Event-by-event fluctuations – theoretical perspective

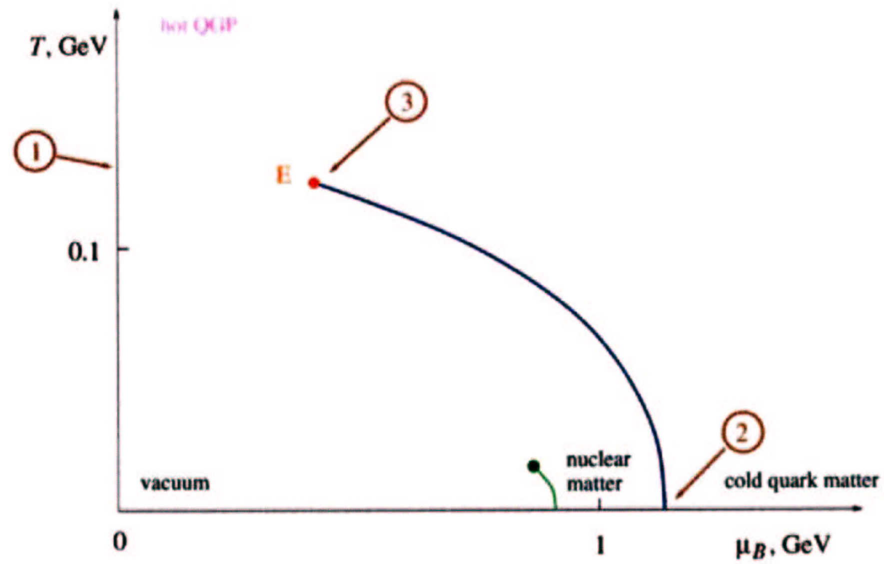
- Two ideas:

  QCD critical point (MS, Rajagopal, Shuryak)

  Frozen QGP charge fluctuations (Asakawa, Heinz, Muller, Jeon, Koch)

- Framework

- Phase diagram of QCD and the critical point



(1) Lattice calculations: no singularity at $\mu = 0$ (crossover).

(2) No lattice calculations here (sign problem). Models: 1st order transition (at $m_q = 0$ – chiral restoration).

(1) + (2) = (3): The 1st order transition must end → critical end-point E. As in water at $p = 221$bar, $T = 373°$C – critical opalescence.

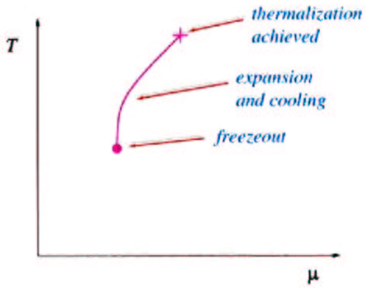
Where is point E? Challenge for theory and experiment.

Heavy ion collision experiments can discover E and leave a mark on the phase diagram of QCD.

- Heavy ion collisions and the QCD phase diagram

"Little Bang"

Time history of a small macroscopic subvolume:

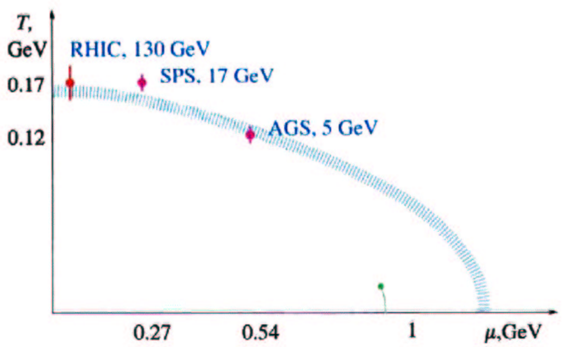


$\tau_H \sim 10 - 20$ fm;
$\tau_{\text{free}} \sim 1$ fm :
grows as $n \to 0$
when $\tau_{\text{free}} \sim \tau_H$ – freezeout

(CMB)

Observed hadron spectra reflect thermodynamics at the time of "last interaction" — freezeout time

(Braun-Munzinger, Stachel) $\to T, \mu$ at freezout



Strategy: scan the phase diagram changing $\sqrt{s}$.

