Dispersive shock waves in interacting one-dimensional systems

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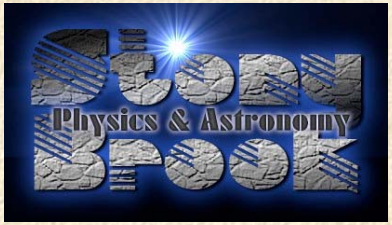
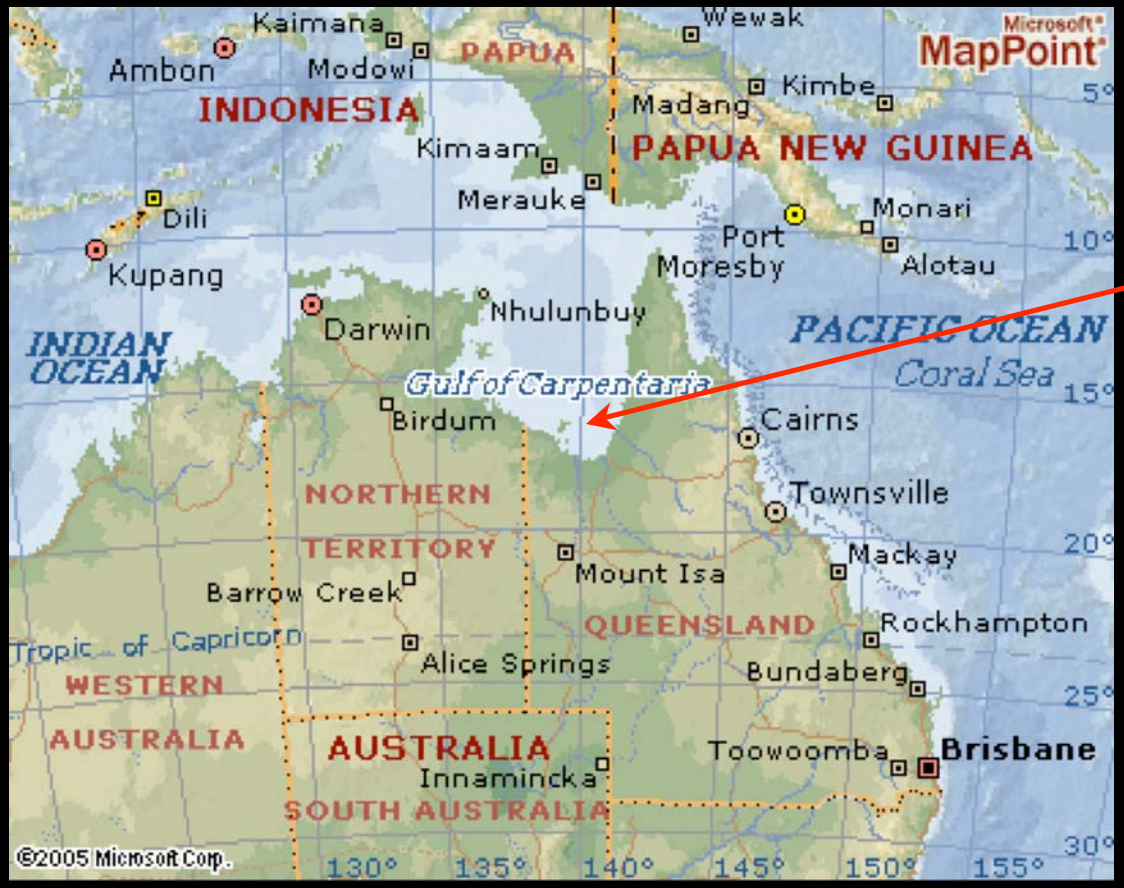
Thanks: A. Guliani, M. Kulkarni, …

