Large θ_{13} and a Novel Origin of CP Violation in SUSY SU(5) x T´

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in collaboration with
M.-C. Chen, Jinrui Huang, Alex Wijangco, under preparation;
M.-C, Chen, Phys. Lett. B681 (2009) 444; Phys. Lett. B652 (2007) 34

Snowmass on the Pacific, KITP, Santa Barbara, CA, May 28-31, 2013

Where Do We Stand?

- Exciting Time in ν Physics: recent hints of large θ_{13} from T2K, MINOS, Double Chooz, and Daya Bay
- Latest 3 neutrino global analysis (including recent results from reactor experiments):

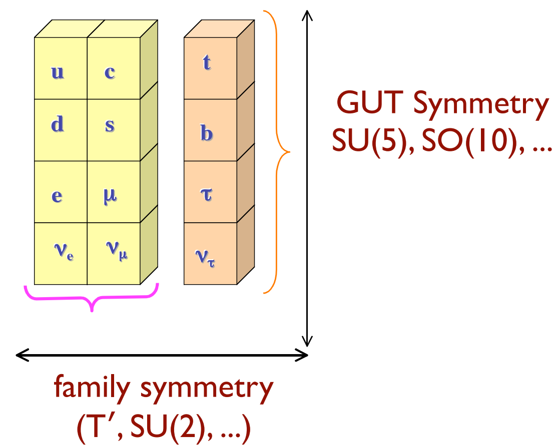
$$P(\nu_a \to \nu_b) = \left|\langle \nu_b | \nu,\ t \rangle\right|^2 \simeq \sin^2 2\theta\ \sin^2\left(\frac{\Delta m^2}{4E} L\right)$$

Fogli, Lisi, Marrone, Montanino, Palazzo, Rotunno, 2012

Parameter	Best fit	1σ range	2σ range	3σ range
$\delta m^2/10^{-5}$ eV^2 (NH or IH)	7.54	7.32 – 7.80	7.15 – 8.00	6.99 – 8.18
$\sin^2\theta_{12}/10^{-1}$ (NH or IH)	3.07	2.91 – 3.25	2.75 – 3.42	2.59 – 3.59
$\Delta m^2/10^{-3}$ eV^2 (NH)	2.43	2.33 – 2.49	2.27 – 2.55	2.19 – 2.62
$\Delta m^2/10^{-3}$ eV^2 (IH)	2.42	2.31 – 2.49	2.26 – 2.53	2.17 – 2.61
$\sin^2\theta_{13}/10^{-2}$ (NH)	2.41	2.16 – 2.66	1.93 – 2.90	1.69 – 3.13
$\sin^2\theta_{13}/10^{-2}$ (IH)	2.44	2.19 – 2.67	1.94 – 2.91	1.71 – 3.15
$\sin^2\theta_{23}/10^{-1}$ (NH)	3.86	3.65 – 4.10	3.48 – 4.48	3.31 – 6.37
$\sin^2\theta_{23}/10^{-1}$ (IH)	3.92	3.70 – 4.31	3.53 – 4.84 ⊕ 5.43 – 6.41	3.35 – 6.63
δ/π (NH)	1.08	0.77 – 1.36	—	—
δ/π (IH)	1.09	0.83 – 1.47	—	—

Origin of Mass Hierarchy and Mixing

- Several models have been constructed based on
  - GUT Symmetry [SU(5), SO(10)] ⊕ Family Symmetry G_F
- Family Symmetries G_F based on continuous groups:
  - U(1)
  - SU(2)
  - SU(3)



- Recently, models based on discrete family symmetry groups have been constructed
  - A_4 (tetrahedron)
  - T´ (double tetrahedron)
  - S_3 (equilateral triangle)
  - S_4 (octahedron, cube)
  - A_5 (icosahedron, dodecahedron)
  - Δ_{27}
  - Q_4

Motivation: Tri-bimaximal (TBM) neutrino mixing

Discrete gauge anomaly constraints:
Araki, Kobayashi, Kubo, Ramos-Sanchez, Ratz, Vaudrevange (2008)

