Kavli Institute of Theoretical Physics, UC Santa Barbara, 5 April 2006

Ultra long-distance interaction between spin qubits

mm~cm

> 100 nm

cond-mat/0603119

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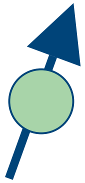
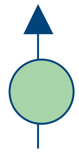
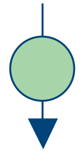
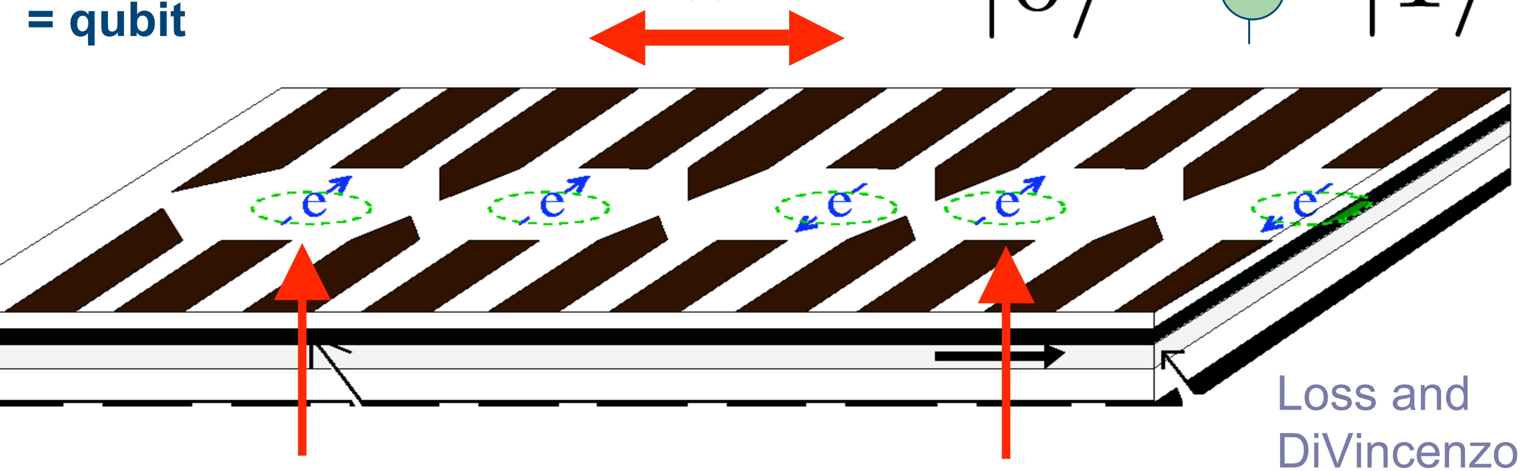
collaboration with

Atac Imamoglu *(ETH Zürich)*

Spin-based qubits

 = qubit

$|0\rangle$ =  $|1\rangle$ = 



Loss and DiVincenzo 1998

how to couple these two qubits?

$$H = \sum_{\langle i,j\rangle} J_{ij}(t)\mathbf{S}_i \cdot \mathbf{S}_j + \sum_i g_i(t)\mu_B \mathbf{B}_i(t) \cdot \mathbf{S}_i$$

