Topological defects and phase transitions for lattice and continuum bosons

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Outline

Part I: a Rokhsar-Kivelson point of lattice bosons
1. relationship to standard models
2. loop representation: three noninteracting types of loops
3. rules for combining topological sectors
4. estimates of critical properties from numerics
   (example of "conformal quantum criticality")
(with C. Xu, P. Fendley, A. Vishwanath)

Part II: (time permitting)
1. Introduction to continuum bosons: atomic BECs
2. Topological defects and topological phase transitions for s=1 bosons: half-vortex unbinding in 2D
(with S. Mukerjee, C. Xu)

Background: RK points

A standard approach to construct model Hamiltonians in 2D to understand critical points, phase transitions, etc.

Canonical example: quantum dimer model on square lattice

Hilbert space:
orthonormal basis of classical dimer coverings
Each site lies on exactly one dimer

Hamiltonian:

$$H = -t(\text{flip plaquettes with parallel dimers}) + V(\text{count flippable plaquettes})$$

$$H = -t \sum \left(|\,=\,\rangle\langle\,\|\,| + |\,\|\,\rangle\langle\,=\,| \right) + V \sum \left(|\,=\,\rangle\langle\,=\,| + |\,\|\,\rangle\langle\,\|\,| \right)$$

Background: RK points

Resulting form of Hamiltonian matrix:

$$H = \begin{pmatrix} n_1 V & -t & 0 & 0 & -t & \dots \\ -t & n_2 V & -t & 0 & 0 & \dots \\ 0 & -t & n_3 V & 0 & 0 & \dots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

Here the diagonal terms counts the number of flippable plaquettes, which is also the number of nonzero off-diagonal elements

At t=V, equal-weight superposition is an exact E=0 eigenstate: quantum critical wavefunction with correlations given by classical critical model (dimer packings)

Degeneracy iff multiple topological sectors not connected by Hamiltonian.

Background: RK points

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Here the diagonal terms counts the number of flippable plaquettes, which is also the number of nonzero off-diagonal elements

At t=V, equal-weight superposition

$$|\psi\rangle \propto \sum_{i \in \text{dimer configs}} |i\rangle$$

is an exact E=0 eigenstate:
quantum critical wavefunction with correlations given by classical critical model (dimer packings)

Degeneracy iff multiple topological sectors not connected by Hamiltonian.

Background: RK points

Such RK points turn out to be useful as starting points for the phase diagram: for example, on the triangular lattice a gapped Z2 spin liquid is obtained near the RK point.

The RK point is a quantum critical point with dynamical critical exponent z=2.

Mapping to a height model in dimer case: continuum wavefunction is

$$|\psi\rangle = \int (dh) e^{-K \int (\nabla h)^2/2} |h\rangle$$

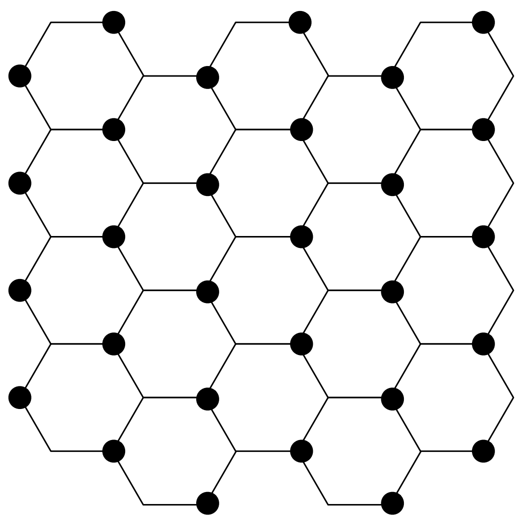
Goal of part I:
understand how to realize more physical RK points with more interesting structure -> possible nonabelian phases

(Most direct realization of dimer model is via lattice decoration->Klein H)

Step I: define Hilbert space from classical model

Consider hard-core bosons or Ising spins on the honeycomb lattice with exactly 3 sites occupied per hexagon.