Tri-bimaximal Neutrino Mixing

- Neutrino Oscillation Parameters $P(\nu_a \to \nu_b) = \left|\langle \nu_b | \nu,\ t\rangle\right|^2 \simeq \sin^2 2\theta\ \sin^2\left(\frac{\Delta m^2}{4E} L\right)$

$$U_{MNS} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

- Latest Global Fit (3σ)

Fogli, Lisi, Marrone, Montanino, Palazzo, Rotunno, 2012

$$\sin^2\theta_{atm} = 0.386\ (0.331 - 0.637) \qquad \sin^2\theta_{\odot} = 0.307\ (0.259 - 0.359)$$

$$\sin^2\theta_{13} = 0.0241\ (0.0169 - 0.0313)$$

- Tri-bimaximal Mixing Pattern

Harrison, Perkins, Scott (1999)

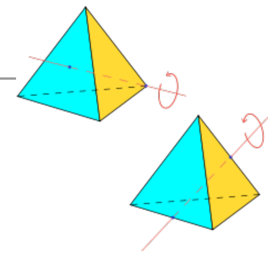
$$U_{TBM} = \begin{pmatrix} \sqrt{2/3} & \sqrt{1/3} & 0 \\ -\sqrt{1/6} & \sqrt{1/3} & -\sqrt{1/2} \\ -\sqrt{1/6} & \sqrt{1/3} & \sqrt{1/2} \end{pmatrix}$$

$$\sin^2\theta_{\mathrm{atm,\,TBM}} = 1/2 \qquad \sin^2\theta_{\odot,\mathrm{TBM}} = 1/3$$

$$\sin\theta_{13,\mathrm{TBM}} = 0.$$

- Leading Order: TBM (from symmetry) + Corrections (dictated by symmetry)

Group Theory of T´



- Smallest Symmetry to realize TBM ⇒ Tetrahedral group A_4
- tetrahedral group A_4: Ma, Rajasekaran (2001); Babu, Ma, Valle (2003)
  - even permutations of four objects: S: (1234) → (4321), T: (1234) → (2314)
  - geometrically -- invariant group of tetrahedron
  - does NOT give rise to CKM mixing: $V_{ckm} = 1$
  - all CG coefficients real
- Double covering of tetrahedral group A_4: Frampton & Kephart, (1994)
  - in-equivalent representations:

A4: $1,\ 1',\ 1'',\ 3$ (vectorial) → TBM for neutrinos

other: $2,\ 2',\ 2''$ (spinorial) → 2 +1 assignments for quarks

  - generators:

$$S^2 = R,\ T^3 = 1,\ (ST)^3 = 1,\ R^2 = 1$$

R=1: $1,\ 1',\ 1'',\ 3$

R= -1: $2,\ 2',\ 2''$

Group Theory of T´

- generators: in 3-dim representations, T-diagonal basis

$$S = \frac{1}{3}\begin{pmatrix} -1 & 2\omega & 2\omega^2 \\ 2\omega^2 & -1 & 2\omega \\ 2\omega & 2\omega^2 & -1 \end{pmatrix} \qquad T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega & 0 \\ 0 & 0 & \omega^2 \end{pmatrix}$$

- product rules:

$$1^0 \equiv 1, \; 1^1 \equiv 1', \; 1^{-1} \equiv 1''$$

$$1^a \otimes r^b = r^b \otimes 1^a = r^{a+b} \qquad \text{for } r = 1, 2 \qquad a, b = 0, \pm 1$$

$$1^a \otimes 3 = 3 \otimes 1^a = 3$$

$$2^a \otimes 2^b = 3 \oplus 1^{a+b}$$

$$2^a \otimes 3 = 3 \otimes 2^a = 2 \oplus 2' \oplus 2''$$

$$3 \otimes 3 = 3 \oplus 3 \oplus 1 \oplus 1' \oplus 1''$$

Group Theory of T´

- intrinsic complex CG coefficients in T′ (complexity independent of choice of basis for generators)
- spinorial x spinorial ⊃ vector:

