Universality classes in growing interfaces: Reaction fronts in disordered flow

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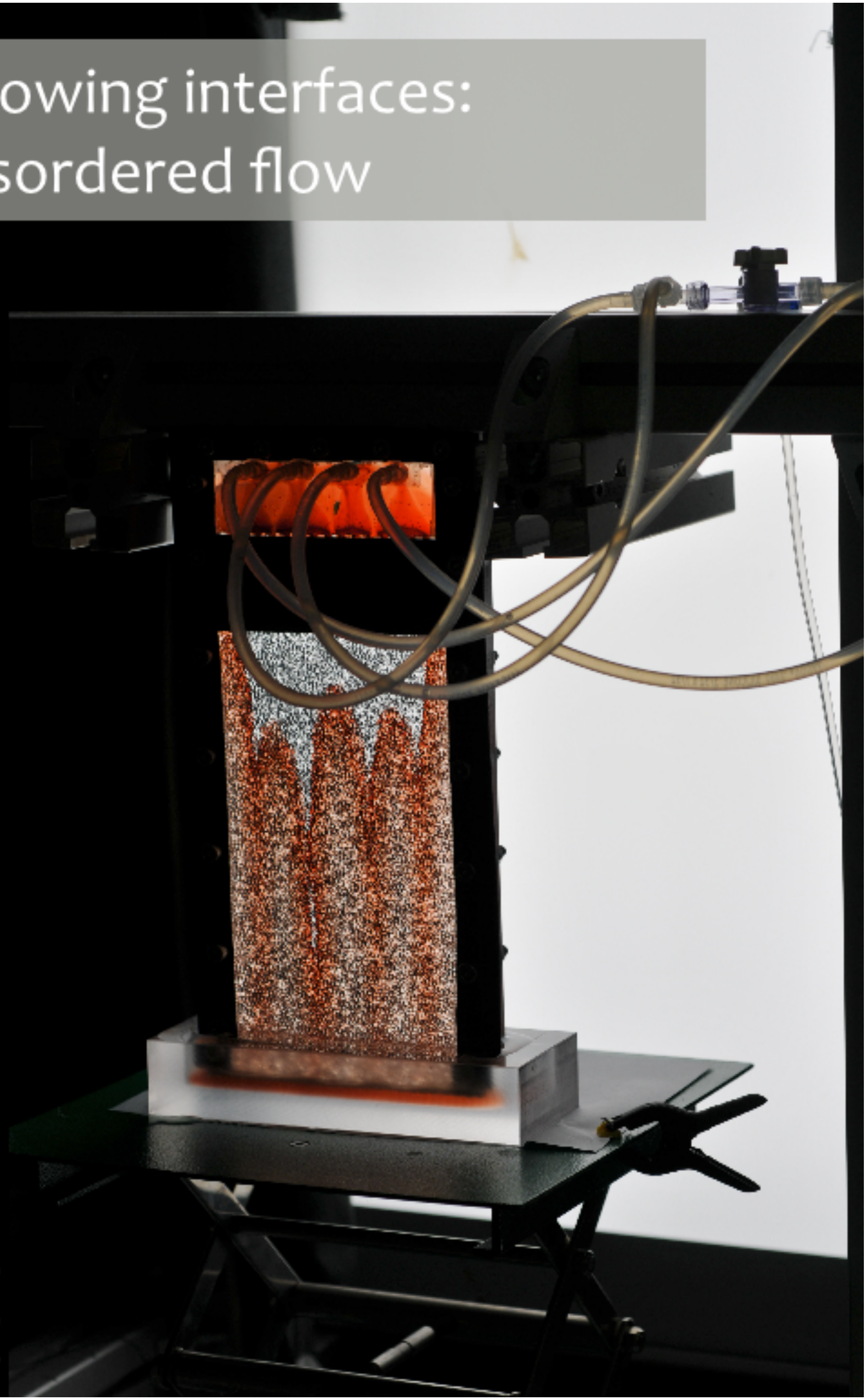

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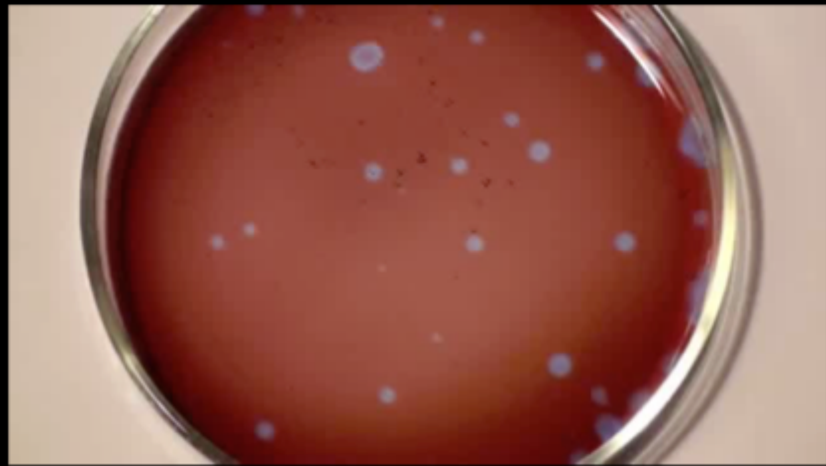
Growing interfaces

Out-of-equilibrium phenomena

Crystal growth in supercooled liquids

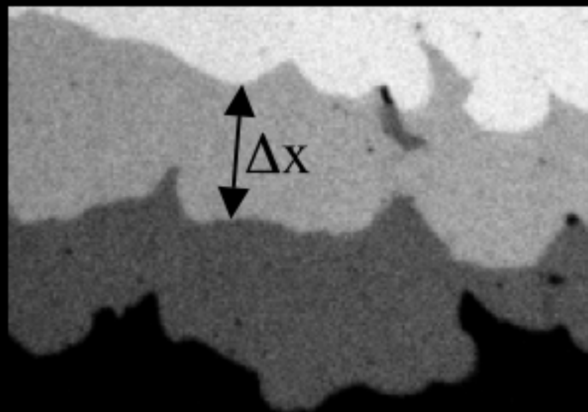
Imbibition fronts

Belousov-Zhabotinsky reaction



video S. Morris

Magnetic domain wall avalanches



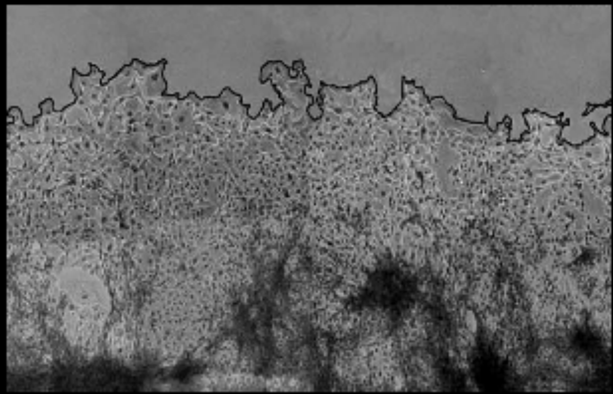
Repain et al., *EPL* **68** (2004)

Growing interfaces

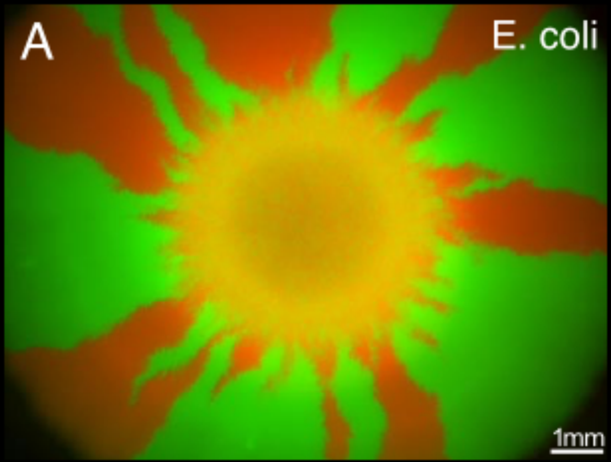
Living systems

Lychen

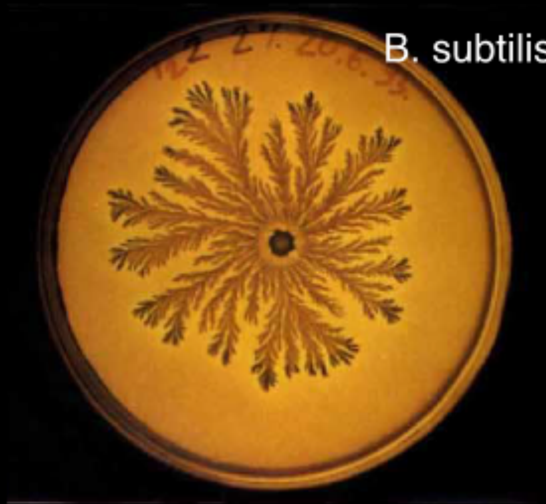
Vero cells colony



Huergo et al. (2010)



O. Hallatschek et al. (2007)



Benjacob et al. (1994)

PLAN

1 - Kardar-Parisi-Zhang model in presence of quenched noise

2 - Experiments with reaction fronts in disordered flow

3 - Transition between different universal behaviors

4 - Conclusion and perpectives

PLAN

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Growing interfaces

Kardar-Parisi-Zhang (KPZ) model:

a generic model for an interface growing along its local normal

$$\frac{\partial h(x,t)}{\partial t} = \nu \nabla^2 h(x,t) + \frac{\lambda}{2}[\nabla h(x,t)]^2 + \eta(x,t) + f$$

Kardar & al., *PRL* **56** (1986)

stochastic noise term:

$$\overline{\eta(x,t)\eta(x',t')} = 2D\delta(x-x')\delta(t-t')$$

random deposition

lateral growth

+ effective stiffness

+ applied force

Growing interfaces

transformation:

$$h(x,t) = ft + \tilde{h}(x,t)$$

$$\frac{\partial \tilde{h}(x,t)}{\partial t} = \nu \nabla^2 \tilde{h}(x,t) + \frac{\lambda}{2}[\nabla \tilde{h}(x,t)]^2 + \eta(x,t)$$

invariant under $\tilde{h}(x,t) \rightarrow -\tilde{h}(x,t)$

Growing interfaces

Predicted exponents: d=1+1

$$\alpha = \frac{1}{2} \qquad \beta = \frac{1}{3}$$

$$w(l,t) \sim l^{\alpha} \qquad w(l,t) \sim t^{\beta}$$

Growing interfaces

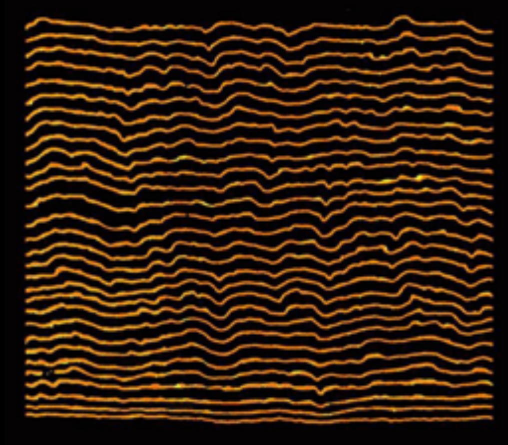
Experimental observations

Predicted exponents: d=1+1

$$\alpha = \frac{1}{2} \qquad \beta = \frac{1}{3}$$

$$w(l,t) \sim l^{\alpha} \qquad w(l,t) \sim t^{\beta}$$

paper combustion front

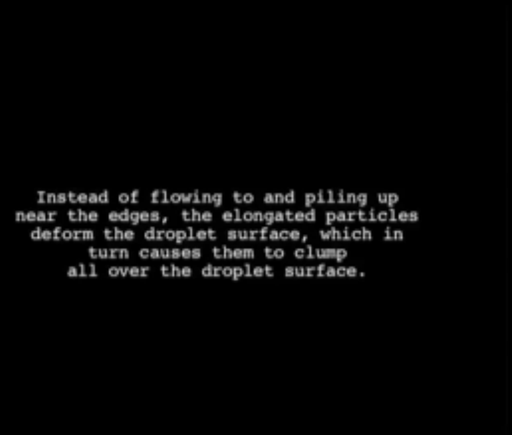


Myllys et al. (1993)

Turbulent liquid crystals

Takeuchi et al. (2010)

colloidal suspension drying

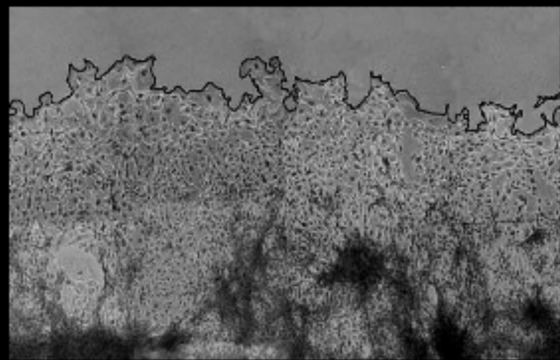


Yunker et al. (2013)

mutant B. subtilis

Wakita et al. (1997)

Vero cells colony



Huergo et al. (2010)

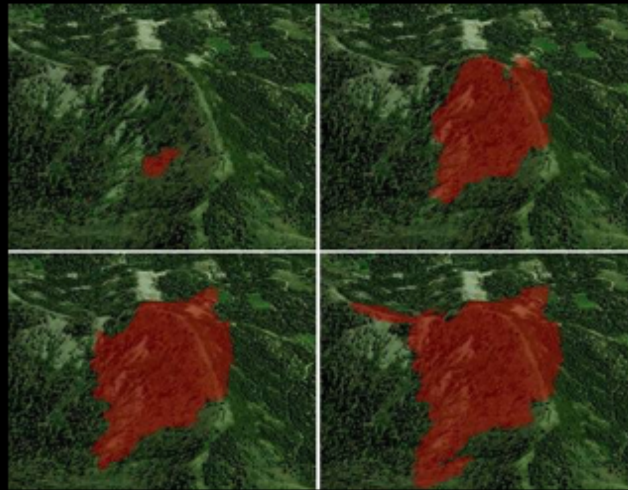
Heterogeneous environment

Quenched noise

Topology

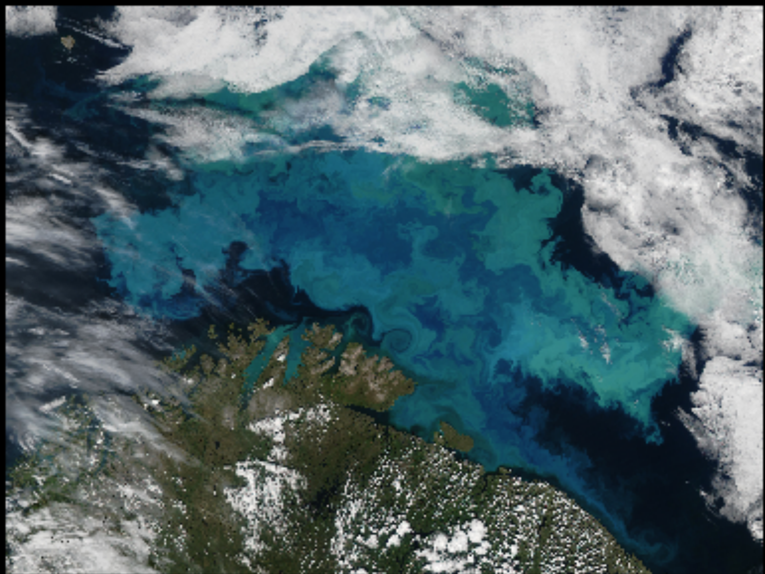
Black Death in Medieval Europe

Forest fire

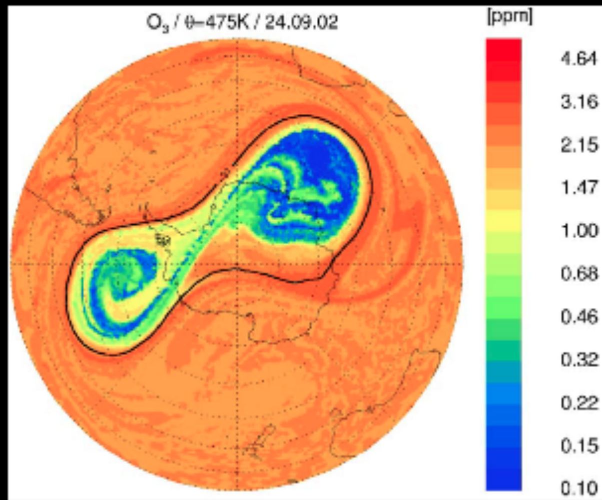


Flow

Plancton bloom in Barent Sea



Ozone hole



Gross et al. (2006)

Heterogeneous environment

positive quenched KPZ model

In a heterogeneous medium, the "noise" acquires a static quenched component

$$\frac{\partial h(x,t)}{\partial t} = \nu \nabla^2 h(x,t) + \frac{\lambda}{2}[\nabla h(x,t)]^2 + \bar{\eta}(x, h(x,t)) + f$$

Kessler & al., *PRA* **43** (1991)

Amaral & al., *PRE* **51** (1995)

quenched noise term:

$$\overline{\bar{\eta}(x,h)\bar{\eta}(x',h')} = 2\bar{D}\delta(x-x')\delta(h-h')$$

$\bar{\eta}(x, h(x,t))$ is no longer invariant with the transformation:

$$h(x,t) = ft + \tilde{h}(x,t)$$

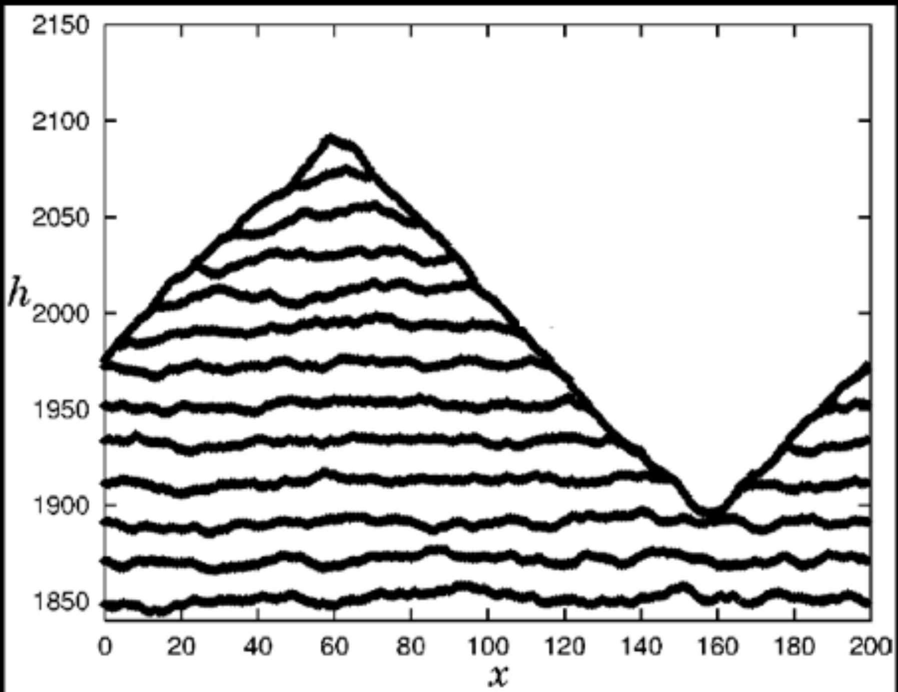
The driving force f is a new parameter of the problem and its relative sign to λ matter!