Example: antiferromagnetic configuration



Connection I: three-color model has 0, 3, 6 per hexagon

The three-color model is a c=2 CFT with an SU(3) nonabelian symmetry (Read, Reshetikhin), but construction of a useful RK point is made difficult by ergodicity problems.

$$S/\hexagon \leq S_{3c} = 0.379$$

Our model

Connection II: faithful dual representation as

half-integers on the sites of triangular lattice, with sum of half-integers around each triangle equal to ±1/2.

More restrictive than triangular Ising antiferromagnet, which has same constraint but each site can only be ±1/2.

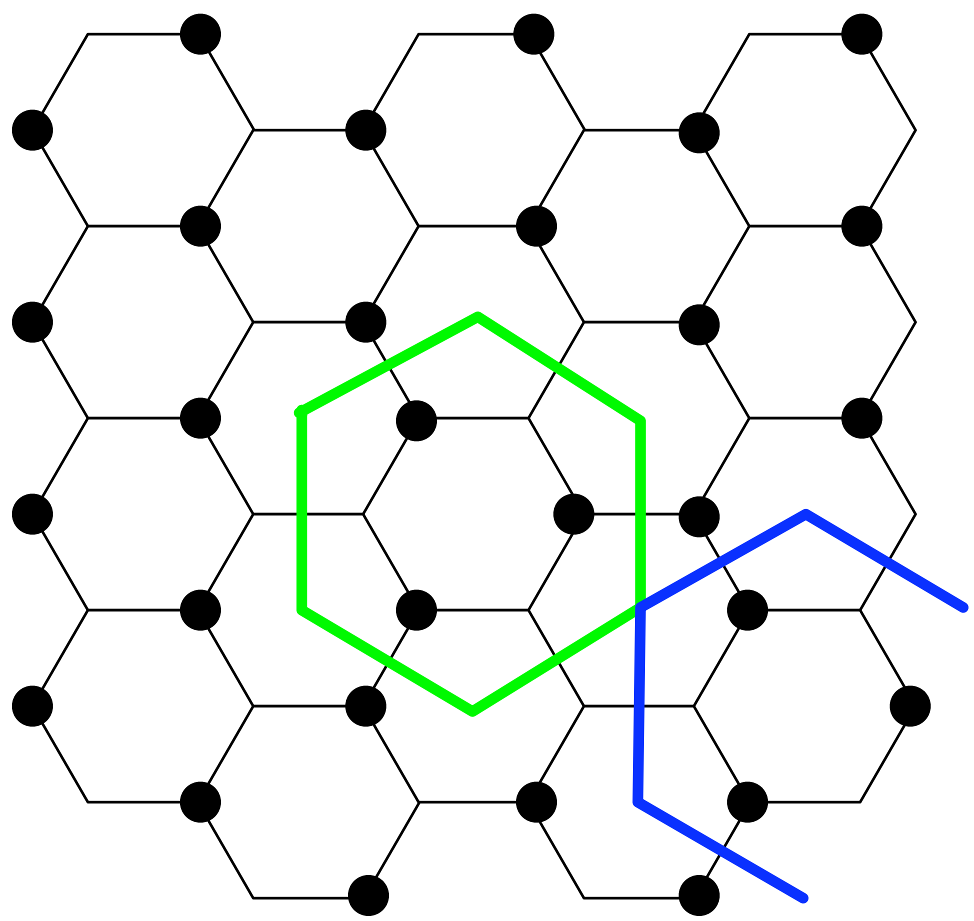
Gives entropy lower bound:

$$S/\hexagon \geq 0.323$$

Our model

Connection III: loop representation
(useful for analysis of sectors in dynamics)
Draw bonds as follows on the dual (triangular) lattice:

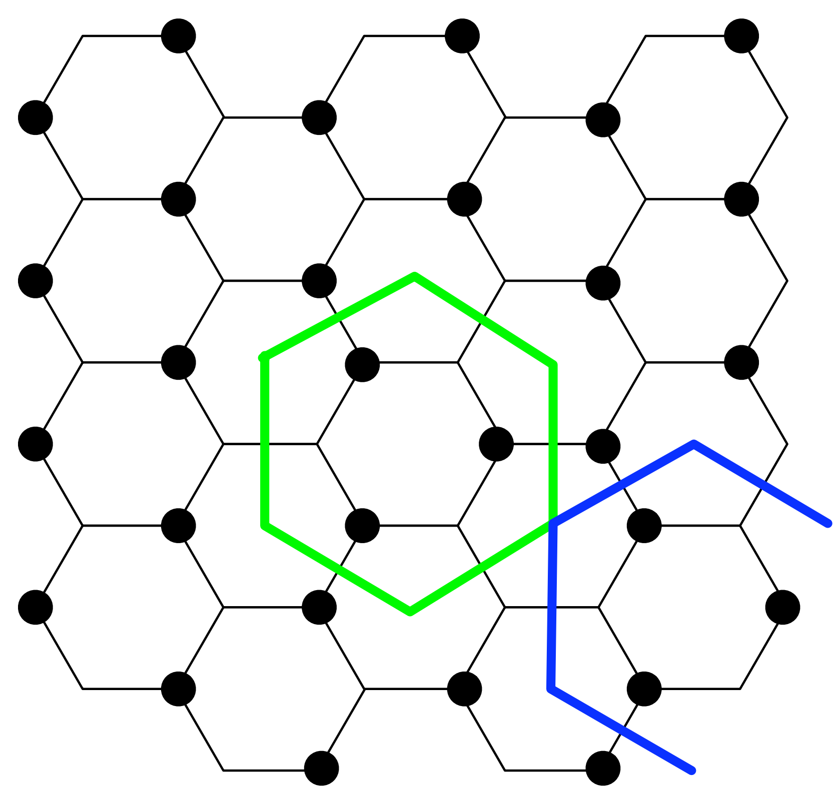
if original lattice bond is ferromagnetic, draw a crossing bond
if original lattice bond is antiferromagnetic, do not cross it



Loops

More precisely, there are three colors of loops on the dual lattice: the triangular lattice has 3 sublattices, and A-B, B-C, and A-C loops are all closed.

Loops of different colors can **touch** but not **cross**.



Equivalent to domain walls of 3 Ising models on the 3 sublattices, +constraint

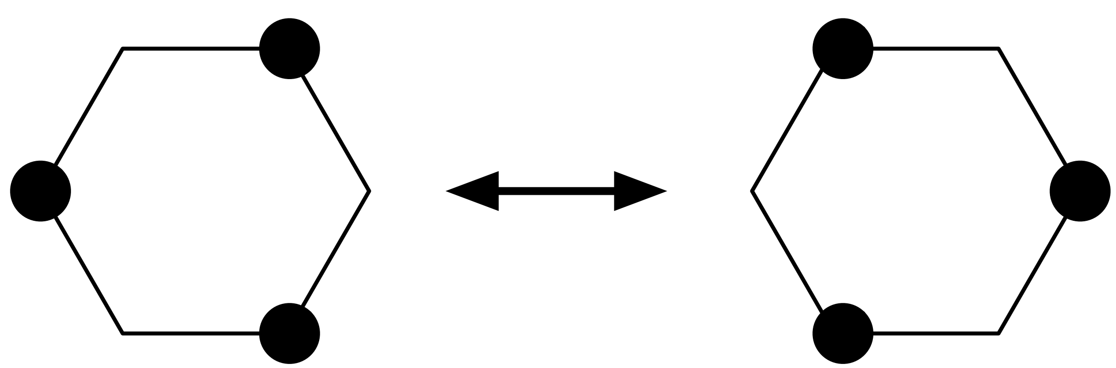
Loops

I. Symmetries of loop representation enable large-size transfer matrix studies, which suggest that the model is critical

$$S/\hexagon \approx 0.364 \qquad\qquad c \approx 2, \quad \Delta \approx 2/3$$

(Open question: is c=2=1+1 or c=2=4/5+6/5?)

II. Loop rules enable counting of sectors under the dynamics



which moves, annihilates, and creates loops, obeying the noncrossing constraint.

