Hadron correlators, spectral functions and dilepton rates from lattice QCD

ITP, April 2002

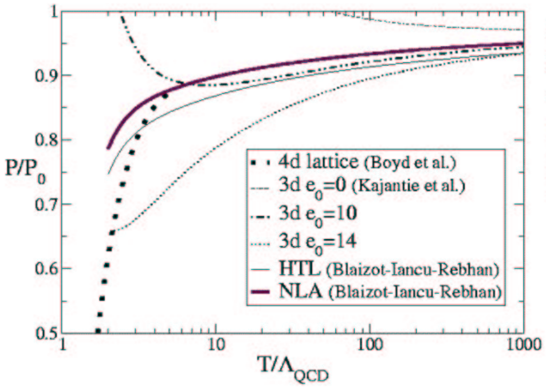
- Bulk thermodynamics: QCD equation of state
- Thermal hadron correlation functions and spectral functions
- Lattice and HTL-results for pseudo-scalar and vector correlation functions
- Dilepton rates
- Conclusions

Bulk thermodynamics: Equation of State

- lattice gauge theory provides basic results on the "phase" transition at $T \neq 0$ and properties of the high temperature phase
  - $T_c = (175 \pm 15)\ MeV$ (for $n_f = 2$)
  - $\epsilon_c = (6 \pm 2) T_c^4 \simeq (0.3 - 1.3)\ GeV/fm^3$

EoS: $T < T_c$ ~ resonance gas

$T \gtrsim 2T_c$ ~ weakly interacting quasi-particles



HTL, dim.red., massive quark gas models,.. match lattice calculations for $T \gtrsim (2-3)\ T_c$

- for large T the EoS probes the large momentum sector of the plasma: $|\vec{q}| \sim 3T$

  only basic quasi-particle feature of HTL propagators is probed:

  $\sigma(\omega, \vec{q}) \sim \delta(\omega^2 - q^2 - m_\infty^2)\ ,\ \ m_\infty \sim g(T)T$

- (hadronic) correlation functions probe the low momentum structure of the thermal medium
- information on

  (thermal modifications of)

  the hadron spectrum is encoded in

  (finite temperature)

  Euclidean correlation functions

$$G_H(\tau,\vec{r},T) = \langle J_H(\tau,\vec{r}) J_H^\dagger(0,\vec{0})\rangle$$

mesonic currents: $J_H = \bar{\psi}\Gamma_H\psi$,

$$\Gamma_H = \gamma_5 \text{ (pseudo-scalar)}, \quad = \gamma_\mu \text{ (vector)},\ldots$$

$T = 0:$ $\quad G_H(\tau,\vec{r},0) \sim \exp(-m_H x), \quad x = \sqrt{\tau^2+\vec{r}^2} \to \infty$

$T > 0:$ $\quad$ spatial correlation functions:

$$G_H(0,\vec{r},T) \sim \exp(-m_{scr}|\vec{r}|)$$

eg: $m_{scr} \neq m_H(T)$

inverse susceptibilities:

$$\chi_H \equiv \int_0^{1/T} d\tau \int d^3r \; G_H(\tau,\vec{r},T) \sim m_\chi^{-2}$$

eg: χ_H diverges; needs subtraction

⇝ Thermal Green's Functions in Euclidean space:

- modifications of spectrum ⇔ changes of spectral function

  most transparent in temporal correlation functions

Reminder: from Minkowski to Euclidean space

$$G_H(\tau,\vec{r},T) = T\sum_n \int d^3p \;\exp[-i(\tau\omega_n - \vec{r}\vec{p})] \; ReD_H^R(i\omega_n,\vec{p},T)$$

$$ReD_H^R(i\omega_n,\vec{p},T) = \frac{1}{\pi}\int d\omega \frac{ImD_H^R(\omega,\vec{p},T)}{\omega - i\omega_n}$$

$$= \int d\omega \; \frac{\sigma_H(\omega,\vec{p},T)}{i\omega_n - \omega}$$

⇓

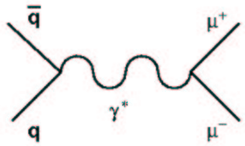
$$G_H(\tau,\vec{p},T) = \int_0^\infty d\omega \; \sigma_H(\omega,\vec{p},T) \; \frac{\cosh(\omega(\tau - 1/2T))}{\sinh(\omega/2T)}$$