Heterogeneous environment

negative quenched KPZ model

$$\frac{\partial h(x,t)}{\partial t} = \nu \nabla^2 h(x,t) - \frac{\lambda}{2}[\nabla h(x,t)]^2 + \bar{\eta}(x, h(x,t)) + f$$

Jeong & al., *PRL* **77**(1996), *PRE* **59** (1999)



Heterogeneous environment

Predicted exponents: **q-KPZ positive**

$\alpha \simeq 0.63 \quad \beta \simeq 0.63$

$w(l,t) \sim l^{\alpha} \quad w(l,t) \sim t^{\beta}$

Heterogeneous environment

Experimental observations

Predicted exponents: **q-KPZ positive**

$\alpha \simeq 0.63 \quad \beta \simeq 0.63$

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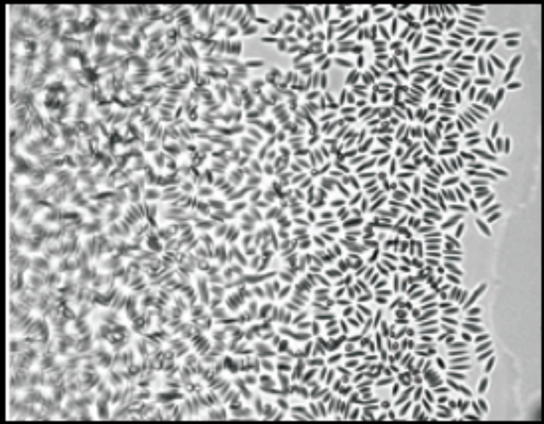
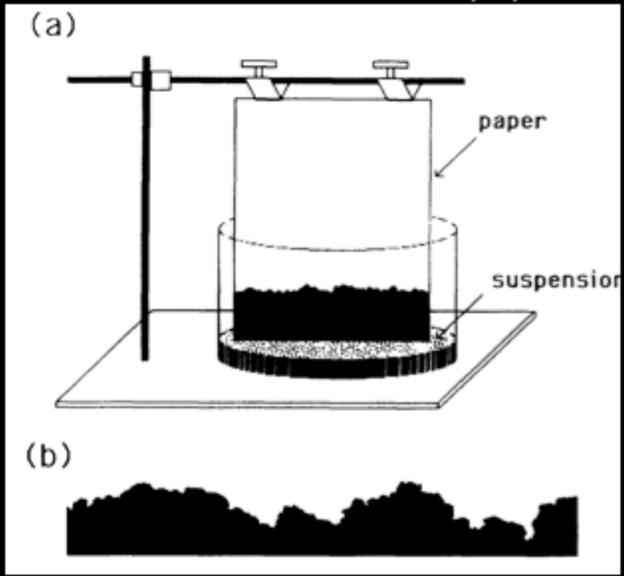
colloidal suspension drying



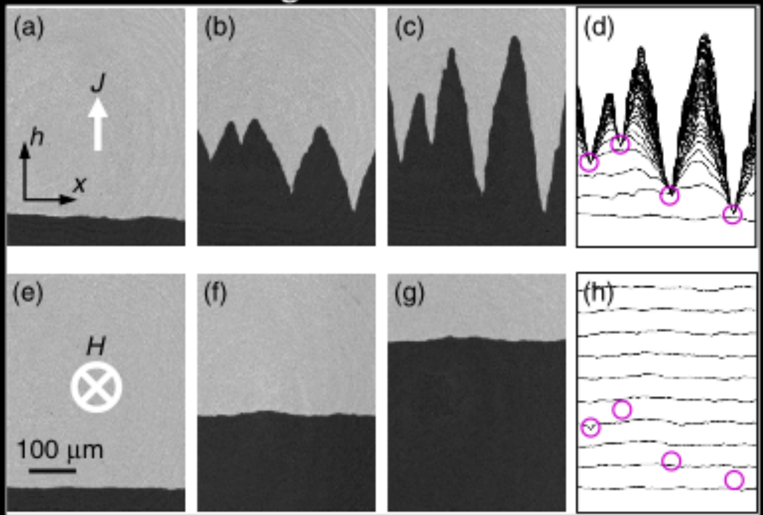
Yunker et al. (2013)

coffee or ink imbibition in paper



Buldyrev et al. (1992)

magnetic media



Moon et al. (2013)

Heterogeneous environment

In the moving phase, quenched KPZ of either sign crosses over to KPZ at large scales.

$$\frac{\partial h(x,t)}{\partial t} = \nu \nabla^2 h(x,t) + \frac{\lambda}{2}\left[\nabla h(x,t)\right]^2 + \bar{\eta}\Big(x, h(x,t)\Big) + f$$

In the limit of large mean interface velocity:

$$v = \overline{\partial_t h(x,t)}$$

$$h(x,t) \rightarrow vt + \tilde{h}(x,t)$$

$$\bar{\eta}(x, vt + \tilde{h}(x,t)) \approx \bar{\eta}(x, vt)$$

→ $\alpha = \frac{1}{2}$ and $\beta = \frac{1}{3}$

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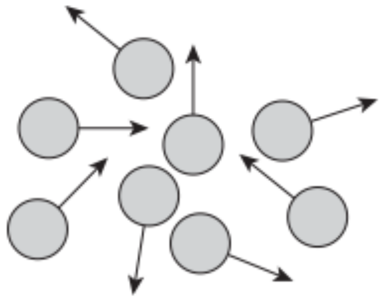
- Reaction Diffusion equation

$$\frac{\partial u}{\partial t} = D\nabla^2 u + f(u)$$

$$u = [B]$$



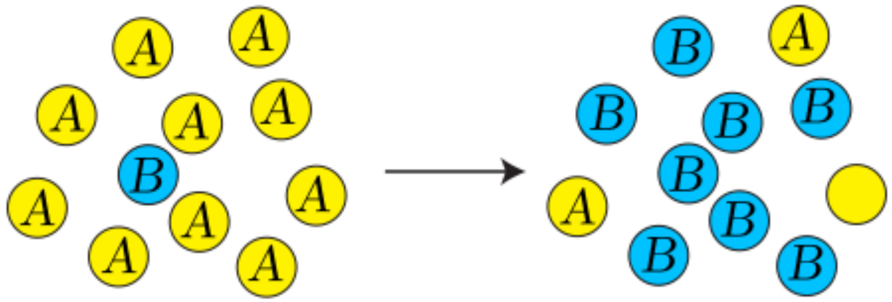
$\longrightarrow$ $D\nabla^2 u$ diffusion term

$\longrightarrow$ $f(u)$ reaction term

autocatalytic process $\longrightarrow$ nonlinearity $f(u) = r\,u(1-u)$

feedback

$$A + B \longrightarrow 2B + C$$



- Fisher-Kolmogorov equation (FKPP model)

[Kolomogorov et al. 1937, R. A. Fisher 1937]

$$\frac{\partial u}{\partial t} = D\frac{\partial^2 u}{\partial x^2} + ru(1-u)$$

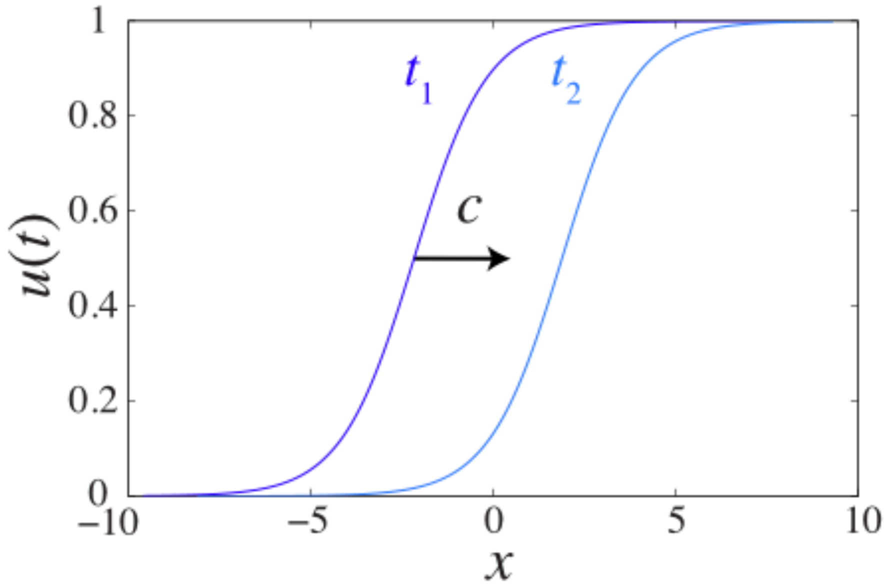
$$X = x \pm ct \quad \longrightarrow \quad c\frac{\partial u}{\partial X} = D\frac{\partial^2 u}{\partial X^2} + f(u)$$

- Progressive wave solutions

$$u(x,t) = u(x \pm ct)$$

FKPP

$$\longrightarrow \quad c = 2\sqrt{rD}$$



Experimental setup

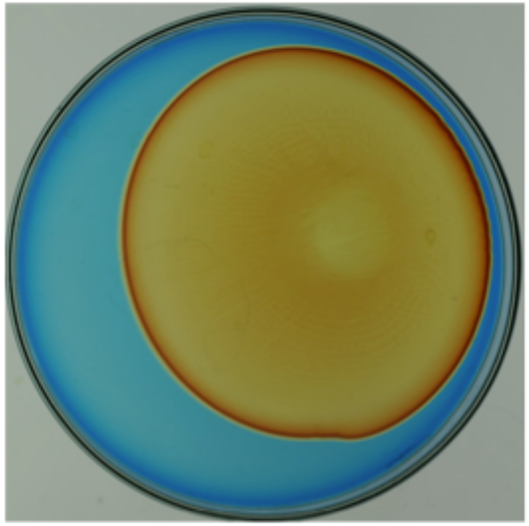
- Iodate acid arseneous reaction (IAA)

$$IO_3^- + 5I^- + 3H_3AsO_3 \rightarrow 6I^- + 3H_3AsO_4$$

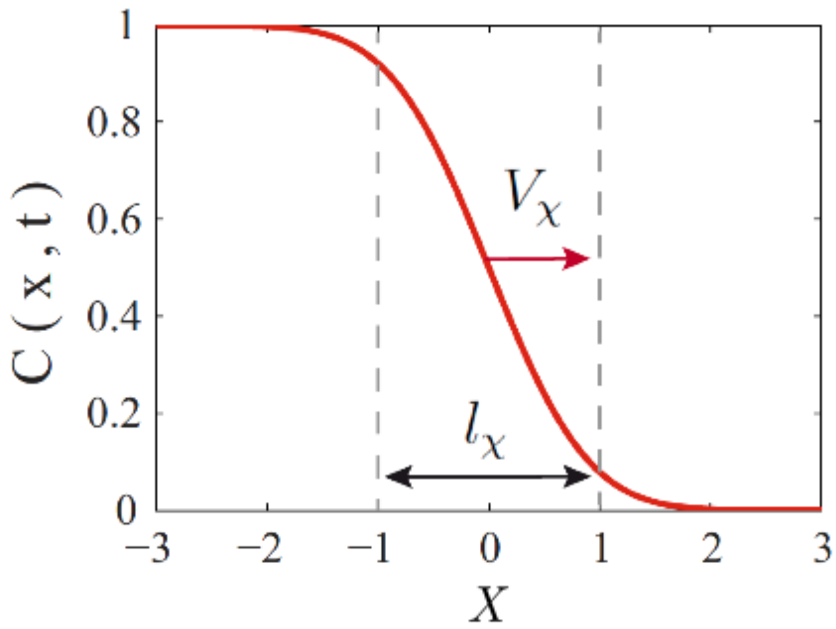
→ resulting from the balance between diffusion and reaction

$$\begin{cases} V_\chi = \sqrt{\dfrac{\alpha D_m}{2}} \simeq 10\mu m/s \\ l_\chi = \sqrt{\dfrac{2D_m}{\alpha}} \simeq 100\mu m \end{cases}$$

$$C(x,t) = \frac{1}{1+\exp[(x - V_\chi t)/l_\chi]}$$

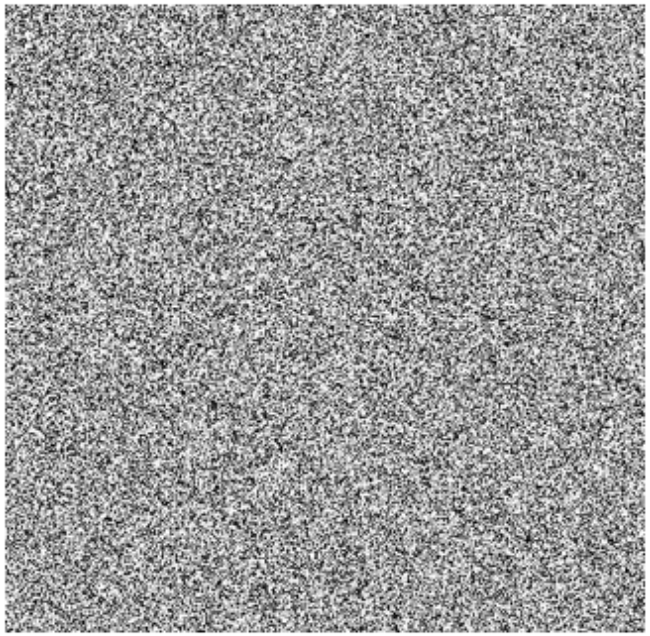


IAA wave front [movie D. Salin]

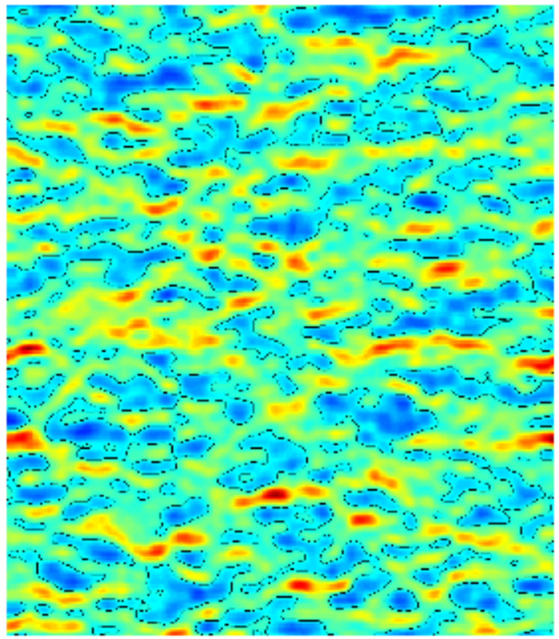


Experimental setup

What's happening in the presence of noise?

