Dynamics and relaxation in integrable quantum systems

New approaches to non-equilibrium and random systems
KITP, 17 February 2016

Jean-Sébastien Caux
Universiteit van Amsterdam

Work done in collaboration with (among others):

A'dam gang: R. van den Berg, R. Vlijm, S. Eliens, J. De Nardis, B. Wouters, S. E. Tapias Arze, M. Panfil, M. Brockmann, D. Fioretto, O. El Araby, E. Quinn
F.H.L. Essler, R. Konik, N. Robinson, M. Haque, E. Ilievski, T. Prosen, …


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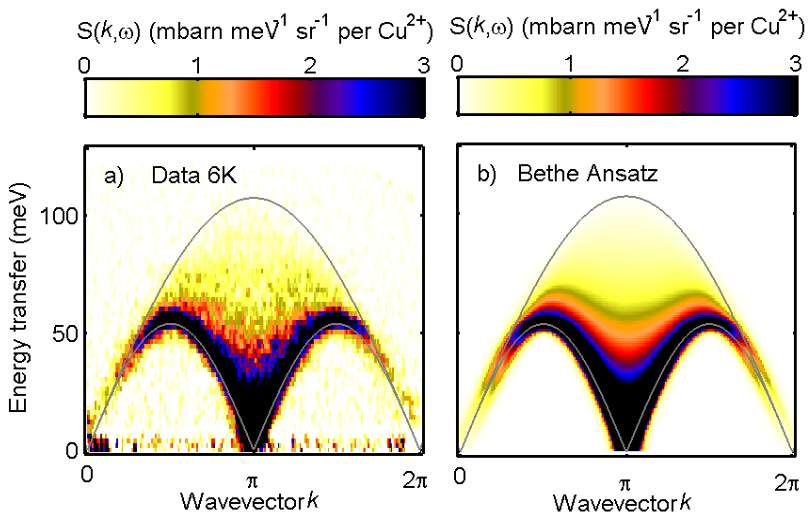
Plan of the talk

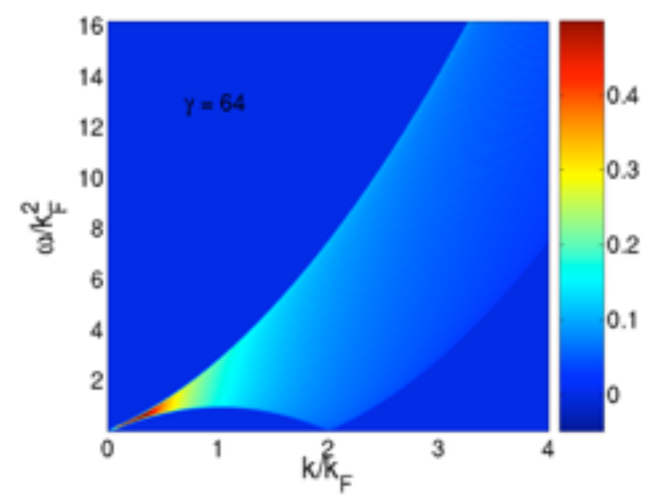
- Out-of-equilibrium dynamics
    - *Quasisoliton dynamics in XXZ*
    - *Quantum Newton's cradle:TG limit*
    - *Interaction quench in Lieb-Liniger*
    - *Anisotropy quench in XXZ*
- Summary & perspectives

The Quench Action

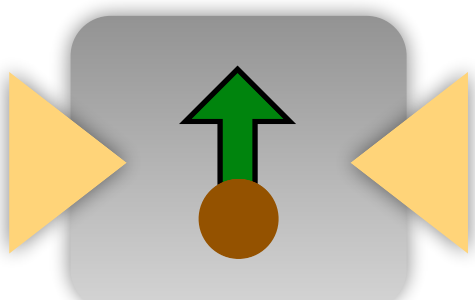
Applications of integrability in many-body physics



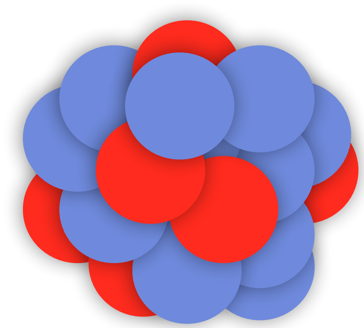
Quantum magnetism



Ultracold atoms

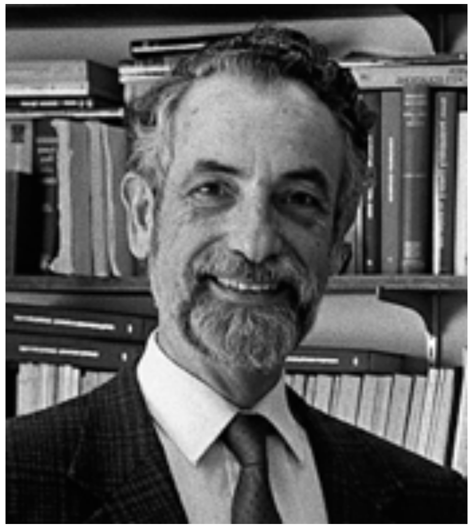


Quantum dots, NV centers



Atomic nuclei

Models discussed in this talk:



- Interacting Bose gas (Lieb-Liniger)

$$\mathcal{H}_N = -\sum_{j=1}^{N} \frac{\partial^2}{\partial x_j^2} + 2c \sum_{1\leq j<l\leq N} \delta(x_j - x_l)$$

- Heisenberg spin-1/2 chain

$$H = \sum_{j=1}^{N} \left[J(S_j^x S_{j+1}^x + S_j^y S_{j+1}^y + \Delta S_j^z S_{j+1}^z) - H_z S_j^z \right]$$

The
Bethe Wavefunction

Michel Gaudin's book *La fonction d'onde de Bethe* is a uniquely influential masterpiece on exactly solvable models of quantum mechanics and statistical physics. Available in English for the first time, this translation brings his classic work to a new generation of graduate students and researchers in physics. It presents a mixture of mathematics interspersed with powerful physical intuition, retaining the author's unmistakably honest tone.

The book begins with the Heisenberg spin chain, starting from the coordinate Bethe Ansatz and culminating in a discussion of its thermodynamic properties. Delta-interacting bosons (the Lieb-Liniger model) are then explored, and extended to exactly solvable models associated with a reflection group. After discussing the continuum limit of spin chains, the book covers six- and eight-vertex models in extensive detail, from their lattice definition to their thermodynamics. Later chapters examine advanced topics such as multicomponent delta-interacting systems, Gaudin magnets and the Toda chain.

MICHEL GAUDIN is recognized as one of the foremost experts in this field, and has worked at Commissariat à l'énergie atomique (CEA) and the Service de Physique Théorique, Saclay. His numerous scientific contributions to the theory of exactly solvable models are well known, including his famous formula for the norm of Bethe wavefunctions.

JEAN-SÉBASTIEN CAUX is a Professor in the theory of low-dimensional quantum condensed matter at the University of Amsterdam. He has made significant contributions to the calculation of experimentally observable dynamical properties of these systems.

Cover illustration: a representation of the Yang-Baxter relation by John Collingwood.

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Gaudin and Caux The **Bethe Wavefunction**

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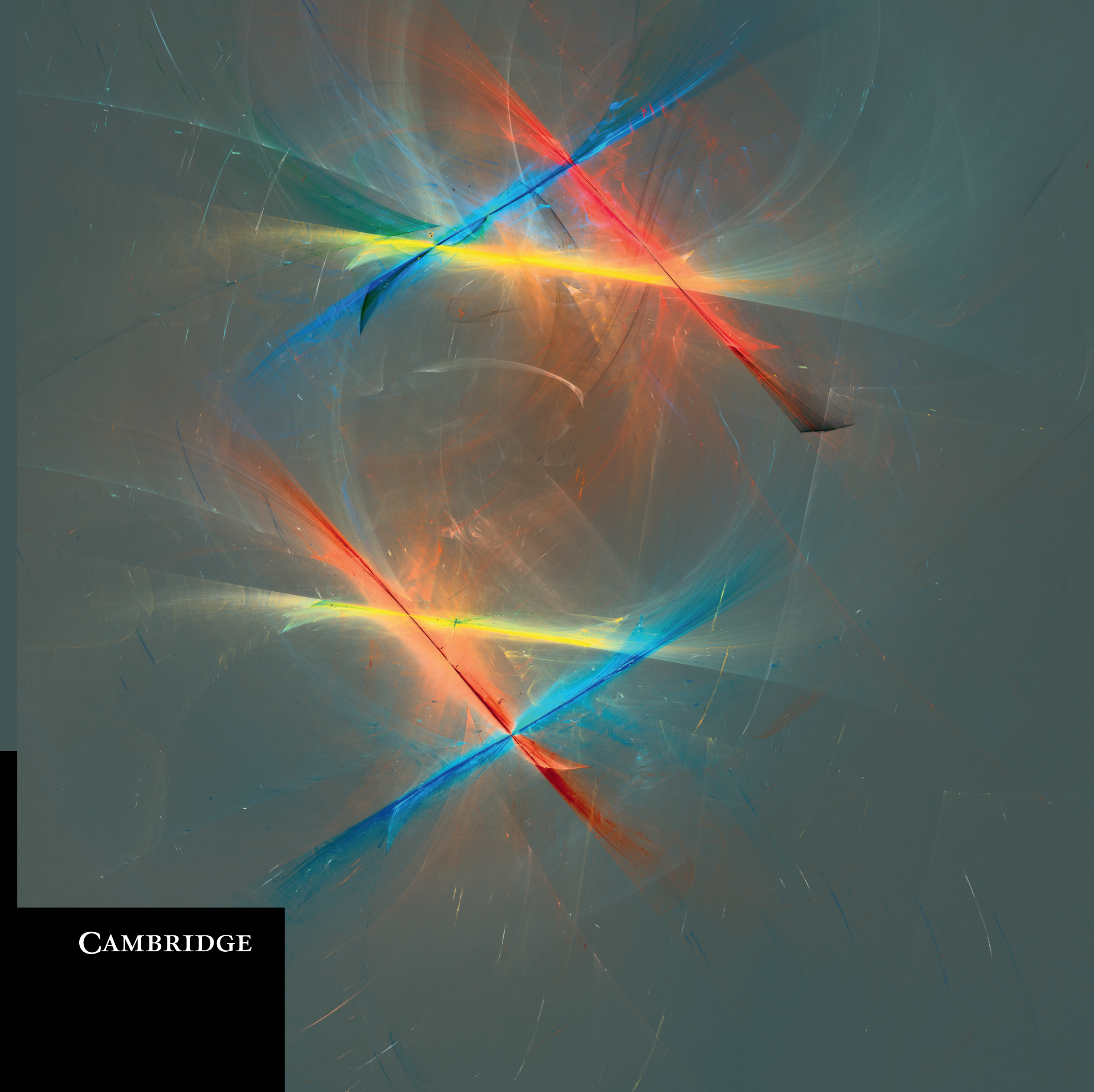
The
Bethe Wavefunction

Michel Gaudin

Translated by Jean-Sébastien Caux



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The general idea, simply stated:

Start with your favourite quantum state
(expressed in terms of Bethe states)

$$\mathcal{O} \rightarrow |\{\lambda\}\rangle$$

Apply some operator on it

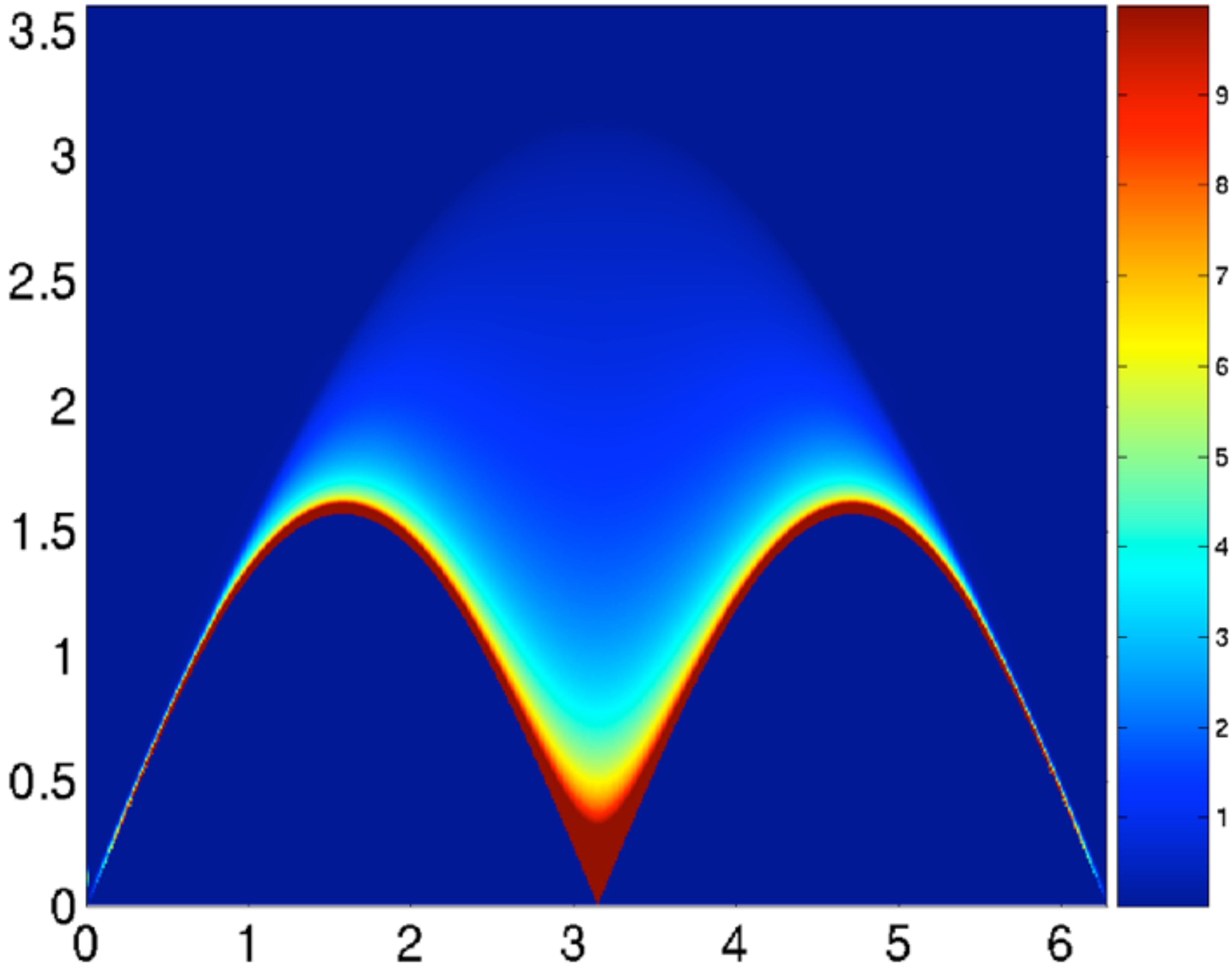
Reexpress the result in the basis of Bethe states:

$$\mathcal{O}|\{\lambda\}\rangle = \sum_{\{\mu\}} F^{\mathcal{O}}_{\{\mu\},\{\lambda\}} |\{\mu\}\rangle$$

using 'matrix elements' $F^{\mathcal{O}}_{\{\mu\},\{\lambda\}} = \langle\{\mu\}|\mathcal{O}|\{\lambda\}\rangle$

Heisenberg spin chain

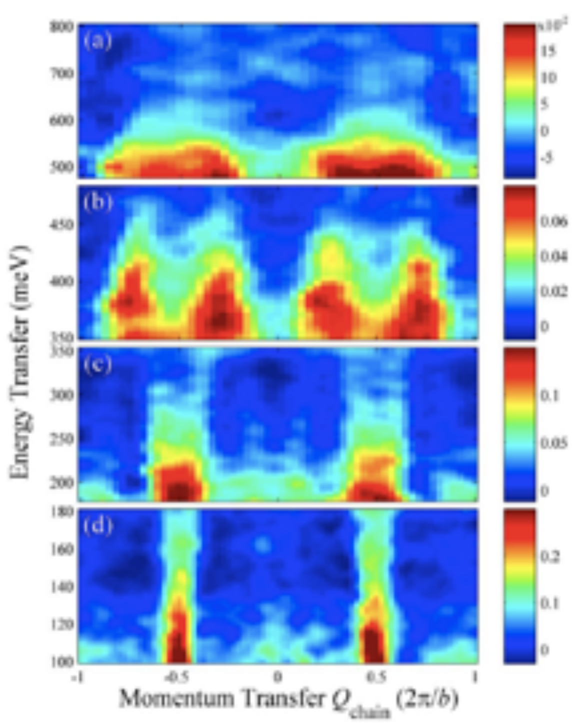
$$S(k,\omega), \quad \Delta = 1, \quad h = 0$$



Quantum spin chains

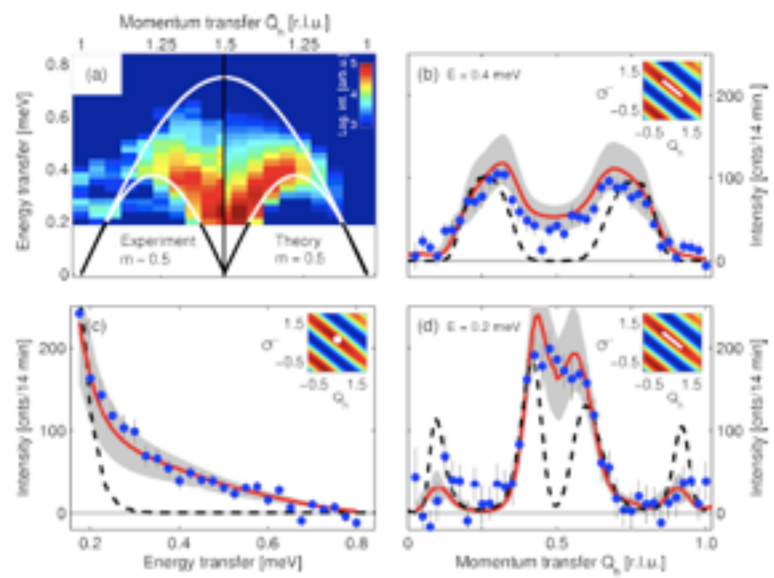
Correlations, experiments (INS, RIXS), prefactors, ...