J. Q. Chen & P. D. Fan, J. Math Phys 39, 5519 (1998)

$$2 \otimes 2 = 2' \otimes 2'' = 2'' \otimes 2' = 3 \oplus 1$$

$$3 = \begin{pmatrix} \left(\frac{1-i}{2}\right)(\alpha_1\beta_2 + \alpha_2\beta_1) \\ i\alpha_1\beta_1 \\ \alpha_2\beta_2 \end{pmatrix}$$

- spinorial x vector ⊃ spinorial:

$$2 \otimes 3 = 2 \oplus 2' \oplus 2''$$

$$2 = \begin{pmatrix} (1+i)\alpha_2\beta_2 + \alpha_1\beta_1 \\ (1-i)\alpha_1\beta_3 - \alpha_2\beta_1 \end{pmatrix}$$

A Novel Origin of CP Violation

M.-C.Chen, K.T. M
Phys. Lett. B681, 444 (2009)

- Conventionally:
  - explicit CP violation: complex Yukawa couplings
  - spontaneous CP violation: complex Higgs VEVs
- Complex CG coefficients in T′ ⇒ explicit CP violation
  - real Yukawa couplings, real Higgs VEVs
  - CP violation determined entirely by complex CG coefficients
  - no additional parameters needed ⇒ extremely predictive model!

Tri-bimaximal Neutrino Mixing

- fermion charge assignments:

$$\begin{pmatrix} \ell_1 \\ \ell_2 \\ \ell_3 \end{pmatrix}_L \sim 3, \quad e_R \sim 1, \quad \mu_R \sim 1'', \quad \tau_R \sim 1'$$

- SM Higgs ~ singlet under T′
- operators for neutrino masses: $\frac{HHLL}{M}\left(\frac{\langle\xi\rangle}{\Lambda}+\frac{\langle\eta\rangle}{\Lambda}\right)$
- two scalar (flavon) fields for neutrino sector: $\xi \sim 3, \quad \eta \sim 1$

$$T' \to G_{TST^2}: \quad \langle\xi\rangle = \xi_0\Lambda \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \qquad T' - \text{invariant:} \quad \langle\eta\rangle = u\Lambda$$

- product rules:

$$3 \otimes 3 = 3 \oplus 3 \oplus 1 \oplus 1' \oplus 1''$$

Tri-bimaximal Neutrino Mixing

- Neutrino Masses: triplet flavon contribution

$$3_S = \frac{1}{3}\begin{pmatrix} 2\alpha_1\beta_1 - \alpha_2\beta_3 - \alpha_3\beta_2 \\ 2\alpha_3\beta_3 - \alpha_1\beta_2 - \alpha_2\beta_1 \\ 2\alpha_2\beta_2 - \alpha_1\beta_3 - \alpha_3\beta_1 \end{pmatrix} \qquad 1 \;= \alpha_1\beta_1 + \alpha_2\beta_3 + \alpha_3\beta_2$$

- Neutrino Masses: singlet flavon contribution $\quad 1 \;= \alpha_1\beta_1 + \alpha_2\beta_3 + \alpha_3\beta_2$
- resulting mass matrix:

$$M_\nu = \frac{\lambda v^2}{M_x}\begin{pmatrix} 2\xi_0 + u & -\xi_0 & -\xi_0 \\ -\xi_0 & 2\xi_0 & u - \xi_0 \\ -\xi_0 & u - \xi_0 & 2\xi_0 \end{pmatrix} \qquad U_{\rm TBM} = \begin{pmatrix} \sqrt{2/3} & 1/\sqrt{3} & 0 \\ -\sqrt{1/6} & 1/\sqrt{3} & -1/\sqrt{2} \\ -\sqrt{1/6} & 1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix}$$

$$V_\nu^{\rm T} M_\nu V_\nu = {\rm diag}(u + 3\xi_0,\, u,\, -u + 3\xi_0)\frac{v_u^2}{M_x}$$

Form diagonalizable:
-- no adjustable parameters
-- neutrino mixing from CG coefficients!