Loops

II. Loop rules enable counting of sectors under the dynamics

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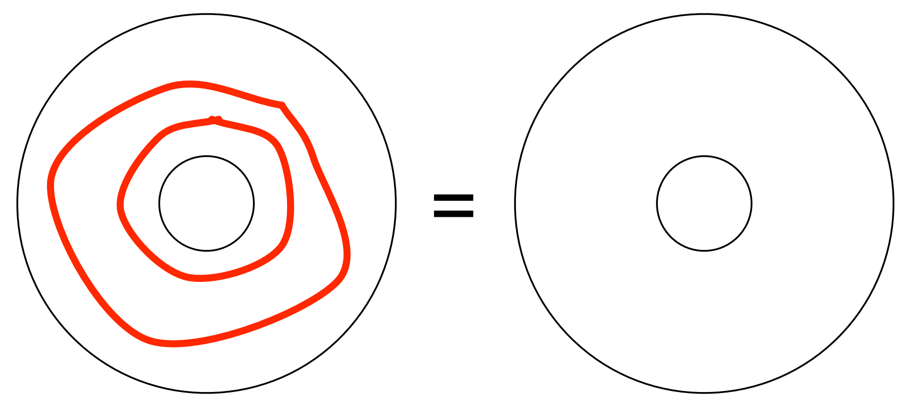
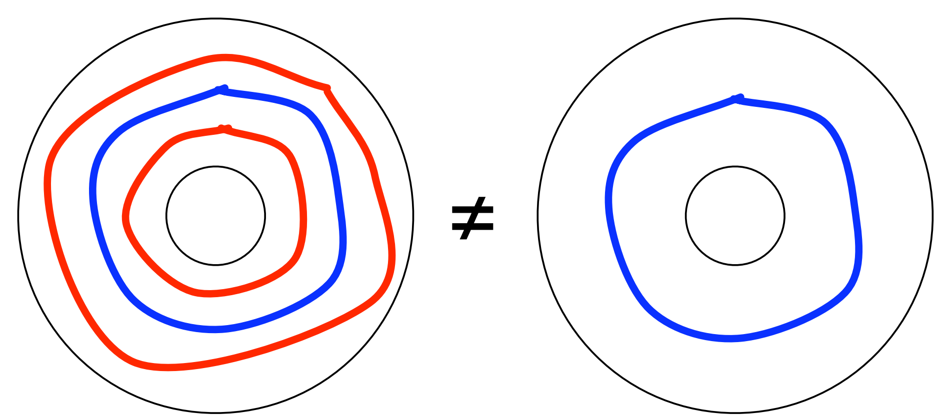
Example: Consider topological sectors on the cylinder (equivalently, an annulus) circled by loops

Two loops of the same color can annihilate



but not if separated by a loop of a different color



Can define nonabelian group action by concatenation of cylinder segments: free group on three elements, modulo $a^2 = b^2 = c^2 = 1$

Defects

At criticality, expect logarithmic (Coulomb) confinement of defects (ends of loops), based on 3-color model example

Staying within simple local Hamiltonians, does the system go into an interesting topological phase describable in terms of loops?

Conclusion of part I:
a local lattice Hamiltonian that generates an RK point
(probably critical according to numerics)
that is a non-fully-packed superposition of loops,
with sectors determined by topological features

Interlude: critical entanglement entropy

How much entanglement is there at these "conformal quantum critical points"?

Partitioning a pure quantum state into subsystems A and B generates an entanglement entropy: away from criticality, expect that if A is finite and B infinite, there is an "area law":

$$S \sim L^{d-1}$$

For clean (Holzhey et al., Vidal) and random (Refael and Moore) critical points in 1D, and for fermions in higher dimensions (Klich and Gioev; Wolf), there is a logarithmic multipler.

Advertisement for talk of E. Fradkin next week:

For conformal quantum critical points, there is an area law term determined by the CFT boundary energy,
plus a possible subleading logarithmic term determined by the CFT central charge and topology of the partition.

