Detecting Fractional Statistics: Interferometry and Noise Correlations in Fractional Quantum Hall Fluids

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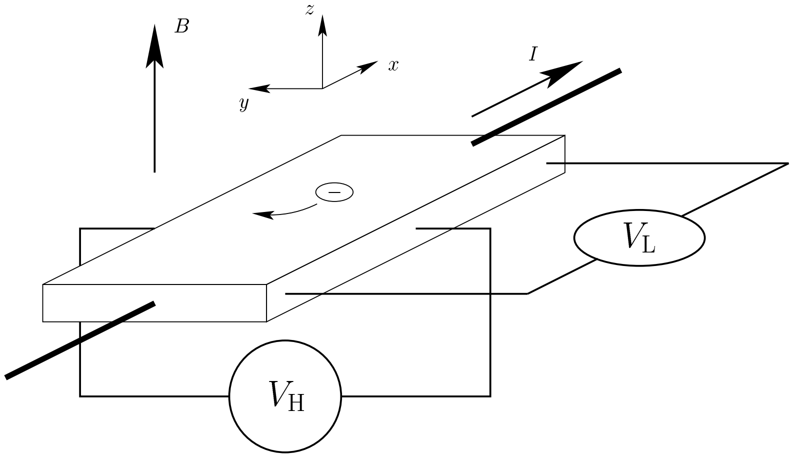
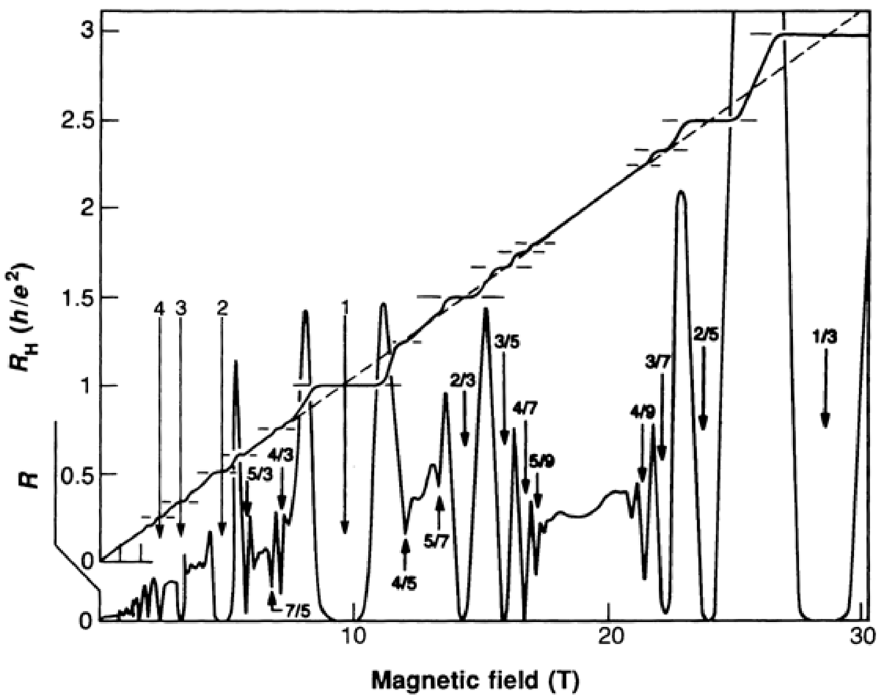
Outline

- FQH Interferometers and fractional statistics
- The Abelian FQH Interferometer(s)
- The non-Abelian FQH Interferometer(s)
- Noise correlations as a probe of fractional statistics
- Goldman's experiment: what does it measure?
- Conclusions

The Fractional Quantum Hall Effect(s)





Eisenstein and Störmer, 1990

Why don't we have yet experimental proof of fractional statistics?

- Fractional statistics is a fundamental prediction of quantum mechanics in two dimensions
- It is a subtle effect involving delicate correlations between slowly moving excitations
- QH experiments for the most part measure transport and charge
- FQH Interferometry experiments are difficult, requiring very clean samples and very low temperatures
- Lack of funding

Statistics and Quantum Mechanics

In Quantum Mechanics the wave-function depends on the positions of the particles and their quantum numbers $i_1 i_2 \ldots$. To make the notation simpler, we just denote the labels $i_1; i_2 \ldots$ by a single one a:

$$\Psi_a(x_1, x_2, \ldots)$$

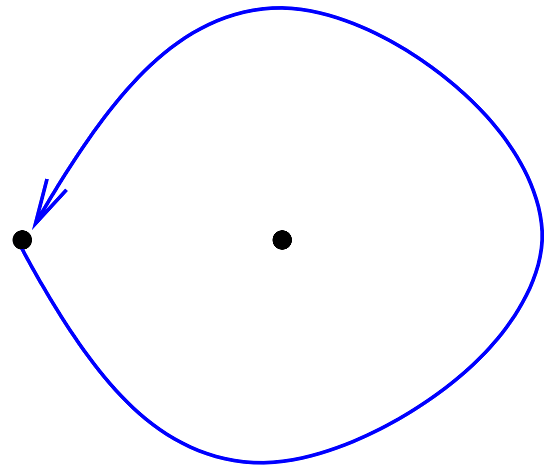
The statistics of the particles comes from the behavior of Ψ under the interchange $x_1 \leftrightarrow x_2$.

