Quantum fluctuation theorems, contextuality and work quasi-probabilities

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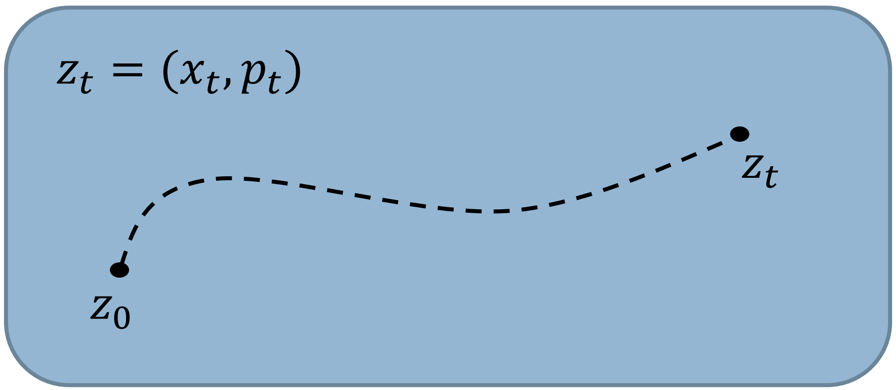
Central question

How do we define work fluctuation in coherent quantum systems?

<u>Classically: Hamiltonian evolution</u>



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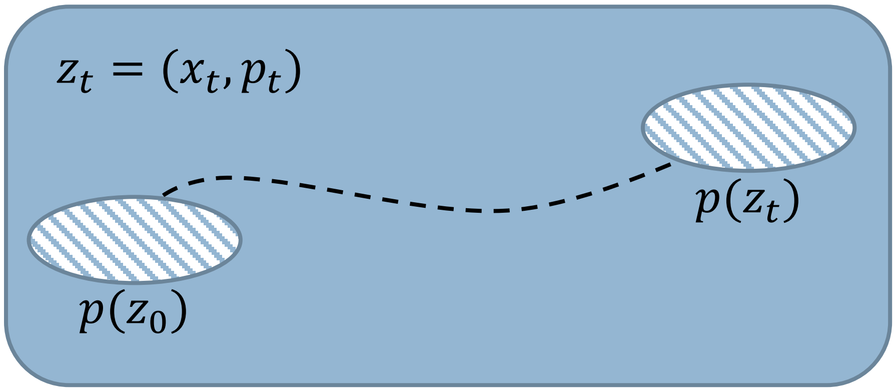

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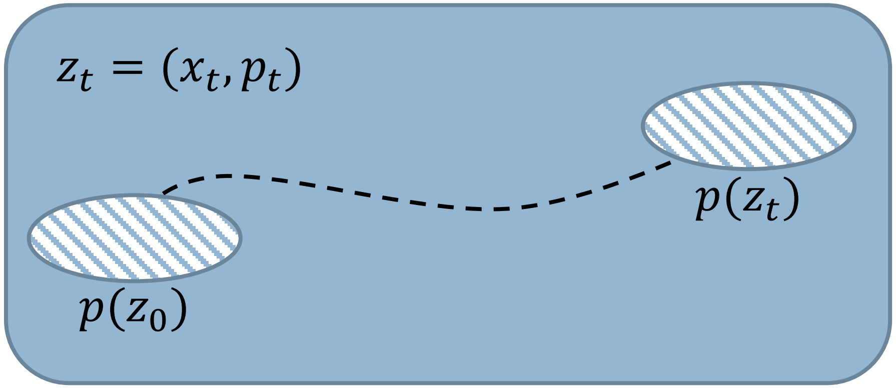

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Quantumly: unitary evolution

$$H = \sum_i E_i |E_i\rangle\langle E_i| \to H' = \sum_j E'_j |E'_j\rangle\langle E'_j|$$

$$|\Psi_0\rangle = \sqrt{p}|0\rangle + \sqrt{1-p}|1\rangle \neq p|0\rangle\langle 0| + (1-p)|1\rangle\langle 1|$$

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Some proposed solutions

1) *Two-point-projective measurement (TPM) scheme*: apply an energy measurement at the beginning and at the end to define a work distribution

$$p(w) = \sum_{i,j} \langle E_i|\rho|E_i\rangle \left|\langle E_j'|U|E_i\rangle\right|^2 \delta(w - (E_j' - E_i))$$