Tri-bimaximal Neutrino Mixing

- charged lepton sector -- without quarks
  - operators for charged lepton masses

$$(\ell\phi)_1 e_R(1) + (\ell\phi)_{1'}\mu_R(1'') + (\ell\phi)_{1''}\tau_R(1')$$

  - scalar sector: flavon triplet for charged lepton masses

$$\begin{aligned} 1 &= \alpha_1\beta_1 + \alpha_2\beta_3 + \alpha_3\beta_2 \\ 1' &= \alpha_3\beta_3 + \alpha_1\beta_2 + \alpha_2\beta_1 \\ 1'' &= \alpha_2\beta_2 + \alpha_1\beta_3 + \alpha_3\beta_1 \end{aligned} \qquad T' \to G_T : \qquad \langle\phi\rangle = \phi_0\Lambda \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

- resulting charged lepton mass matrix = I
- leptonic mixing matrix = tri-bimaximal

$$V_{MNS} = V_{e,L}^{\dagger} V_\nu = \mathcal{I} \cdot U_{TBM} = U_{TBM}$$

- in our model: SU(5) GUT corrections from charged lepton sector

The Model

M.-C.Chen, K.T.M., under preparation;
Phys. Lett. B652, 34 (2007); Phys. Lett. B681, 444 (2009)

- Symmetry: SUSY SU(5) x T′
- Particle Content $10(Q,\, u^c,\, e^c)_L$ $\quad \overline{5}(d^c, \ell)_L$ $\quad \omega = e^{i\pi/6}.$

	T_3	T_a	$\overline{F}$	N	H_5	$H'_{\overline{5}}$	Δ_{45}	ϕ	ϕ'	ψ	ψ'	ζ	ζ'	ξ	η	η''	S
SU(5)	10	10	$\overline{5}$	1	5	$\overline{5}$	45	1	1	1	1	1	1	1	1	1	1
T'	1	2	3	3	1	1	$1'$	3	3	$2'$	2	$1''$	$1'$	3	1	$1''$	1
Z_{12}	ω^5	ω^2	ω^5	ω^7	ω^2	ω^2	ω^5	ω^3	ω^2	ω^6	ω^9	ω^9	ω^3	ω^{10}	ω^{10}	ω^{10}	ω^{10}
Z'_{12}	ω	ω^4	ω^8	ω^5	ω^{10}	ω^{10}	ω^3	ω^3	ω^6	ω^7	ω^8	ω^2	ω^{11}	1	1	1	ω^2

- additional $Z_{12} \times Z'_{12}$ symmetry:
  - predictive model: only 11 operators allowed up to at least dim-7
  - vacuum misalignment: neutrino sector vs charged fermion sector
  - mass hierarchy: lighter generation masses allowed only at higher dim
  - forbids Higgsino mediated proton decay

The Model

- Superpotential: only 11 operators allowed

$$\mathcal{W}_{\rm Yuk} = \mathcal{W}_{TT} + \mathcal{W}_{TF} + \mathcal{W}_{\nu}$$

$$\mathcal{W}_{TT} = y_t H_5 T_3 T_3 + \frac{1}{\Lambda^2} H_5 \left[y_{ts} T_3 T_a \psi \zeta + y_c T_a T_b \phi^2 \right] + \frac{1}{\Lambda^3} y_u H_5 T_a T_b \phi'^3$$

up type quarks

$$\mathcal{W}_{TF} = \frac{1}{\Lambda^2} y_b H'_{\overline{5}} \overline{F} T_3 \phi \zeta + \frac{1}{\Lambda^3} \left[y_s \Delta_{45} \overline{F} T_a \phi \psi \zeta' + y_d H_{\overline{5}'} \overline{F} T_a \phi^2 \psi' \right]$$

down type quarks & charged leptons

$$\mathcal{W}_{\nu} = \lambda_1 N N S + \frac{1}{\Lambda^3} \left[H_5 \overline{F} N \zeta \zeta' \left(\lambda_2 \xi + \lambda_3 \eta + \lambda_4 \eta'' \right) \right]$$

neutrino masses

Λ : scale above which T' is exact

Reality of Yukawa couplings: ensured by degrees of freedom in field redefinition

The Model

- Abelian subgroups of T´

G_T $\qquad T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega & 0 \\ 0 & 0 & \omega^2 \end{pmatrix}$ $\qquad \omega = e^{2\pi i/3}$