In $3+1$ dimensions the only allowed symmetry of the wave function under exchange requires that the particles are either fermions and bosons

$$\Psi_a(x_1, x_2, \ldots) = \pm\Psi_a(x_2, x_1, \ldots)$$

Statistics and Adiabatic Evolution

In $2+1$ dimensions there are more possibilities. We will regard the identical particles as having a hard core and we will consider an adiabatic time evolution which corresponds to an exchange process:



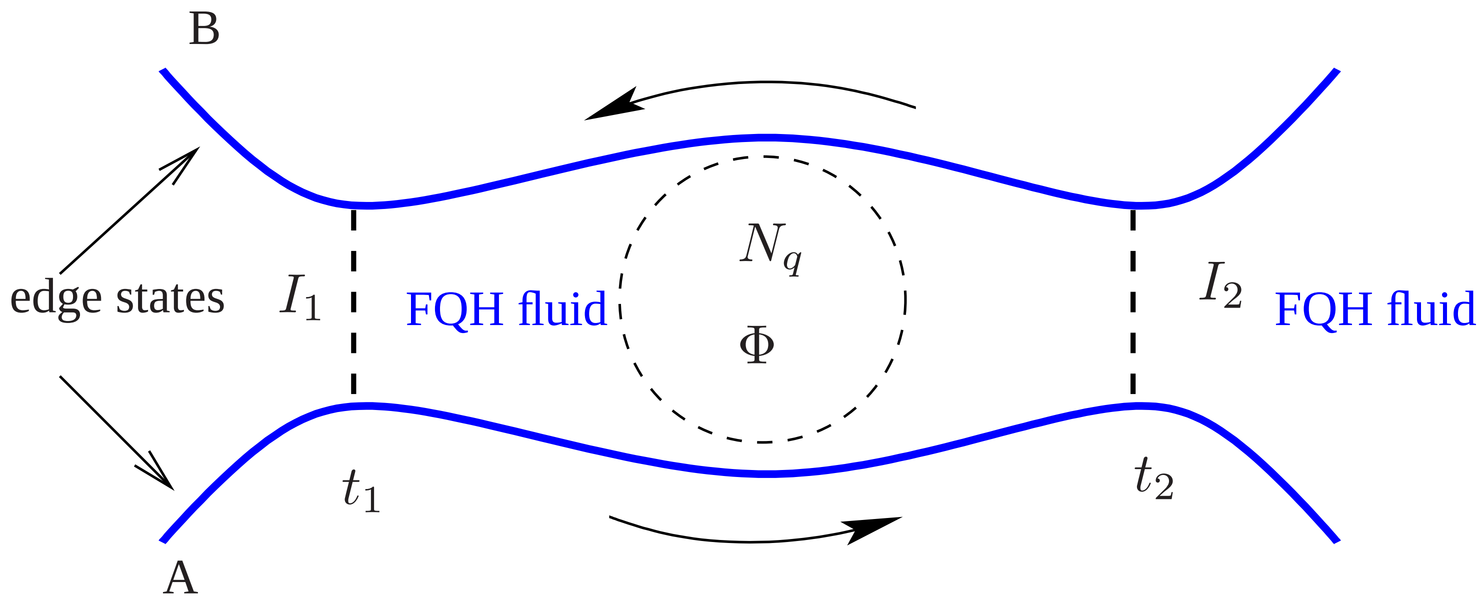
- $3+1$ dimensions: this path is topologically trivial
- $2+1$ dimensions: this path is topologically non- trivial $\Rightarrow$ Braids!

For Laughlin (and Jain) states

$$\Psi_a(x_1, x_2, \ldots) = e^{i\theta}\Psi_a(x_2, x_1, \ldots), \quad \theta = \frac{\pi}{m}$$

Anyons with Abelian (braid) fractional statistics!

FQH Interferometers and Fractional Statistics



Chamon, Freed, Kivelson, Sondhi and Wen (1997)

- Internal tunneling only!
- If we hold the electron number (and therefore the quasihole number) in the central region fixed, then the conductance will oscillate as a function of Φ with period $\frac{e}{e^*}\Phi_0$, where e^* is the quasihole charge.
- If, on the other hand, we vary N_q, we can probe the statistics.

