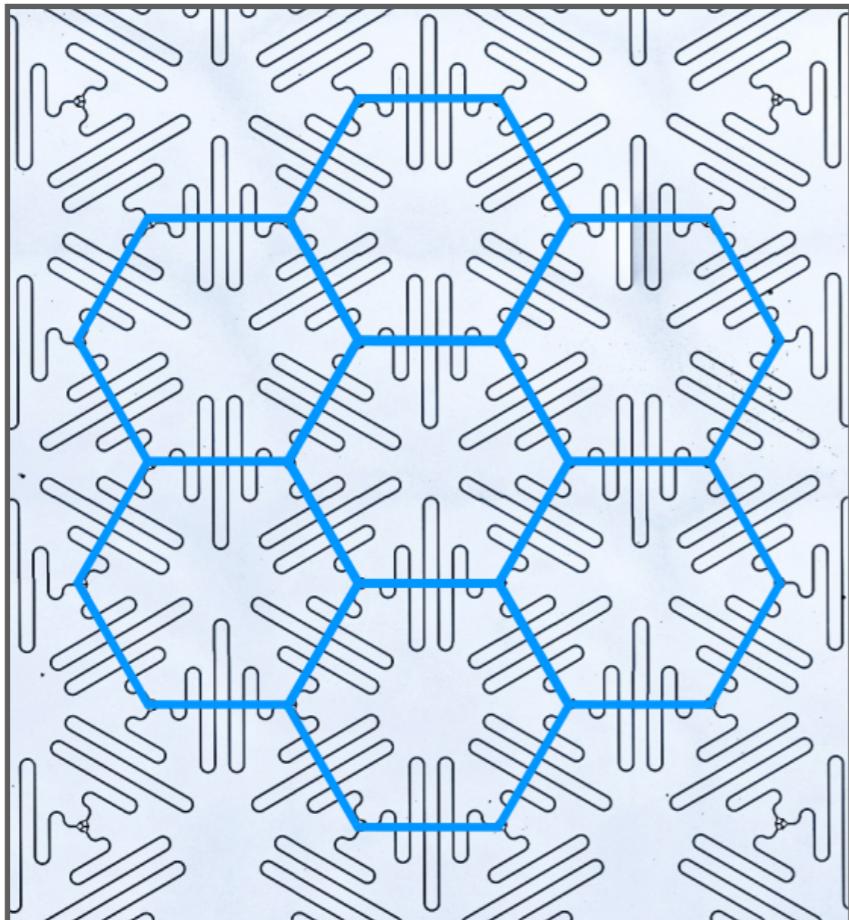
- An *edge* on each resonator

Layout X

- A *vertex* on each resonator

Layout and Effective Lattices

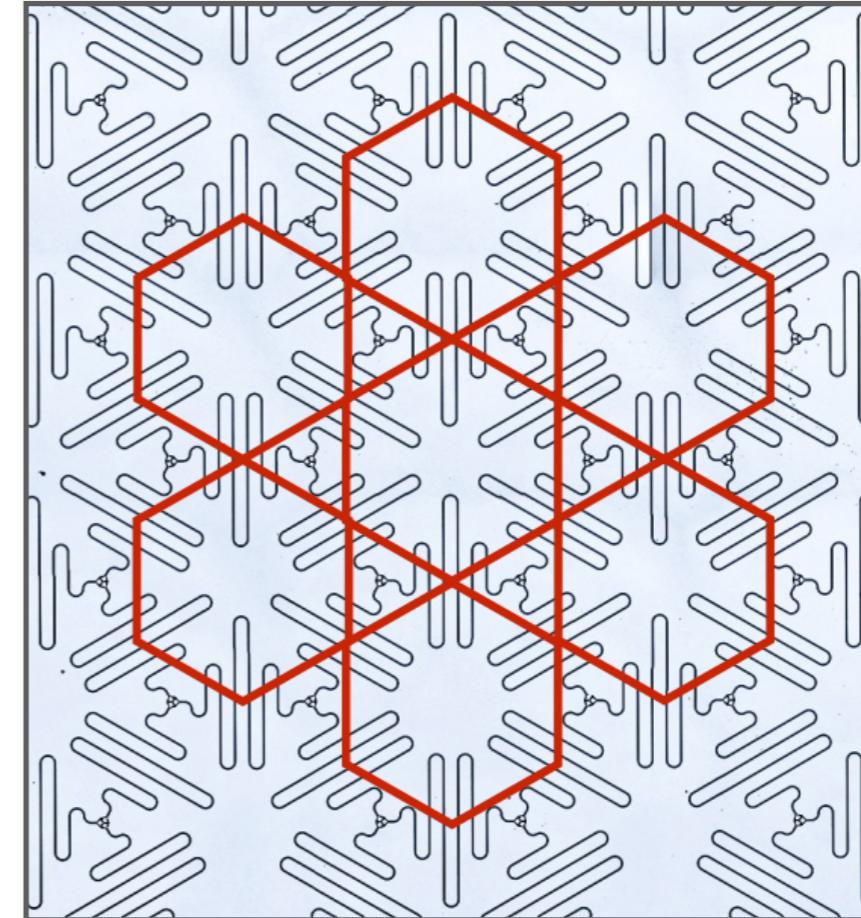
Resonator Lattice



- An *edge* on each resonator

Layout X

Effective Photonic Lattice



- A *vertex* on each resonator

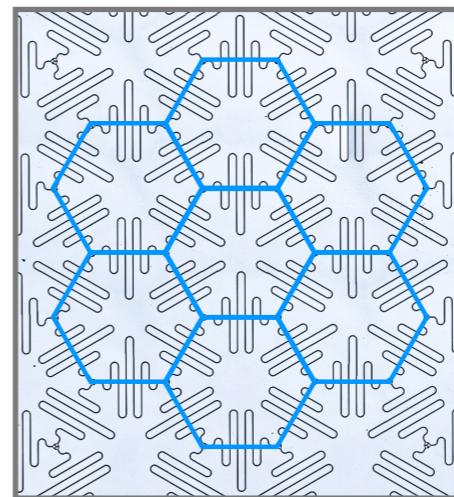
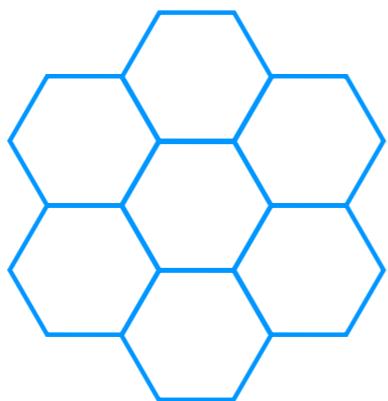
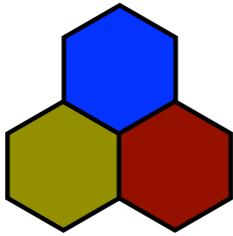
Line Graph $L(X)$

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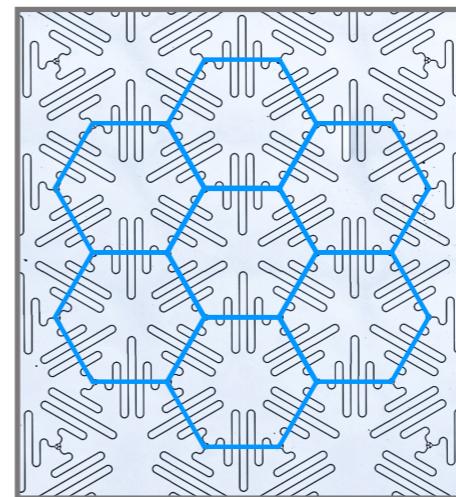
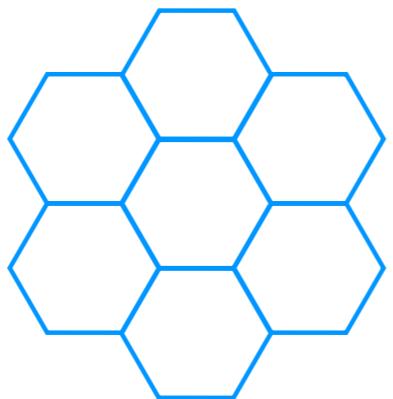
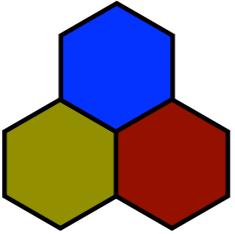
Projecting to Flat 2D

$n = 6$
flat

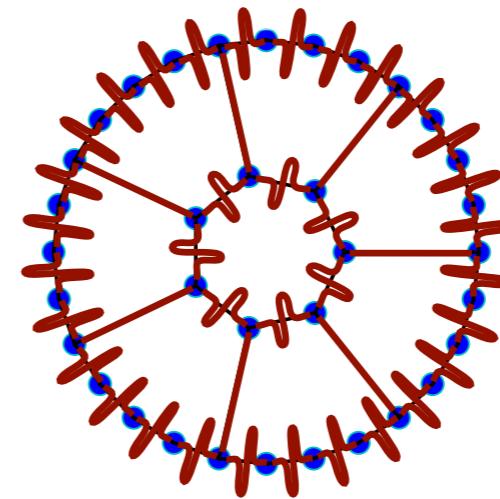
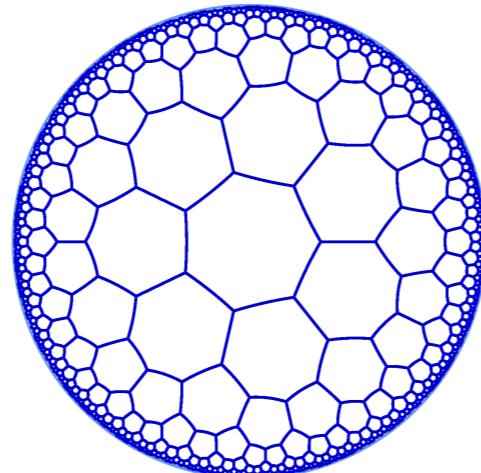
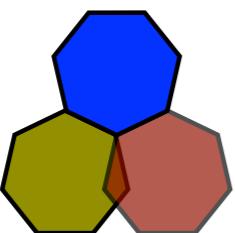


Projecting to Flat 2D

$n = 6$
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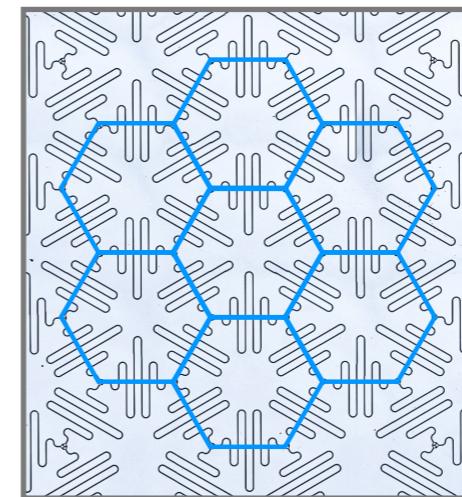
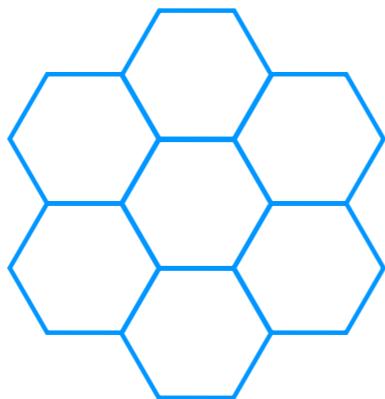
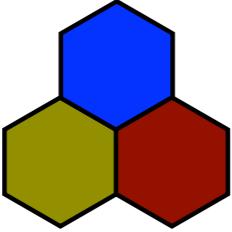


$n = 7$
hyperbolic

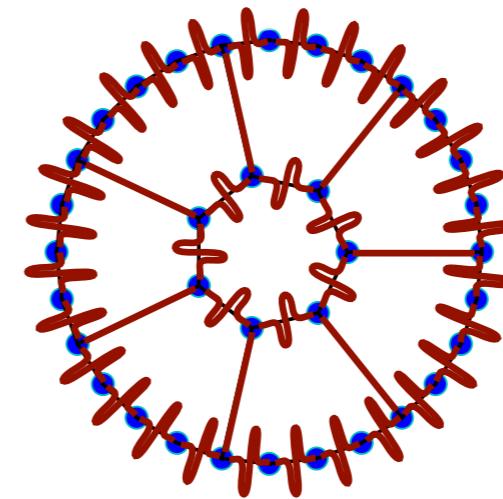
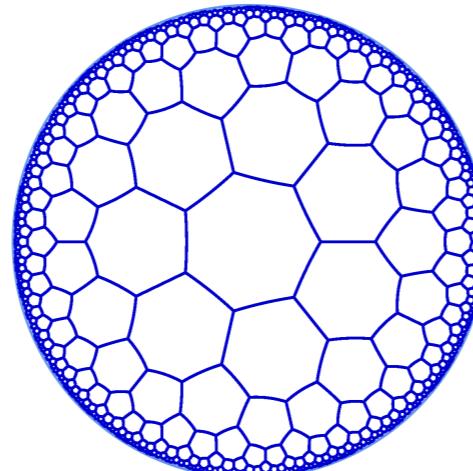
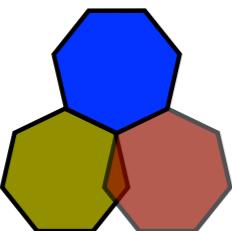


Projecting to Flat 2D

$n = 6$
flat



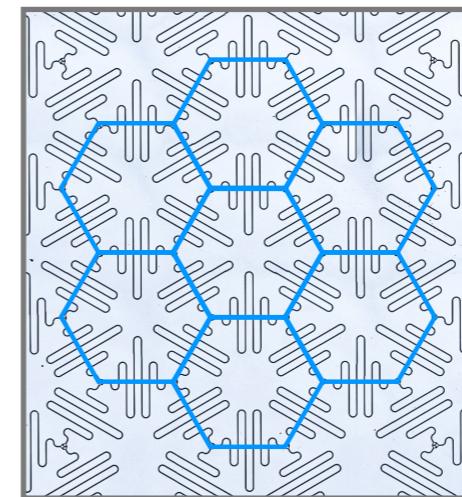
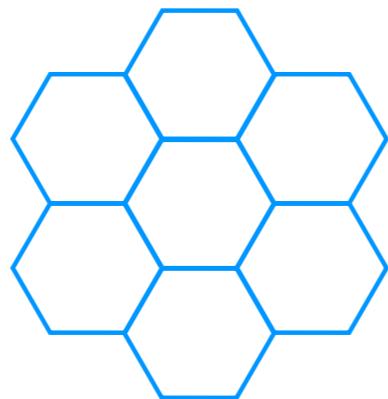
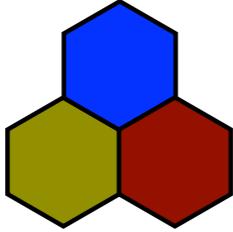
$n = 7$
hyperbolic



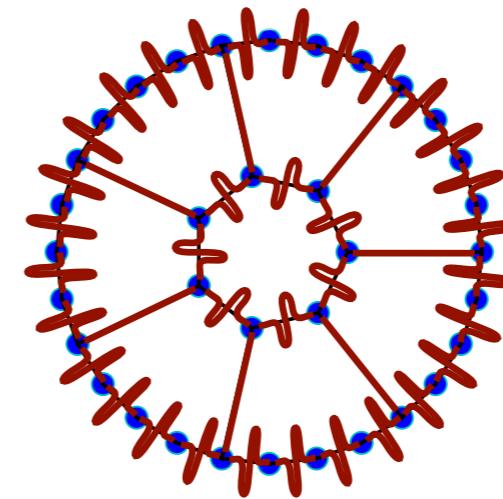
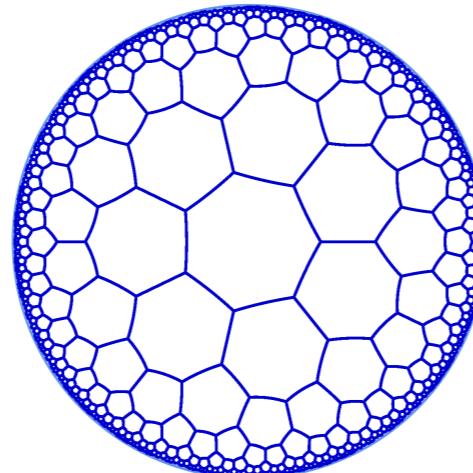
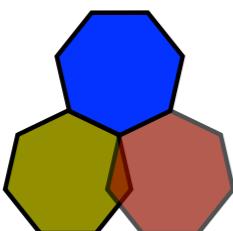
- Distance is not preserved.

Projecting to Flat 2D

$n = 6$
flat



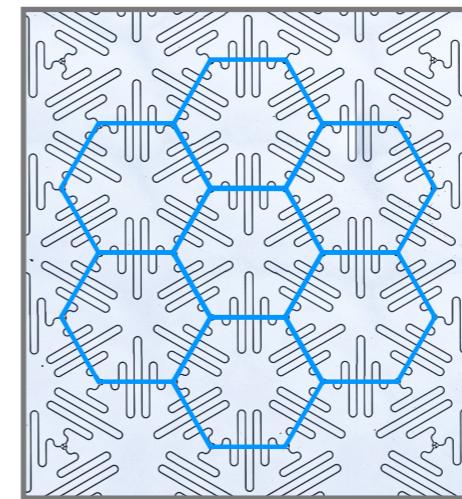
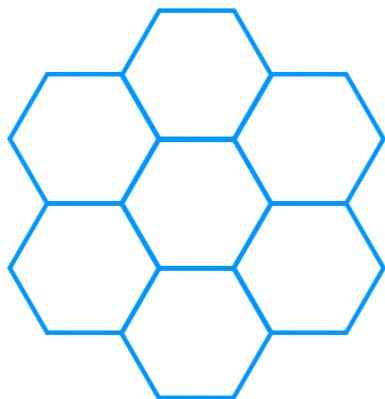
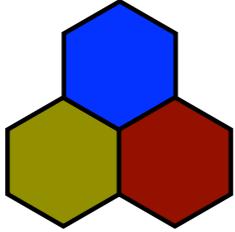
$n = 7$
hyperbolic



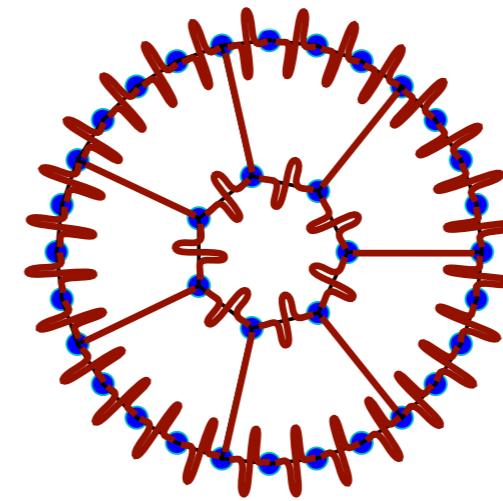
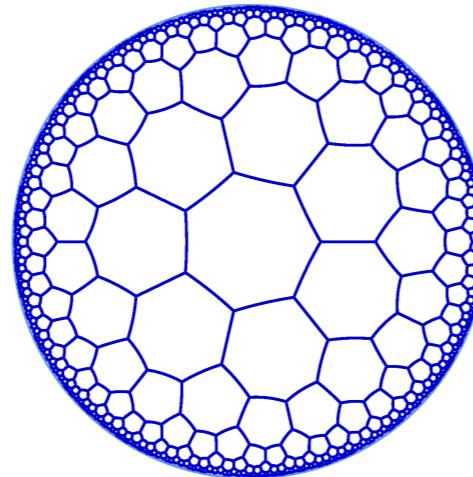
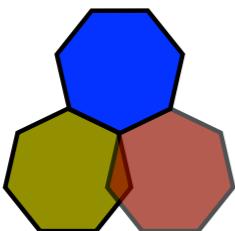
- Distance is not preserved.
- t is preserved.