$G_S,\ G_{TST^2}$ $\qquad S = \frac{1}{3}\begin{pmatrix} -1 & 2\omega & 2\omega^2 \\ 2\omega^2 & -1 & 2\omega \\ 2\omega & 2\omega^2 & -1 \end{pmatrix}$ $\qquad TST^2 = \frac{1}{3}\begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix}$

- T´ Breaking
  - neutrino sector ⟶ exact tri-bimaximal mixing

$T' \to G_{TST^2}:$ $\qquad \langle \xi \rangle = \xi_0 \Lambda \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$

$T' - \textbf{invariant:}$ $\qquad \langle \eta \rangle = u\Lambda$ $\qquad \langle S \rangle = S_0$

$T' \to G_S:$ $\qquad \langle \eta'' \rangle = \eta''_0 \Lambda$

The Model

- charged fermion sector

$$T' \to G_{TST^2}: \qquad \langle \phi' \rangle = \phi'_0 \Lambda \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

$$T' \to G_T: \qquad \langle \phi \rangle = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \phi_0 \Lambda \, , \; \langle \psi \rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \psi_0 \Lambda$$

$$T' \to \textbf{nothing}: \qquad \langle \psi' \rangle = \psi'_0 \Lambda \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$T' \to G_S: \qquad \langle \zeta \rangle = \zeta_0 \Lambda, \quad \langle \zeta' \rangle = \zeta'_0 \Lambda$$

Neutrino Sector

- Operators:

$$\mathcal{W}_\nu = \lambda_1 NNS + \frac{1}{\Lambda^3}\left[H_5\overline{F}N\zeta\zeta'\left(\lambda_2\xi + \lambda_3\eta + \lambda_4\eta''\right)\right]$$

- symmetry breaking

$T' \to G_{TST^2}$: $\langle\xi\rangle = \begin{pmatrix}1\\1\\1\end{pmatrix}\xi_0\Lambda$ $\quad$ T' – invariant: $\langle\eta\rangle = \eta_0\Lambda$ $\quad$ $\langle S\rangle = S_0$

$T' \to G_S$: $\langle\eta''\rangle = \eta_0''\Lambda$

- resulting mass matrices

$$M_{RR} = \begin{pmatrix}1 & 0 & 0\\ 0 & 0 & 1\\ 0 & 1 & 0\end{pmatrix} s_0\Lambda$$

$$M_D = \begin{pmatrix}2\xi_0+\eta_0 & -\xi_0 & -\xi_0+\eta_0''\\ -\xi_0 & 2\xi_0+\eta_0'' & -\xi_0+\eta_0\\ -\xi_0+\eta_0'' & -\xi_0+\eta_0 & 2\xi_0\end{pmatrix}\zeta_0\zeta_0' v_u$$

$$M_\nu = -M_D M_{RR}^{-1} M_D^T$$

only vector representations
$\Rightarrow$ all CG are real

$\eta_0'' = 0$: M_ν diagonalized by TBM;

$\eta_0'' \neq 0$ $\Rightarrow$ deviation from TBM

[Note: m2 → (1,1,1) unchanged]