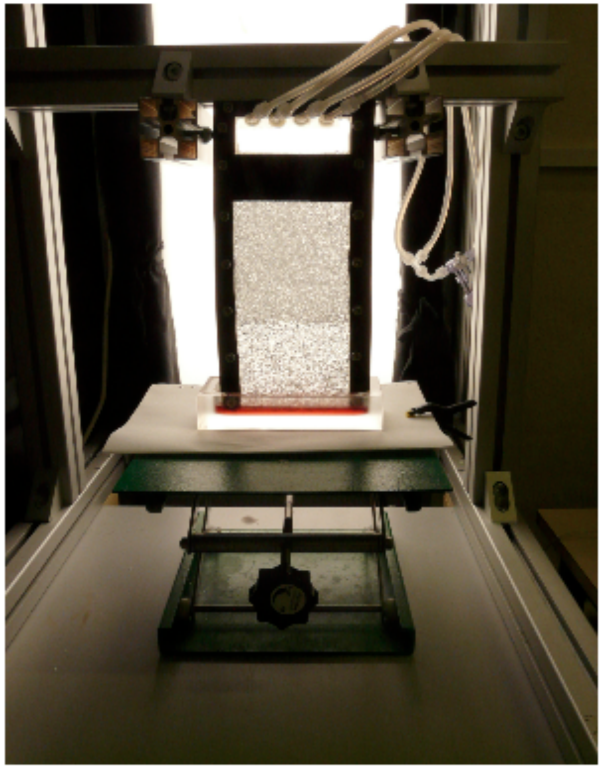


Disordered reactive flow field



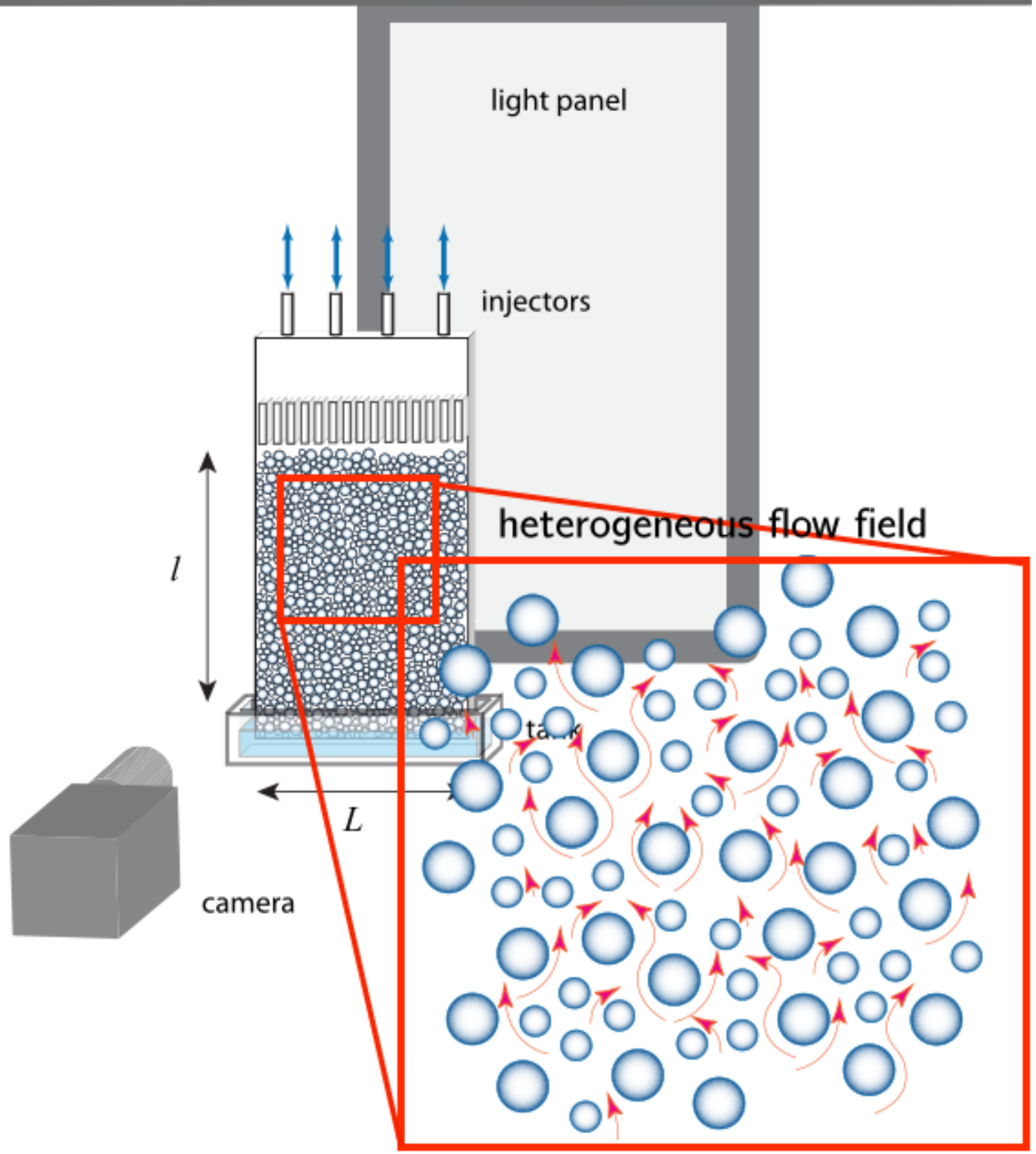
Experimental setup

- Spatially disordered flow



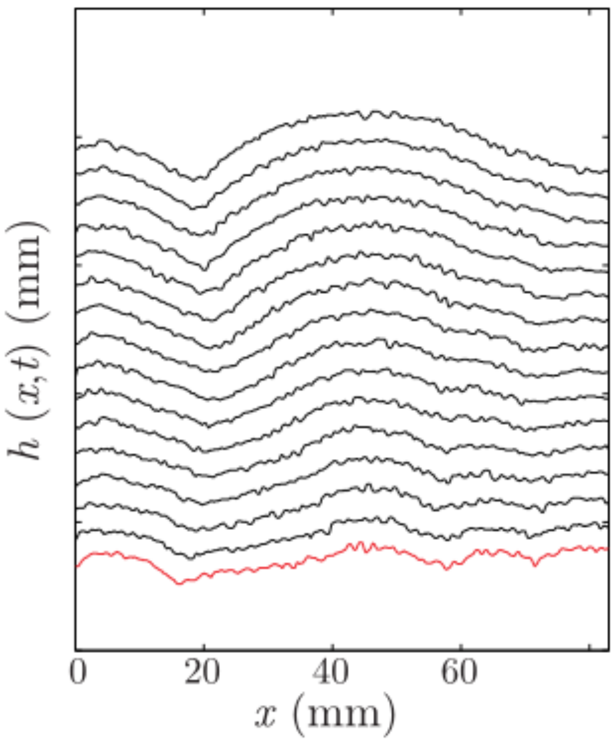
flow through a granular medium

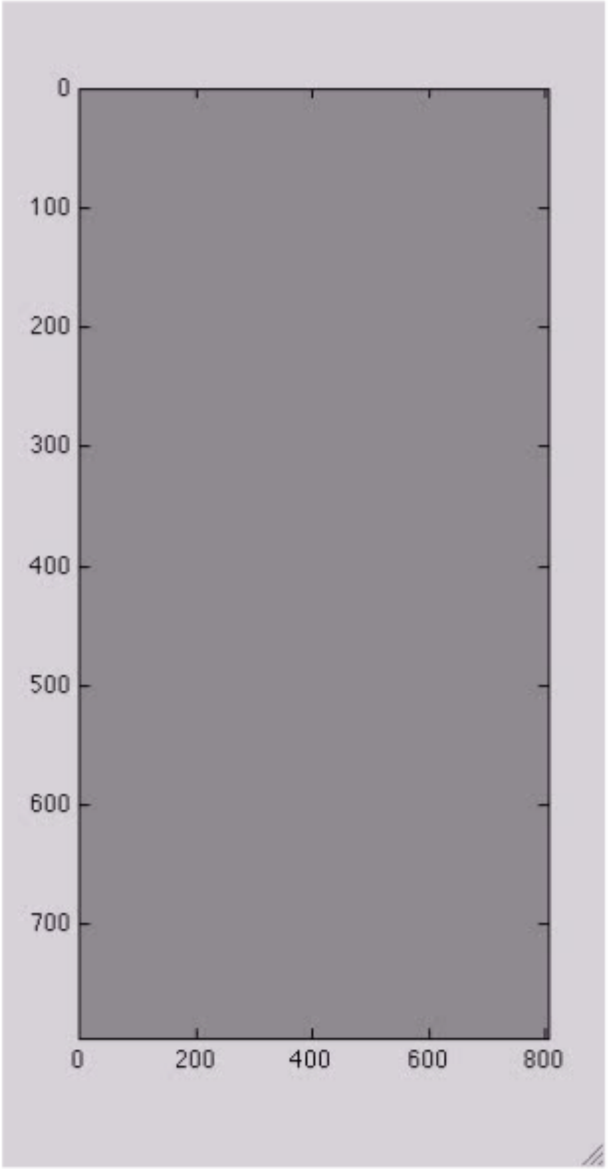
1.5 mm and 2 mm diameter glass beads



Experimental setup

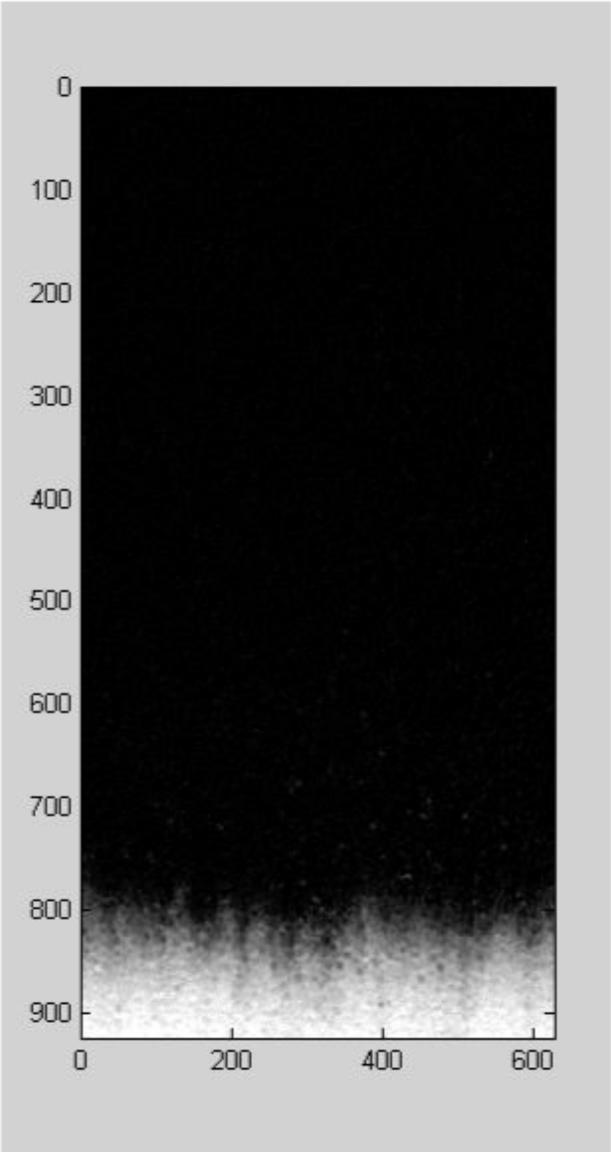
- **Reaction front propagation without disordered flow**





Experimental setup

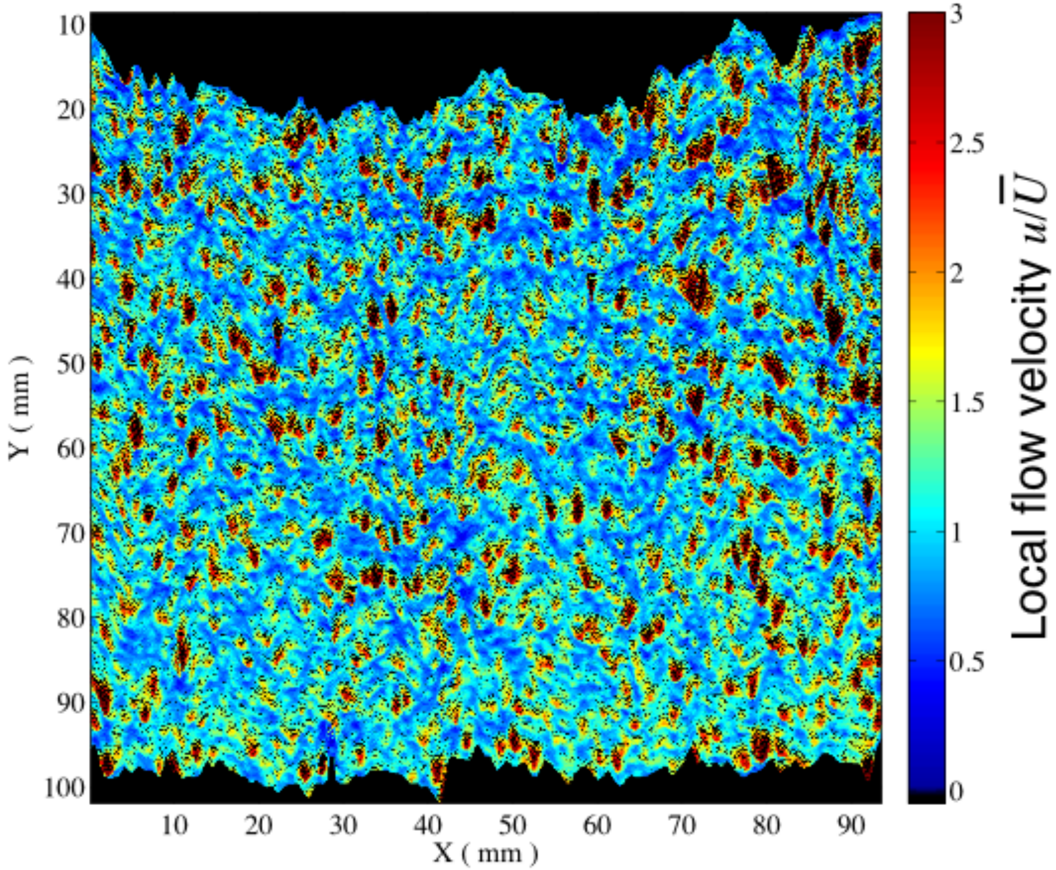
Tracers dispersion experiments: **measurements of the local flow velocity**



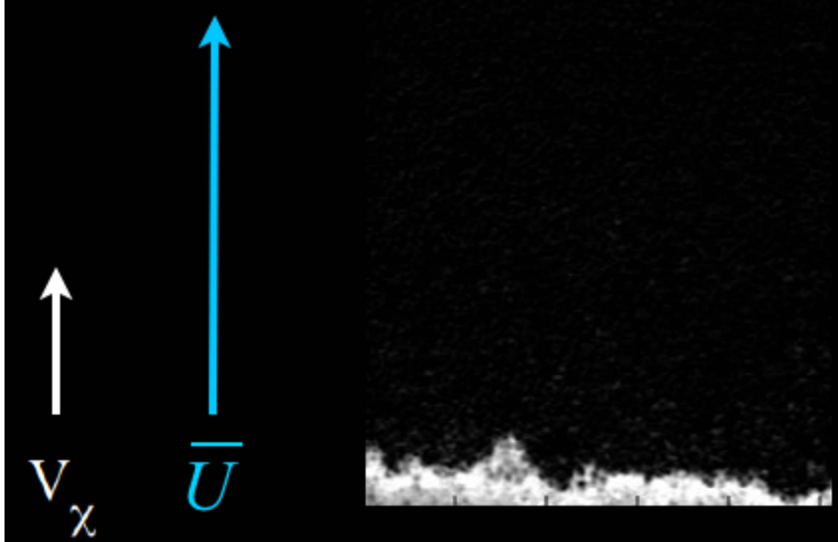
Fluctuations correlation length:

$$d_{\parallel} = 1.8 \pm 0.1 \text{ mm}$$

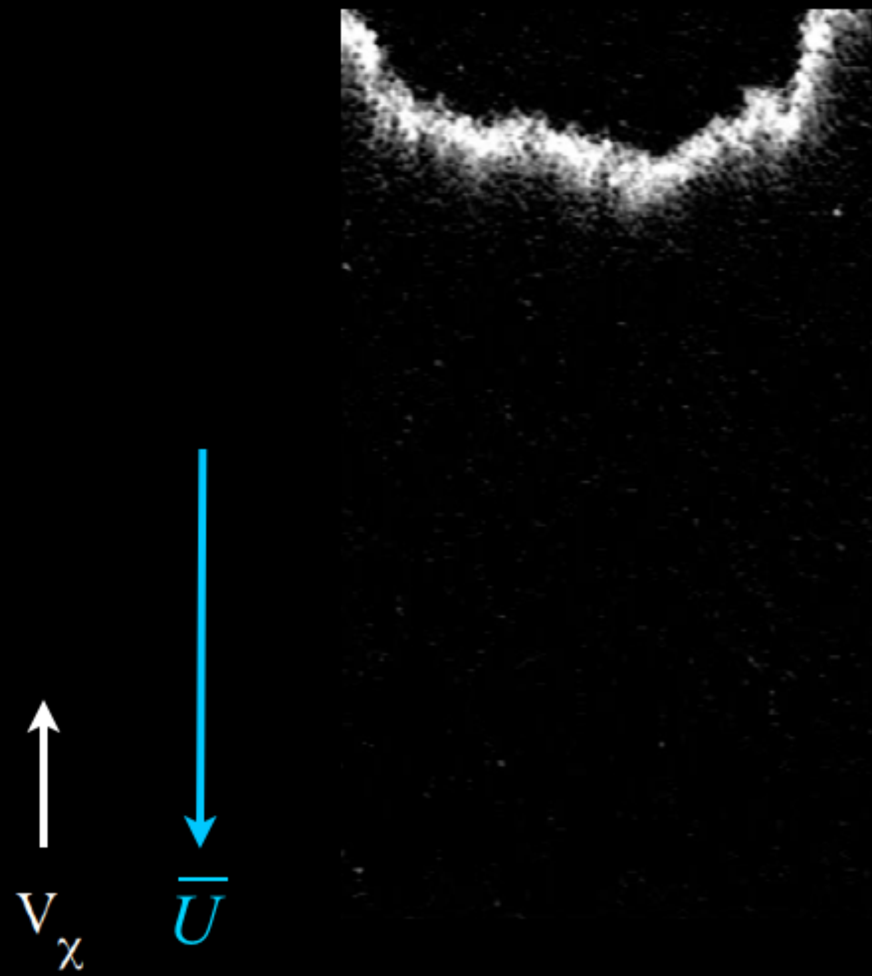
Disordered flow of mean velocity U



Supportive flow

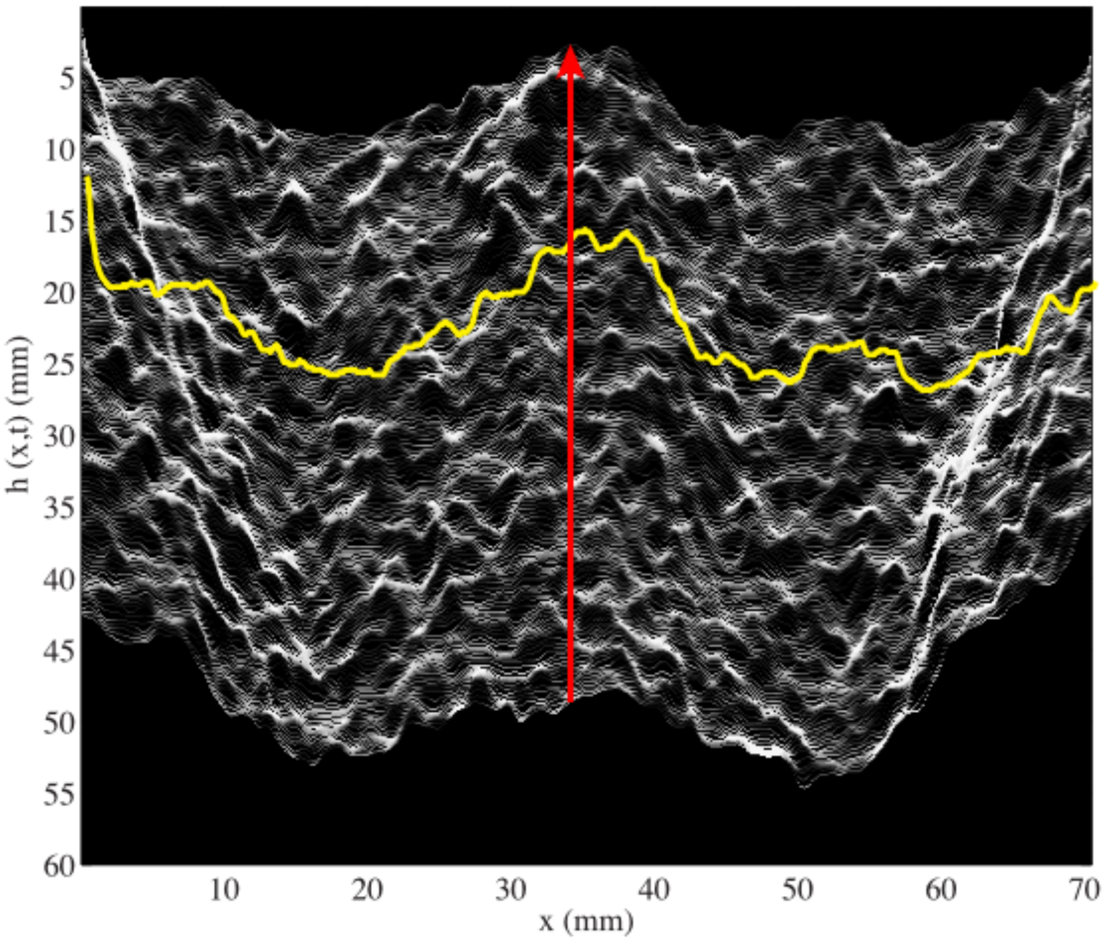


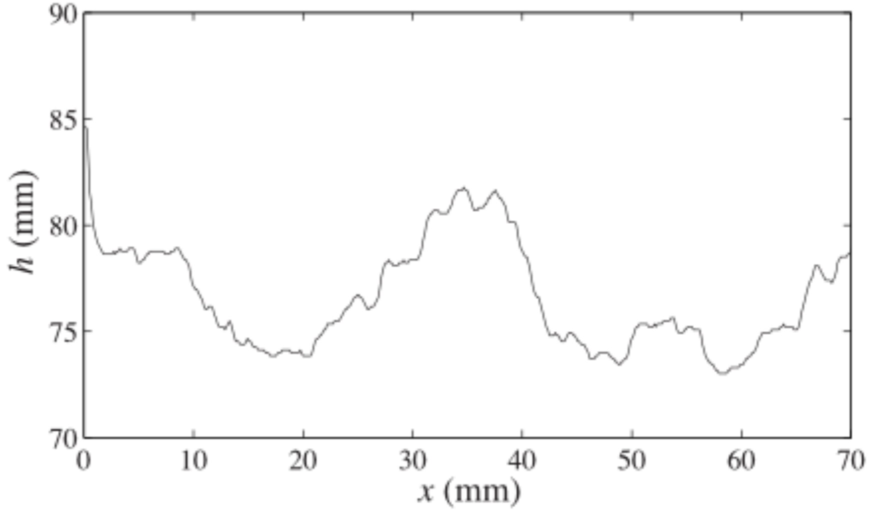
Adverse flow

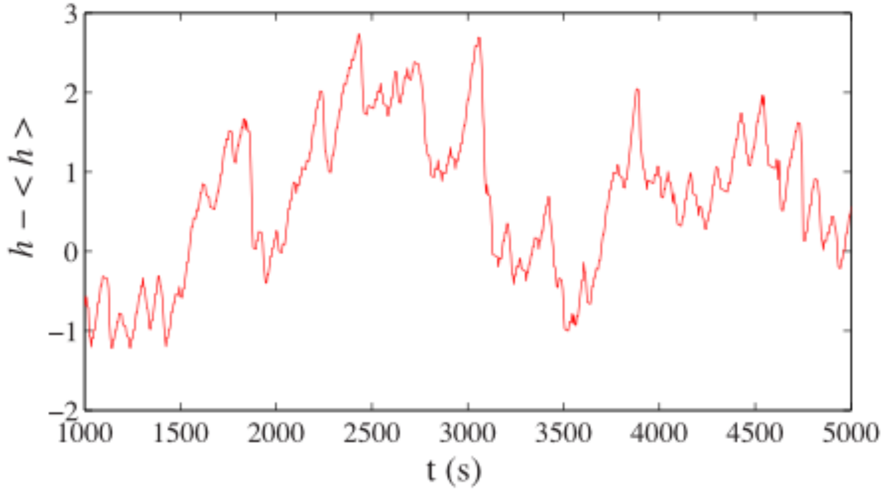


Upstream propagating fronts

- Front height spatiotemporal fluctuations measurements







Upstream propagating fronts

roughness exponent

$$w(\Delta x, t) = \left\langle \sqrt{\langle [h(x,t) - \langle h \rangle_{\Delta x}\ ]^2 \rangle_{\Delta x}} \right\rangle_L \sim \Delta x^{\alpha}$$

growth exponent

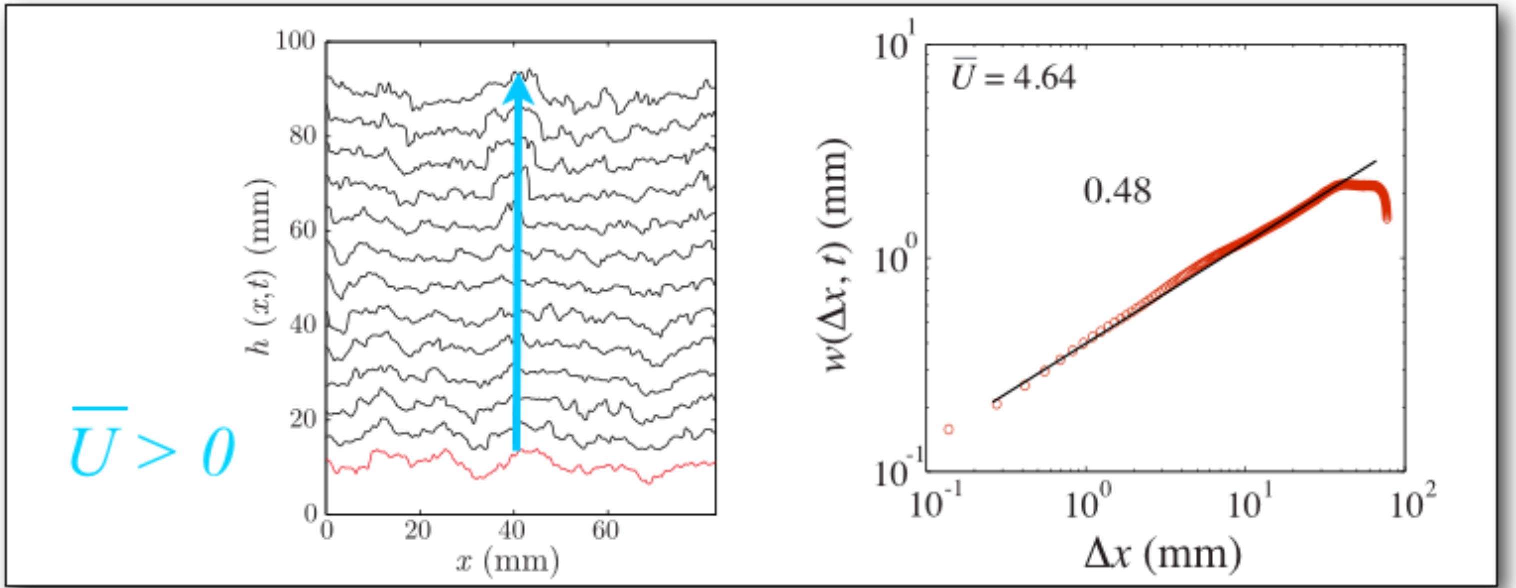
$$w(x, \Delta t) = \left\langle \sqrt{\langle [h(x,t) - \langle h \rangle_{\Delta t}]^2 \rangle_{\Delta t}} \right\rangle_T \sim \Delta t^{\beta}$$