⇑

accessible to LGT-calculations at discrete points:

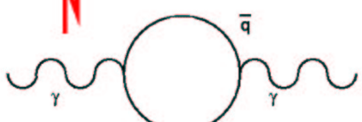
$\tau T = k/N_\tau \;, \;\; k = 0,\; 1, \ldots, \;\; N_\tau - 1$

Thermal dilepton rate and vector spectral function



rate $\sim |q\bar{q} \to \gamma^*|^2 \cdot |l^+ l^- \to \gamma^*|^2$

L.D. McLerran, T. Toimela, PR D31 (85) 545.



q photon self-energy

⇕

propagation of a $q\bar{q}$-pair with
the quantum numbers of a vector meson

spectral representation of
dilepton rate

$$\frac{dW}{d\omega d^3p} = \frac{5\alpha^2}{27\pi^2} \frac{1}{\omega^2 (e^{\omega/T} - 1)} \sigma_V(\omega, \vec{p}, T)$$

similarly: real photons

$$\omega \frac{dW}{d^3p} = \frac{5\alpha}{18\pi^2} \frac{1}{(e^{\omega/T} - 1)} \sigma_V(\omega, |\vec{p}| = \omega, T)$$

Warmup:

Correlators and spectral functions in the infinite temperature limit

free spectral function ($m_q = 0$):

$$\sigma_{PS}(\omega) = 0.5\sigma_V = \frac{N_c}{8\pi^2} \omega^2 \tanh(\omega/4T)$$

$\omega/T < 1: \ \sigma(\omega) \sim \omega^3$

free correlation function ($\vec{p} \equiv 0$):

$$G_{PS}/T^3 = G_V/2T^3 = \pi N_c \, (1 - 2\tilde{\tau}) \frac{1 + \cos^2(2\pi\tilde{\tau})}{\sin^3(2\pi\tilde{\tau})} + 2N_c \frac{\cos(2\pi\tilde{\tau})}{\sin^2(2\pi\tilde{\tau})} \quad , \quad \tilde{\tau} = \tau T$$

$G_V(\tau T = 1/2, T)/2T^3 \equiv 1$

"free" dilepton rate (Born rate), $\vec{p} = 0$:

$$\frac{dW^{Born}}{d\omega d^3p} = \frac{5\alpha_{em}^2}{36\pi^4} \frac{1}{(e^{\omega/2T} + 1)^2}$$

$\omega = 0$: finite rate

Note:

$$G_H(\tau T = 1/2, \vec{0}, T)/T^3 = \int_0^\infty d\tilde{\omega} \, \frac{\sigma_H(\omega, \vec{0}, T)}{\sinh(\tilde{\omega}/2)}, \ \tilde{\omega} = \omega/T$$

most sensitive to small ω region

Correlators and spectral functions in HTL-resummed perturbation theory

E. Braaten, R.D. Pisarski, T.C. Yuan, Phys. Rev. Lett. 64 (1990) 2242
F.K, M.G. Mustafa, M.H. Thoma PL B497 (2001) 249

- mesonic correlation functions probe structure of HTL-resummed quark propagator

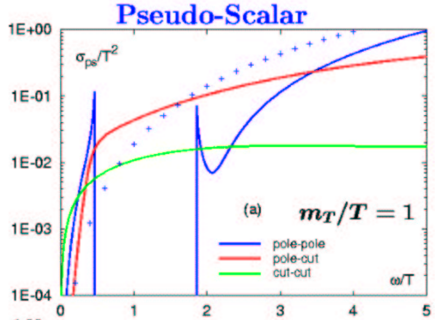
$$\rho_{\rm HTL}(\omega,\vec{q}) = \frac{1}{2}\rho_+(\omega,q)(\gamma_0 - i\,\hat{q}\cdot\vec{\gamma}) + \frac{1}{2}\rho_-(\omega,q)(\gamma_0 + i\,\hat{q}\cdot\vec{\gamma})$$

with

$$\rho_\pm(\omega,q) = \frac{\omega^2-q^2}{2m_T^2}[\delta(\omega-\omega_\pm)+\delta(\omega+\omega_\mp)] + \beta_\pm(\omega,q)\Theta(q^2-\omega^2)$$

