Exploring Novel Phases in Cold Quantum Gases

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Atomic quantum gases

Bose-Einstein condensation

- Gross-Pitaevskii equation
- non-linear dynamics

Quantum degenerate *dilute* atomic gases of fermions and bosons

control and tunability

Rotating condensates

- vortices
- fractional quantum Hall

Molecules

- Feshbach resonances
- BCS-BEC crossover
- dipolar gases

Optical lattices

- quantum information
- Hubbard models
- strong correlations
- exotic phases

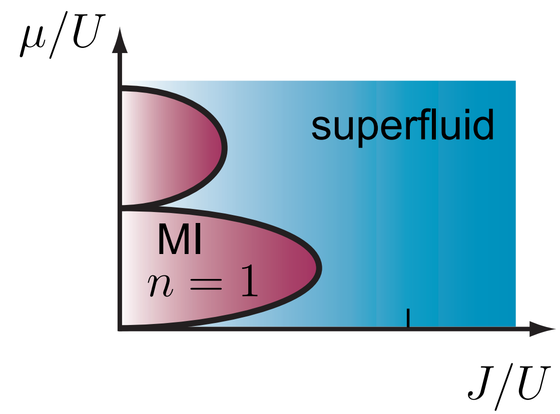
Atomic gases in an optical lattice

Preparation

- lattice loading schemes
- controlled single particle manipulations (entanglement)
- decoherence of qubits

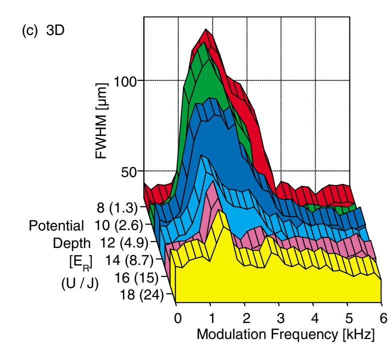
Thermodynamics

- Hubbard models
- design of Hamiltonians
- strongly correlated many-body systems

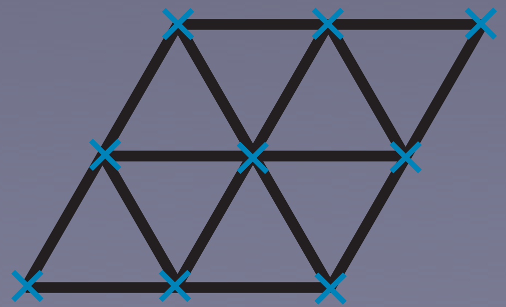


Measurement

- momentum distribution
- structure factor
- pairing gap
- ...



