PAUL SCHERRER INSTITUT

Wir schaffen Wissen – heute für morgen

Paul Scherrer Institut (Villigen) and ICTP (Trieste)

Markus Müller

Manifestations of KPZ-scaling in localized quantum particles: Magnetoresistance, conductance fluctuations and the role of exchange statistics

KITP, Non-eq. dynamics of stochastic and quantum integrable systems, Feb 2016

Outline and Scope

- Finding traces of KPZ in disordered quantum systems
  - Understand structure of quantum wavefunctions
  - KPZ equation with complex potential; negative weights
  - Amazing robustness of KPZ uiversality
- Effects of quantum statistics in insulators?
  - Opposite magnetoresistance due to quantum statistics
  - Bosons are harder to localize than fermions
- Experimental traces of KPZ behavior in disordered Cooper pair insulators

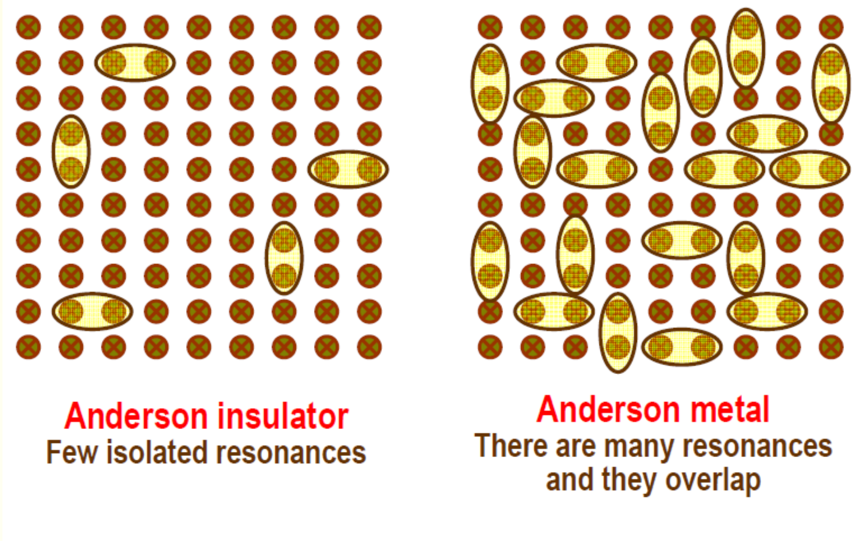
Anderson localization

Anderson localization (1958) [single particle]

$$H = \sum_i \varepsilon_i n_i - t \sum_{\langle i,j \rangle} \left(c_i^+ c_j + \text{h.c.} \right)$$

Resonance = Δε < hopping t

Delocalization transition
(insulator → metal)
= Percolation of resonances



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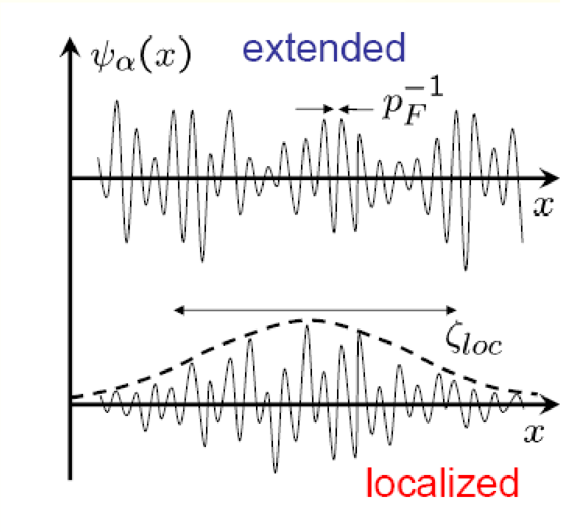
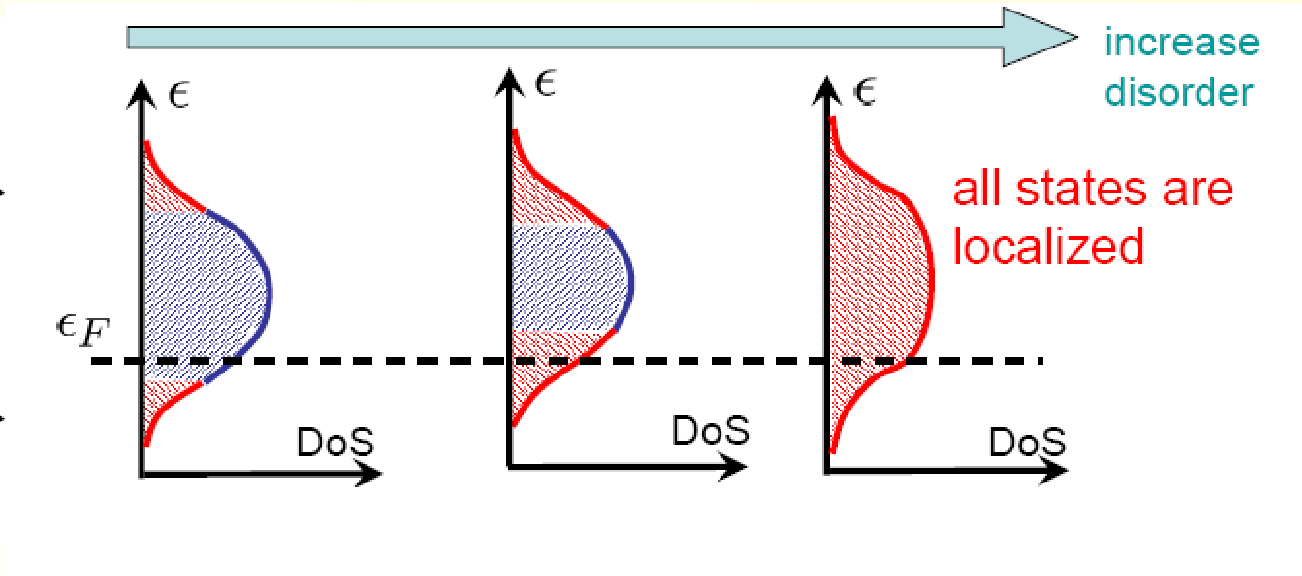
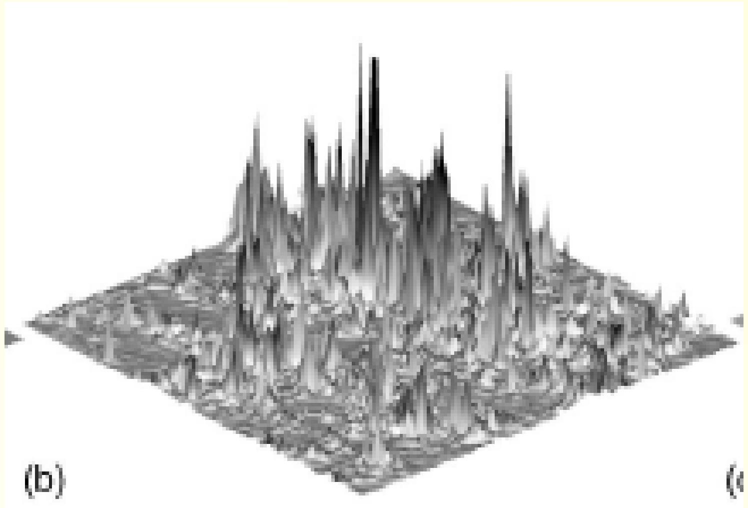




Quantum wavefunctions in d=2 (d>1)

Strongly localized electrons



- Strong inhomogeneity: Optimal disorder paths dominate over diffusive spreading

- Dictionary: Growth, Directed polymer ↔ Q-wavefunction
  Direction of Growth ↔ Distance from origin
  Height function; free energy ↔ $\log(|\psi|^2)$

Localization: Not only single particles!

X. Yu, MM, Ann. Phys. '13

Similar localization properties of excitations above ground states of disordered quantum systems!

e.g.:

- localized "spin waves" in disordered magnets
- excitations in Bose glasses (= "Dirty bosons")

→ Dirty bosons versus dirty fermions?

Examples of "dirty bosons"

- Superconductors with preformed pairs

  Exp. systems: InOx, PbTe, and other negative U systems

- Granular superconductors /

  Josephson junction arrays

- Cold bosonic atoms in disordered potentials

- Disordered quantum spin systems

  (Ising, XY, Heisenberg)

Effects of quantum statistics in insulators?

How are hard core bosons different from free fermions?

(only difference: phase picked up upon exchange of two particles)

Disordered insulators
Simplest model: Hopping+disorder

Model

$$H = \sum_i \varepsilon_i n_i - \sum_{\langle i,j \rangle} t_{ij}(b_j^\dagger b_i + b_i^\dagger b_j), \quad n_i = b_i^\dagger b_i.$$

Fermions

$$\{b_i, b_j\} = 0, , \quad \{b_i^\dagger, b_j\} = \delta_{ij}$$

P. W. Anderson (1958)
.....

Fock space: Fully antisymmetric wavefunctions
with 0 or 1 particle on any given site

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$$\{b_i, b_j\} = 0, , \quad \{b_i^\dagger, b_j\} = \delta_{ij}$$

P. W. Anderson (1958)
.....

Hard core bosons
($\leftrightarrow$ spin ½)

$$[b_i, b_j] = 0, \quad [b_i^\dagger, b_j] = \delta_{ij}(2n_i - 1)$$

Krauth, Trivedi, Randeria;
Feigelman, Ioffe, Kravtsov;
Ioffe, Mézard, Feigelman;
Syzranov, Moor, Efetov;
Yu, MM

Fock space: Fully symmetric wavefunctions
with 0 or 1 particle on any given site

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$$\{b_i, b_j\} = 0, , \quad \{b_i^\dagger, b_j\} = \delta_{ij}$$