Interference and Braiding

- A quasihole which is injected at point A on the bottom edge and tunnels at the first point-contact arrives at point B in state $|\psi\rangle$.
- A quasihole which tunnels at the second point contact is in the state $e^{i\alpha}\, B_{N_q}|\psi\rangle$, where B_{N_q} is the braiding operator for the quasihole to encircle the quasiholes in the central region and $e^{i\alpha}$ is the additional Aharonov-Bohm and dynamical phase acquired along the second path.
- The current which is measured at B will be proportional to

$$\frac{1}{2}\left(|t_1|^2 + |t_2|^2\right) + \text{Re}\left\{t_1^* t_2\, e^{i\alpha}\, \langle\psi|B_{N_q}|\psi\rangle\right\}$$

- $\langle\psi|B_{N_q}|\psi\rangle$ is given by the expectation value of the Wilson lines representing the world-lines of the quasiholes in the effective Chern-Simons field theory
- In the non-Abelian case, it measures the Jones polynomial $V_{N_q}(e^{i\pi/4})$ of these loops! (Fradkin, Nayak, Tsvelik and Wilczek, (1998).)
- For a non Abelian state with N_q odd, the interference amplitude vanishes! Bonderson, Kitaev and Shtengel (2006); Stern and Halperin (2006)

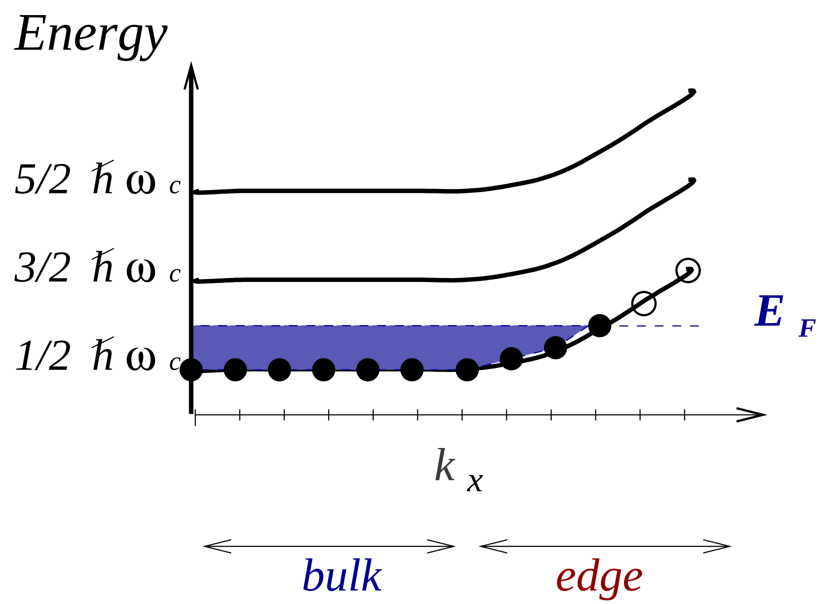
Edge States: Hydrodynamic picture

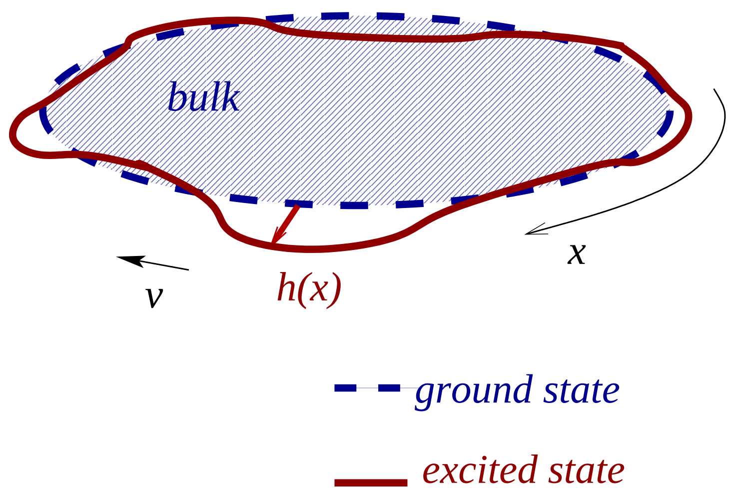
- The surface wave of edge distortions is the only gapless excitation.
- Dissipationless chiral Luttinger liquid. (Wen, 1990; Stone 1991)
- 1D density ripple $J(x) = \rho h(x)$ is related to chiral boson ϕ_+ through bosonization

$$J_+(x) \equiv -\frac{\sqrt{\nu}}{2\pi}\partial_x\phi_+$$

$$\psi_+^\dagger = \frac{1}{\sqrt{2\pi}} e^{\frac{i}{\nu}\phi_+}$$

$$\mathcal{L} = \frac{1}{4\pi}\partial_x\phi_+(\partial_t - \nu\partial_x)\phi_+$$