What about bosons in the continuum?

Motivation: QHE topological phases appear in the continuum
(Bosonic analogues in rotated systems: Cooper, Read, Rezayi)
Are there other topological phases of bosons in the continuum?

Lattice models of bosons can in principle be realized using BECs in optical lattices, but achieving significant intersite interactions is difficult.

Goal for this part:
show that even the simplest phases of nonrotated s=1 bosons have some nontrivial topological defects and associated phase transitions

$$
\begin{aligned}
H \quad = \quad & \int d\mathbf{r}\Big(\frac{\hbar^2}{2M}\nabla\psi_a^+\cdot\nabla\psi_a + U(\mathbf{r})\psi_a^+\psi_a \\
& +\frac{c_0}{2}\psi_a^+\psi_b^+\psi_b\psi_a + \frac{c_2}{2}\psi_a^+\psi_{a'}^+\mathbf{F}_{ab}\cdot\mathbf{F}_{a'b'}\psi_{b'}\psi_b\Big)
\end{aligned}
$$

s=1 bosons

$$H = \int d\mathbf{r}\Big(\frac{\hbar^2}{2M}\nabla\psi_a^+ \cdot \nabla\psi_a + U(\mathbf{r})\psi_a^+\psi_a$$
$$+\frac{c_0}{2}\psi_a^+\psi_b^+\psi_b\psi_a + \frac{c_2}{2}\psi_a^+\psi_{a'}^+\mathbf{F}_{ab}\cdot\mathbf{F}_{a'b'}\psi_{b'}\psi_b\Big)$$

$$\psi = \sqrt{n_0}\zeta$$

Two simple T=0 mean-field phases (Ho, 1998)

c2>0 polar

c2<0 ferromagnetic

$$\zeta_P = e^{i\theta}U\begin{pmatrix}0\\1\\0\end{pmatrix}, \quad \zeta_F = e^{i\theta}U\begin{pmatrix}1\\0\\0\end{pmatrix}.$$

Here theta is a phase and U is a spatial rotation matrix (SO(3)).

s=1 bosons

What are the order parameter manifolds in these phases?

Need to find cosets

$$M = \frac{G}{H}, \quad G = U(1)_G \times SO(3)_R, \quad H = \text{residual symmetry}$$

Results: $M_F = SO(3)$ (which also appears in 3He)

$$M_P = \frac{S^1 \times S^2}{\mathbb{Z}_2} \qquad \mathbb{Z}_2 = \text{ identify } (\theta, \hat{\mathbf{n}}) \leftrightarrow (\theta + \pi, -\hat{\mathbf{n}})$$

$$\zeta_P = e^{i\theta} \begin{pmatrix} \frac{e^{i\alpha}}{\sqrt{2}} \sin\beta \\ \cos\beta \\ \frac{e^{-i\alpha}}{\sqrt{2}} \sin\beta \end{pmatrix} \qquad \pi_1(M) = \pi_2(M) = \pi_3(M) = \mathbb{Z}$$

s=1 bosons

Concentrate on the polar state in 2D:

Locally the manifold looks like a spin part on the sphere (which flows to high temperature), and a phase part with no flow in perturbation theory.

Thus at small finite temperature the system is in a "nematic superfluid" phase: the spin part is disordered, and the phase part is only defined modulo π.

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This phase supports **half-vortices** (vortices with half the total boson circulation of a vortex in a single-component superfluid).

It has power-law correlations

$$\langle e^{2i\theta(0)} e^{-2i\theta(r)} \rangle \sim \frac{1}{|r|^{\Delta}}$$

s=1 bosons

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This phase supports **half-vortices** (vortices with half the total boson circulation of a vortex in a single-component superfluid), as if bosons had paired.