P. W. Anderson (1958)
.....

Hard core bosons
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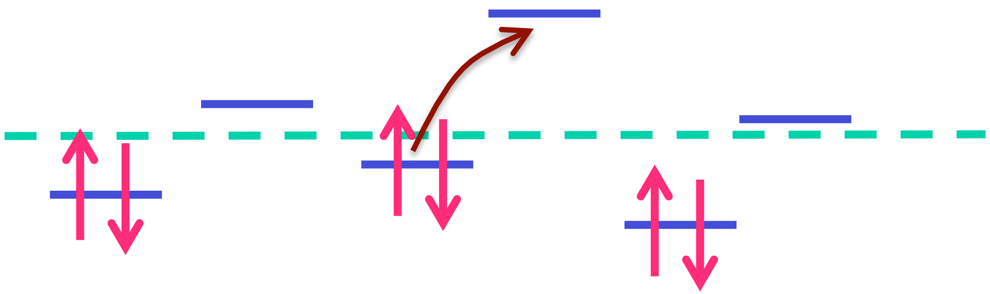
Example: Localized Anderson pseudospins
= doubly occupied or empty orbitals

M. Ma and P. A. Lee (1985),
Kapitulnik and Kotliar (1985)

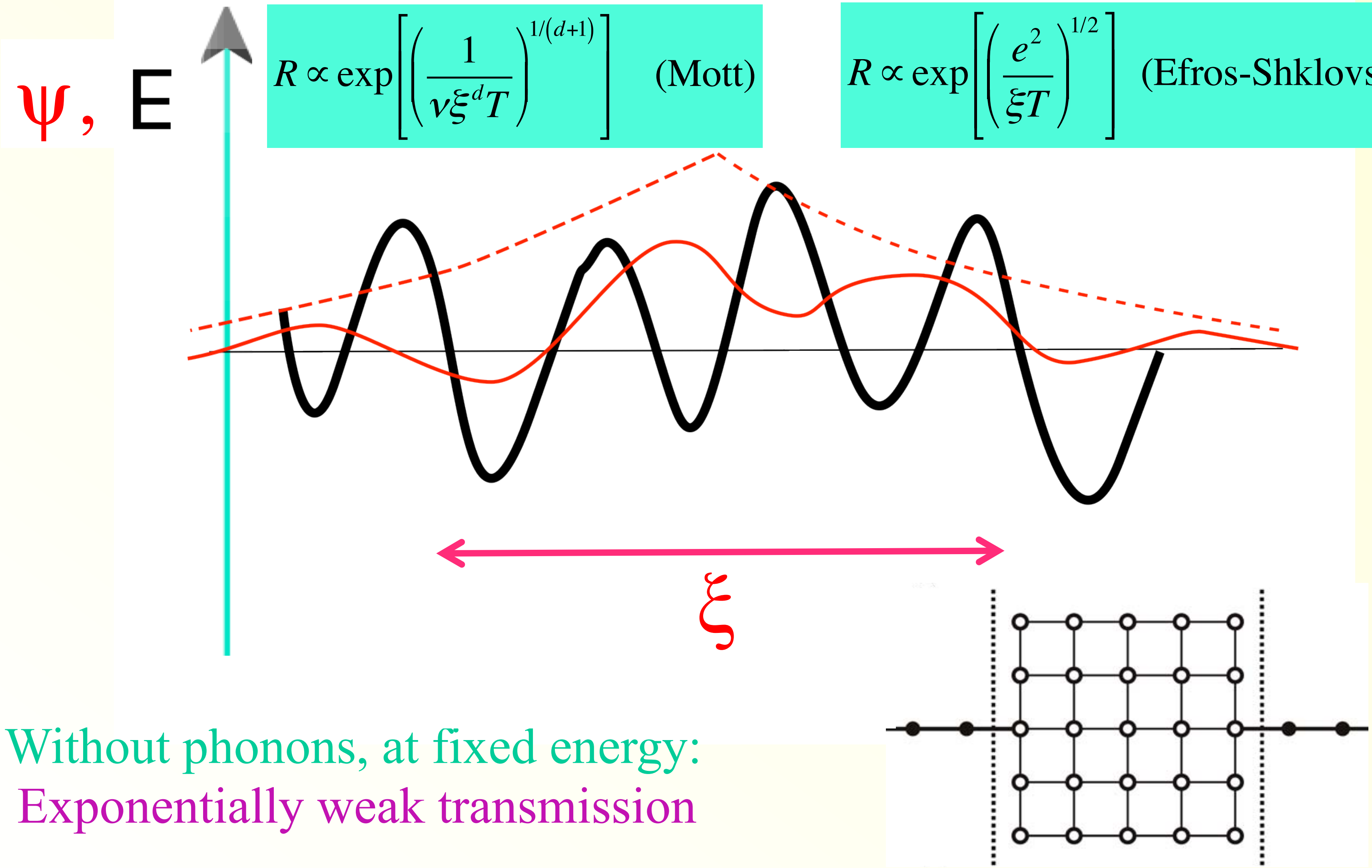


Localization length

Strong insulators: Hopping transport! - Localization length ξ?

$$R \propto \exp\left[\left(\frac{1}{\nu \xi^d T}\right)^{1/(d+1)}\right] \quad \text{(Mott)}$$

$$R \propto \exp\left[\left(\frac{e^2}{\xi T}\right)^{1/2}\right] \quad \text{(Efros-Shklovskii)}$$



Without phonons, at fixed energy:
Exponentially weak transmission

Localization length

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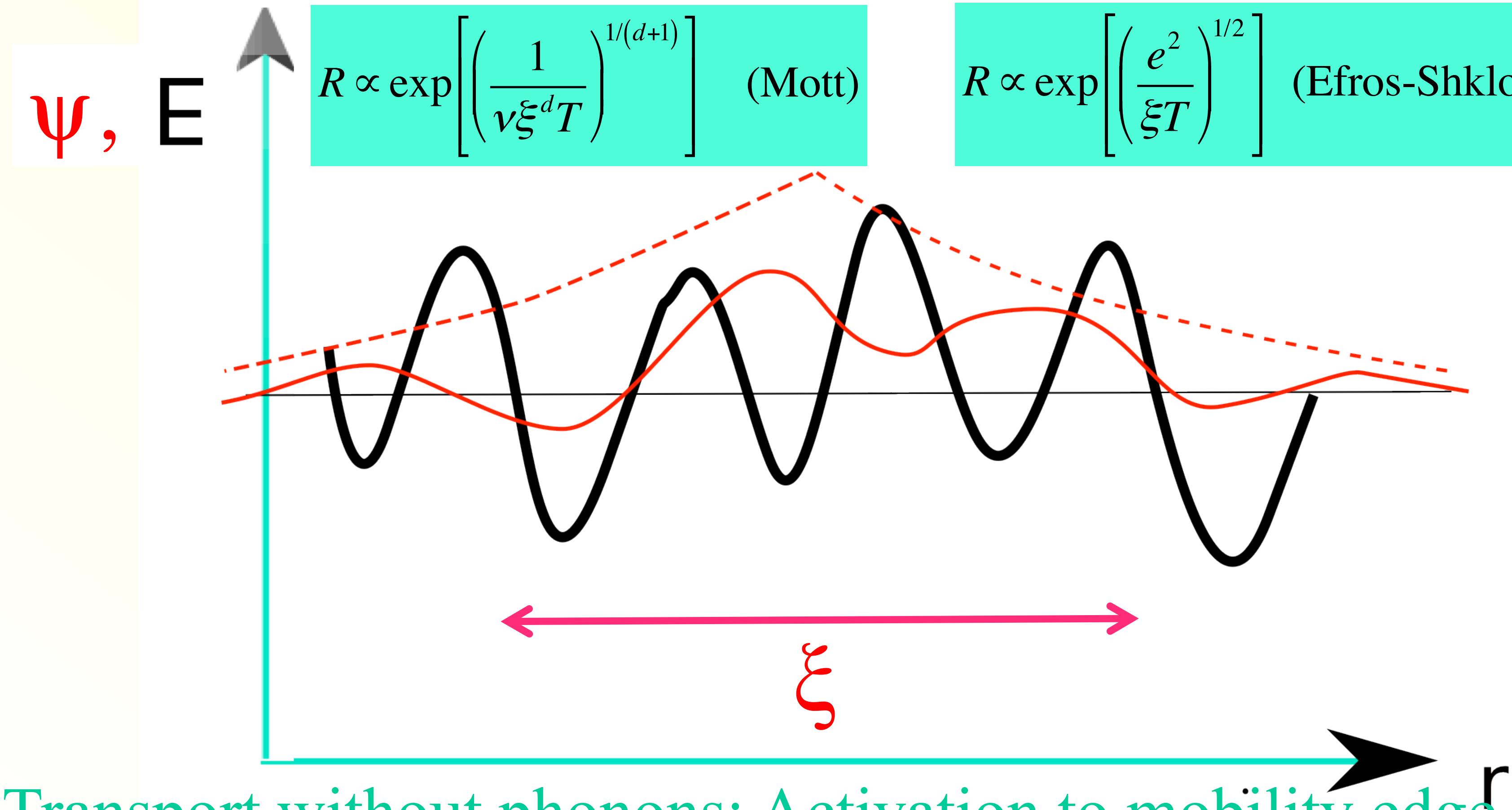

Transport without phonons: Activation to mobility edge

Mobility edge ω_c ⟷ $\xi(\omega \to \omega_c) \to \infty$

Localization length

Fermions $$G^R_{i,0}(t-t') = -i\Theta(t-t')\langle\{b_i(t), b_0^\dagger(t')\}\rangle$$

Bosons $$G^R_{i,0}(t-t') = -i\Theta(t-t')\langle[b_i(t), b_0^\dagger(t')]\rangle$$

Generalized localization length (also interacting)

$$\xi(\omega)^{-1} = -\lim_{\vec{r}_i\to\infty} \overline{\ln[|G^R_{i,0}(\omega)/G^R_{0,0}(\omega)|]/|\vec{r}_i - \vec{r}_0|}.$$

Free fermions: no features near E_F: $\xi(\omega) \sim$ const.

What about bosons?