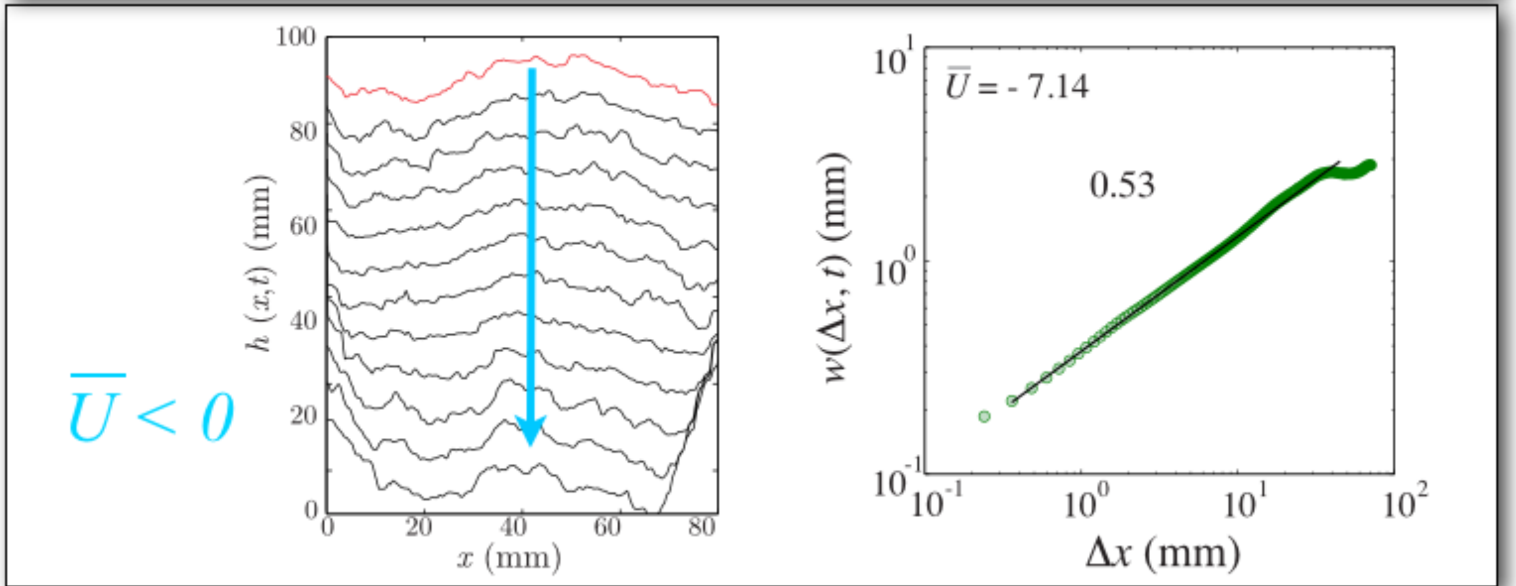
Fast propagation

Roughness exponent

power law:

$$w(\Delta x, t) \sim \Delta x^{\alpha}$$



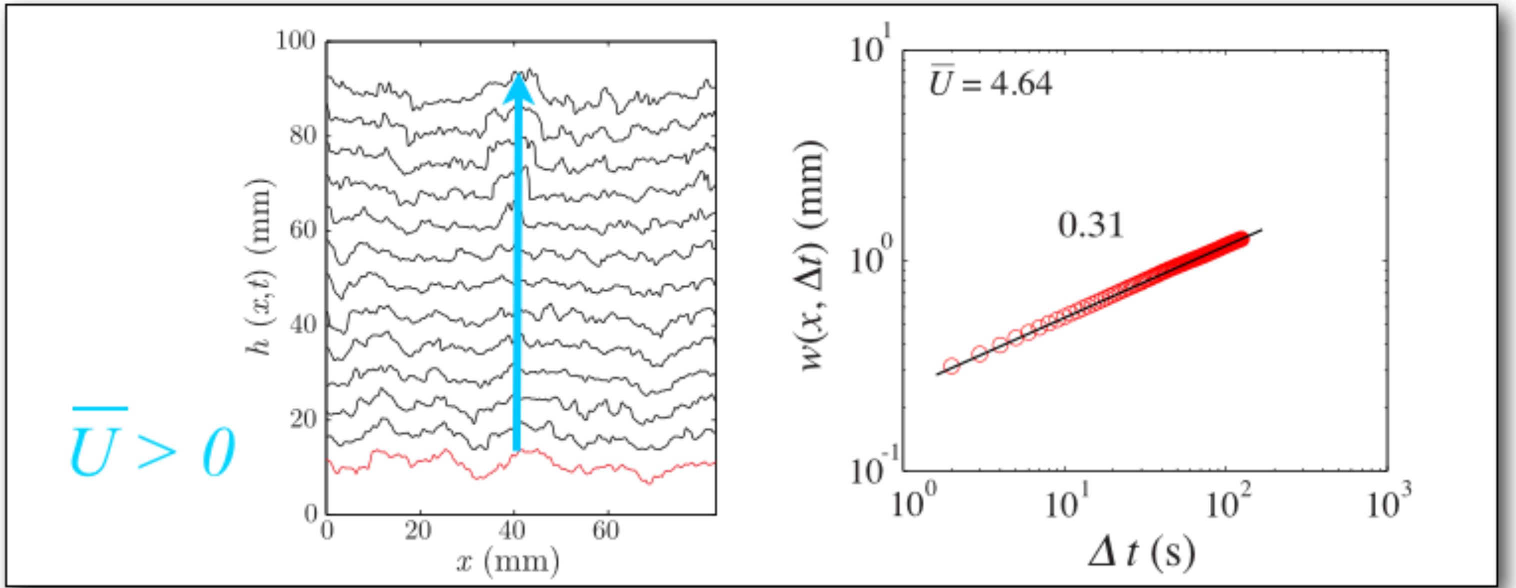


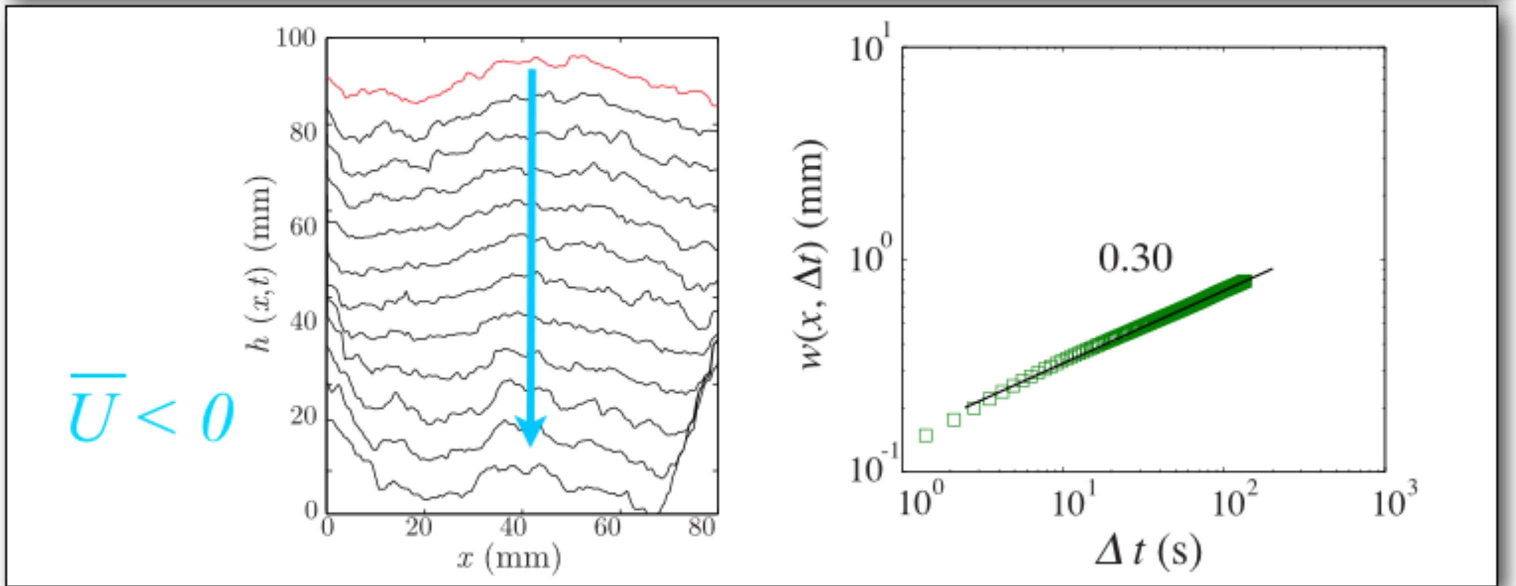
Fast propagation

Growth exponent

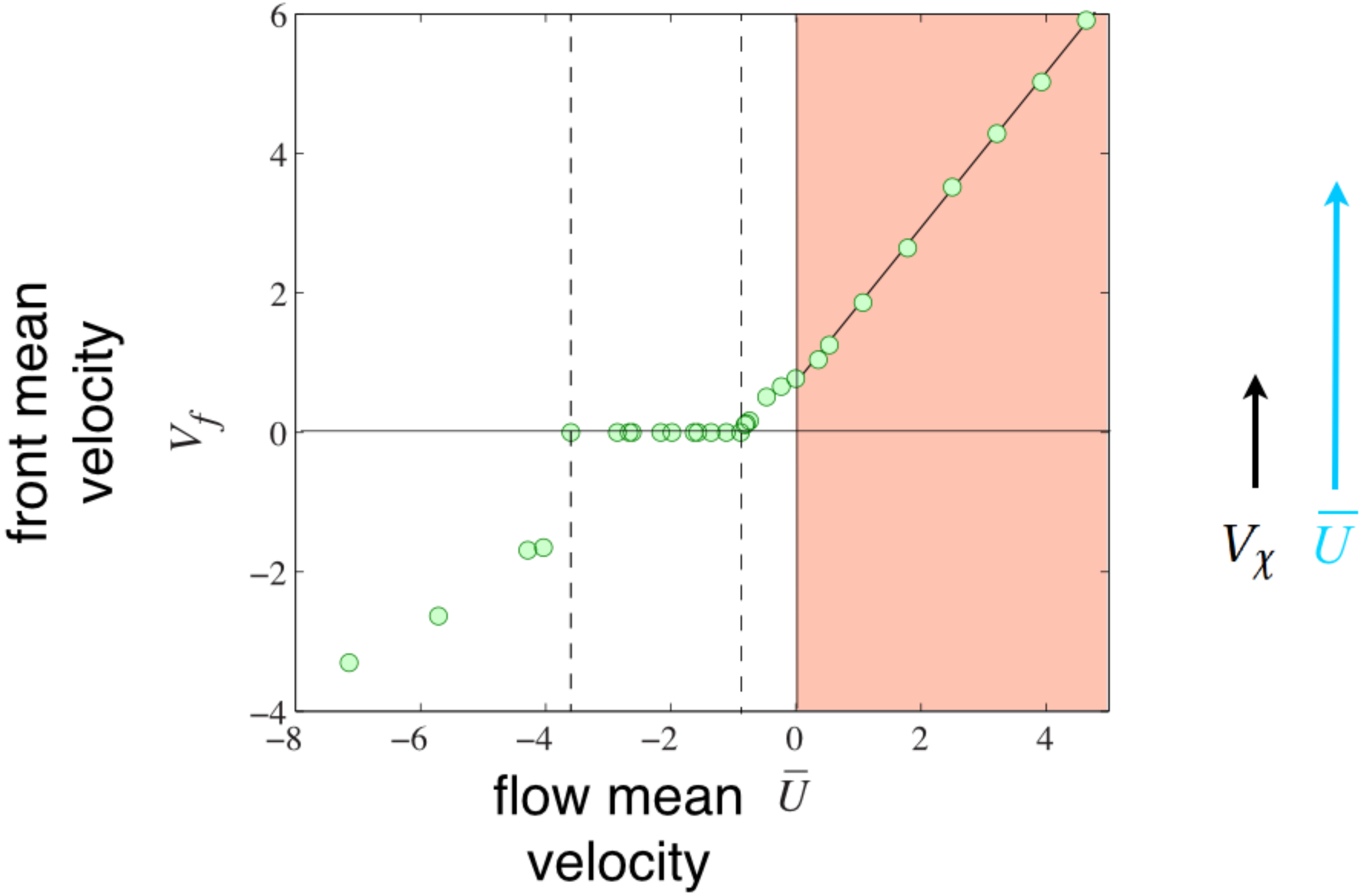
power law:

$$w(x, \Delta t) \sim \Delta t^{\beta}$$





Propagation regimes



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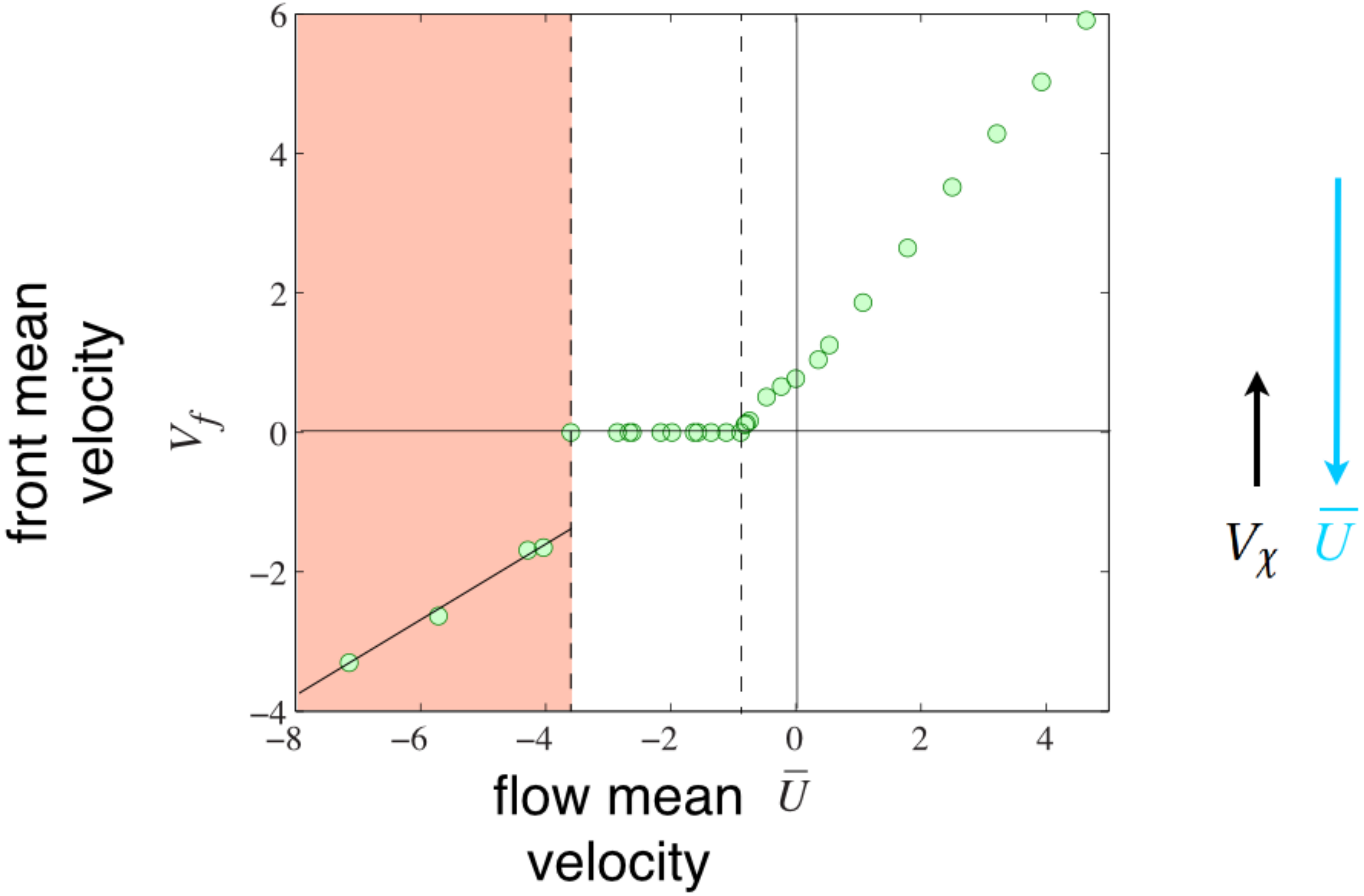
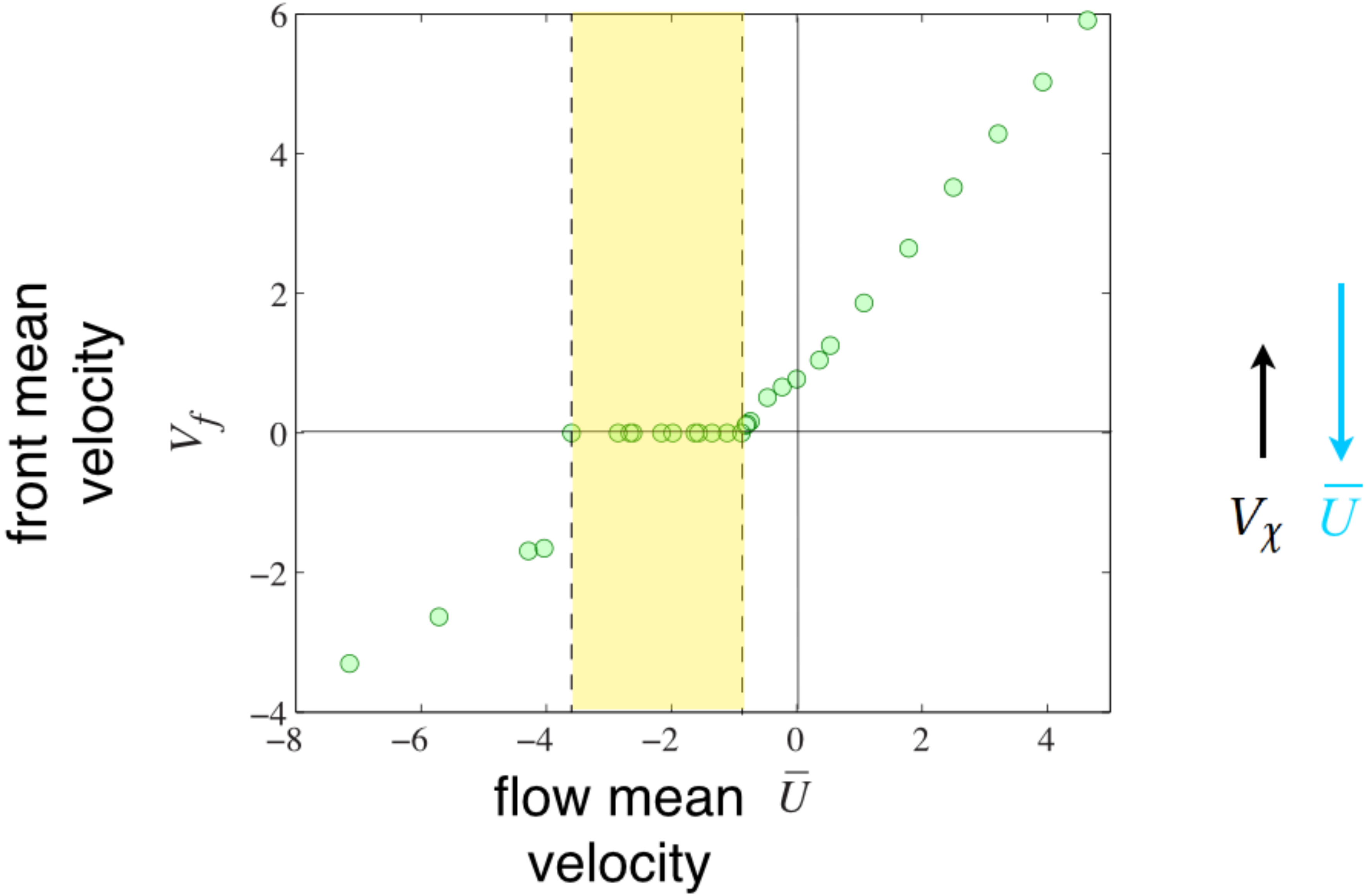
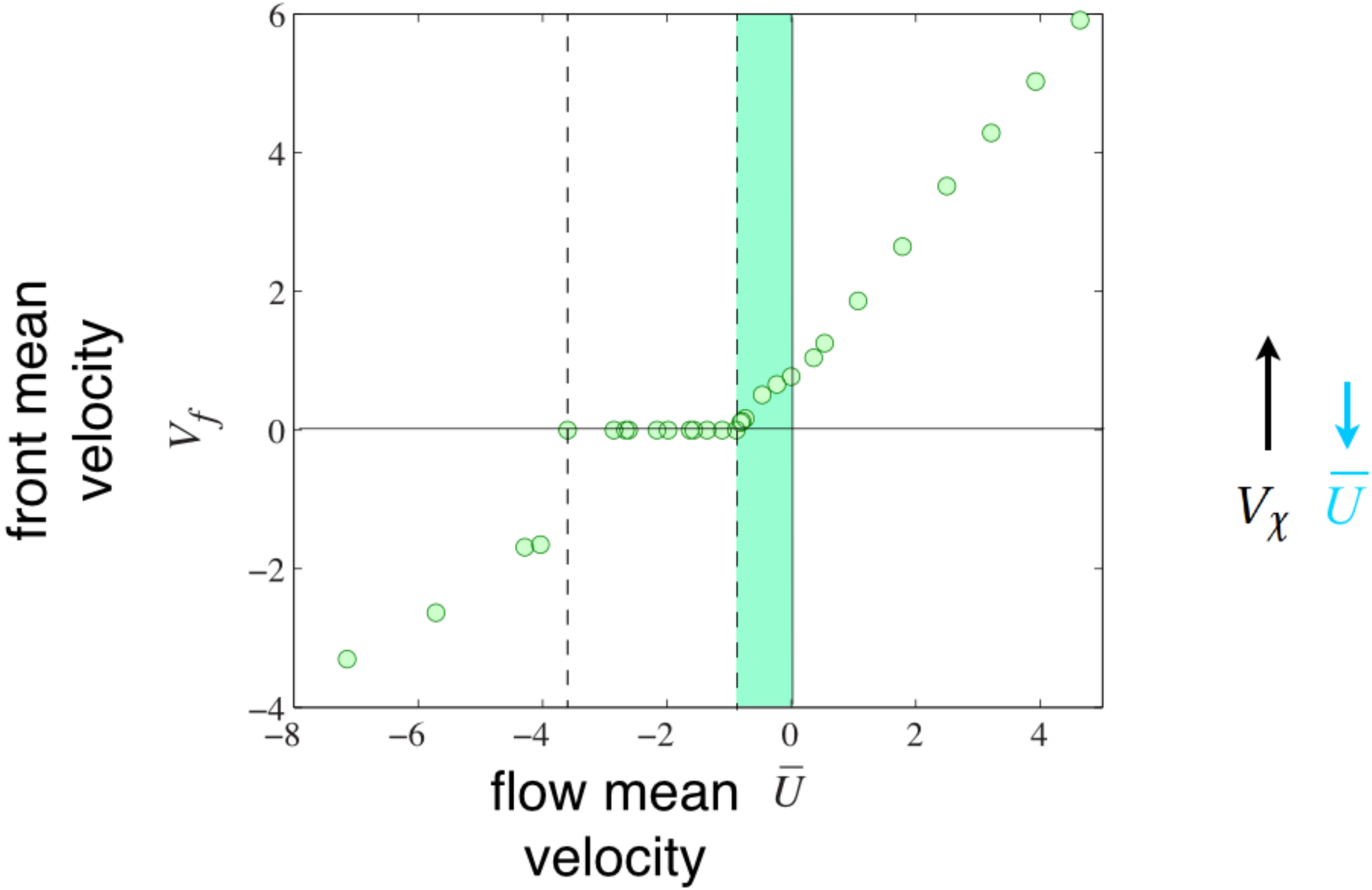


