Semiclassical gravity and quantum de Sitter

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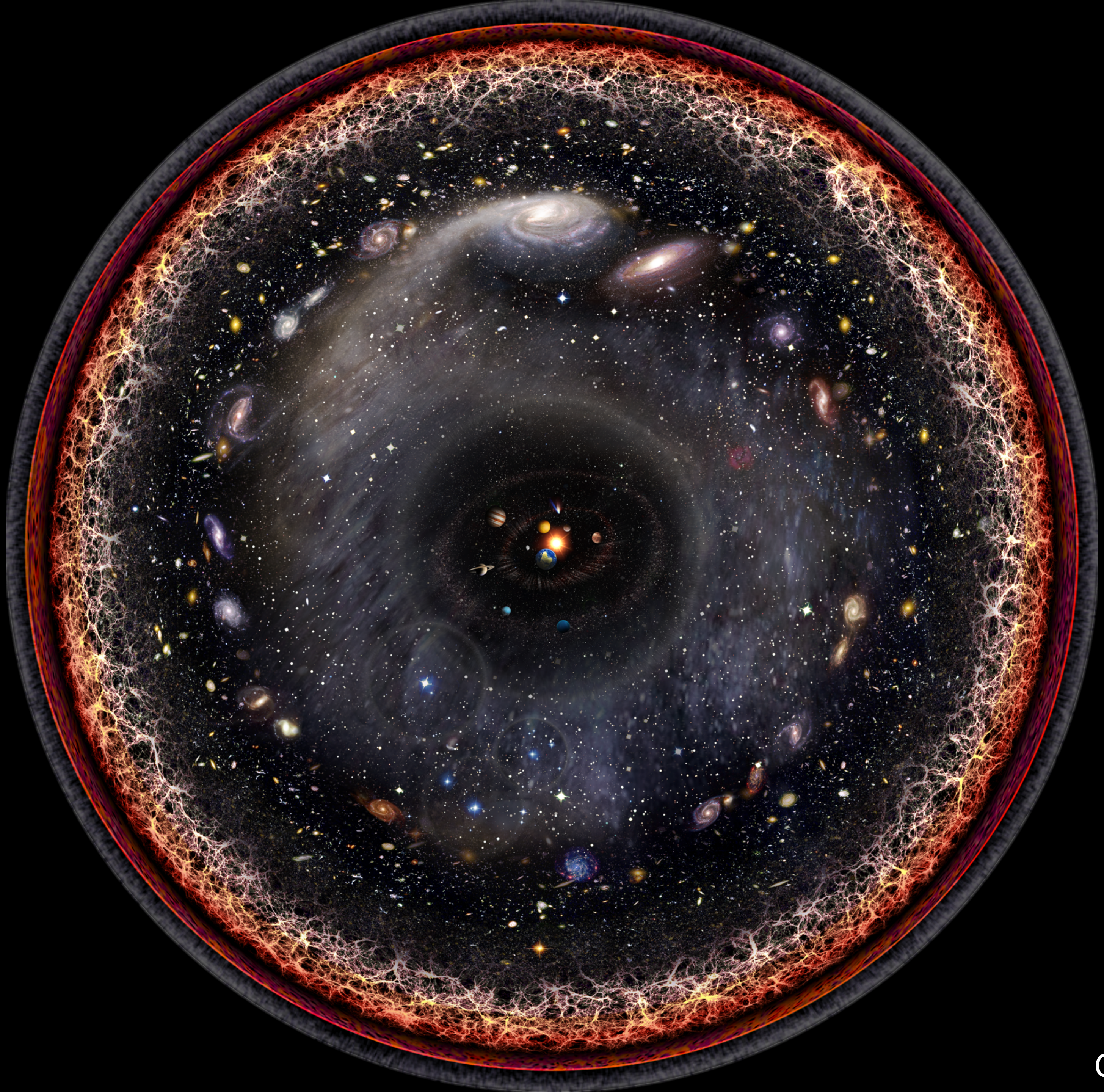
Work with J. Feldbrugge, J-L. Lehners, A. Di Tucci



Credit: Pablo Carlos Budassi

astonishing simplicity: just 5 numbers

			Measurement Error
today	Expansion rate:	67.8±0.9 km s^{-1} Mpc^{-1}	1%
	(Temperature)	2.728 ± 0.004 K	.1%
	(Age)	13.799 ±0.038 bn yrs	.3%
energy	Baryon-entropy ratio	6±.1x10^{-10}	1%
	Dark matter-baryon ratio	5.4± 0.1	2%
	Dark energy density	0.69±0.006 x critical	2%
geometry	Scalar amplitude	4.6±0.006 x 10^{-5}	1%
	Scalar spectral index n_s (scale invariant = 0)	-.033±0.004	12%

$$+m_\nu\text{'s}; \quad \text{but } \Omega_k,\ 1+w_{DE},\ \frac{dn_s}{d\ln k},\ \left\langle \delta^3 \right\rangle, \left\langle \delta^4 \right\rangle .., r = \frac{A_{gw}}{A_s}$$ consistent with zero

Nature has found a way to create a huge hierarchy of scales, apparently more economically than in any current theory

A fascinating situation, demanding new ideas

One of the most minimal is to revisit quantum cosmology

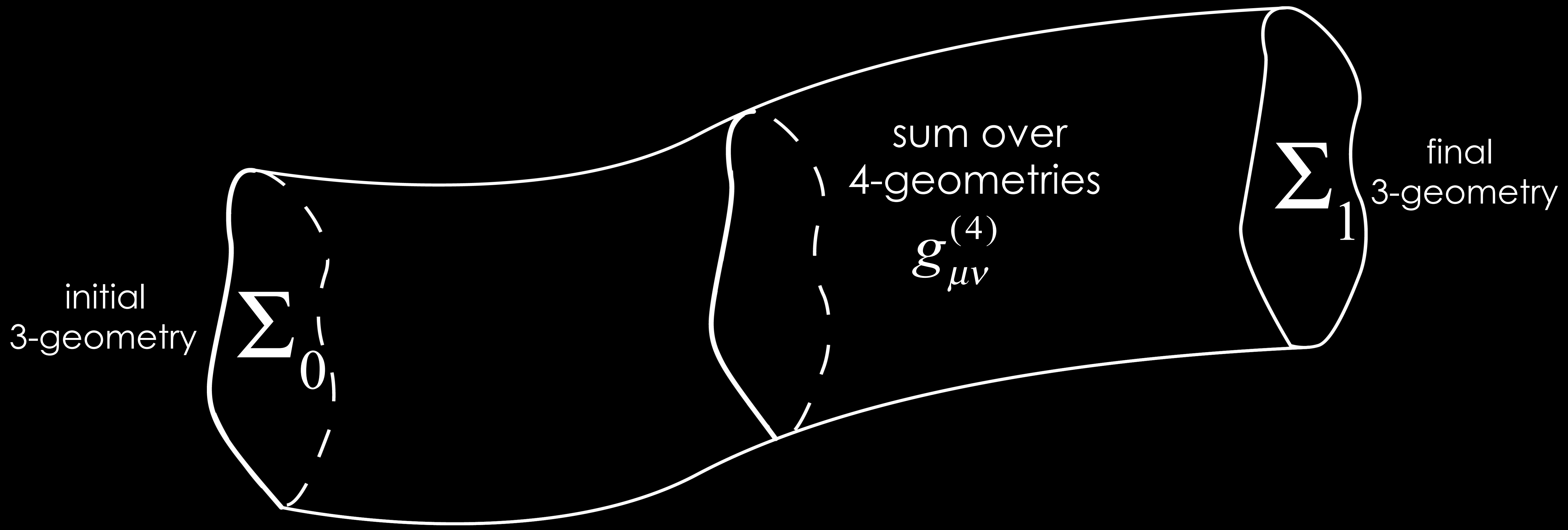
The simplest of all cosmological models is de Sitter; interesting both for today’s dark energy and for inflation

quantum cosmology reconsidered

w/ S. Gielen 1510.00699, *Phys. Rev. Lett.* 117 (2016) 021301,
1612.0279, *Phys. Rev.* D 95 (2017) 103510.
w/ J. Feldbrugge J-L. Lehners, 1703.02076, *Phys. Rev.* D 95 (2017) 103508,
1705.00192, *Phys. Rev. Lett*, 119 (2017) 171301,
1708.05104, *Phys. Rev. D,* in press (2017).
w/A. Di Tucci, J. Feldbrugge and J-L. Lehners , in preparation (2017)

Quantum geometrodynamics

Wheeler, Feynman,
De Witt, Teitelboim …



fundamental object:
Feynman propagator

$$\langle \Sigma_1 | \Sigma_0 \rangle \equiv \langle 1 | 0 \rangle$$

Basic object: phase space Lorentzian path integral

$$ADM: ds^2 \equiv (-N^2 + N_i N^i)dt^2 + 2N_i dt dx^i + h^{(3)}_{ij} dx^i dx^j$$

$$\langle 1 | 0 \rangle = \int DN \int DN^i \int_{\Sigma_0}^{\Sigma_1} Dh^{(3)}_{ij} \int D\pi^{(3)}_{ij} e^{\frac{i}{\hbar}S}$$