Sr_2CuO_3



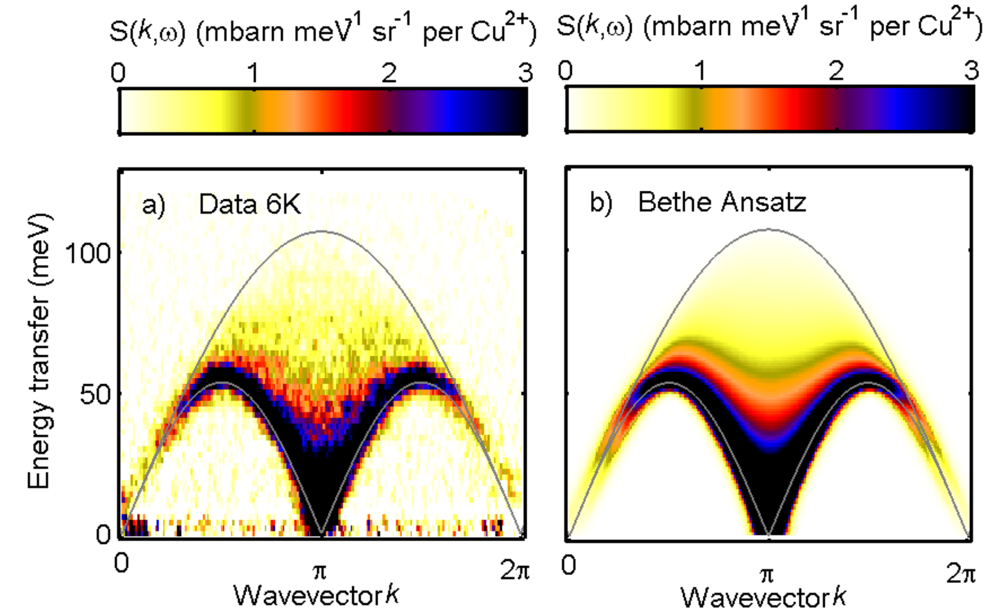
Walters, Perring, Caux, Savici, Gu, Lee, Ku, Zaliznyak, NATURE PHYSICS 2009

$(C_5D_{12}N)_2CuBr_4$

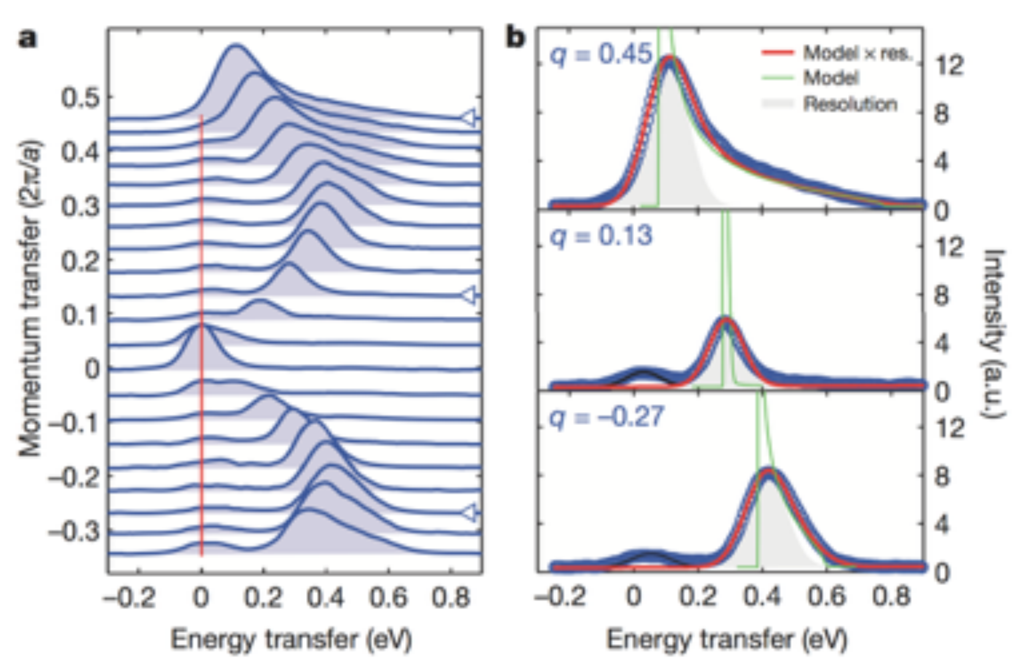


Thielemann, Rüegg, Rønnow, Läuchli, Caux, Normand, Biner, Krämer, Güdel, Stahn, Habicht, Kiefer, Boehm, McMorrow, Mesot, PRL 2009

$KCuF_3$



Lake, Tennant, Caux, Barthel, Schollwöck, Nagler, Frost, PRL 2013



Sr_2CuO_3

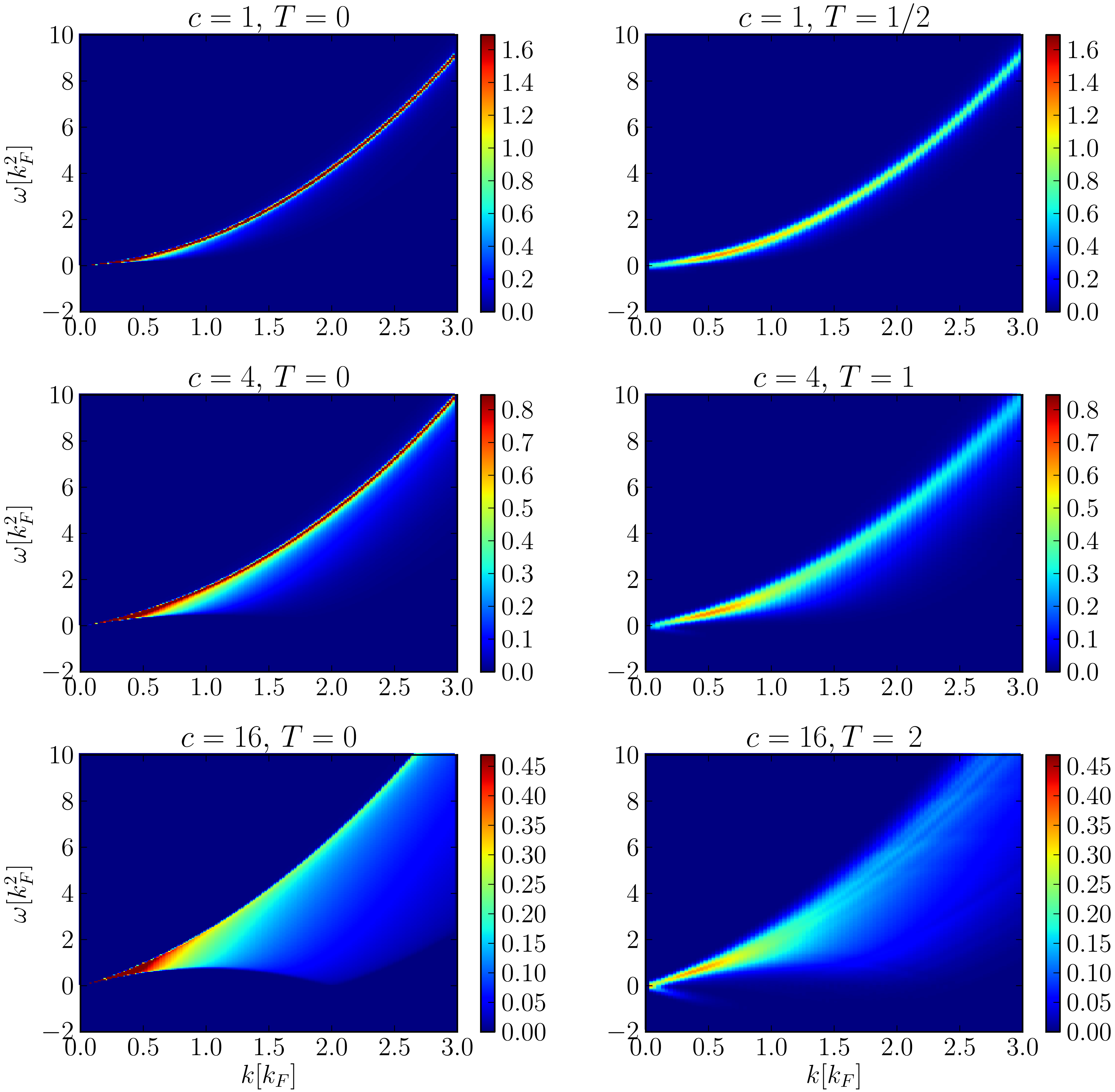
(RIXS)

Schlappa, Wohlfeld, Zho, Mourigal, Haverkort, Strocov, Hozoi, Monney, Nishimoto, Singh, Revcolevschi, Caux, Patthey Rønnow, van den Brink, Schmitt, NATURE 2012

Repulsive Lieb-Liniger gas

Dynamical structure factor at finite T

M. Panfil and J-SC, PRA 89 (2014)

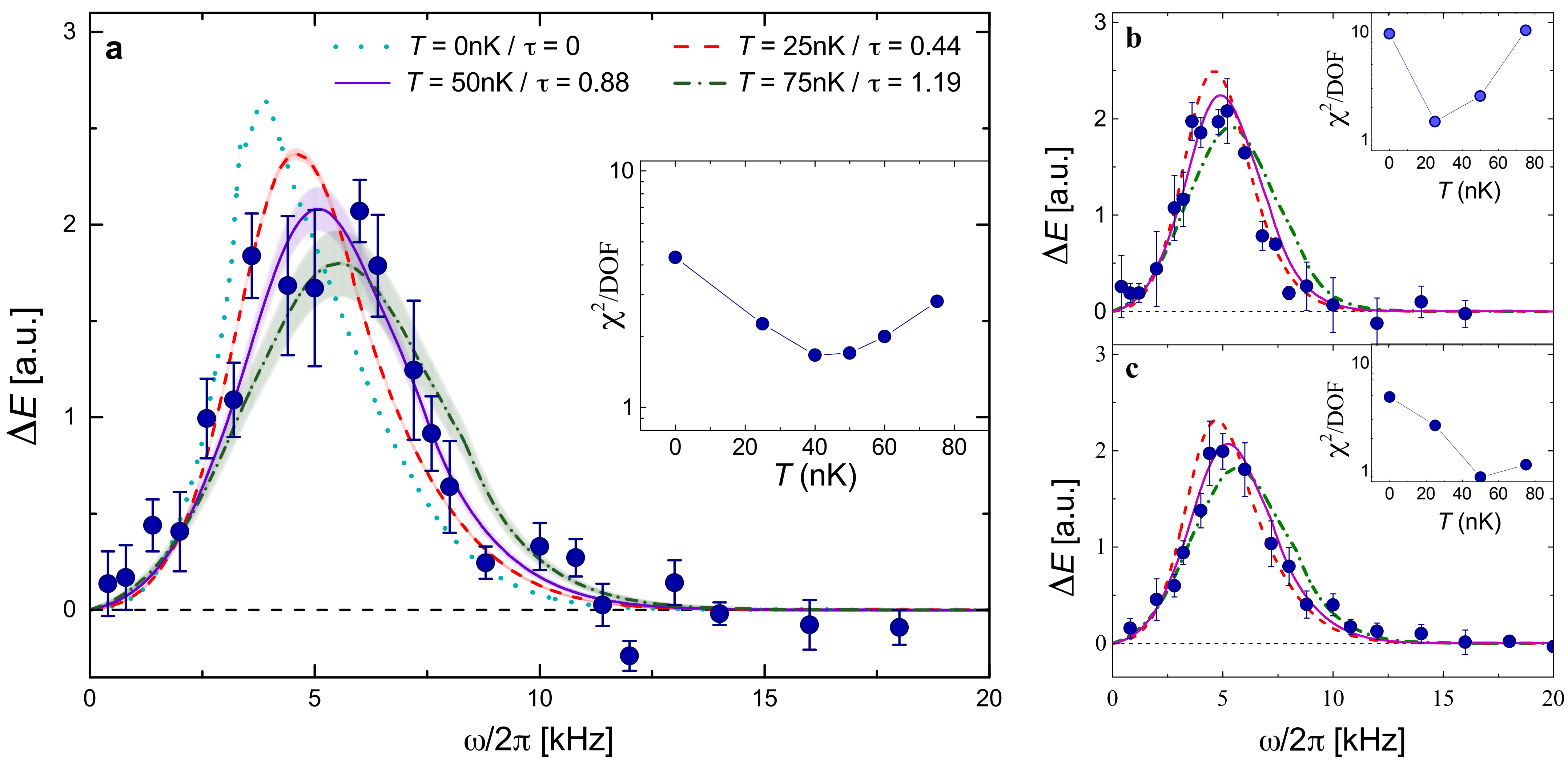


Cold atoms

PHYSICAL REVIEW A **91**, 043617 (2015)

Dynamical structure factor of one-dimensional Bose gases: Experimental signatures of beyond-Luttinger-liquid physics