Projecting to Flat 2D

$n = 6$
flat

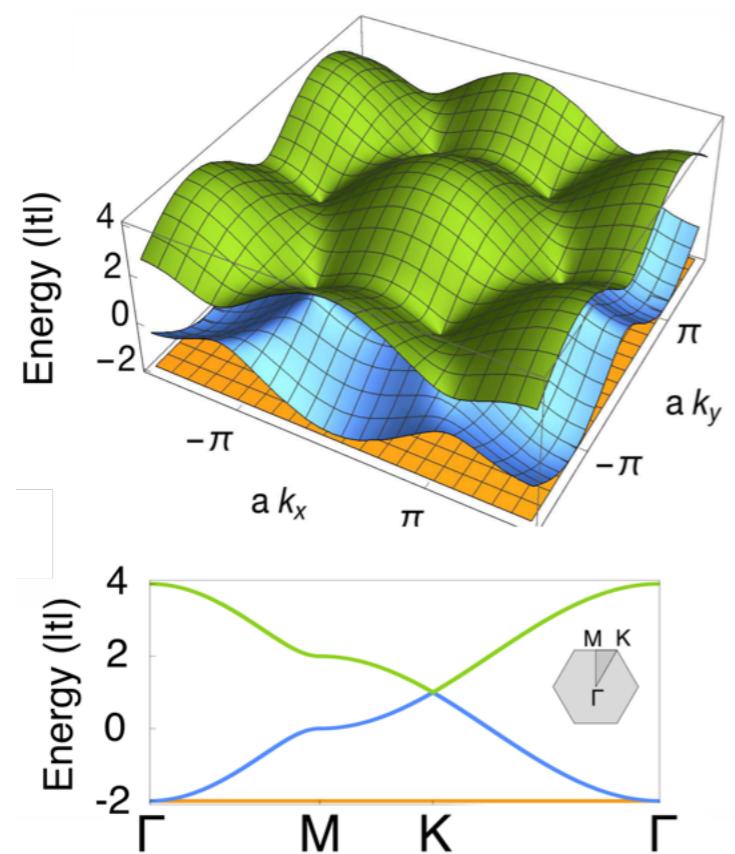


$n = 7$
hyperbolic

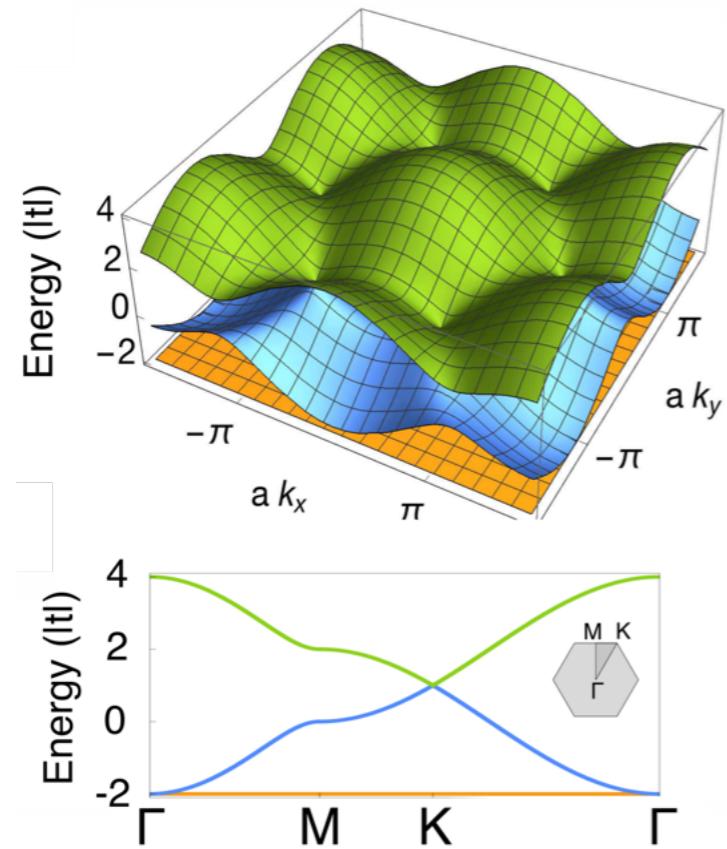


- Distance is not preserved.
- t *is* preserved.
- H *is* preserved.

Band Structure Calculations

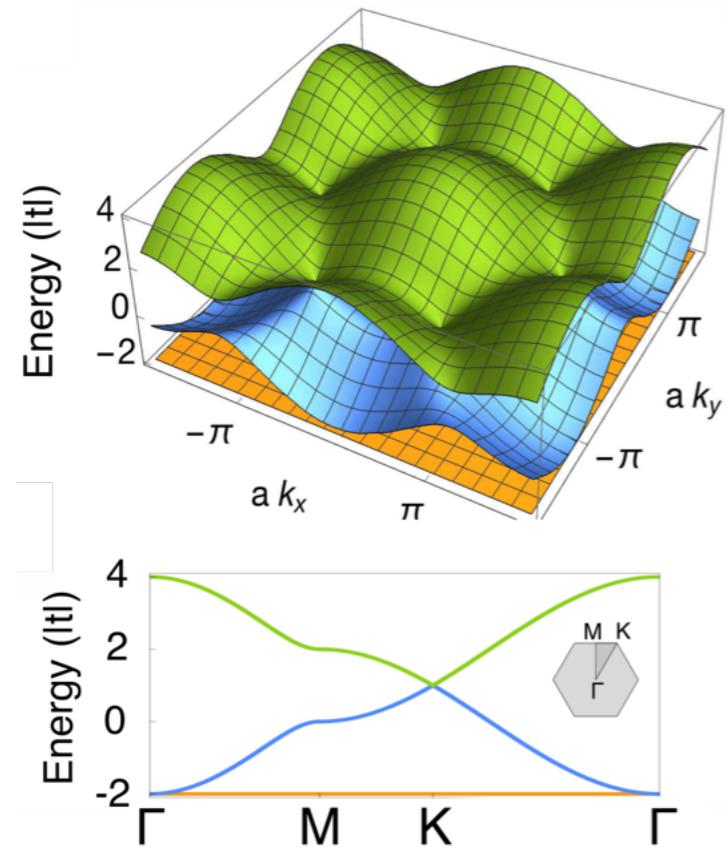


Band Structure Calculations



Hyperbolic geometry is non-commutative

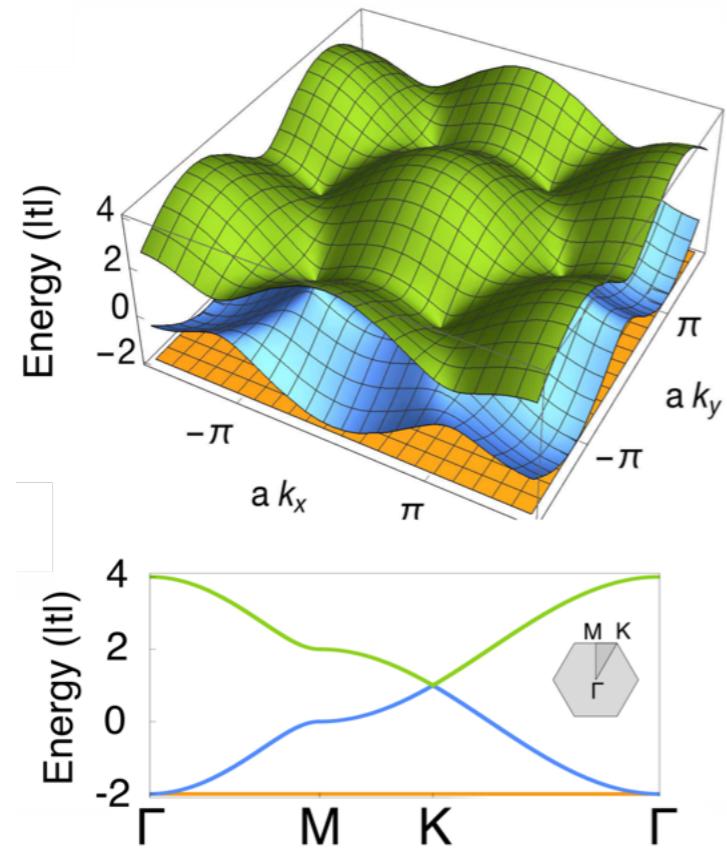
Band Structure Calculations



Hyperbolic geometry is non-commutative

- No Bravais lattice

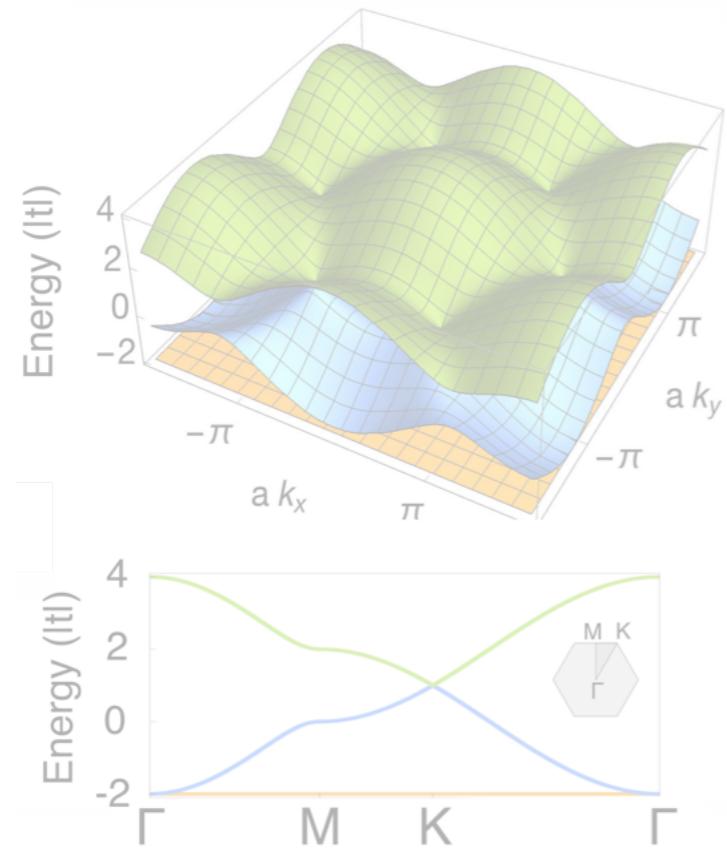
Band Structure Calculations



Hyperbolic geometry is non-commutative

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- No Bloch theory

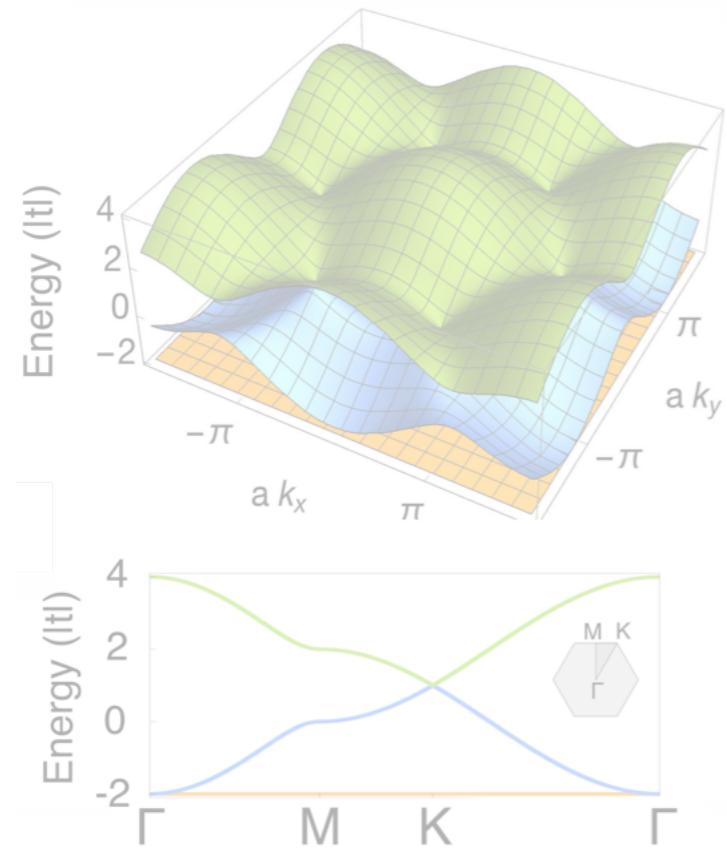
Band Structure Calculations



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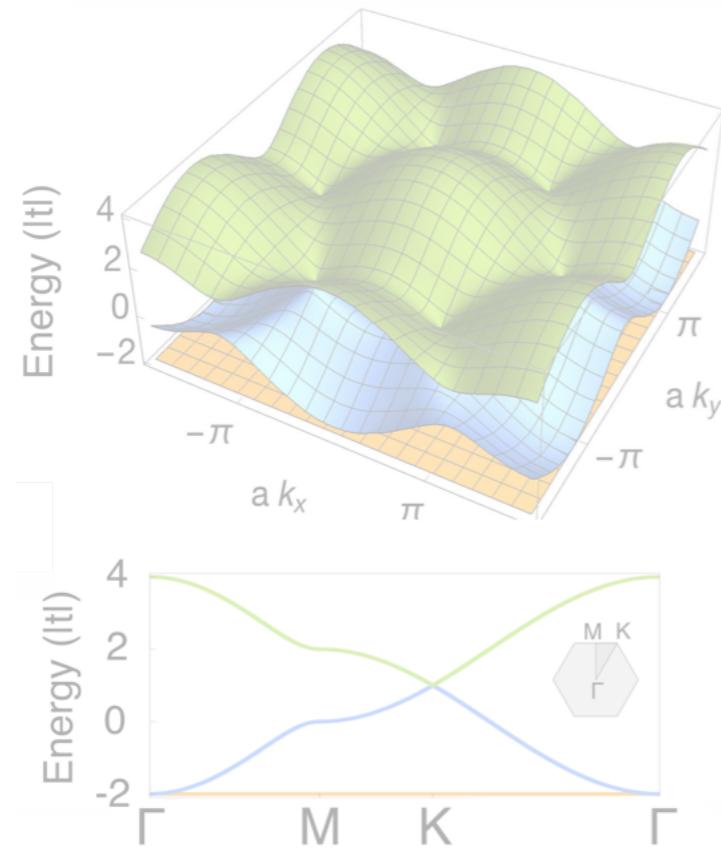
Band Structure Calculations



Hyperbolic geometry is non-commutative

- No Bravais lattice
- No Bloch theory
- Graph theory

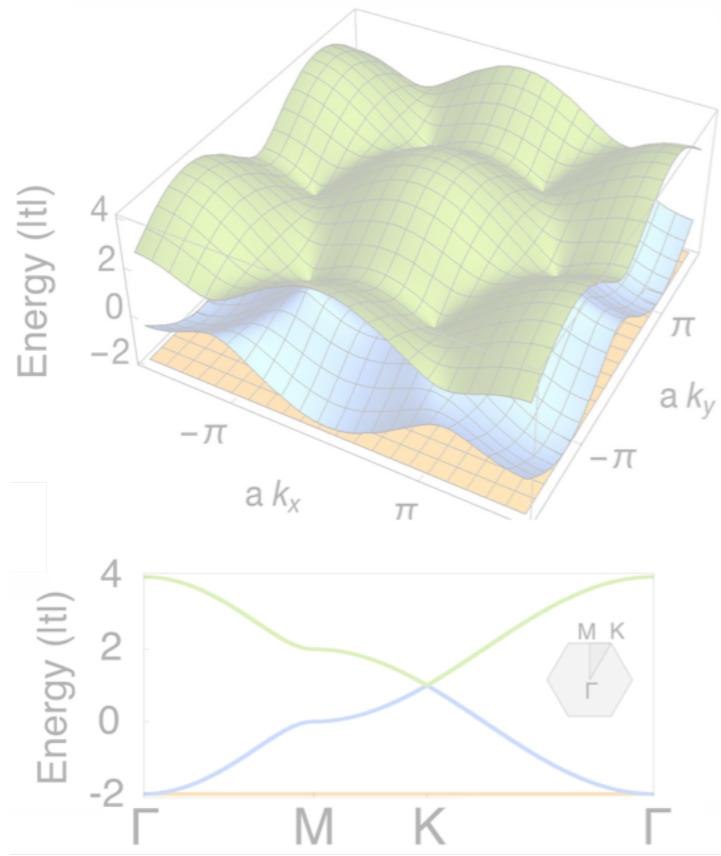
Band Structure Calculations



Hyperbolic geometry is non-commutative

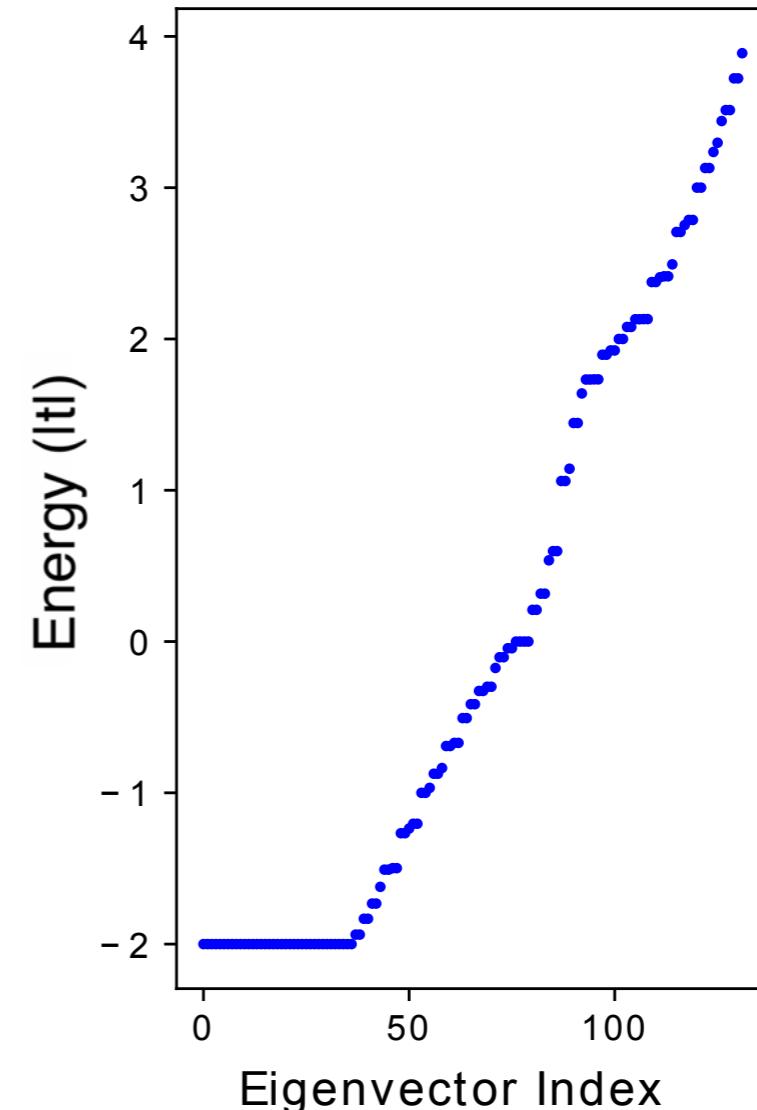
- No Bravais lattice
- No Bloch theory
- Graph theory
- Brute force TB numerics