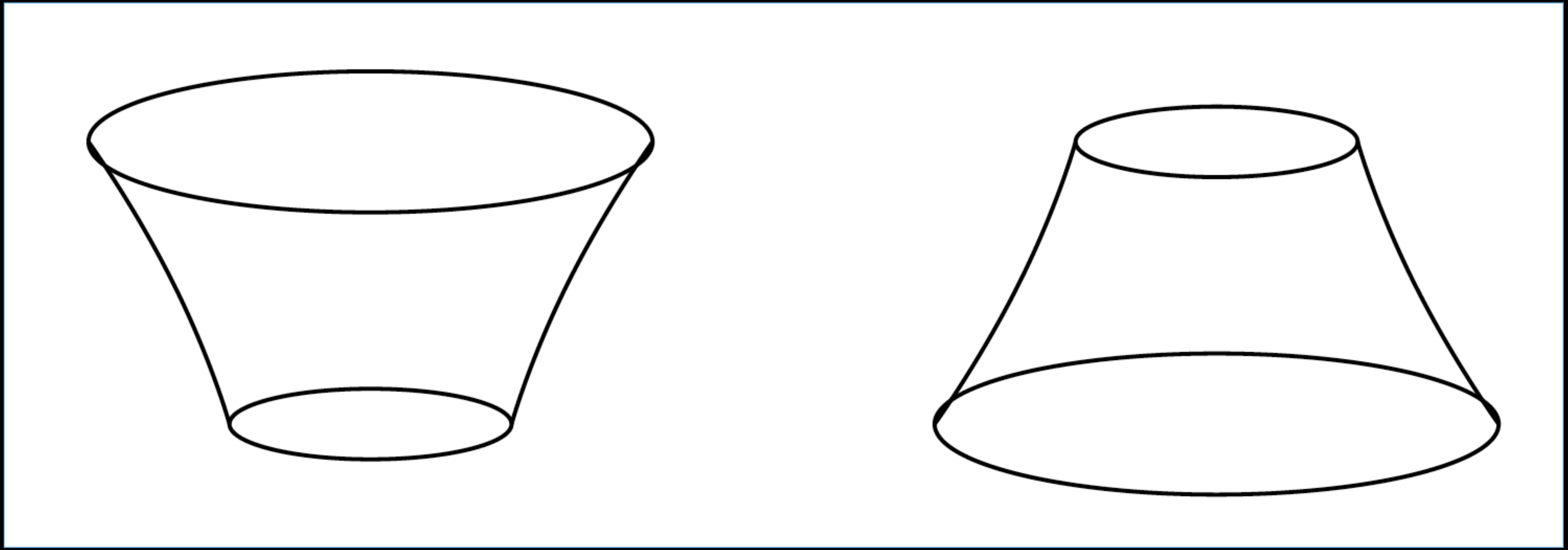
$$S = \int_0^1 dt \int d^3x (\pi^{(3)}_{ij} \dot{h}^{(3)}_{ij} - N_i H^i - NH)$$

Basic references:
C. Teitelboim (now Bunster), "Causality and Gauge Invariance in Quantum Gravity and Supergravity," Phys. Rev. Lett. 50, 705 (1983); see also Phys. Rev. D25, 3159 (1983); D28, 297 (1983).

Perhaps the most impressive fact which emerges from a study of the quantum theory of gravity is that it is an extraordinarily economical theory. It gives one just exactly what is needed in order to analyze a particular physical situation, but not a bit more. Thus it will say nothing about time unless a clock to measure time is provided, and it will say nothing about geometry unless a device (either a material object, gravitational waves, or some other form of radiation) is introduced to tell when and where the geometry is to be measured.[50] In view of the strongly operational foundations of both the quantum theory and general relativity this is to be expected. When the two theories are united the result is an operational theory *par excellence*.[51]

B.S. DeWitt, *Phys. Rev.* 160, 1967 (p 1140)

Some basic points, *e.g.*, causal propagator, defined by integrating only over positive lapse N allows you to distinguish an expanding from a contracting universe.



Final: 1

Initial: 0

Wheeler, Teitelboim, …

theories of initial conditions for inflation

Wave function of the Universe

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(Received 29 July 1983)

The quantum state of a spatially closed universe can be described by a wave function which is a functional on the geometries of compact three-manifolds and on the values of the matter fields on these manifolds. The wave function obeys the Wheeler-DeWitt second-order functional differential equation. We put forward a proposal for the wave function of the "ground state" or state of minimum excitation: the ground-state amplitude for a three-geometry is given by a path integral over all compact positive-definite four-geometries which have the three-geometry as a boundary.

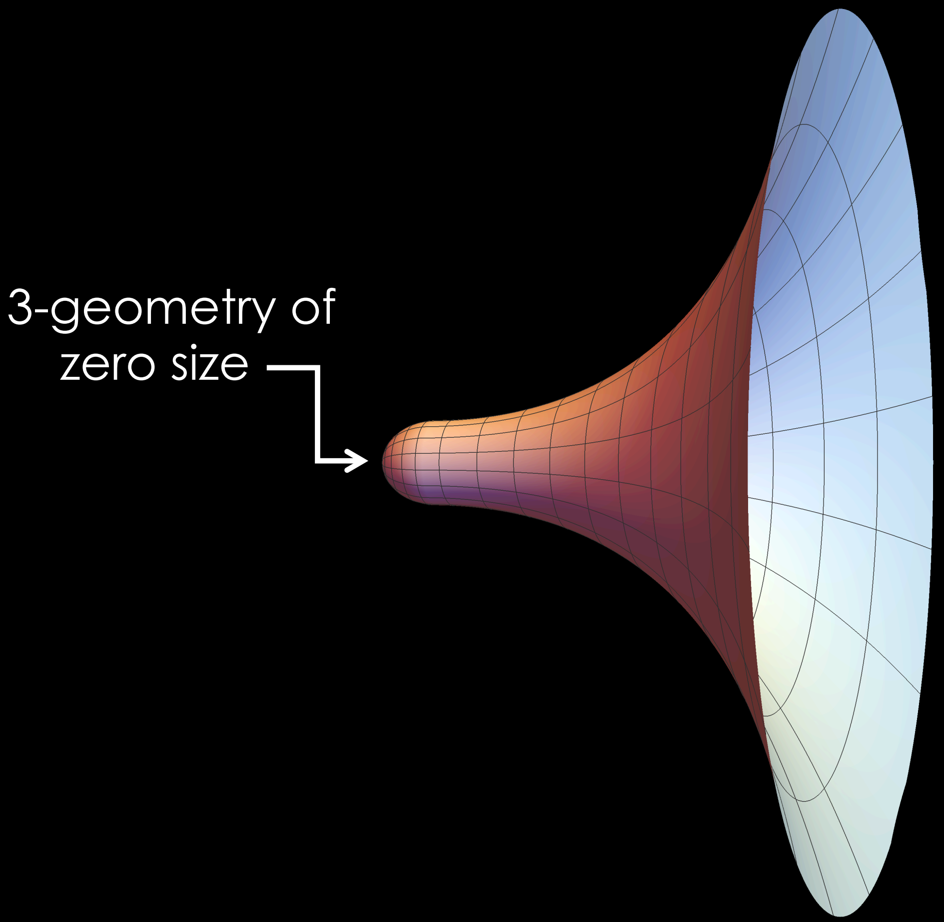
One can interpret the functional integral over all compact four-geometries bounded by a given three-geometry as giving the amplitude for that three-geometry to arise from a zero three-geometry, i.e., a single point. In other words, the ground state is the amplitude for the Universe to appear from nothing.[4] In the following we shall elaborate on this construction and show in simple models that it indeed supplies reasonable wave functions for a state of minimum excitation.

[4]For related ideas, see A. Vilenkin, Phys. Lett. 117B, 25 (1982); Phys. Rev. D 27, 2848 (1983).