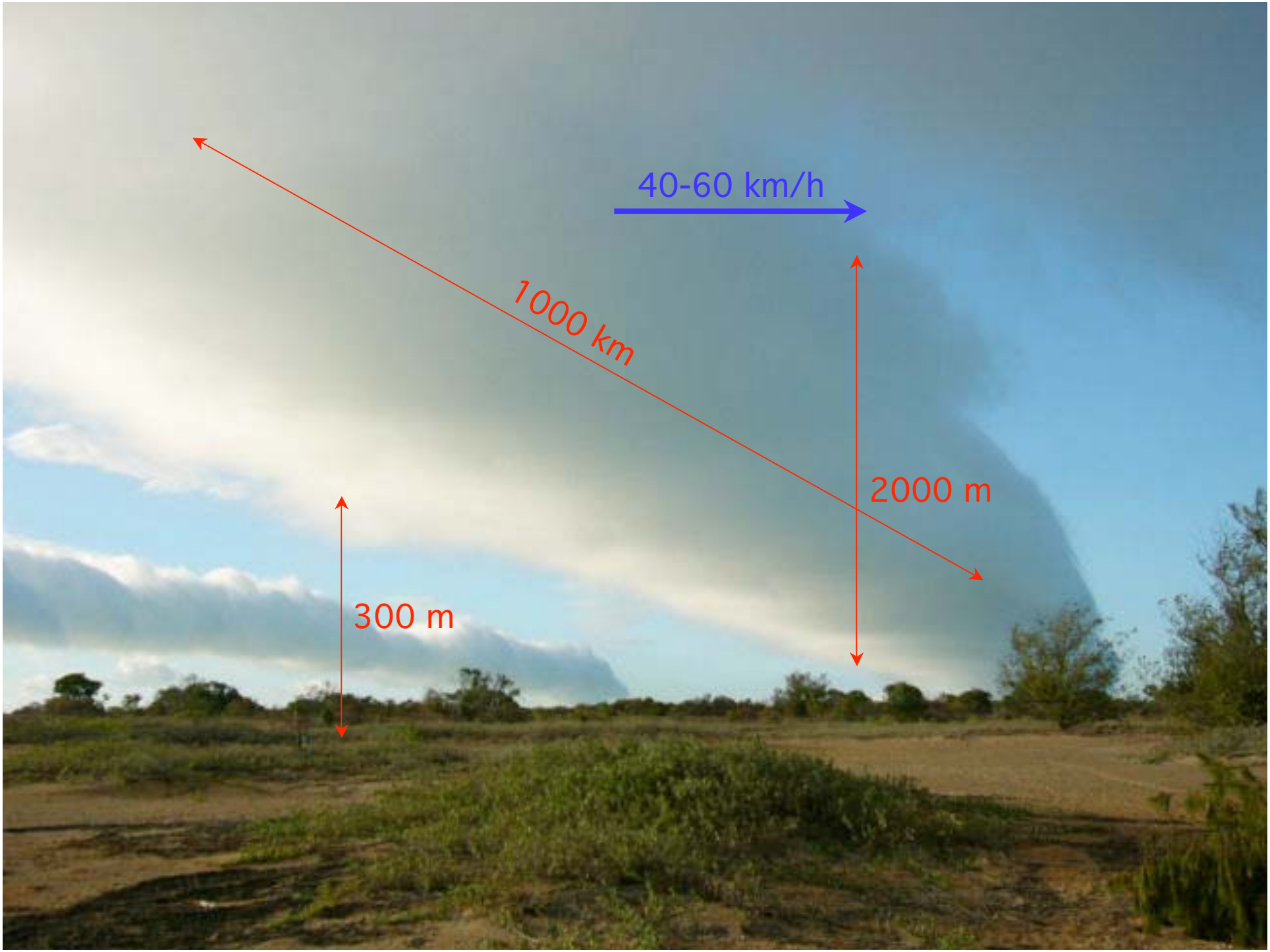


Morning Glory

Rolling clouds in the Gulf of Carpenteria, Northern Australia



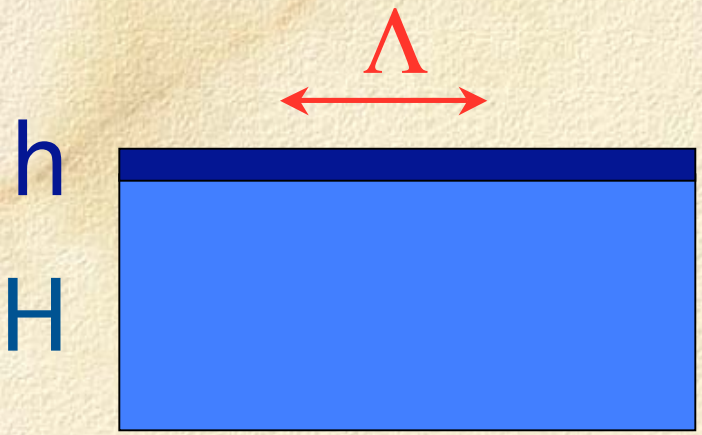
www.dropbears.com


40-60 km/h
1000 km
2000 m
300 m

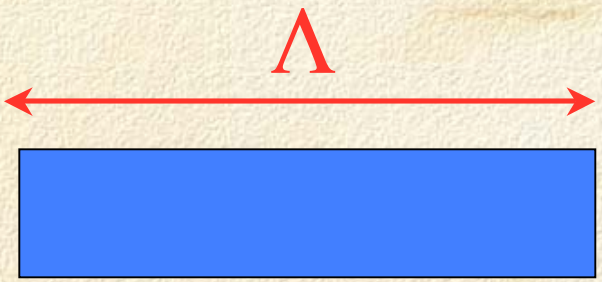
Rolling clouds in the Gulf of Carpentaria