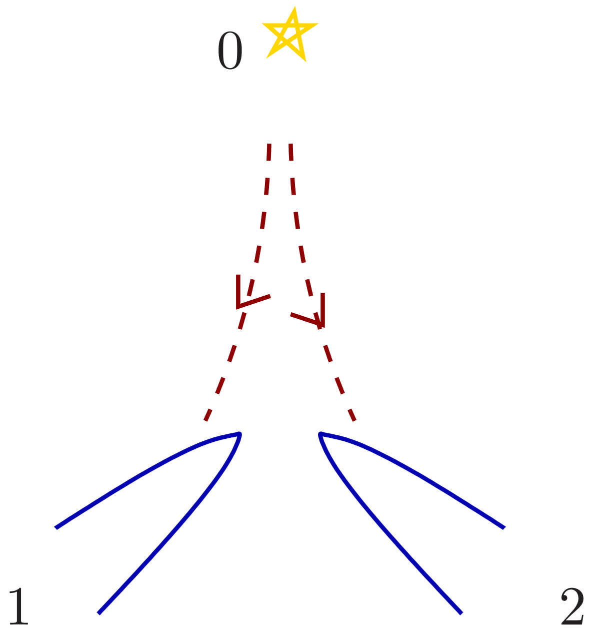


Fractional statistics in QH Jain States

	Boson	$\nu = \frac{1}{\text{odd}}$ (Laughlin)	$\nu = \frac{p}{2np+1}$ (Jain)	Fermion
phase	$1 = e^0$	$e^{i\nu\pi}$	$e^{i\theta} (\frac{\theta}{\pi} = \frac{2n}{2np+1} + 1)$	$-1 = e^{i\pi}$
charge		νe	$Q = \frac{-e}{2np+1}$	$-e$

$2n = \#$ of attached flux quanta, $p =$ effective filling factor

Hanbury-Brown & Twiss (1956): Photons

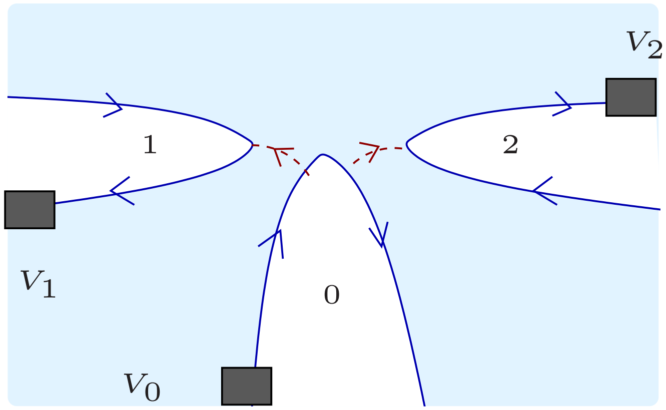


Intensity-intensity correlation

$\Longrightarrow$ Bunching

T- junction in Jain states

Kim, Lawler, Vishveshwara, Fradkin (2005)



Cross correlations

$$S(t) = \langle \Delta I_1(t) \Delta I_2(0) \rangle$$

$\Rightarrow$ Non-equilibrium $V_1 - V_0 = V_2 - V_0 = V, T > 0$

Related works on Laughlin states at $T = 0$:

I. Safi et. al., S. Vishveshwara

Tunneling Hamiltonian

$$\mathcal{L}_{int,l}(t) = \sum_{\epsilon=\pm} -\Gamma_l e^{i\epsilon\omega_0 t} V_l^{(\epsilon)}(t)$$

$$V_l^{(\epsilon)}(t) = (F_0 F_l^{-1})^{\epsilon} e^{i\epsilon\varphi_0(t)} e^{-i\epsilon\varphi_l(t)}$$

$\omega_0 = e^* V/\hbar$: Josephson frequency, q.p. for edge l with unitary Klein factors F_l

$$\psi_l^\dagger \propto F_l e^{i\varphi_l}, F_l F_m = e^{-i\alpha_{lm}} F_m F_l$$

$\alpha_{02} = \alpha_{21} = \alpha_{01} = \theta$, $\alpha_{lm} = -\alpha_{ml}$

Edge states for the Jain sequence

- Chiral boson Lagrangian (charge mode ϕ_c, topological modes ϕ_N)
  López and Fradkin, 1999

$$\mathcal{L}_0 = \frac{1}{4\pi\nu}\partial_x\phi_c(-\partial_t\phi_c - \partial_x\phi_c) + \frac{1}{4\pi}(\partial_x\phi_N\partial_t\phi_N)$$