Revised Vilenkin proposal (framed in terms of Lorentzian path integral):
Phys Rev. D30, 509 (1984); Phys Rev D50, 2581 (1994), gr-qc/9403010

Earlier versions: Lemaitre, Fomin, Tryon, Brout-Englert-Gunzig …

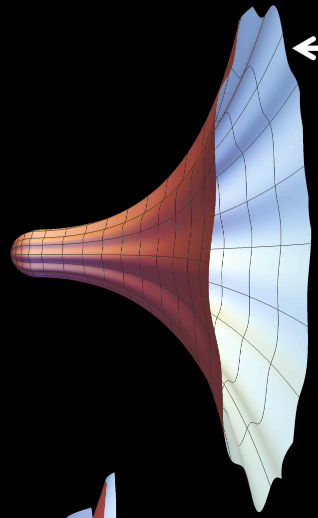
no boundary proposal



A very beautiful idea: the laws of physics determine their own initial conditions simplest model: Einstein gravity plus cosmological constant

$$S = \int \left(\tfrac{1}{2} R - \Lambda\right) + \text{surface terms} \quad (8\pi G \equiv 1)$$

linearized perturbation h_{Tij} (g-wave)

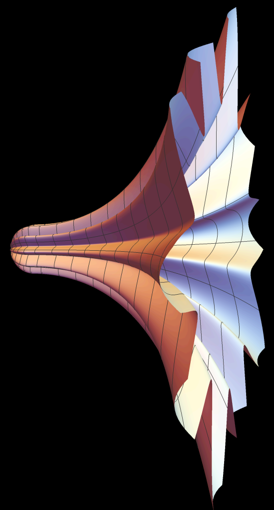


Usual claim:

$$\Psi \propto e^{+\frac{12\pi^2}{\hbar\Lambda}\left(1 - l(l+1)(l+2)h_T^2\right)}$$

Our claim:

$$\Psi \propto e^{-\frac{12\pi^2}{\hbar\Lambda}\left(1 - l(l+1)(l+2)h_T^2\right)}$$



perturbations completely out of control: there is no meaningful one-point amplitude for a 3-geometry "persistence of nothing"

Some overlap with previous work: Vilenkin (bg), Rubakov (perts), Ambjorn/Loll (bg), Sorkin (bg)…

We evaluate the Lorentzian gravitational path integral carefully, using cosmological perturbation theory and P-L/Cauchy to determine relevant saddles

Integrate out background (zero mode), then fluctuations, then lapse N

Background:

$$ds^2 = -\bar{N}^2 dt^2 + a^2 d\Omega_3^2; \qquad S = 2\pi^2 \int_0^1 dt \left[-N^{-1} 3a\dot{a}^2 + N(3a - \Lambda a^3) \right]$$

redefine* $\bar{N} \equiv N a^{-1},\ q \equiv a^2 \Rightarrow\ S = 2\pi^2 \int_0^1 dt \left[-N^{-1} \tfrac{3}{4} \dot{q}^2 + N(3 - \Lambda q) \right]$ quadratic in q

(Halliwell)

$ds^2 = -N^2 q^{-1} dt^2 + q\ d\Omega_3^2;$ work in gauge N=const

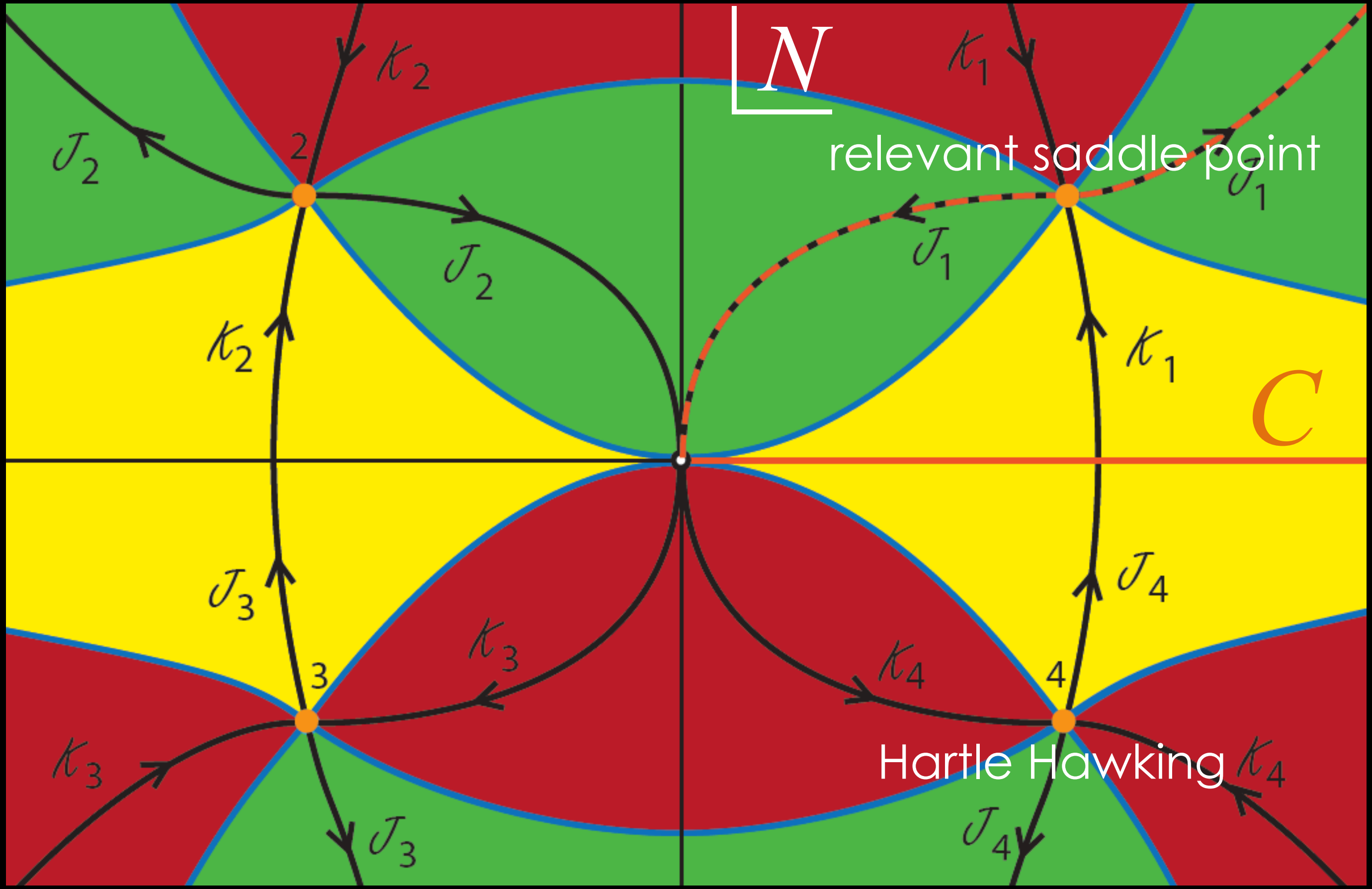
"no boundary" classical solution: $q_{cl}(t) = \tfrac{1}{3}\Lambda N^2 t^2 + (-\tfrac{1}{3}\Lambda N^2 + q_1)t: \quad q_{cl}(0) = 0,\ q_{cl}(1) = q_1$

Classical action: $$\boxed{S_{cl}(q_1; N) = 2\pi^2 \left[\tfrac{1}{36} \Lambda^2 N^3 + (3 - \tfrac{1}{2}\Lambda q_1) N - \tfrac{3}{4} q_1^2 N^{-1} \right]}$$

*properly defined FPI is invariant under such redefinitions (see Gielen +NT): do not affect leading semiclassical exponent

$$\langle 1|0\rangle_F = \int_{0^+}^{\infty} dN \sqrt{\frac{3\pi i}{2\hbar N}}\, e^{\frac{i}{\hbar}S_{cl}(q_1;N)}$$

4 saddles, related by
$N \to -N$ and
complex conjugation



P-L theory:

every saddle σ defines a "Lefshetz thimble" J_σ (complete steepest descent contour) upon which integral is absolutely convergent. Generically, each J_σ intersects a steepest ascent contour K_σ

intersection number $\rightarrow \langle J_\sigma K_{\sigma'}\rangle = \delta_{\sigma\sigma'}$

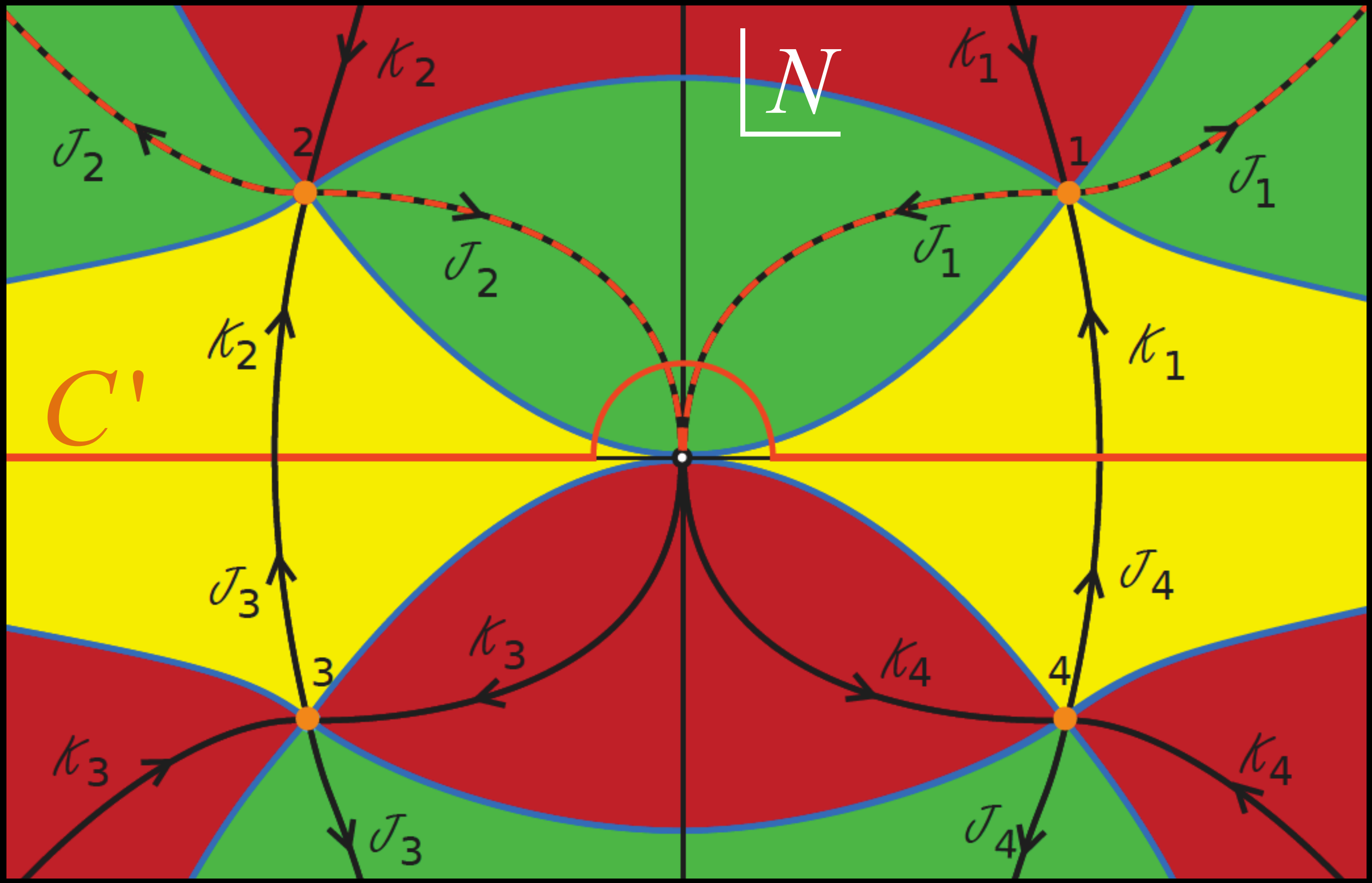
One can deform the defining contour C into one passing along a number of thimbles,