FRAMEWORK

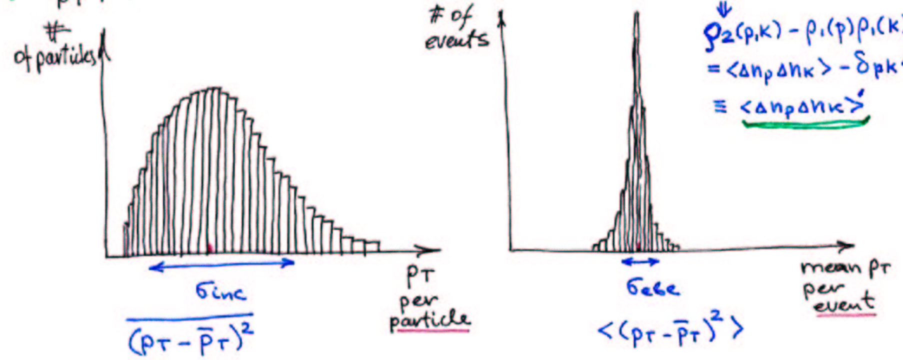
- $\langle n_p n_k \rangle$

n_p - number of particles in bin p for given event
$\langle \ldots \rangle$ - average over events

$$\langle n_p n_k \rangle - \langle n_p \rangle \langle n_k \rangle \equiv \langle \Delta n_p \Delta n_k \rangle = \underbrace{\delta_{pk} \langle n_p \rangle}_{\text{trivial statistical}} + \underbrace{\text{correlations}}_{\text{dynamical}}$$

$$\rho_1(p) = \langle n_p \rangle$$
$$\rho_2(p,k) = \langle n_p n_k \rangle - \delta_{pk} \langle n_p \rangle$$
$$\Downarrow$$
$$\rho_2(p,k) - \rho_1(p)\rho_1(k) = \langle \Delta n_p \Delta n_k \rangle - \delta_{pk} \langle n_p \rangle \equiv \underline{\langle \Delta n_p \Delta n_k \rangle'}$$

- Φ_{p_T} / F



$\Phi_{p_T} \equiv \sqrt{\langle N \rangle}\, \sigma_{ebe} - \sigma_{inc}$ (=0 if all particles statist. indep.)

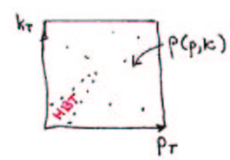
$F = \dfrac{\langle N \rangle \sigma_{ebe}^2}{\sigma_{inc}^2}$ (less sensitive to flow - easier to predict)

$\Phi_{p_T} = \sigma_{inc} (\sqrt{F} - 1)$

$$F = \frac{1}{\langle N \rangle} \sum_p \sum_k \langle \Delta n_p \Delta n_k \rangle \frac{(p_T - \bar{p}_T)(k_T - \bar{k}_T)}{\sigma_{inc}^2} = \underbrace{1}_{\delta_{pk}\langle n_p \rangle} + \underline{\text{correlations}}$$

STAR (Trainor, Reid)

$$\rho(p,k) = \frac{\langle n_p n_k \rangle'}{\langle n_p \rangle \langle n_k \rangle} = 1 + \frac{\langle \Delta n_p \Delta n_k \rangle'}{\langle n_p \rangle \langle n_k \rangle}$$



- Why event-by-event fluctuations?

Criticality is always due to a divergent correlation length (= vanishing mass).