Locator expansion and forward scattering

Fermions

J. Hubbard (1963): Equation of motion for Green's function!

$$\left(i\frac{d}{dt}-\varepsilon_i\right)G^R_{i,0}(t)$$

$$=\delta(t)\delta_{i,0}+i\Theta(t-t')\left\langle\left\{\sum_{j\in\partial i}t_{ij}b_j(t),b_0^\dagger(t')\right\}\right\rangle$$

$$=\delta(t)\delta_{i,0}-\sum_{j\in\partial i}t_{ij}G^R_{j,0}(t)$$

$$i\frac{d}{dt}b_i(t)$$

$$\{b_i,b_j\}=0,\qquad \{b_i^\dagger,b_j\}=\delta_{ij}$$

Locator expansion and forward scattering

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$$\left(i\frac{d}{dt} - \varepsilon_i\right) G^R_{i,0}(t)$$

$$i\frac{d}{dt}b_i(t)$$

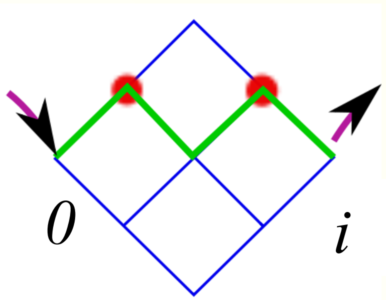
$$= \delta(t)\delta_{i,0} + i\Theta(t-t')\left\langle\left\{\sum_{j\in\partial i} t_{ij} b_j(t), b_0^\dagger(t')\right\}\right\rangle$$

$$= \delta(t)\delta_{i,0} - \sum_{j\in\partial i} t_{ij} G^R_{j,0}(t)$$

Fourier transform → Anderson-Feynman sum over paths *Anderson (1958)*

Forward scattering approximation: Sum over shortest paths!

Spivak, Shklovskii, Nguyen (1983)



$$\frac{G^R_{i,0}(\omega)}{G^R_{0,0}(\omega)} = \sum_{\mathcal{P}=\{j_0=0,\dots,j_\ell=i\}} \prod_{p=1}^{\ell} t_{j_{p-1},j_p} \frac{1}{\varepsilon_{j_p} - \omega}$$

Locator expansion and forward scattering

Fermions

Magnetoresistance: **negative** *(Nguyen, Spivak, Shklovskii)*

Path amplitudes: real with random signs!

B-field: $t_{ij} \longrightarrow t e^{-i\phi_{ij}}$ makes destructive interference less likely $\rightarrow$ ξ increases, R decreases

Forward scattering approximation: Sum over shortest paths!

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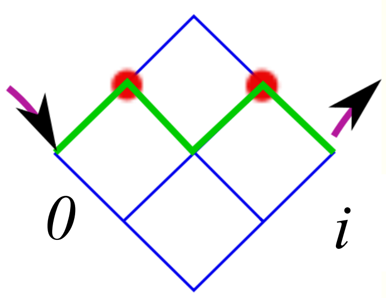

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Locator expansion and forward scattering

Bosons
(hard core)

MM (EPL '13)
X. Yu, MM, Ann. Phys '13

Equation of motion for Green's function!

$$\left(i\frac{d}{dt} - \varepsilon_i\right) G^R_{i,0}(t) = \delta(t)\delta_{i,0}(1 - 2\langle n_0\rangle)$$

$$+ i\Theta(t - t')\left\langle \left[(-1)^{n_i(t)} \sum_{j\in\partial i} t_{ij} b_j(t), b_0^\dagger(t')\right]\right\rangle$$

$$\approx \delta(t)\delta_{i,0}(1 - 2\langle n_0\rangle) - \mathrm{sgn}(\varepsilon_i) \sum_{j\in\partial i} t_{ij} G^R_{j,0}(t)$$

$$[b_i, b_j] = 0, \quad [b_i^\dagger, b_j] = \delta_{ij}(2n_i - 1)$$

Locator expansion and forward scattering

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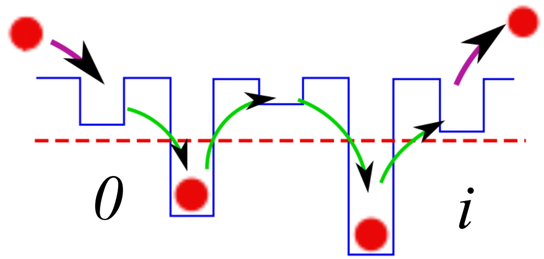
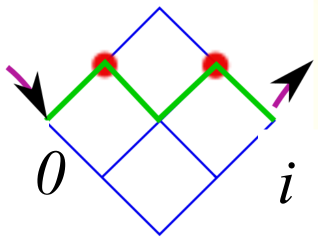
Forward scattering: Sum over shortest paths, lowest order in t!





Sign difference Bosons/Fermions:
Loop of two paths:
Ring exchange of particles

$$\frac{G^R_{i,0}(\omega)}{G^R_{0,0}(\omega)} = \sum_{\mathcal{P}=\{j_0=0,\ldots,j_\ell=i\}} \prod_{p=1}^{\ell} t_{j_{p-1},j_p} \frac{\mathrm{sgn}(\varepsilon_{j_p})}{\varepsilon_{j_p} - \omega}$$

Locator expansion and forward scattering

Bosons (hard core)

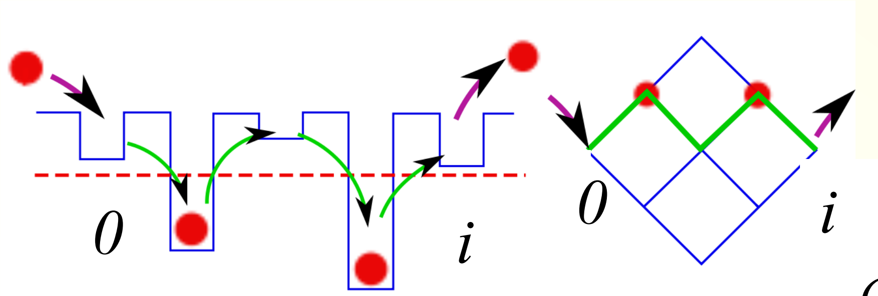
Magnetoresistance: positive

cf also Zhou, Spivak (1991)
Syzranov et al (2012)

Path amplitudes: **all** positive at (ω → 0) !

B-field: $t_{ij} \longrightarrow te^{-i\phi_{ij}}$ destroys constructive interference, ξ decreases, R **increases**.

Forward scattering: Sum over shortest paths, lowest order in t!



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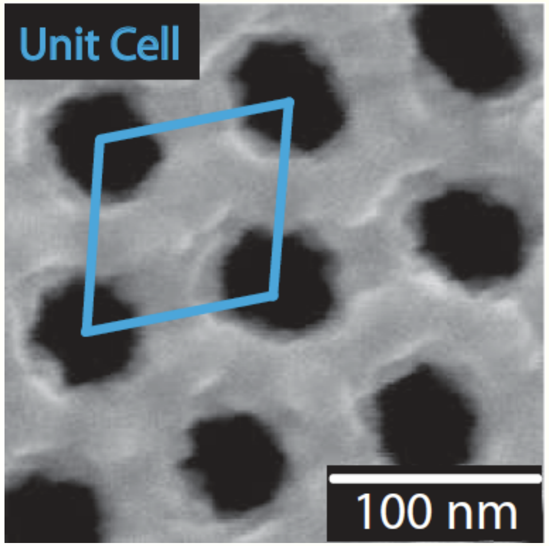
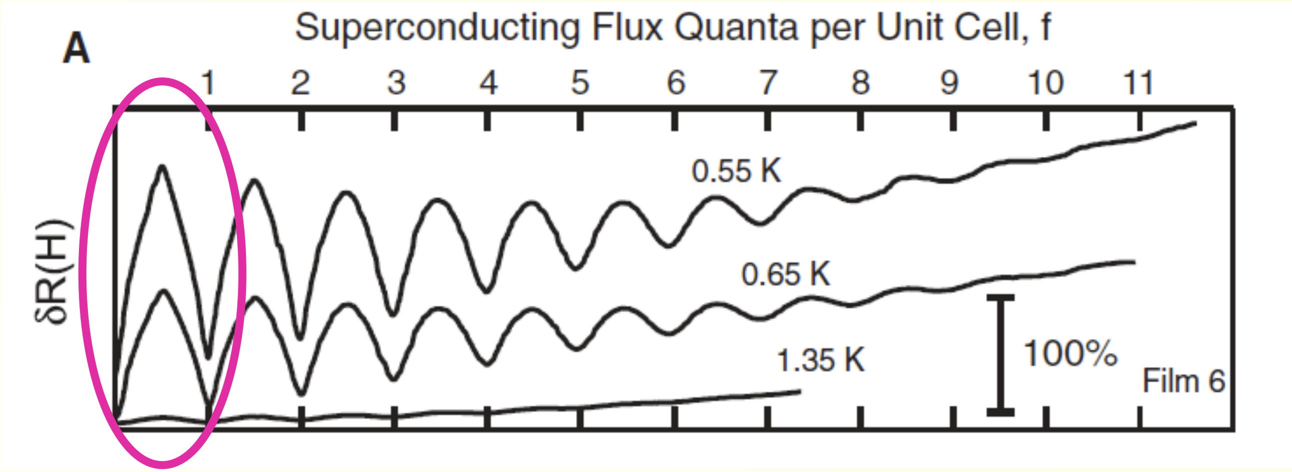




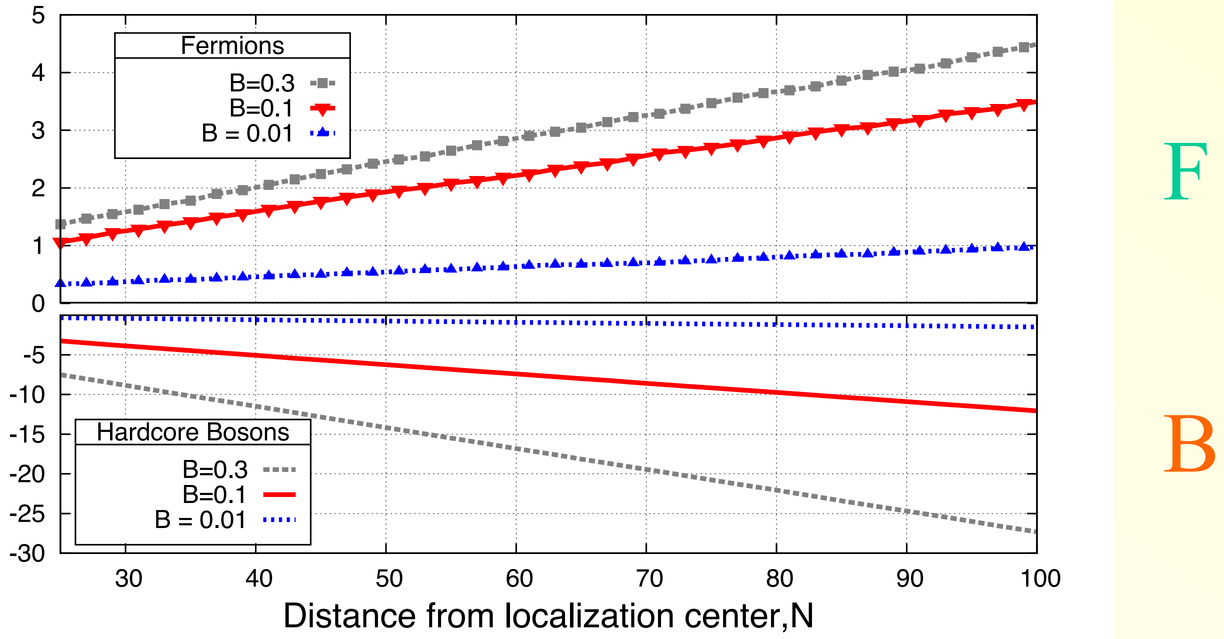
J. Valles et al. (2007-11): Patterned Bi films - Oscillations start with pos. MR: smoking gun for bosons!