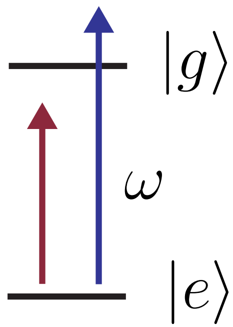
Bose-Hubbard
tool box



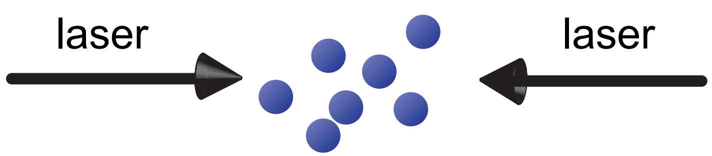
Optical lattices

- AC Stark shift

off-resonant
laser



- standing laser configuration



$$V(\mathbf{x}) = V_0 \sin^2 \mathbf{k}\,\mathbf{x} + \ldots$$

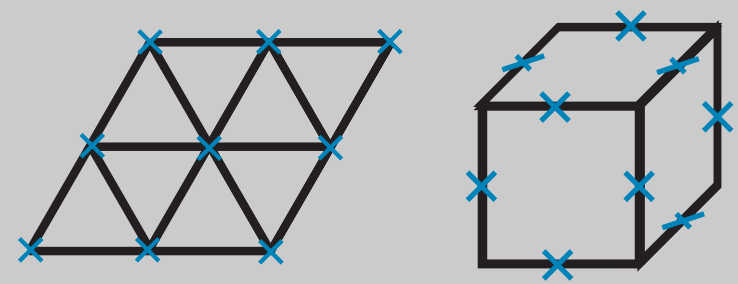
- characteristic energies

$$E_r = \frac{\hbar^2 \mathbf{k}^2}{2m} \sim 10\text{kHz}$$

$$V_0 / E_r \sim 50$$

- high stability of the
  optical lattice

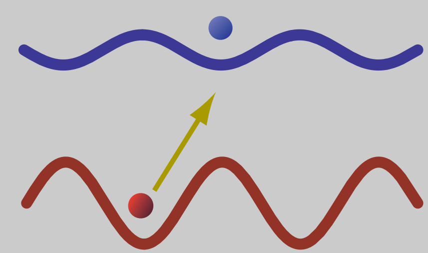
1D, 2D, and 3D Lattice
structures



Internal states

- spin dependent
  optical lattices

- alkaline earth
  atoms

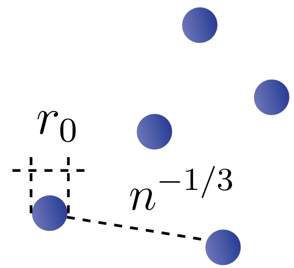


Control of interaction

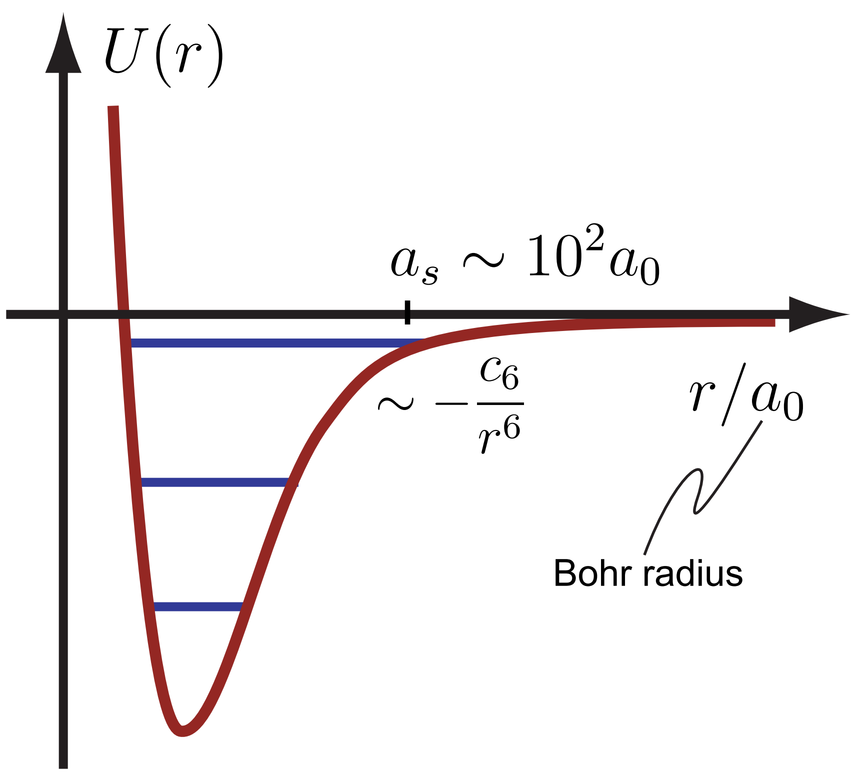
Interaction potential:

- effective range

$$r_0^3 n \ll 1$$



- pseudo-potential approximation



Scattering properties

- scattering amplitude: $f(k) = -\frac{1}{1/a_s + ik}$

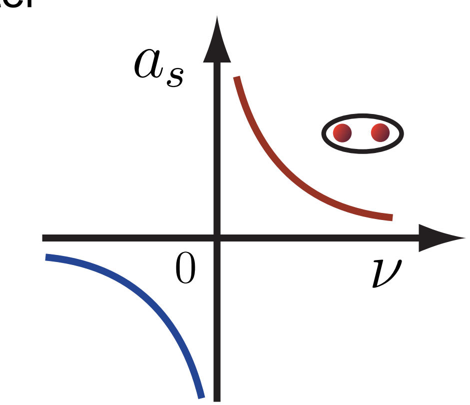
- bound state energy $a_s > 0$: $E_{\mathrm{M}} = -\frac{\hbar^2}{m a_s^2}$

Tuning of scattering length

- changing the first "bound state" energy via an external parameter

  - magnetic Feshbach resonance

  - optical Feshbach resonance



Microscopic Hamiltonian

$$H = \int dx\, \psi^+(x) \left(-\frac{\hbar^2}{2m}\Delta + V(x) \right) \psi(x) + \frac{g}{2} \int dx\, \psi^+(x)\psi^+(x)\psi(x)\psi(x)$$

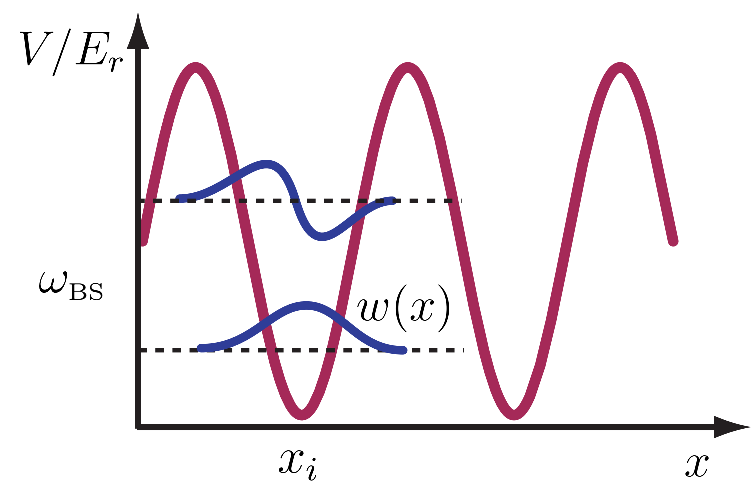
optical lattice

$$g = \frac{4\pi\hbar^2 a_s}{m}$$: interaction strength

- strong opitcal lattice $V > E_{\mathrm{r}}$
- express the bosonic field operator in terms of Wannier functions
- restriction to lowest Bloch band