In QCD it is $m_\sigma \to 0$ (σ – fluctuation of the magnitude of $\langle\bar\psi\psi\rangle$)

σ's we do not see after freezeout, because $\sigma \to \pi\pi$ in vacuum

However, at freezeout, fluctuations of the σ field ($\sim 1/m_\sigma^2$) create correlations in the pion momenta distributions (due to $\sigma\pi\pi$ coupling)

Such correlations can be measured using e-b-e fluctuations (of p_T)

Effect of light σ on $\langle \Delta n_p \Delta n_k \rangle$

Consider $\pi^+\pi^-$ only for simplicity (most of observed particles)

THERMODYNAMICS

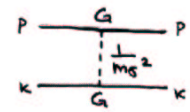
WE NEED $\langle \Delta n_p \Delta n_k \rangle$ FOR INTERACTING PION GAS (σ-exchange)

$\langle \Delta n_p \Delta n_k \rangle' \sim A_{pk}(0)$ - forward scattering amplitude

$$\langle \Delta n_p \Delta n_k \rangle' = \rho_2(p,k) - \rho_1(p)\rho_1(k) = f_p f_k \left(e^{-\beta E_2(p,k)} - 1\right) \approx$$
$$\approx f_p f_k(-\beta E_2(p,k)) = f_p f_k \beta A_{pk}(0)$$

- attraction: $E_2 < 0 \quad \Rightarrow \rho_2 > \rho_1 \cdot \rho_1 \Rightarrow \langle \Delta n_p \Delta n_k \rangle' > 0$

NEAR CRITICAL POINT ($m_\sigma \to 0$) $A_{pk}(0)$ $\pi\pi \to \pi\pi$ becomes large:



$G\sigma\pi^+\pi^-$ coupling

$A \sim \dfrac{G^2}{m_\sigma^2}$ - singular contibution

ESTIMATE:

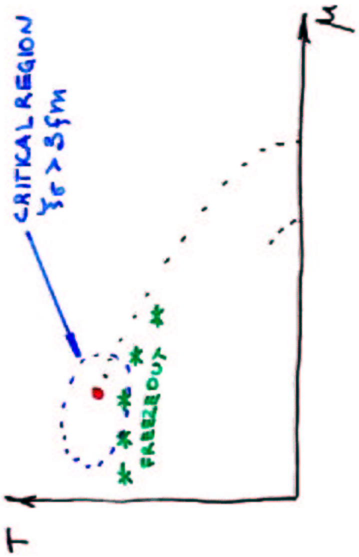
$$F - 1 \approx 0.1 \left(\frac{G}{300\,\text{MeV}}\right)^2 \left(\frac{\xi_\sigma}{3\,\text{fm}}\right)^2, \quad \xi_\sigma \equiv \frac{1}{m_\sigma}$$

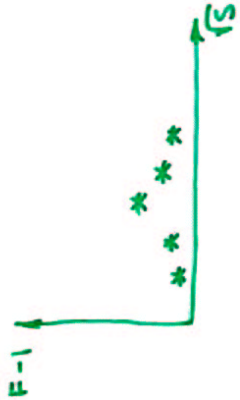
or $\Phi_{p_T} \approx (15\,\text{MeV}) \left(\dfrac{G}{300\,\text{MeV}}\right)^2 \left(\dfrac{\xi_\sigma}{3\,\text{fm}}\right)^2$

STRATEGY

Q: WHAT DISTINGUISHES σ contribution to F from possible other contr.?

A: NONMONOTONIC BEHAVIOR ON $\sqrt{s}$ (a parameter controling $\mu_{freezeout}$)





ALSO NOTE:

- $\langle \Delta n_p^\alpha \Delta n_k^\beta \rangle \sim \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}^{\alpha\beta}$ ← tor — "charge blind" fluctuations (compare to ρ^0) $\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$
- $\langle \Delta n_p \Delta n_k \rangle' \sim f_p f_k \frac{1}{\omega_p}\frac{1}{\omega_k}\frac{G^2}{m_\sigma^2}$ dominated by low p, k

FRAMEWORK

ACCEPTANCE (rapidity)

$$F-1 = \frac{1}{\langle N \rangle} \sum_{pk} \langle \Delta n_p \Delta n_k \rangle' \frac{\Delta p_T \Delta k_T}{\sigma_{inc}^2}$$