N. Fabbri,[1,*] M. Panfil,[2,3] D. Clément,[4] L. Fallani,[1,5] M. Inguscio,[1,5,6] C. Fort,[1] and J.-S. Caux[3]

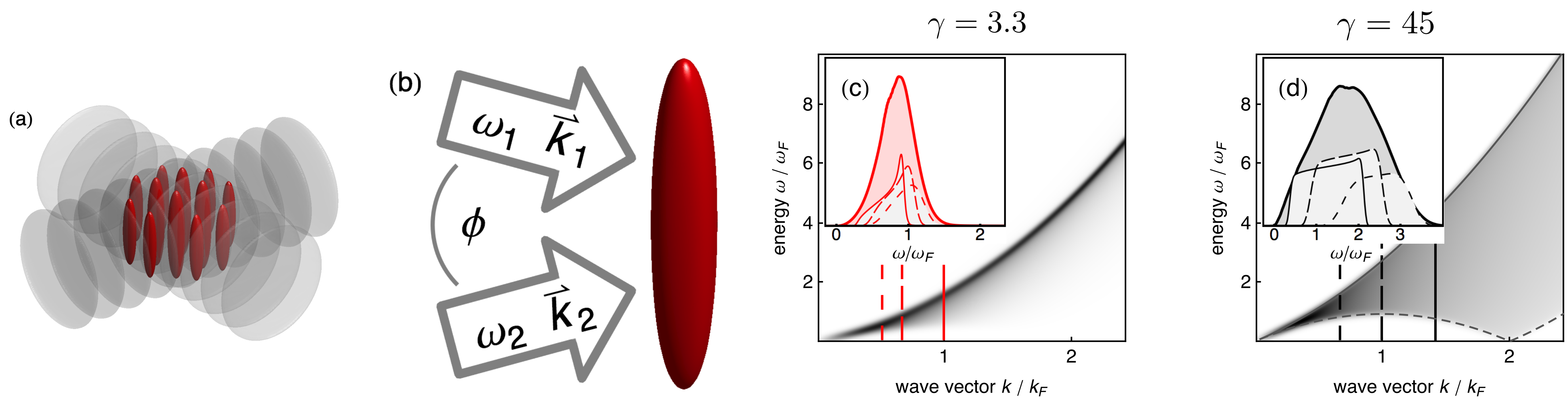


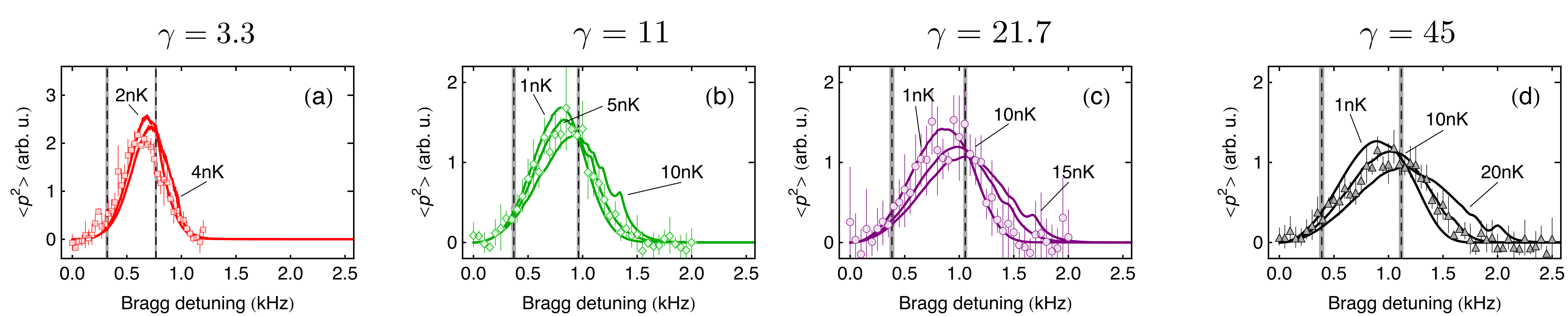
Cold atoms

PRL **115**, 085301 (2015) PHYSICAL REVIEW LETTERS week ending 21 AUGUST 2015

Probing the Excitations of a Lieb-Liniger Gas from Weak to Strong Coupling

F. Meinert,[1] M. Panfil,[2] M. J. Mark,[1,3] K. Lauber,[1] J.-S. Caux,[4] and H.-C. Nägerl[1]

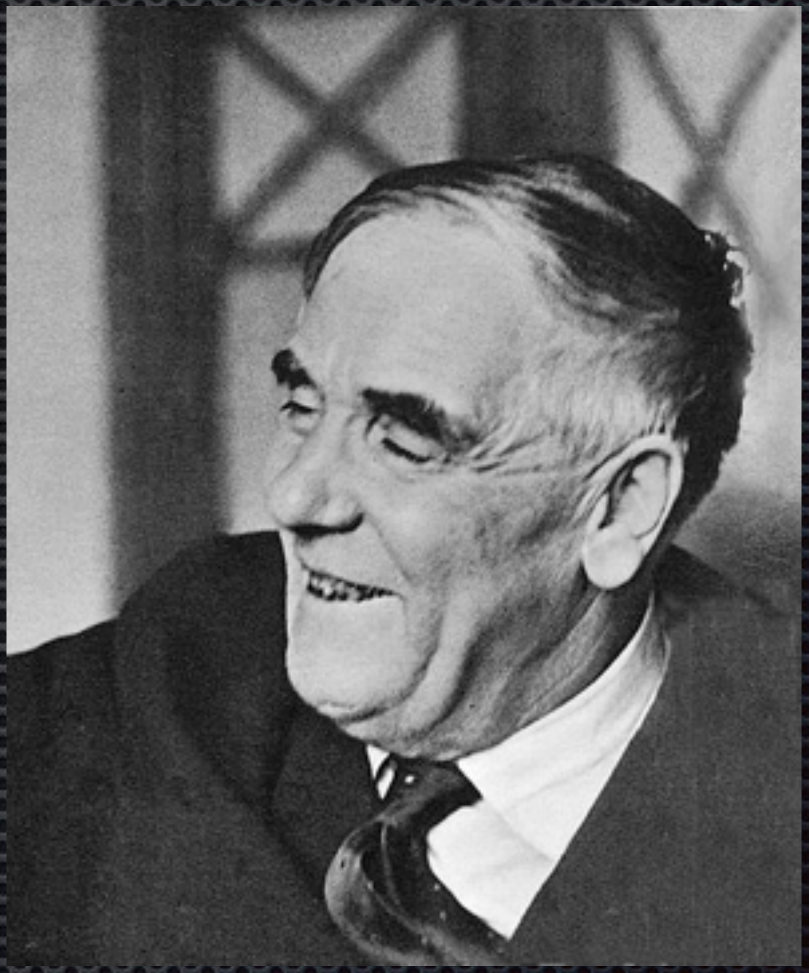




Out-of-equilibrium dynamics from integrability

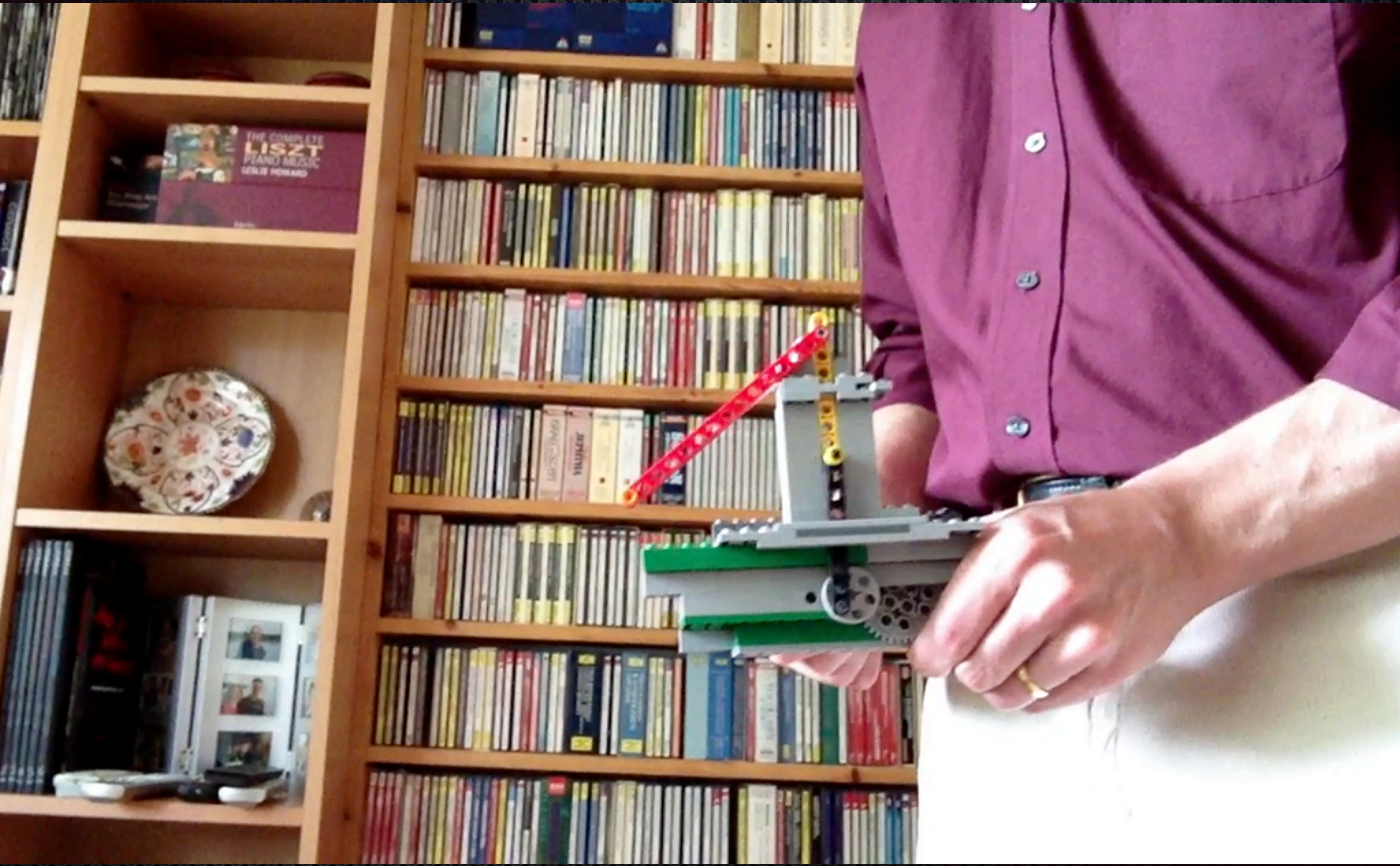
The simple pendulum on its head

Kapitza pendulum, 1951



Pyotr L. Kapitza
(8/7/1894-8/4/1984)

The Kapitza pendulum



Out-of-equilibrium using integrability

Highly excited initial (eigen)states:

- The super Tonks-Girardeau gas
- Split Fermi sea in Lieb-Liniger
- Quasisolitons

Quenched states:

- Interaction quench in Richardson
- Domain wall release in Heisenberg
- Geometric quench
- Interaction turnoff in Lieb-Liniger
- Release of trapped Lieb-Liniger
- BEC to Lieb-Liniger quench
- Quantum Newton's cradle: TG
- Néel to XXZ quench

Driven systems:

- Spin echo in quantum dots

Quasisoliton dynamics in spin chains

Solitons (classical)

John Scott Russell:
wave of translation (1834)

(Herriot-Watt University)

Solitons (classical)

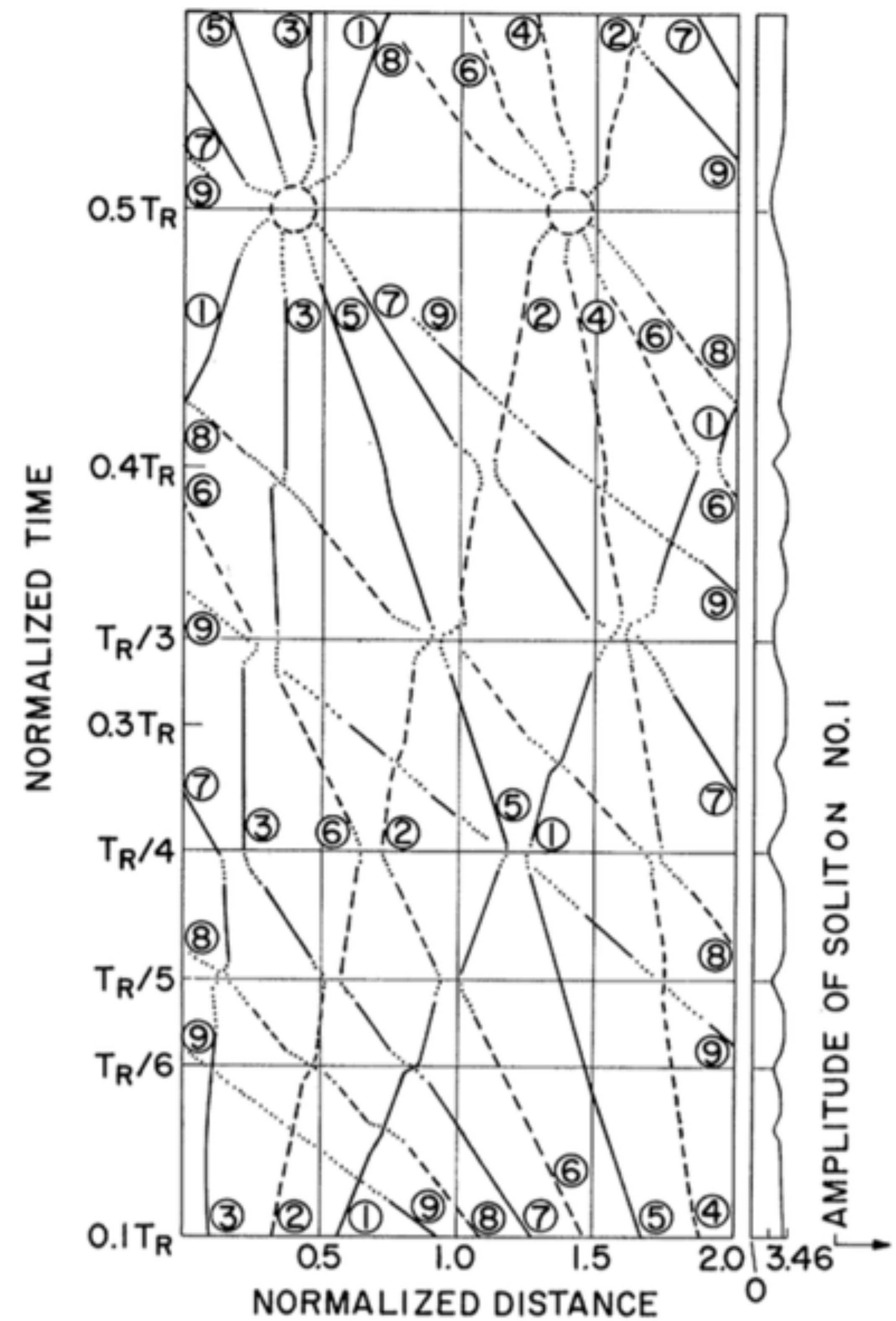
(Boussinesq)
Korteweg-de Vries equation

$$\partial_t u + u\partial_x u + \delta^2 \partial_x^3 u = 0$$

First simulations:
Fermi-Pasta-Ulam-(Tsingou)
absence of ergodicity

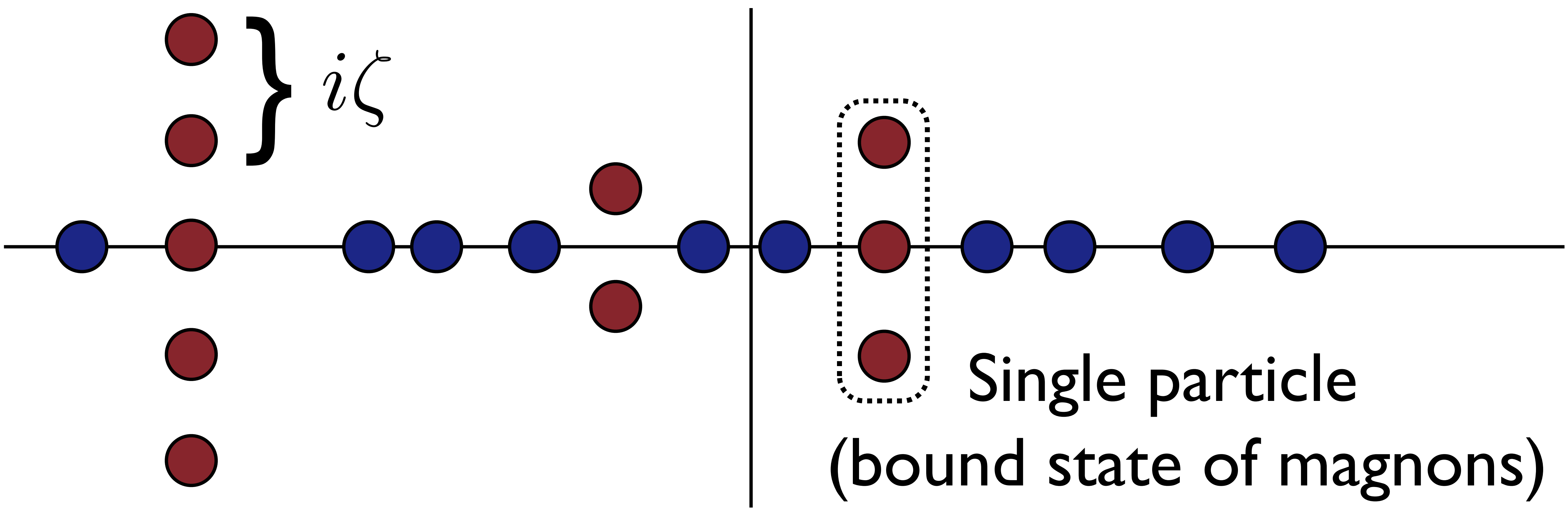
Further simulations:
Zabusky & Kruskal 1965
concept of a soliton

<u>Classical inverse scattering</u>



“Particle content” of XXZ: nontrivial

Solution of Bethe equations: rapidities + strings



$$\lambda_\alpha^{j,a} = \lambda_\alpha^j + i\frac{\zeta}{2}(n_j + 1 - 2a) + i\delta_\alpha^{j,a}$$

Classification of strings: Bethe, Takahashi, Suzuki, ...

String wavepackets

See M. Ganahl, E. Rabel, F. H. L. Essler, and H. G. Evertz 2012 Vlijm, Ganahl, Fioretto, Brockmann, Haque, Evertz and Caux, 2015

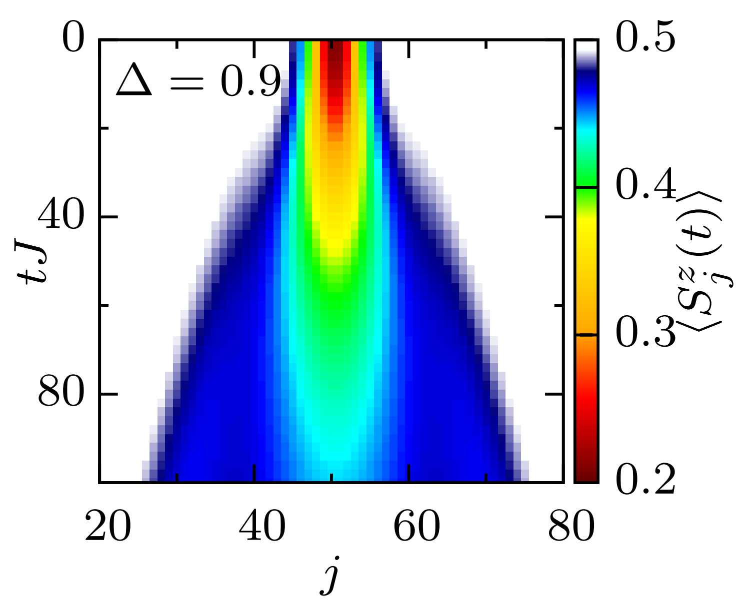
In the eigenbasis, time evolution of a generic state: simple!