(Jaksch et al PRL ‘98)

$$\psi(\mathbf{x}) = \sum_i w(\mathbf{x} - \mathbf{x}_i) b_i$$



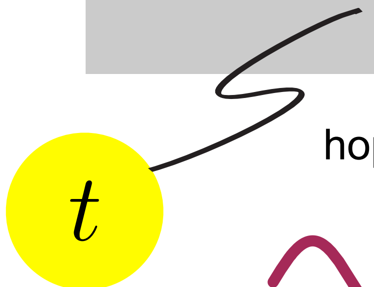
Bose-Hubbard Model

Bose-Hubbard model (Fisher et al PRB ‘81)

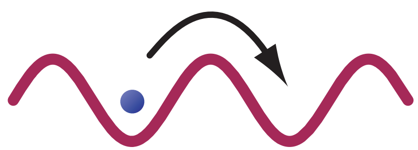
$$H_{\mathrm{BH}} = -J \sum_{\langle i,j \rangle} b_i^+ b_j + U/2 \sum_i b_i^+ b_i^+ b_i b_i$$

$$U \sim E_{\mathrm{r}} a_s / \lambda$$

$$J \sim E_{\mathrm{r}} e^{-2\sqrt{V/E_{\mathrm{r}}}}$$



hopping energy

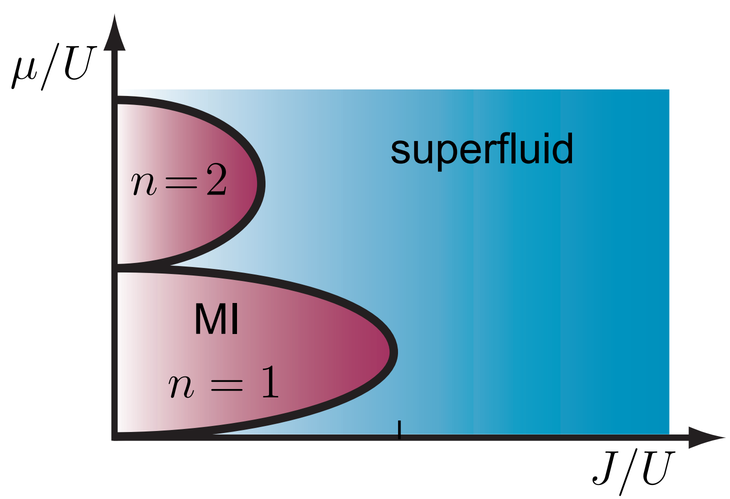


interaction energy

Phase diagram

Mott insulator

- fixed particle number
- incompressible
- excitation gap

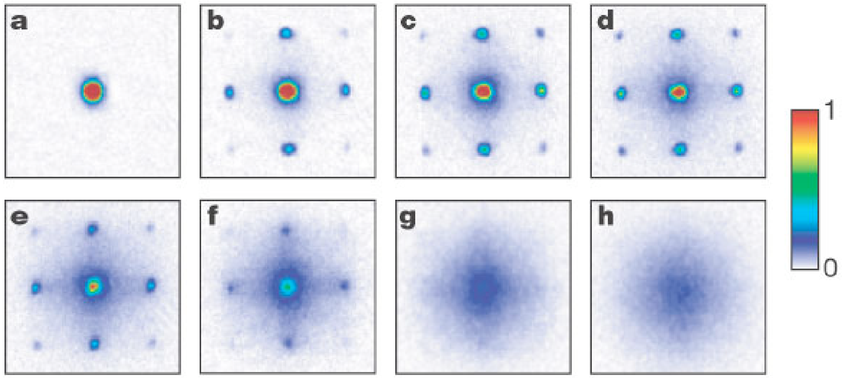


Superfluid

- long-range order
- finite superfluid stiffness
- linear excitation spectrum

Experiments

Long-range order:

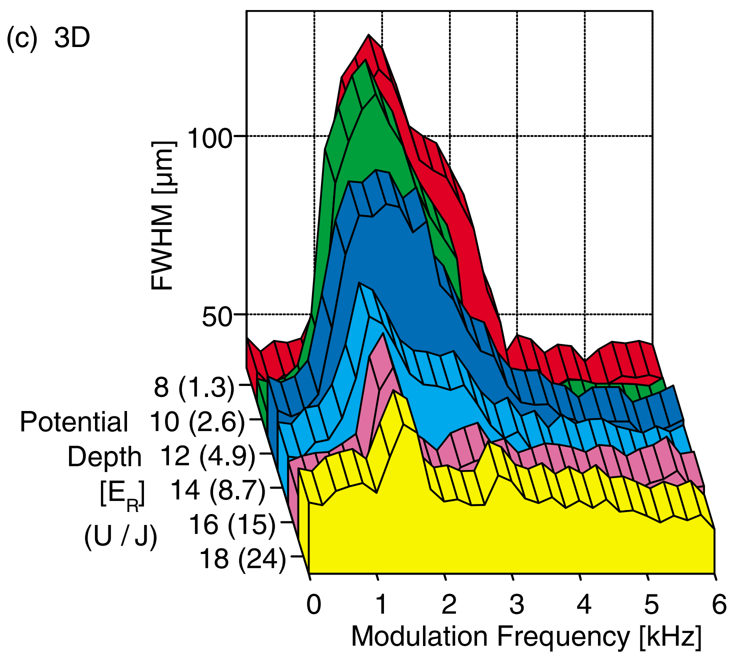


(Greiner et al., 02)

Disappearance of coherence for strong optical lattices (Greiner et al. '02)

$$\frac{V}{E_r} > 13$$

Structure factor

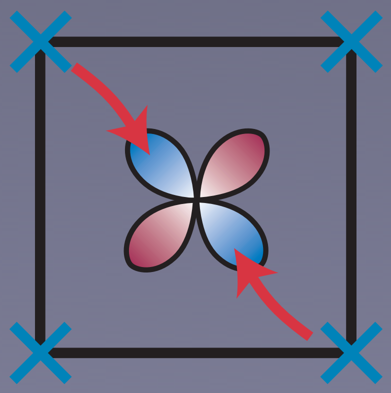


(Esslinger et al., 04)

Appearance of well defined two particle excitations

Ring exchange and Exotic phases in cold gases

(H.P. Büchler, M. Hermele, S.D. Huber, M.P.A Fisher, and P. Zoller, PRL '05)



Exotic phases

What is an exotic phase?

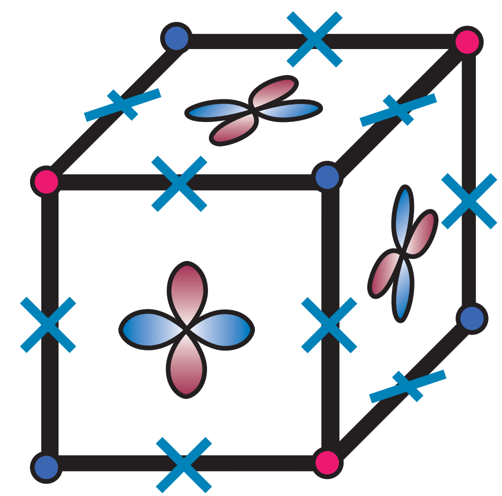
- the low energy properties of the system are dominated by a symmetry not present in the microscopic model

⇨ emergent symmetry

- local conserved quantity, local gauge symmetries

- fractional excitations

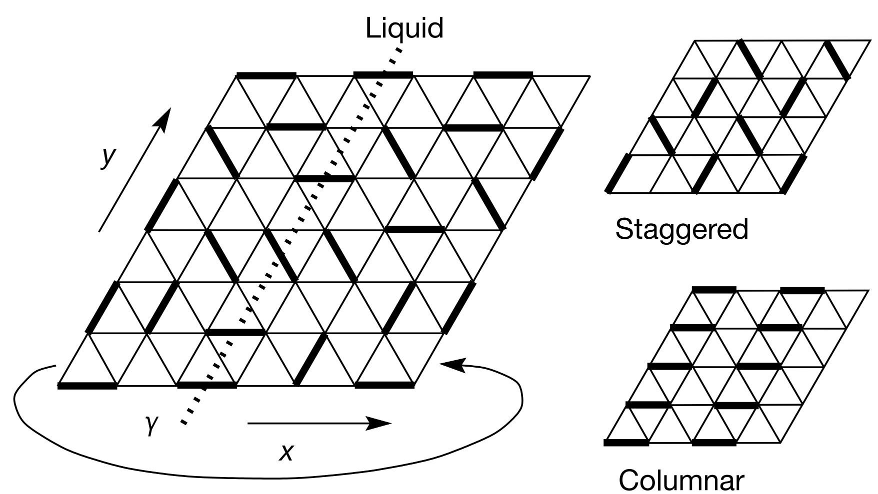
- spin-liquid, resonating valence bond (RVB), Coulomb phase



Applications

- topological protected quantum memory and quantum computing
  (Kitaev et al, '97, Ioffe et al., Nature '02)

- how often do they appear in nature?