- Quasi particle at $x = 0$:

$$\psi^\dagger(t) \propto e^{i(\frac{1}{p}\phi_c + \sqrt{1+\frac{1}{p}}\phi_N)} \equiv e^{i\varphi(t)}$$

$$\langle\psi(t)\psi^\dagger(0)\rangle = e^{\langle\varphi(t)\varphi(0)\rangle} = C(t)e^{-i\frac{\theta}{2}\mathrm{sgn}(t)}, \quad C(t) \equiv \left|\frac{\frac{\pi\tau_0}{\beta}}{\sinh(\frac{\pi}{\beta}t)}\right|^K$$

$\frac{K}{2} = \frac{1}{2p(2np+1)}$: scaling dimension, $\beta = 1/k_B T$

Perturbative calculation of the Cross Noise Correlations

- $S^{\tilde{\epsilon}}(t)$ to lowest nontrivial order

$$\propto \tilde{\epsilon} \int dt_i^2 \qquad \cos[\omega_0(t - t_1 - \tilde{\epsilon}t_2)](C(t-t_1)C(t_2))^2$$
$$\times \left\{ \left(\frac{C(t-t_2)C(t_1)}{C(t)C(t_1-t_2)} \right)^{\tilde{\epsilon}} \sum_{\eta_1,\eta_2} \chi(\theta) - 1 \right\}$$

$\eta = +/-$: forward/backward Keldysh time contour

$\tilde{\epsilon} = +/-$: relative tunneling orientation

- The phase sum $\sum_{\eta_1,\eta_2} \chi(\theta) = \sum_{\eta_1,\eta_2} \eta_1\eta_2 e^{i\Phi_{\tilde{\epsilon}}^{\eta_1,\eta_2}[R_\zeta]}$

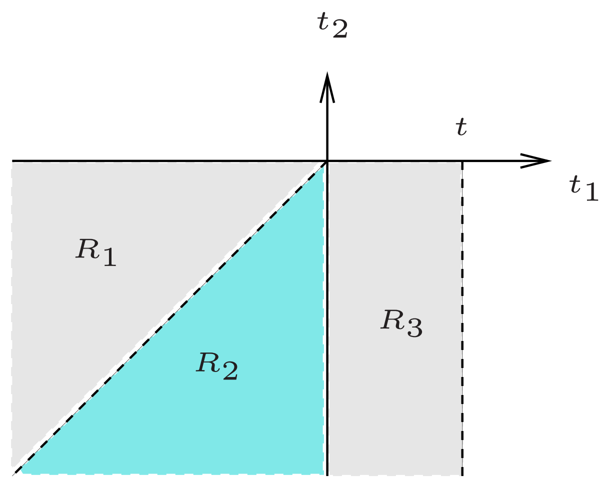
1) comes from contour ordering

2) carries the information of statistics

$\Rightarrow S(t) = \mathcal{A}(\omega_0 t; T/T_0, K) + \cos\theta \; \mathcal{B}(\omega_0 t; T/T_0, K)$

Anatomy of the phase factor

$R_1(t_1 < t_2 < 0)$ and $R_2(t_2 < t_1 < 0)$ allow virtual exchanges.