$\leftarrow (k_T - \bar{k}_T)$

Define a correlator:

$$C_{p_T}(y_1, y_2) \equiv \sum_{pk} \delta(y_1 - y_p)\,\delta(y_2 - y_k) \langle \Delta n_p \Delta n_k \rangle' \frac{\Delta p_T \Delta k_T}{\sigma_{inc}^2}$$

i.e. count particles with "weight" $\frac{\Delta p_T}{\sigma_{inc}}$

Then

$$(F-1)_{y_{acc}} = \frac{1}{\langle N \rangle_{y_{acc}}} \int_0^{y_{acc}} dy_1 \int_0^{y_{acc}} dy_2\, C_{p_T}(y_1, y_2)$$

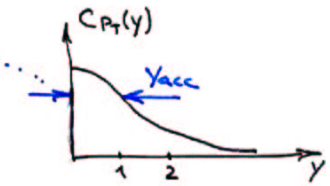
to simplify – boost invariance : $C_{p_T}(y_1, y_2) = C_{p_T}(y_1 - y_2)$

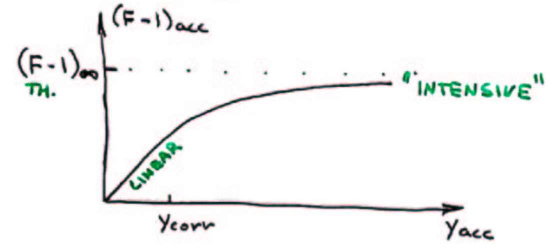
and

$$(F-1)_{y_{acc}} = \frac{2}{dN/dy} \int_0^{y_{acc}} dy \left(1 - \frac{y}{y_{acc}}\right) C_{p_T}(y)$$

Two limits of y_{acc}:

$$\rightarrow \frac{1}{dN/dy} \begin{cases} C_{p_T}(0) \cdot y_{acc} & ; \; y_{acc} \ll y_{corr} \\ 2\int_0^\infty dy\, C_{p_T}(y) & : \; y_{acc} \gg y_{corr} \end{cases}$$





Experiments A and B: (e.g. PHENIX/STAR)

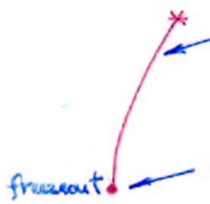
$$\frac{(F-1)_A}{(F-1)_B} = \begin{cases} y_A / y_B & : \; y_{A,B} \ll y_{corr} \\ 1 & ; \; y_{A,B} \gg y_{corr} \end{cases}$$

(Frozen) Charge fluctuations

Asakawa, Heinz, Müller
Jeon, Koch

Fluctuations which do not have time to equillibrate in hadronic phase

Conserved quantity, such as Q.



$\langle \Delta Q^2 \rangle_{QGP}$

$\langle \Delta Q^2 \rangle_{HADRON} \approx 2-3 \langle \Delta Q^2 \rangle_{QGP}$ (per same entropy)

$Q(\text{quark}) < Q(\text{pion})$

Q conserved ⇒ can change only by diffusion

(Equation for harmonics of charge distribution in y

$\dot{f}_k = \gamma k^2 f_k + \text{noise}_k \quad , \quad k \sim \frac{1}{\lambda}$)

$\Delta Q^2 = \sum_k c_k^2 f_k^2$

F.T. Δy

⇒ Long wave harmonics of ΔQ relax slowly

⇒ for $\Delta y \gg 1$ $\quad \langle \Delta Q^2 \rangle_{freezeout} < \langle \Delta Q^2 \rangle_{HADRON}$

Experiment (prelim): $\langle \Delta Q^2 \rangle_{freezeout} \simeq \langle \Delta Q^2 \rangle_{HADRON}$

Conclusion: Δy not wide enough (diff. wins)?

Acceptance: analysis similar to p_T fluctuations: $C_Q(y)$ - balance function

What can we learn from E-b-e?

- By discovering critical point we map a distinct feature of the QCD phase diagram

- Using charge fluctuations we may be able to look back into the history of the collision, and see QGP.