Classical Benjamin-Ono equation

(internal waves in deep stratified fluid)



$$h \ll \Lambda \ll H$$



$$H \ll \Lambda$$

Benjamin-Ono

$$u_t + uu_x + u^H_{xx} = 0$$

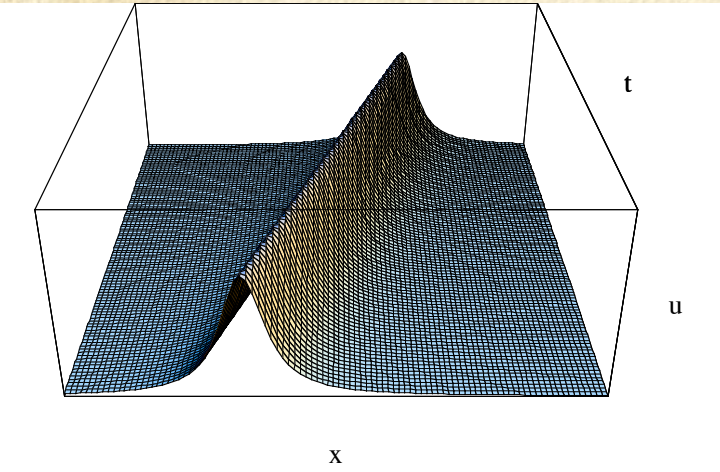
KdV

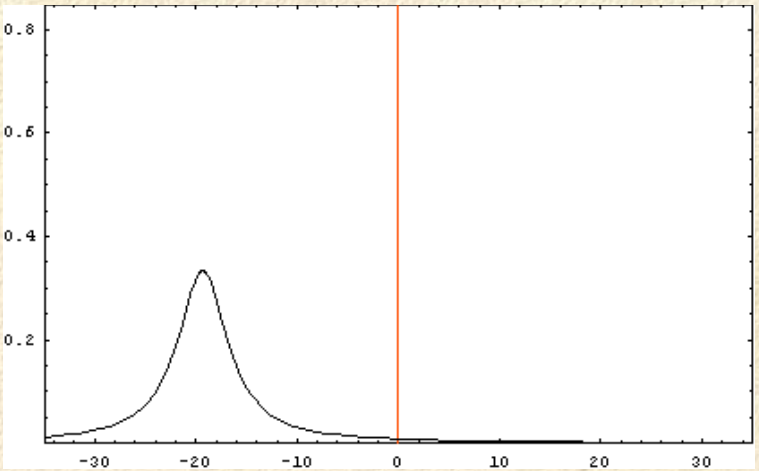
$$u_t + uu_x + u_{xxx} = 0$$

Benjamin 1967
Ono 1975

$$u^H(x) = \frac{1}{\pi} \int_{p.v.} dy \, \frac{u(y)}{y - x} = i(u^+ - u^-)$$

One-soliton solution of Benjamin-Ono equation

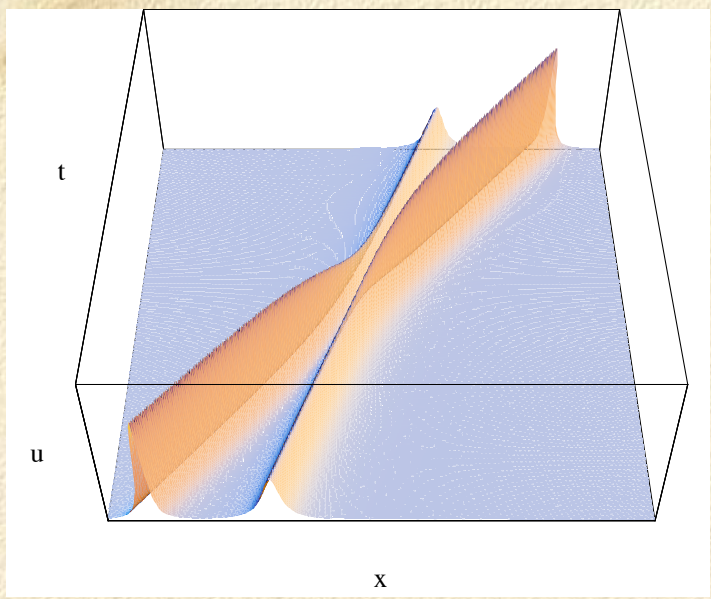


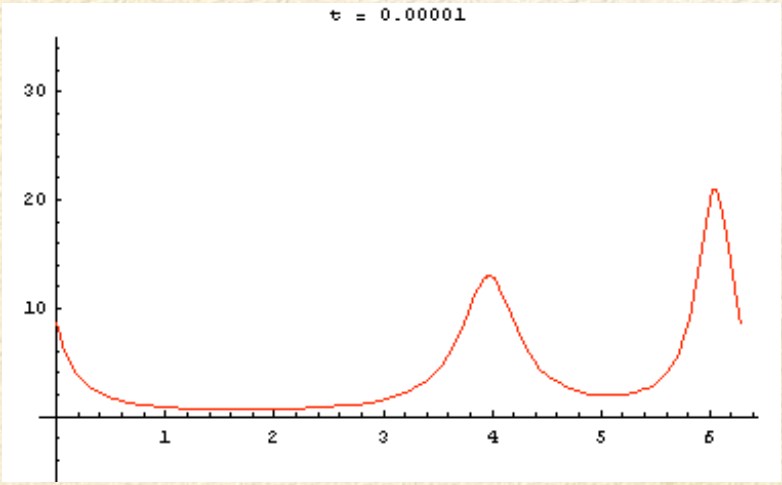


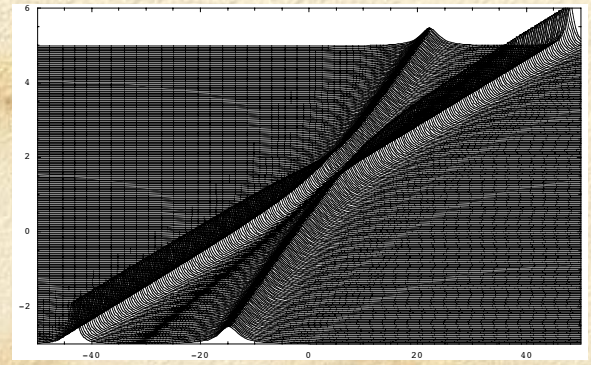
$$\int_{-\infty}^{+\infty} u(x,t)dx = 2\pi$$

$$u(x,t) = \frac{2V}{V^2(x - x_0 - Vt)^2 + 1}$$

Two-soliton solution of Benjamin-Ono equation (N=2)







$$u_t + uu_x + \alpha u_{xx}^H = 0$$

$$u(x,t) = -\sum_{j=1}^{N} \frac{2i\alpha}{x - z_j(t)} + c.c. \qquad \ddot{z}_j = -\frac{\partial}{\partial z_j} \sum_{n<m} \frac{(2\alpha)^2}{(z_n - z_m)^2}$$