It has power-law correlations

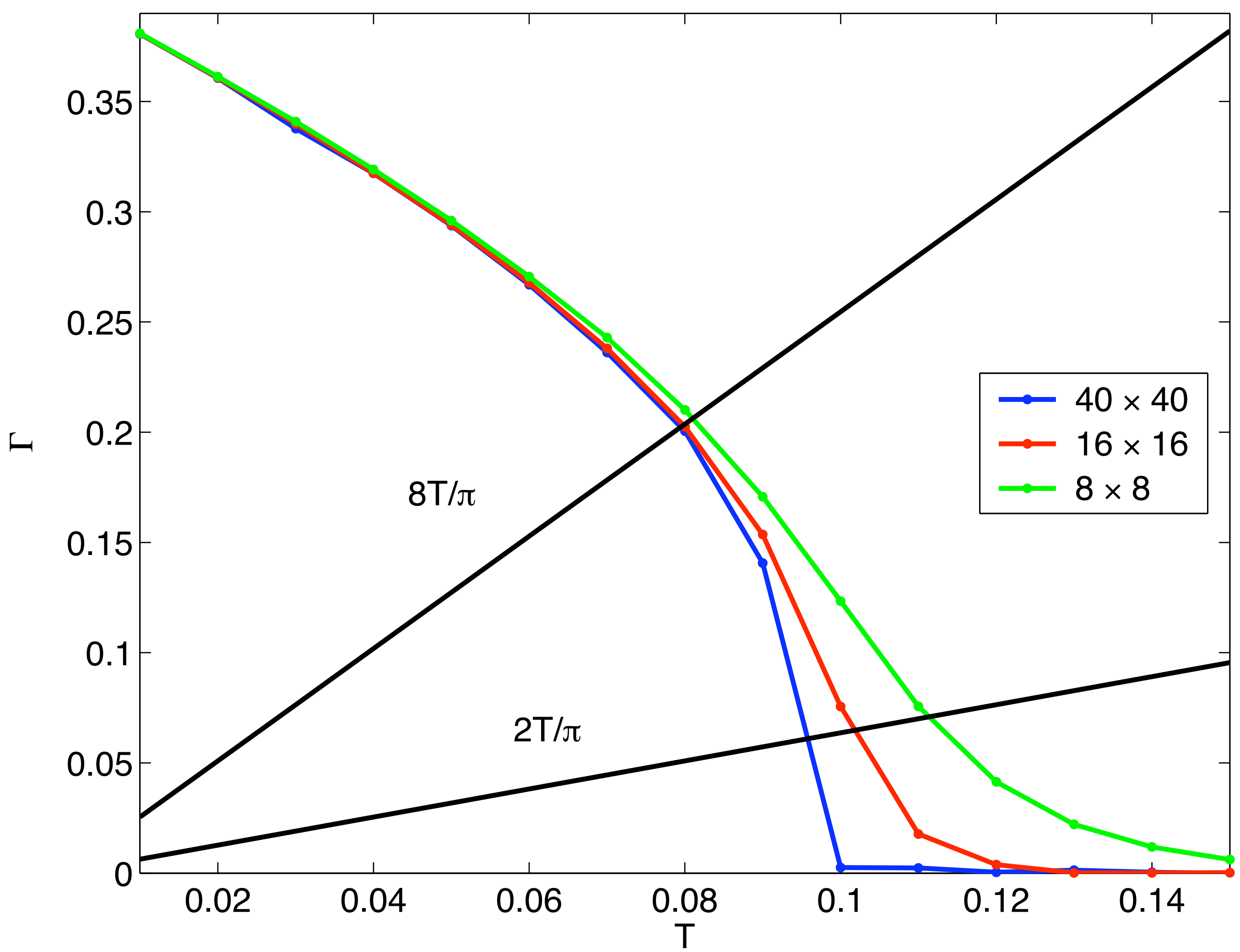
$$\langle e^{2i\theta(0)} e^{-2i\theta(r)} \rangle \sim \frac{1}{|r|^{\Delta}}$$

Smoking gun for numerics (and possibly experiments)

The transition out of this phase should be a Kosterlitz-Thouless transition driven by unbinding of half-vortices.

If so, there is a universal stiffness jump four times the usual value:
(cf. cond-mat/0605102)

$$\rho_\infty(T_c) = \frac{8T_c}{\pi}$$



THE END

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