$$C = \sum_\sigma n_\sigma J_\sigma \Leftrightarrow n_\sigma = \langle C K_\sigma \rangle$$

i.e., a saddle contributes iff its steepest *ascent* contour intersects C

Above gives the Feynman (causal) propagator: one can also integrate over $C' = (-\infty, \infty)$ which just gives the real part of the Feynman propagator

From $\hat{H}\langle 1|0\rangle_F = -i\hbar\delta(\Sigma_1 - \Sigma_0)$, it follows that $\hat{H}\,\mathrm{Re}[\langle 1|0\rangle_F] = 0$. So the contour integral over C' gives a solution of the WdW equation



basic issues with the Euclidean path integral

Usual Wick rotation $N = -iN_E$ renders exponent $\frac{i}{\hbar}S \equiv -\frac{1}{\hbar}S_E$ real but it is an odd function of N_E so, semiclassically, the integral over $-\infty < N_E < \infty$ diverges (in any D)

Integrating over a half-line does not provide a "wavefunction of the universe" satisfying the homogeneous WdW equation

Furthermore, in D=4, divergences at $N_E \to 0^\pm$ and $N_E \to \pm\infty$ have opposite signs so that (for any $q_1 > 0$) the half-line integral diverges

Perturbations:

$$ds^2 = -N^2 q^{-1} dt^2 + q(\gamma_{ij}^{S_3} + h_{ij}^T) dx^i dx^j;$$

$$S = S^{(0)} + S^{(2)}; \;\; S^{(2)} = \pi^2 \int_0^1 dt [N^{-1} q^2 \dot{h}_{Tl}^2 - Nl(l+2) h_{Tl}^2]$$

redefine: $\chi_l = q\ h_{Tl} \Rightarrow eom \quad -\ddot{\chi}_l + \frac{1}{4t^2}(\gamma^2 - 1)\chi_l == 0,\ t \to 0$

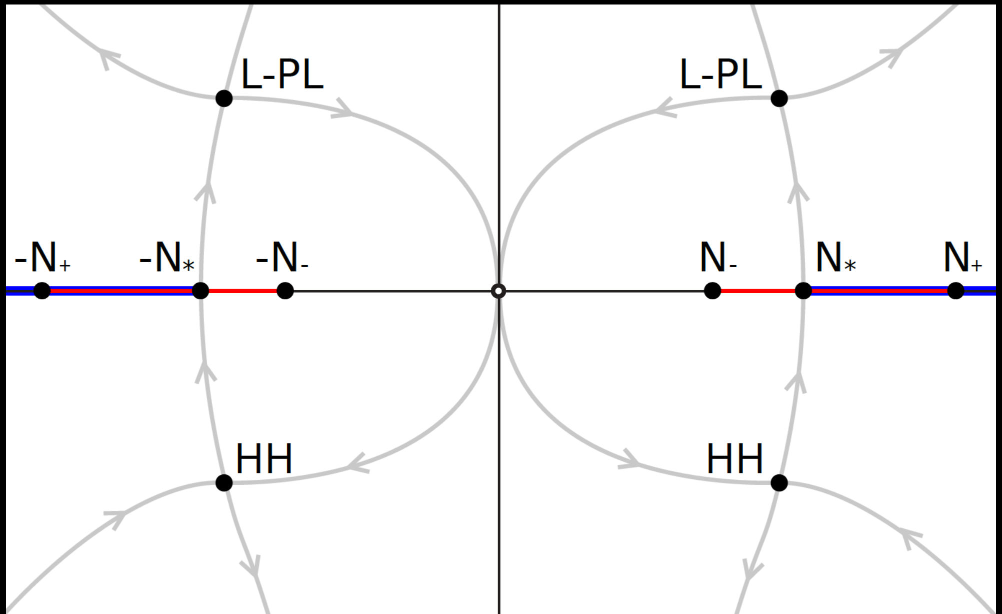
Can show $\mathrm{Re}[\gamma] > 0$ everywhere in complex N-plane (ensures finite action) except on two branch cuts (arise only because of infinite dimensionality)

$-N_+ < N < -N_-,\ N_- < N < N_+$ where

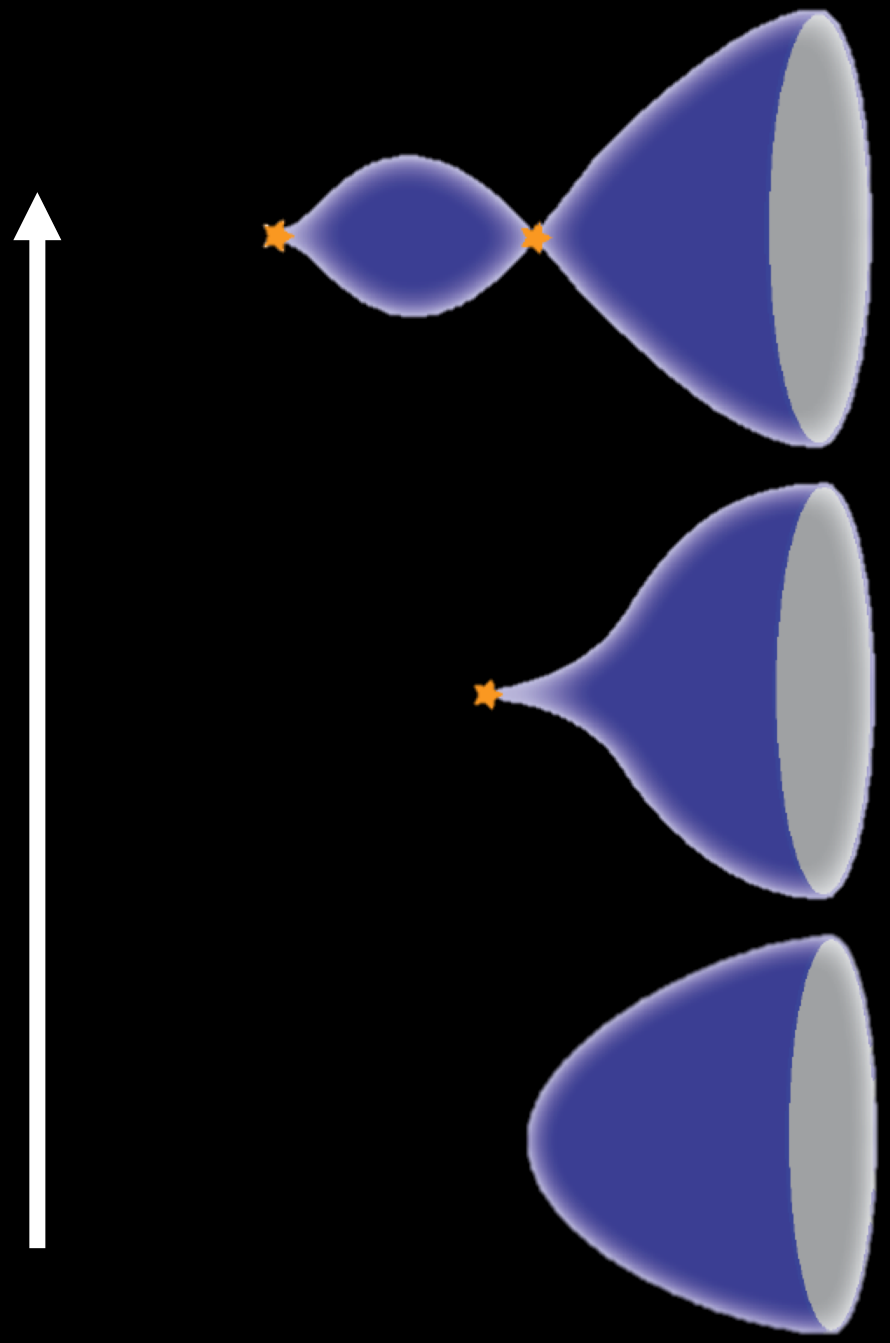
$N_\pm =$

$$\frac{3}{\Lambda}\sqrt{2l(l+2) + q_1 \frac{\Lambda}{3} \pm 2\sqrt{l(l+2)(l(l+2) + q_1 \frac{\Lambda}{3})}}$$