M. Campisi, P. Hanggi, and P. Talkner, *Rev. Mod. Phys. 83, 771* (2011).

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2) *Operator of work*: measure ρ in the eigenbasis of $U^\dagger H' U - H$

G.N. Bochkov *et al.*, Sov. Phys. JETP, 45, 125 (1977), A. Allahverdyan *et al.* Phys. Rev. E **71**, 066102 (2005)

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A. Allahverdyan, Phys. Rev. E 90, 032137 (2014), M.L. Phys. Rev. Lett. 120, 040602 (2018)

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Approach	Condition 1	Condition 2	Condition 3
TPM [4, 5, 41–46]	🙂	🙂	☹
Operator of work [33–40]	🙂	☹	🙂
Gaussian measurements [66–68]	🙂	😐	😐
Full-Counting statistics [55, 84, 85]	☹	🙂	🙂
Post-selection schemes [72, 86]	😐	🙂	😐
Weak values quasi-probability [54, 86–89]	☹	🙂	🙂
Consistent Histories [88, 90, 91]	☹	☹	🙂
Quantum Hamilton-Jacobi [71]	☹	☹	🙂
POVM depending on the initial state [92]	☹	🙂	🙂

Fluctuating work in coherent quantum systems: proposals and limitations in "Thermodynamics in the quantum regime - Recent Progress and Outlook" (**arXiv:1805.10096**) E. Bäumer, ML, M. Perarnau-Llobet, R. Sampaio

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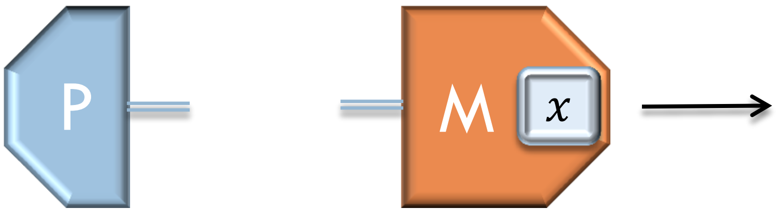
Why? A necessary condition for quantum advantages is to generate non classical statistics (not formalism dependent).

How? In communication and cryptography advantages are ultimately related to Bell inequalities violations. In thermodynamics?

Contextuality

Statistics collected in any prepare and measure scenario

Operational theory



$p(x|P,M)$

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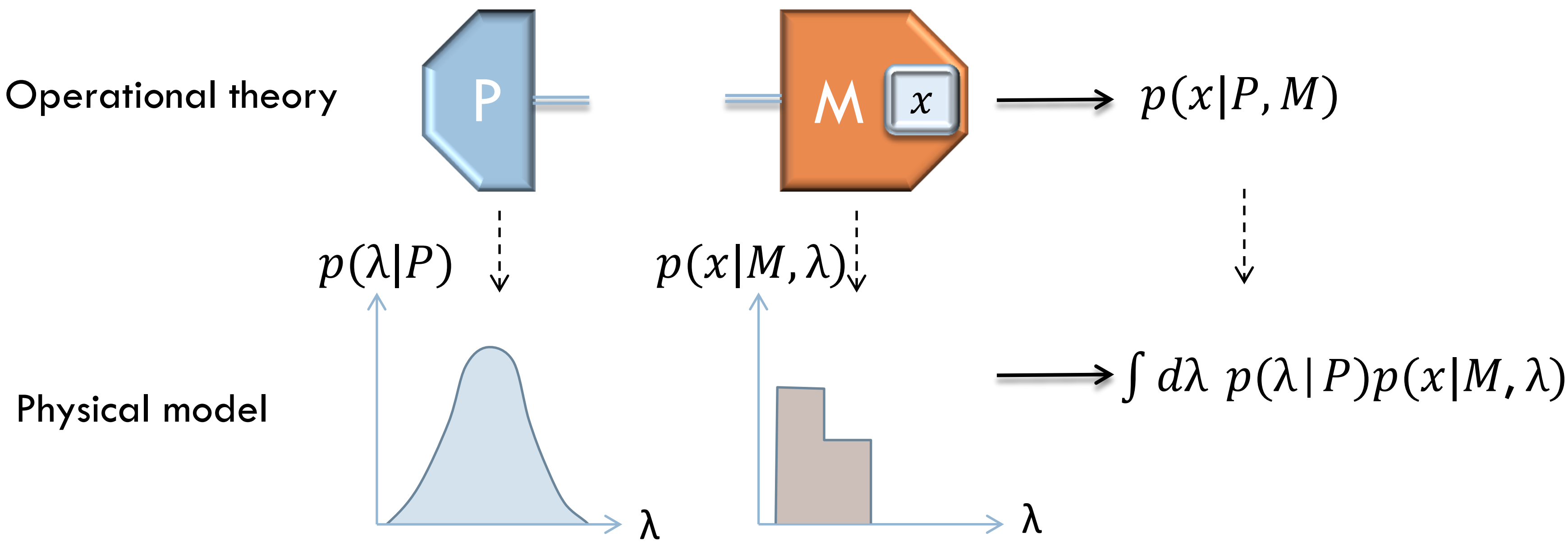