Multi-soliton solutions of Benjamin-Ono equation

$$u_t + uu_x + \alpha u_{xx}^H = 0$$

Hirota form

$$u = 2i\alpha\partial_x \ln \tau^* / \tau$$

$$\left(iD_t - \alpha D_x^2\right)\tau^* \bullet \tau = 0$$

Pole ansatz

$$u(x,t) = -\sum_{j=1}^{N} \frac{2i\alpha}{x - z_j(t)} + c.c.$$

$$\ddot{z}_j = -\frac{\partial}{\partial z_j} \sum_{n<m} \frac{(2\alpha)^2}{(z_n - z_m)^2}$$

Explicit solution

$$\tau = \det M$$

$$M_{jk} = \left(i(x - \alpha a_j t - x_{j0}) + 1/a_j\right)\delta_{jk} + \frac{2}{a_j - a_k}(1 - \delta_{jk})$$

Parameters of N-soliton solution (j=1,2,...,N): a_j, x_{j0}

$$\tau \propto \Pi(x - z_j(t))$$

Calogero-Sutherland model

$$H = -\frac{1}{2}\sum_{i=1}^{N}\frac{\partial^2}{\partial x_i^2} + \sum_{i<j}\frac{\lambda(\lambda-1)}{(x_i-x_j)^2}$$