Bosons vs fermions?

Bosons (hard core)		Fermions
Strongly positive	← Mag. Resistance →	Weakly negative
Shrinks	← Wavefunction →	Expands

$$N\left[\frac{1}{\xi(B)}-\frac{1}{\xi(0)}\right]$$

$$\left(=\Delta\left[\log\left|G_N\right|\right]\right)$$



Bosons: Change in inv loc length is ~7 times bigger than fermions!
Exponentially strong effect on resistance!

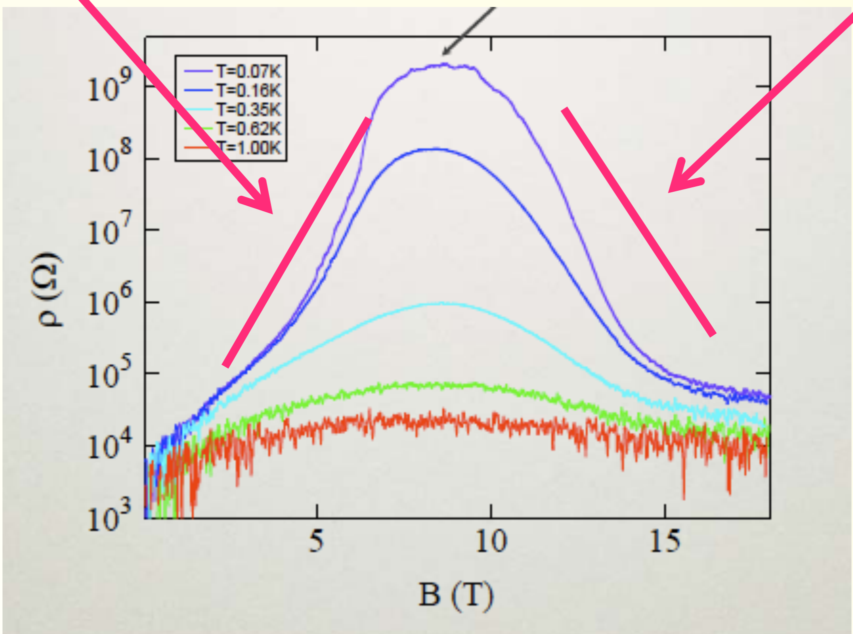
Magnetoresistance peak

Hebard+Palaanen, Gantmakher et al., Shahar et al, Baturina et al, W. Wu, Valles et al., Goldman et al.

A key ingredient to MR peak in superconducting films:

Local pairs = bosons
→ exponentially positive MR

Unpaired fermions
→ exponentially negative MR



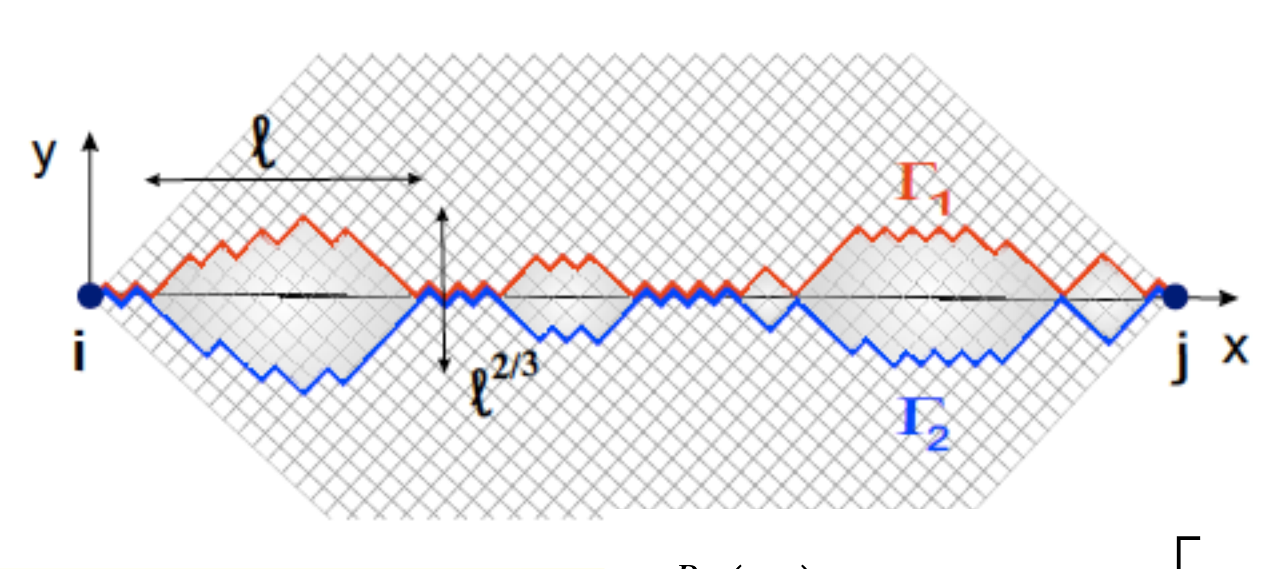
Sambandamurthy, Shahar et al. (2005) - InO_x

ξ(B) more quantitatively?

Magnetoresistance quantitaively

A. Gangopadhyay, V. Galitski, MM (PRL 2013)

Relevant paths form droplets:



Quantum Green's function

$$\frac{G^R_{i,0}(\omega)}{G^R_{0,0}(0)} = \sum_{\Gamma:\ 0\to i} \prod_{p\in\Gamma} \frac{t\cdot\left[\operatorname{sgn}\left(\varepsilon_p\right)\right]^B}{\varepsilon_p-\omega}$$

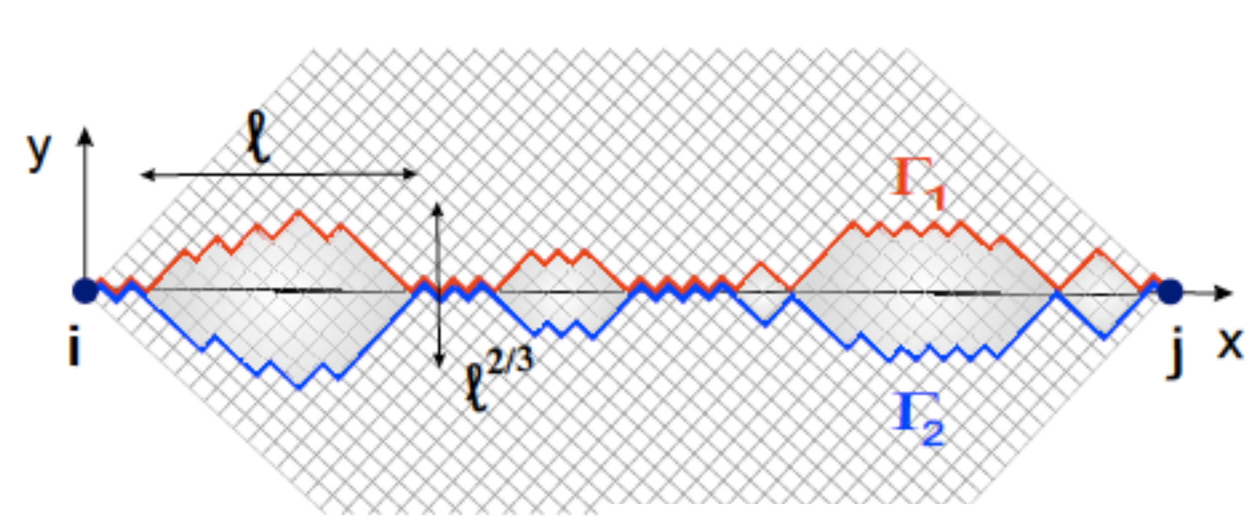
Essentially like directed polymers in random media!

(Monthus, Garel; Ortuno, Prior, Somoza; 2009)

Forward propagation ↔ directed polymers

A. Gangopadhyay, V. Galitski, MM (PRL 2013)

Relevant paths form droplets:



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↔

Directed polymer partition function

$$Z_{i,0} = \sum_{\Gamma:\,0\to i} \prod_{p\in\Gamma} \exp\left[-\frac{V(p)}{T}\right]$$

Weak fields: Complex KPZ

*A. Gangopadhyay, V. Galitski, MM, PRL **111**, 026801 (2013)*

Sum over forward paths:

$$S_{ji}(B) \equiv \frac{1}{t^{\mathrm{dist}(ij)}} \frac{G^R_{j,i}(\omega)}{G^R_{i,i}(\omega)} \Bigg|_{\omega\to\varepsilon_i} = \sum_\Gamma e^{i\Phi_\Gamma(B)} J_\Gamma(\omega = \varepsilon_i)$$

amp $\quad J_\Gamma(\omega) = \prod_{k\in\Gamma\setminus\{i\}} \frac{\mathrm{sgn}(\varepsilon_k)}{\varepsilon_k - \omega}$

phase $\quad \Phi_\Gamma(B) = \int_\Gamma d\mathbf{r}\cdot\mathbf{A}$

Recursion relation: ($\omega = 0$)

$$S_{x+1,y}(B) = V_{x+1,y}[e^{i\phi_-} S_{x,y-1}(B) + e^{i\phi_+} S_{x,y+1}(B)],$$

$$\phi_\pm = \int_{\Gamma_\pm} \mathbf{A}\cdot d\mathbf{r} \qquad \Gamma_\pm: (x, y\pm 1) \to (x+1, y)$$

Continuum limit for weak disorder and B-field: $\quad (B = \nabla\times A)$

$$D_x S = D_y^2 S + V(x,y) S$$

Covariant derivative: $\quad D_{\alpha=(x,y)} \equiv \partial_\alpha - iA_\alpha(x,y)$

In Landau gauge: $\quad \vec{A}(x,y) = (0, Bx)$

$$\partial_x S = \partial_y^2 S + [V(x,y) + iBx] S$$

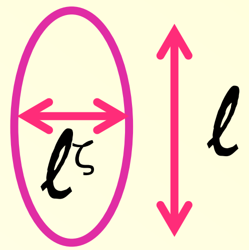
Complex KPZ equation! – Field theoretic treatment?