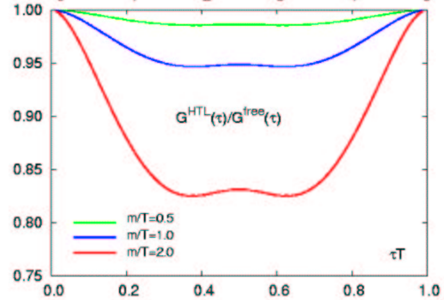
pole cut

Pseudo-Scalar



+ + + free ($T=\infty$) result

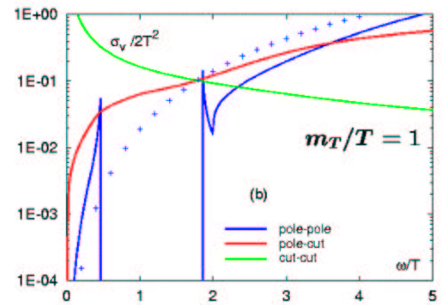
pole contribution dominates for $\omega/T > 3$



$G^{HTL}(\tau) < G^{free}(\tau)$

thermal mass effect dominates

HTL: Vector correlators and spectral functions



cut contribution dominates for $\omega/T < 2$:

$\sigma_V^{cut} \sim 1/\omega$

dilepton rate $\sim 1/\omega^4$

- HTL-resummed vector correlator diverges for all τT due to singular cut-cut contributions in σ_V

- meson correlator:

$$G_V(\tau,\vec{0},T) = \int_0^\infty d\omega\, \sigma_V(\omega,\vec{0},T)\, \frac{\cosh(\omega(\tau-1/2T))}{\sinh(\omega/2T)}$$

⇒ need $\sigma_V \rightarrow 0$ for $\omega \rightarrow 0$ to get finite G_V

including multiple scattering effects (LPM) might reduce the divergence of σ_V to $\sigma_V \sim 1/\sqrt{\omega}$: this is not enough!

P. Arnold, G.D. Moore, L.G. Yaffe, JHEP 0111 (2001) 057
(so far only for real photons)

Lattice results on thermal meson correlation functions

FK, E. Laermann, P. Petreczky, S. Stickan, I. Wetzorke, Phys. Lett. B530 (2002) 147

- quenched QCD;
- Wilson fermions with non-perturbatively improved clover coefficients and meson currents

$$J_H^{ren}(x) = 2\kappa Z_H(g^2,\kappa) J_H^{bare}(x), \quad H = V,\ PS$$

M. Lüscher et. al., Nucl. Phys. B491 (1997) 344

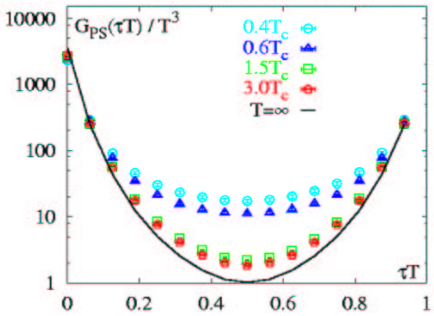
- large lattices: $N_\sigma^3 \times N_\tau$, $16 \le N_\sigma \le 64$, $N_\tau = 12,\ 16$
- calculations above T_c for $T/T_c = 1.5,\ 3.0$ performed at $\kappa_c(T=0)$, i.e. for massless quarks!
- analysis based on (40-60) configurations (for $N_\sigma = 64$)
- some results below T_c: $T/T_c = 0.4, 0.6, 0.9$

preliminary!