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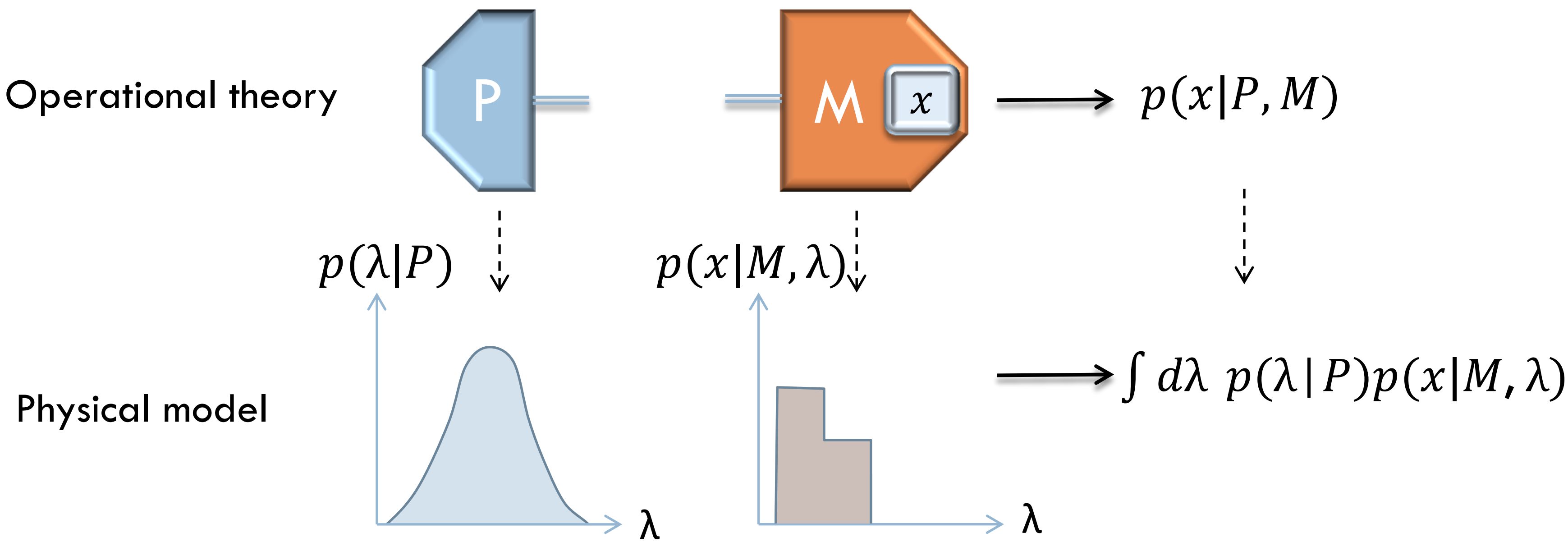

A physical model is non-contextual if

For every operationally indistinguishable P, P': $p(\lambda|P) \equiv p(\lambda|P')$

For every operationally indistinguishable M, M': $p(x|M,\lambda) \equiv p(x|M',\lambda)$