Propagation regimes



Propagation regimes



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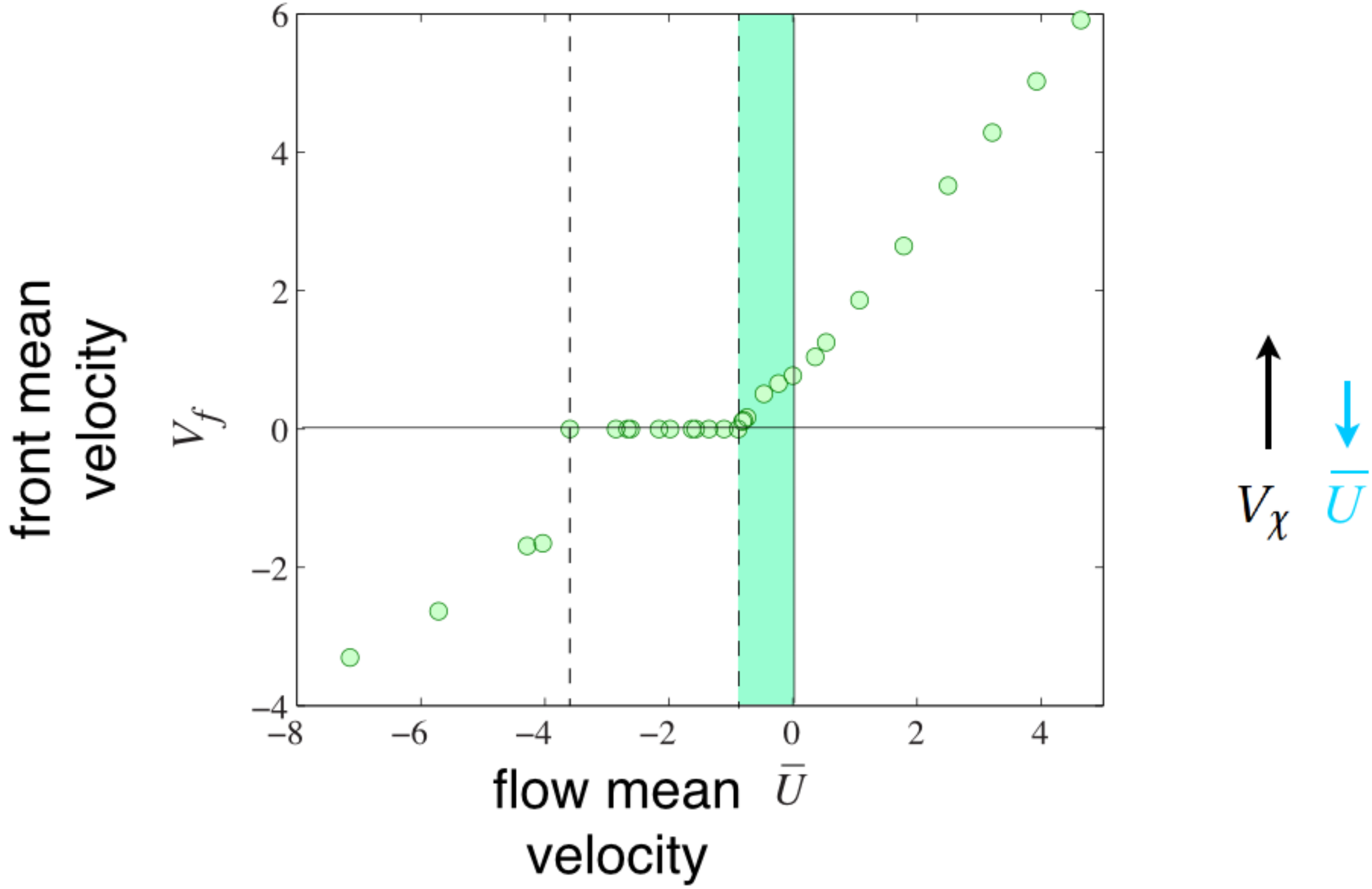
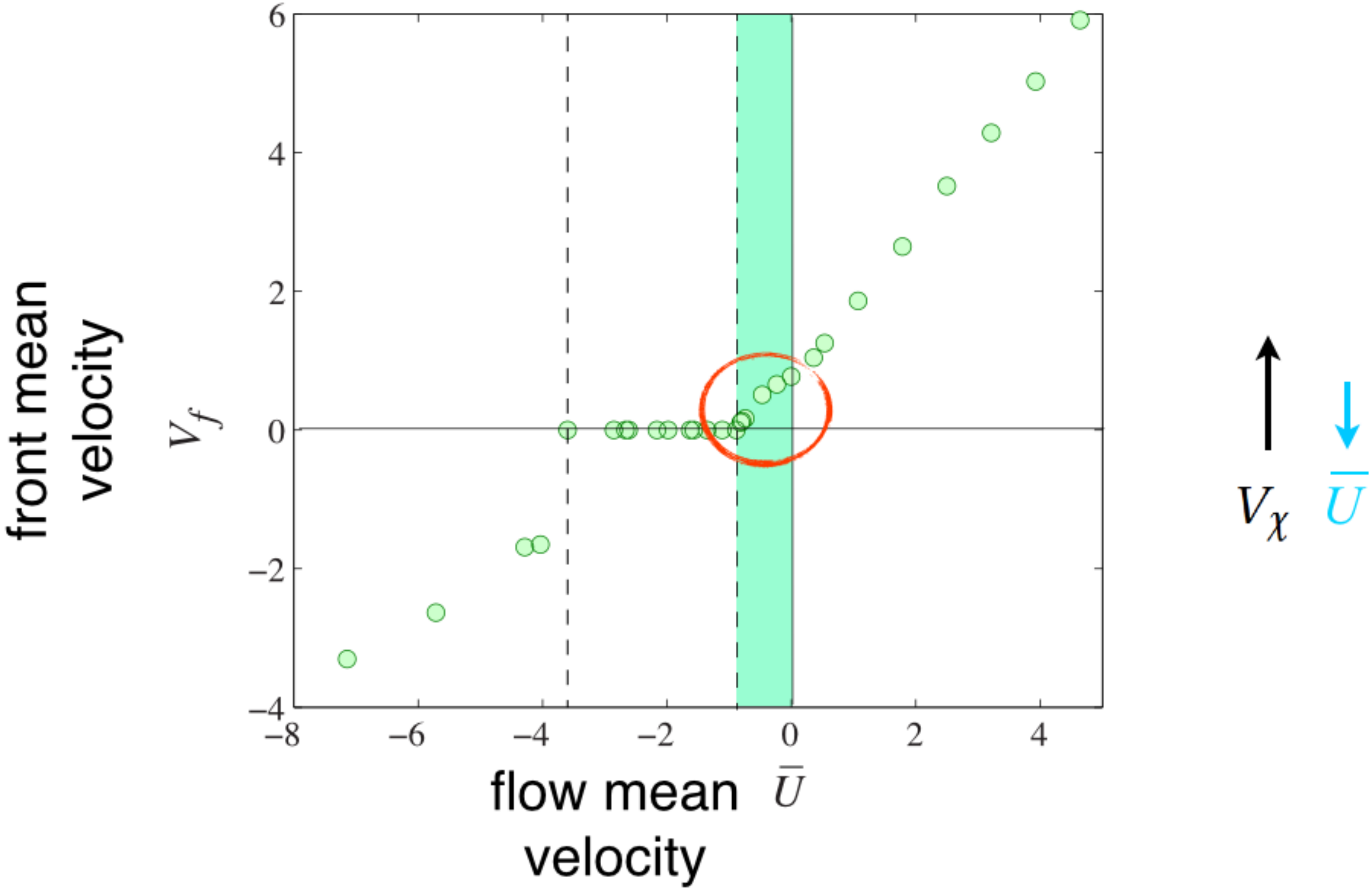
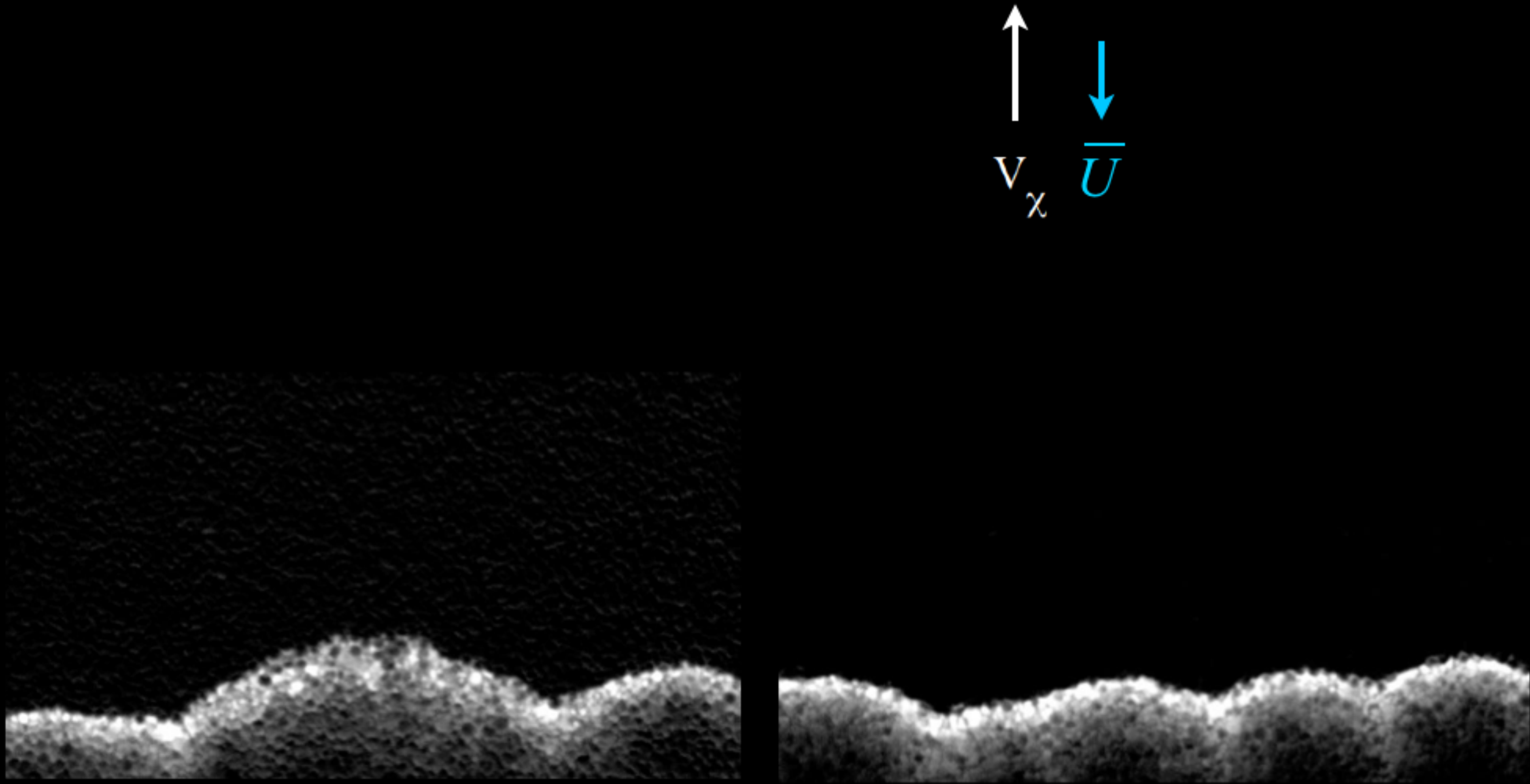


Propagation regimes



Adverse flow



Upstream propagating fronts

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growth exponent

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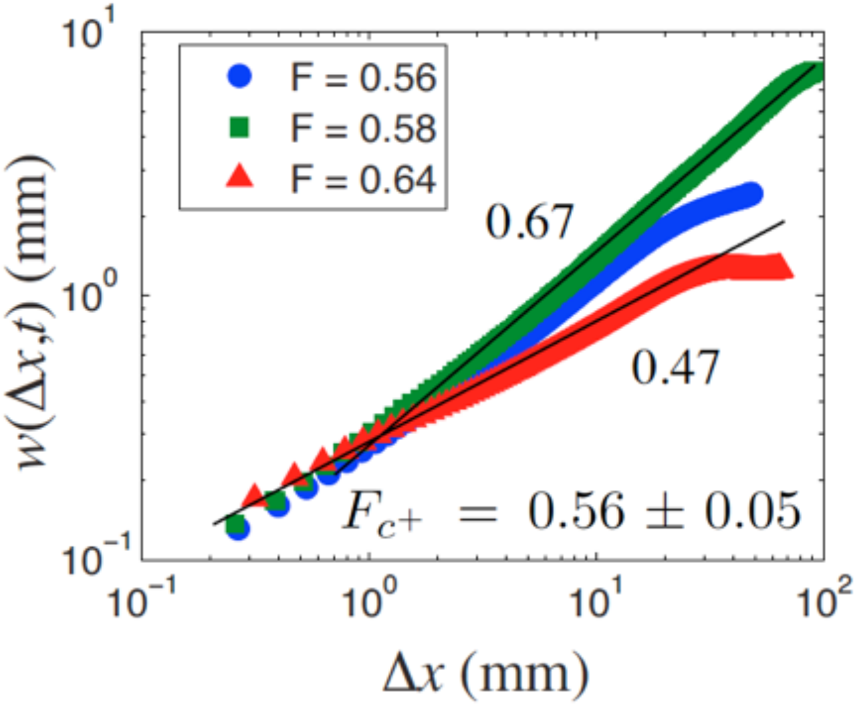
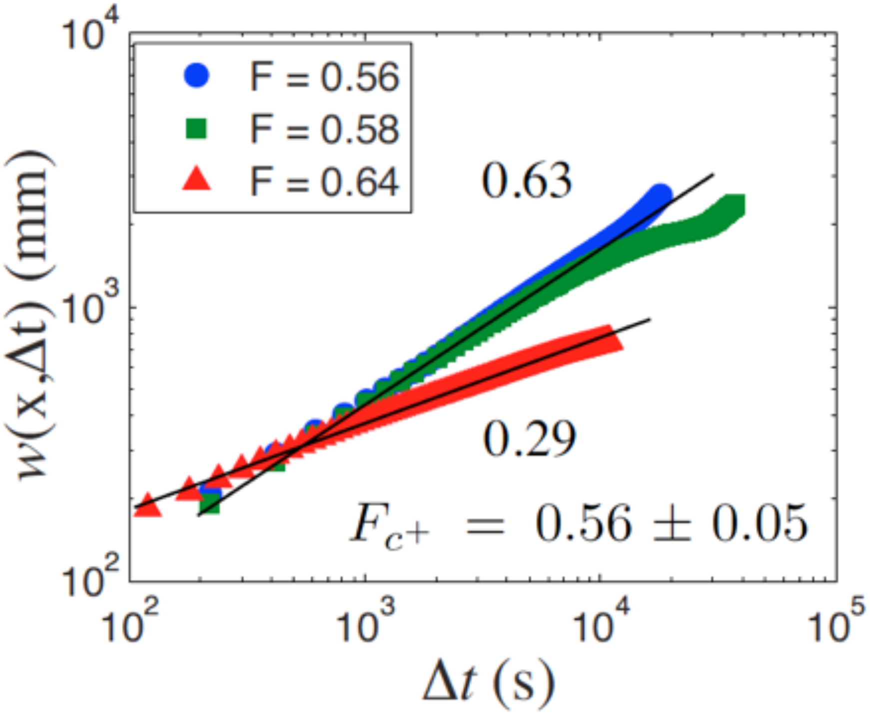
Control parameter:

$$F = \frac{\bar{U} + V_\chi}{V_\chi} + f_0$$





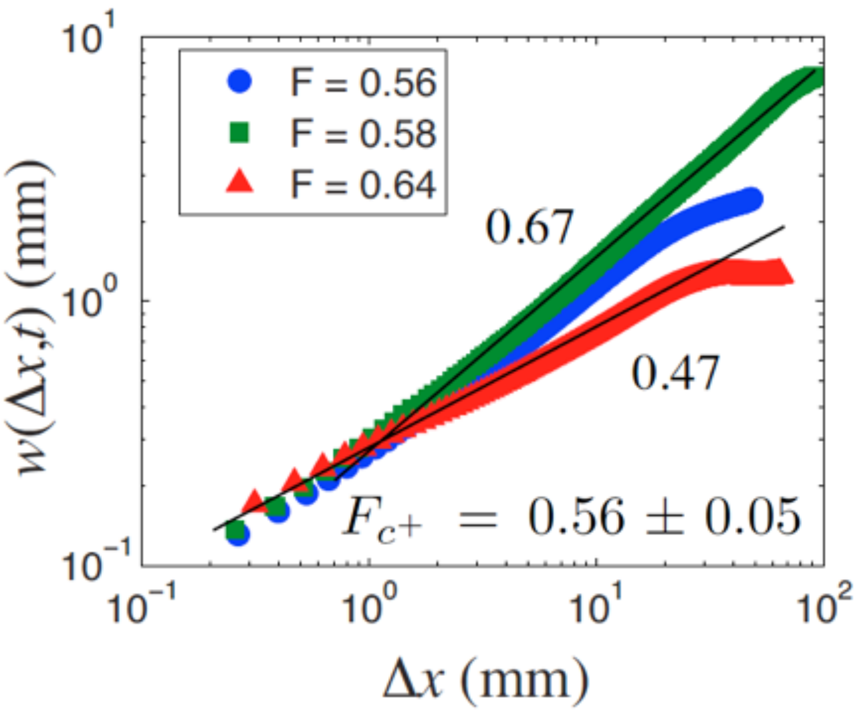
Upstream propagating fronts

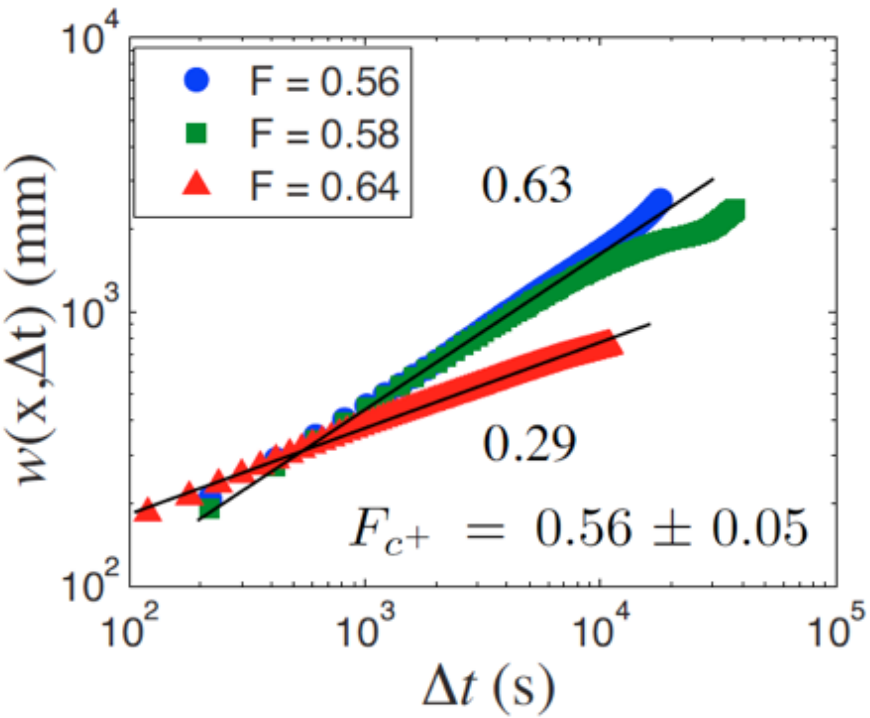
quenched KPZ model predicted exponent

$\alpha \simeq 0.63 \quad \beta \simeq 0.63$

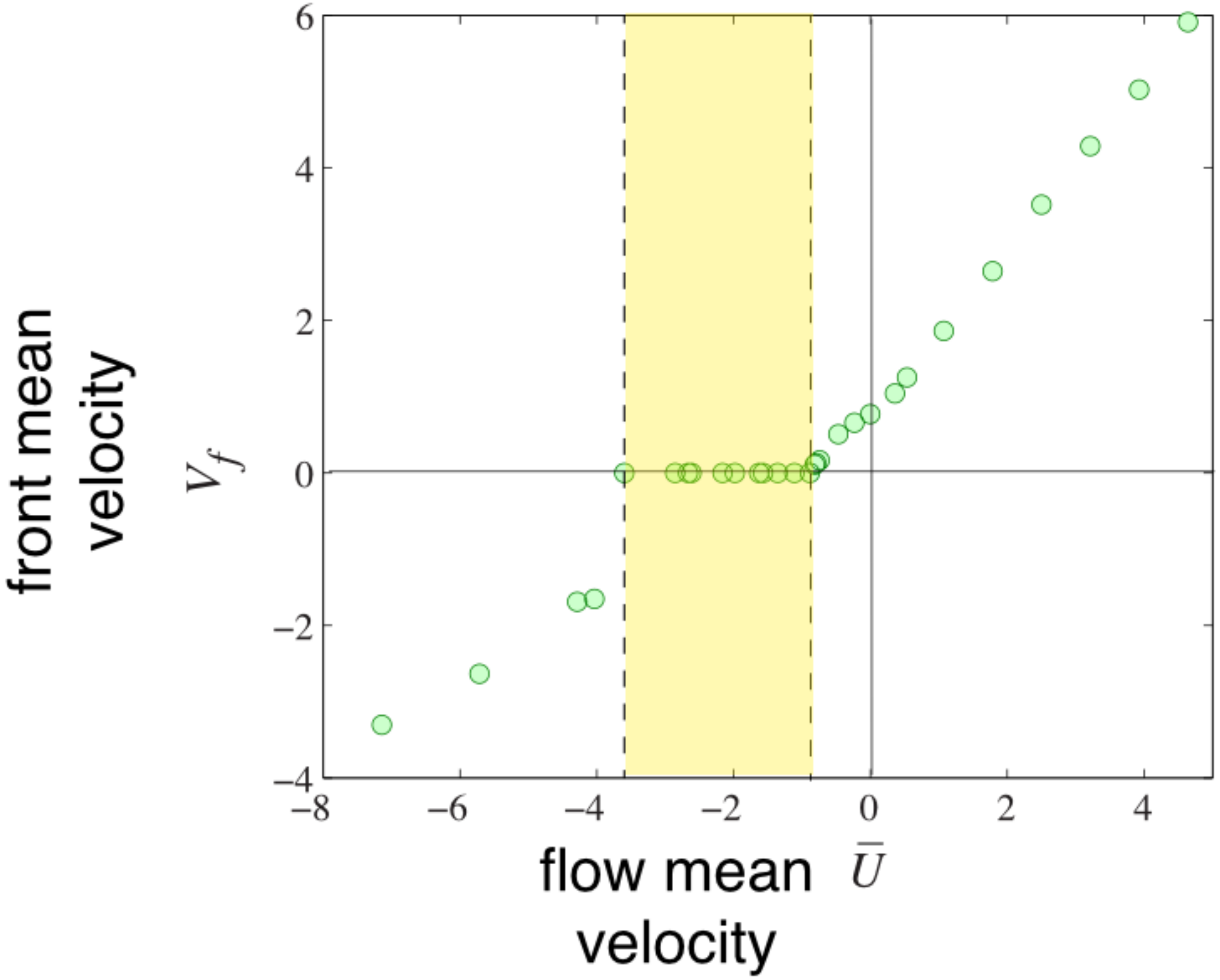
Control parameter:

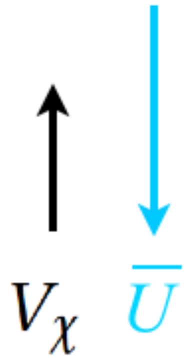
$$F = \frac{\bar{U}+V_\chi}{V_\chi} + f_0$$



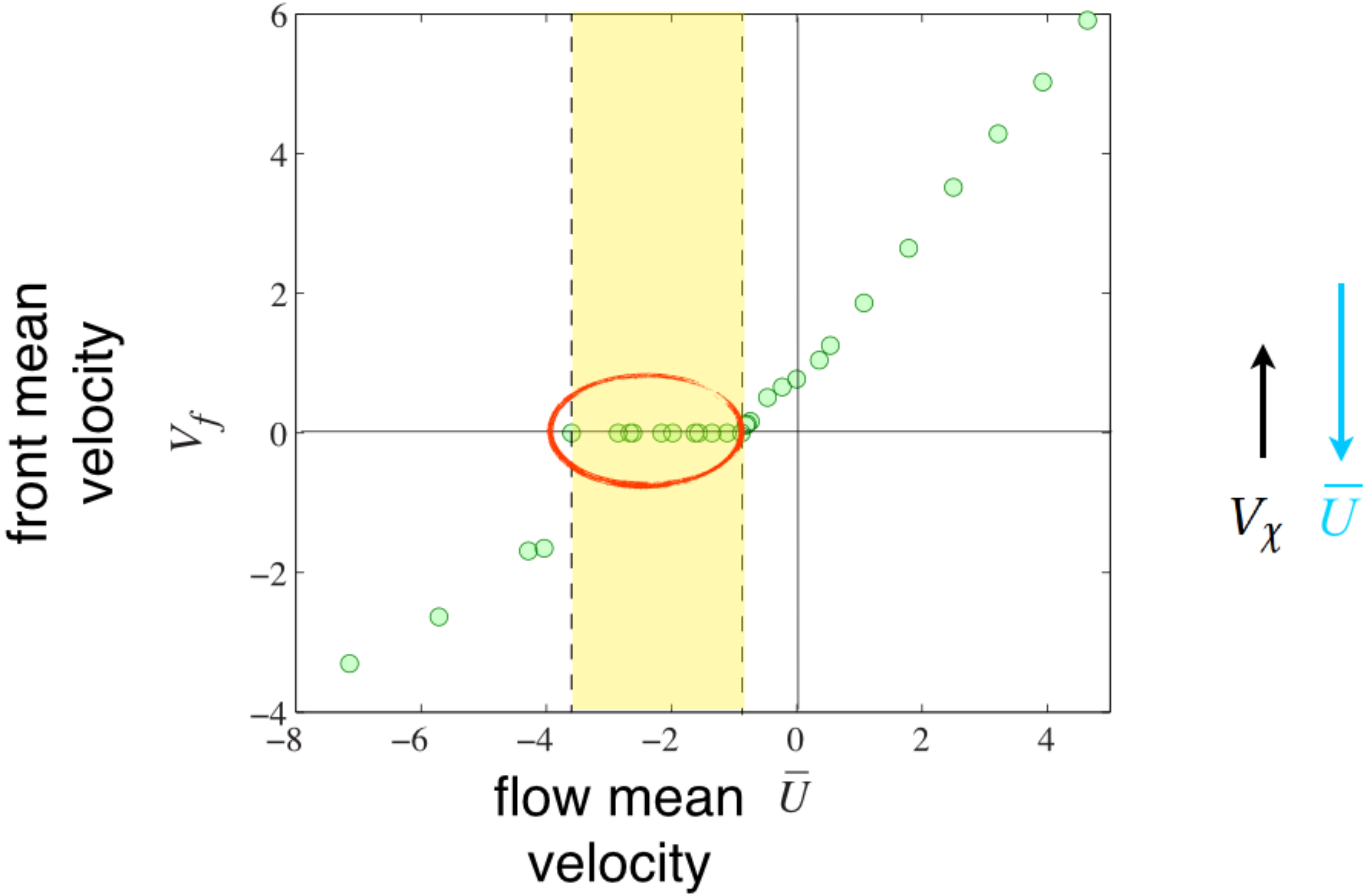


Propagation regimes



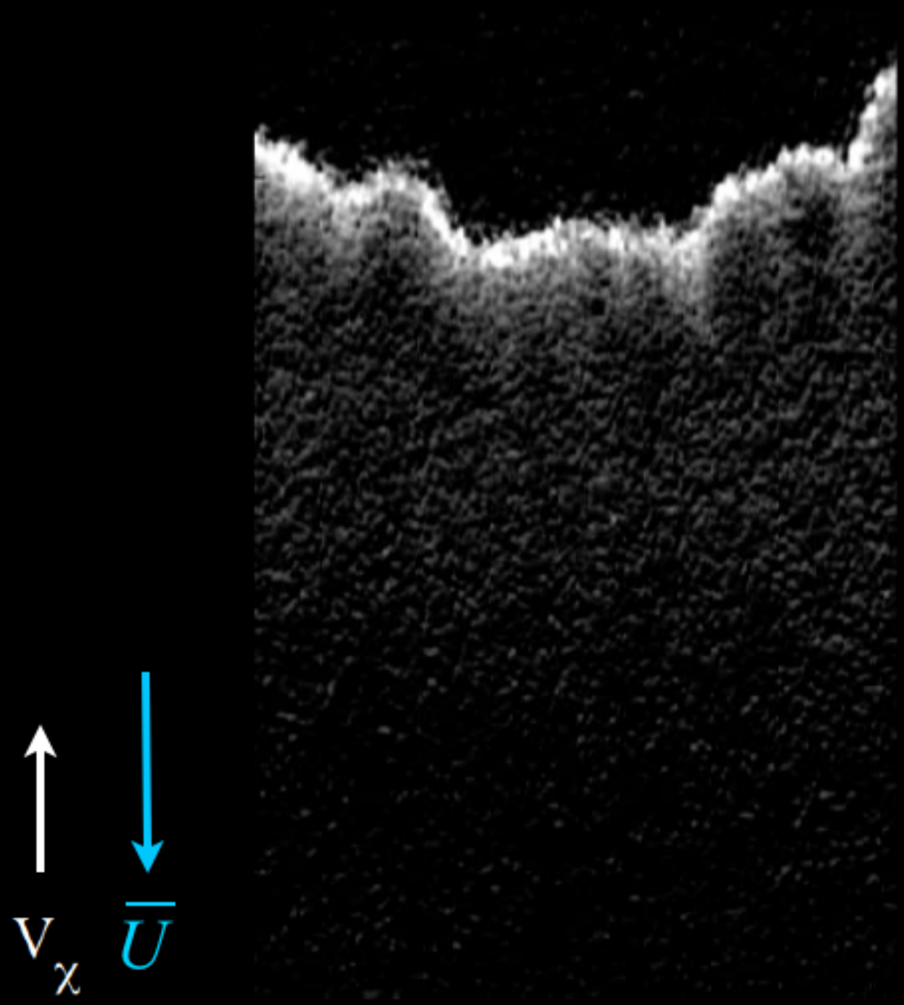


Propagation regimes



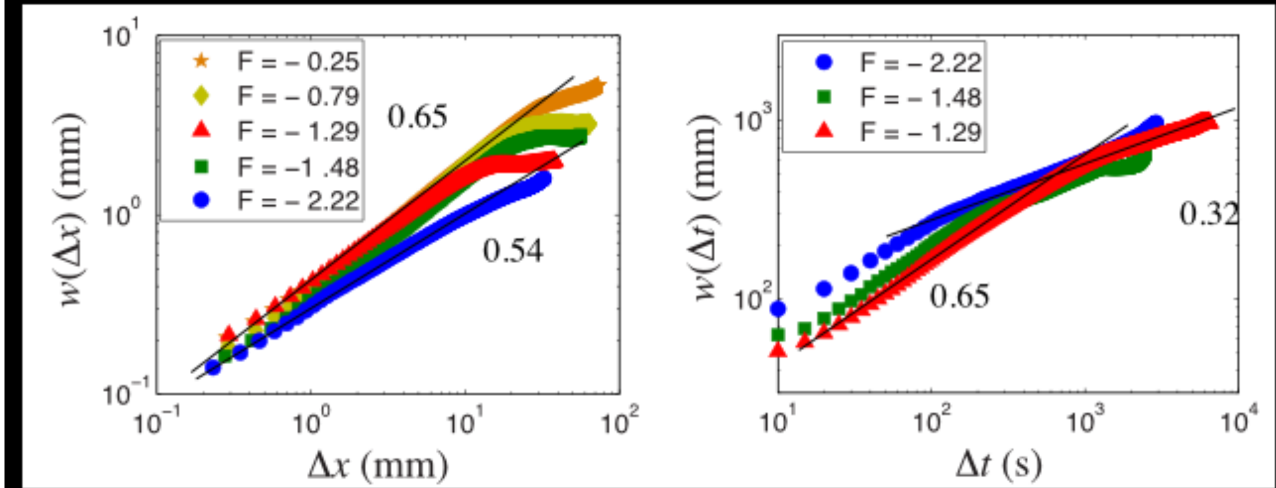
Adverse flow

backward



Backward propagating fronts

scaling of the front before the formation of the sawtooth pattern



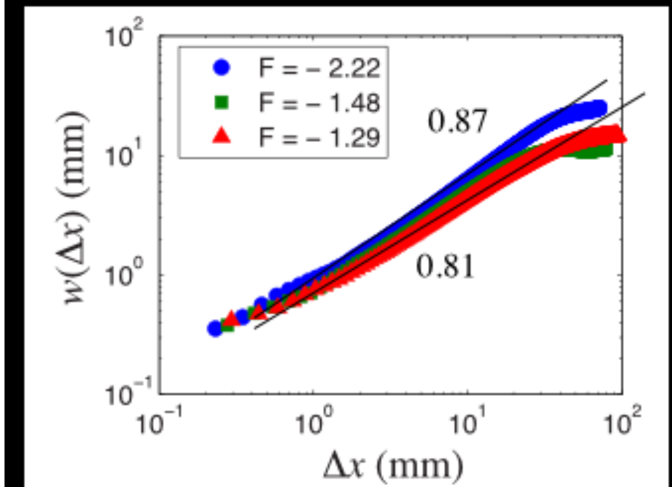
Predicted exponents:

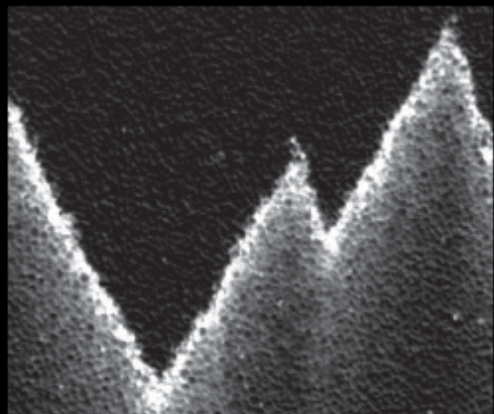
q-KPZ positive

$\alpha \simeq 0.63$ $\beta \simeq 0.63$

$w(l,t) \sim l^{\alpha}$ $w(l,t) \sim t^{\beta}$

roughness of the final sawtooth pattern





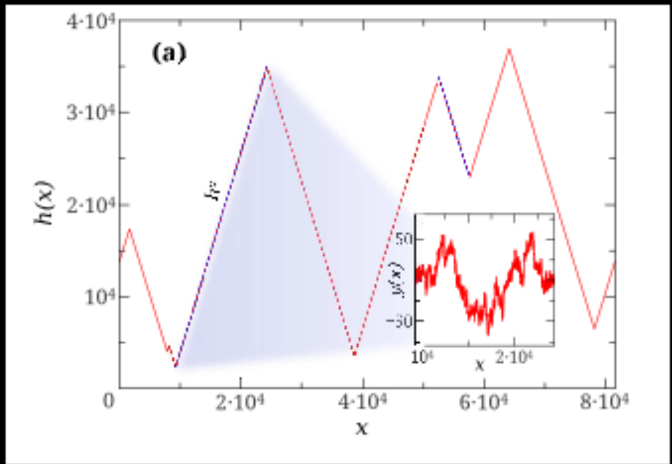
self-similar front

$\alpha \to 1$

Scaling in the negative qKPZ is altered by the large scale structures

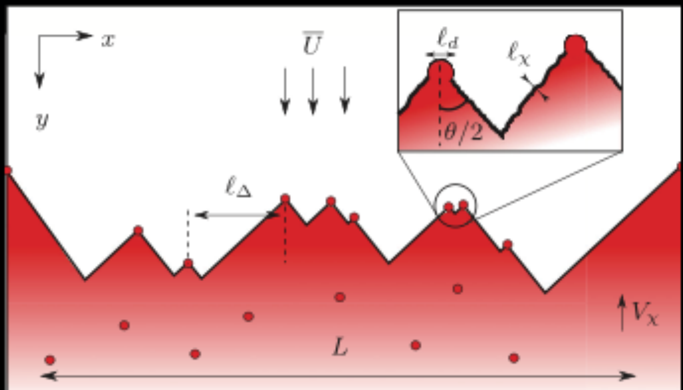
Backward propagating fronts

Alternative analysis



roughness of the front after subtracting the slope

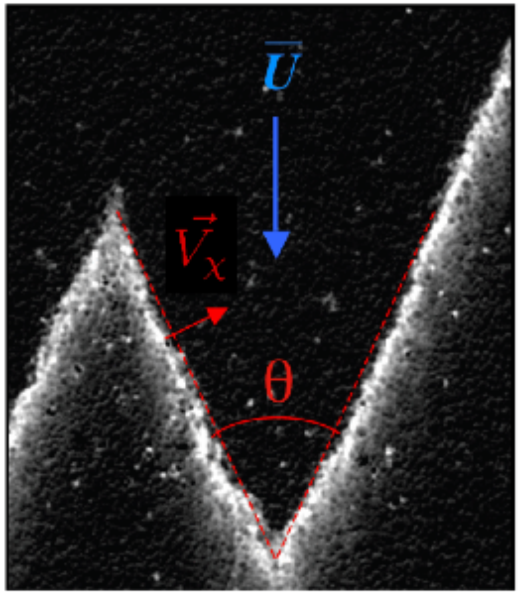
Moglia et al. , *Stat. Mech* (2014)



small-gradient qKPZ may not be quantitatively accurate. A more precise scenario was proposed based on the PNG model and extreme-value statistics.