Thermal meson correlation functions from LGT

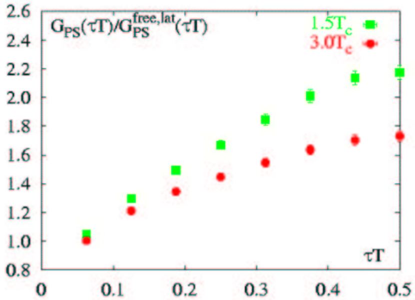
- results in the pseudo-scalar (and scalar) channel

$64^3 \times 16$ lattices



light Goldstone pion below T_c

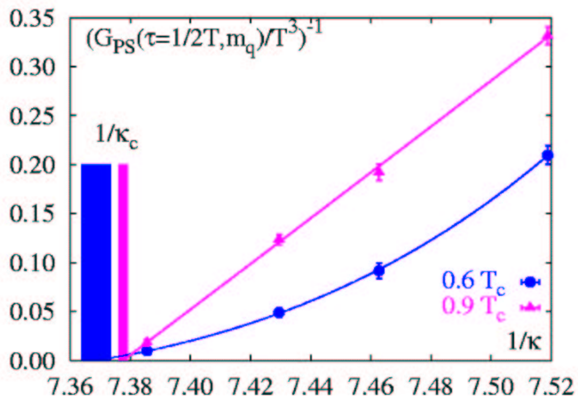
large deviations from free ($T=\infty$) result above T_c



$G_{PS}(\tau) > G_{PS}^{free}(\tau)$

quite different from HTL; soft modes dominate

- **the pion is a Goldstone particle up to T_c**



$$G_{PS}(\tau T = 1/2, \vec{0}, T)/T^3 = \int_0^\infty d\tilde{\omega}\, \frac{\sigma_{PS}(\omega, \vec{0}, T)}{\sinh(\tilde{\omega}/2)},\ \tilde{\omega} = \omega/T$$

$$\sim T/(m_{PS} \sinh[m_{PS}/2T])$$

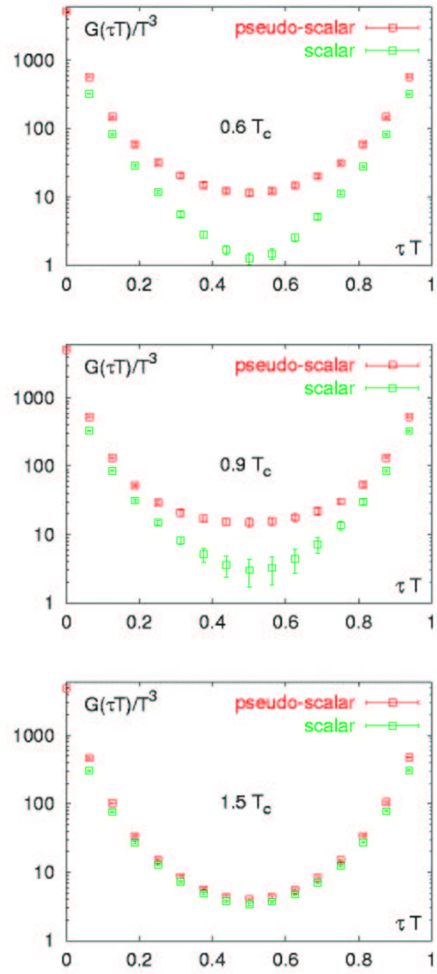
$$\sim (\kappa^{-1} - \kappa_c^{-1})^{-1}$$

$m_{PS}(\kappa, T) \to 0$ for $\kappa \to \kappa_c$, $T < T_c$

- **effective $U_A(1)$ symmetry restoration above T_c**

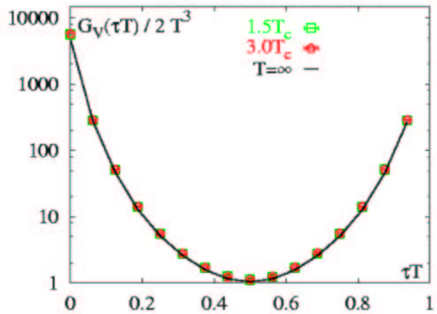
$$\pi : J_{PS} \sim \bar{q}\gamma_5\tau q \quad \overset{}{\Leftrightarrow} \quad \delta : J_S \sim \bar{q}\tau q$$