$$N_* = \sqrt{N_+ N_-} = \sqrt{\frac{3}{\Lambda} q_1}$$

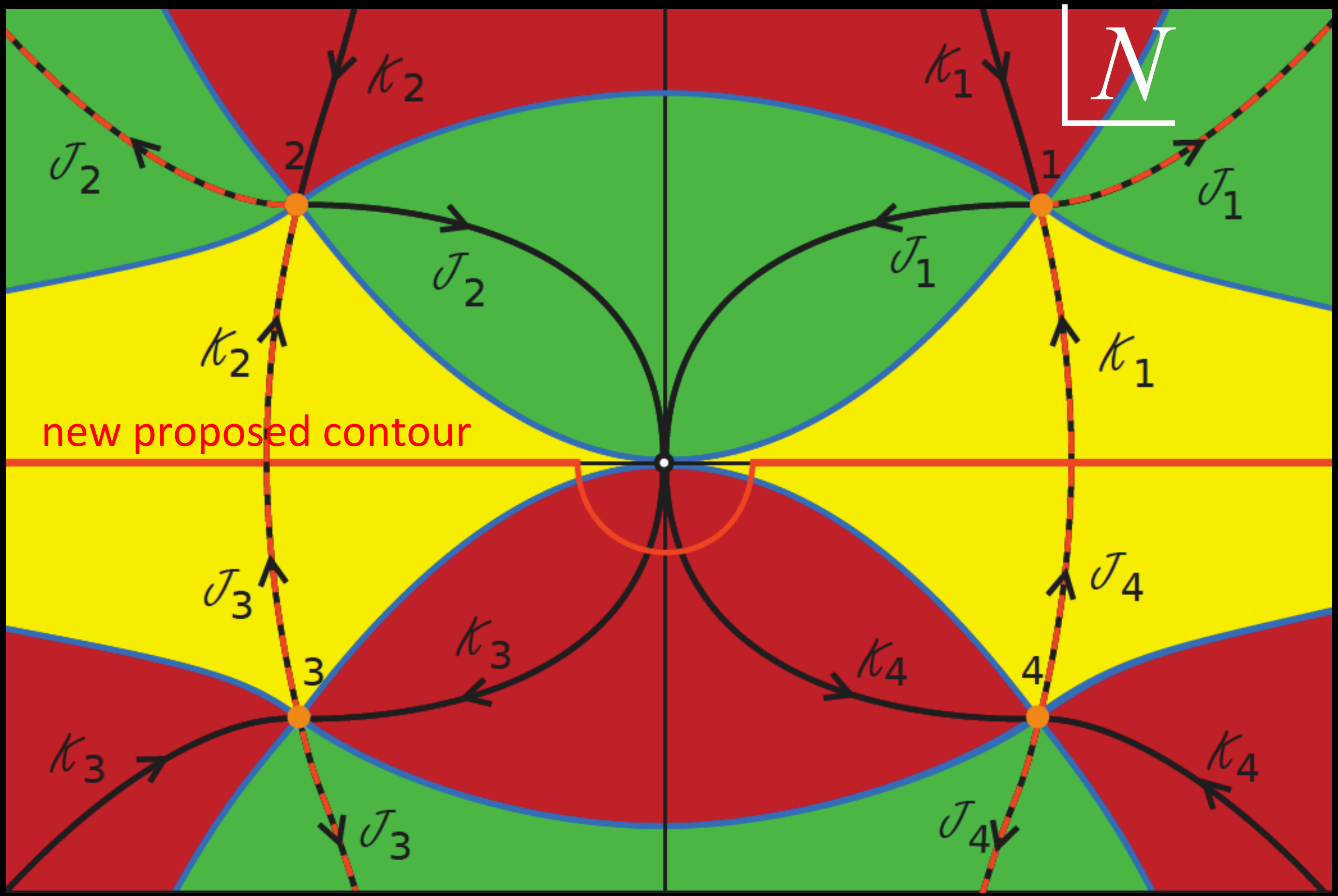


increasing real N



Rescue attempt

J. Diaz Dorronsoro, J.J Halliwell, J.B. Hartle, T. Hertog, O. Janssen
"The Real No Boundary Wavefunction in Lorentzian Quantum Cosmology," *Phys. Rev.* D 96 (2017) 043505, arXiv 1705.05340