- virtual exchange of qp's

  $\Rightarrow \chi[R_2;\eta] = e^{i\theta\eta}\chi[R_1;\eta]$

- virtual exchange of p-h's

  $\Rightarrow \chi[R_2;\eta] = e^{-i\theta\eta}\chi[R_1;\eta]$

- Phase factor sum in R_1 and R_2

$$\sum_{\eta=\pm}\chi[R_1;\eta](=e^{i\theta\eta}) \propto \sin\theta$$

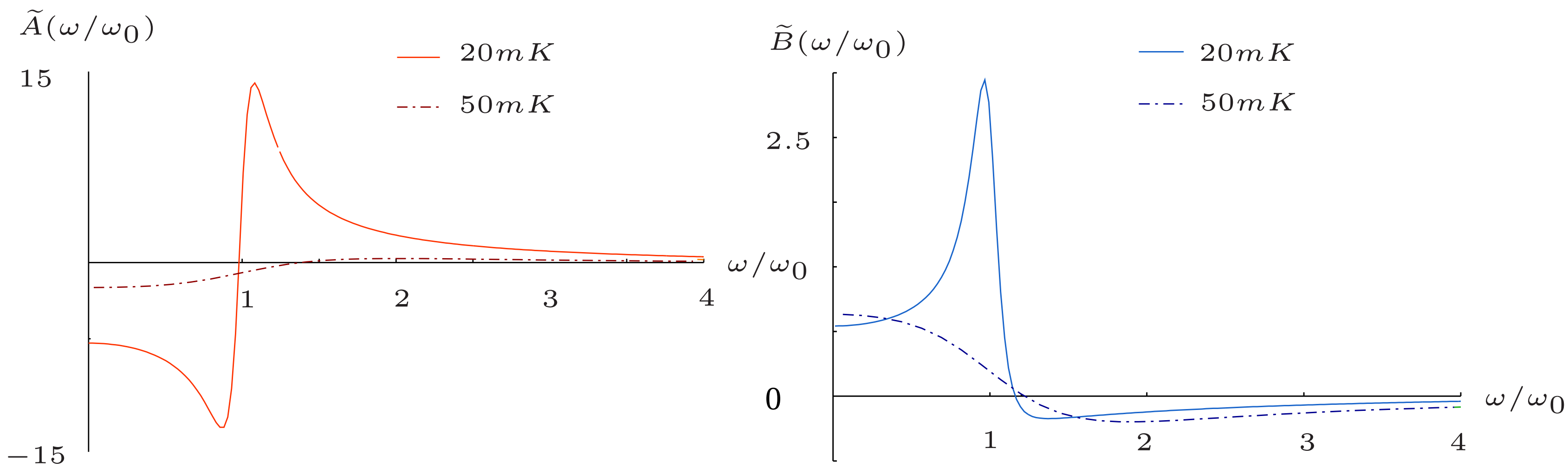
$$\sum_{\eta=\pm}\chi[R_2;\eta](=e^{i(\theta+\theta)\eta}) \propto \sin\theta\,\cos\theta$$