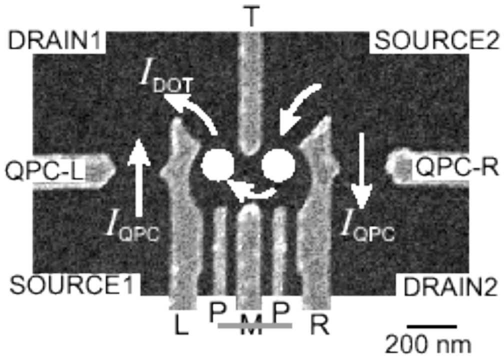
exchange coupling
local two-qubit gates

Zeeman term, g-factor
one-qubit gates

1. SWAP fault-tolerant?
2. teleport (using previously prepared entanglement) FT √
3. long-distance interaction FT √

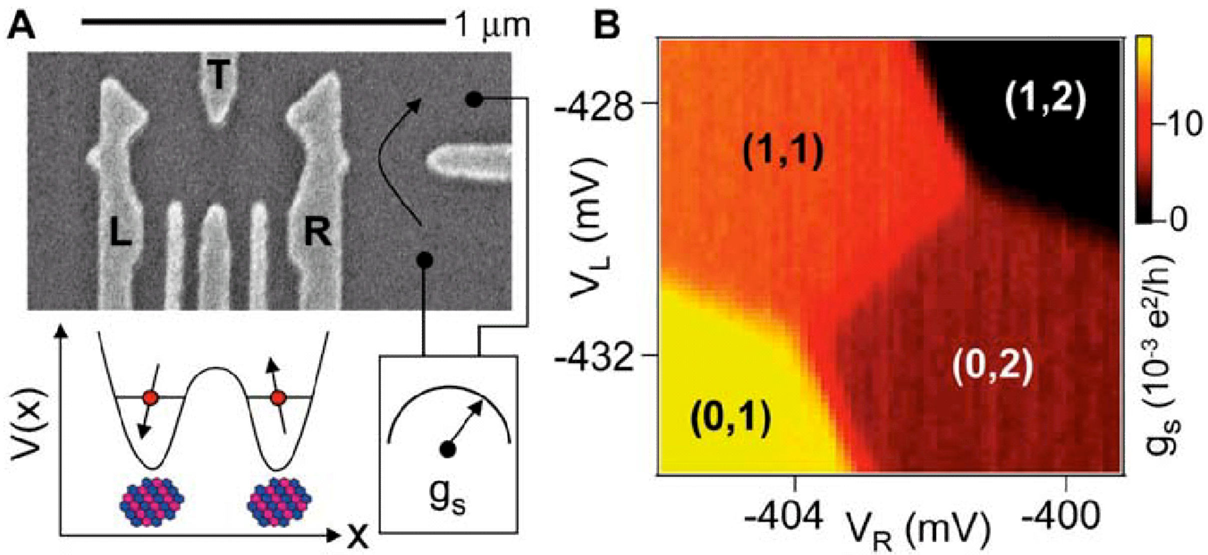
Double quantum dot two-spin qubits

Elzerman *et al.*,
Phys. Rev. B (2003).



⊗ B

Petta *et al.*, Science **309**, 2180 (2005).



- charge detection

- spin echo: T_2 = 1-10 μs
- $\sqrt{SWAP}$ in 180 ps

Levy, Phys. Rev. Lett. **89**, 147902 (2002).
Taylor *et al.*, Nature Phys. **1**, 177 (2005).

$$|S\rangle = (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)/\sqrt{2} = |0\rangle$$
$$|T_0\rangle = (|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle)/\sqrt{2} = |1\rangle$$

Circuit Quantum Electrodynamics

Strong coupling of a single photon to a superconducting qubit using circuit quantum electrodynamics

A. Wallraff[1], D. I. Schuster[1], A. Blais[1], L. Frunzio[1], R.- S. Huang[1,2], J. Majer[1], S. Kumar[1], S. M. Girvin[1] & R. J. Schoelkopf[1]

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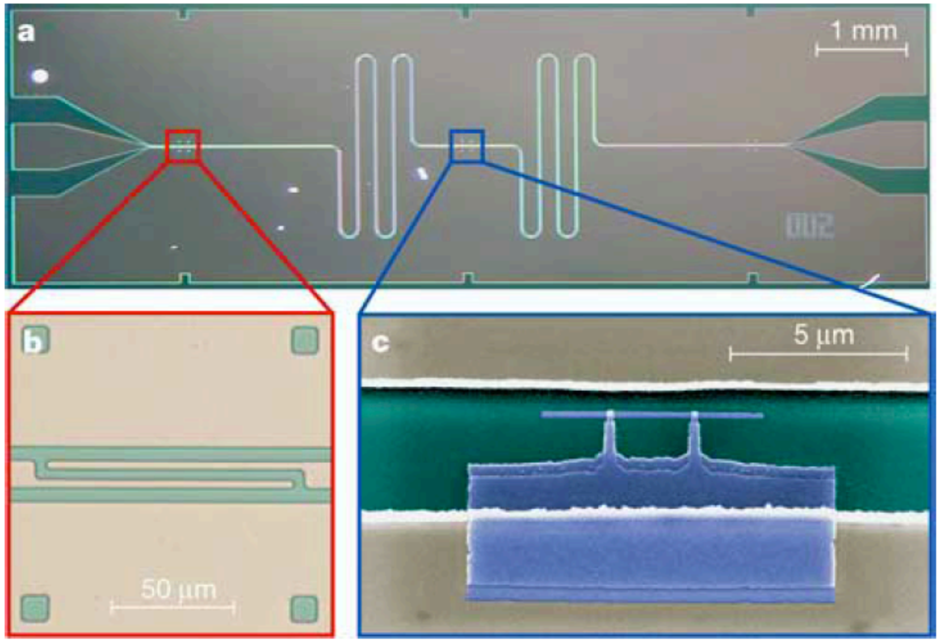
[2]*Department of Physics, Indiana University, Bloomington, Indiana 47405, USA*