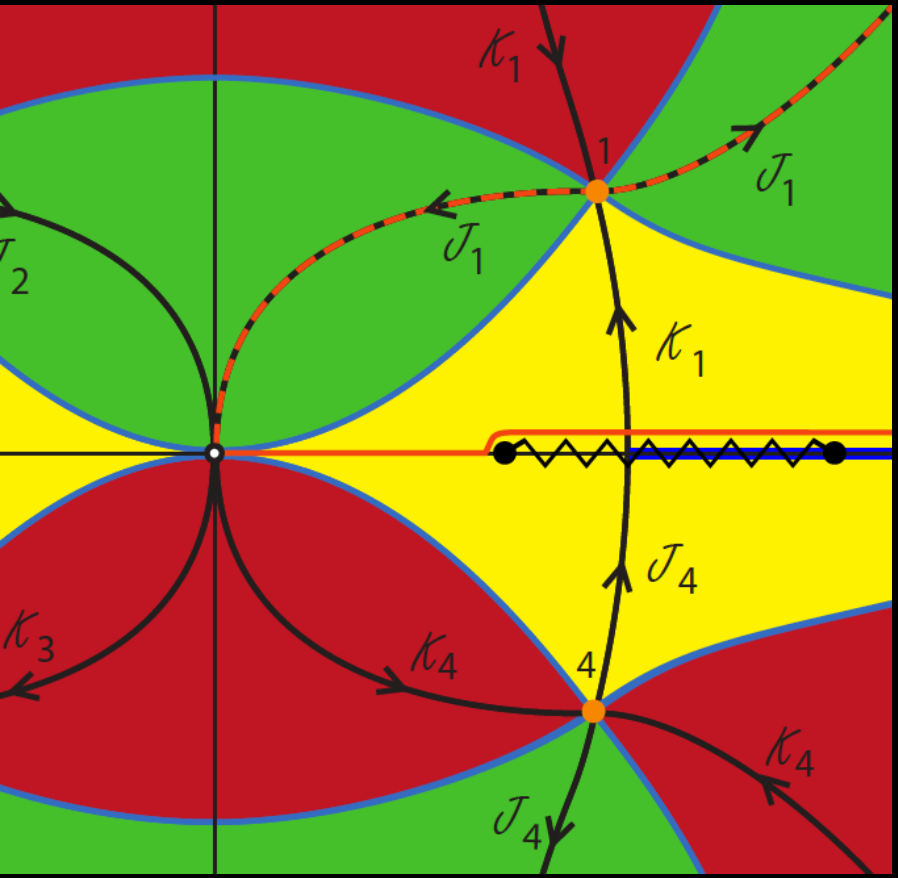
1. not Lorentzian

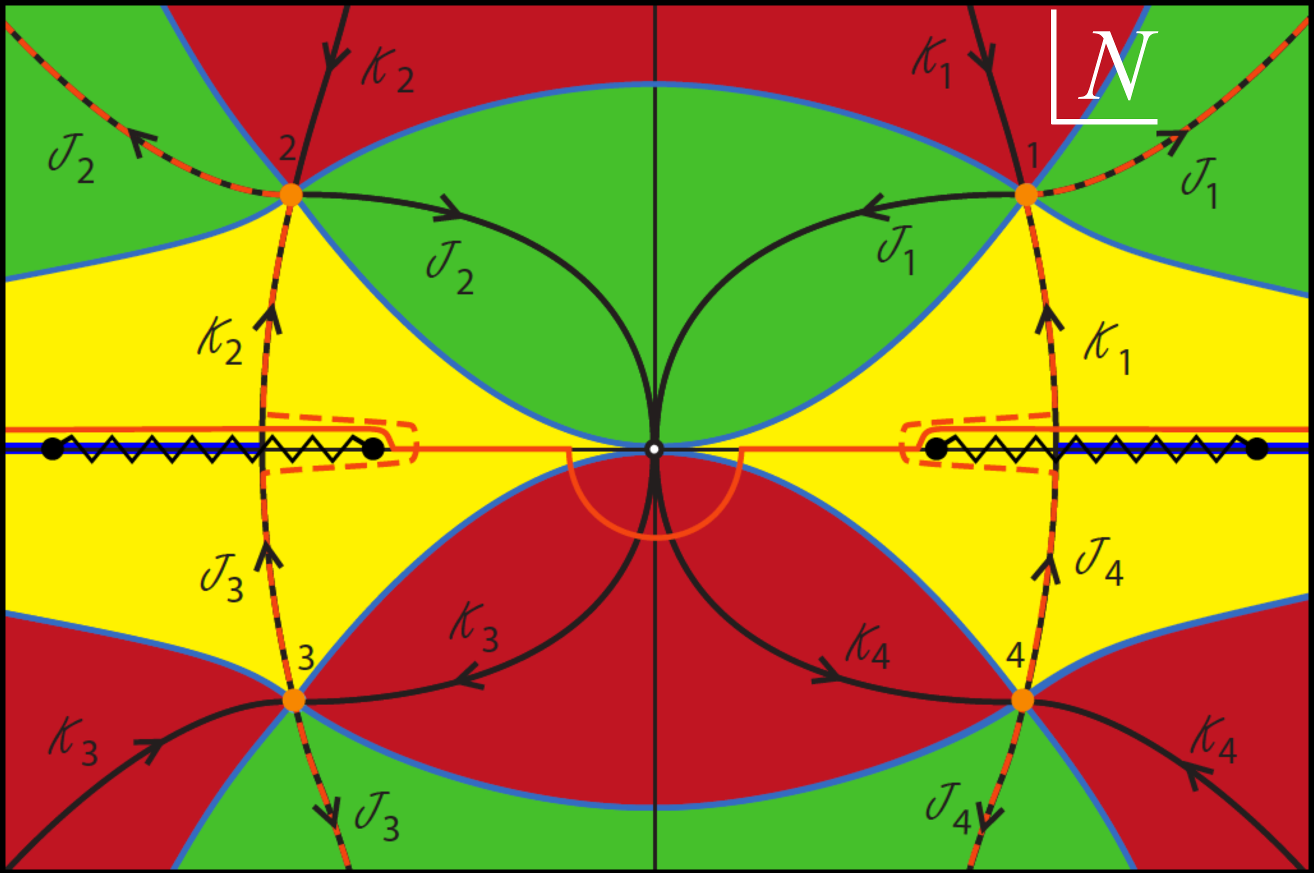
2. contributions from all four saddles

3. Nonperturbative contributions render perturbations out of control

Response: J. Feldbrugge, J-L. Lehners and NT, "No rescue for the no boundary proposal: pointers to the future of quantum cosmology," *Phys. Rev. D in press*, arXiv 1708.05104

We have seen how non-analyticity arises in the exponent from integrating out pertns: cannot then apply Picard-Lefschetz theory for the remaining integral over N
However, Cauchy's theorem still applies: we just distort the contour in advance to avoid any branch cut which arises from integrating out fluctuations





Theorem:

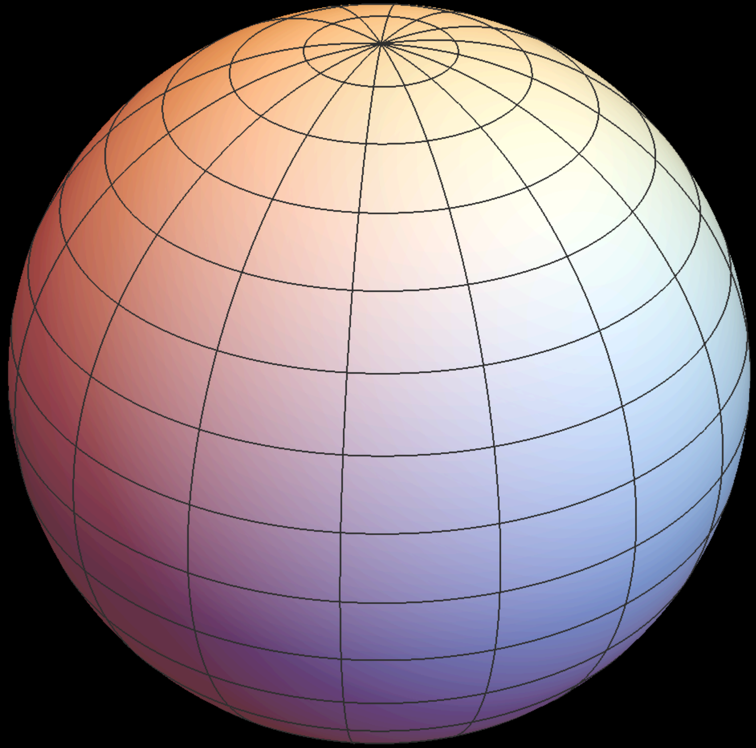
no contour for the lapse avoids contributions from the upper two saddles

quantum fluctuations are out of control

Interpretation:

There is no meaningful one-point function for a 3-geometry (for 4d gravity with positive Λ)

Persistence of nothing



If we consider the limit $q_1 = q_0 = 0$, then the small N_E divergence disappears and the Euclidean path integral over the background becomes well defined

There is a saddle with $N_s = -\frac{6i}{\Lambda}; \quad S_E = \frac{24\pi^2}{\hbar\Lambda}$

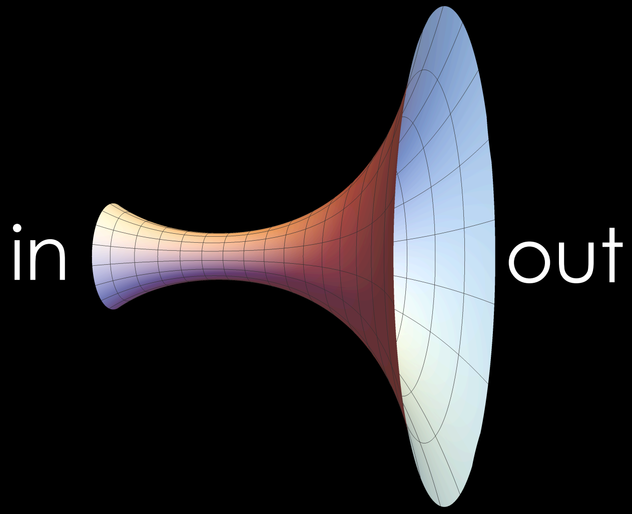
The Euclidean action for the tensor fluctuations is positive definite so that the nothing-nothing "self-energy" amplitude is real

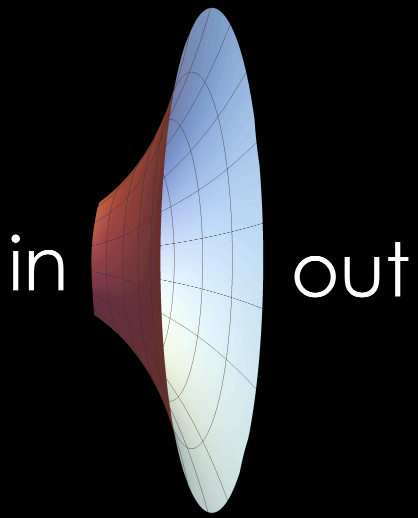
We take this to mean that "nothing" is stable

quantum de Sitter

Lorentzian in-out amplitudes may be constructed semi-classically

For classically allowed q_0 and q_1, both larger than the de Sitter throat, there are always just two, real saddle point solutions