Droplet arguments for magnetoresponse

A. Gangopadhyay, V. Galitski, MM (PRL 2013)

Disorder dominates entropy! (Larkin-Ovchinnikov)
→ interfering loops are NOT random walks!

Roughness of interfering regions ("magnetic length")

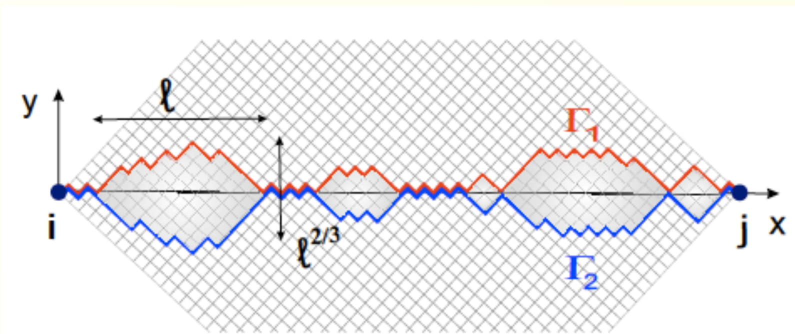


$$B\ell_B \ell_B^{\zeta} = 1 \quad \to \quad \ell_B = B^{-1/(1+\zeta)} \qquad \zeta = 2/3$$

Probability of significant interference at scale ℓ_B

$$\text{Contrast: } I = G_{\text{subdom. branch}} \Big/ G_{\text{dominant branch}} \propto \exp\left[-\Delta F\left(\ell_B\right)\right]$$

$$\text{Prob}\left(I = O(1)\right) \sim \text{Prob}\left(\Delta F\left(\ell_B\right) = O(1)\right) \sim \ell_B^{-\theta} \qquad \theta = 1/3$$



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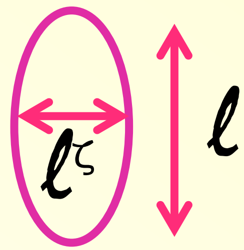

$$B\ell_B \ell_B^{\xi} = 1 \;\rightarrow\; \ell_B = B^{-1/(1+\zeta)} \qquad \zeta = 2/3$$

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$$\Delta\left(\frac{L}{\xi}\right) \sim \frac{L}{\ell_B}\left[\text{Prob}\left(I = O(1)\right) + cst\ \text{Prob}\left(I = O(1)\right)^2 + \ldots\right]$$

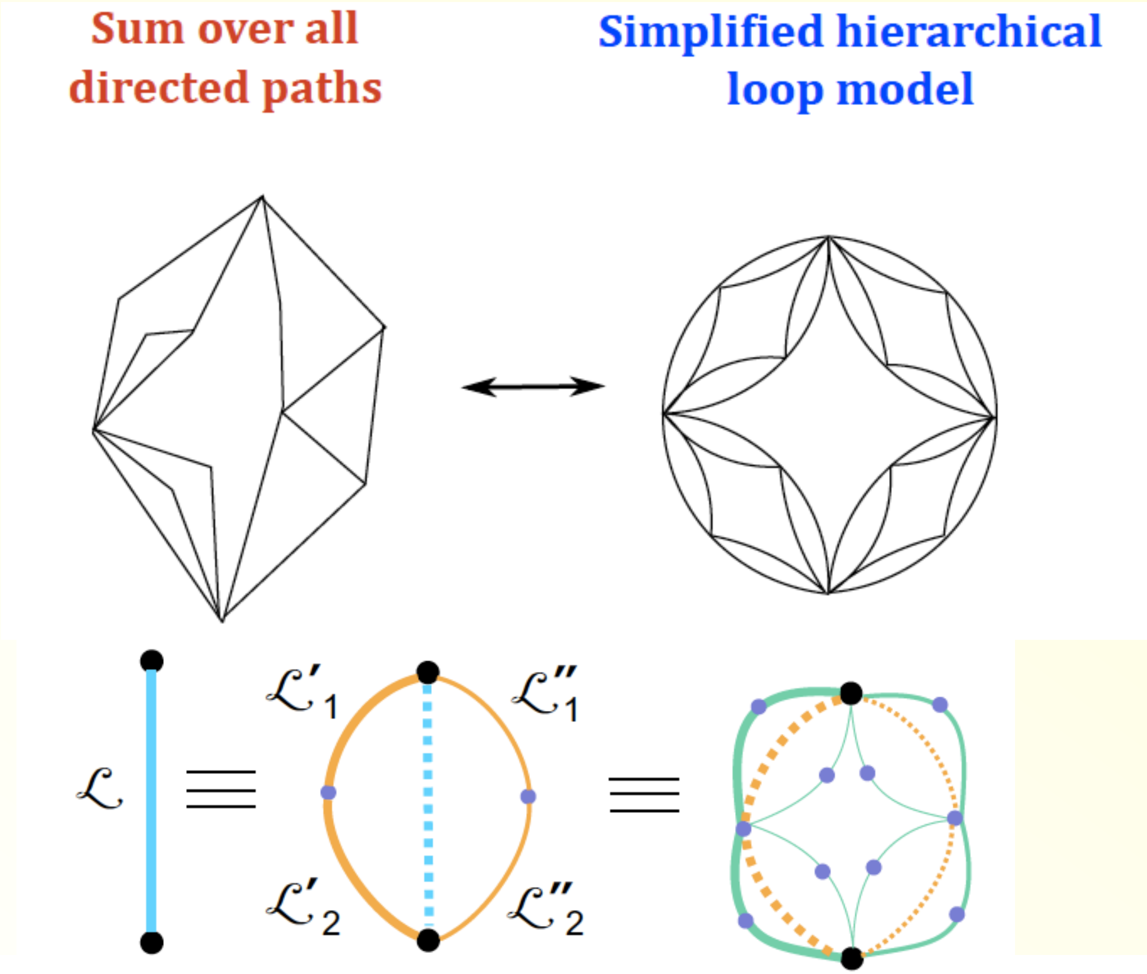
$$\Delta\left(\frac{1}{\xi}\right) \sim \frac{1}{\ell_B^{1+\theta}}\left(1 + \frac{const}{\ell_B^{\theta}} + \ldots\right) \sim B^{\chi}\left(1 + O\left(B^{\alpha}\right)\right)$$

$$\chi = \frac{1+\theta}{1+\zeta} = \frac{4}{5};$$

$$\alpha = \frac{\theta}{1+\zeta} = \frac{1}{5}$$

Simplified hierarchical model

A. Gangopadhyay, V. Galitski, MM (PRL 2013)



(cf. *Hwa, Fisher+Huse's* droplet theory for directed polymers, 1994)

$$S_{\mathcal{L}}^{k} = S_{\mathcal{L}'_1}^{k+1} S_{\mathcal{L}'_2}^{k+1} + e^{-f_{\mathcal{L}} L_k^{\theta}} e^{i a_{\mathcal{L}} B L_k^{1+\zeta}} S_{\mathcal{L}''_1}^{k+1} S_{\mathcal{L}''_2}^{k+1}$$

Interference sum *S* recursively defined – Use virial expansion!

So far

KPZ-traces in $\xi(B)$

Other hall marks of KPZ scaling in 2d?

Phononless conductance!

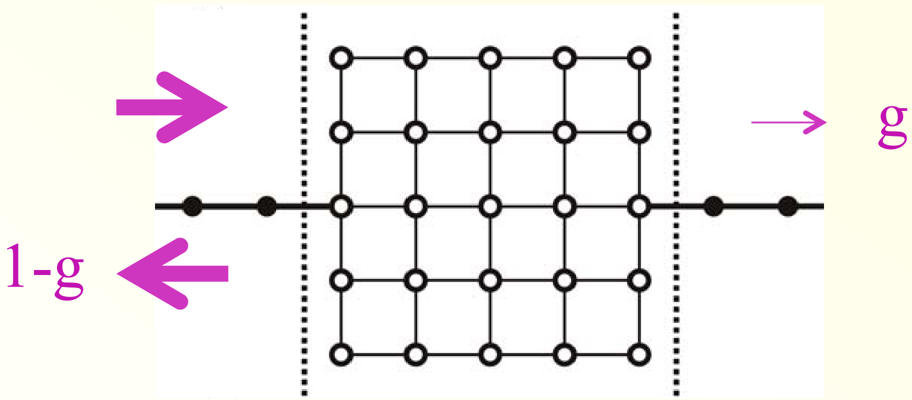
Anderson localization and directed polymers

A.M.Somoza, J. Prior, M. Ortuno,
*PRB **73**, 184201 (2006).*
C. Monthus and T. Garel,
*PRB **80**, 024203 (2009).*

Phononless conductance

Typical set-up: g measures transmission from left to right lead:

$$g \approx \exp\left[-\frac{2L}{\xi}\right]$$



$$\xi = -\lim_{L\to\infty} \frac{2L}{\langle \log(g) \rangle}$$

Distribution of conductance: like partition function of directed polymer!

$$\log(g) = -\frac{2L}{\xi} + \alpha\left(\frac{L}{\xi}\right)^{\theta_{DP}} \chi_{DP}; \qquad \theta_{DP} = 2\zeta - 1 \begin{cases} = & 1/3 \quad d = 1+1 \\ \approx & 0.244 \quad d = 1+2 \end{cases}$$