Band Structure Calculations

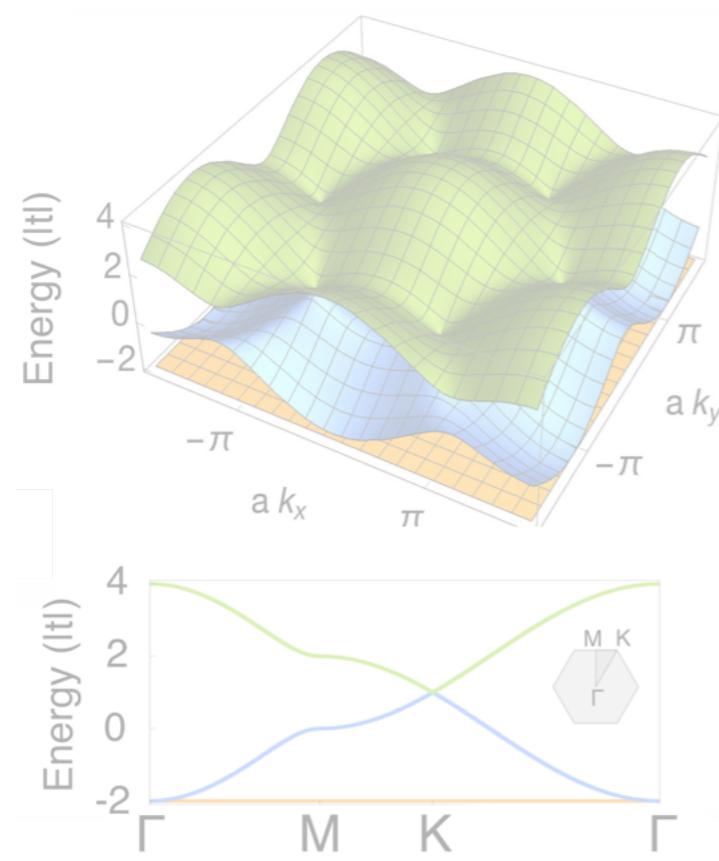


Hyperbolic geometry is non-commutative

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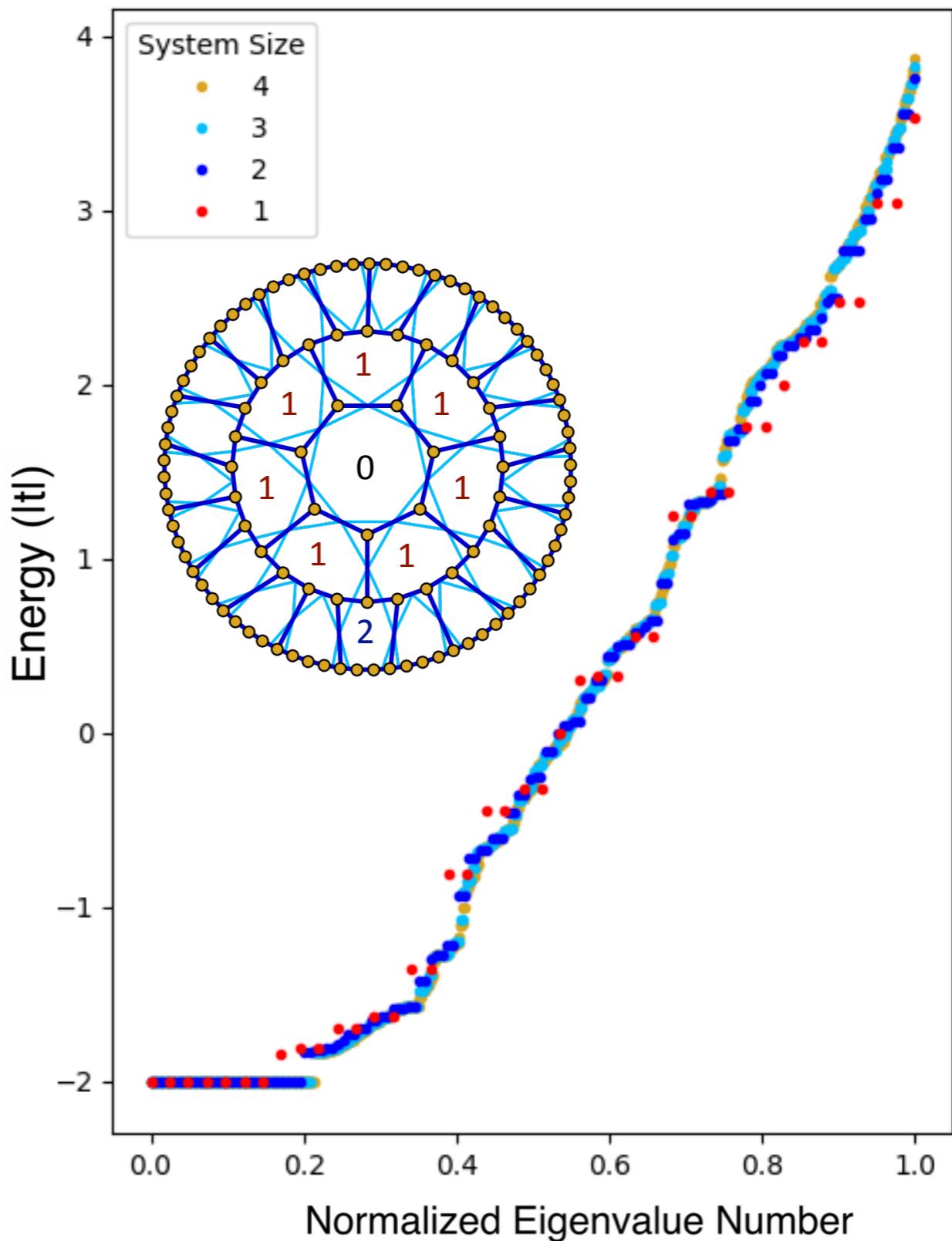


Band Structure Calculations

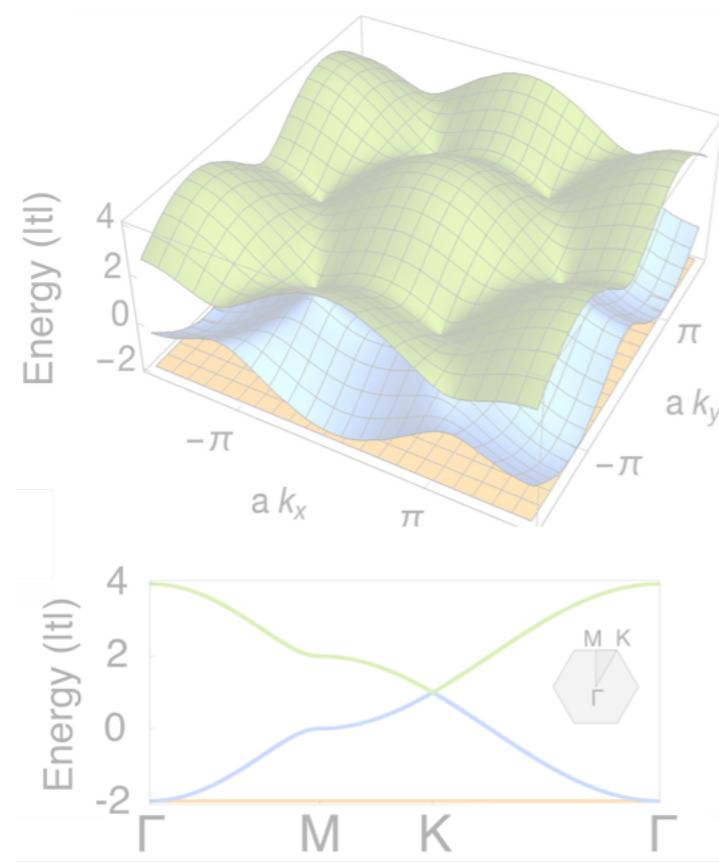


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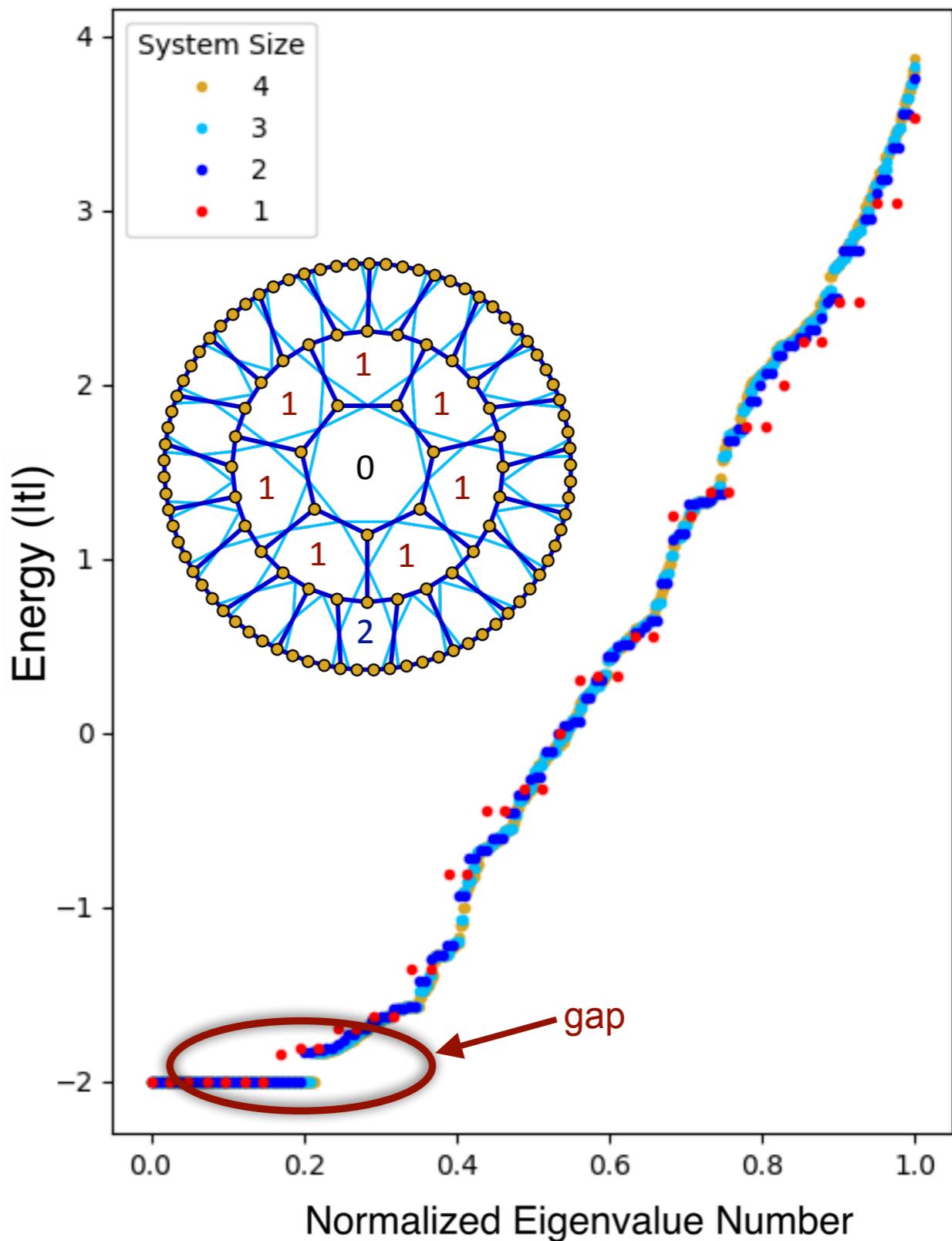


Band Structure Calculations

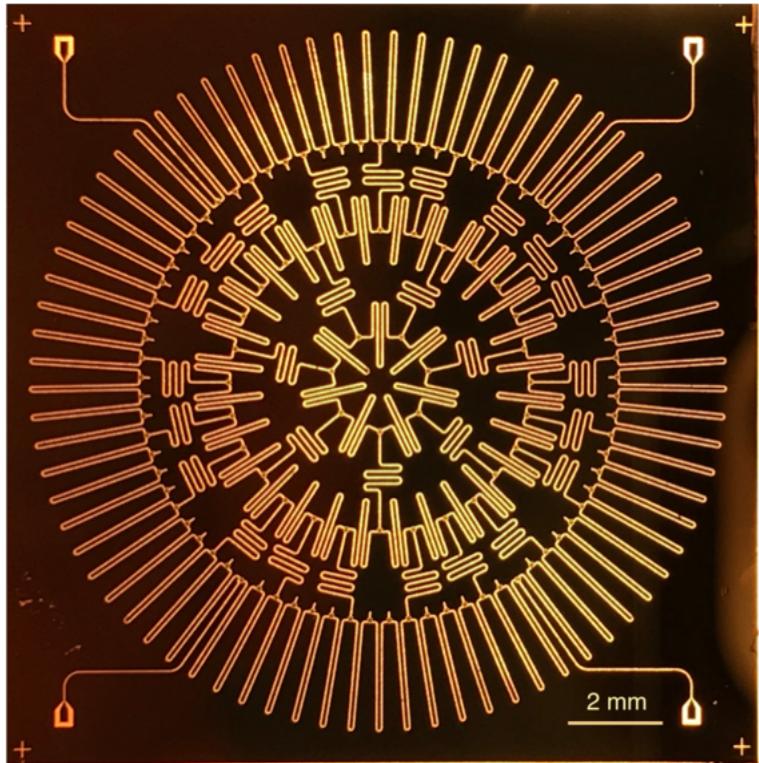


Hyperbolic geometry is non-commutative

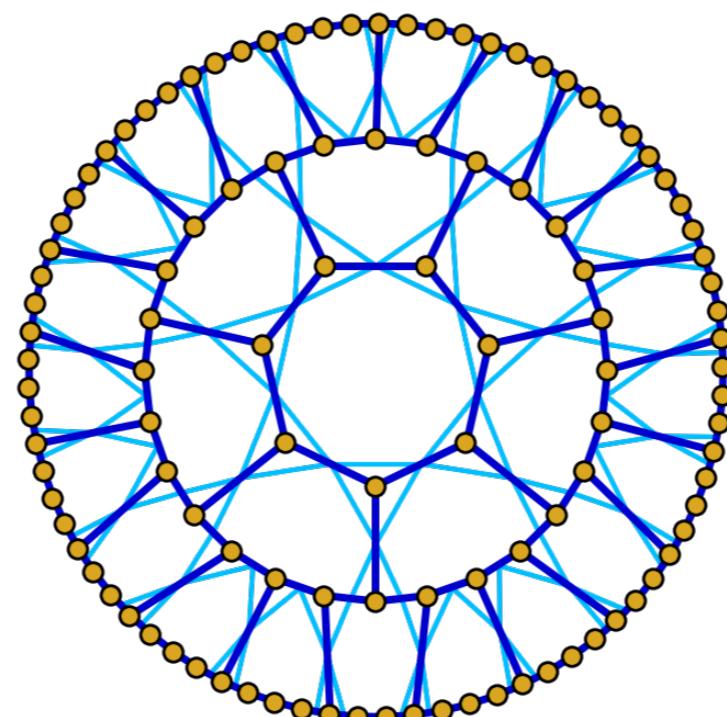
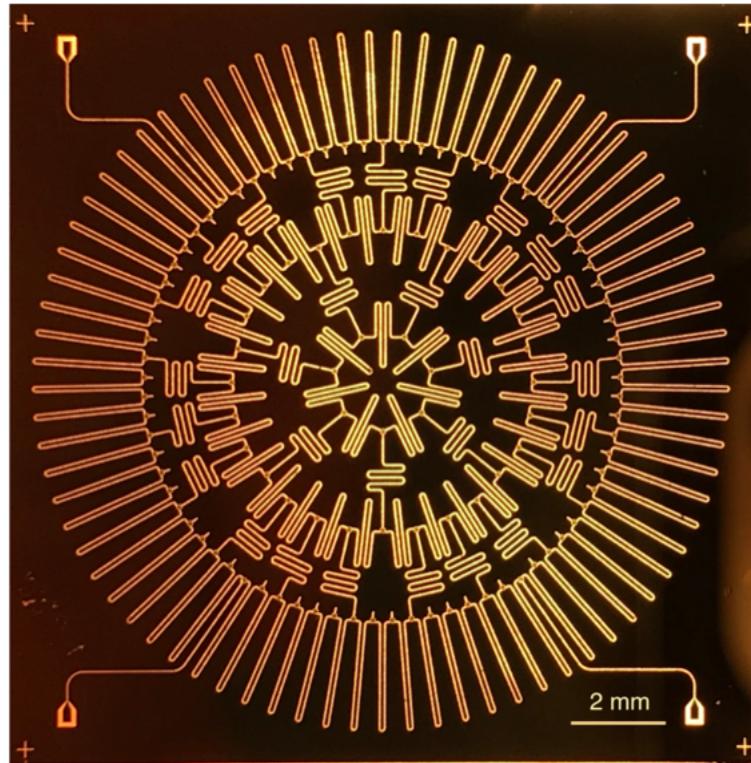
- No Bravais lattice
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- Brute force TB numerics



Heptagon-Kagome Device

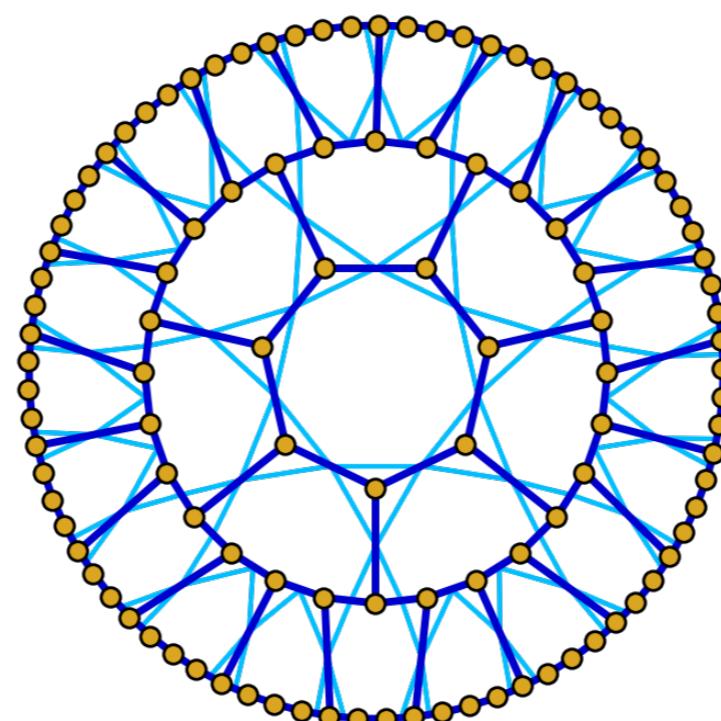
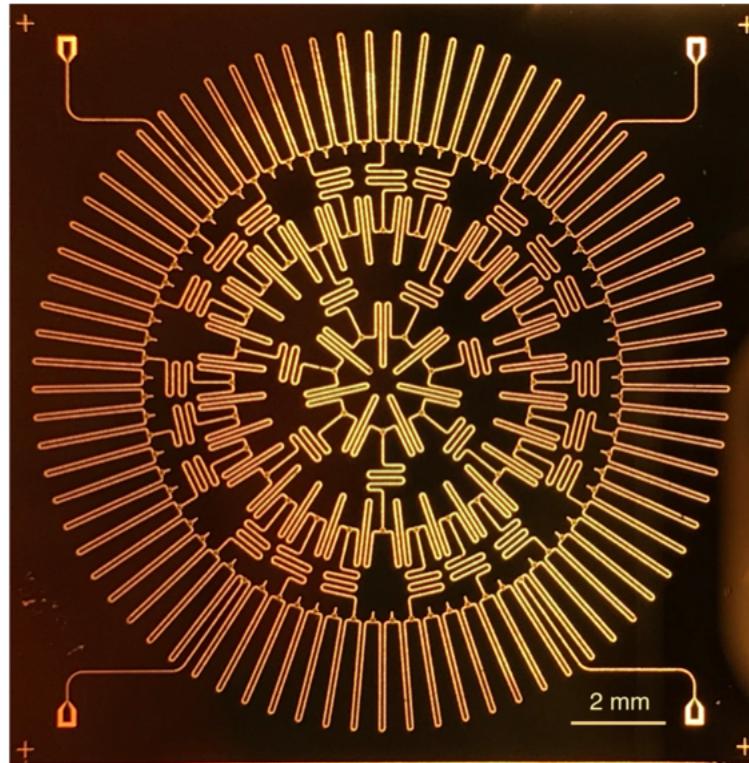