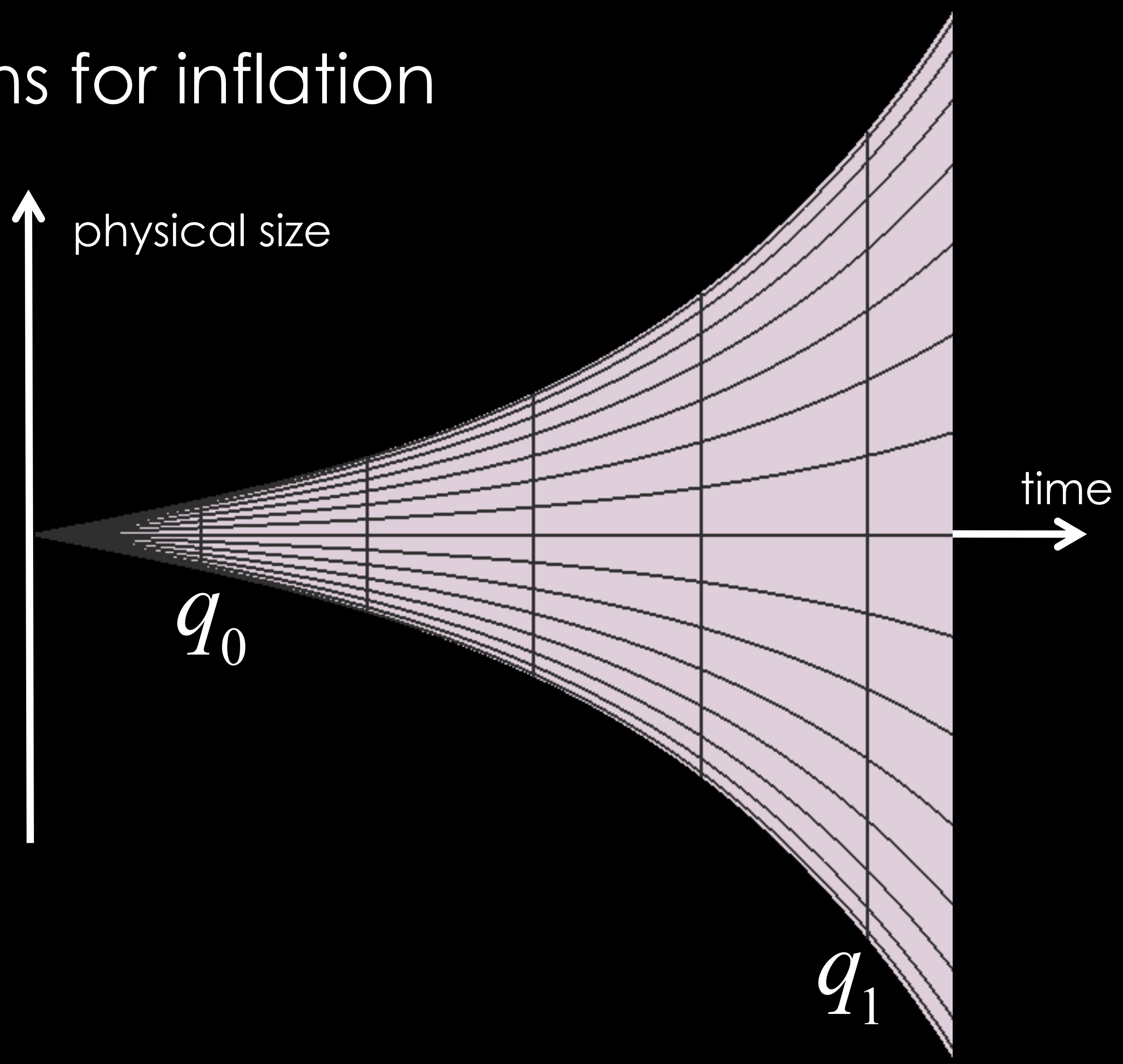
These interfere in interesting (and calculable) ways

We have been able to find the linearized mode solutions analytically for general N, as well as to compute the corresponding classical action

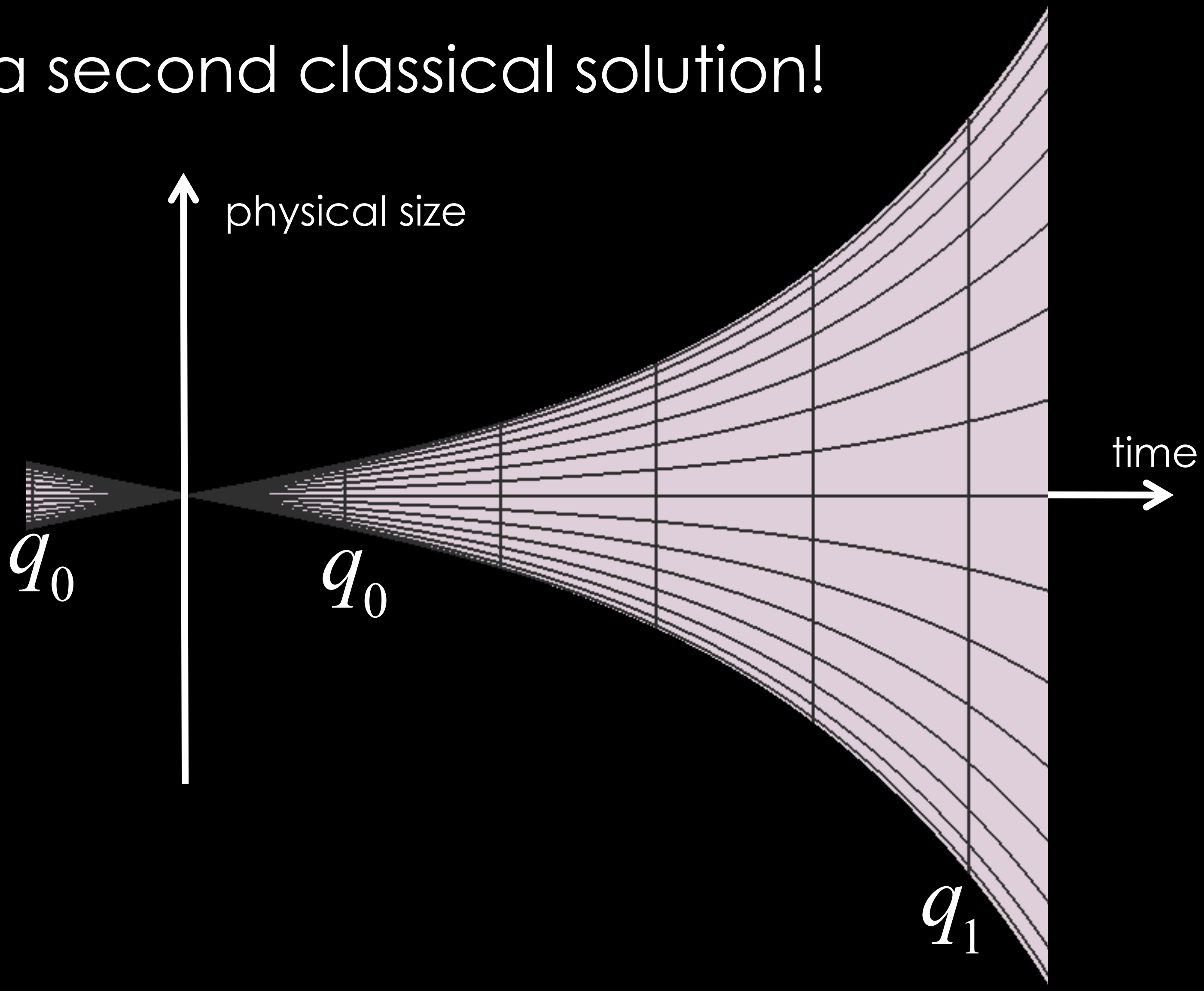
We also have developed numerical techniques to include nonlinear backreaction based on systematically improving the complex linear solutions

This provides a fascinating laboratory in which to study real-time quantum phenomena using semiclassical methods, for example the growth of perturbations in the collapsing phase, leading to the creation of black holes which then evaporate in the expanding phase

implications for inflation



there is a second classical solution!



Quantum incompleteness of inflation

To define the "in" vacuum, a common technique is to take the limit $\eta_0 \to -\infty e^{-i\varepsilon}$

(where $a = e^{Ht_P} = -\frac{1}{H\eta}$)

e.g. S. Weinberg, arXiv: 0805.3781

However, we have $\eta_0 = -\frac{1}{H\sqrt{q_0}}$ so in quantum geometrodynamics this amounts to

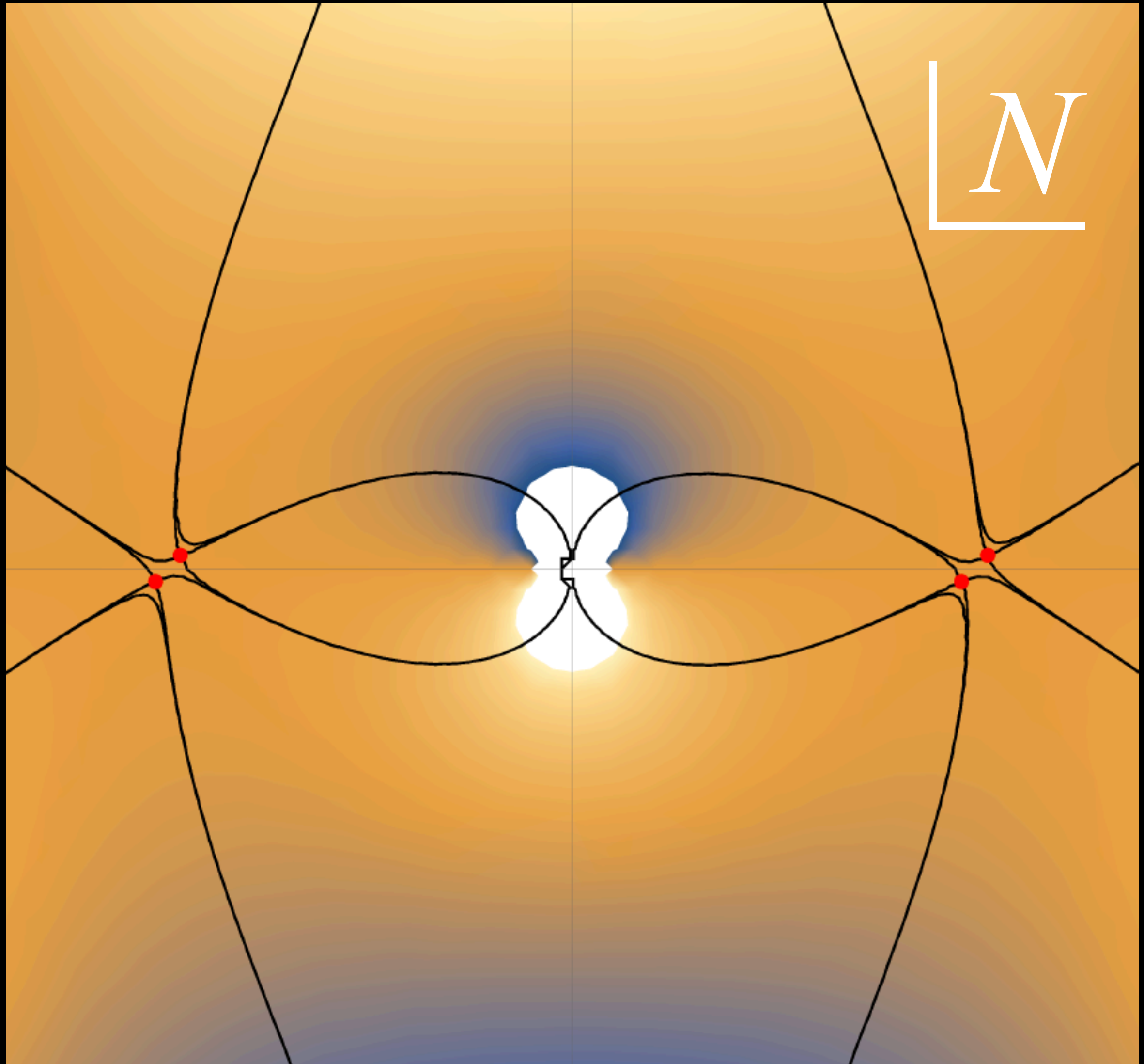
performing a small rephasing of q_0 in the opposite sense