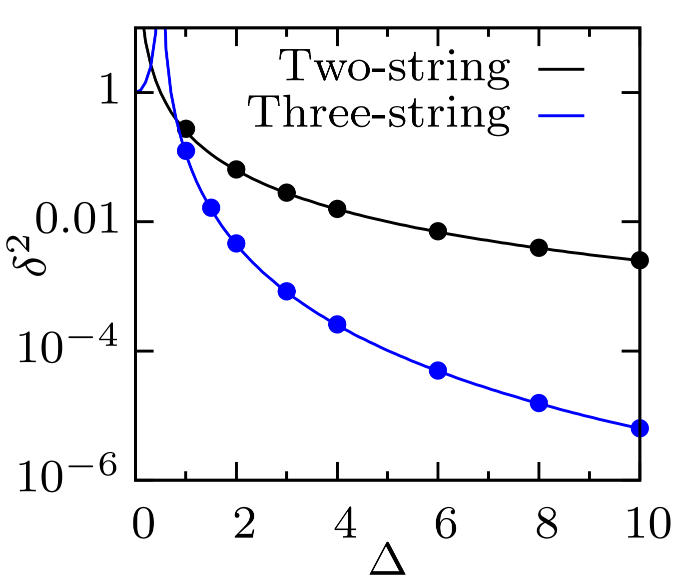
$$|\Psi(t)\rangle = e^{-iHt}|\Psi(0)\rangle = \sum_{\{\lambda\}} e^{-iE_{\{\lambda\}}t} C_{\{\lambda\}} |\{\lambda\}\rangle$$

Localized wavepacket:

$$|\Psi(0)\rangle = \mathcal{N}_0 \sum_p e^{-ip\overline{x} - \frac{\alpha^2}{4}(p-\overline{p})^2} |\lambda^{(n)}(p)\rangle$$

Dispersion:





width ~ t:

$$\Delta x(t) = \sqrt{\frac{\alpha^2}{4} + \frac{\delta_n^2 t^2}{\alpha^2}}$$

function of anisotropy:

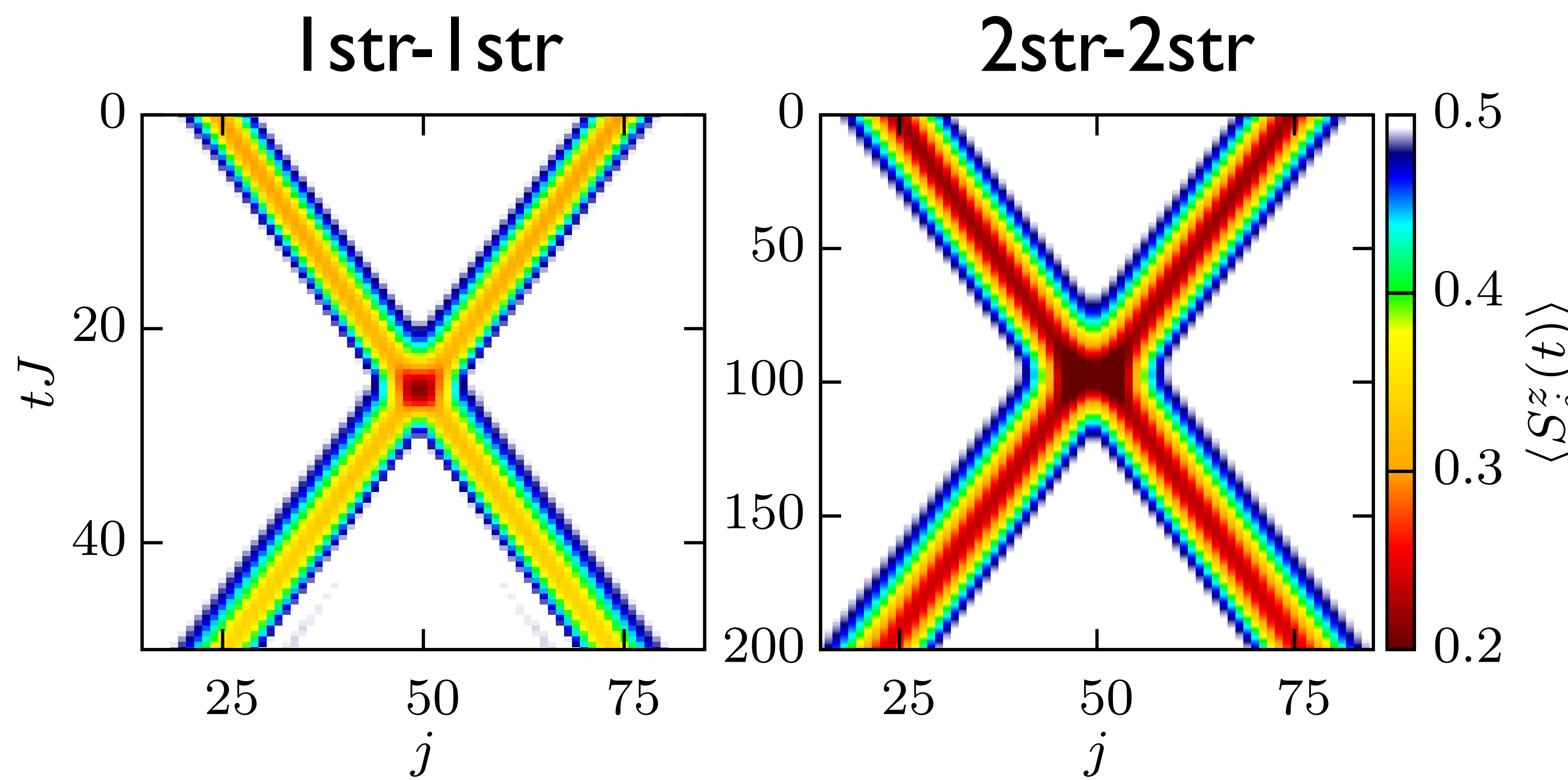
$$\delta_n^2 = J^2 \left(\frac{\phi_2(0)}{\phi_{2n}(0)}\right)^2 \cos^2(\overline{p})$$

Quasisoliton scattering (quantum)

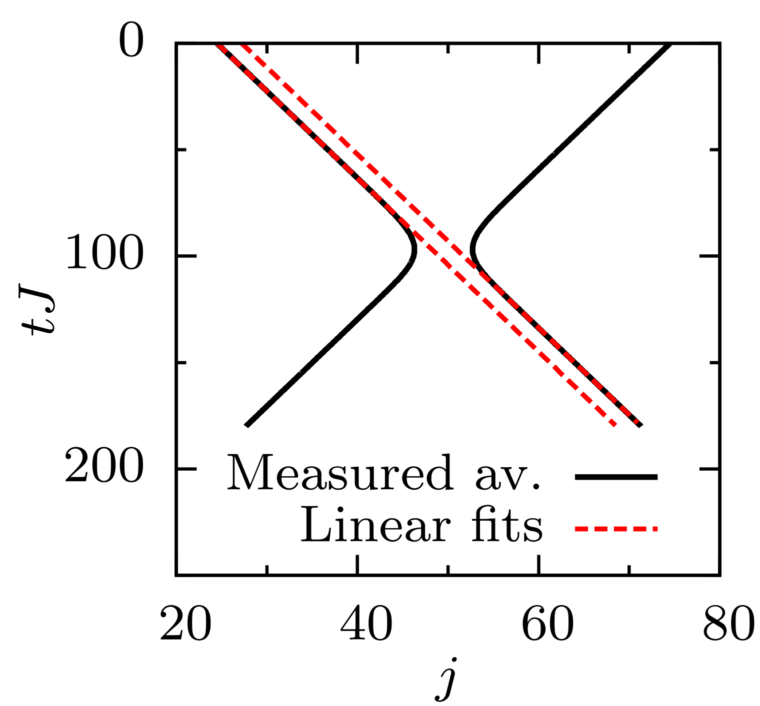
Vlijm, Ganahl, Fioretto, Brockmann, Haque, Evertz and Caux, 2015

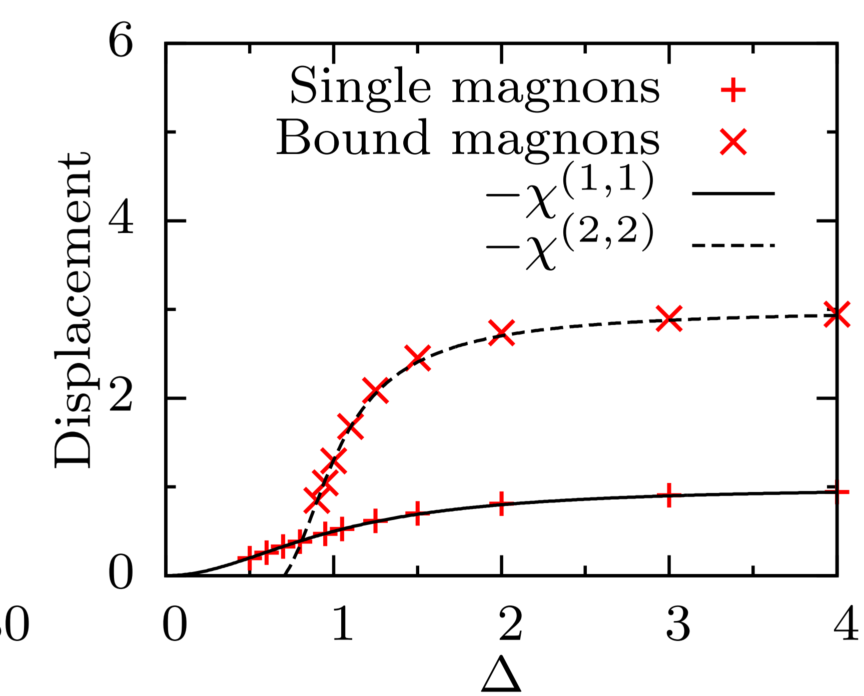
‘Worldlines’ of colliding wavepackets:

$\Delta = 2$



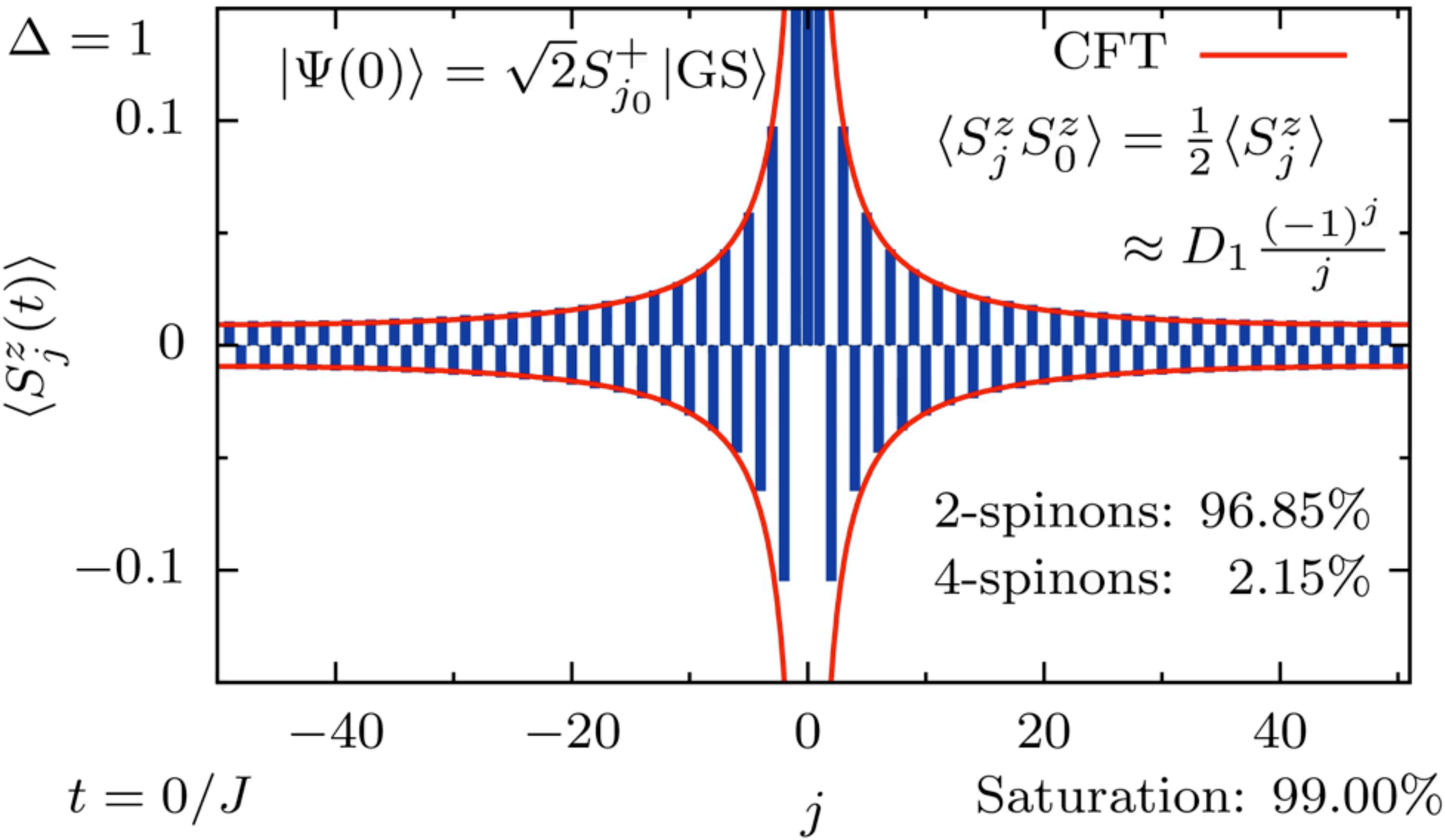
Displacement as a function of anisotropy (fixed incoming momenta)