$$
\begin{aligned}
\nu_r &= (2\pi)^{-1}/\sqrt{LC} \approx 6\,\mathrm{MHz} \\
\nu_{\mathrm{Rabi}} &\equiv g/\pi = 2dE_0/h \approx 10\,\mathrm{MHz} \\
\kappa/2\pi &= \nu_r/Q \approx 0.8\,\mathrm{MHz} \\
Q &\approx 10^4 \\
\gamma/2\pi &\equiv T_2^{-1} \approx 0.7\,\mathrm{MHz} \\
g &> \kappa, \gamma \quad \text{(strong coupling)}
\end{aligned}
$$

$$
\begin{aligned}
H &= h\nu_r \left(a^\dagger a + \frac{1}{2} \right) \\
&\quad + \frac{\epsilon}{2}\sigma_z + \frac{\Delta}{2}\sigma_x \\
&\quad + \hbar g \left(a\sigma_+ + a^\dagger \sigma + - \right)
\end{aligned}
$$

	optics	circuit
d/e	0.1 Å (alkali atoms) 10 nm (Rydberg)	$\sim 100\,\mathrm{nm}$
E_0	$0.002\,\mathrm{V/m}$	$0.2\,\mathrm{V/m}$

Cavity setup and result

$$H = \sum_{i=1,2}\left(\frac{\bar{\varepsilon}_i}{2}\sigma_z^i + g_i\sigma_x^i(a+a^\dagger) + h\nu_r\left(a^\dagger a + \frac{1}{2}\right)\right)$$

Schrieffer-Wolff transformation
(eliminate virtual photons)

$$H_{\text{eff}} = \sum_{i=1,2}\frac{\tilde{\varepsilon}_i}{2}\sigma_z^{(i)} + g_{\text{eff}}\left(\sigma_+^1\sigma_-^2 + \sigma_-^1\sigma_+^2\right)$$

CNOT gate
Imamoglu *et al.* 1999

$$g_{\text{eff}} = g_1 g_2\left(\frac{1}{\bar{\varepsilon}_1 - h\nu_r} + \frac{1}{\bar{\varepsilon}_2 - h\nu_r}\right)$$