$$U_A(1)$$

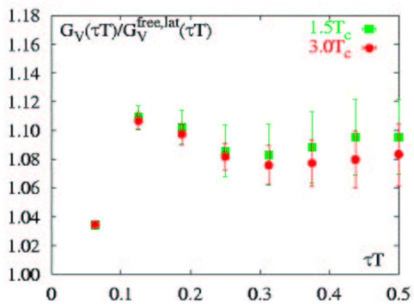


- **results in the vector channel**

$64^3 \times 16$ lattices



G_V stays close to G_V^{free} above T_c



$G_V(\tau) > G_V^{free}(\tau)$

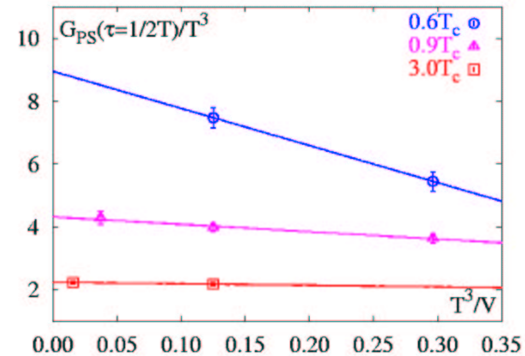
$$\frac{G_V(1/2T,T)}{G_V^{free,lat}(1/2T,T)} = \begin{cases} 1.09 \pm 0.03\,, & T/T_c = 1.5 \\ 1.08 \pm 0.03\,, & T/T_c = 3 \end{cases}$$

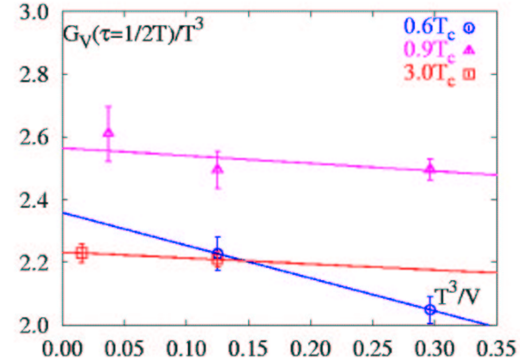
– rules out simple quasi-particle models

– no severe infrared (and ultraviolet) problems

– suggests $\sigma_V \to 0$ for $\omega \to 0$

- **volume dependence at $\tau T = 1/2$ is weak above T_c**





Reconstruction of the spectral function

M. Asakawa, T. Hatsuda, Y. Nakahara,
Prog. Part. Nucl. Phys. 46 (2001) 459

$$G_H(\tau,\vec{0},T) = \int_0^\infty d\omega\ \sigma_H(\omega,\vec{0},T)\ \frac{\cosh(\omega(\tau-1/2T))}{\sinh(\omega/2T)}$$

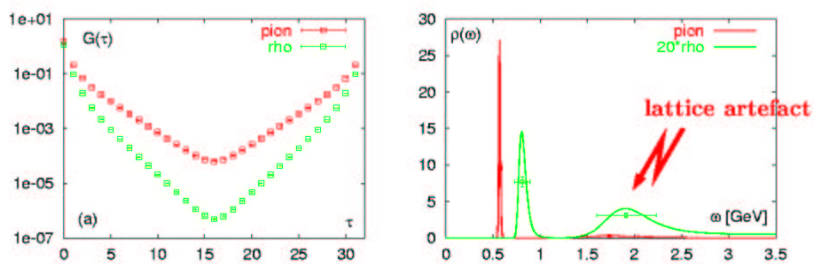
try to explore Maximum Entropy Method:

maximize $Q = \alpha S - \chi^2/2$ with

entropy $S = \int d\omega[\sigma(\omega) - m(\omega) - \sigma(\omega)\ln(\sigma(\omega)/m(\omega))]$