$$\sum_{\eta=\pm}\chi[R_1;\eta](=\eta e^{i\theta\eta}) \propto \sin\theta$$

$$\sum_{\eta=\pm}\chi[R_2;\eta](=\eta e^{i(\theta-\theta)\eta}) = 0$$

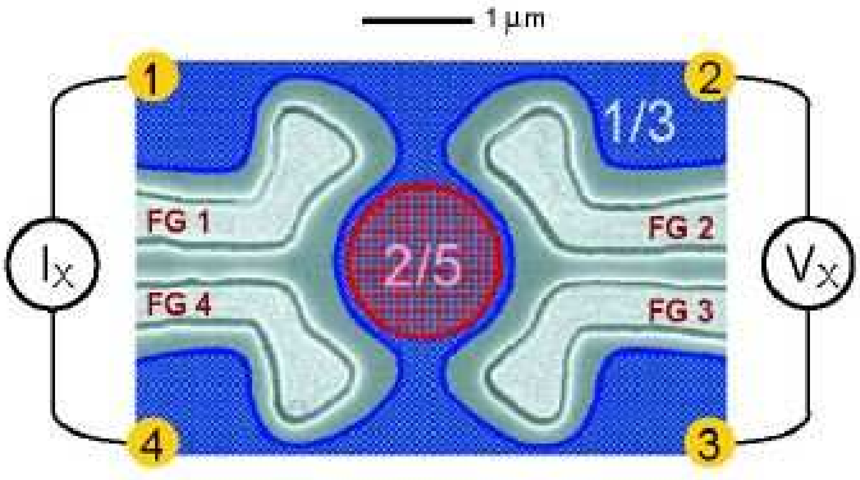
Frequency spectrum

Direct term $\widetilde{A}(\omega)$ v.s. exchange term $\widetilde{B}(\omega)$



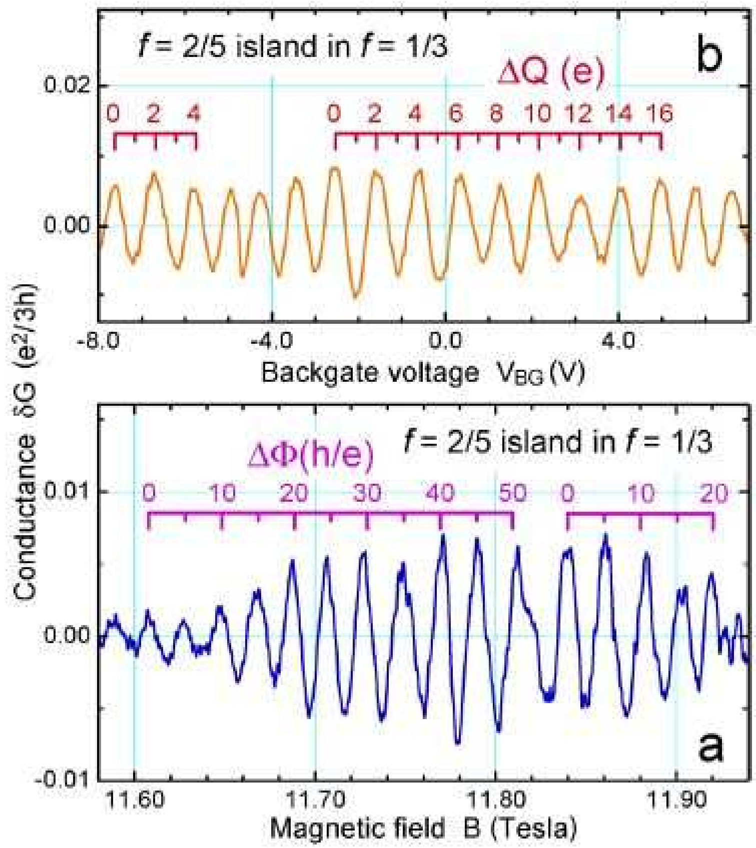
- $\widetilde{S}(\omega/\omega_0;T) = \widetilde{A} + \cos\theta\, \widetilde{B}$.
- "Bunching" Laughlin qp ($\theta < \pi/2$) v.s."anti-bunching" non-Laughlin qp ($\theta > \pi/2$).

Goldman's Interferometer Experiment



Camino, Zhou, Goldman (2005)

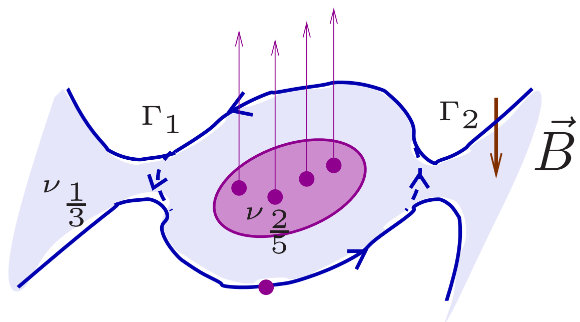
Superperiod oscillation with $\Delta\Phi = 5\phi_0$.



Does this experiment measure fractional statistics?

Oscillations of the tunneling conductance, Eun-Ah Kim, cond-mat/0604359

(Related works Kim and Fradkin (2003); Chamon, Freed, Kivelson, Sondhi and Wen (1997)



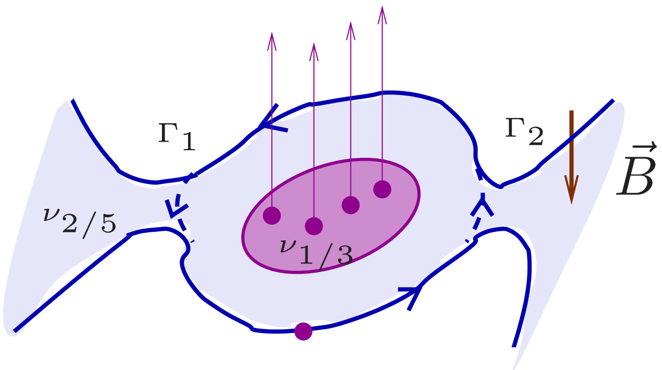
Assumptions:

- The $\nu = 1/3$ fluid is an open system (connects to the leads)
- No direct tunneling between outer edge and the inner puddle.
- Coherent propagation of $1/3$ quasihole along outer edge.
- Both FQH fluids are incompressible and self-consistently adjust their area with B
- The $\nu = 1/3$ quasiholes have fractional statistics with $\theta = \pi/3$.
- Perturbative calculation of the conductance oscillations in powers of Γ_1 and Γ_2

Hierarchical Picture of the Incompressible $2/5$ FQH liquid

- $1/3$ qp's condense to form a puddle of $2/5$ state
- Incompressibility of $2/5$ state $\Rightarrow$ flux superquantization
- $\dfrac{(\text{total charge of puddle } Q)/e}{Bs/\phi_0} = \nu_{2/5}$
- $B \Uparrow$ require $Q \Uparrow$:N extra $1/3$ qp condense to the puddle of area s
- $\nu_{1/3}\frac{B\ s}{\phi_0} + \frac{1}{3}N = \nu_{2/5}\frac{B\ s}{\phi_0}$, (Jain, Kivelson, Thouless,1993)
  $N = \left[\frac{|B|s}{5\phi_0}\right]$

Interference conditions



- Two independent periods:
  - Aharanov-Bohm phase due to flux through the area S
    $2\pi \frac{e^*}{e} \frac{|B|S}{\phi_0} = 2\pi \frac{1}{3} \frac{|B|S}{\phi_0}$
  - Statistical phase due to N qp's in the puddle of area s
    $-2\theta N = -\frac{2\pi}{3} N = -\frac{2\pi}{3} \left[\frac{|B|s}{5\phi_0}\right]$