Calogero 1969
Sutherland 1971

$$\Psi_{ground} \sim \prod_{i<j}(x_i - x_j)^{\lambda}$$

λ=1 - free fermions

$$k_F = \pi\rho_0$$

$$v_s = \lambda\pi\rho_0$$

$$\epsilon(\rho) = \frac{1}{6}(\lambda\pi\rho)^2$$

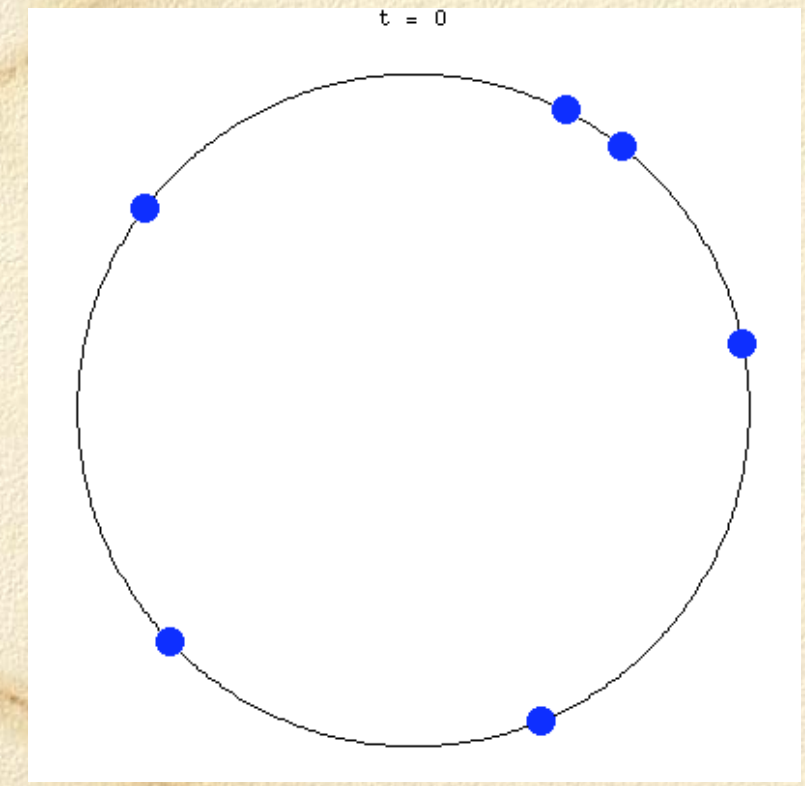
Exactly solvable
by Bethe Ansatz

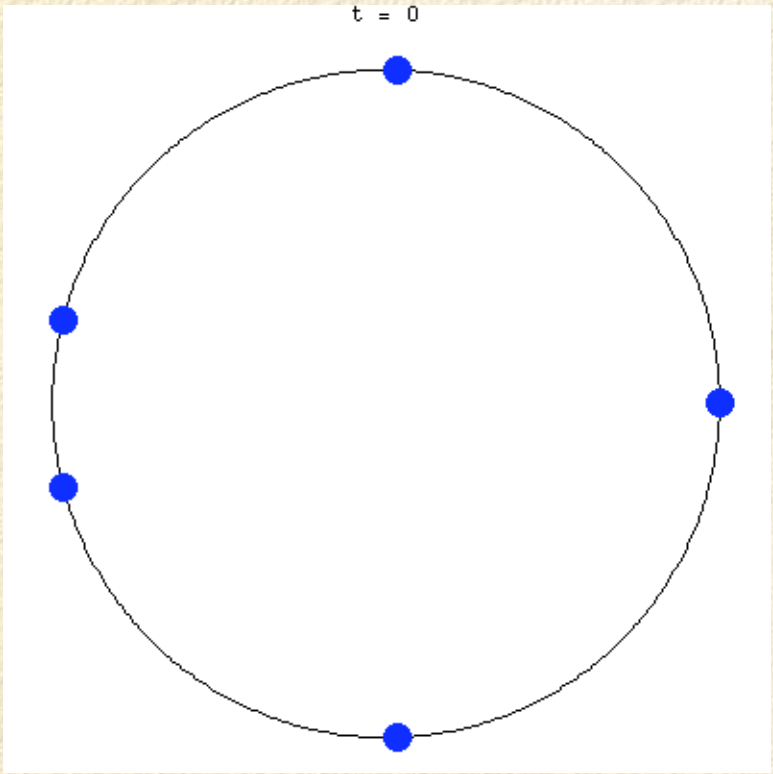