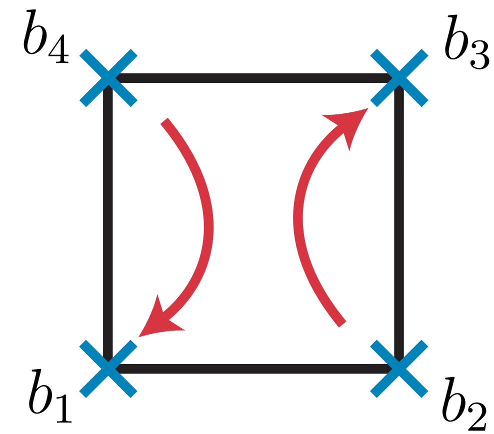


Ring exchange

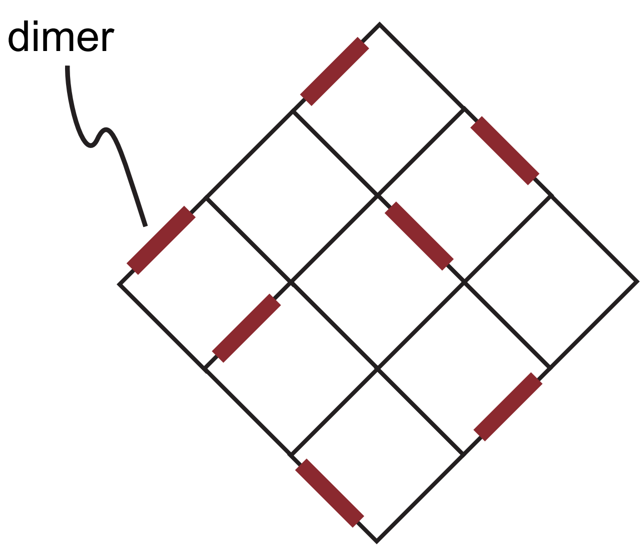
Ring exchange

- bosons on a lattice

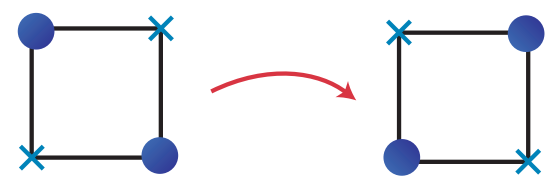
$$H_{\mathrm{R-E}} = K\left[b_1^{+}b_2b_3^{+}b_4 + b_1b_2^{+}b_3b_4^{+}\right]$$



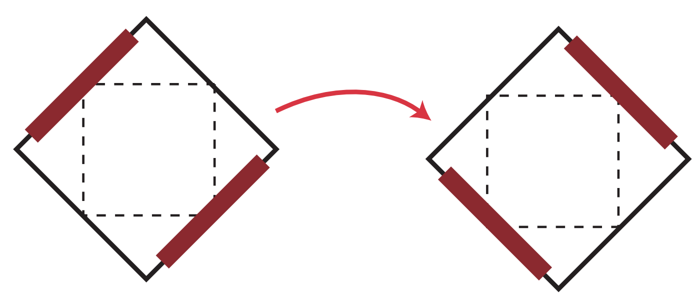
Relation to dimer models



- ring exchange energy



- kinetic energy in the dimer model

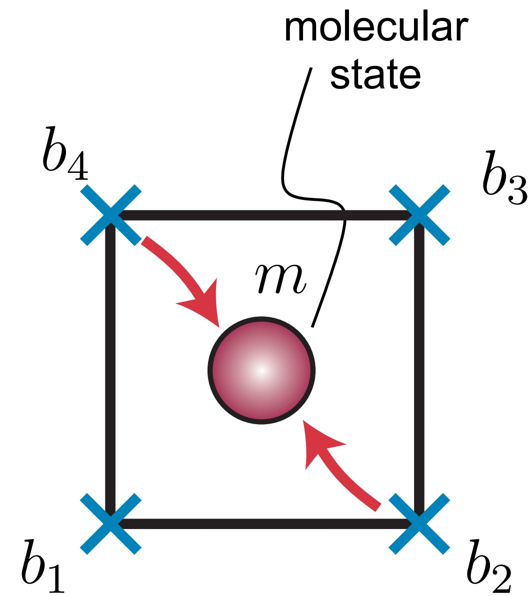


Scattering Resonance

Geometric scattering resonance

- bosons in an optical lattice
- coupling to an artificial ‘molecular’ state
  (analogy of Feshbach resonance)
- tuning the energy of the ‘molecule’

⇒ scattering resonance with large interactions



Effective coupling Hamilton

detuning — coupling (Rabi frequency) — symmetry of the molecule

$$H = \nu\, m^{+} m + g \sum_{i \neq j} c_{ij} \left[m^{+}\, b_i b_j + m\, b_i^{+} b_j^{+} \right]$$