- $\Delta \frac{\gamma}{2\pi} = \left(5 \frac{e^*}{e} \frac{S}{s} - \frac{\theta}{2\pi}\right) \Delta \left[\frac{|B|s}{5\phi_0}\right] = \left(\frac{5}{3} \frac{S}{s} - \frac{1}{3}\right) \Delta \left[\frac{|B|s}{5\phi_0}\right] =$ integer

- $S/s = 1.43 \sim 7/5 \Rightarrow \Delta|B|s = 5\phi_0$:
  consistent with the experiment

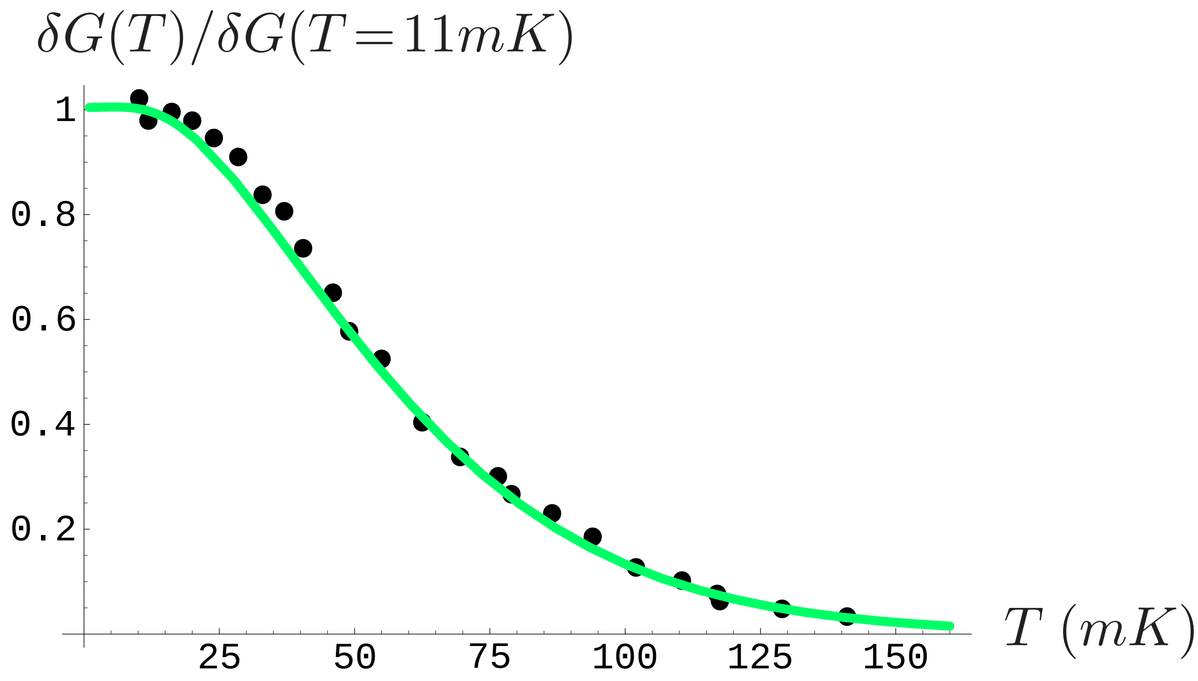
- The existence of periodic Aharonov-Bohm oscillations with periods *larger* than the fundamental quantum of flux is a consequence of fractional statistics in the $\nu = 1/3$ FQH fluid!

Temperature dependence of the conductance oscillation

Perturbative calculation to leading order in Γ

$$H_t = \frac{\Gamma_1}{2} e^{-i\omega_J t} \psi^\dagger_{R,1} \psi_{L,1} + e^{i\gamma} \frac{\Gamma_2^*}{2} e^{i\omega_J t} \psi^\dagger_{L,2} \psi_{R,2} + h.c.$$

$$G(\omega_0, v/R, T) = \bar{G}(\omega_0/T) + \cos\gamma\, \delta G(\omega_0, v/R, T),\, \omega_0 = e^* V/\hbar$$



Kim's fit to data from Goldman's group

Conclusions

- Interferometers can provide direct evidence for Abelian and non-Abelian Fractional Statistics
- We have shown that noise cross current correlations provide for a direct way to measure the statistical angle of the quasiholes an Abelian FQH state
- Evidence for bunching and anti-bunching behavior in different states of the Jain series
- These experiments are feasible within current technology
- We are currently working on the extension to the non Abelian case
- Goldman's recent experiment appears to be consistent with fractional statistics; more detailed experiments are needed