input default model: $m(\omega) = \sigma_H^{free}(\omega) \sim \omega^2\ (T=0)$

$T \simeq 0$ lattice: $16^3 \times 32$

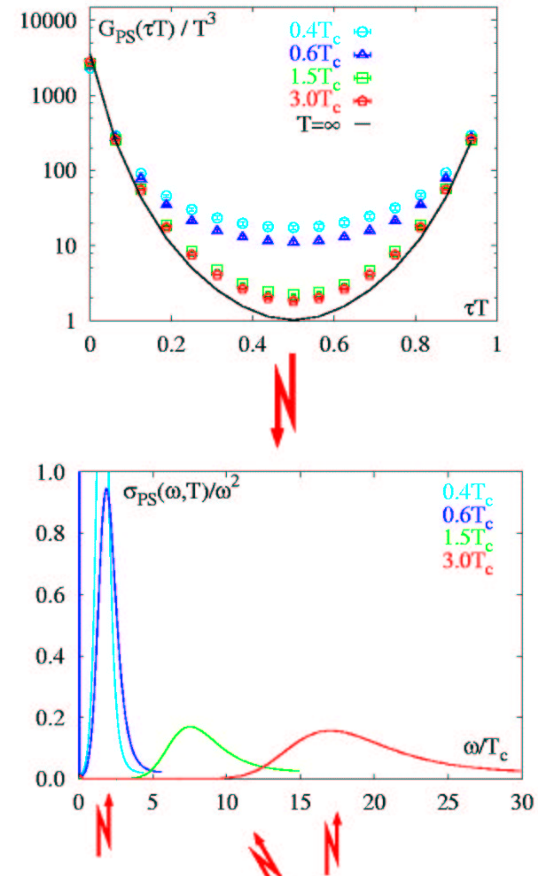


$\sigma_H(\omega) \equiv \rho_H(\omega)\omega^2$

lattice artefact: heavy Wilson doubler

CP-PACS, Phys.Rev.Lett. 84 (2001) 238

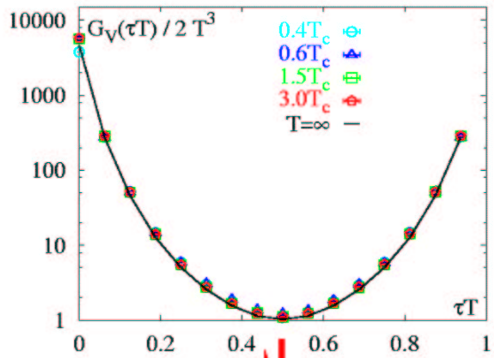
pseudo-scalar spectral functions



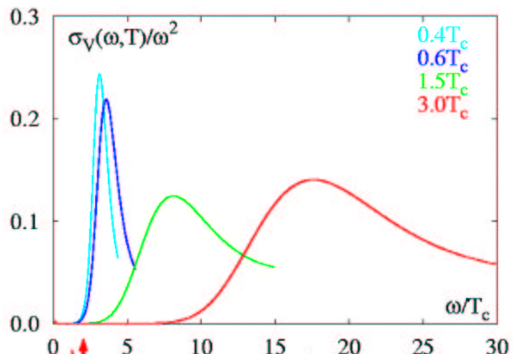
κ adjusted to give identical m_{PS}

$\kappa \equiv \kappa_c$, i.e. massless $(T=0)$ quarks

Vector spectral functions



little deviations from G_V^{free} already for $T \gtrsim 0.4 T_c$!

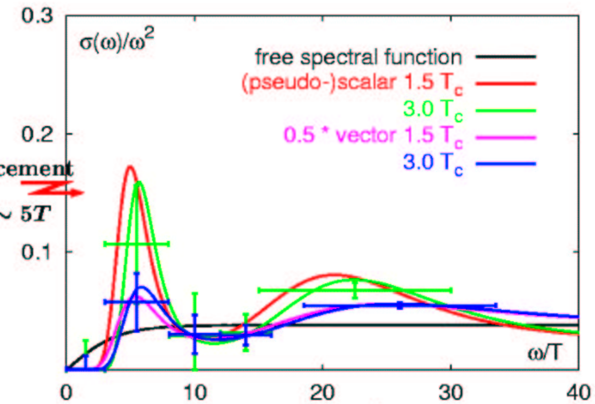


κ adjusted to give identical m_{PS}

$\kappa \equiv \kappa_c$, i.e. massless ($T = 0$) quarks

(Pseudo-)scalar and vector spectral functions above T_c

enhancement for $\omega \sim 5T$



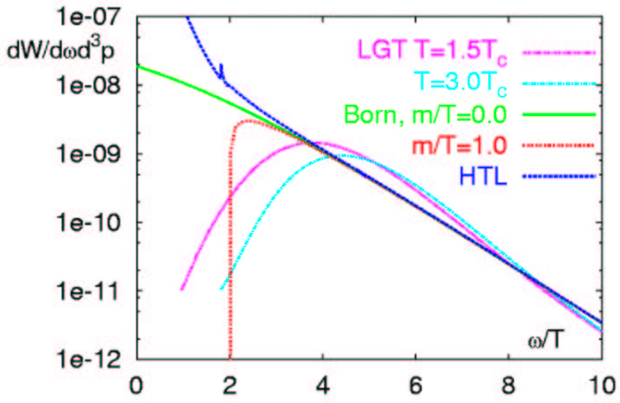
suppression relative to free spec. fctn. for $\omega \lesssim 3T$