Up Quark Sector

- Operators: $$\mathcal{W}_{TT} = y_t H_5 T_3 T_3 + \frac{1}{\Lambda^2} H_5 \left[y_{ts} T_3 T_a \psi \zeta + y_c T_a T_b \phi^2 \right] + \frac{1}{\Lambda^3} y_u H_5 T_a T_b \phi'^3$$
- top mass: allowed by T′
  - lighter family acquire masses thru operators with higher dimensionality
  - dynamical origin of mass hierarchy
- symmetry breaking:

$$T' \to G_T \quad \langle \phi \rangle = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \phi_0 \Lambda \;, \; \langle \psi \rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \psi_0 \Lambda \qquad \boldsymbol{T' \to G_S :} \; \langle \zeta \rangle = \zeta_0 \Lambda$$

dim-6

$$T' \to G_{TST^2} : \qquad \langle \phi' \rangle = \phi'_0 \Lambda \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

dim-7

no contributions to elements involving 1st family; true to all levels

- Mass matrix:

$$M_u = \begin{pmatrix} i\phi_0'^3 & \frac{1-i}{2}\phi_0'^3 & 0 \\ \frac{1-i}{2}\phi_0'^3 & \phi_0'^3 + (1-\frac{i}{2})\phi_0^2 & y'\psi_0\zeta_0 \\ 0 & y'\psi_0\zeta_0 & 1 \end{pmatrix} y_t v_u$$

both vertor and spinorial reps involved
⇒ complex CG

Down Quark & Charged Lepton Sectors

- operators: $$\mathcal{W}_{TF} = \frac{1}{\Lambda^2} y_b H'_{\overline{5}} \overline{F} T_3 \phi \zeta + \frac{1}{\Lambda^3} \left[y_s \Delta_{45} \overline{F} T_a \phi \psi \zeta' + y_d H_{\overline{5}'} \overline{F} T_a \phi^2 \psi' \right]$$
- generation of b-quark mass: breaking of T′ : dynamical origin for hierarchy between m_b and m_t
- lighter family acquire masses thru operators with higher dimensionality
- symmetry breaking:

$$T' \to G_T : \quad \langle \phi \rangle = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \phi_0 \Lambda \, , \; \langle \psi \rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \psi_0 \Lambda$$

$$T' \to \text{nothing:} \quad \langle \psi' \rangle = \psi'_0 \Lambda \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$T' \to G_S : \quad \langle \zeta \rangle = \zeta_0 \Lambda, \quad \langle \zeta' \rangle = \zeta'_0 \Lambda$$

- mass matrix:

$$M_d = \begin{pmatrix} 0 & (1+i)\phi_0\psi'_0 & 0 \\ -(1-i)\phi_0\psi'_0 & \psi_0\zeta'_0 & 0 \\ \phi_0\psi'_0 & \phi_0\psi'_0 & \zeta_0 \end{pmatrix} y_b v_d \phi_0 \qquad M_e = \begin{pmatrix} 0 & -(1-i)\phi_0\psi'_0 & \phi_0\psi'_0 \\ (1+i)\phi_0\psi'_0 & -3\psi_0\zeta'_0 & \phi_0\psi'_0 \\ 0 & 0 & \zeta_0 \end{pmatrix} y_b v_d \phi_0$$

complex CG

- consider 2nd, 3rd families only: TBM exact
- Georgi-Jarlskog relations: $m_d \simeq 3 m_e \qquad m_\mu \simeq 3 m_s$ → corrections to TBM

Model Predictions

M.-C.Chen, K.T.M.,
Phys. Lett. B652, 34 (2007); Phys. Lett. B681, 444 (2009)

- Charged Fermion Sector (7 parameters)

$$M_u = \begin{pmatrix} ig & \frac{1-i}{2}g & 0 \\ \frac{1-i}{2}g & g+(1-\frac{i}{2})h & k \\ 0 & k & 1 \end{pmatrix} y_t v_u \qquad M_d,\ M_e^T = \begin{pmatrix} 0 & (1+i)b & 0 \\ -(1-i)b & (1,-3)c & 0 \\ b & b & 1 \end{pmatrix} y_b v_d \phi_0$$