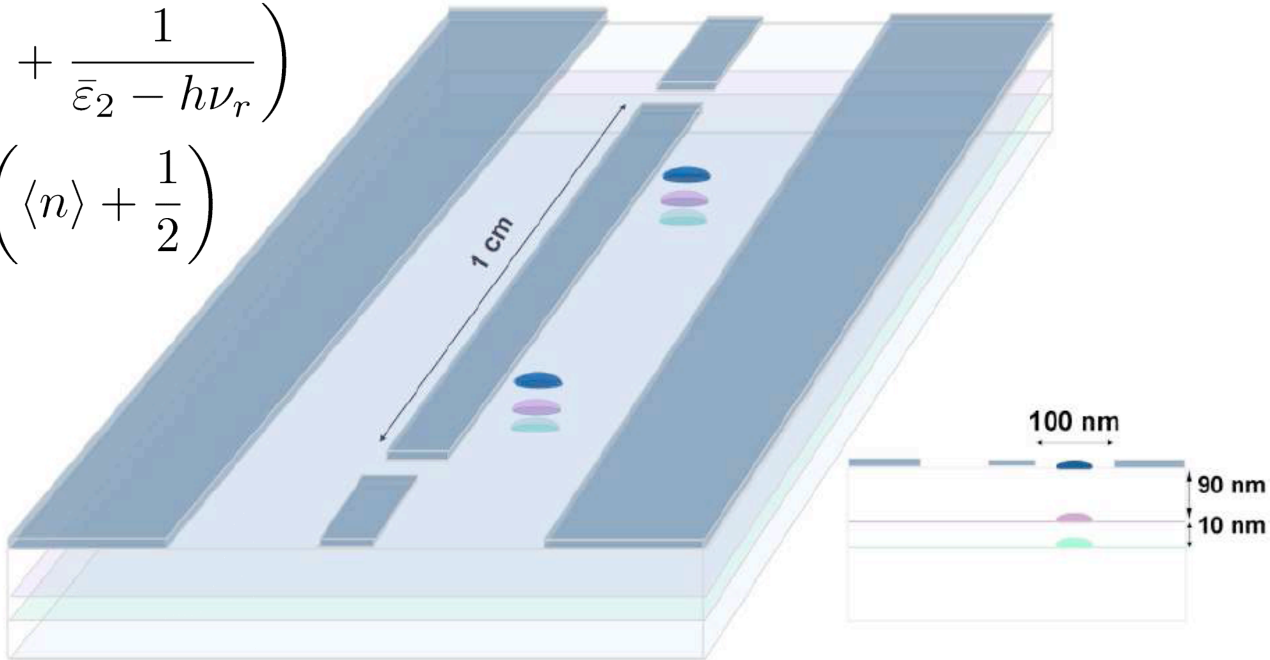
$$\frac{\tilde{\varepsilon}_i}{2} = \frac{\bar{\varepsilon}_i}{2} + \frac{g_i^2}{\bar{\varepsilon}_i - h\nu_r}\left(\langle n\rangle + \frac{1}{2}\right)$$