Dispersion of quasiparticles

$$\varepsilon_p(k) = \lambda k_F|k| + \frac{1}{2}k^2$$

$$\varepsilon_h(k) = \lambda k_F|k| + \frac{1}{2}\lambda k^2$$

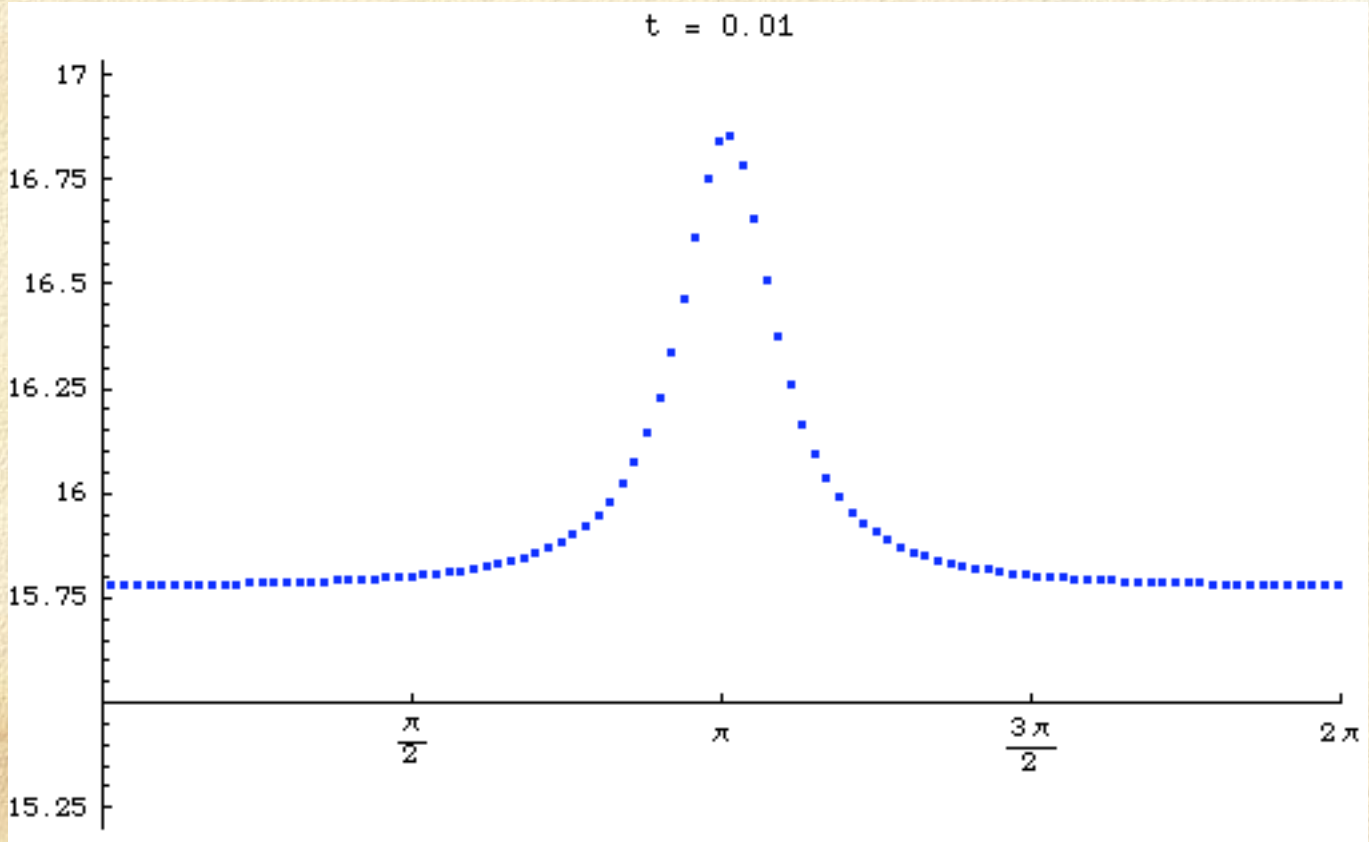
Exact solution for 5 particles





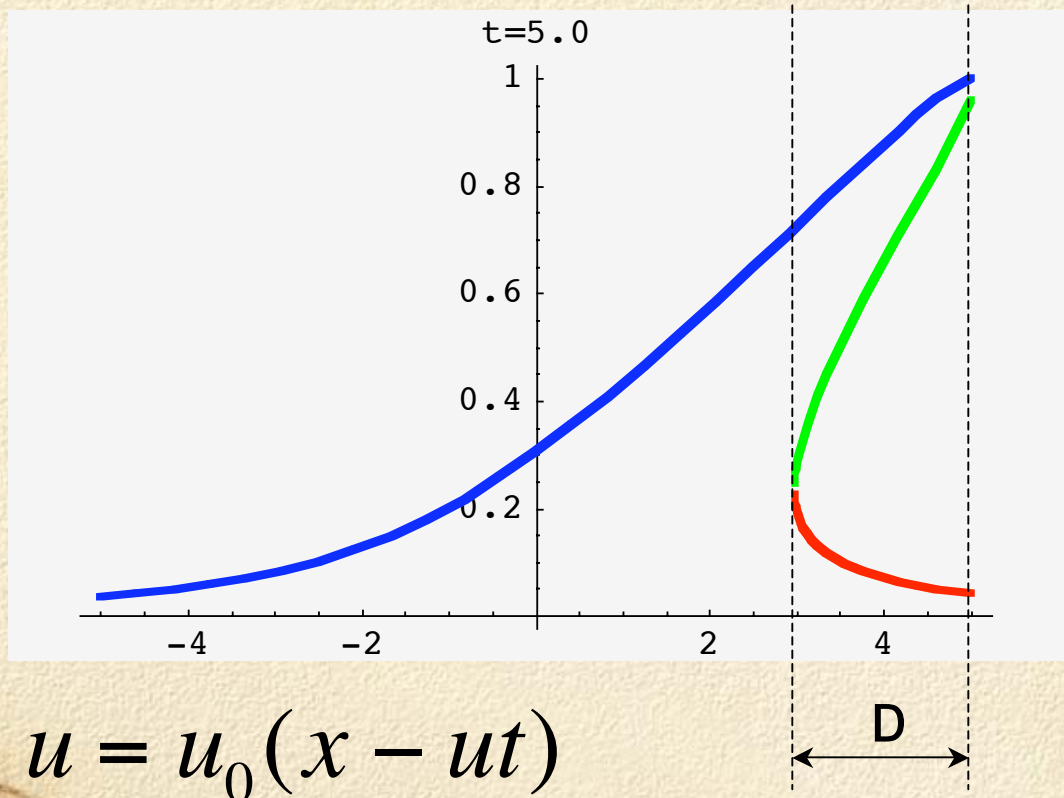
Collective motion in Sutherland model

N=100



Gradient catastrophy in the dispersionless limit