R. W. Spekkens, Phys. Rev. A 71, 052108 (2005)

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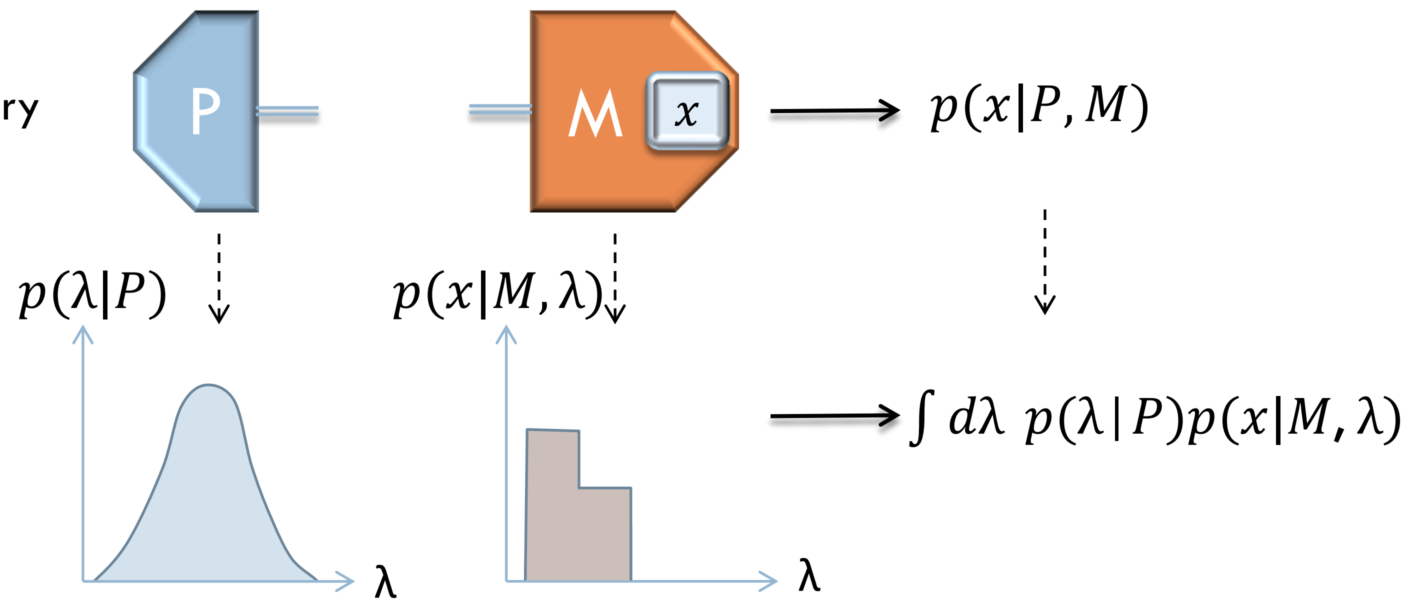

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If $p(x|P,M)$ does not admit a non-contextual model we call it non-classical

Result 1

Suppose $\{E'_j - E_i\}$ is non degenerate. For every scheme such that

1. $p(w)$ a probability distribution, linear in ρ

2. $p(w)$ recovers the TPM distribution for initially incoherent states

there exists a *non-contextual* model for preparation ρ and measurement of $p(w)$.

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a. If (2) is to be kept, negativities in $p(w)$ are ~necessary (but by no means sufficient) for the scheme to collect non-classical statistics.

b. There have been proposals of $p(w)$ with negativities, but mostly discounted.

Result 2

Consider the work quasi-probability (proposed in PRE 90, 032137)

$$p_{weak}(w) = Re\,Tr\big[\rho|E_i\rangle\langle E_i|U^\dagger|E_j'\rangle\langle E_j'|U\big]$$

accessed through a weak measurement of $|E_i\rangle\langle E_i|$ with postselection $|E_j'\rangle$.

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a. Weak measurement: prepare broad Gaussian pointer, interact through $H_{int} \propto |E_i\rangle\langle E_i|\otimes p$, measure displacement.

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b. Based on the fact that ``generalised anomalous weak values" provide proofs of contextuality (generalisation of Phys. Rev. Lett. 113, 200401).

Conclusions

1. In the absence of degeneracies:

Recover TPM for incoherent + non-classical statistics

$\Rightarrow$ negativity of the work distribution (or dependence on decomposition)

2. Negativity of weak value quasi-probability

$\Rightarrow$ non-classical statistics

<u>Future direction</u>

Prove a quantum advantage in a thermodynamic task:
based on quantum contextuality?

Thanks for listening!

[Phys. Rev. Lett. **120**, 040602]