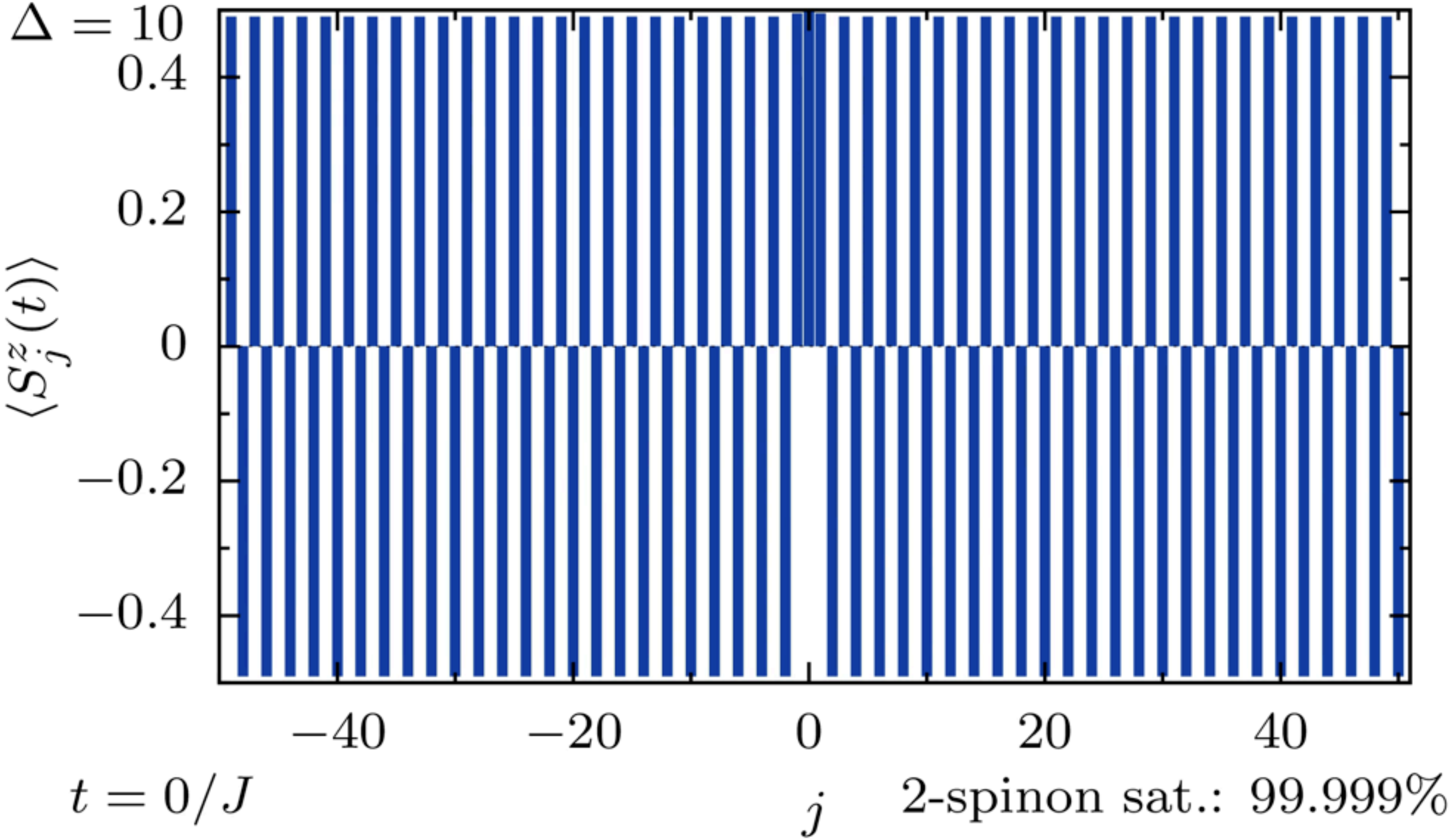
Spinon dynamics in real space/time

Vlijm, Caux, arXiv:1602.03745



Spinon dynamics in real space/time

Vlijm, Caux, arXiv:1602.03745

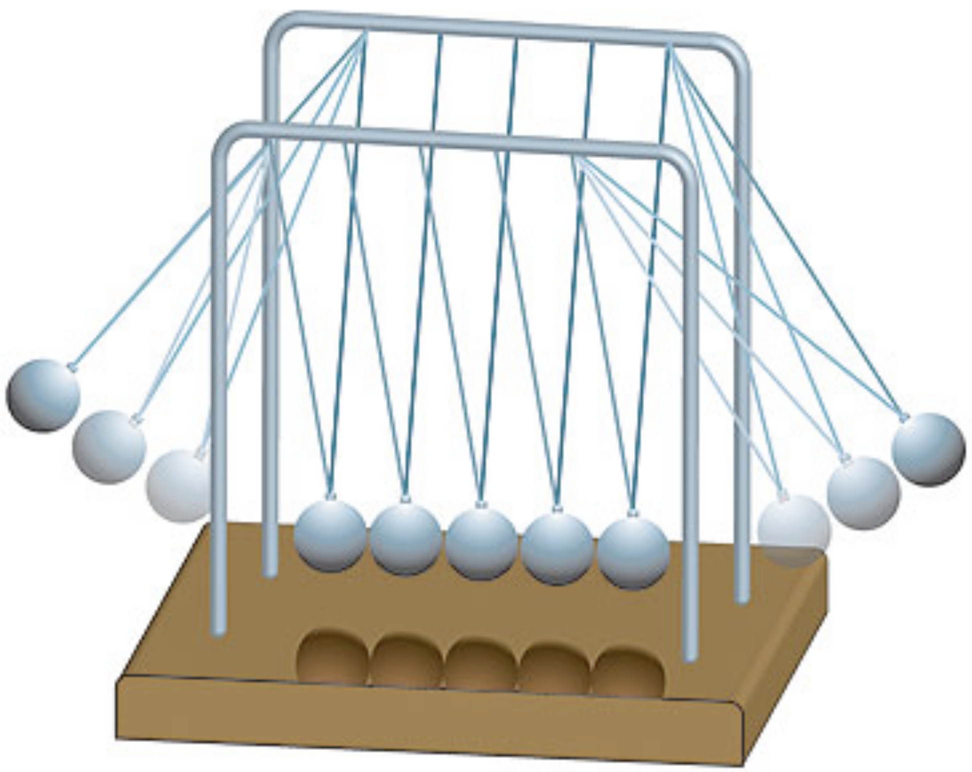


Quantum quenches

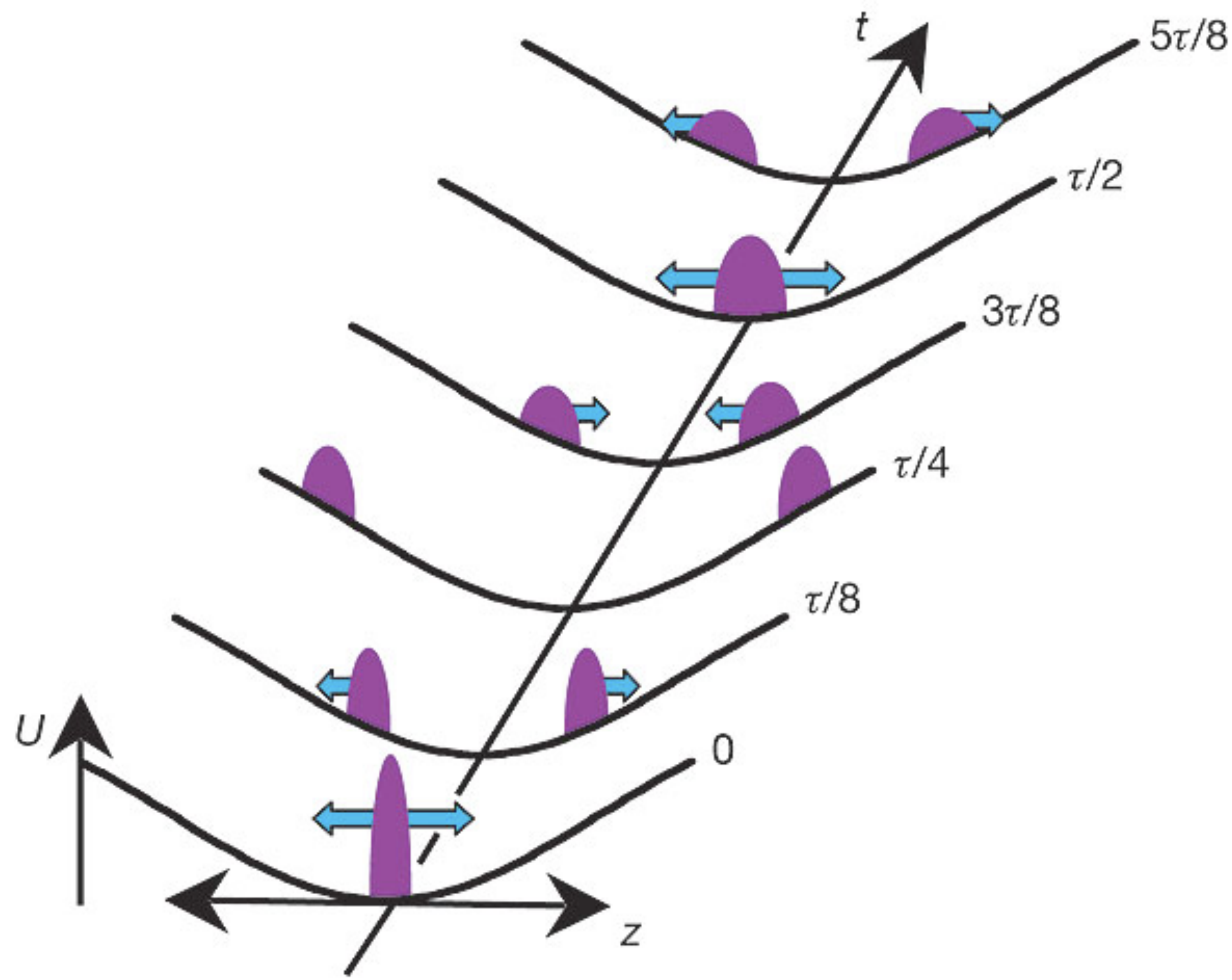
Quantum quenches:

Quantum
Newton’s cradle

David Weiss's quantum Newton's cradle experiment



Ergodicity in interacting quantum systems close to an integrable model



Quantum Newton's cradle: strongly-interacting limit

van den Berg, Wouters, Eliëns, De Nardis, Konik and Caux, 2015

Kapitza-Dirac pulse:

$$\hat{U}_B(q, A) = \exp\left(-iA \int dx \, \cos(qx) \hat{\Psi}^\dagger(x) \hat{\Psi}(x) \right)$$

Tonks-Girardeau limit: bosonic wavefns from fermionic ones

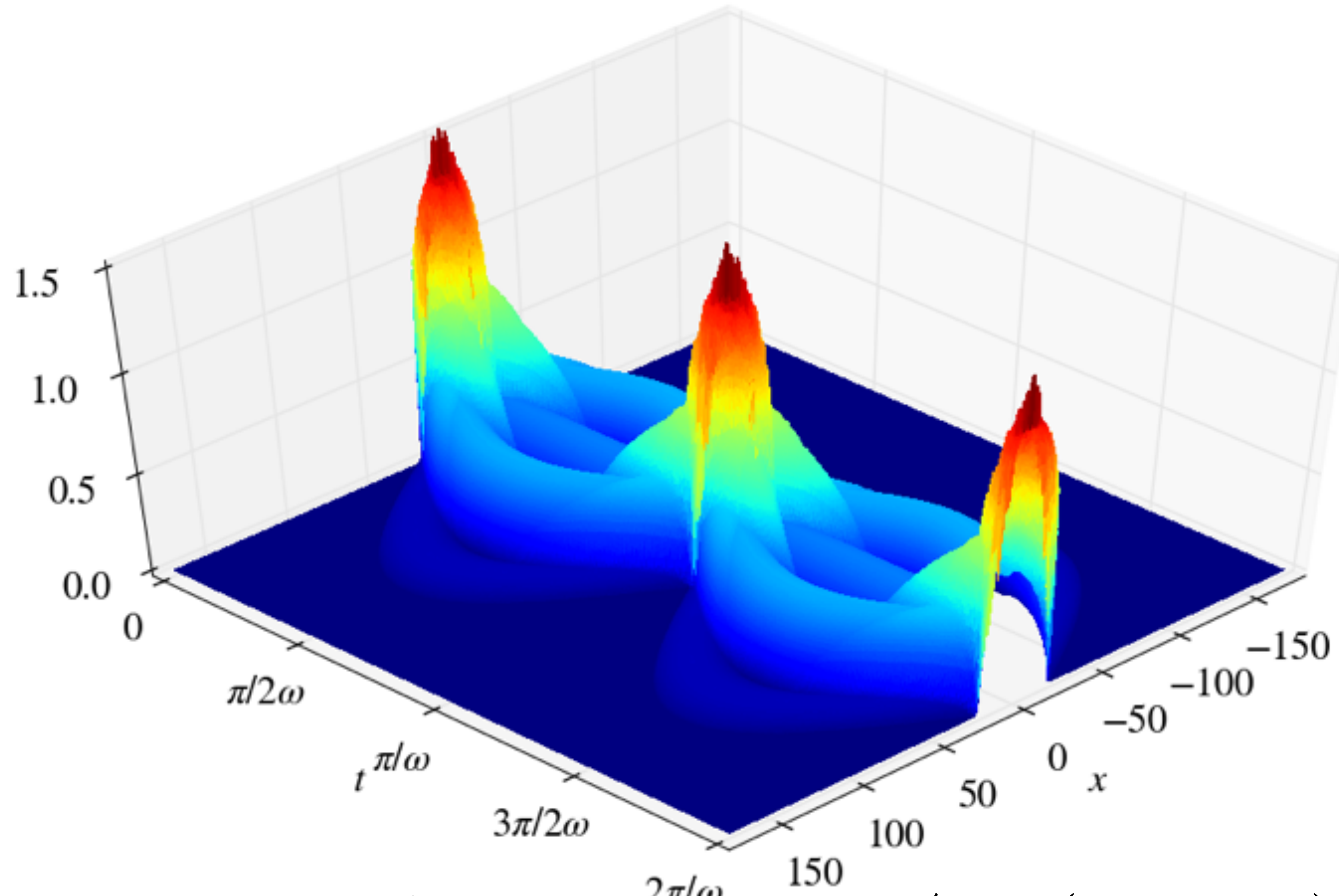
$$\psi_B(\boldsymbol{x}; t) = \prod_{1 \le i < j \le N} \mathrm{sgn}(x_i - x_j) \psi_F(\boldsymbol{x}; t)$$

Slater determinant of single-particle states

Quantum Newton's cradle: TG limit, trap geometry

van den Berg, Wouters, Eliëns, De Nardis, Konik and Caux, 2015

Local density $\langle \hat{\rho}(x,t) \rangle = \langle \hat{\Psi}^\dagger(x,t) \hat{\Psi}(x,t) \rangle$

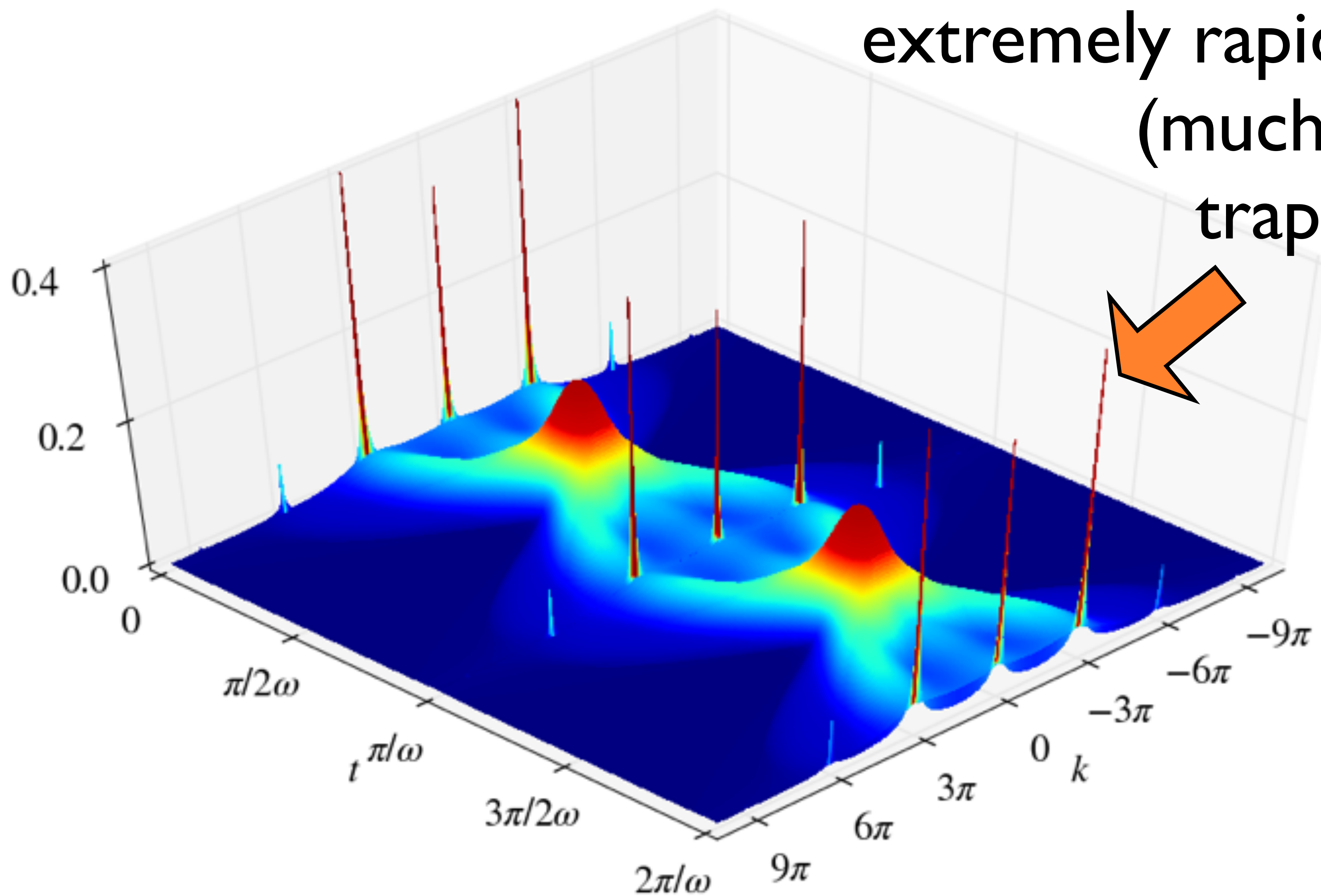


$q = 3\pi, \quad A = 1.5, \quad \omega = 10/N \quad (N = 50)$

Quantum Newton's cradle: TG limit, trap geometry

van den Berg, Wouters, Eliëns, De Nardis, Konik and Caux, 2015

Momentum distn fn $\langle \hat{n}(k,t) \rangle = \frac{1}{2\pi} \int dx dy e^{i(x-y)k} \langle \hat{\Psi}^\dagger(x,t) \hat{\Psi}(y,t) \rangle$

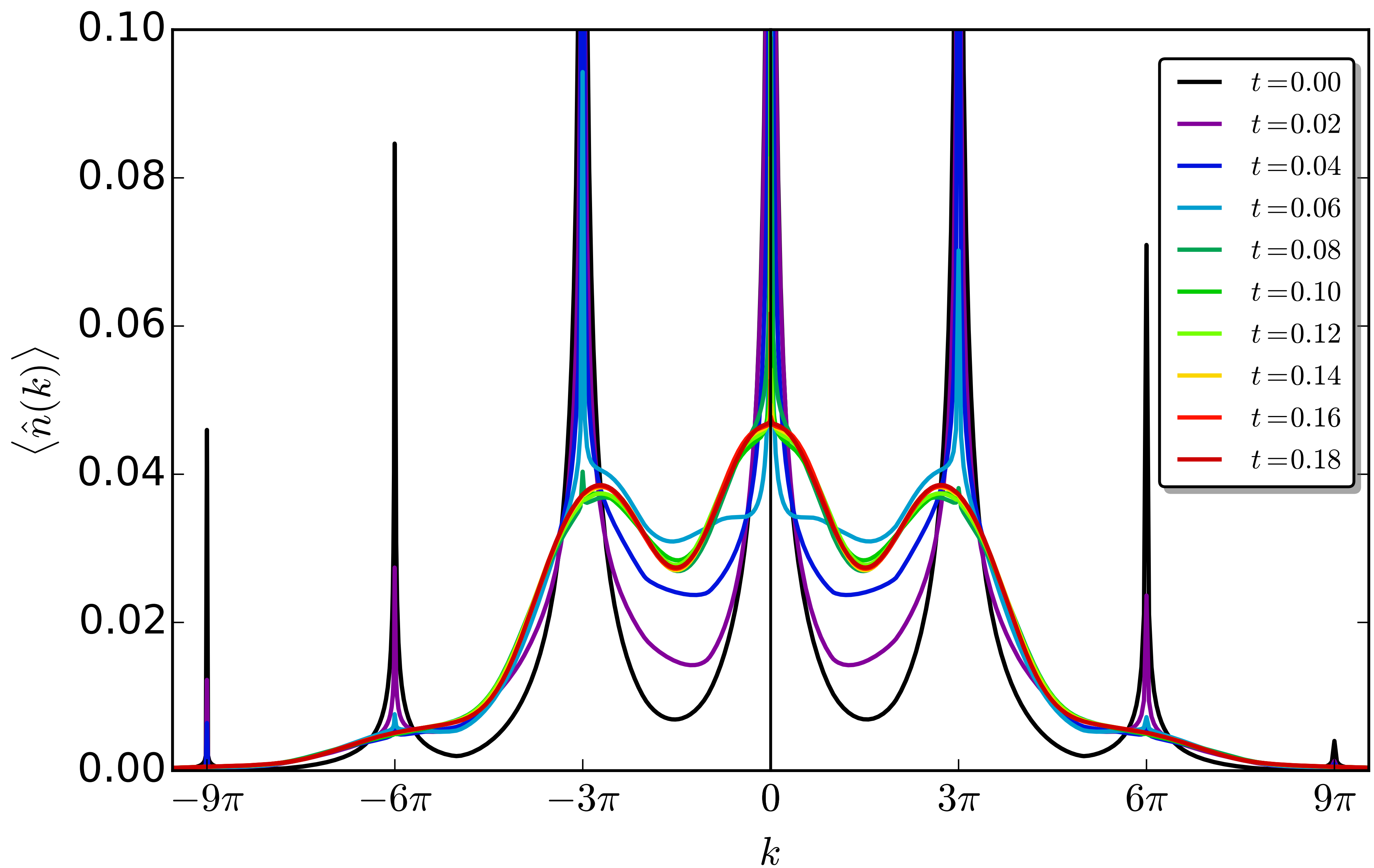


extremely rapid relaxation (much faster than trap oscillation)

$$q = 3\pi, \quad A = 1.5, \quad \omega = 10/N \quad (N = 50)$$

Quantum Newton's cradle: TG limit, ring geometry

van den Berg, Wouters, Eliëns, De Nardis, Konik and Caux, 2015



$$q = 3\pi, A = 1.5$$

Quantum quenches:

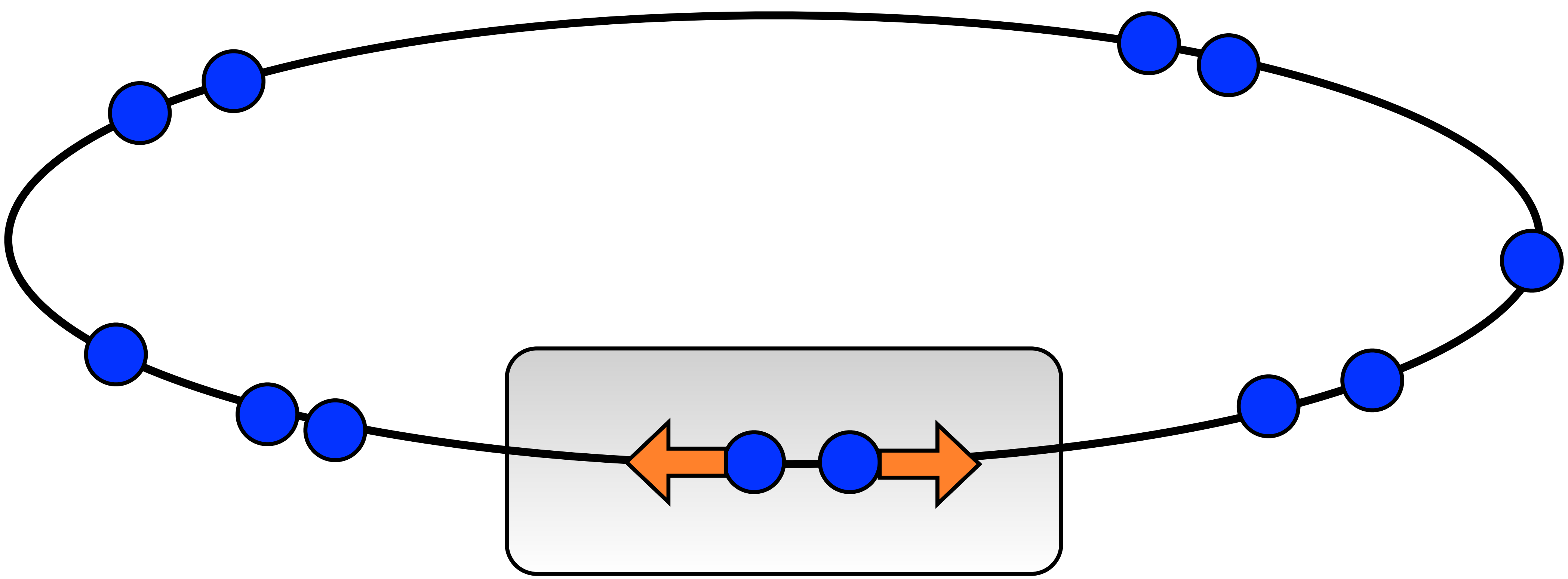
BEC to repulsive Lieb-Liniger quench

Quench from BEC to repulsive gas

Start from GS of noninteracting theory,

$$|0_N\rangle \equiv \frac{1}{\sqrt{L^N N!}} \left(\psi^\dagger_{k=0}\right)^N |0\rangle$$

Turn repulsive interactions on from t=0 onwards:



particles 'repel away' from each other,
system heats up, momentum distribution broadens, ...

This is a difficult problem to treat...

1) Generalized Gibbs ensemble logic

Kormos, Shashi, Chou and Imambekov, arxiv:1204.3889

Conserved charges:

$$\hat{Q}_n : \qquad \hat{Q}_n|\{\lambda\}_N\rangle = Q_n|\{\lambda\}_N\rangle$$

$$Q_n(\{\lambda\}_N) = \sum_{j=1}^{N} \lambda_j^n$$

Davies 1990; Davies and Korepin

→ GGE inapplicable, charges take infinite values!

J-S C + J. Mossel, unpublished

2) GGE on lattice, q-deformed model

→ Works, partial results only (using a few charges)

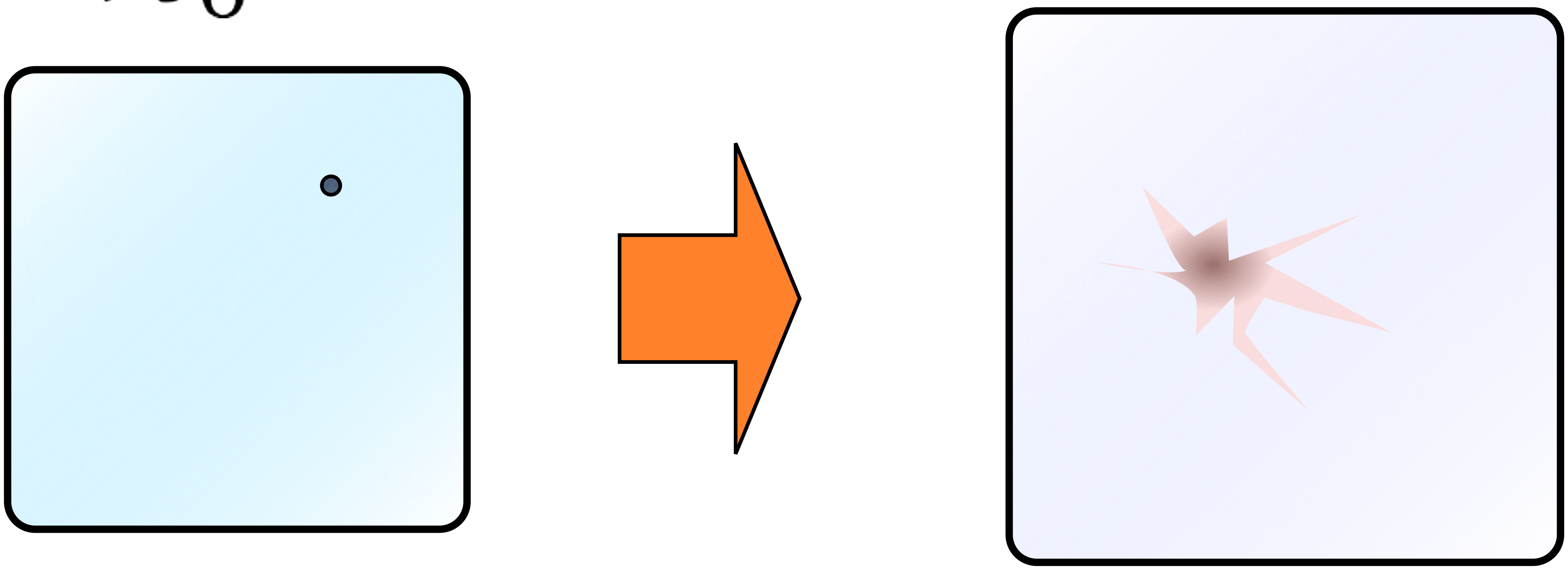
Kormos, Shashi, Chou, JSC, Imambekov, PRA 2014

The ‘quench action’ approach

J-SC & F.H.L. Essler, PRL 2013

in pictures...

$\mathcal{H}_0$ Initial state: $\mathcal{H}$

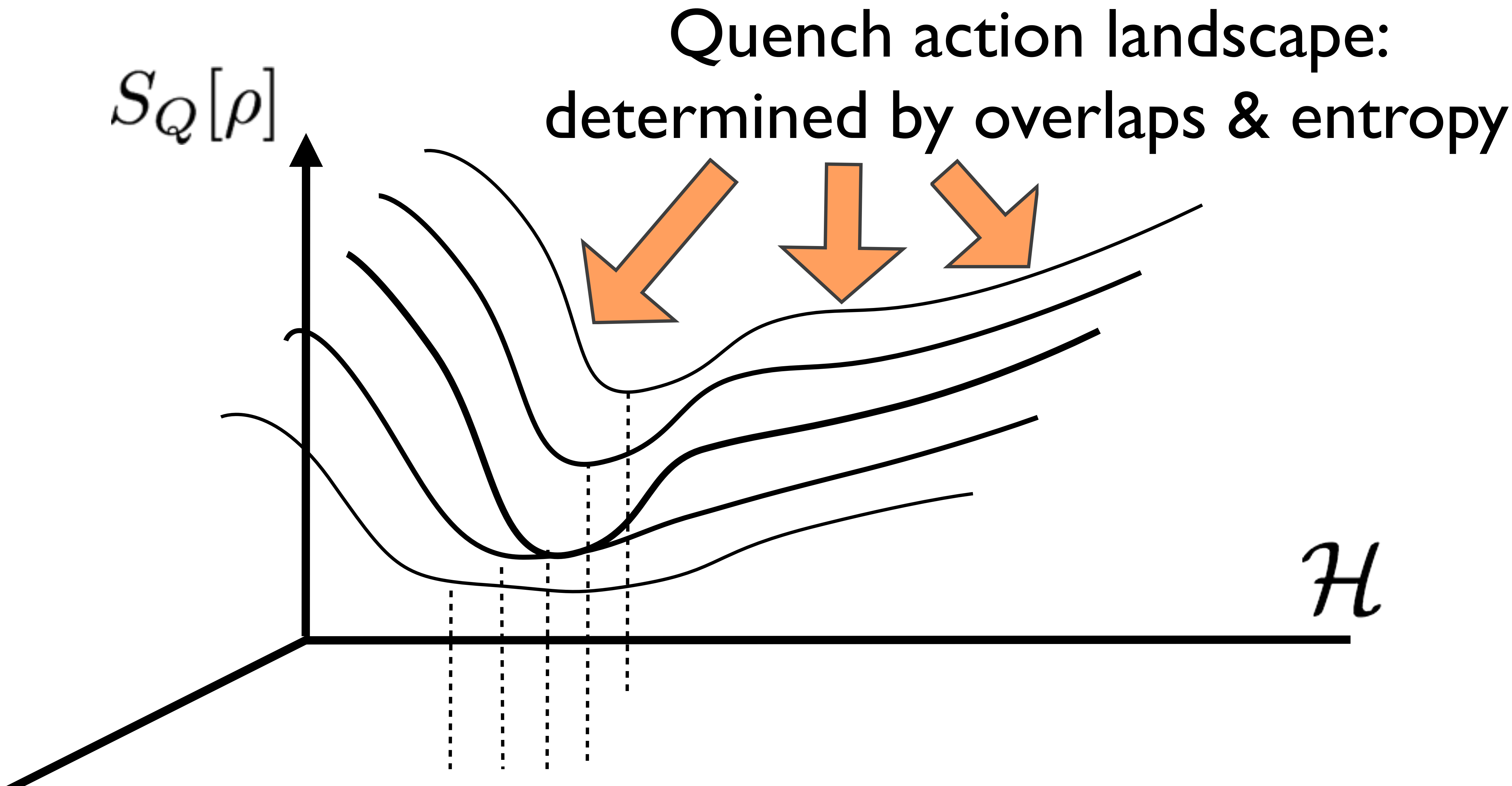


in pre-quench Hilbert space basis

in post-quench Hilbert space basis

The 'quench action' approach

J-SC & F.H.L. Essler, PRL 2013

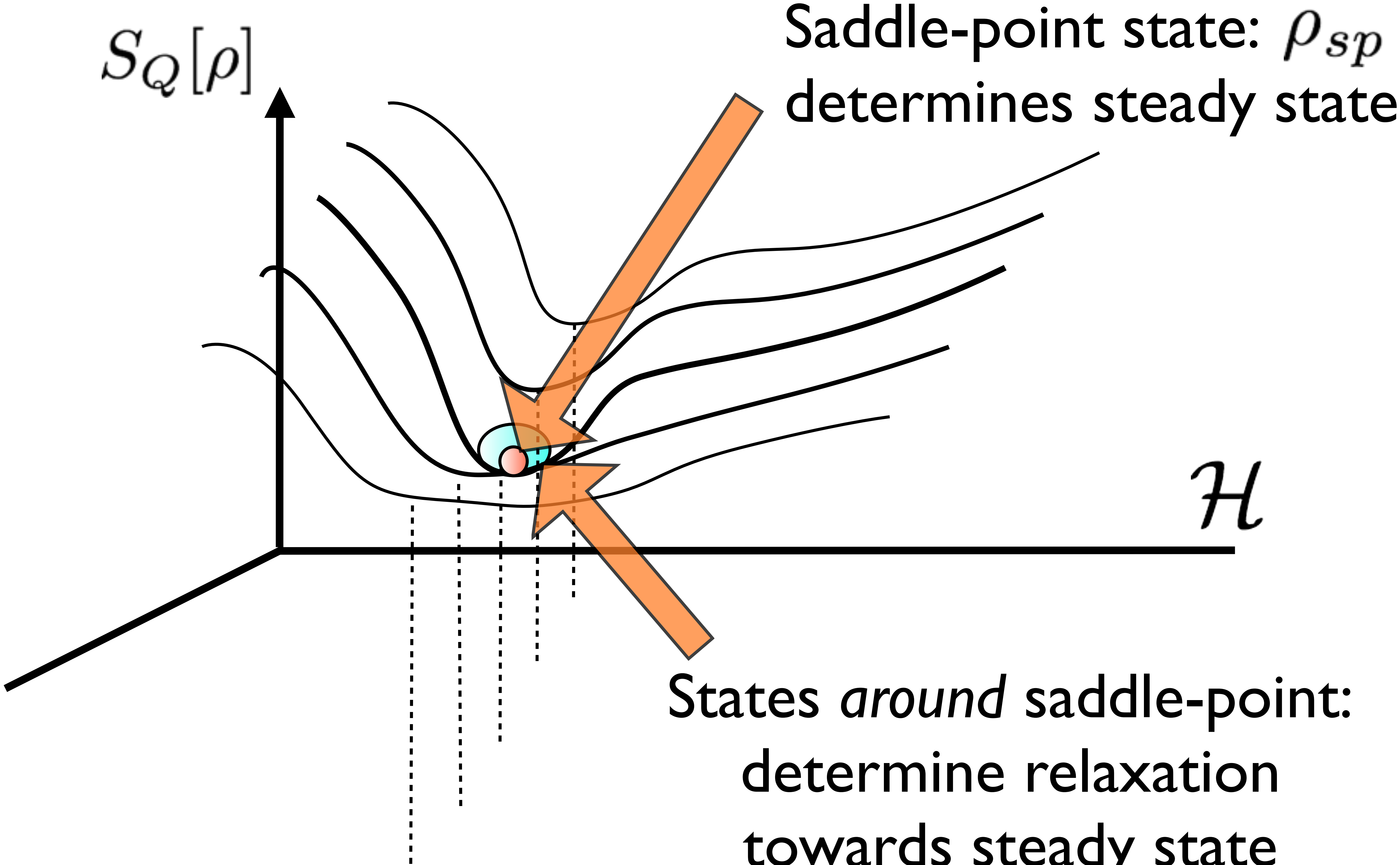


Variational approach, implemented by a 'Generalized thermodynamic Bethe Ansatz'

J. Mossel and J-SC, JPA 2012; J-SC & R. Konik, PRL 2012,
see also Fioretto & Mussardo NJP 2010, Pozsgay JSTAT 2011

The ‘quench action’ approach

J-SC & F.H.L. Essler, PRL 2013



The ‘quench action’ approach

J-SC & F.H.L. Essler, PRL 2013

Generic time-dependent expectation values:

$$\lim_{Th} \bar{\mathcal{O}}(t) = \lim_{Th} \frac{1}{2} \sum_{\{\mathbf{e}\}} \Big[e^{-\delta S_{\{\mathbf{e}\}}[\rho_{sp}] - i\omega_{\{\mathbf{e}\}}[\rho_{sp}]t} \langle \rho_{sp} | \mathcal{O} | \rho_{sp}; \{\mathbf{e}\} \rangle + e^{-\delta S^*_{\{\mathbf{e}\}}[\rho_{sp}] + i\omega_{\{\mathbf{e}\}}[\rho_{sp}]t} \langle \rho_{sp}; \{\mathbf{e}\} | \mathcal{O} | \rho_{sp} \rangle \Big]$$

Main message: the *full* time dependence is recoverable using a minimal amount of data

- saddle-point distribution (from GTBA)
- excitations in vicinity of sp state (easy)
- differential overlaps
- selected matrix elements

Back to BEC-LL quench

Explicit result:

J. De Nardis, B. Wouters, M. Brockmann & J-SC, PRA 89, 2014

M. Brockmann JPA 2014

$$\langle \{\lambda_j\}_{j=1}^{N/2}, \{-\lambda_j\}_{j=1}^{N/2} | 0 \rangle = \sqrt{\frac{(cL)^{-N} N!}{\det_{j,k=1}^{N} G_{jk}}} \frac{\det_{j,k=1}^{N/2} G_{jk}^{Q}}{\prod_{j=1}^{N/2} \frac{\lambda_j}{c} \sqrt{\frac{\lambda_j^2}{c^2} + \frac{1}{4}}}$$

(reminiscent of Gaudin formula)

with matrix $G_{jk}^{Q} = \delta_{jk} \Big(L + \sum_{l=1}^{N/2} K^{Q}(l_j, l_l) \Big) - K^{Q}(l_j, l_k)$

$$K^{Q}(\lambda, \mu) = K(\lambda - \mu) + K(\lambda + \mu) \qquad K(\lambda) = \frac{2c}{\lambda^2 + c^2}$$

Quench action approach to BEC-LL quench

J. De Nardis, B. Wouters, M. Brockmann & J-SC, PRA 89, 2014

We are now in position to apply the quench action logic!