Heptagon-Kagome Device



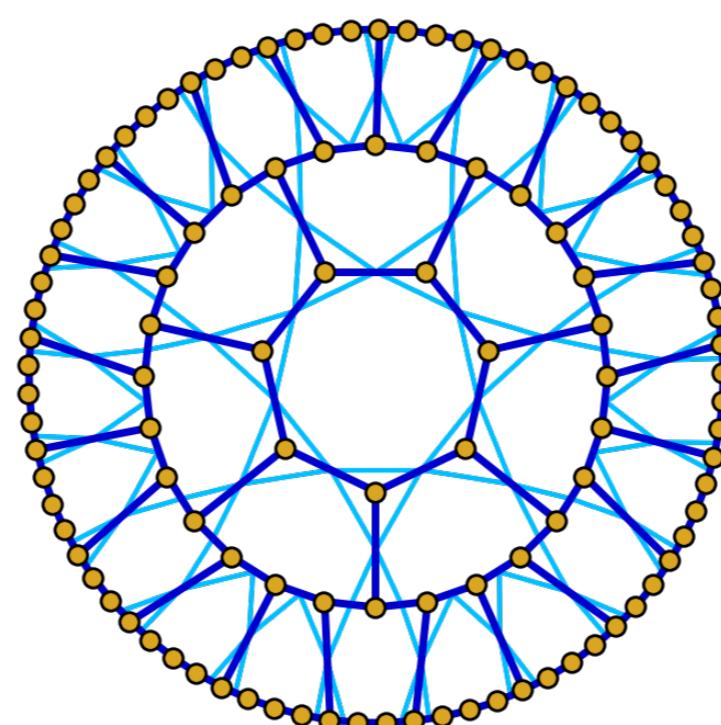
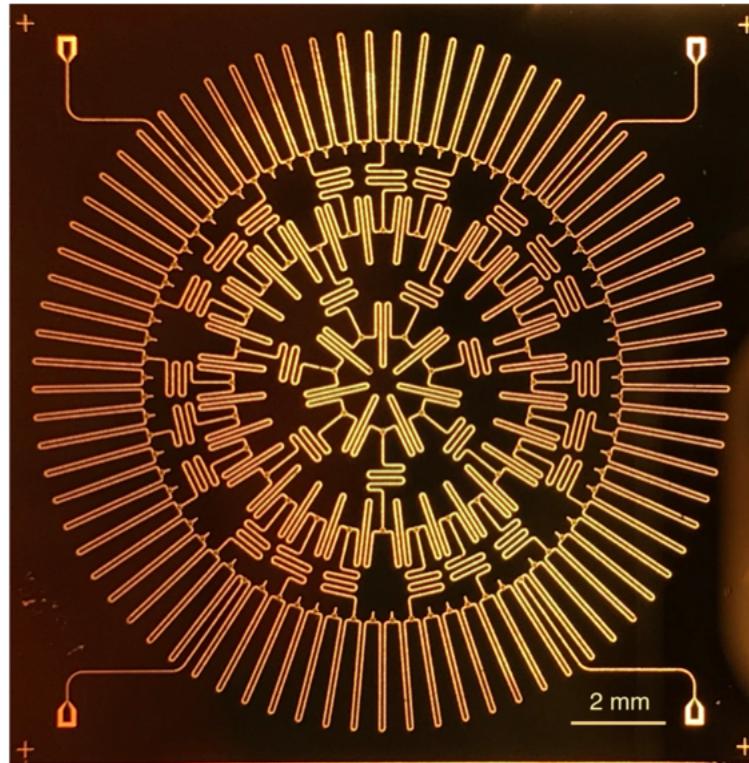
- 2 shells

Heptagon-Kagome Device



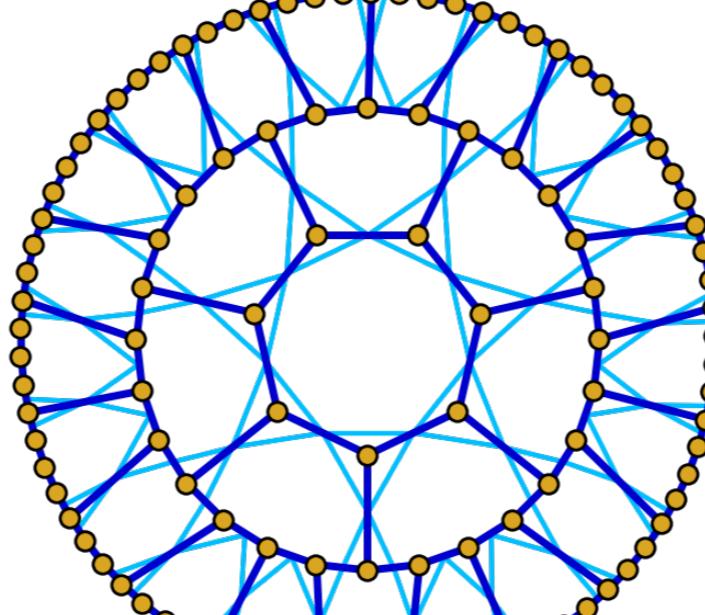
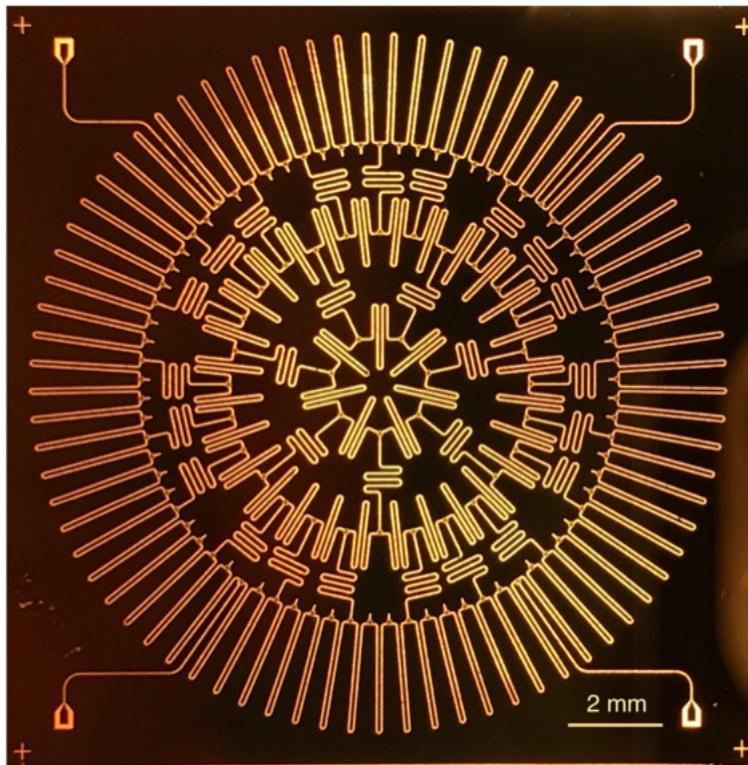
- 2 shells
- Operating frequency: 16 GHz

Heptagon-Kagome Device

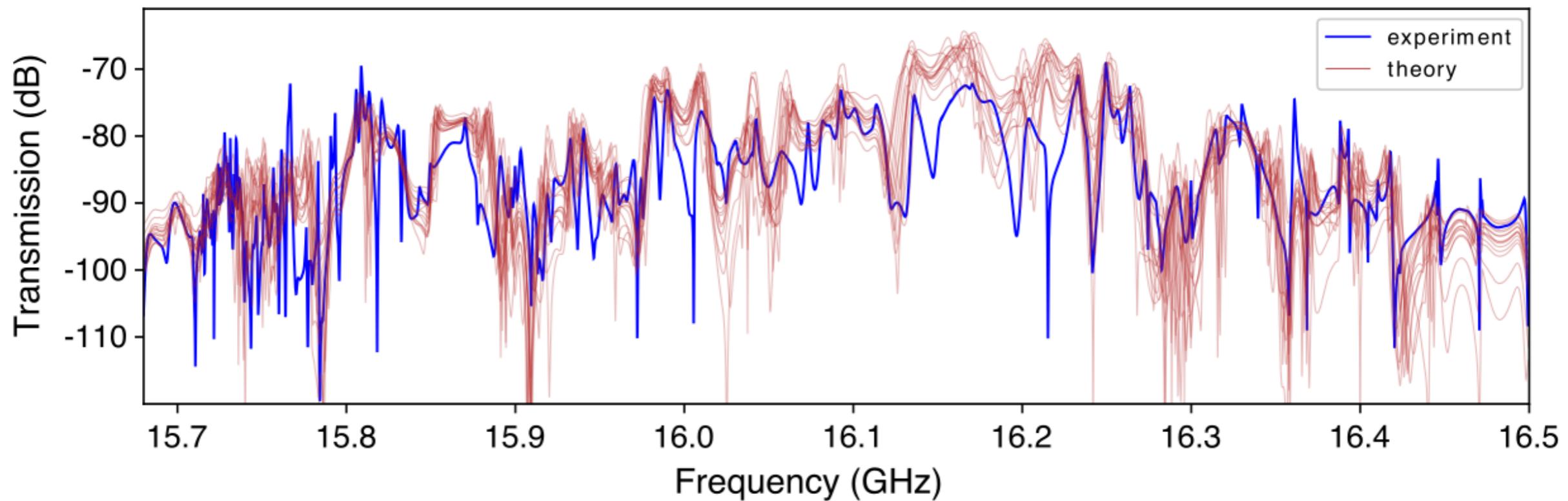


- 2 shells
- Operating frequency: 16 GHz
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Heptagon-Kagome Device

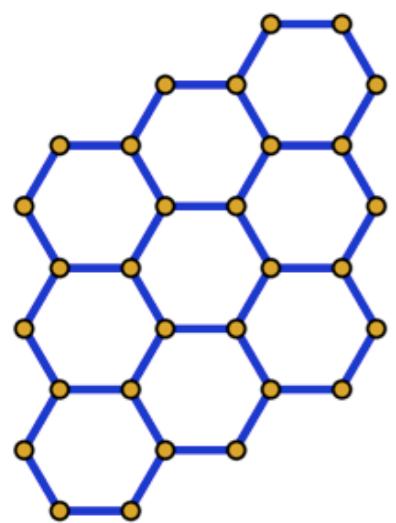


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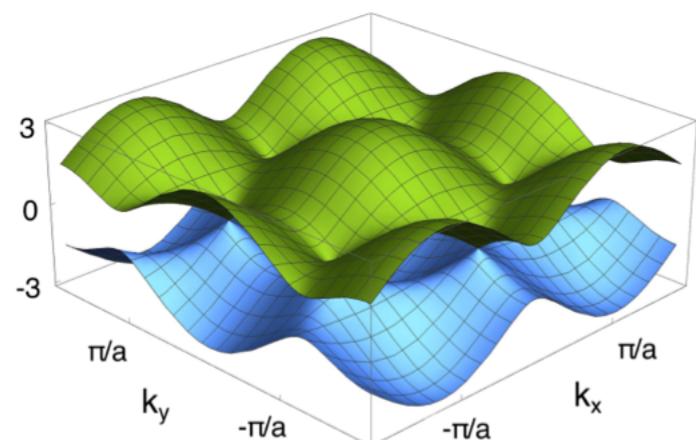


Band Structure Correspondence

Layout X

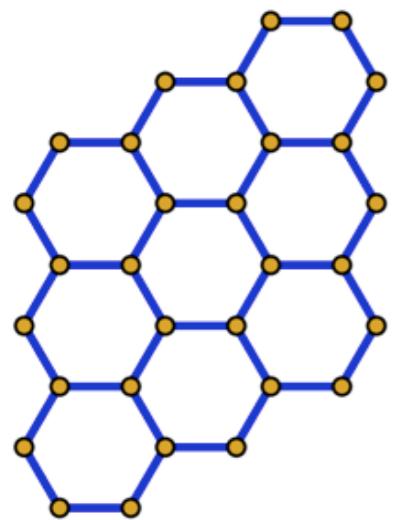


Energy ($|t|$)

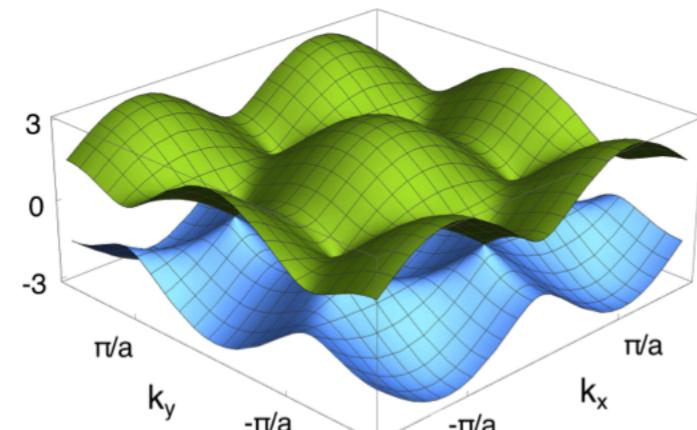


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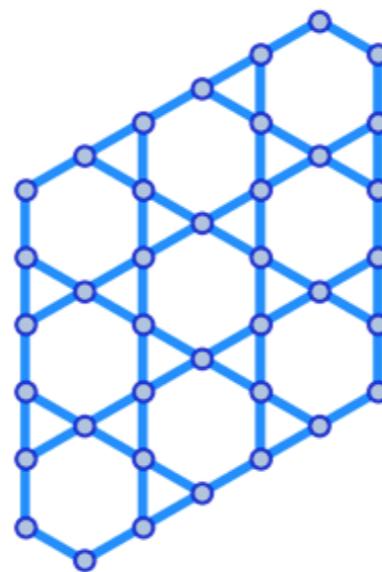
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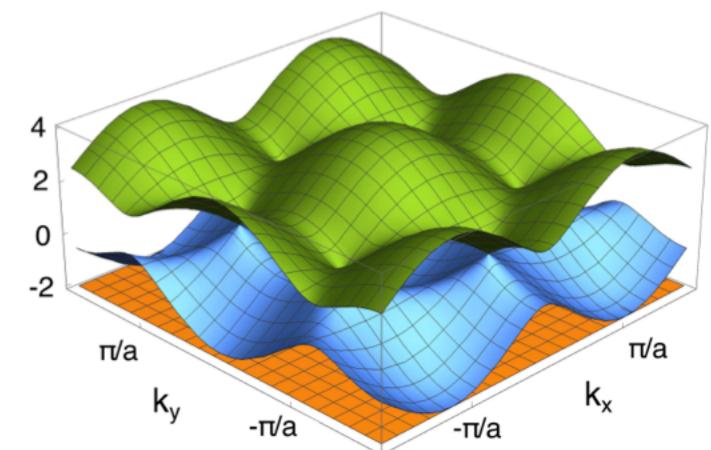
Energy ($t\epsilon$)



Line Graph $L(X)$

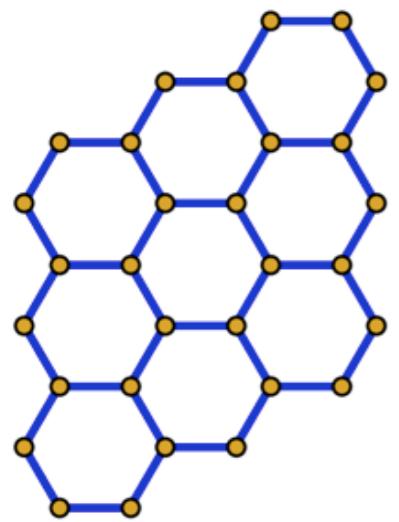


Energy ($t\epsilon$)

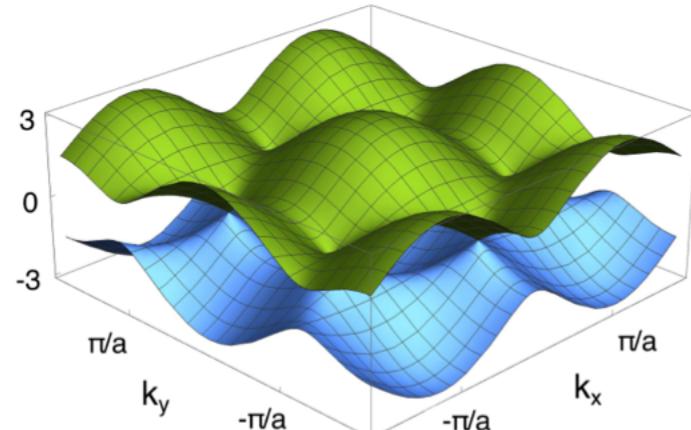


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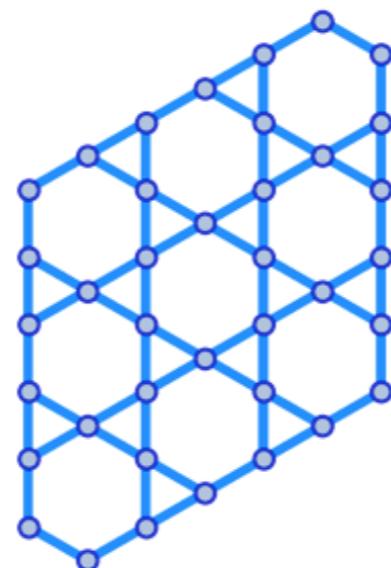
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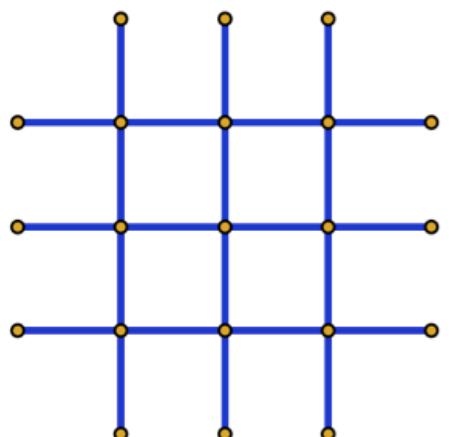
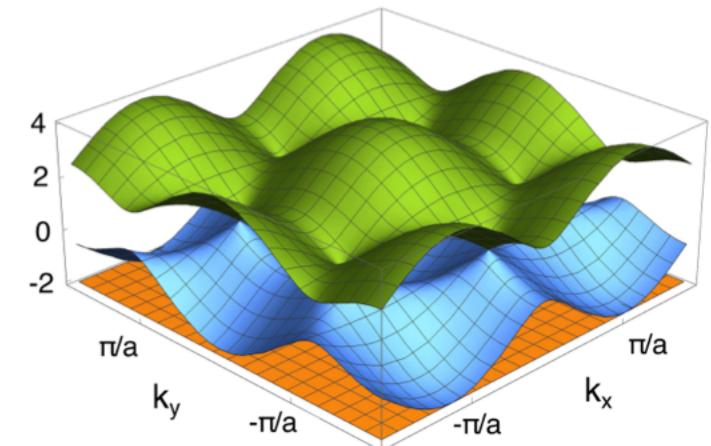
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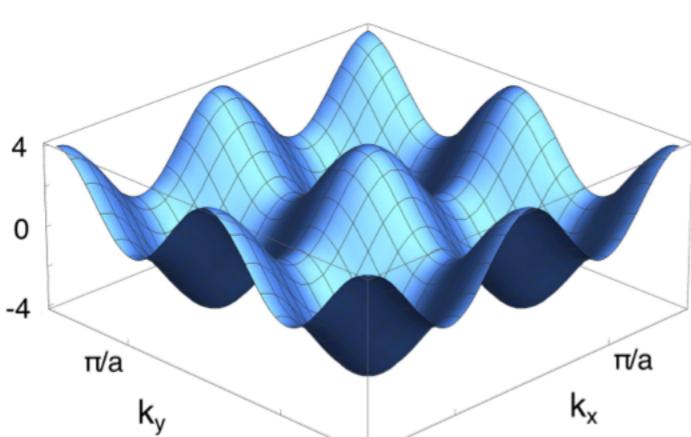
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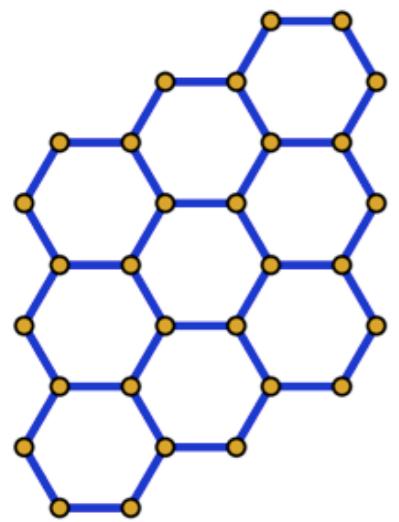


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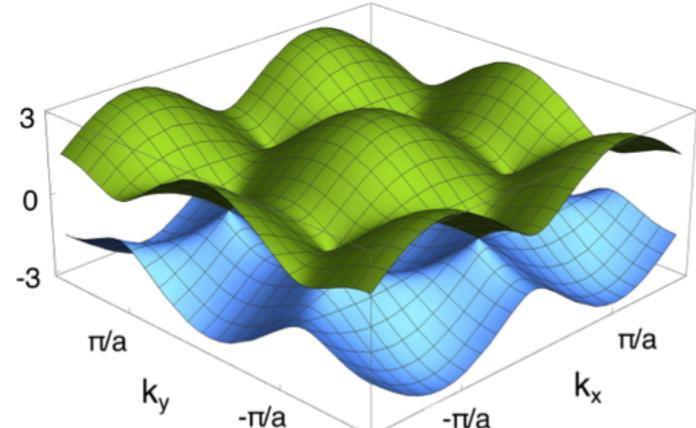