$$\langle n\rangle = \langle a^\dagger a\rangle < 0.06$$

$$T \sim 100\,\text{mK}$$

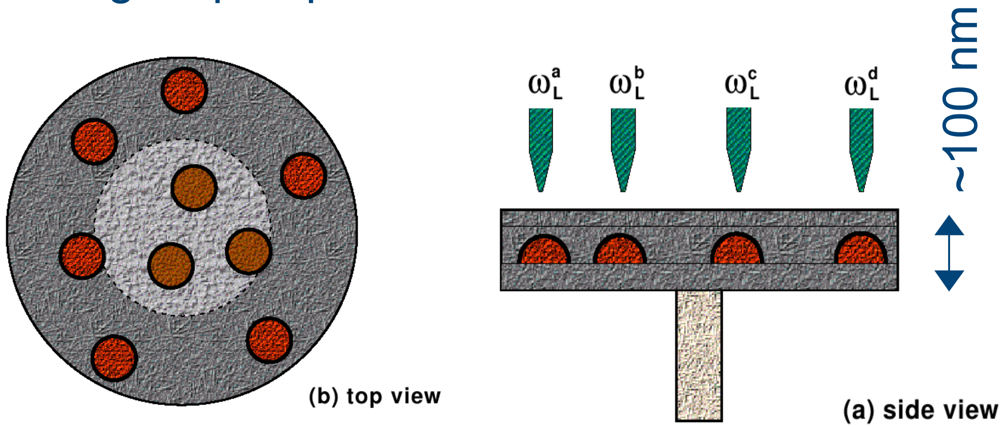
$$h\nu_r/k_B \sim 300\,\text{mK}$$

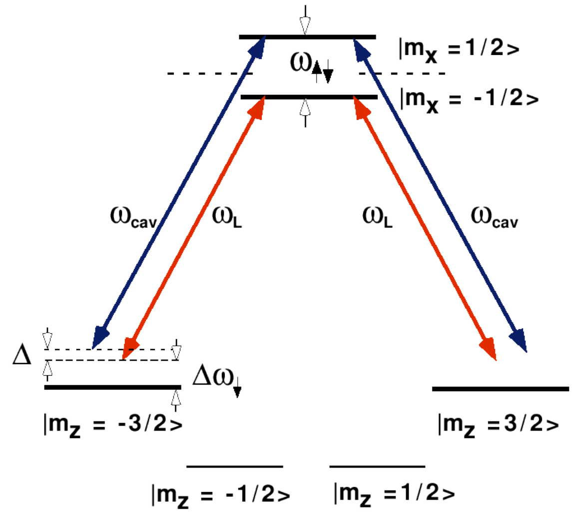
$$g_i \approx 60\,\text{MHz}$$



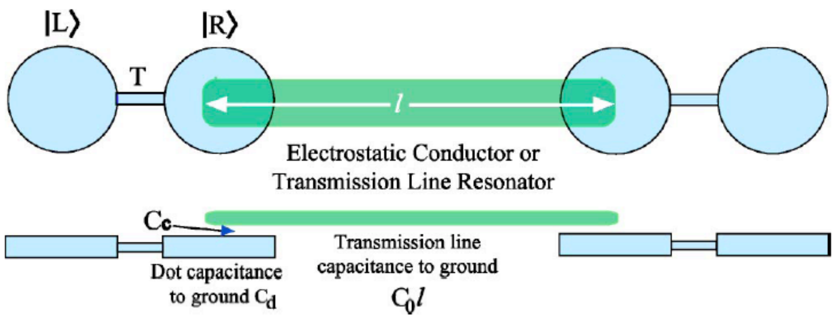
Related work

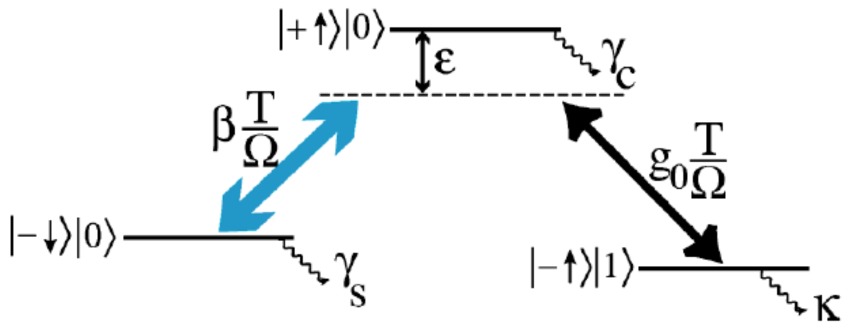
Imamoglu, Awschalom, Burkard, DiVincenzo, Loss, Sherwin, Small, PRL (1999):
use optical semiconductor cavity to generate two-photon Raman transitions for single-spin qubits





Childress, Sorensen, and Lukin, Phys. Rev. A (2004):
use opt. cavity + ESR to generate two-photon Raman transitions for single-spin qubits





How can this work?

1. **Spin conservation**: forbids **electric** dipole transition

   inhomogeneous magnetic field breaks spin conservation

   Overhauser field due to nuclear spins (even unpolarized) provides inhomogeneous magnetic field

$$g \propto g\mu_B(B_1 - B_2) = \delta h$$

2. **Orbital symmetry**

   for symmetric double quantum dots (ε=0), the singlet state at finite tunneling is a mixture of (1,1)S and (2,0)S + (0,2)S, while (2,0)S - (0,2)S not present

   electric dipole operator is odd under parity operation **P**

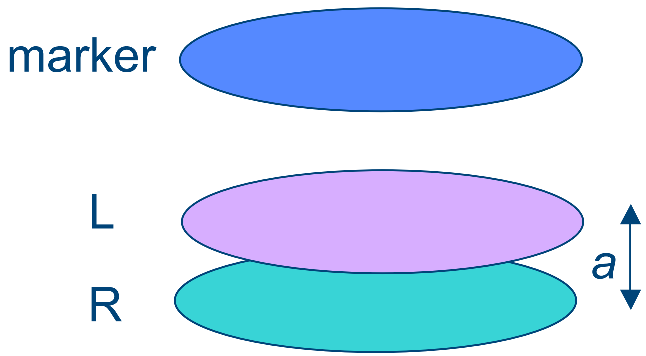
   the non-vanishing dipole transition is between (2,0)S - (0,2)S and (1,1)S, but at ε=0, the state (2,0)S - (0,2)S is completely decoupled from all other states

   Apply bias $\varepsilon \neq 0$ to allow for electric dipole transitions.

$$g \propto \varepsilon$$

Double quantum dot: model

$$H_{\mathrm{el}} = H_D + H_T + H_{\mathrm{int}}$$



quantum dots (L,R):