Need thermodynamic limit form of overlaps:

$$\lim_{Th} \langle \lambda, -\lambda | 0 \rangle = \exp\left(-\frac{L}{2} n \left(\log \frac{c}{n} + 1 \right) \right)$$
$$\times \exp \left\{ -\frac{L}{2} \int_0^\infty d\lambda \rho(\lambda) \log \left[\frac{\lambda^2}{c^2} \left(\frac{\lambda^2}{c^2} + \frac{1}{4} \right) \right] + \mathcal{O}(L^0) \right\}$$

Quench action now defined, saddle-point solution via generalized thermodynamic Bethe ansatz

Quench action solution to BEC-LL quench

J. De Nardis, B. Wouters, M. Brockmann & J-SC, PRA 89, 2014

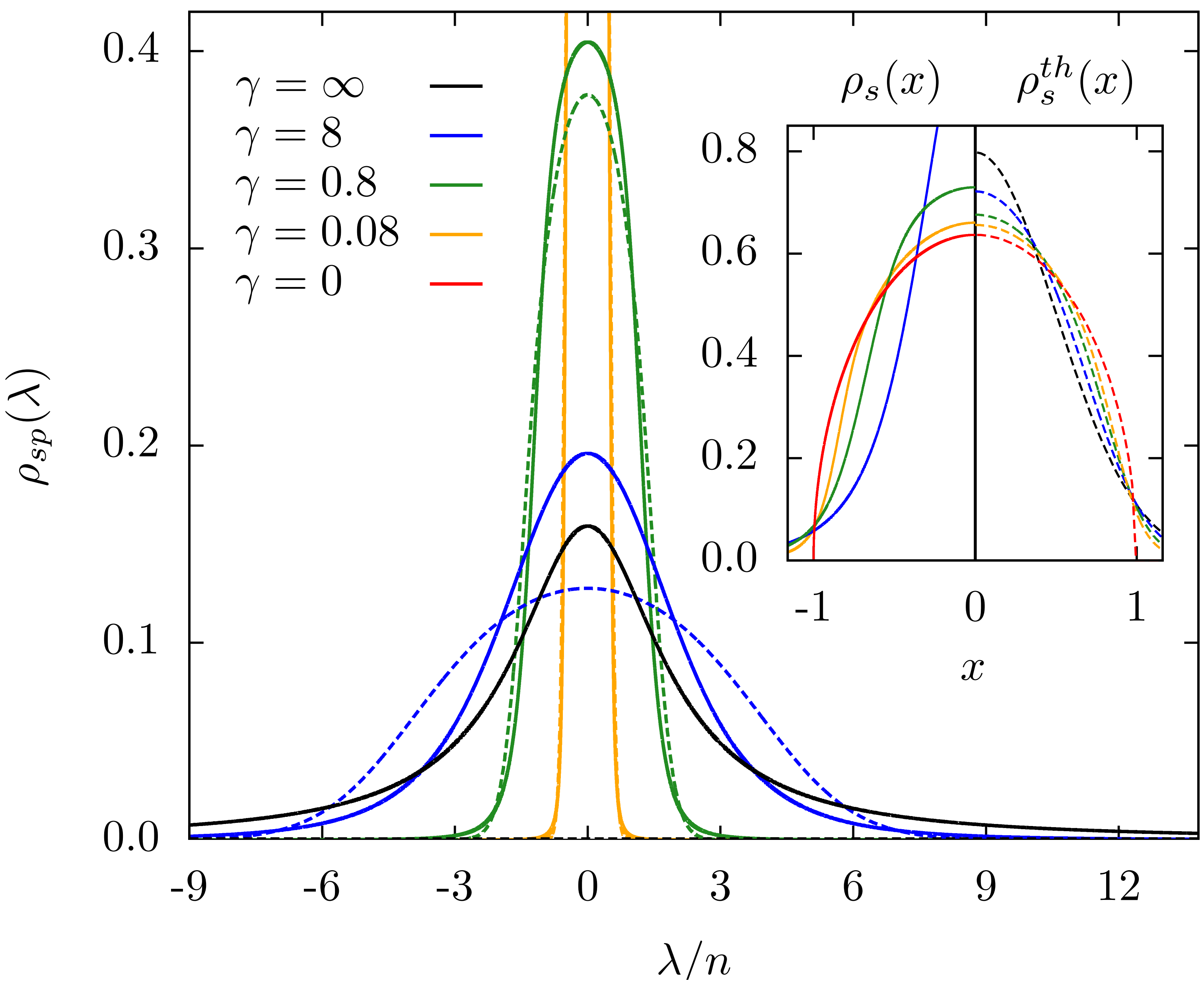
It is in fact possible to give a closed form solution of the GTBA for the saddle-point state, for any value of the interaction:

$$\rho(\lambda) = -\frac{\gamma}{2\pi}\frac{\partial a(\lambda)}{\partial \gamma}(1 + a(\lambda))^{-1}$$

$$a(\lambda) = \frac{2\pi/\gamma}{\frac{\lambda}{c}\sinh\left(\frac{2\pi\lambda}{c}\right)} I_{1-2i\frac{\lambda}{c}}\left(\frac{4}{\sqrt{\gamma}}\right) I_{1+2i\frac{\lambda}{c}}\left(\frac{4}{\sqrt{\gamma}}\right)$$

Quench action solution to BEC-LL quench

J. De Nardis, B. Wouters, M. Brockmann & J-SC, PRA 89, 2014



Subplot: scaled fn

$$\rho_s(x) = \sqrt{\gamma}\rho(c\sqrt{\gamma}x/2)/2$$

Large c:

$$\rho(\lambda) = \frac{1}{2\pi}\frac{4n^2}{\lambda^2 + 4n^2}$$

Small c: semicircle

$$\rho(\lambda) \sim \frac{1}{\pi\sqrt{\gamma}}\sqrt{1 - \frac{\lambda^2}{4\gamma n^2}}$$

Asymptotics as from q-bosons: $2\pi\rho(\lambda) \sim \frac{n^4\gamma^2}{\lambda^4} + \frac{n^6\gamma^3(24-\gamma)}{4\lambda^6} + \dots$

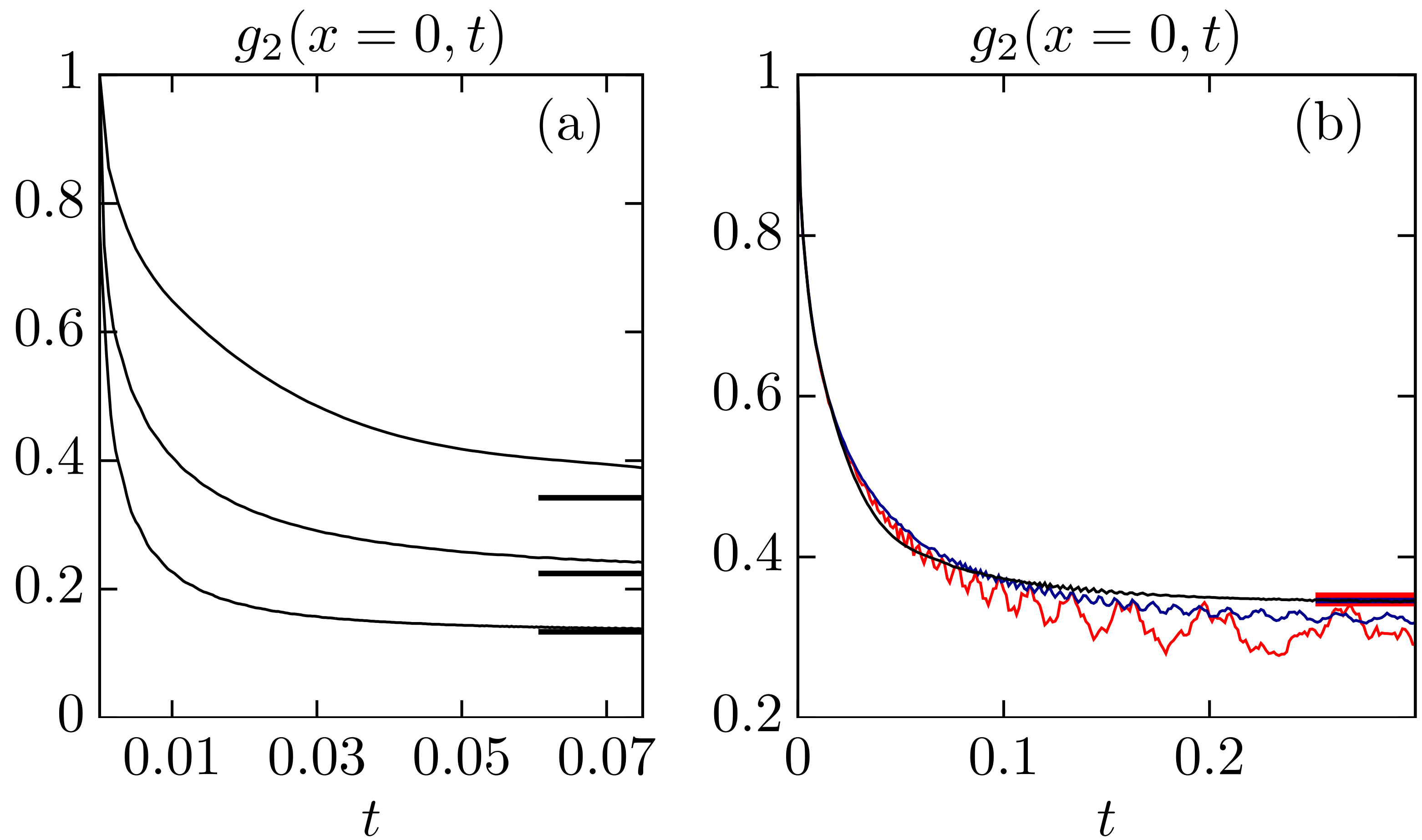
Tail explains divergences of evalues of conserved charges

BEC-LL quench: time dependence

De Nardis, Piroli and Caux, JPA 2015

Time evolution of local density moment:

$$g_2(x=0,t)n^2 = \langle \mathrm{BEC}| e^{i\hat{H}_{LL}t} : \hat{\rho}^2(0) : e^{-i\hat{H}_{LL}t} |\mathrm{BEC}\rangle$$



$\gamma = 4, 8, 16$

(top to bottom)

$\gamma = 4, \;\; N = 6, 8, \infty$

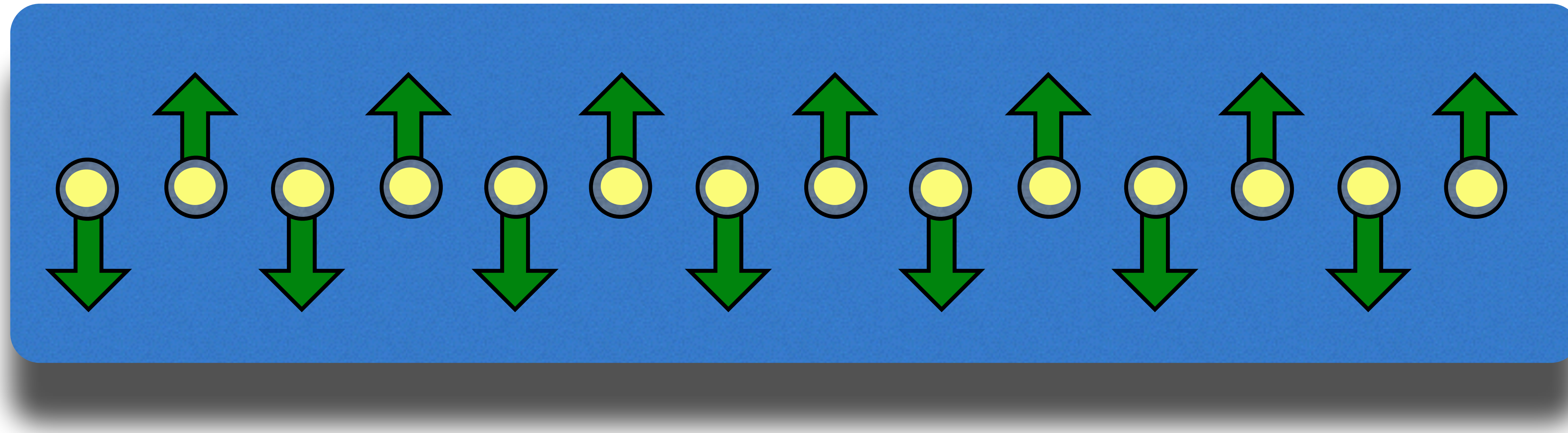
Generically: power law in time for observables

Quantum quenches:

Néel to XXZ quench

Quench from Néel to XXZ

Start from Néel state:



From t=0 onwards, evolve with XXZ Hamiltonian

$$H = \sum_{j=1}^{N} \left[J(S_j^x S_{j+1}^x + S_j^y S_{j+1}^y + \Delta S_j^z S_{j+1}^z) - H_z S_j^z \right]$$

Can one treat this problem exactly?

Quench action approach to Néel-XXZ quench

First step: exact overlaps of Néel state with XXZ eigenstates

Tsuchiya JMP1998; Kozlowski & Pozsgay JSTAT 2012

Gaudin-like form again!

M. Brockmann, J. De Nardis, B. Wouters & J-SC JPA 2014

$$\frac{\langle \Psi_0 | \{\pm\lambda_j\}_{j=1}^{M/2} \rangle}{\| \{\pm\lambda_j\}_{j=1}^{M/2} \|} = \sqrt{2} \left[\prod_{j=1}^{M/2} \frac{\sqrt{\tan(\lambda_j + i\eta/2)\tan(\lambda_j - i\eta/2)}}{2\sin(2\lambda_j)} \right] \sqrt{\frac{\det_{M/2}(G^+_{jk})}{\det_{M/2}(G^-_{jk})}}$$

$$G^{\pm}_{jk} = \delta_{jk} \left(N K_{\eta/2}(\lambda_j) - \sum_{l=1}^{M/2} K^+_\eta(\lambda_j, \lambda_l) \right) + K^{\pm}_\eta(\lambda_j, \lambda_k)$$

$$K^{\pm}_\eta(\lambda, \mu) = K_\eta(\lambda - \mu) \pm K_\eta(\lambda + \mu) \qquad K_\eta(\lambda) = \frac{\sinh(2\eta)}{\sin(\lambda + i\eta)\sin(\lambda - i\eta)}$$

Quench action approach to Néel-XXZ quench

Second step: generalized TBA

B. Wouters, J. De Nardis, M. Brockmann, D. Fioretto, M.Rigol & J-SC, PRL 2014

$$\ln \eta_n(\lambda) = -2\,h\,n - \ln W_n(\lambda) + \sum_{m=1}^{\infty} a_{nm} * \ln\left(1+\eta_m^{-1}\right)(\lambda)$$

where $\eta_n(\lambda) \equiv \rho_{n,h}(\lambda)/\rho_n(\lambda)$ $\quad a_n(\lambda) = \frac{1}{\pi}\frac{\sin n\eta}{\cosh n\eta - \cos 2\lambda}$

and the effective driving terms (pseudo-energies) are

$$W_n(\lambda) = \begin{cases} \frac{1}{2^{n+1}\sin^2 2\lambda}\frac{\cosh n\eta-\cos 2\lambda}{\cosh n\eta+\cos 2\lambda}\prod_{j=1}^{\frac{n-1}{2}}\left(\frac{\cosh(2j-1)\eta-\cos 2\lambda}{(\cosh(2j-1)\eta+\cos 2\lambda)(\cosh 4\eta j-\cos 4\lambda)}\right)^2 & \text{if } n \text{ odd,} \\ \frac{\tan^2\lambda}{2^n}\frac{\cosh n\eta-\cos 2\lambda}{\cosh n\eta+\cos 2\lambda}\frac{1}{\prod_{j=1}^{\frac{n}{2}}(\cosh 2(2j-1)\eta-\cos 4\lambda)^2}\prod_{j=1}^{\frac{n-2}{2}}\left(\frac{\cosh 2j\eta-\cos 2\lambda}{\cosh 2j\eta+\cos 2\lambda}\right)^2 & \text{if } n \text{ even.} \end{cases}$$