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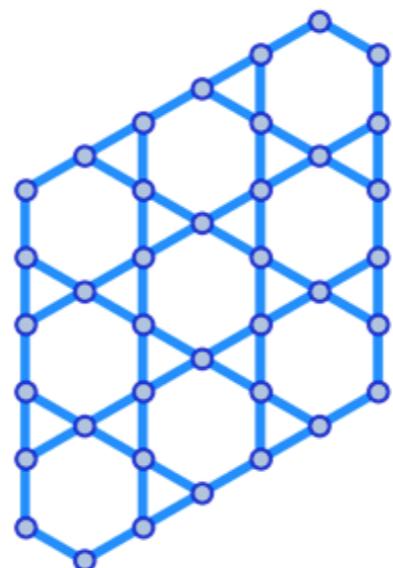
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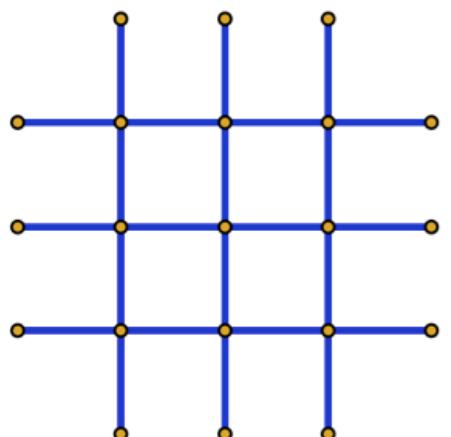
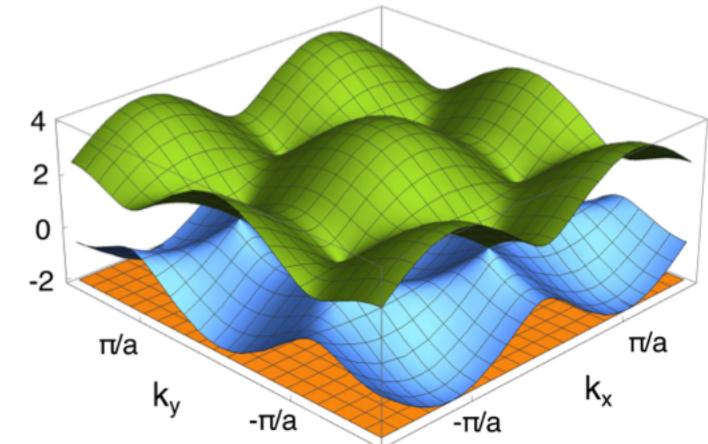
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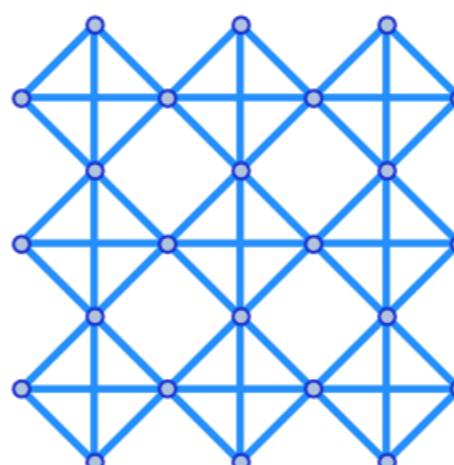
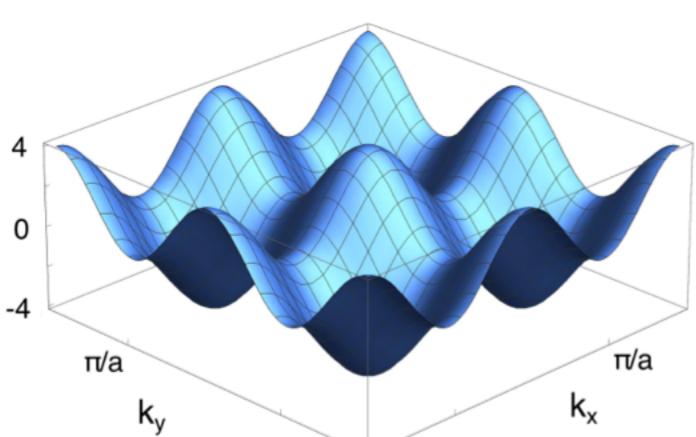
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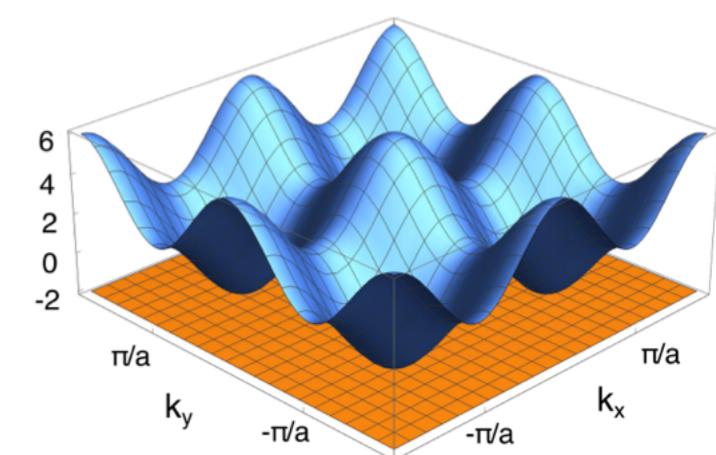
Energy ($|t|$)



Energy ($|t|$)



Energy ($|t|$)



Band Structure Correspondence

Layout Tight-Binding Hamiltonian

- Bounded self-adjoint operator on X

$$H_X$$

Band Structure Correspondence

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Effective Hamiltonian

- Bounded self-adjoint operator on $L(X)$

$$\bar{H}_s(X) = H_{L(X)}$$

Band Structure Correspondence

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Incidence Operator

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$$E_{\bar{H}_s} = \begin{cases} d - 2 + E_{H_X} \\ -2 \end{cases}$$

Incidence Operator

- From X to $L(X)$

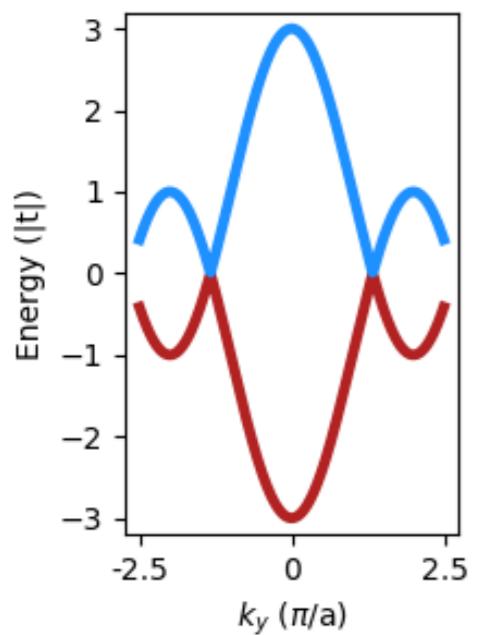
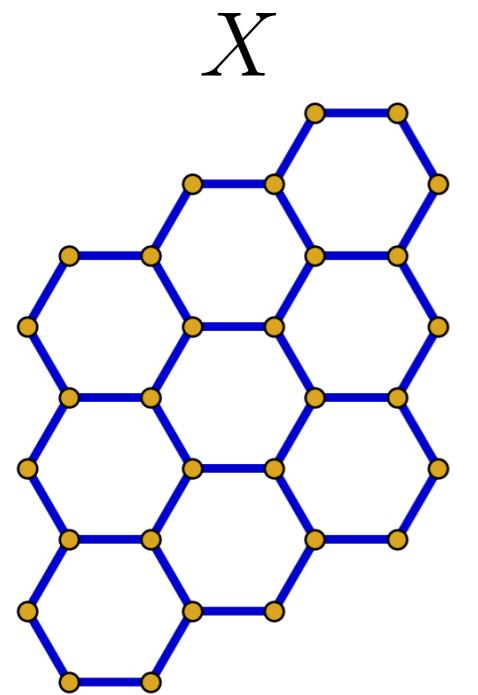
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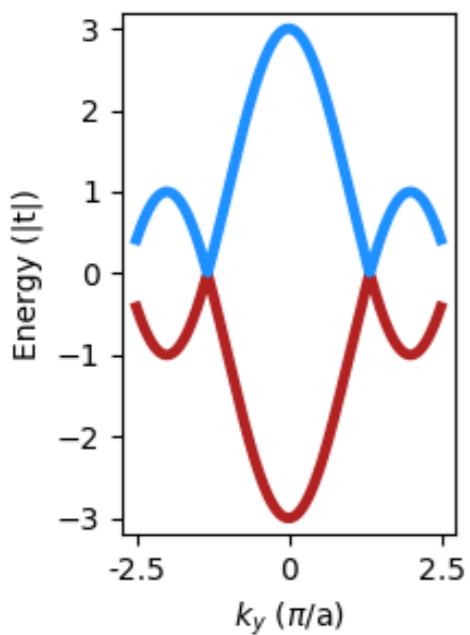
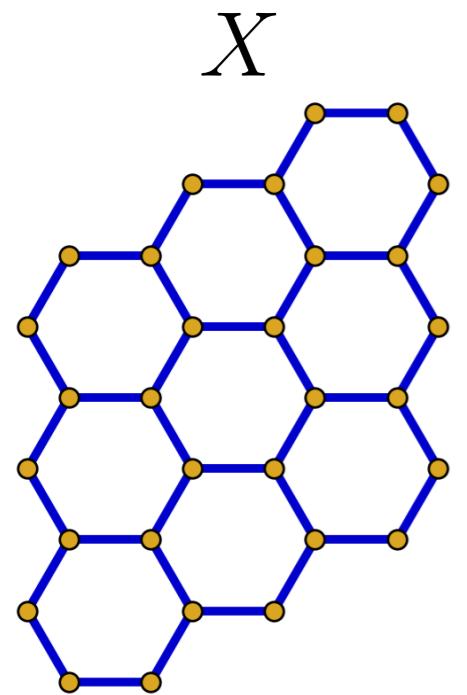
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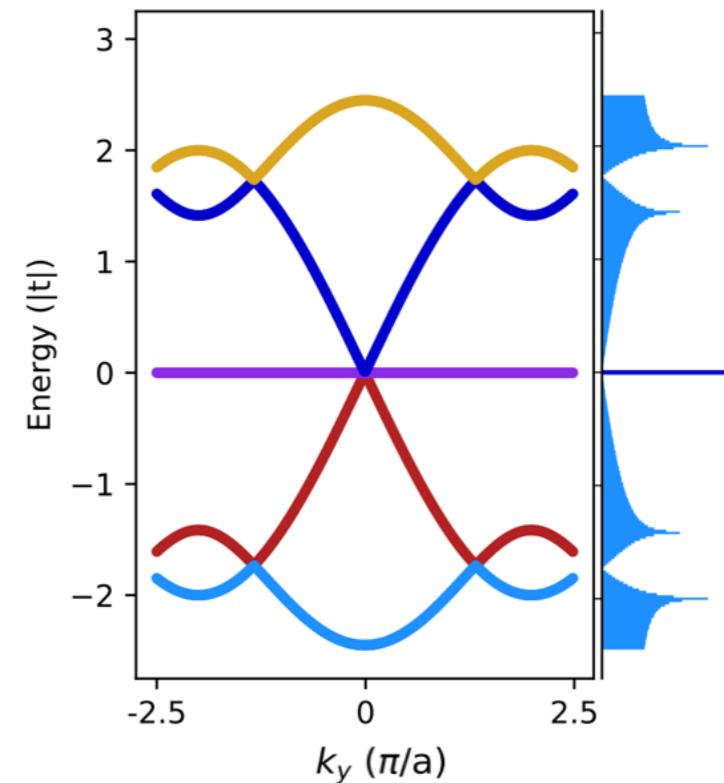
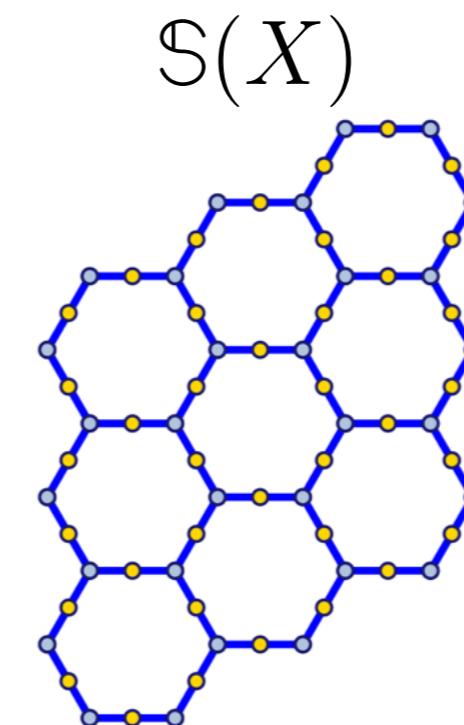
Subdivision Graphs and Optimally Gapped Flat Bands



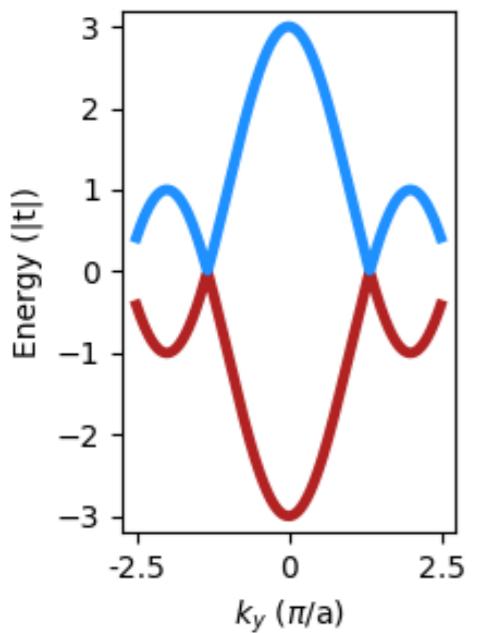
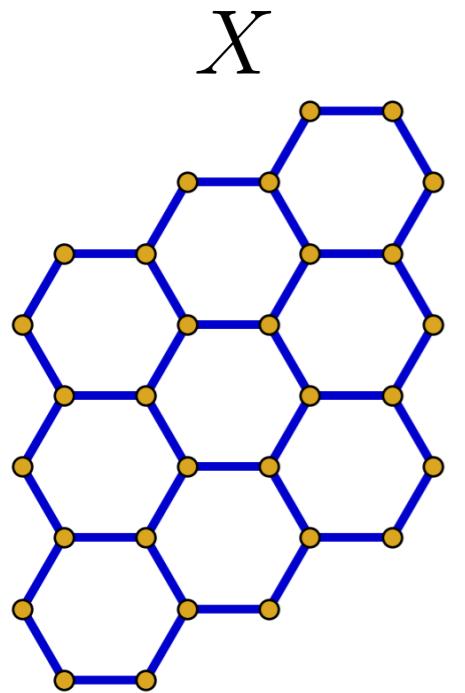
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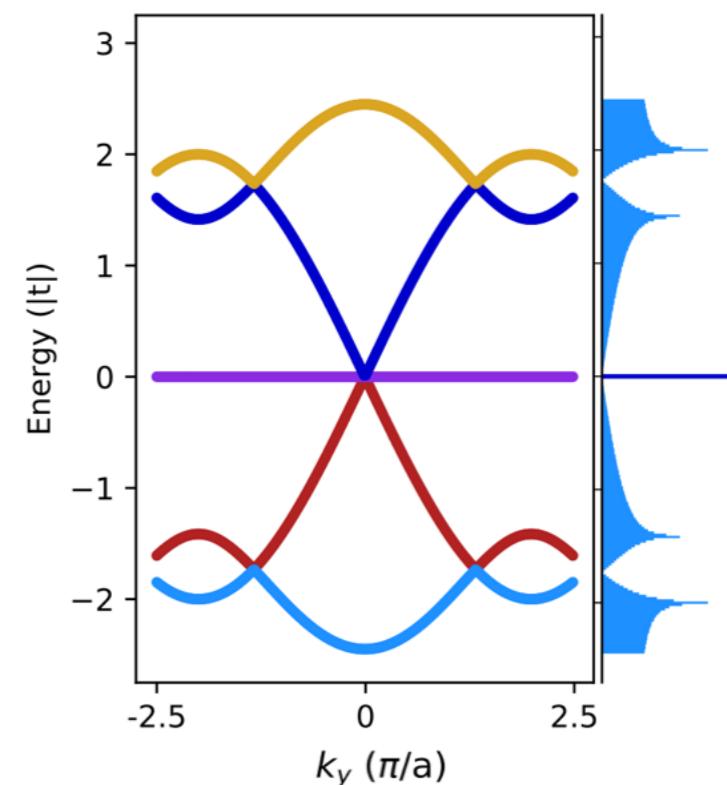
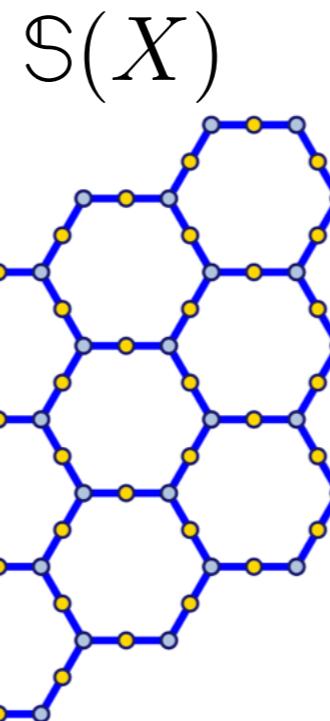
$$E_{\mathbb{S}(X)} = \begin{cases} \pm \sqrt{E_X + 3} \\ 0 \end{cases}$$