T. Gueudré, A. K. Dubey, L. Talon, and A. Rosso, *PRE* (2014)

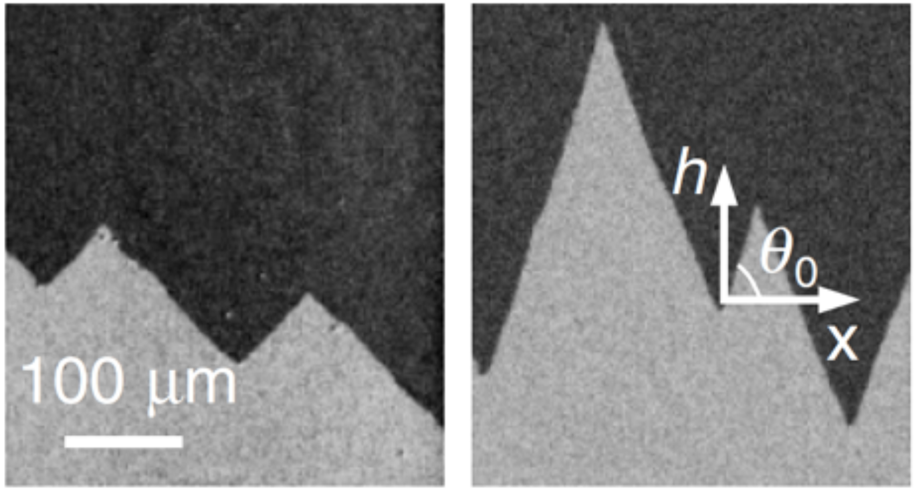
Backward propagating fronts



[Atis & al., *PRL* **110** (2013)]

$$V_\chi + \overline{U}\sin(\theta/2) = 0$$

- **Magnetic domain wall frozen steady states**



[Moon & al., *PRL* **110** (2013)]

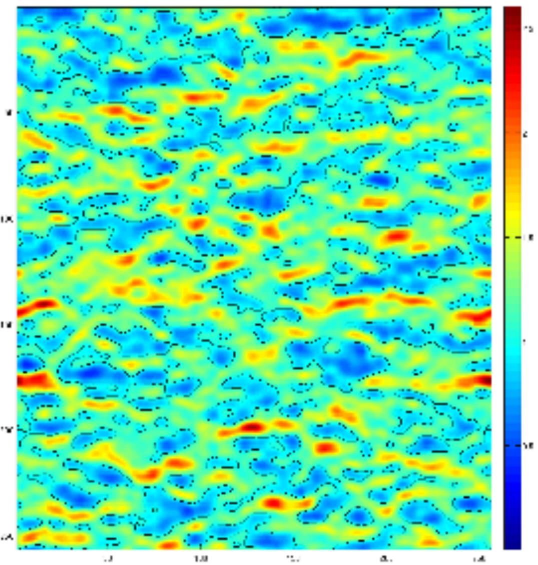
$$\epsilon(= H/J\cos\theta_0)$$

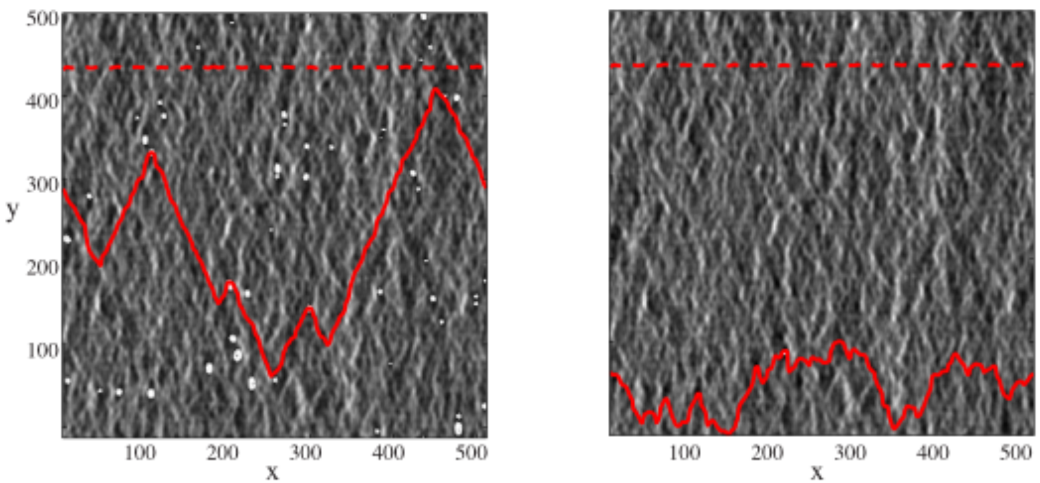
H : applied magnetic filed

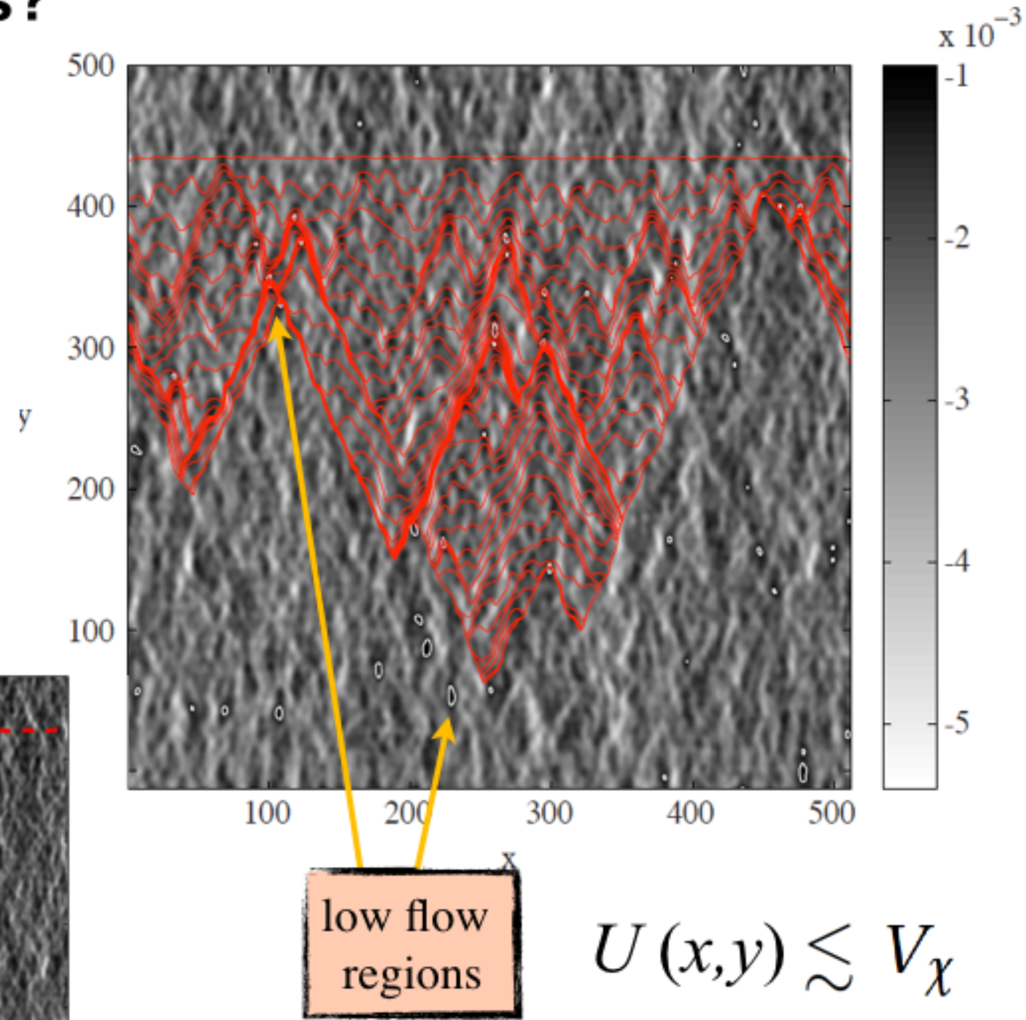
J : applied electric current

Backward propagating fronts

- **What is pinning reaction fronts?**







$$U\,(x,y) \lesssim V_\chi$$

[Saha & al., *EPL* **101** (2013)]

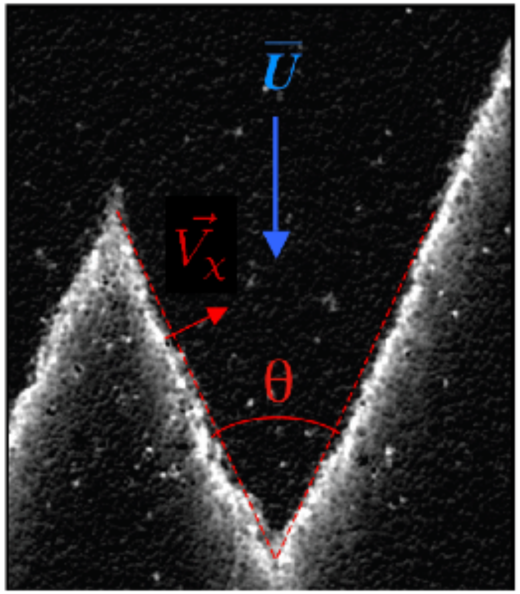
Backward propagating fronts

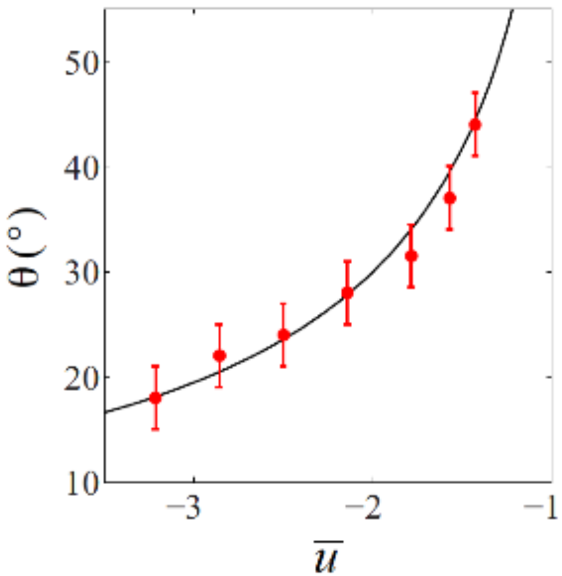
approximation eikonal $l_\chi << l_d$

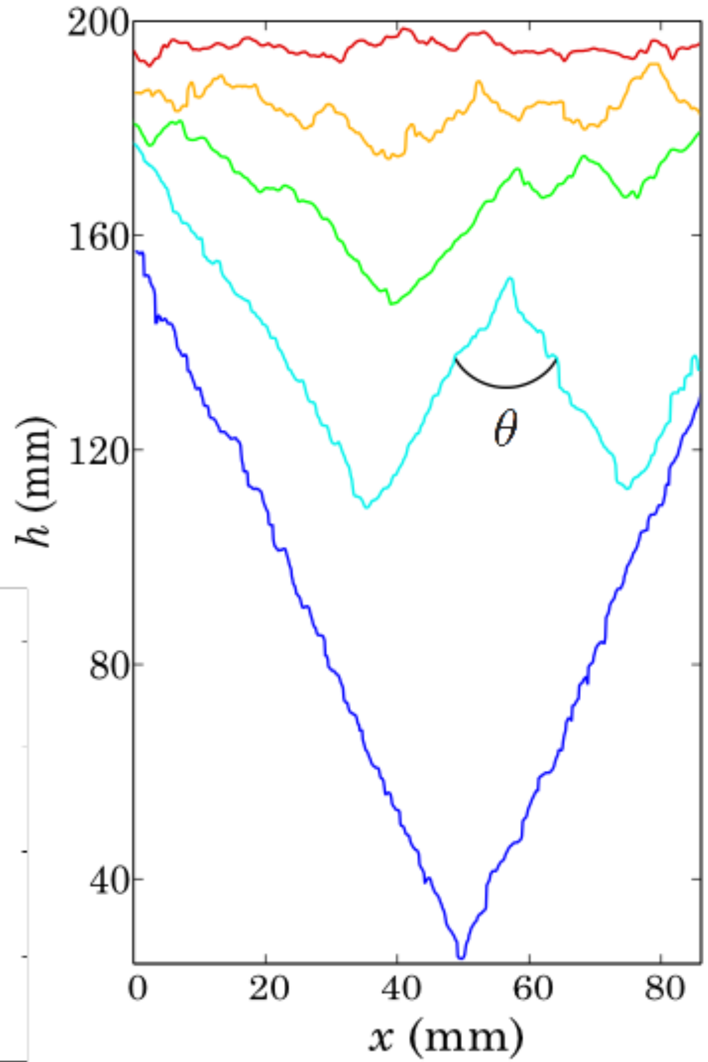
$$\vec{V_f}(\vec{r}) \cdot \vec{n} = V_\chi + \vec{U}(\vec{r}) \cdot \vec{n} + D_m \kappa$$

final static fronts

$$V_\chi + \overline{U} \sin(\theta/2) = 0$$





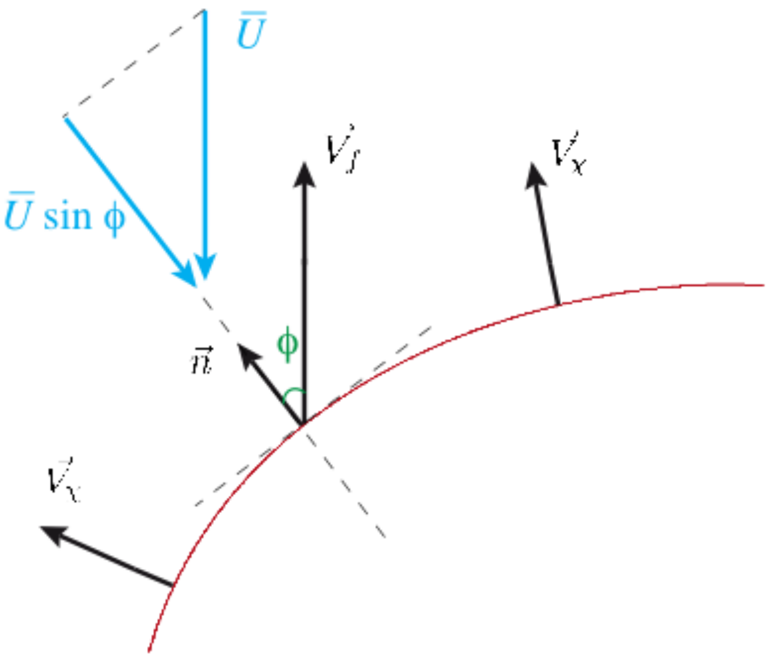


[Atis & al., *PRL* **110** (2013)]

Advection - Reaction - Diffusion equation in thin front limit

Eikonal equation:

$$\vec{V_f} \cdot \vec{n} = V_\chi + D_m \kappa + \vec{U}(\vec{r}) \cdot \vec{n}$$



$$\vec{V_f} = \begin{pmatrix} 0 \\ V_f \end{pmatrix}, \quad \vec{U} = \begin{pmatrix} U_x \\ U_y \end{pmatrix}, \quad \vec{n} = \begin{pmatrix} -\sin\phi \\ \cos\phi \end{pmatrix}$$

$$V_f = \frac{\partial h}{\partial t} \text{ et } \kappa = \frac{\partial^2 h/\partial x^2}{(1+(\partial h/\partial x)^2)^{3/2}} ,$$

$$\tan\phi = \nabla_x h$$

$$\cos\phi = \frac{1}{\sqrt{1+(\nabla_x h)^2}}$$

with $s = \nabla h$ and $\vec{U}(\vec{r}) = \bar{U}\vec{e}_y + \delta\vec{U}(\vec{r})$

$$\frac{\partial h}{\partial t} = \sqrt{1+s^2}\left[D_m \partial_x^2 h/(1+s^2)^{3/2} + V_\chi + \left(\bar{U} + \delta U_y(\vec{r}) - s\delta U_x(\vec{r})\right)/\sqrt{1+s^2}\right]$$

$$\frac{\partial h}{\partial t} = \sqrt{1+s^2}\left[D_m \partial_x^2 h/(1+s^2)^{3/2} + V_\chi + \left(\overline{U} + \delta U_y(\vec{r}) - s\delta U_x(\vec{r})\right)/\sqrt{1+s^2}\right]$$

small gradients limite $|\nabla h| \ll 1$:

$$\frac{\partial h}{\partial t} \simeq \frac{D_m \nabla^2 h}{1+(\nabla h)^2} + V_\chi \sqrt{1+(\nabla h)^2} + \overline{U} + \delta U_y(\vec{r})$$

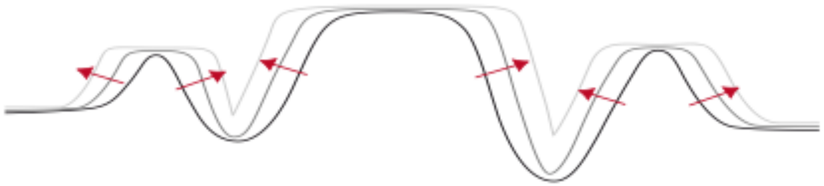
the flow is highly anisotropic $\delta U_x \ll \delta U_y$

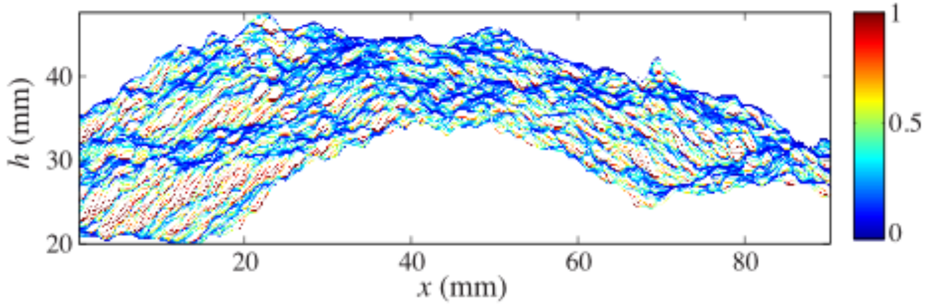
$$\frac{\partial h}{\partial t} \simeq D_m \nabla^2 h + \frac{V_\chi}{2}(\nabla h)^2 + \delta U_y(\vec{r}) + \overline{U} + V_\chi$$