Scattering Resonance

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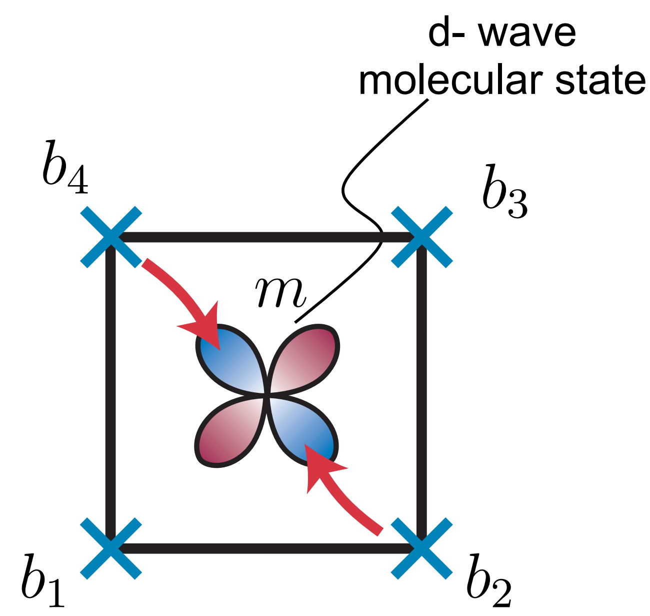


Effective coupling Hamilton

detuning

coupling
(Rabi frequency)

symmetry of
the molecule

$$H = \nu\, m^+ m + g \sum_{i \neq j} c_{ij} \left[m^+\, b_i b_j + m\, b_i^+ b_j^+ \right]$$

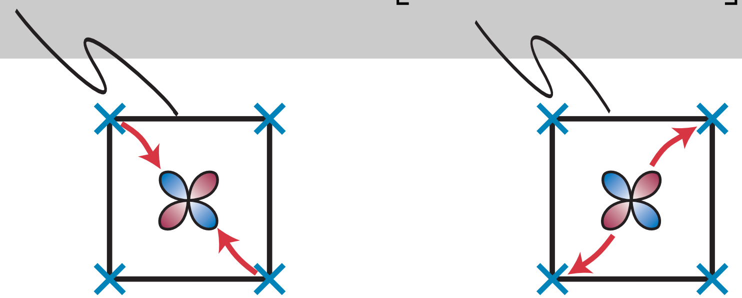
d-wave
symmetry

⇒

$$m^+ \left[b_1 b_3 - b_2 b_4 \right] + c.c.$$

Ring exchange

$$H = \nu\, m^{+} m + g m^{+}\left[b_1 b_3 - b_2 b_4\right] + g m \left[b_1^{+} b_3^{+} - b_2^{+} b_4^{+}\right]$$

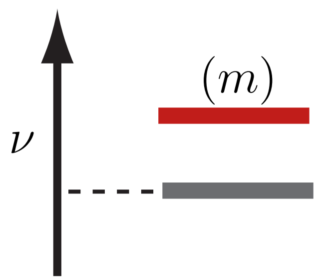


Relation to Ring exchange

- integrating out the molecule

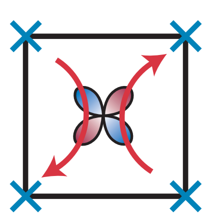
$\nu \gg g$

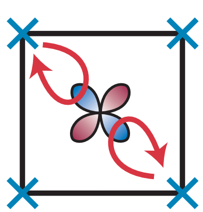
- molecule only virtually populated



$$H = K\left[b_1^{+} b_2 b_3^{+} b_4 + b_1 b_2^{+} b_3 b_4^{+} - n_1 n_3 - n_2 n_4\right]$$

$$K = \frac{g^2}{\nu}$$

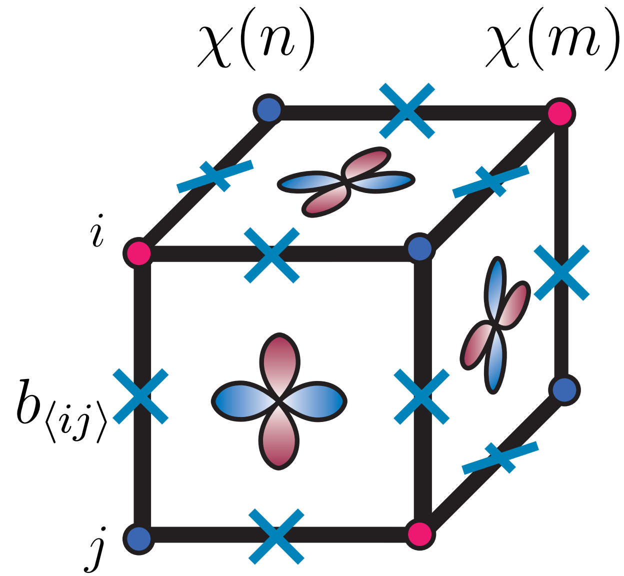




Lattice gauge theory

3D setup

- atoms on links of a 3D cubic lattice
- molecular site in the center of each face
- half-filling: 3/2 bosons on each cube