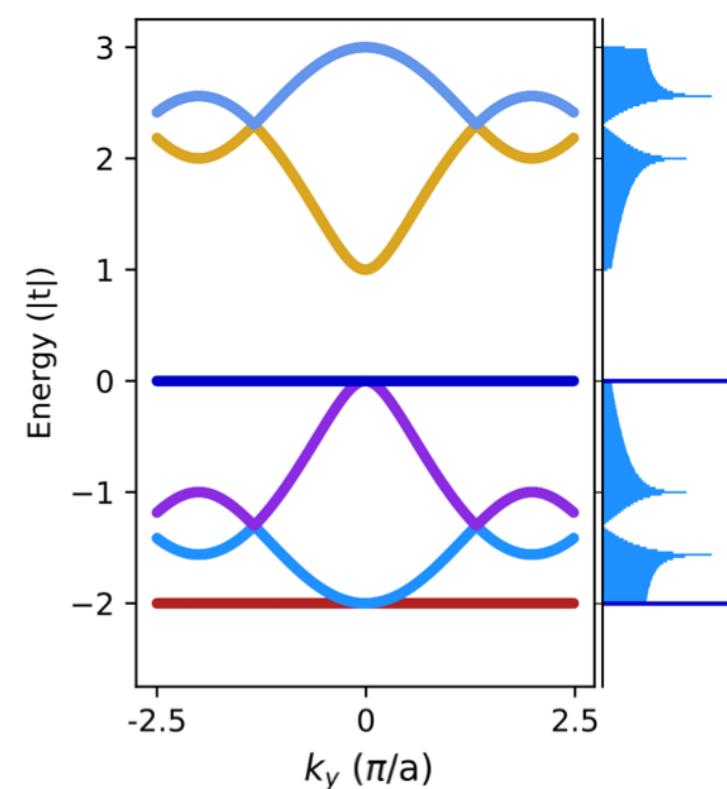
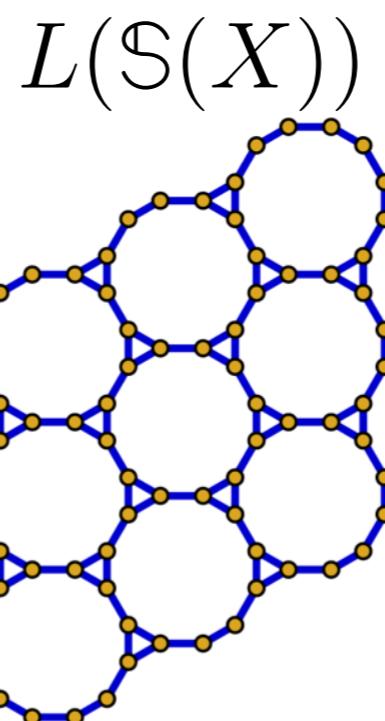
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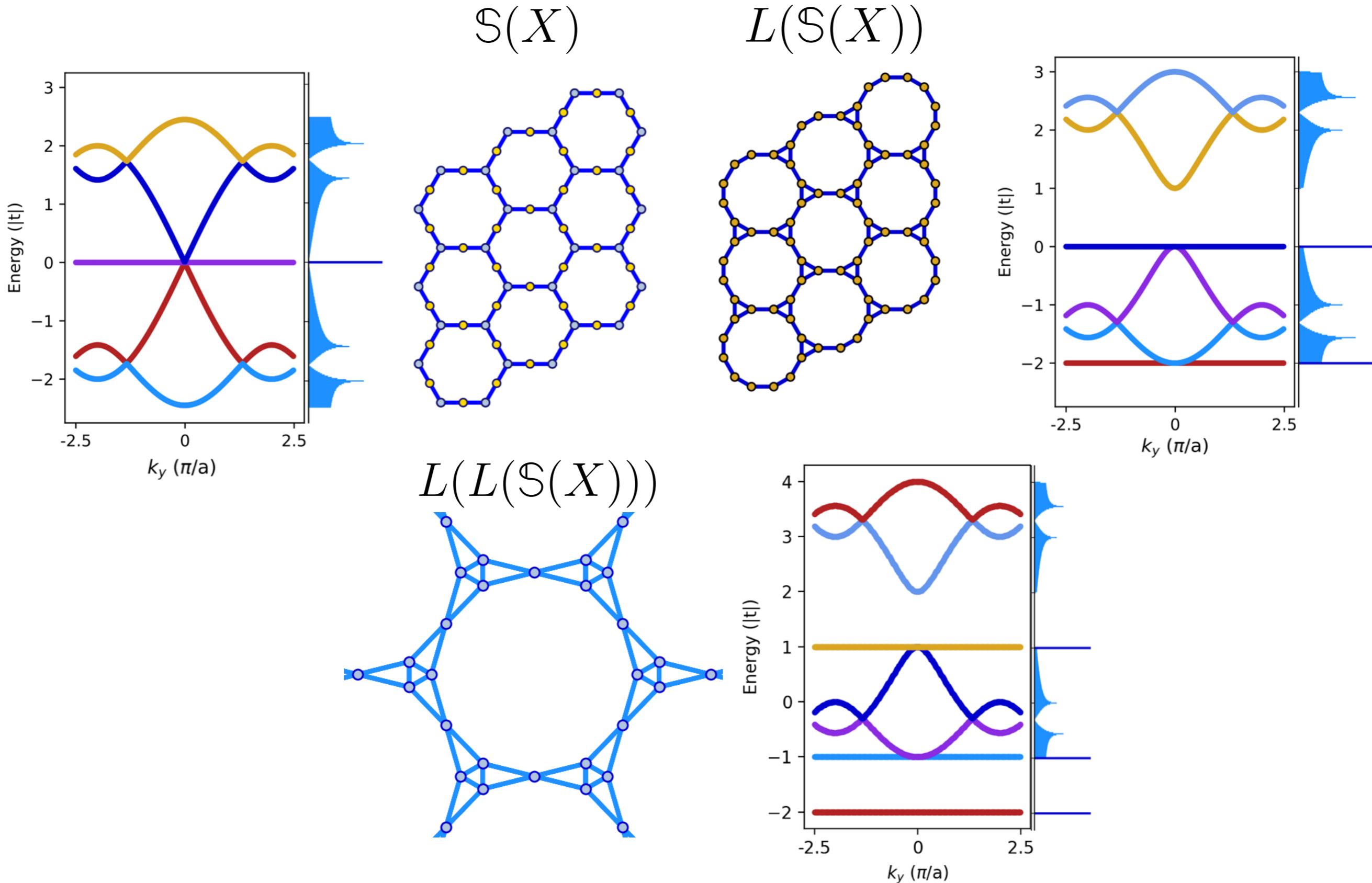
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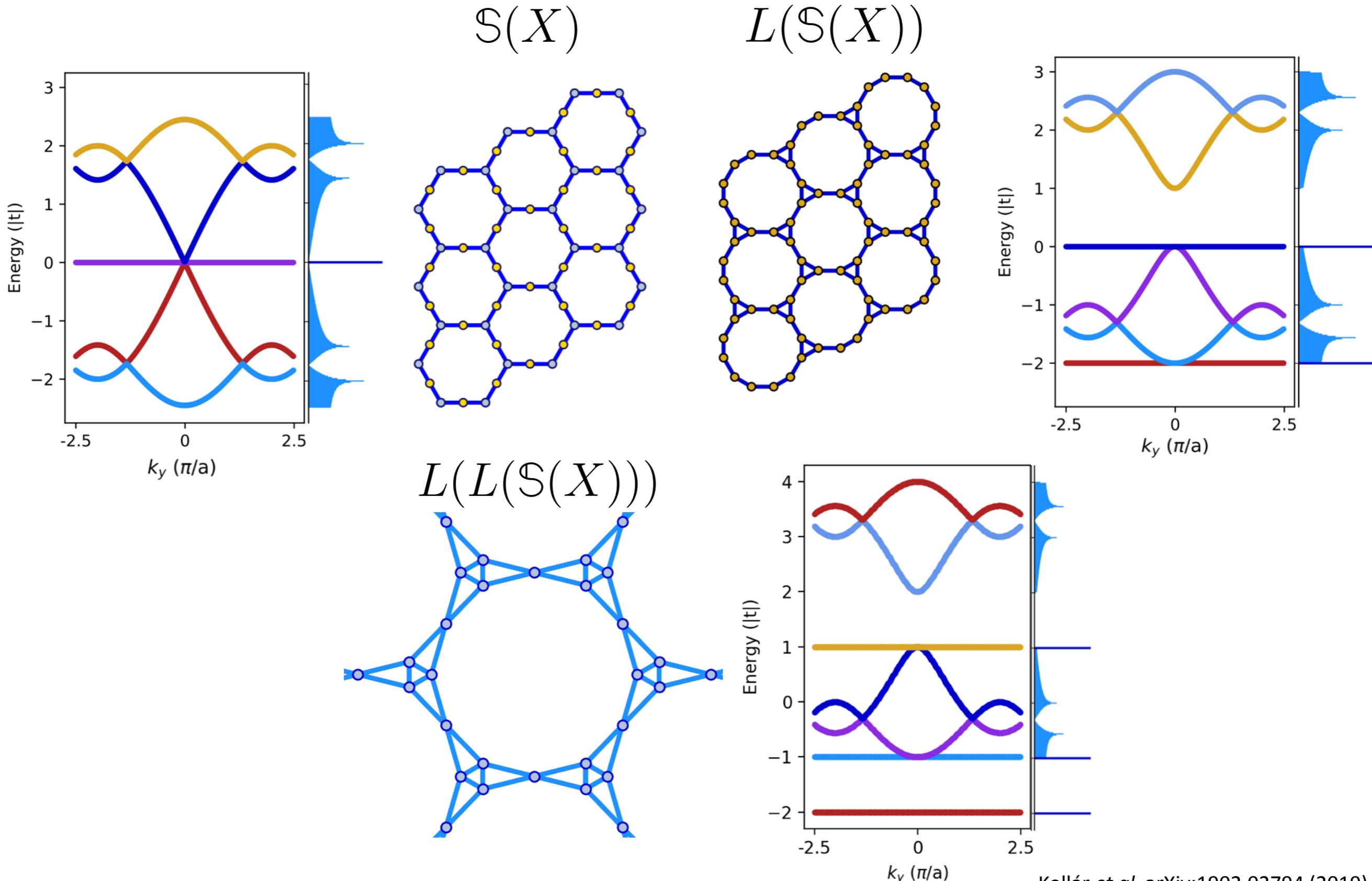
$$E_{L(\mathbb{S}(X))} = \begin{cases} \frac{1 \pm \sqrt{1+4(E_X+3)}}{2} \\ 0 \\ -2 \end{cases}$$



Subdivision Graphs and Optimally Gapped Flat Bands



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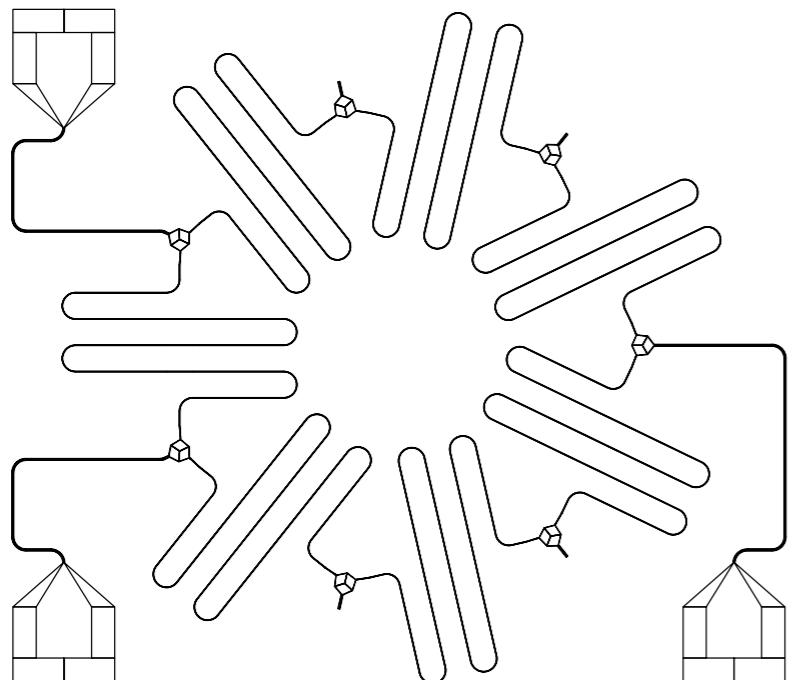


Outline

- Coplanar Waveguide (CPW) Lattices
 - Interacting photons
- Hyperbolic lattices
 - Projection to flat space
 - Deformable resonators
- Flat-band lattices
 - Line graphs
 - Maximal gaps
- Non-linear lattices
 - Limit cycles
 - Chaos

Small-Scale Lattice Device

Heptagonal Ring

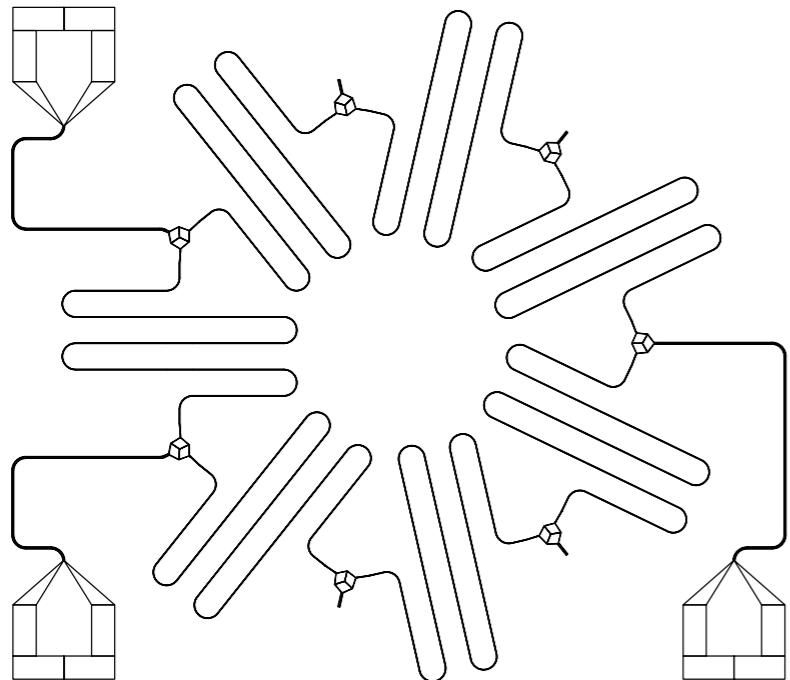


Niobium TiN

- Kinetic Inductor
- Kerr-nonlinear

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Heptagonal Ring



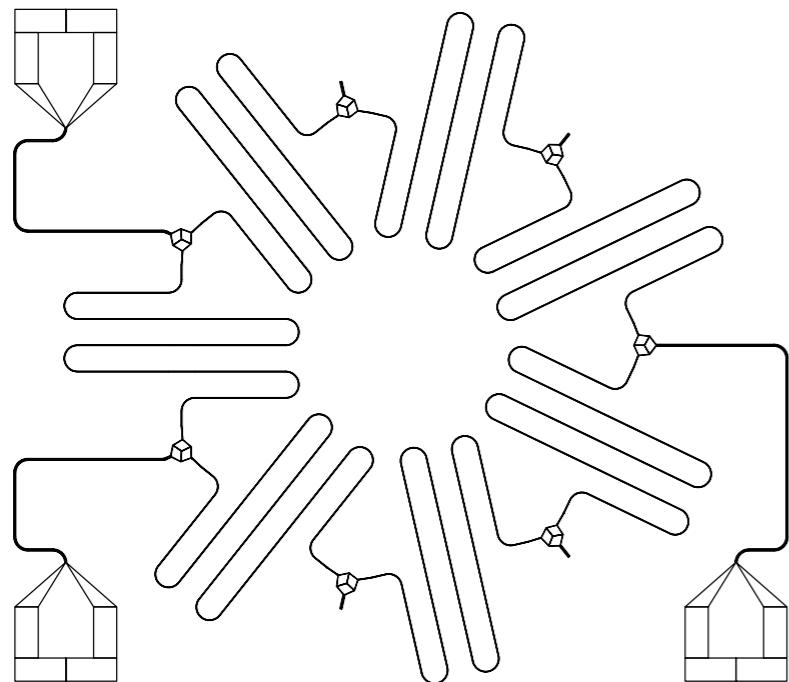
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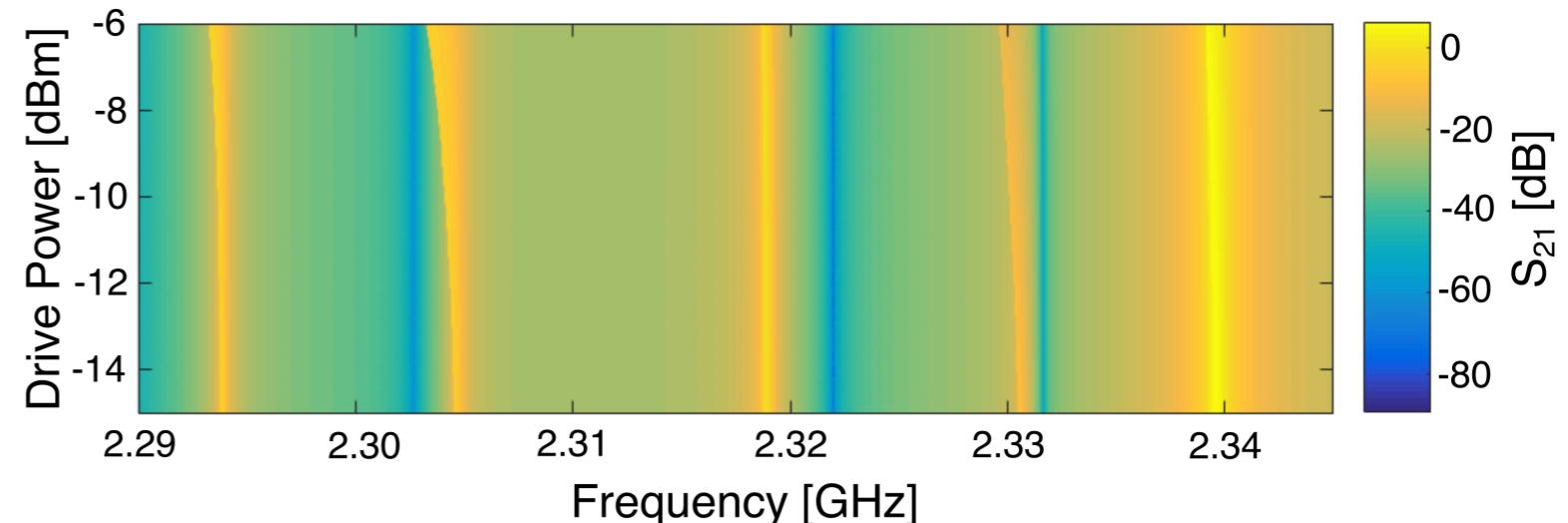
$$\begin{aligned} H_{TB} = & \omega_0 \sum_i a_i^\dagger a_i - t \sum_{\langle i,j \rangle} (a_i^\dagger a_j + a_j^\dagger a_i) \\ & + U \sum_i a_i^\dagger a_i^\dagger a_i a_i \end{aligned}$$