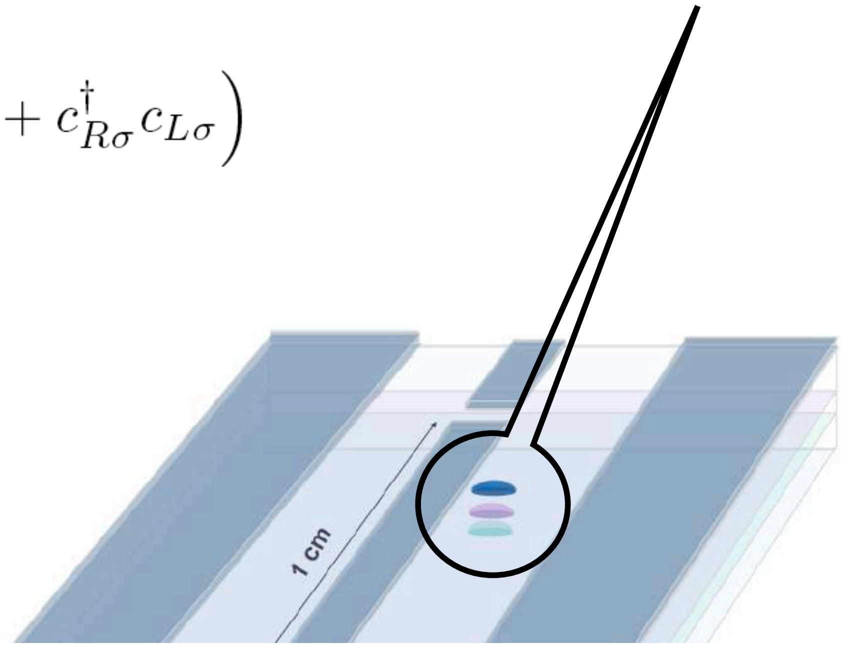
$$H_D = \sum_{\substack{\alpha=L,R \\ \sigma,\sigma'=\uparrow,\downarrow}} c^{\dagger}_{\alpha\sigma'} \left(\varepsilon_\alpha + \frac{\hbar}{2} g \mu_B \mathbf{B}_\alpha \cdot \boldsymbol{\sigma}_{\sigma'\sigma} \right) c_{\alpha\sigma}$$

tunneling between dots:

$$H_T = t \sum_{\sigma=\uparrow,\downarrow} \left(c^{\dagger}_{L\sigma} c_{R\sigma} + c^{\dagger}_{R\sigma} c_{L\sigma} \right)$$

Coulomb repulsion on each dot:

$$H_{\mathrm{int}} = U \sum_{\alpha=L,R} c^{\dagger}_{\alpha\uparrow} c_{\alpha\uparrow} c^{\dagger}_{\alpha\downarrow} c_{\alpha\downarrow}$$



The two-electron double quantum dot

$$|T_0\rangle = \frac{1}{\sqrt{2}}\left(c^\dagger_{L\uparrow}c^\dagger_{R\downarrow} + c^\dagger_{L\downarrow}c^\dagger_{R\uparrow}\right)|0\rangle$$

$$|T_\sigma\rangle = c^\dagger_{L\sigma}c^\dagger_{R\sigma}|0\rangle$$

$$|S\rangle \equiv |S(1,1)\rangle = \frac{1}{\sqrt{2}}\left(c^\dagger_{L\uparrow}c^\dagger_{R\downarrow} - c^\dagger_{L\downarrow}c^\dagger_{R\uparrow}\right)|0\rangle$$

$$|D_\pm\rangle = \frac{1}{\sqrt{2}}\left(c^\dagger_{L\uparrow}c^\dagger_{L\downarrow} \pm c^\dagger_{R\uparrow}c^\dagger_{R\downarrow}\right)|0\rangle$$

S(2,0) S(0,2)