Tracy-Widom distribution in conductance

*J. Prior, A.M.Somoza, and M. Ortuno, EPJB **70**, 513(2009).*
*A.M. Somoza, P. Le Doussal. M. Ortuno, PRB **91**, 155413 (2015).*

2d bulk systems: Distribution of conductance g is Tracy-Widom!

$$\log(g) = -\frac{2L}{\xi} + \alpha\left(\frac{L}{\xi}\right)^{1/3} \chi_{TW}$$

Amazing robustness of Tracy-Widom law: holds also

- with negative weights (finite energies, fermions)
- with complex weights (B-field)
- with loops (full Anderson problem instead of forward scattering)

Those only determine ξ and possibly the number α (varies very little numerically!) – despite the fact that the average conductance follows different RG flow with size:

$$\beta(g) = \frac{d\log(g)}{d\log(L)}; \quad \beta_{B=0}(g) \neq \beta_{B\neq 0}(g)$$

$$\beta(g) = \log(g) + \frac{\alpha\overline{\chi_{TW}}}{3}\left[-\frac{1}{2}\log(g)\right]^{1/3}$$

Tracy-Widom distribution in conductance

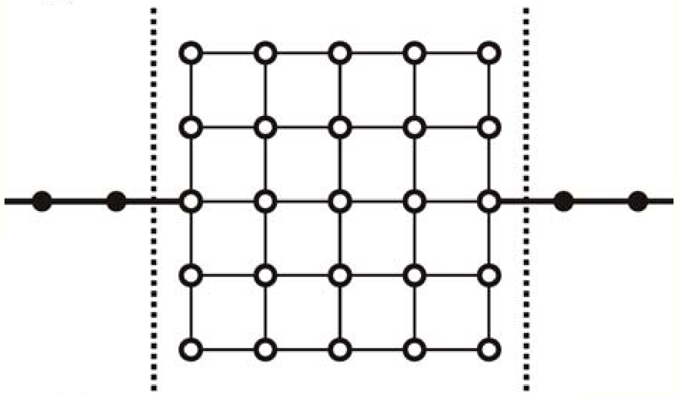
*J. Prior, A.M.Somoza, and M. Ortuno, EPJB **70**, 513(2009).*
*A.M. Somoza, P. Le Doussal. M. Ortuno, PRB **91**, 155413 (2015).*

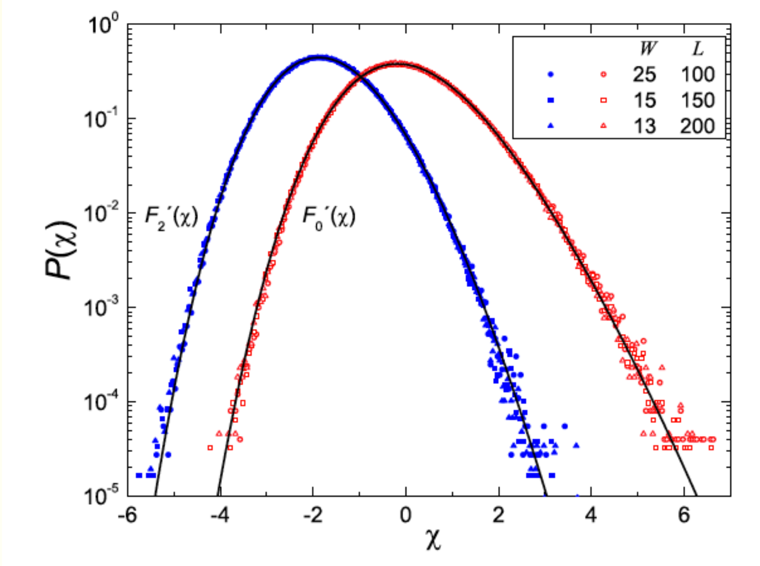
2d bulk systems: Distribution of conductance g is Tracy-Widom!

$$\log(g) = -\frac{2L}{\xi} + \alpha\left(\frac{L}{\xi}\right)^{1/3} \chi_{TW}$$

Only aspect determining the type of Tracy-Widom distribution:
Geometrical boundary conditions of the conductance (like dir. Pol!):
1) Full plane, point contacts:

$$P(\chi_{TW}) = F_2'(\chi_{TW})$$





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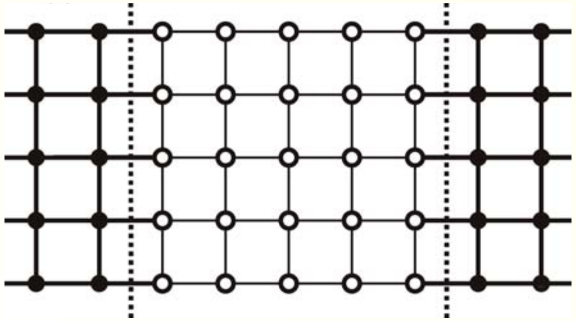
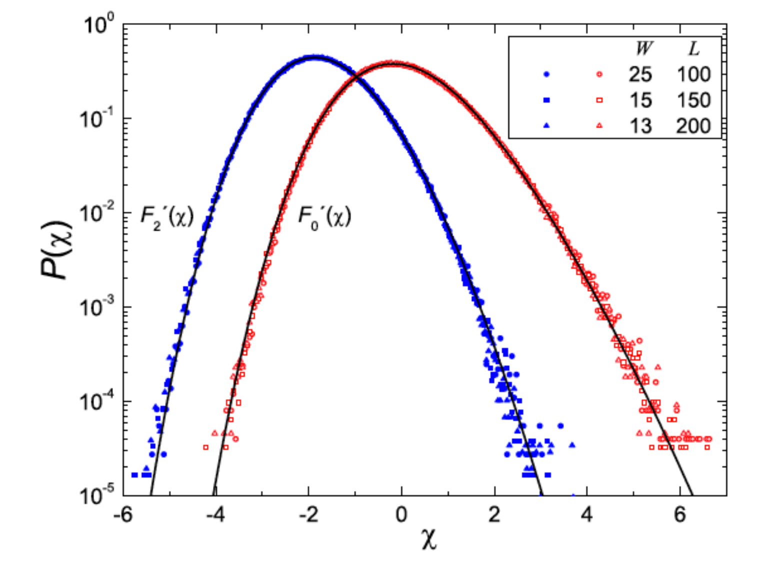
Only aspect determining the type of Tracy-Widom distribution:
Geometrical boundary conditions of the conductance (like dir. Pol!):
2) Full plane, wide contacts:

$$P(\chi_{TW}) = F_0'(\chi_{TW})$$





Tracy-Widom distribution in conductance

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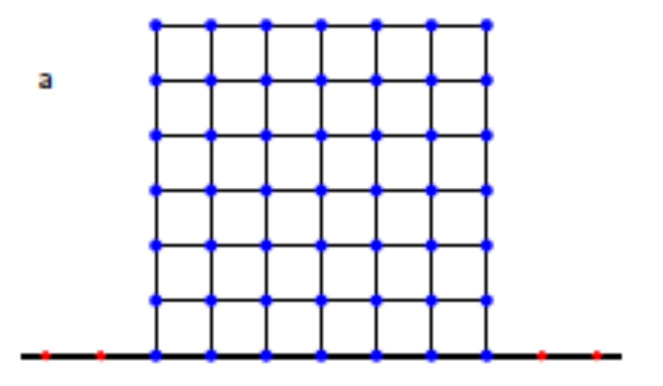
2d bulk systems: Distribution of conductance *g* is Tracy-Widom!

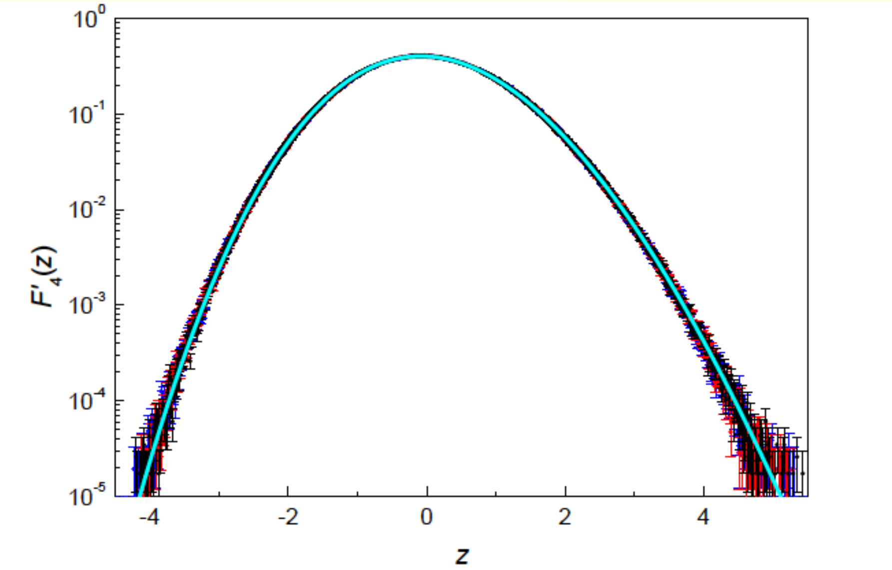
$$\log(g) = -\frac{2L}{\xi} + \alpha\left(\frac{L}{\xi}\right)^{1/3} \chi_{TW}$$

Only aspect determining the type of Tracy-Widom distribution:
Geometrical boundary conditions of the conductance (like dir. Pol!):

3) Half plane, point contacts:

$$P(\chi_{TW}) = F_4'(\chi_{TW})$$





Tracy-Widom distribution in conductance

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2d bulk systems: Distribution of conductance g is Tracy-Widom!

$$\log(g) = -\frac{2L}{\xi} + \alpha \left(\frac{L}{\xi}\right)^{1/3} \chi_{TW}$$

Only aspect determining the type of Tracy-Widom distribution:
Geometrical boundary conditions of the conductance (like dir. Pol!):

Open challenge: why is Tracy-Widom so robust?

Energy dependence of ξ?