Small-Scale Lattice Device

Heptagonal Ring



Transmission



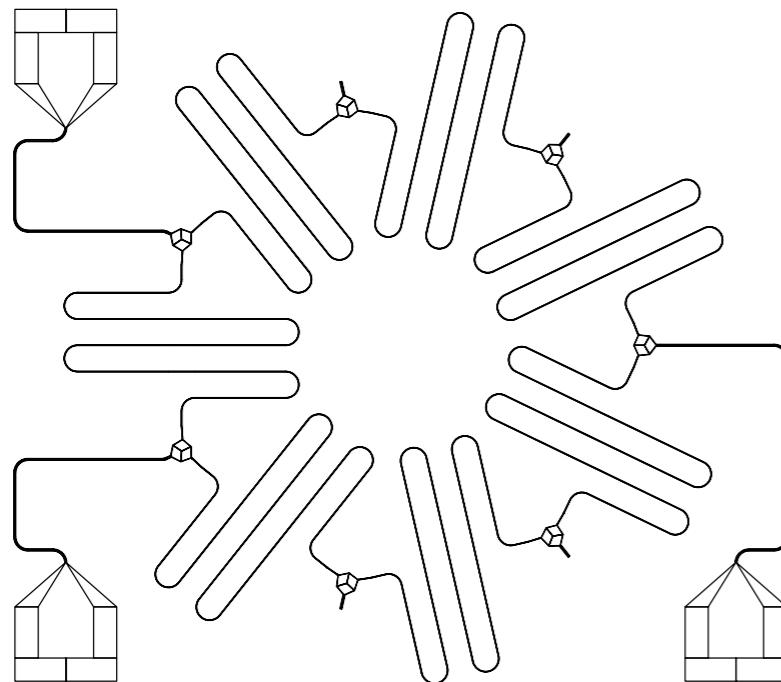
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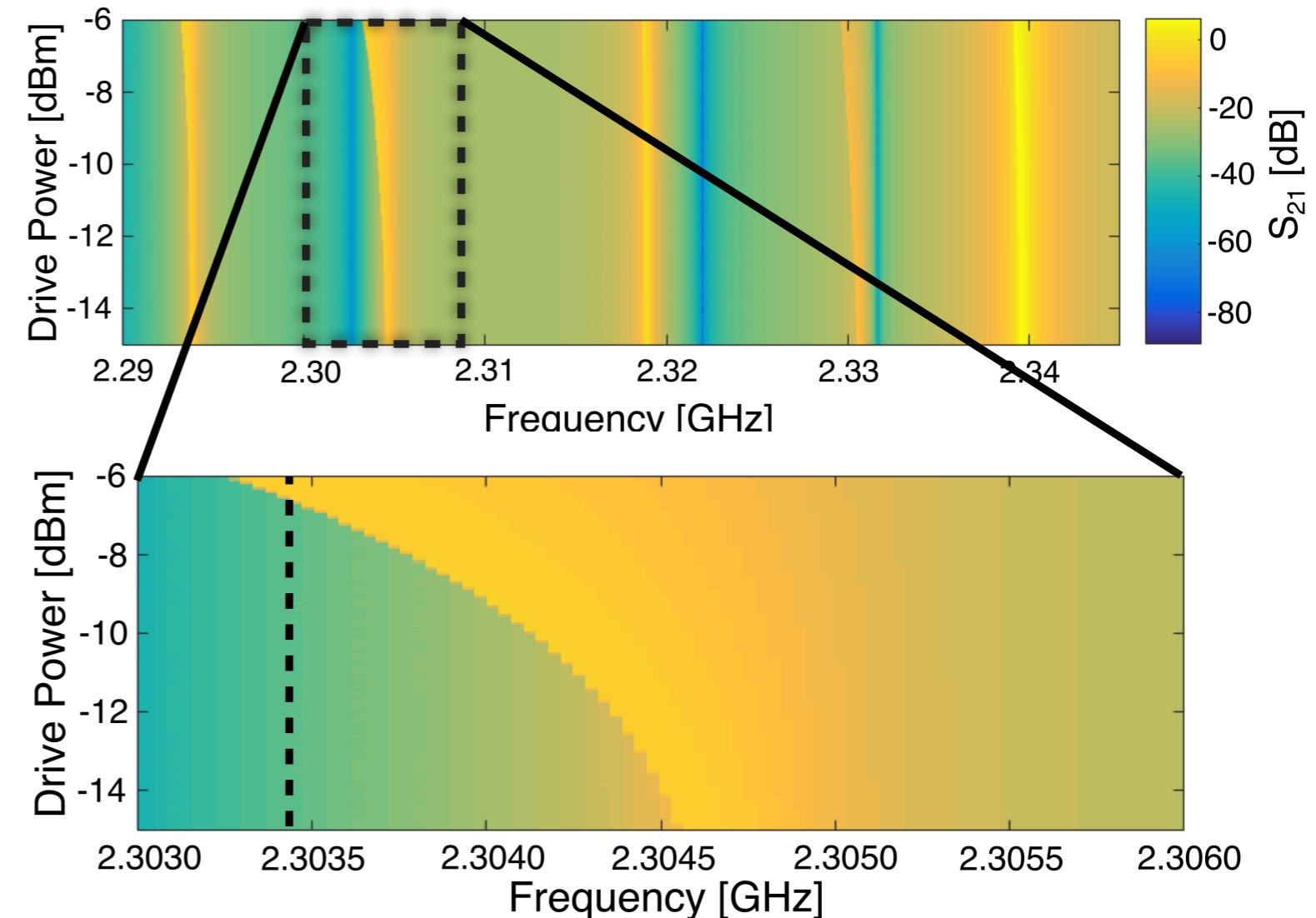
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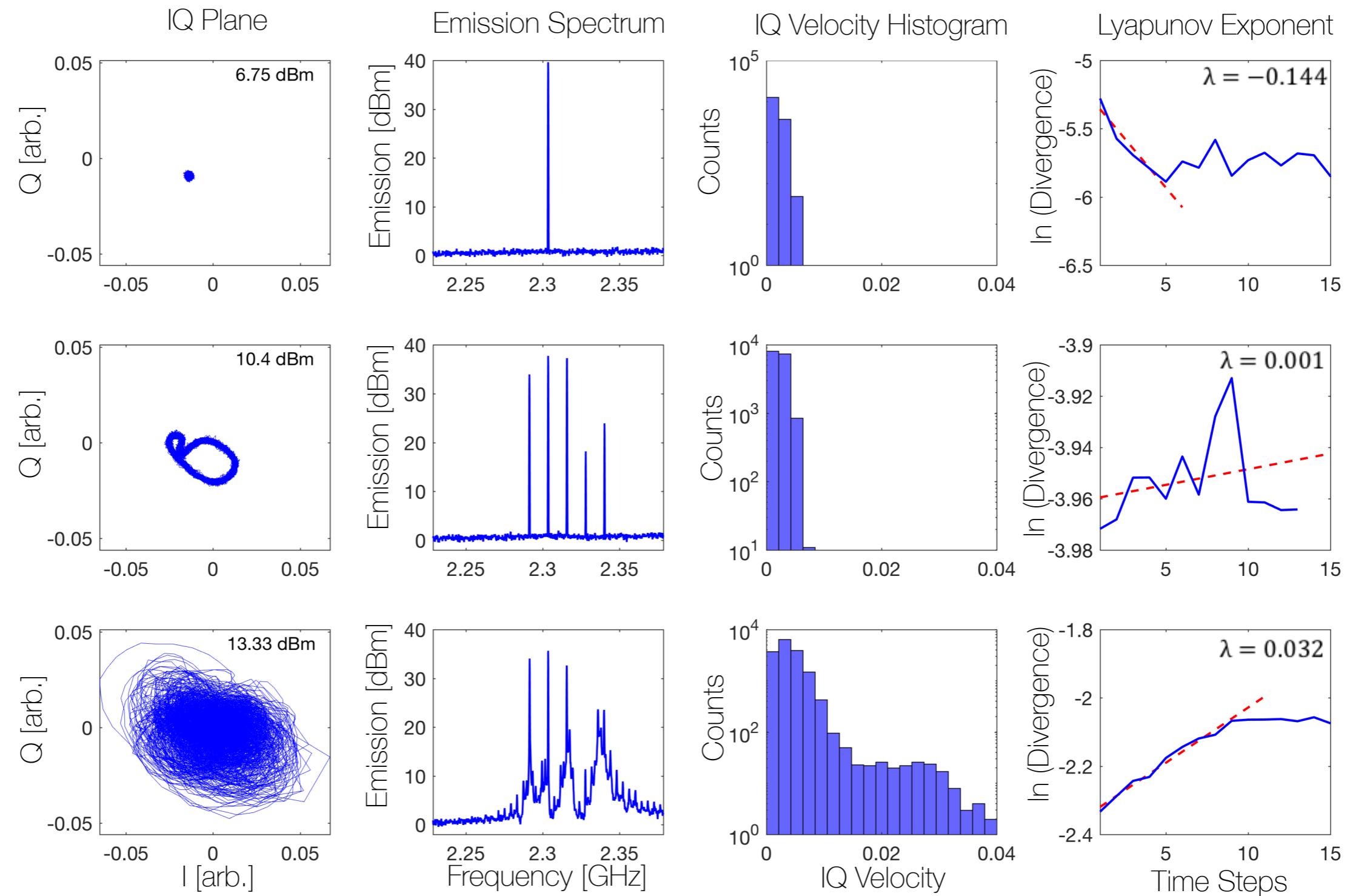
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Linear and Non-linear Response

Increasing Power



Equations of Motion and Linear Stability

Equation of Motion

$$d\psi = -iH_{TB}\psi - \kappa\psi + D - iR(\psi)$$

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$$Re(\tau) > 0$$

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- Rewrite drive and Kerr terms

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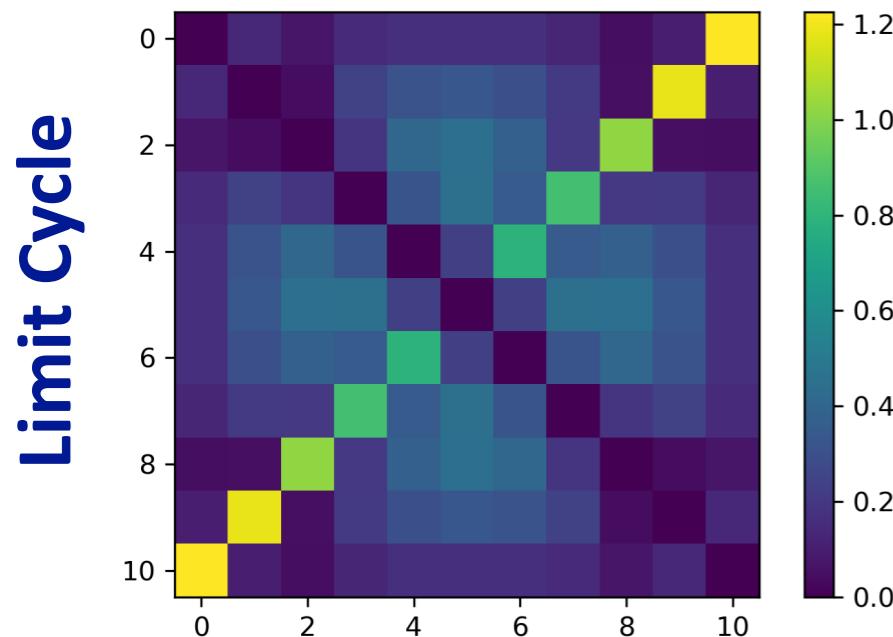
$$d\psi = (-iH_{TB} - \kappa + \tilde{D} - i\tilde{R})\psi$$

- Effective Hamiltonian

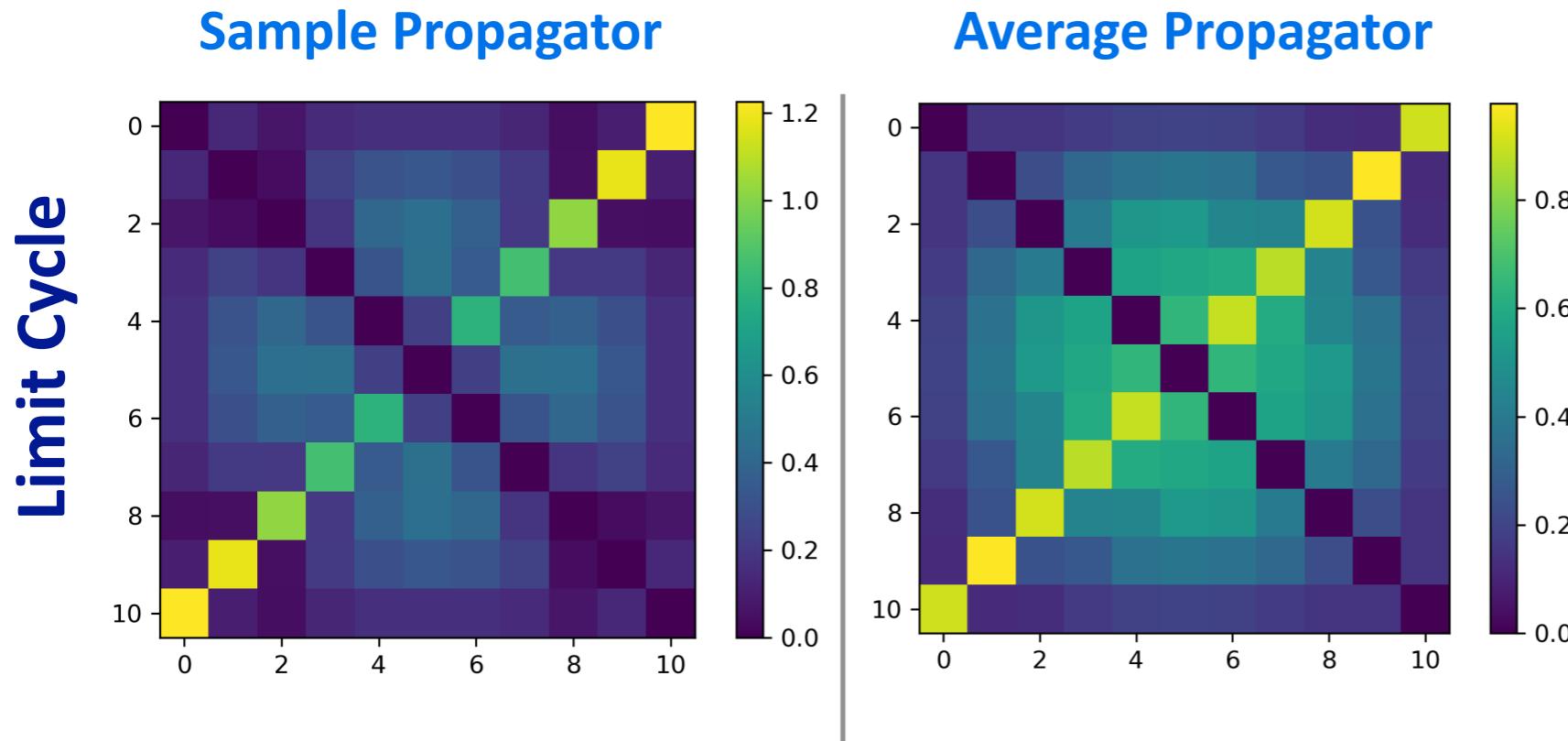
$$H_{eff} = H_{TB} - i\kappa + i\tilde{D} + \tilde{R}(\psi)$$

Propagator, “Effective” Hamiltonian

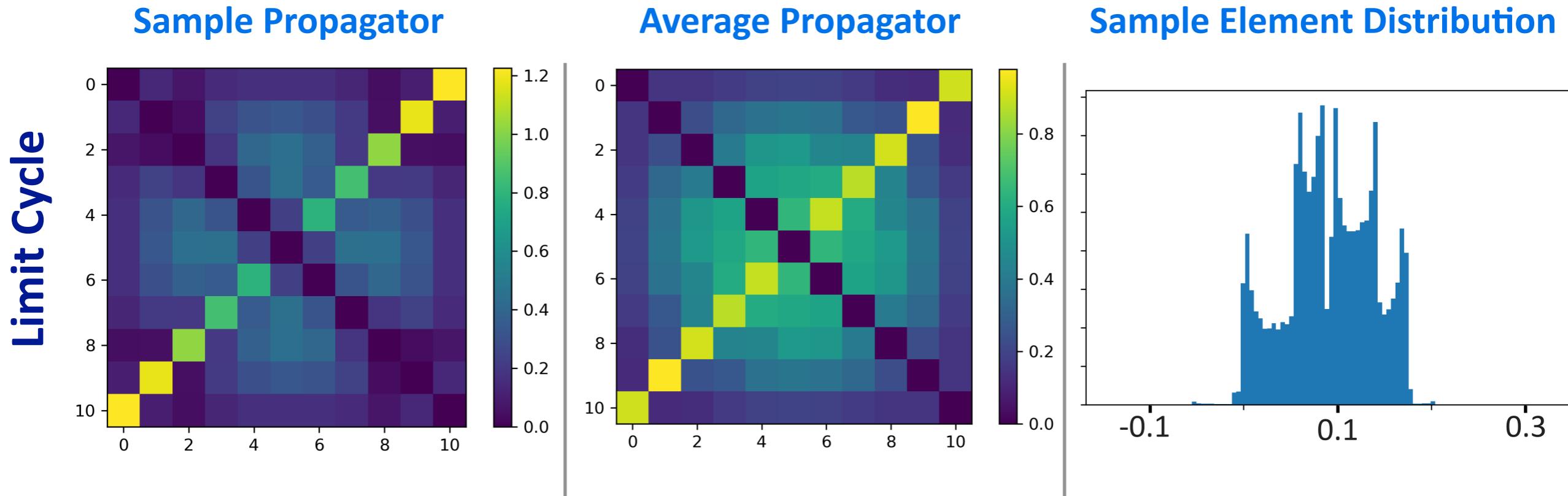
Sample Propagator



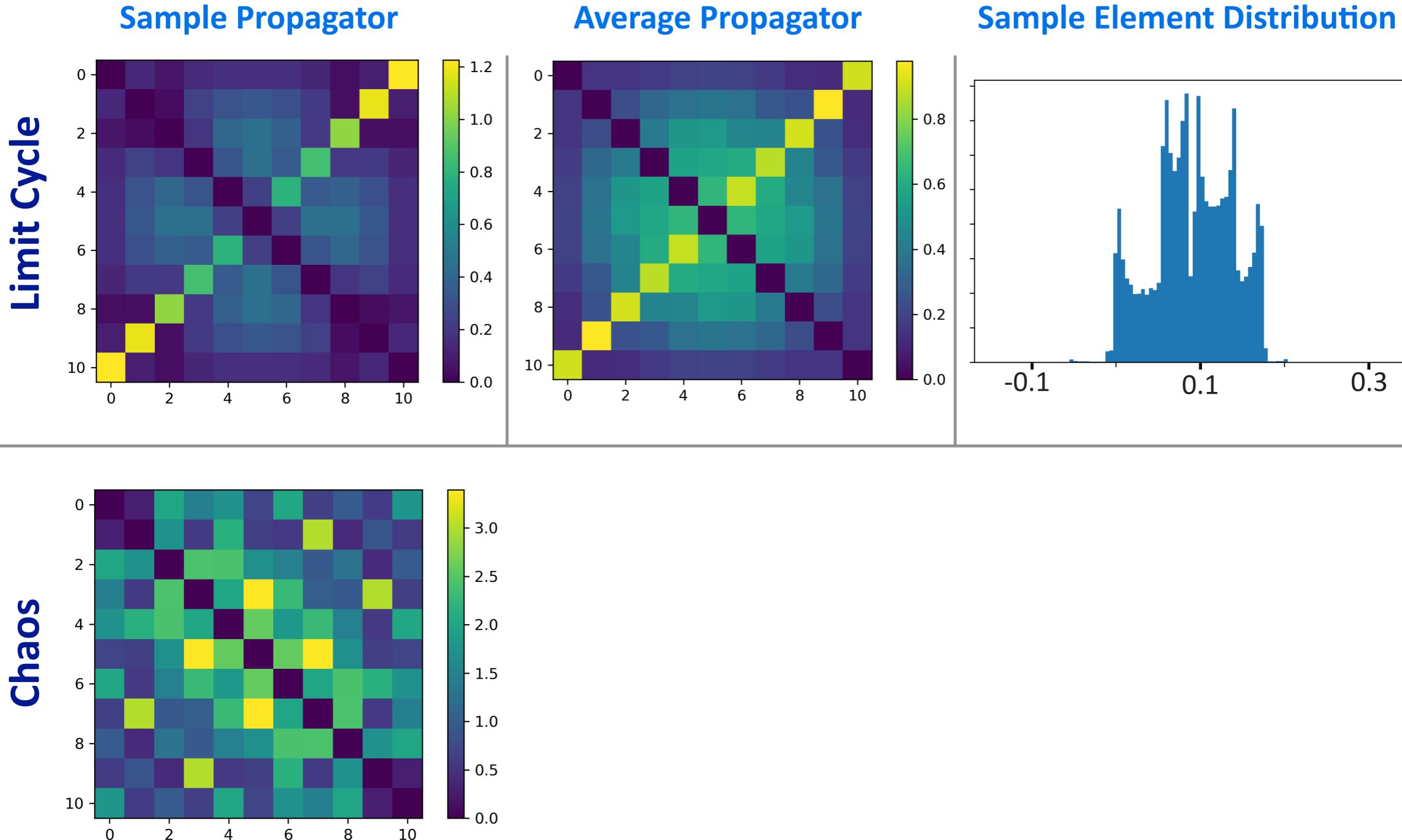
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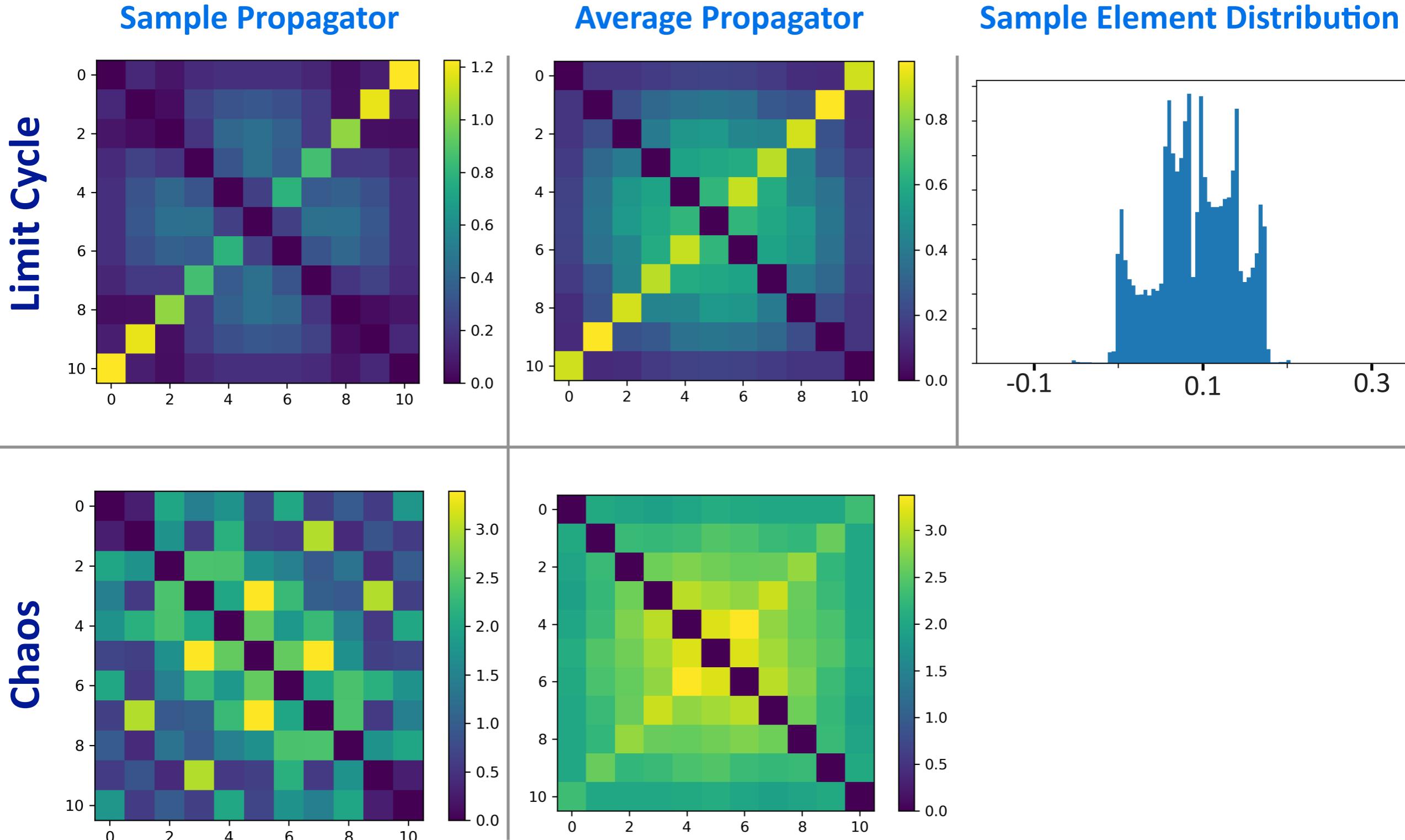
Propagator, “Effective” Hamiltonian