$$u_t + uu_x = 0$$

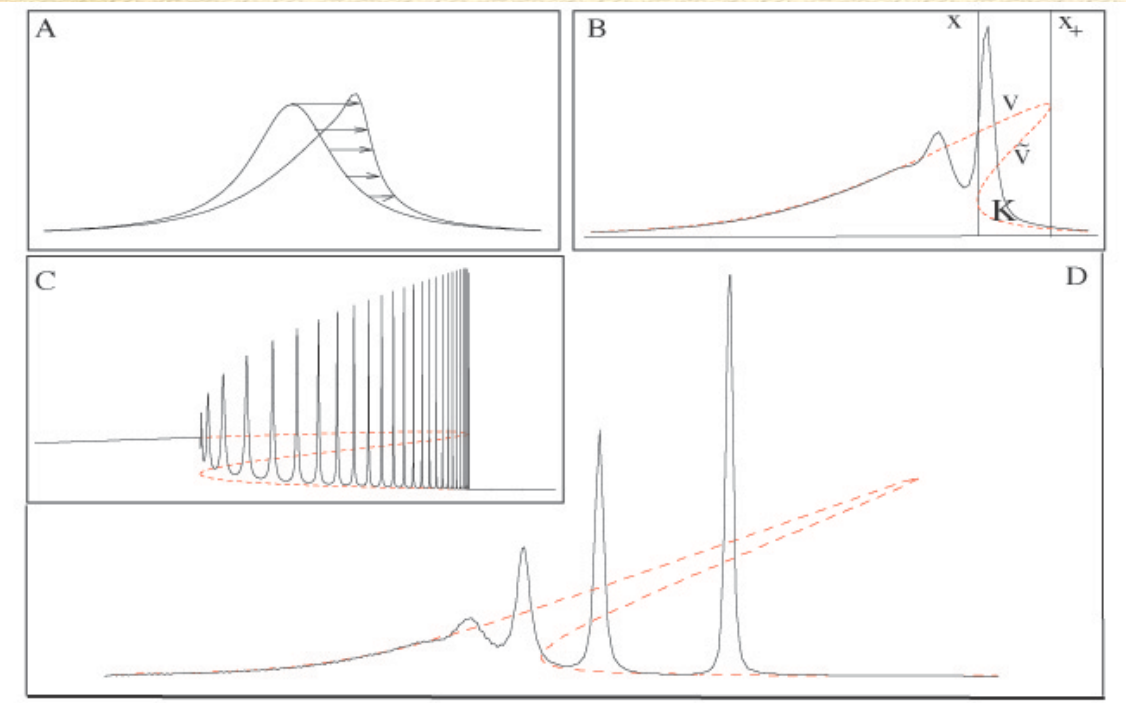


$$u = u_0(x - ut)$$

Solution of Riemann equation

Non-dissipative shock waves

Solution of Riemann equation + Modulated periodic solution of BO



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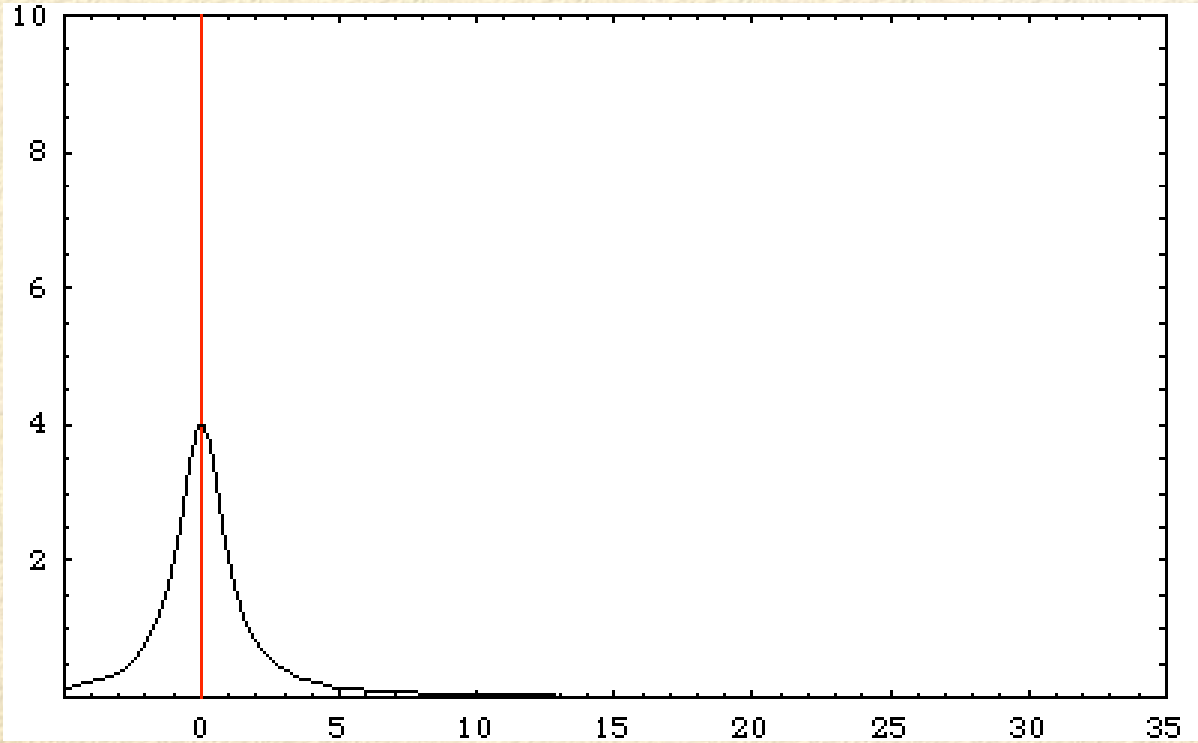
Whitham theory