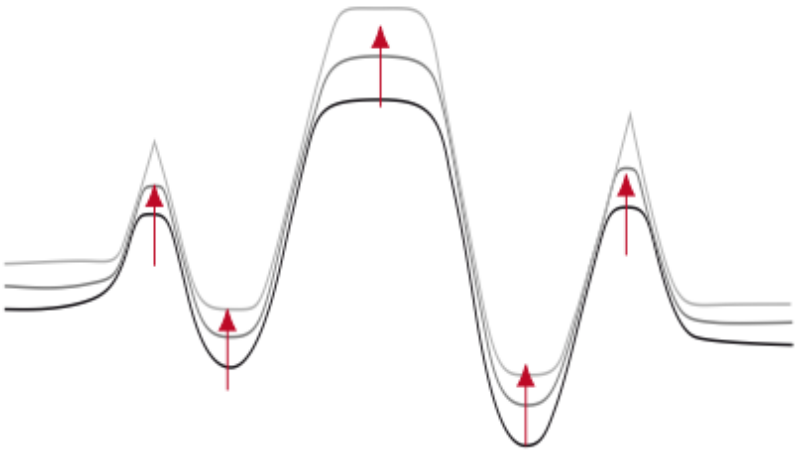
with $F = V_\chi + \overline{U}$

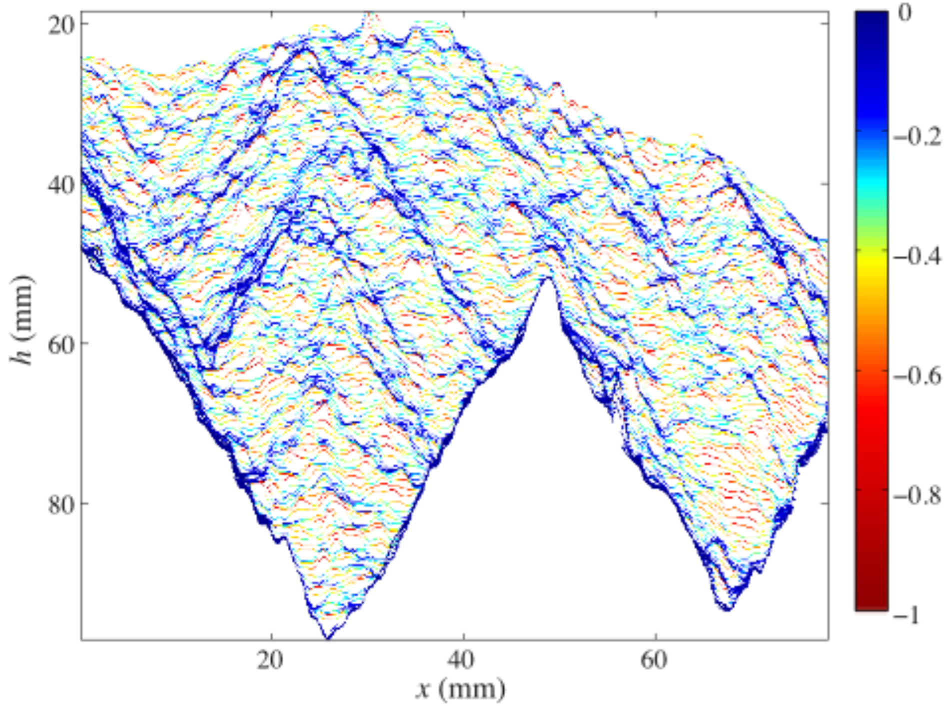
qKPZ equation :

$$\frac{\partial h(x,t)}{\partial t} = \nu \nabla^2 h(x,t) + \frac{\lambda}{2}\left[\nabla h(x,t)\right]^2 + \bar{\eta}\Big(x, h(x,t)\Big) + f$$

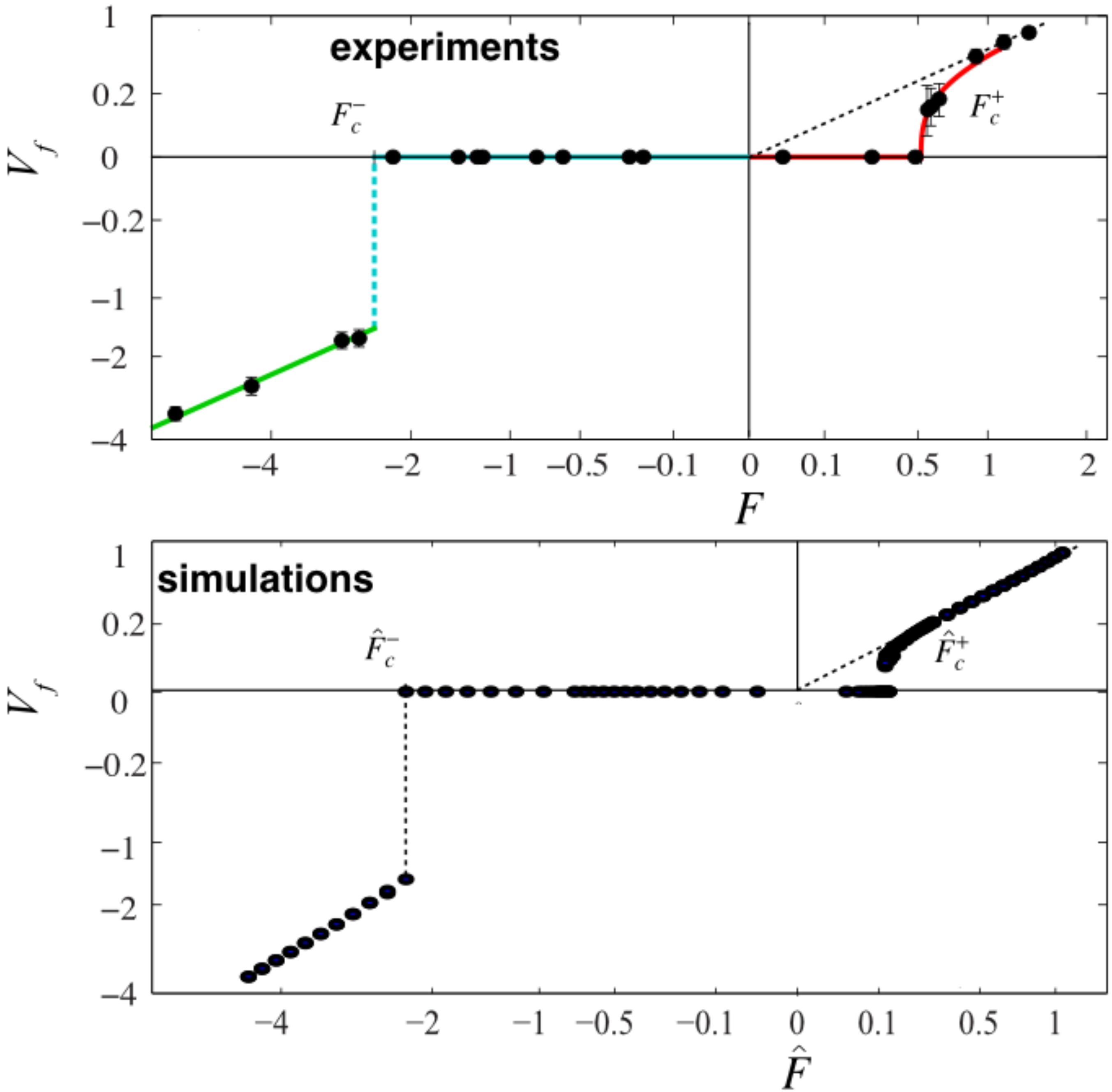


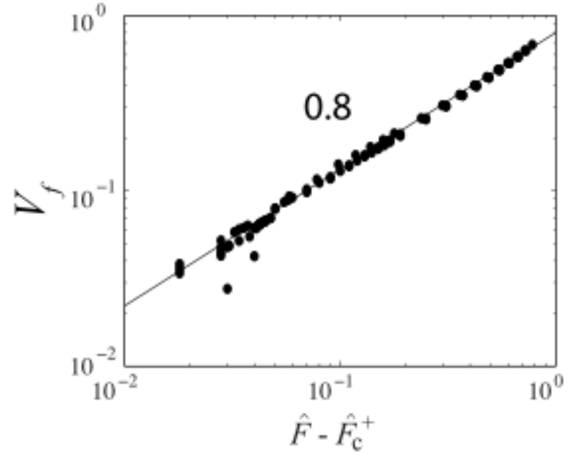






$$\frac{\partial h(x,t)}{\partial t} = D_m \nabla^2 h(x,t) - \frac{V_\chi}{2}[\nabla h(x,t)]^2 + \delta U_y(x, h(x,t)) + f$$





Amaral & al., *PRE* **51** (1995

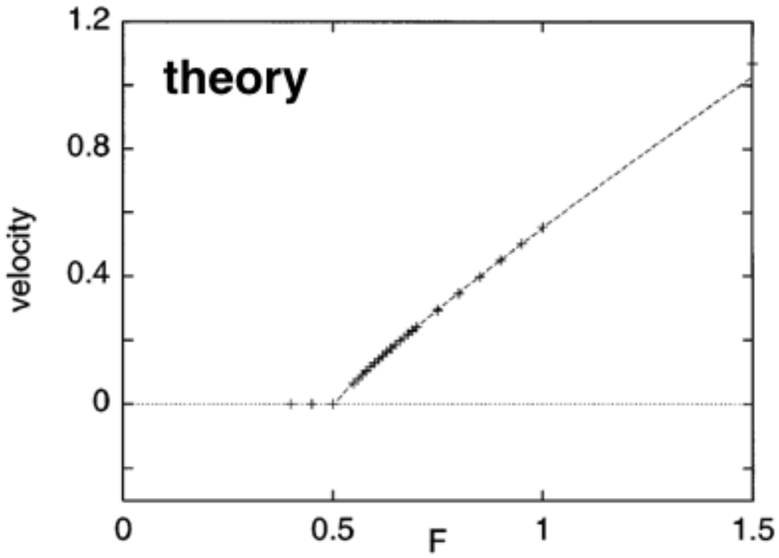
[S. Atis, K. D. Awadhesh, D. Salin, L. Talon, P. Le Doussal, K. Wiese, *PRL* (2015)]

Control parameter:

$$F = \frac{\bar{U} + V_\chi}{V_\chi} + f_0$$

positive quenched KPZ model

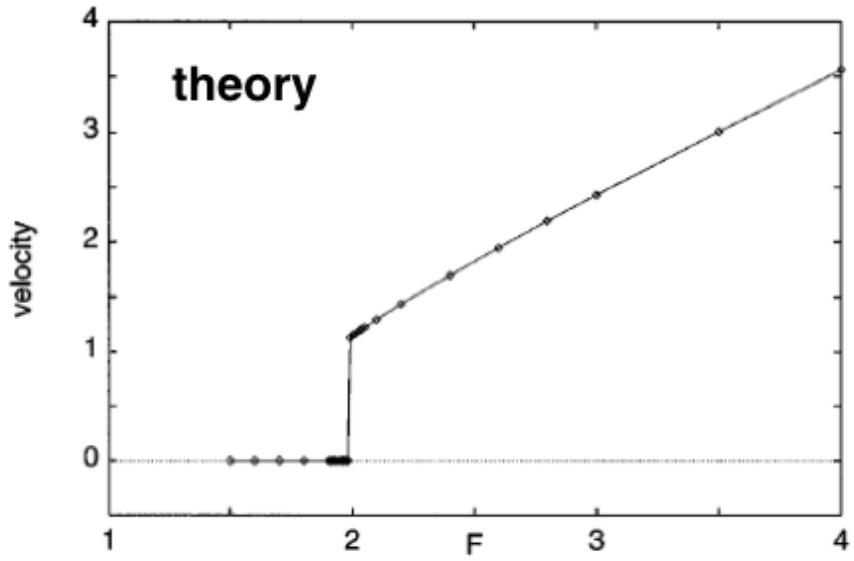
- **2nd order transition predicted**



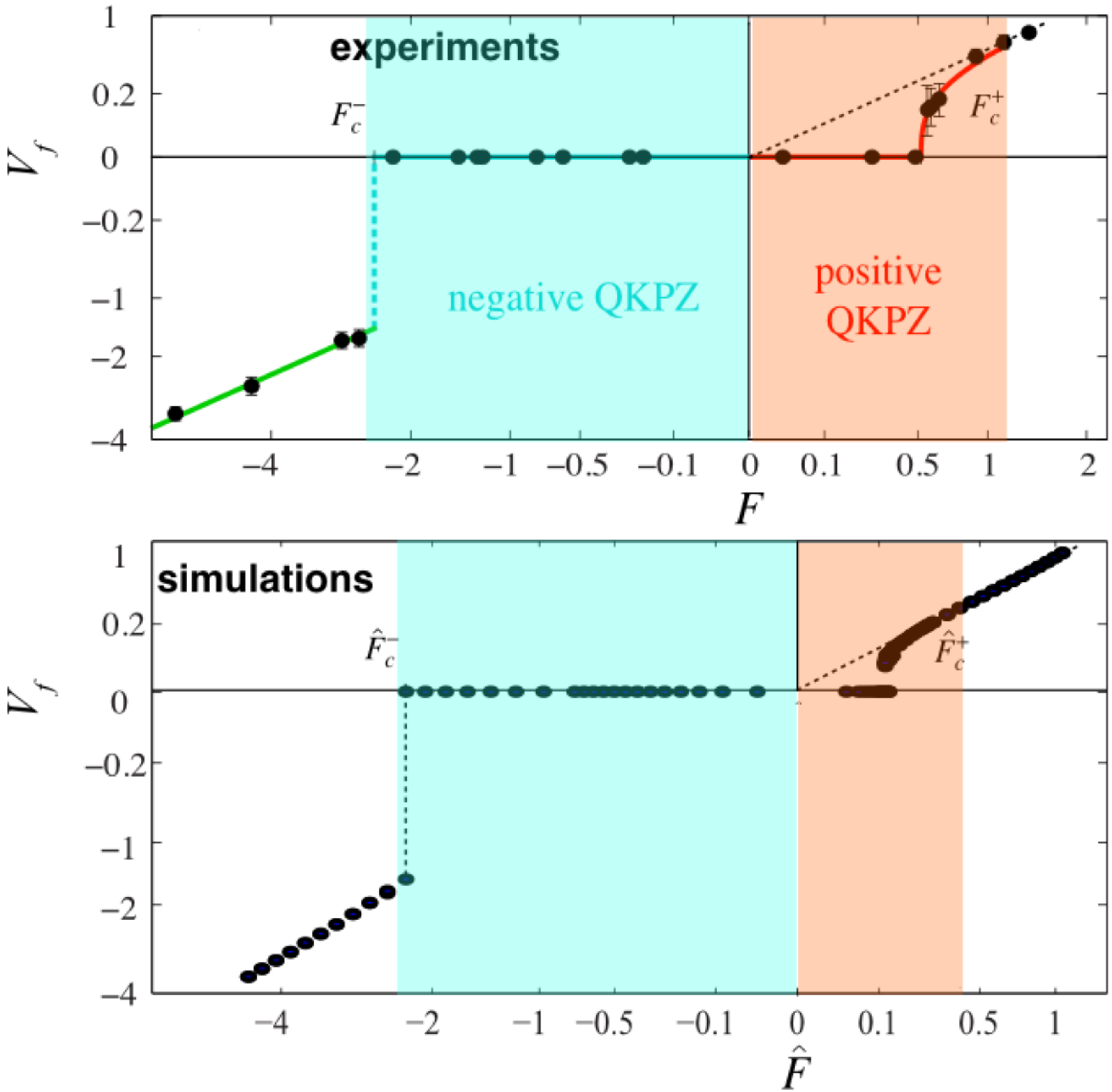
Amaral et al., *PRE* 51 (1995)

negative quenched KPZ model

- **1st order transition predicted**



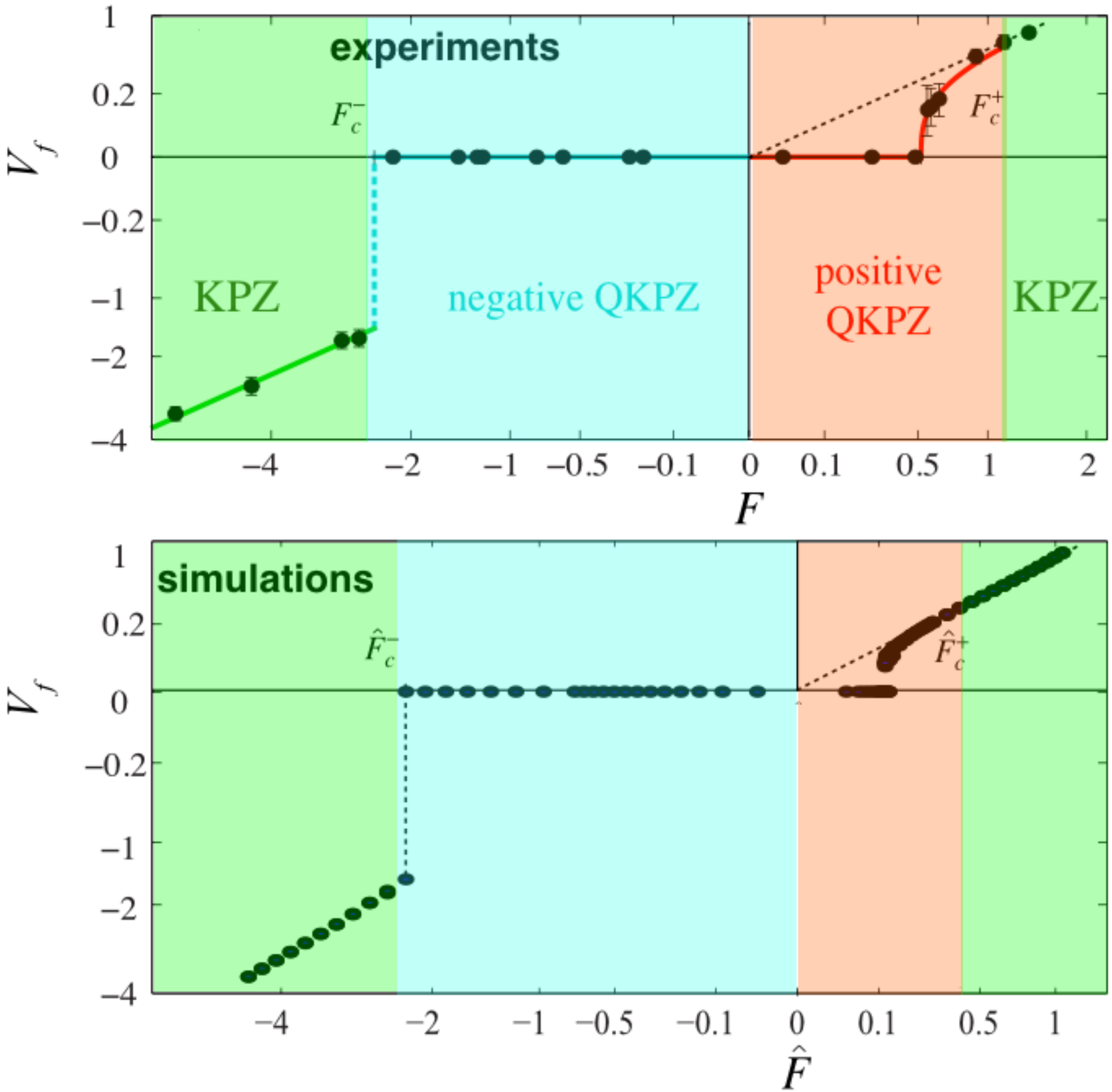
Jeong & al., *PRL* 77 (1996)



S. Atis, K. D. Awadhesh, D. Salin, L. Talon, P. Le Doussal, K. Wiese, *PRL* (2015)

Control parameter:

$$F = \frac{\bar{U}+V_\chi}{V_\chi} + f_0$$



S. Atis, K. D. Awadhesh, D. Salin, L. Talon, P. Le Doussal, K. Wiese, *PRL* (2015)

Control parameter:

$$F = \frac{\bar{U}+V_\chi}{V_\chi} + f_0$$

PLAN

1 - Kardar-Parisi-Zhang model in presence of quenched noise

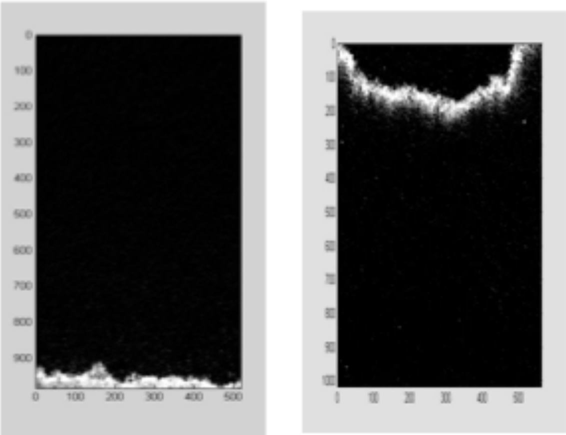
2 - Experiments with reaction fronts in disordered flow

3 - Transition between different universal behaviors

4 - Conclusion and perpectives

Conclusion

3 universality classes



KPZ behavior for moving phase

Positive qKPZ growth process for upward propagating fronts

Negative qKPZ growth with static sawtooth pattern formation for backward propagating fronts

The flow rate is the unique control parameter

Conclusion & perspectives

- **Bacteria colonies dynamics in complex flows**
- **Reaction front pinning control in microfluidic devices**

Conclusion & perspectives

Avalanches phenomena at the depinning transition