Carrying this through consistently, one finds that the relevant Lorentzian saddle (to the N-integral) is the one in the upper-half N-plane, giving unbounded perturbations

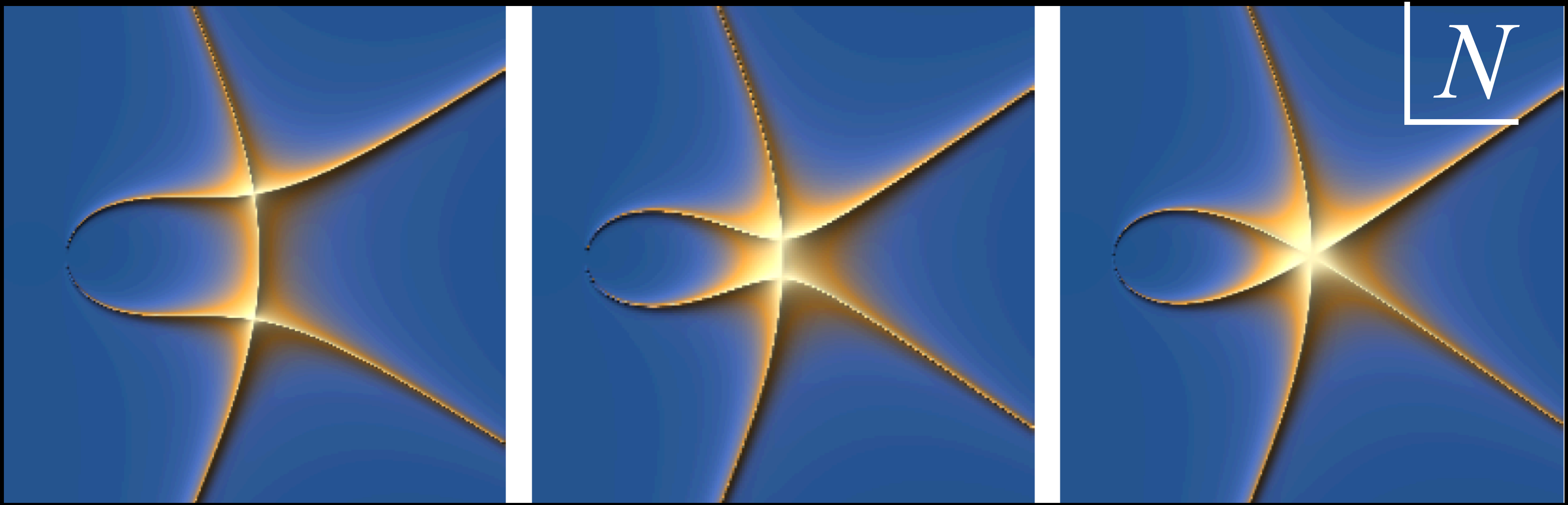
So there is a tension between quantum geometrodynamics and inflation, meaning that the "Bunch-Davies" vacuum is potentially susceptible to quantum gravitational effects

This quantum incompleteness is closely related to the classical, geodesic incompleteness of inflation

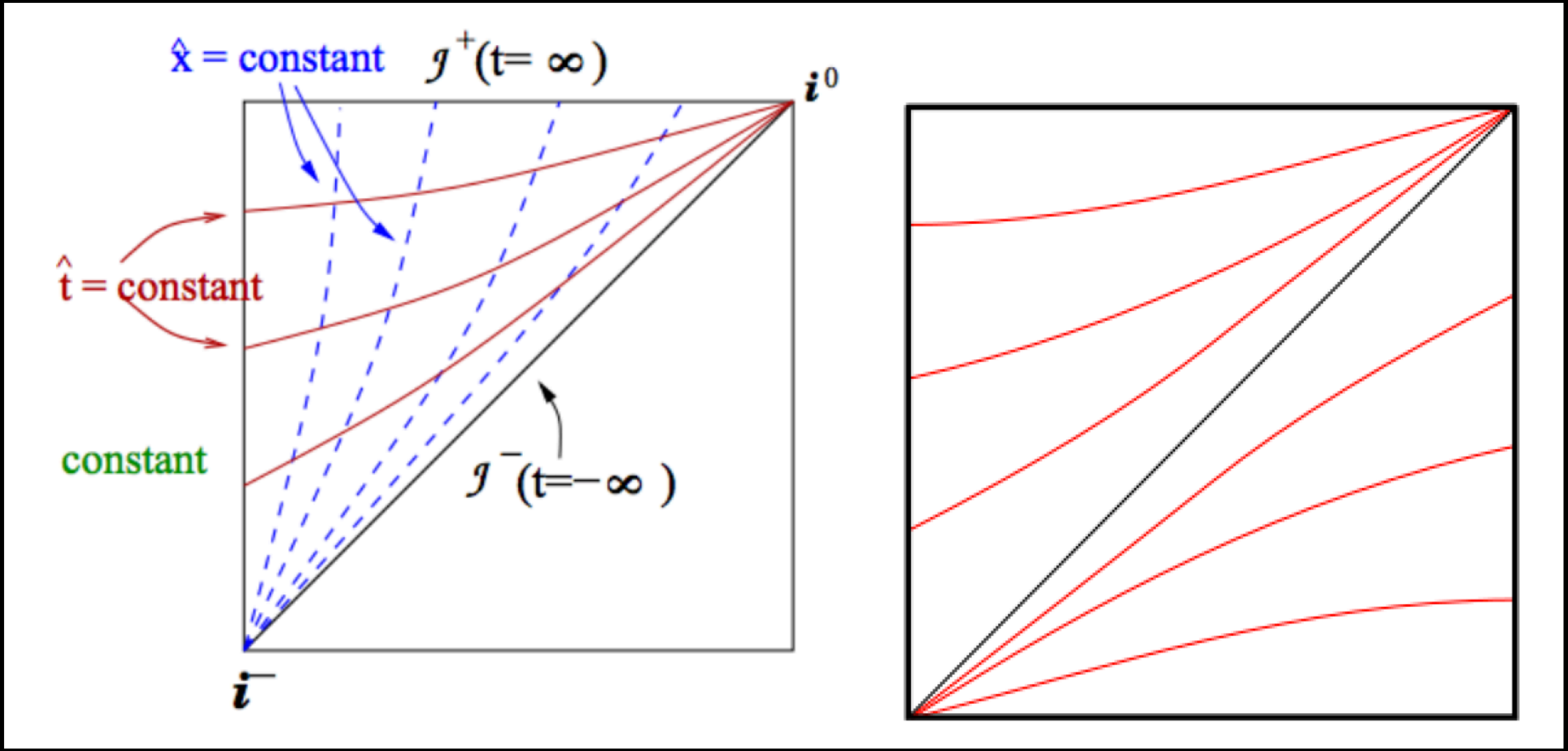
de Sitter flat slicing with $q_0 \to 0$ from uhp



the same conclusion is obtained by taking the flat universe limit of the closed no-boundary universe



de Sitter in flat (inflationary) slicing



summary

- Picard-Lefschetz-Cauchy deformation allows us to obtain unambiguous predictions from the Lorentzian path integral for gravity in the semiclassical limit.
- The (path integral formulation) of the no boundary proposal is still an attractive idea but seems to be mathematically problematic. The Lorentzian semiclassical path integral version yields perturbations which are out of control.
- Inflation and the "Bunch-Davies" vacuum are subject to similar nonperturbative corrections, emphasizing their quantum mechanically incompleteness
- Quantizing the background is important! Intriguing connection between the zero modes (IR) and the QFT vacuum for inhomogeneous perturbations (UV)
- Techniques potentially of wide applicability, *e.g.*, to black holes & holography
- Pointers to new, much simpler and more predictive scenarios for cosmology

thank you!