A. Gurevich, L. Pitaevsky (1973)

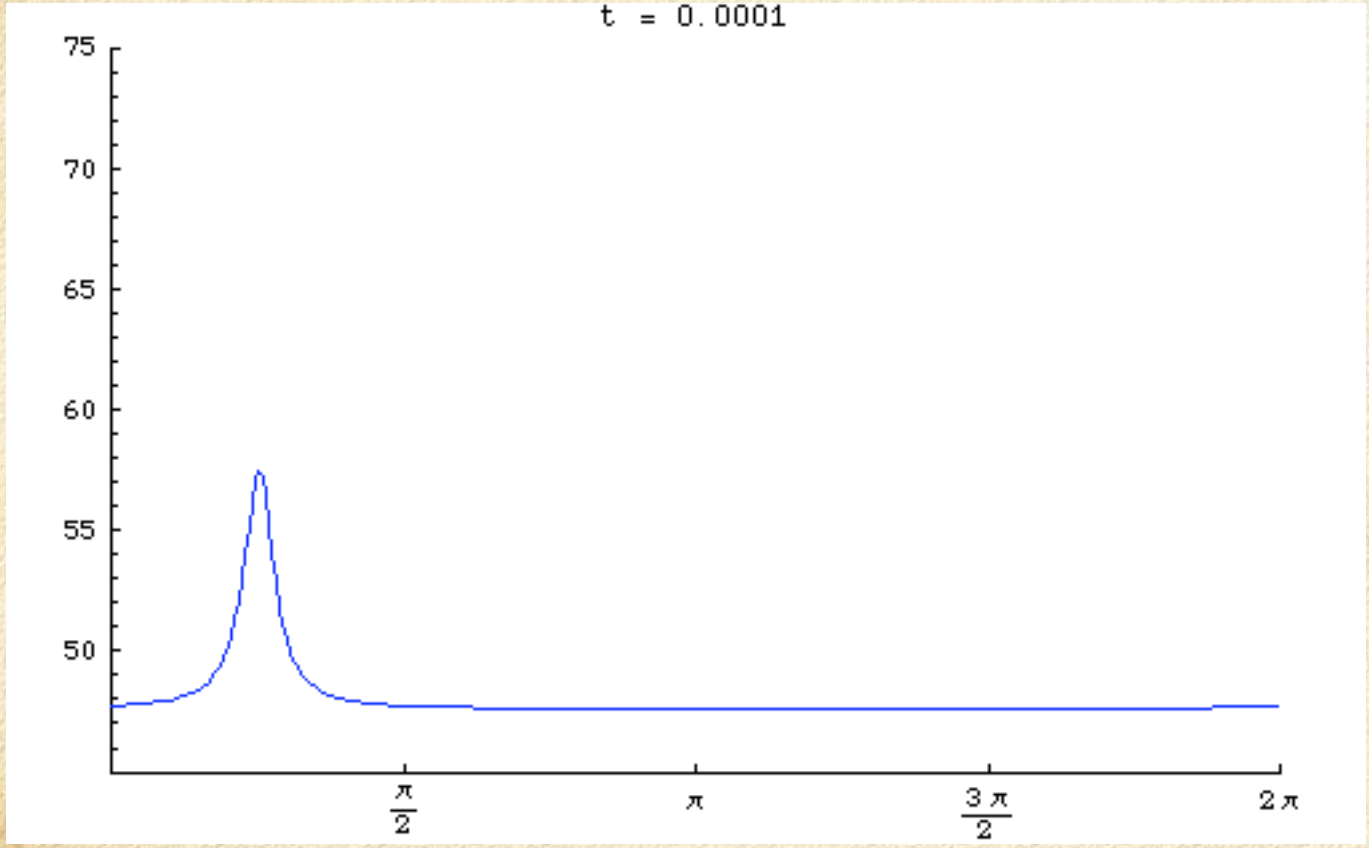
S. Dobrokhotov, I. Krichever (1991)

Y. Matsuno (1998)

Nondissipative shock waves



Shock waves in Sutherland model



Dynamics of 300 particles on a circle. Initial profile: Lorentzian with area 3.