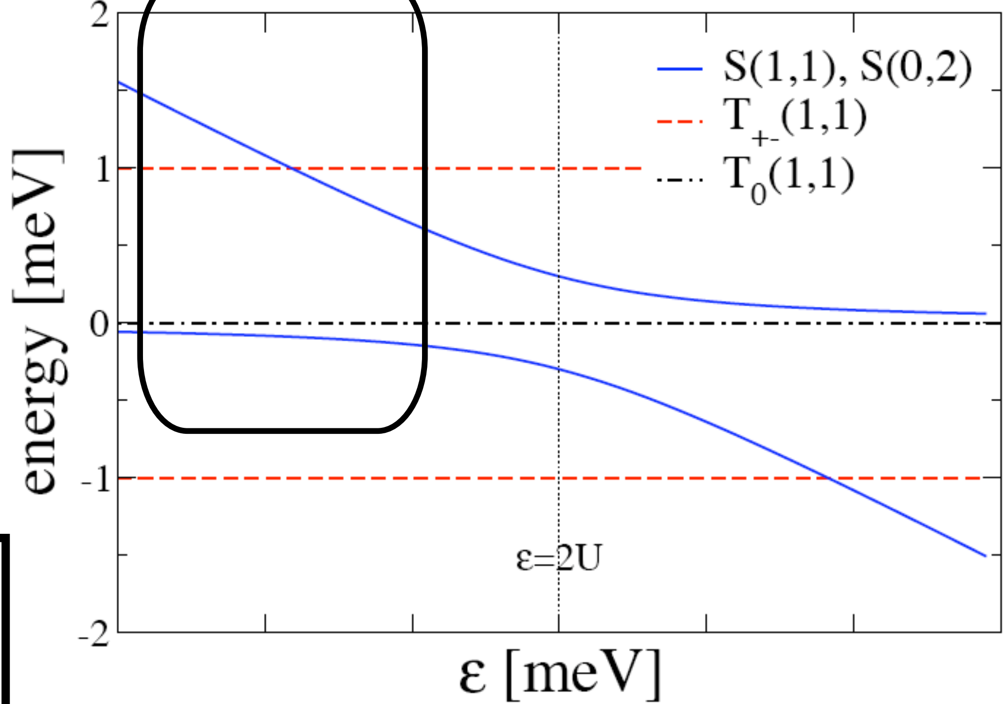
Schrieffer-Wolff eliminate $|D_\pm>$

$$H = \begin{pmatrix} 0 & \delta h/2 & 0 & 0 \\ \delta h/2 & 0 & 2t & 0 \\ 0 & 2t & U & \varepsilon/2 \\ 0 & 0 & \varepsilon/2 & U \end{pmatrix}$$

magnetic field gradient

energy bias

on-site Coulomb energy



Elimination of doubly occupied states

$$H = \begin{pmatrix} 0 & \delta h/2 & 0 & 0 \\ \delta h/2 & 0 & 2t & 0 \\ 0 & 2t & U & \varepsilon/2 \\ 0 & 0 & \varepsilon/2 & U \end{pmatrix}$$

Schrieffer-Wolff

$$\tilde{H} = e^{-S} H e^{S} \simeq H_0 + [H_T, S]/2 \qquad [H_0, S] = -H_T$$

$$S = \frac{4t}{(4U^2-\varepsilon^2)^2 - 2\delta h^2(4U^2+\varepsilon^2)} \begin{pmatrix} 0 & s \\ -s & 0 \end{pmatrix}$$

$$s = \begin{pmatrix} \delta h(4U^2+\varepsilon^2-\delta h^2) & -4\delta h U\varepsilon \\ 2U(4U^2-\varepsilon^2-\delta h^2) & -\varepsilon(4U^2-\varepsilon^2+\delta h^2) \end{pmatrix}$$

$$\varepsilon = \delta h = 0 \qquad J = 4t^2/U$$

$$\tilde{H} \simeq \begin{pmatrix} \tilde{H}_S & 0 \\ 0 & \tilde{H}_D \end{pmatrix} \qquad J = \frac{16t^2U(4U^2-\varepsilon^2-\delta h^2)}{(4U^2-\varepsilon^2)^2 - 2\delta h^2(4U^2+\varepsilon^2) + \delta h^4}$$

$$\tilde{H}_S \simeq \begin{pmatrix} 0 & \delta\tilde{h}/2 \\ \delta\tilde{h}/2 & -J \end{pmatrix}$$

effective singlet-triplet qubit Hamiltonian

$$\delta\tilde{h} = \delta h\left(1 - \frac{J(4U^2+\varepsilon^2-\delta h^2)}{4U(4U^2-\varepsilon^2-\delta h^2)}\right)$$