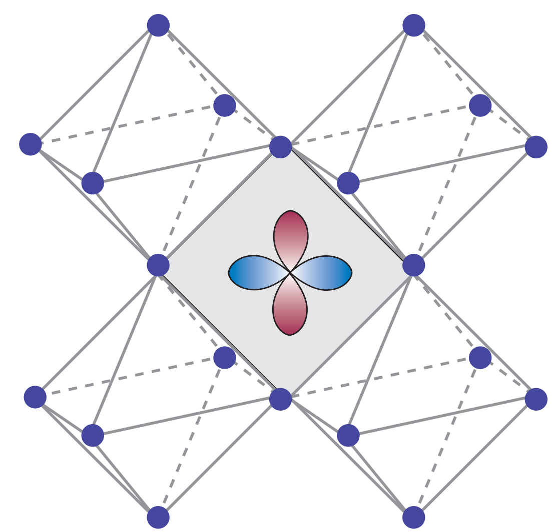


3D lattice gauge theory

- lattice of corner-sharing octahedra
- ring exchange leaves the number of bosons on each octahedra invariant
- gauge transformation

n red corner
m blue corner

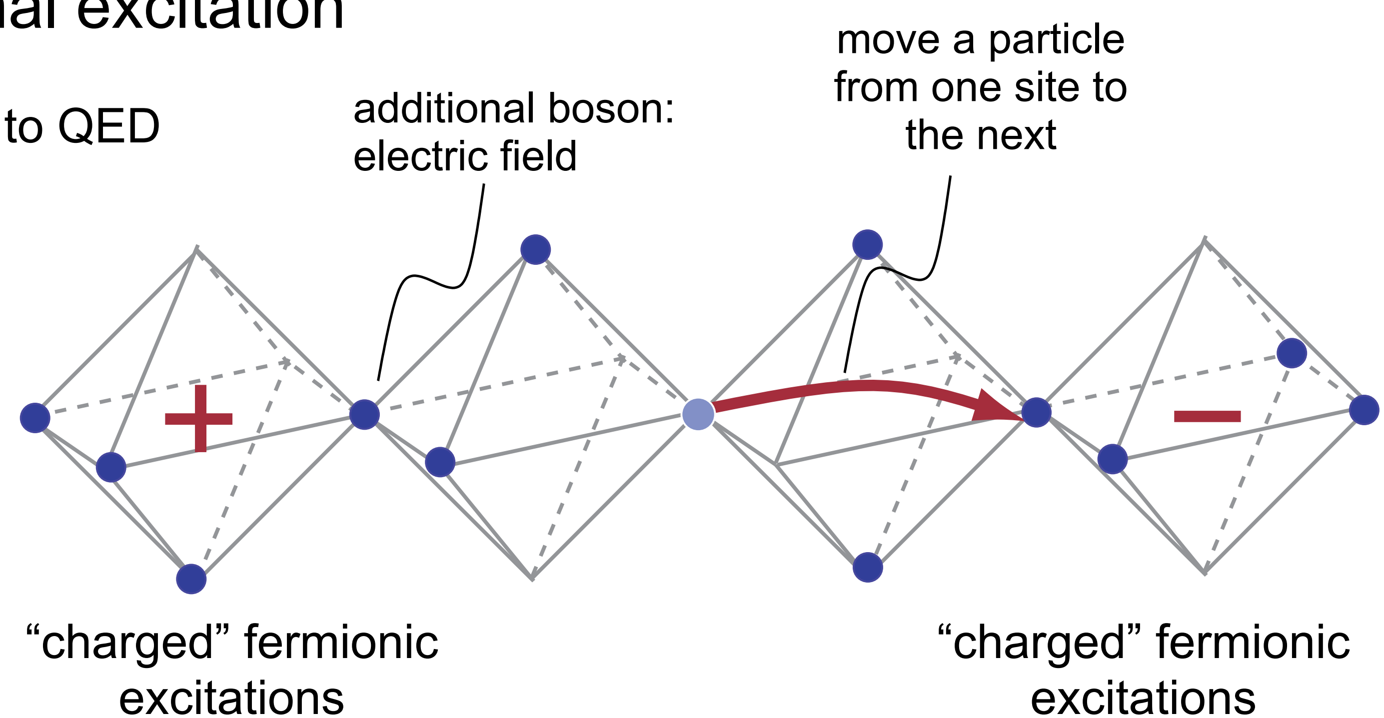
$$b_{\langle nm \rangle} \longrightarrow b_{\langle nm \rangle} e^{i[\chi(n) - \chi(m)]}$$