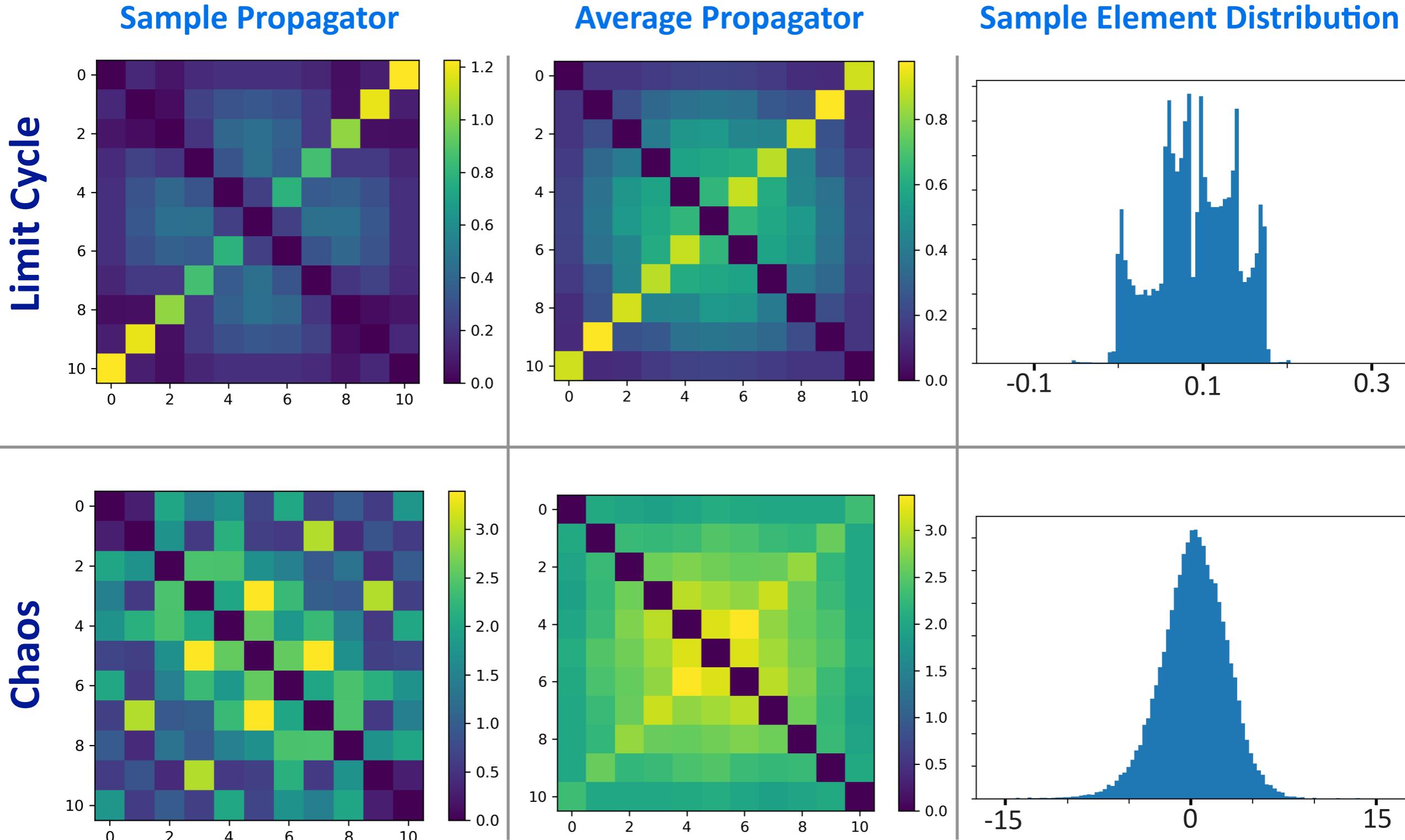
Propagator, “Effective” Hamiltonian



Propagator, “Effective” Hamiltonian

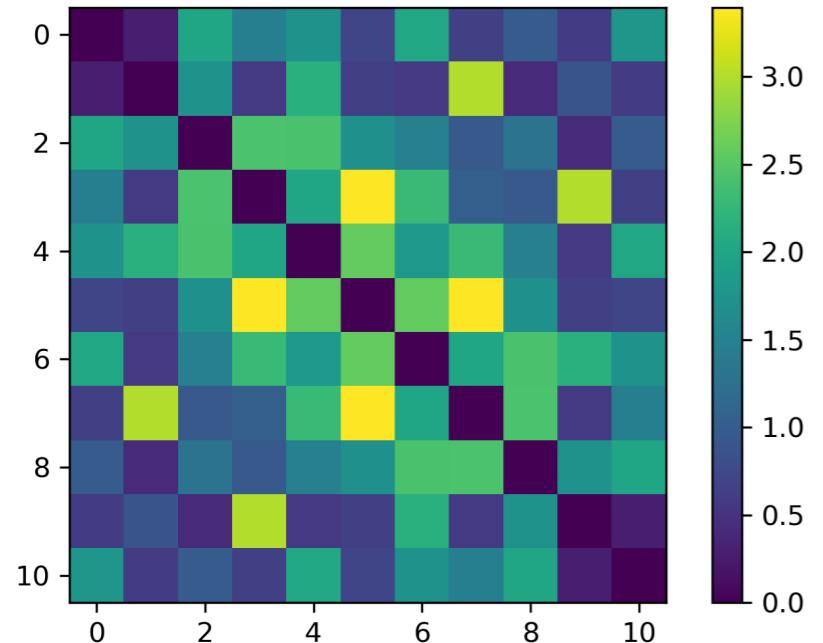


Propagator, “Effective” Hamiltonian

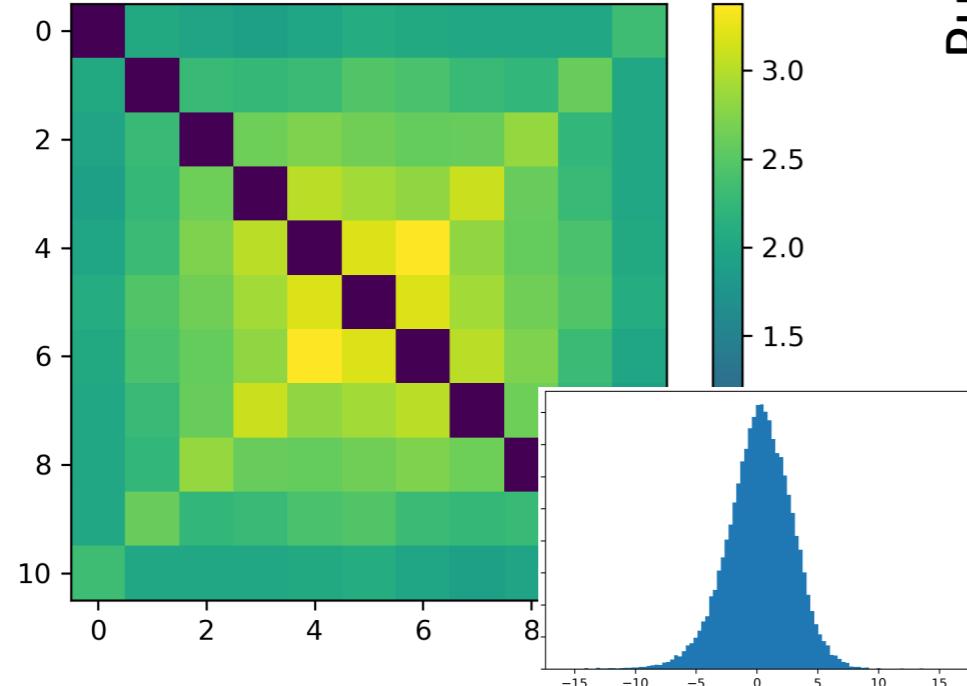


Chaotic “Effective” Hamiltonian

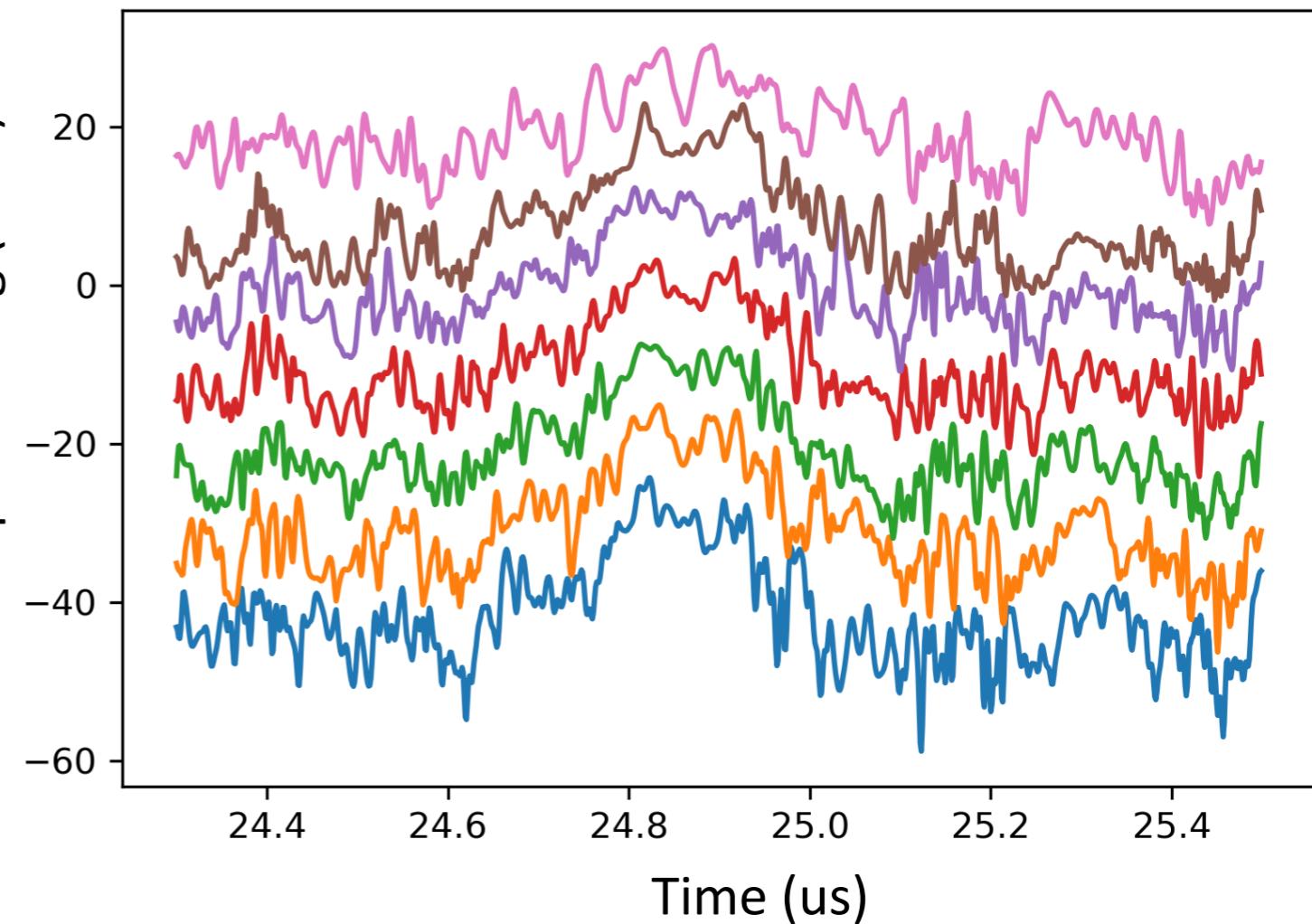
Sample Propagator



Average Propagator



Eigenenergies



Conclusion and Outlook

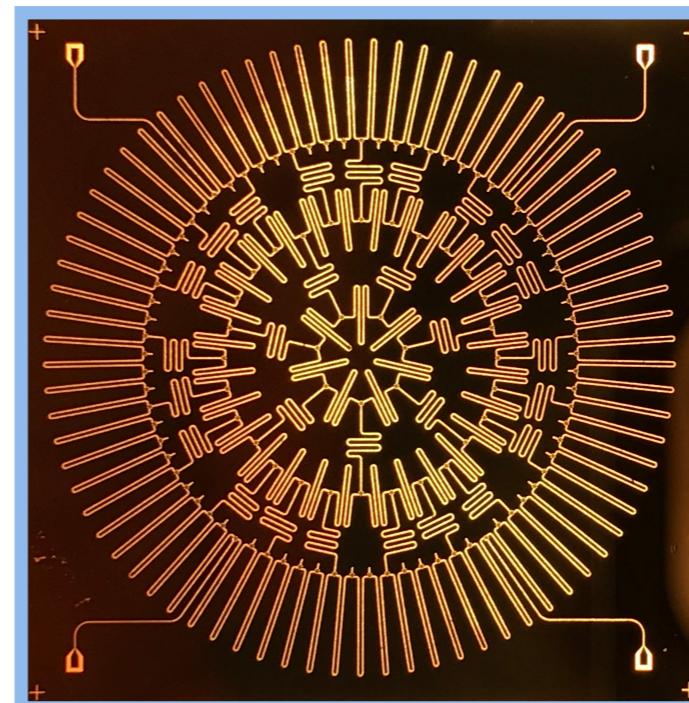
- Circuit QED lattices
 - Artificial photonic materials
 - Interacting photons
- Hyperbolic lattices
 - Unusual band structures
 - On-chip fabrication
- Flat-band lattices
 - 0, -2
 - Optimal gaps
- Kerr device
 - Limit Cycles
 - Chaos

Conclusion and Outlook

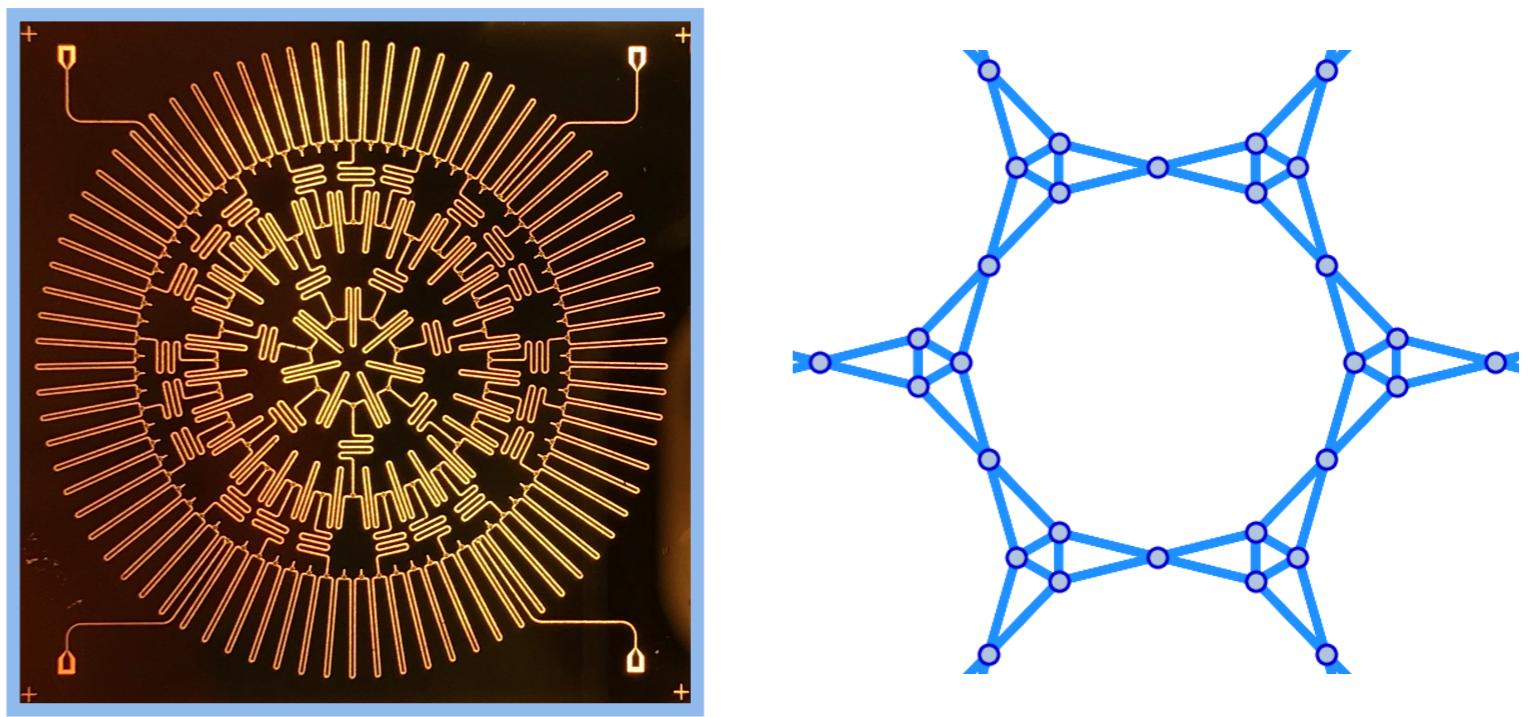
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Conclusion and Outlook

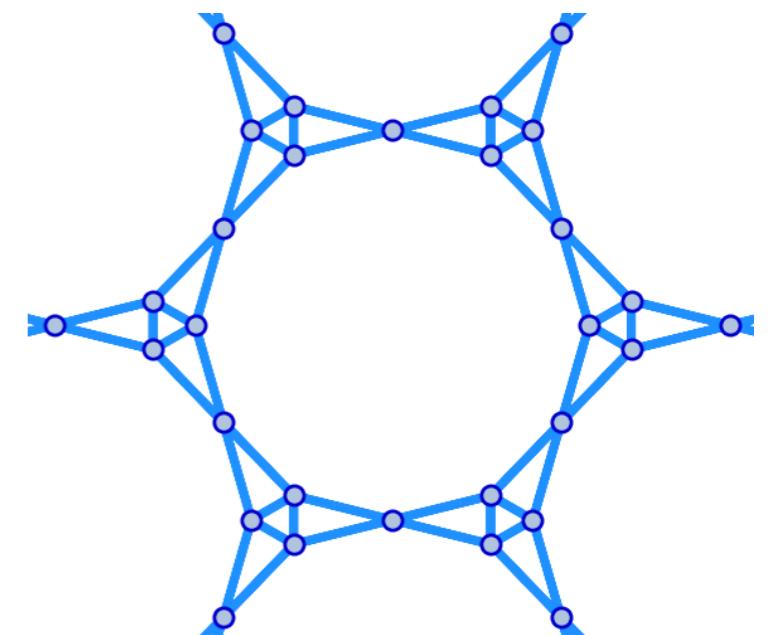
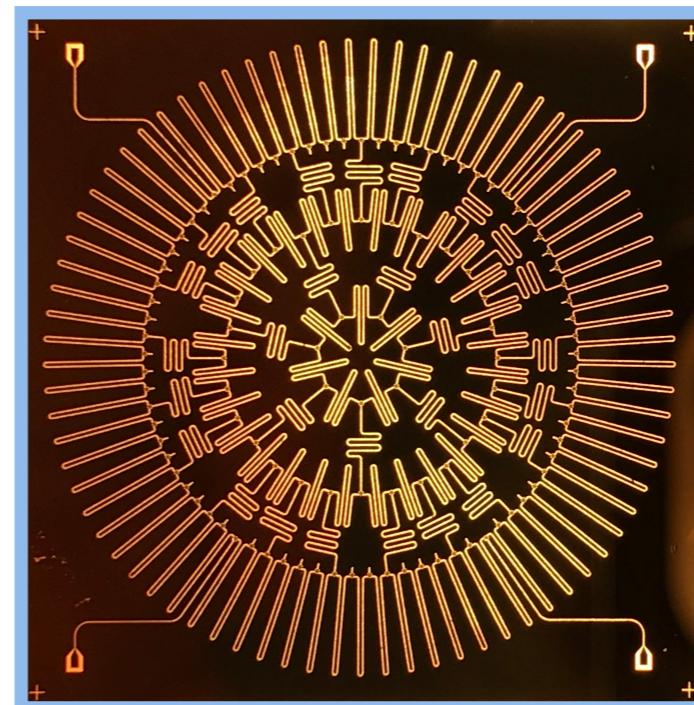
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Conclusion and Outlook

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● Outlook

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- Many-body physics in flat bands
- Photon-mediated spin models



Lattice Simulators in Circuit QED