Dipole matrix element

dipole Hamiltonian

$$H_{\rm dip} = -\frac{e}{m}\mathbf{A}_0\cdot\mathbf{p} = -\frac{e}{m}\left(\frac{\hbar}{2\epsilon_0\epsilon V\omega}\right)^{1/2}\boldsymbol{\epsilon}\cdot\mathbf{p}\,(a+a^\dagger)$$

determine dipole matrix element

$$\tilde{H}_S \simeq \begin{pmatrix} 0 & \delta\tilde{h}/2 \\ \delta\tilde{h}/2 & -J \end{pmatrix}$$

$$g = -\frac{e}{m}\left(\frac{\hbar}{2\varepsilon_0\varepsilon V\omega}\right)^{1/2}\langle\bar{T}_0|\boldsymbol{\epsilon}\cdot\mathbf{p}|\bar{S}\rangle$$

eigenstates of $\tilde{H}_S$

Schrieffer-Wolff transformation

$$\tilde{H}_{\rm dip} \simeq H_{\rm dip} + [H_{\rm dip}, S]$$

orbital symmetry: g=0 for ε=0

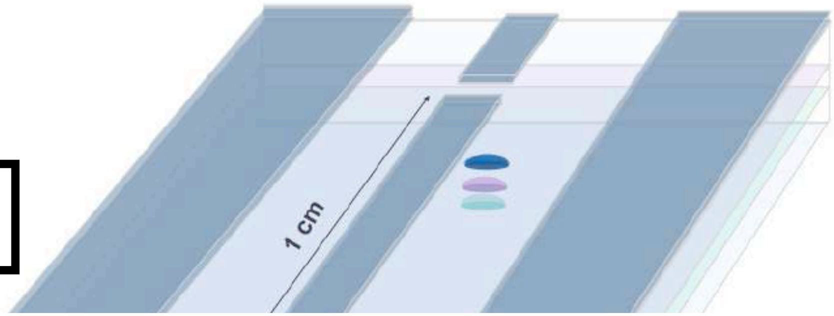
result

spin conservation: g=0 for δh=0

$$g = eaE_0\,\frac{J}{\hbar\omega}\,\frac{\varepsilon\,\delta h}{4U^2-\varepsilon^2-\delta h^2}$$

d=ea

cavity frequency: $\omega = 2\pi\,\nu_r$



Conclusions & open questions

$$H_{\text{eff}} = \sum_{i=1,2} \frac{\tilde{\varepsilon}_i}{2} \sigma_z^{(i)} + g_{\text{eff}} \left(\sigma_+^1 \sigma_-^2 + \sigma_-^1 \sigma_+^2 \right)$$

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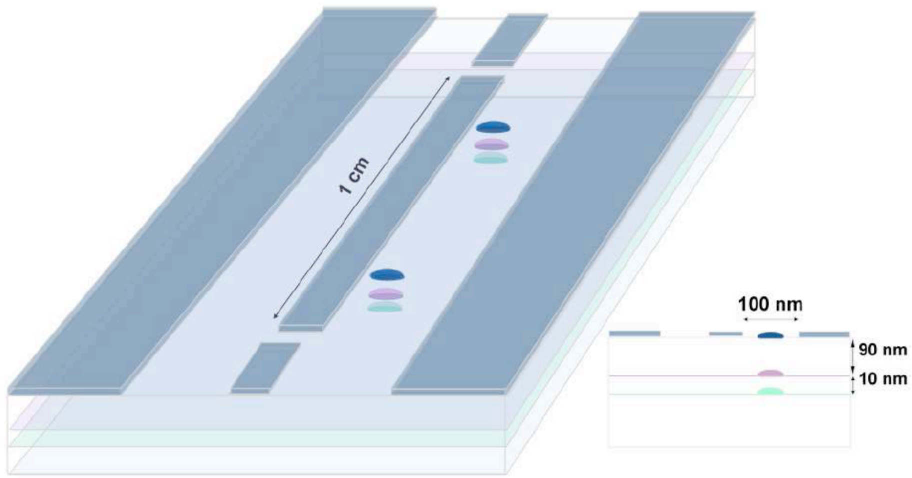

$$g = eaE_0 \frac{J}{\hbar\omega} \frac{\varepsilon\, \delta h}{4U^2 - \varepsilon^2 - \delta h^2}$$

- **coltrollable interaction between spin qubits up to ~cm**
- **interaction can be switched with bias ε**
- **relaxation and decoherence due to coupling to cavity?**
- **interface between spin and superconducting qubits?**
- **electrostatic gating?**