lattice artefact: enhancement due to Wilson doublers

- $\sigma(\omega) \sim \omega^\alpha$ with $\alpha < 2$ seems to be ruled out (remember HTL: $\alpha = -1$)
- evidence for "thermal mass cut-off" below $\omega \simeq (2-3)T$ needs confirmation (need larger N_τ)

Dilepton Rate

$$\frac{dW}{d\omega d^3p} = \frac{5\alpha^2}{27\pi^2}\frac{\sigma_V(\omega,T)}{(e^{\omega/T}-1)\omega^2} \quad , \quad \frac{dW^{Born}}{d\omega d^3p} = \frac{5\alpha^2}{36\pi^4}\frac{1}{(e^{\omega/2T}+1)^2}$$



only small enhancement over Born rate for $\omega \gtrsim 4T$

differences show up for $\omega \lesssim 3T$

- diverging or large rate seems to be ruled out by LGT results; However, details of the low ω-behaviour need confirmation on even larger lattices

Conclusions

- pseudo-scalar channel:
  - Goldstone pion below T_c
  - strong deviations from free quark propagation above T_c
  - approximate $U_A(1)$ above T_c

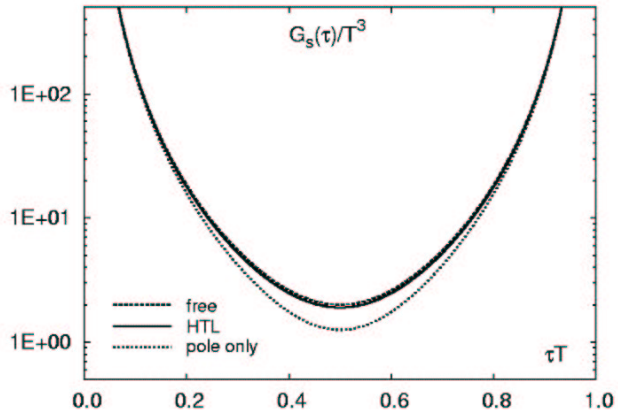
- vector channel:
  - $G_V(\tau,T)/T^3$ close to free ($T=\infty$) correlator
  - $G_V(\tau T = 1/2, T) = 2.17 \pm 0.06$ provides stringent constraint for model calculations of σ_V

- dilepton rate:
  - MEM analysis yields spectral functions
  - no indications for diverging rate at low ω
  - larger temporal lattices needed to resolve details of low ω behaviour

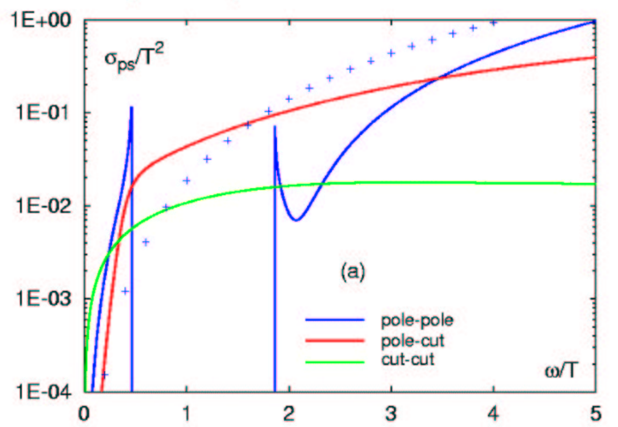
Thermal Meson Correlators in HTL-Approximation

FK, M. Mustafa, M.Thoma, PL B497 (2001) 249

(Pseudo)-Scalar Propoagator



(Pseudo)-Scalar Spectral Functions



Pseudo-scalar and vector spectral functions