(without magnetic field)

Bosons vs fermions

X. Yu, MM, Ann. Phys. '13

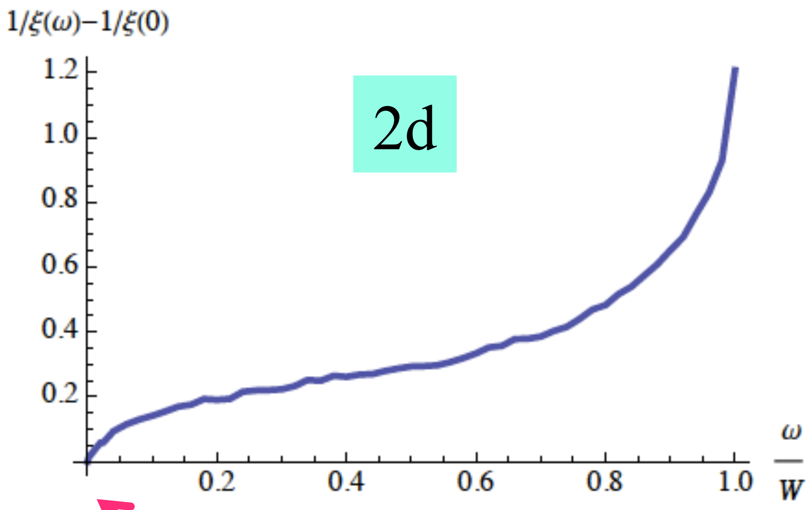
Interference in finite dimensions: leading terms

(F)

$$\frac{G^R_{i,0}(\omega)}{G^R_{0,0}(\omega)} = \sum_{\mathcal{P}=\{j_0=0,\ldots,j_\ell=i\}} \prod_{p=1}^{\ell} t_{j_{p-1},j_p} \frac{1}{\varepsilon_{j_p} - \omega}$$

(B - XY)

$$\frac{G^R_{i,0}(\omega)}{G^R_{0,0}(\omega)} = \sum_{\mathcal{P}=\{j_0=0,\ldots,j_\ell=i\}} \prod_{p=1}^{\ell} t_{j_{p-1},j_p} \frac{\mathrm{sgn}(\varepsilon_{j_p})}{\varepsilon_{j_p} - \omega}$$



Delocalization **strongest** at lowest energies: **ξ(0) > ξ(ω)!**

→ Bosons delocalize best at low energy!

Bosons vs fermions

X. Yu, MM, Ann. Phys. '13

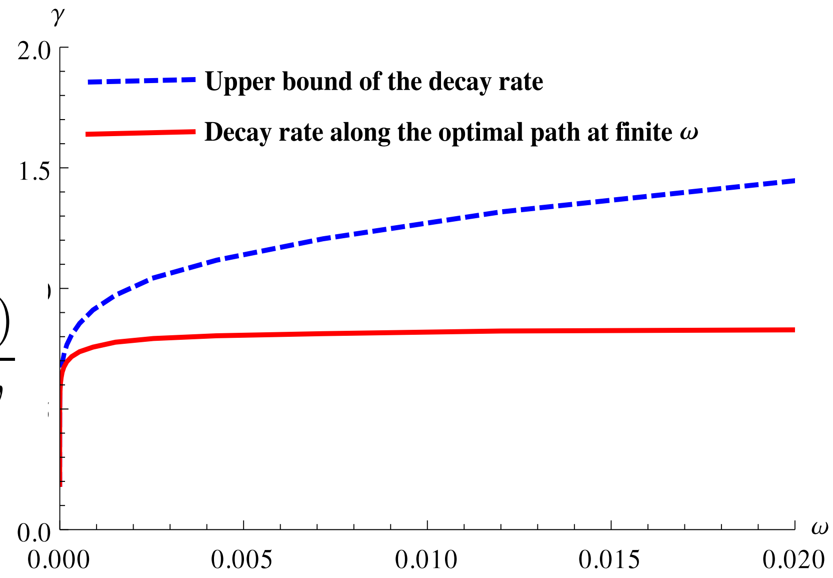
Interference in finite dimensions: leading terms

(B - Ising) + singular non-perturbative corrections (at $\omega \neq 0$)

$$G_{l,0}(\omega) = G_{0,0}(\omega) \sum_{\mathcal{P}=\{j_0=0,\ldots,j_L=l\}} \prod_{p=1}^{L=\mathrm{dist}(l,0)} \frac{4J|\epsilon_{j_p}|}{(2\epsilon_{j_p})^2 - \omega^2}$$

(B - XY)

$$\frac{G^R_{i,0}(\omega)}{G^R_{0,0}(\omega)} = \sum_{\mathcal{P}=\{j_0=0,\ldots,j_\ell=i\}} \prod_{p=1}^{\ell} t_{j_{p-1},j_p} \frac{\mathrm{sgn}(\varepsilon_{j_p})}{\varepsilon_{j_p} - \omega}$$



Effect is even much stronger in quantum Ising models

[Due to symmetry protection of small denominators] *(X. Yu, MM '13)*

→ "Activated" scaling $\xi \sim \log(1/\omega)$.

Real, interacting insulators in d=2?

Add: long range Coulomb interactions
+ magnetic field

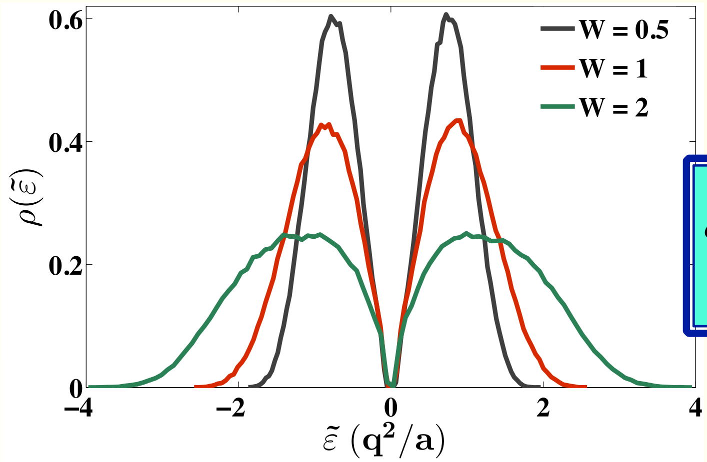
Obtain qualitative results from approximate treatment

Magneto-oscillations of mobility edge

T. Nguyen and MM (2014)

$$H = -t \sum_{\langle i,j \rangle} \left(b_i^+ b_j + \text{h.c.} \right) + \sum_i \varepsilon_i n_i + \frac{1}{2} \sum_{\langle i,j \rangle} \frac{e^2}{r_{ij}} n_i n_j$$

Efros-Shklovskii Coulomb gap for effective potentials:

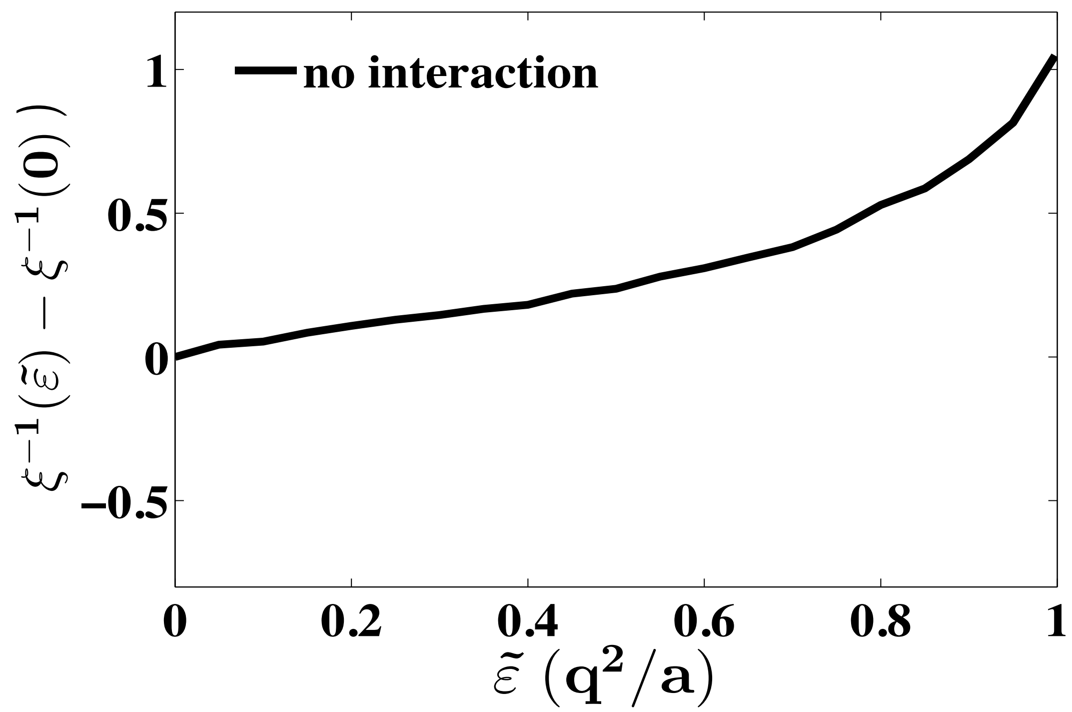


$$\tilde{\varepsilon}_i = \varepsilon_i + \sum_{j \in \partial i} J_{ij} n_j$$

Magneto-oscillations of mobility edge

T. Nguyen and MM (2014)

$$H = -t\sum_{\langle i,j\rangle}\left(b_i^{+}b_j + \text{h.c.}\right) + \sum_i \varepsilon_i n_i + \frac{1}{2}\sum_{\langle i,j\rangle}\frac{e^2}{r_{ij}}n_i n_j$$