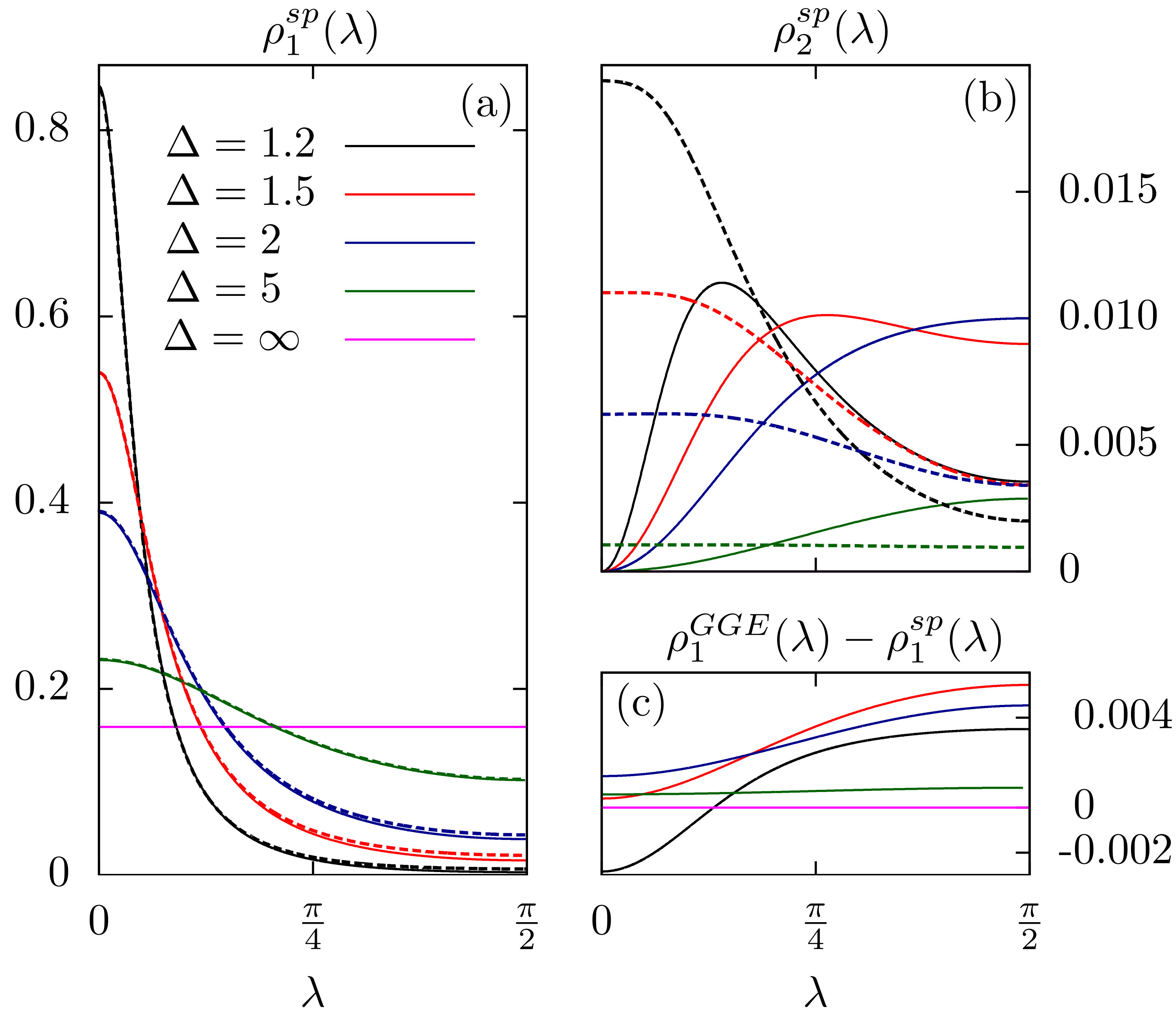
Solution of this GTBA gives steady-state
(analytically!)

The steady state: Néel to XXZ

Solid lines: quench action

Dashed lines: GGE (local charges)

QA and (local)GGE have different saddle-point densities

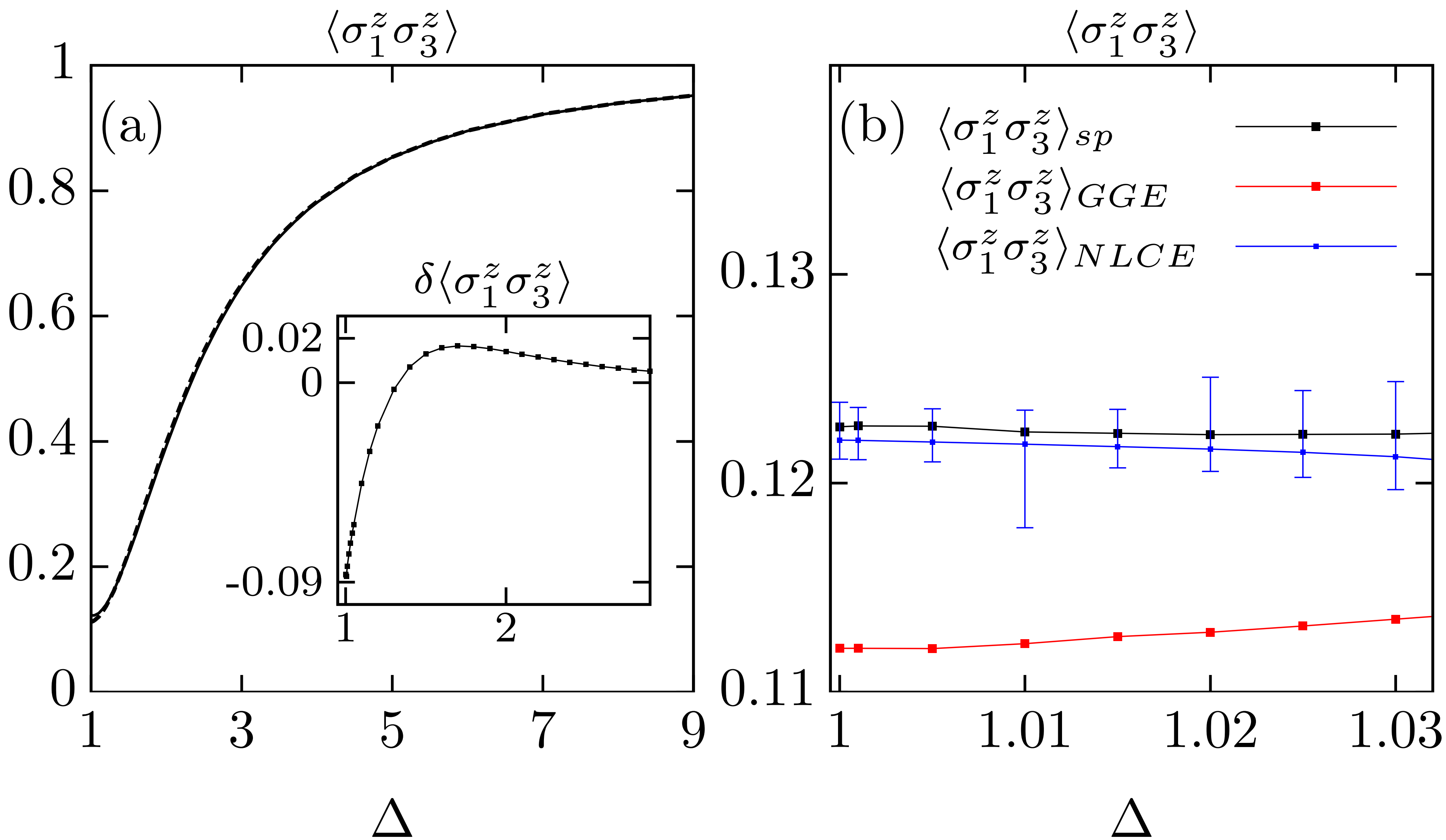


Large Delta expansion:

$$\rho_1^{GGE} - \rho_1^{sp} = \frac{1}{4\pi\Delta^2} + O(\Delta^{-3}),$$

$$\rho_2^{GGE} - \rho_2^{sp} = \frac{1 - 3\sin^2(\lambda)}{3\pi\Delta^2} + O(\Delta^{-3}).$$

Difference in distribution: impact on correlations



Numerical verification using NLCE (M. Rigol)

Large Delta expansions:

$$\langle \sigma_1^z \sigma_2^z \rangle_{QA} = -1 + \frac{2}{\Delta^2} - \frac{7}{2\Delta^4} + \frac{77}{16\Delta^6} + \dots$$

$$\langle \sigma_1^z \sigma_2^z \rangle_{GGE} = -1 + \frac{2}{\Delta^2} - \frac{7}{2\Delta^4} + \frac{43}{8\Delta^6} + \dots$$

Not convinced?

Look at other results by Budapest group

B. Pozsgay, M. Mestyán, M.A. Werner, M. Kormos, G. Zaránd, G. Takács, PRL 2014

- reobtain our Néel results
- also consider initial dimer state
- obtain numerical (iTEBD) evidence for correlations being different in dimer case

There remains no doubt about the correctness of the quench action results because…

‘Revalidating’ the GGE for Néel to XXZ

Quasilocal charges in XXZ

Previously discovered in XXX, XXZ(gapless)

Prosen 2011; Prosen and Ilievski 2013; Ilievski and Prosen 2013; Prosen 2014

Pereira, Pasquier, Sirker and Affleck, JSTAT 2014

Mierzejewski, Prelovšek and Prosen 2015

Here : need generalization to XXZ(gapped)

Ilievski, Medenjak and Prosen, arXiv:1506.05049

Ilievski, De Nardis, Wouters, Caux, Essler, Prosen 2015

Starting point: q-deformed L-operator

$$L(z,s) = \frac{1}{\sinh\eta}\Big(\sinh(z)\cosh(\eta s_s^z)\otimes\sigma^0 + \cos(z)\sinh(\eta s_s^z)\otimes\sigma^z + \sinh(\eta)(s_s^-\otimes\sigma^+ + s_s^+\otimes\sigma^-)\Big)$$

Auxiliary spins obey q-deformed su(2)

$$[s_s^+, s_s^-] = [2s_s^z]_q \quad [s_s^z, s_s^\pm] = \pm s_s^\pm \qquad\qquad [x]_q = \sinh(\eta x)/\sinh(\eta)$$

in 2s+1-dim irrep

$$s_s^z|k\rangle = k|k\rangle, \quad s_s^\pm|k\rangle = \sqrt{[s+1\pm k]_q[s\mp k]_q}|k\pm 1\rangle$$

Quasilocal charges in XXZ(gpd)

Higher-spin transfer matrices:

$$T_s(z) = \mathrm{Tr}_a \left[L_{a,1}(z,s) \ldots L_{a,N}(z,s) \right]$$

lead to spin-s conserved charges

$$X_s(\lambda) = \tau_s^{-1}(\lambda) T_s(z_\lambda^-) T_s'(z_\lambda^+), \qquad z_\lambda^\pm = \pm\frac{\eta}{2} + i\lambda$$

in which $\tau_s(\lambda) = f(-(s+\frac{1}{2})\eta + i\lambda) f((s+\frac{1}{2})\eta + i\lambda) \qquad f(z) = (\sinh(z)/\sinh(\eta))^N$

More convenient for ThLim: $\widehat{X}_s(\lambda) := T_s^{(-)}(z_\lambda^-) T_s^{(+)\prime}(z_\lambda^+)$

built from transfer matrix with $L^{(\pm)}(z,s) = L(z,s)\sinh(\eta)/[\sinh(z \pm s\eta)]$

Families of quasilocal charges:

$$H_s^{(n+1)} = \frac{1}{n!} \partial_\lambda^n \widehat{X}_s(\lambda) \Big|_{\lambda=0} \qquad s = \frac{1}{2}, 1, \frac{3}{2}, \ldots$$

A complete GGE for XXZ

Ilievski, De Nardis, Wouters, Caux, Essler, Prosen 2015

Throughout the gapped regime (including XXX limit), the GGE density matrix is given by

$$\hat{\varrho}_{\mathrm{GGE}} = \frac{1}{Z} \exp\left[- \sum_{n,s=1}^{\infty} \beta_n^s H_{s/2}^{(n)} \right]$$

Steady state: fixed by initial conditions through the generalized remarkable **'string-charge' correspondence**

$$\rho_{2s,h}^{\Psi_0}(\lambda) = a_{2s}(\lambda) + \frac{1}{2\pi} \left[\Omega_s^{\Psi_0}(\lambda + \tfrac{i\eta}{2}) + \Omega_s^{\Psi_0}(\lambda - \tfrac{i\eta}{2}) \right]$$

$$\Omega_s^{\Psi_0}(\lambda) = \lim_{\mathrm{th}} \frac{\langle \Psi_0 | \widehat{X}_s(\lambda) | \Psi_0 \rangle}{N}$$

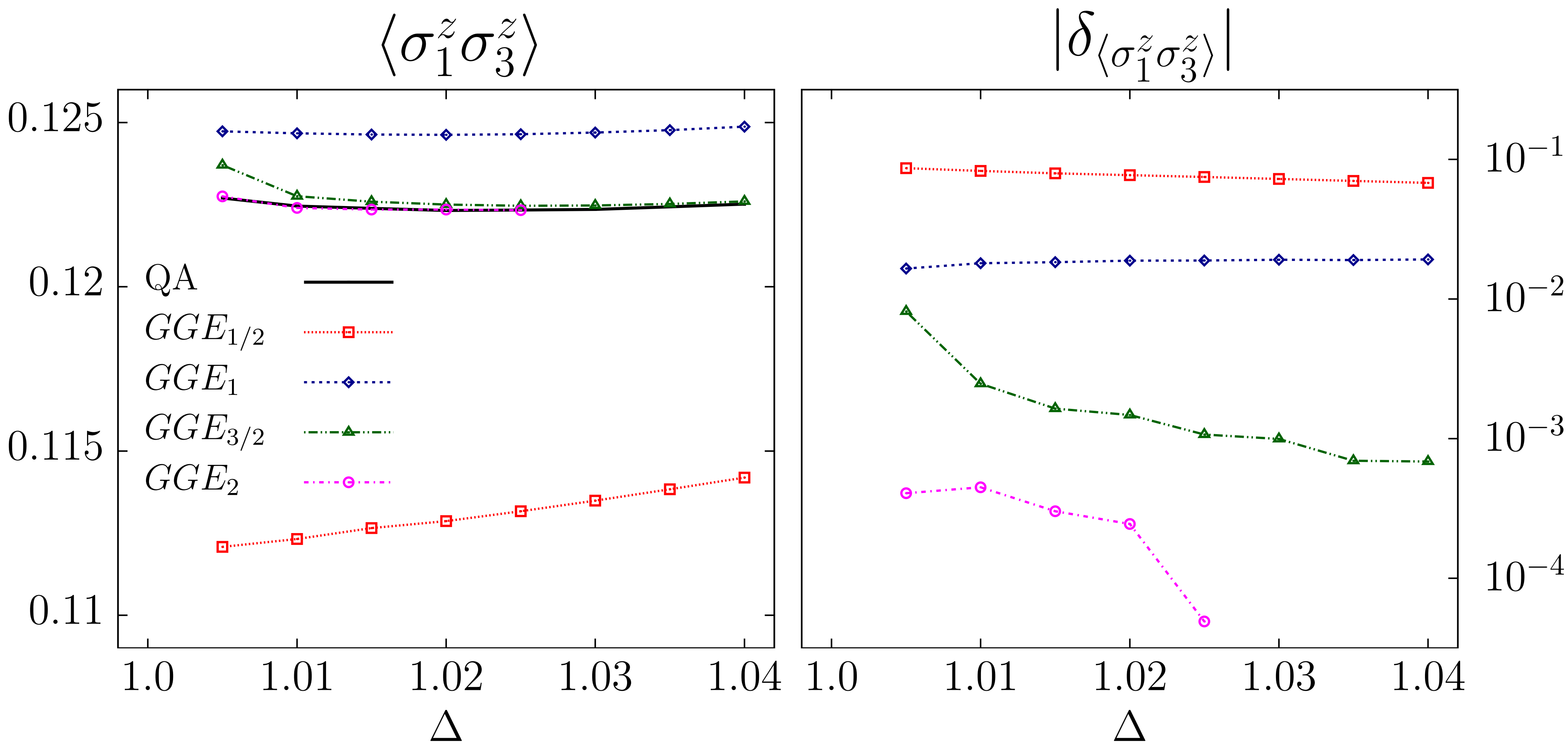
$$s = \frac{1}{2}, 1, \frac{3}{2}, \ldots$$

Fixing the Néel-to-XXZ GGE

Ilievski, De Nardis, Wouters, Caux, Essler, Prosen PRL 2015

Implementing the construction for the Néel-to-XXZ quench makes the GGE converge to correct QA answer

Effect on some simple steady-state correlations:



Summary & perspectives

- Integrability out of equilibrium
  - *real-time dynamics in experimentally accessible setups*
  - *quasisoliton scattering*
  - *pulsed systems*
- Quench action logic
  - *exact approach to out-of-equilibrium problems*
  - *gives access to full time evolution with minimal data*
- Food for thought for GGE users
  - *BEC to LL: exact solution from QA (inaccessible to GGE)*
  - *Néel to XXZ: exact solution from QA*
  - *GGE with local charges gives different steady state!*
  - *GGE needs to include quasilocal charges to reproduce QA*

- Take-home message:

equilibration is steered by new physics