Lattice gauge theory

Fractional excitation

- mapping to QED



Confined phase

- all excitations are gapped
- electric charges are linear confined

Quantum phase transition

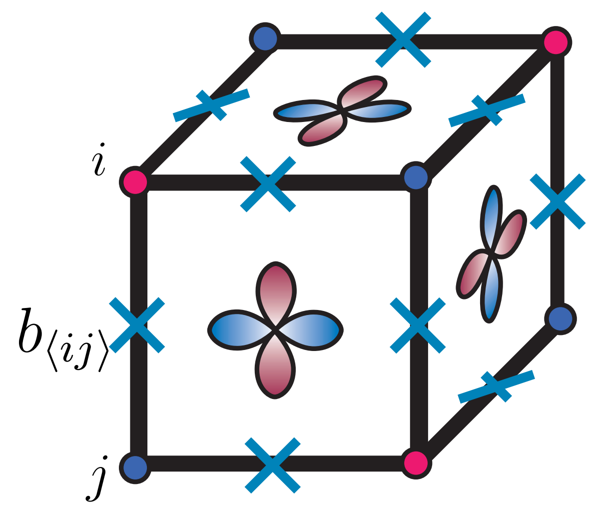
Coulomb phase

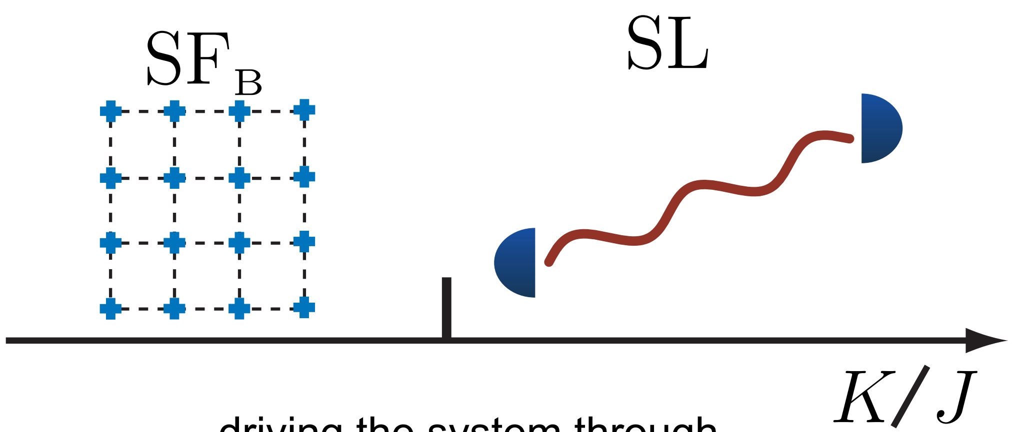
- spin liquid phase
- electric charges interact via Coulomb potential
- two artificial light modes

Phase diagram

Superfluid phase

- broken U(1) symmetry
- finite superfluid stiffness





driving the system through
the scattering resonance

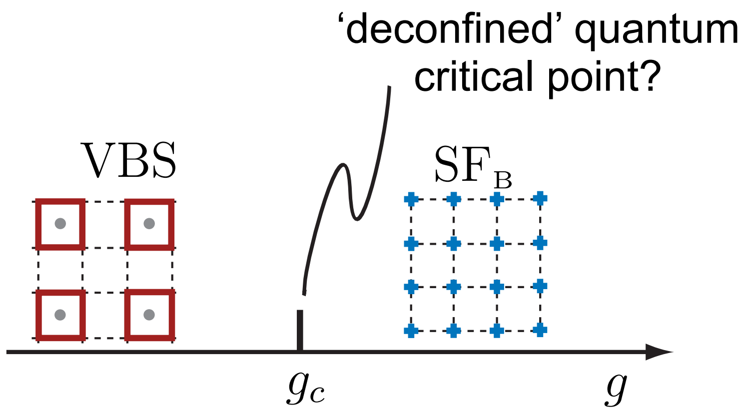
Exotic phase

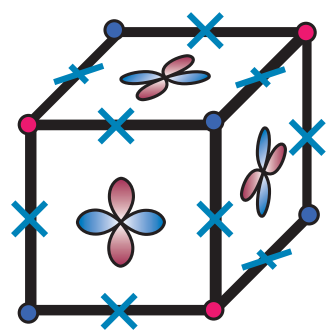
- Coulomb phase of the U(1) lattice gauge theory
- no broken symmetries
- artificial QED: linear light mode
- fractional excitations with Coulomb interaction

Conclusion

Quantum Simulator

- study of systems with ring exchange interaction
- sign problem in Monte-Carlo simulations





Design of Novel Materials

- realization of spin liquids
- artificial QED
- topological order

Spectroscopic Measurement

- measurement of correlation in bosonic systmes
- detection of non-conventional order parameters