Magneto-oscillations of mobility edge

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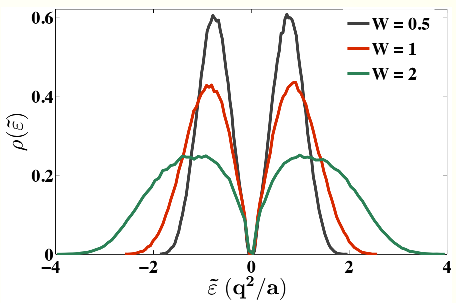
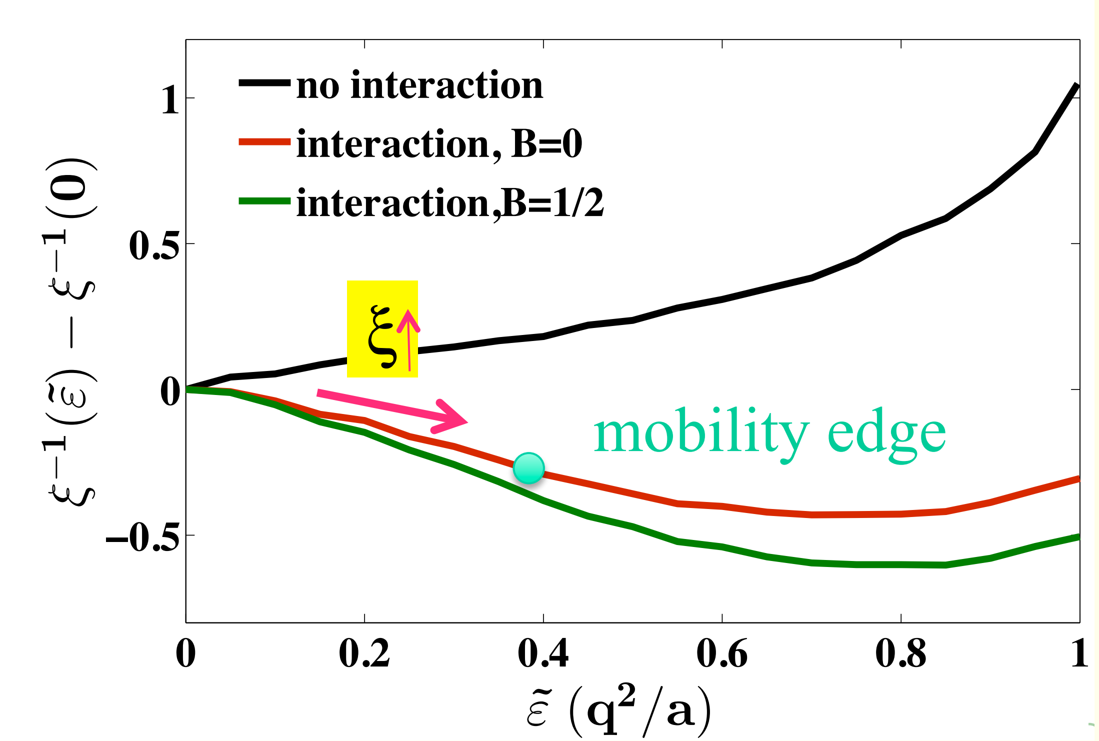




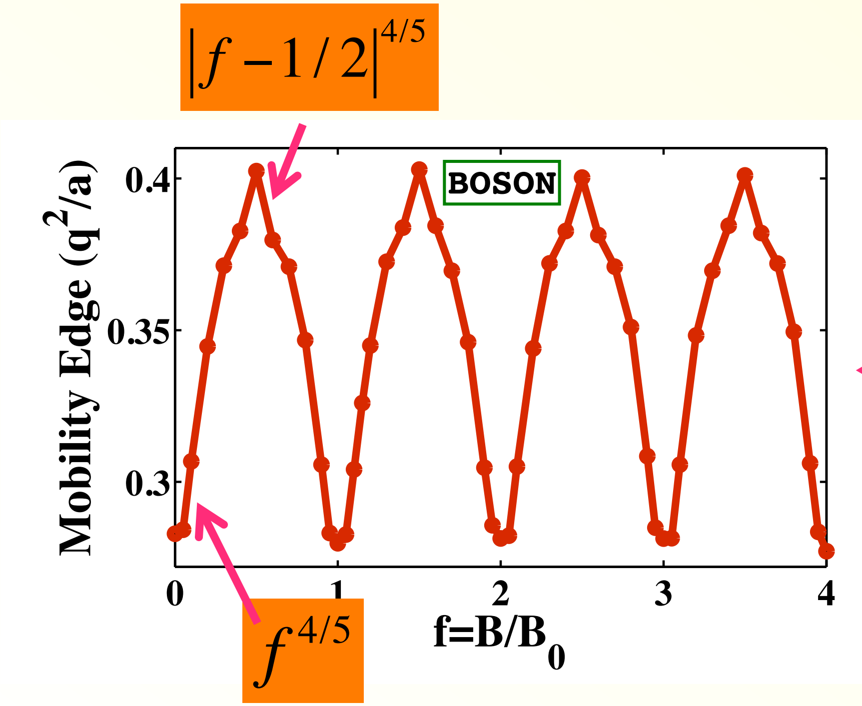
Apply magnetic flux: oscillation of mobility edge!

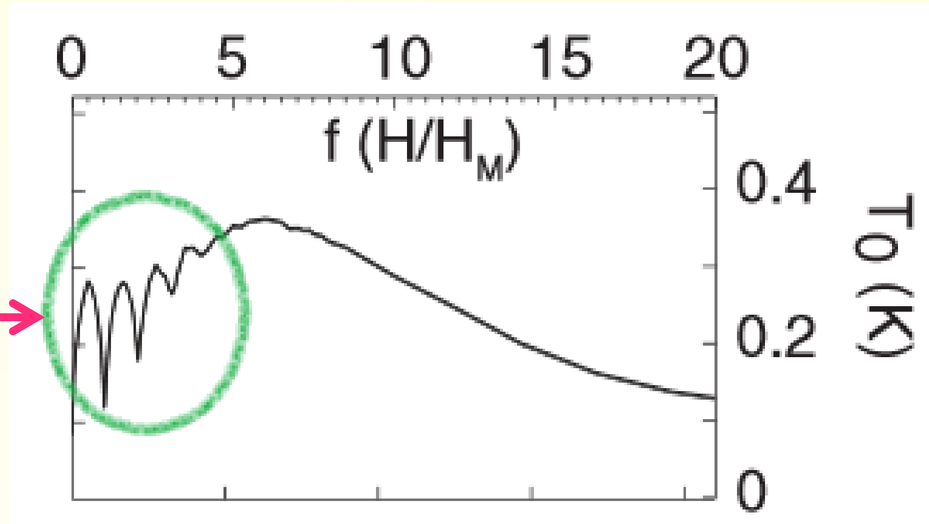
Magneto-oscillations of boson mobility edge

T. Nguyen and MM (2014)

Substantial oscillations with cusps

J. Valles group,
*PRL **103**, 157001 (2009)*





$$R_\square = R_0 \exp\left(\frac{T_0}{T}\right)$$

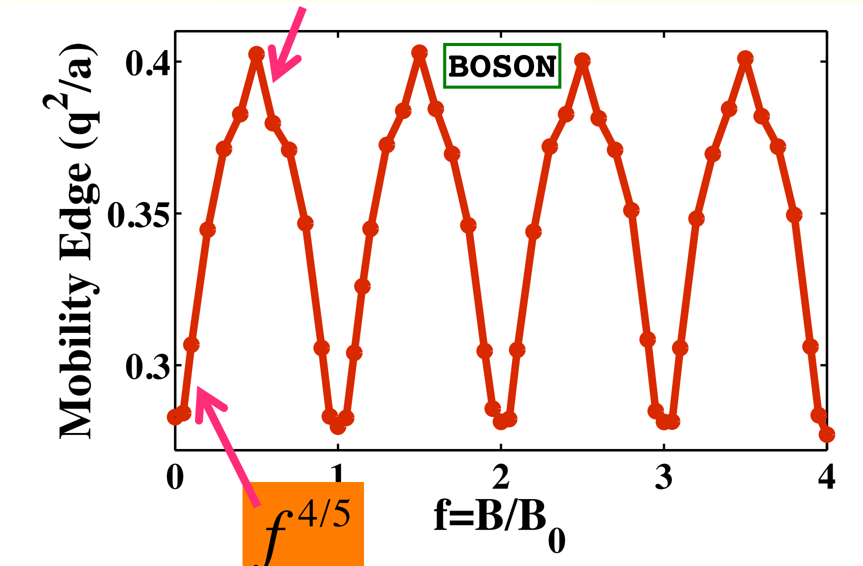
If no phonons: expect transport by activation to mobility edge!

Magneto-oscillations of boson mobility edge

T. Nguyen and MM (2014)

Substantial oscillations with cusps

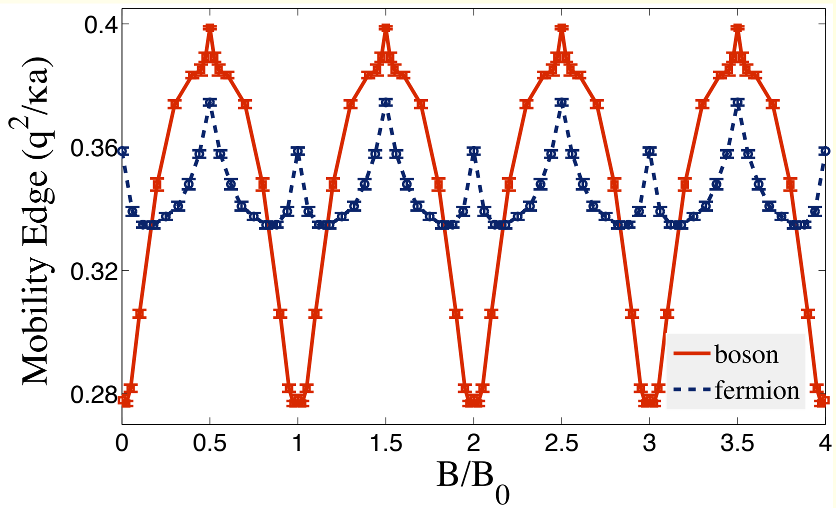
$|f - 1/2|^{4/5}$



$f^{4/5}$

Comparison with fermions:

- opposite, downward lobes
- smaller amplitude
- two maxima per period



If no phonons: expect transport by activation to mobility edge!

Conclusions

- Wavefunction tails in 2d realize KPZ physics
  - despite negative or complex weights.

  Scaling exponents of MR and Tracy-Widom distribution of amplitudes: like for positive weights!

- ξ of bosons shrinks in B-field (destroys positive interference)
  → Positive magnetoresistance in insulators, unlike fermions.

- Hard core bosons localize less than [hard core] fermions in all d>1!

- Effect of Coulomb gap: → Mobility edge
  → Magneto-oscillations of mobility edge:
  Cuspy features in exp: non-trivial KPZ scaling at small B field!