$k \rightarrow V_{cb}$, $b \rightarrow V_{ub}$

$$\theta_c \simeq \left| \sqrt{m_d/m_s} - e^{i\alpha}\sqrt{m_u/m_c} \right| \sim \sqrt{m_d/m_s},$$

$\Rightarrow$

$$\theta_{12}^e \simeq \sqrt{\frac{m_e}{m_\mu}} \simeq \frac{1}{3}\sqrt{\frac{m_d}{m_s}} \sim \frac{1}{3}\theta_c$$

Georgi-Jarlskog relations $\Rightarrow V_{d,L} \neq I$
SU(5) $\Rightarrow M_d = (M_e)^T$
$\Rightarrow$ corrections to TBM related to θ_c

- model parameters:

7 parameters in charged fermion sector

$$\begin{aligned} b &\equiv \phi_0\psi_0'/\zeta_0 = 0.00304 \\ c &\equiv \psi_0\zeta_0'/\zeta_0 = -0.0172 \\ k &\equiv y'\psi_0\zeta_0 = -0.0266 \\ h &\equiv \phi_0^2 = 0.00426 \\ g &\equiv \phi_0'^3 = 1.45 \times 10^{-5} \end{aligned}$$

$$y_t/\sin\beta = 1.25$$

$$y_b\phi_0\zeta_0/\cos\beta \simeq 0.011$$

$$\tan\beta = 10$$

Numerical Results

- Experimentally: $m_u : m_c : m_t = \theta_c^{7.5} : \theta_c^{3.7} : 1 \qquad m_d : m_s : m_b = \theta_c^{4.6} : \theta_c^{2.7} : 1$
- CKM Matrix and Quark CPV measures:

$$|V_{CKM}| = \begin{pmatrix} 0.974 & 0.227 & 0.00412 \\ 0.227 & 0.973 & 0.0412 \\ 0.00718 & 0.0408 & 0.999 \end{pmatrix}$$

predicting: 9 masses, 3 mixing angles, 1 CP Phase; all agree with exp within 3σ

$$A = 0.798 \qquad \overline{\rho} = 0.299 \qquad \overline{\eta} = 0.306$$

$$\beta \equiv \arg\left(\frac{-V_{cd}V_{cb}^*}{V_{td}V_{tb}^*}\right) = 23.6^o, \ \sin 2\beta = 0.734 \ ,$$

$$\alpha \equiv \arg\left(\frac{-V_{td}V_{tb}^*}{V_{ud}V_{ub}^*}\right) = 110^o \ ,$$

$$\gamma \equiv \arg\left(\frac{-V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right) = \delta_q = 45.6^o \ ,$$

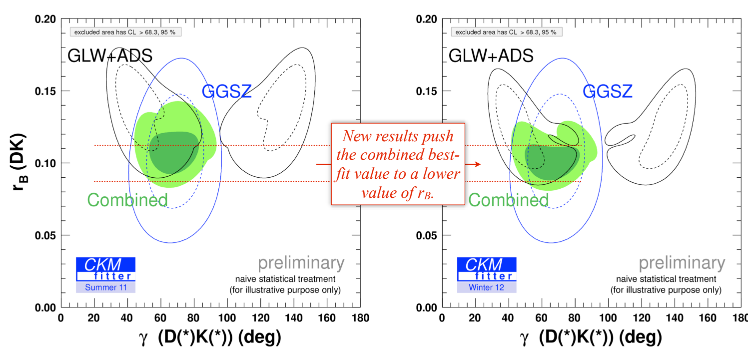
$$J \equiv \text{Im}(V_{ud}V_{cb}V_{ub}^*V_{cs}^*) = 2.69 \times 10^{-5} \ ,$$

CPV entirely from CG coefficients

Direct measurements @ 3σ (CKMFitter, ICHEP2012)

$$\sin 2\beta = 0.691^{+0.060}_{-0.047} \qquad \gamma\ (\text{degree}) = 66^{+36}_{-30} \qquad \alpha\ (\text{degree}) = 89^{+21}_{-13}$$

Recent LHCb result on gamma angle:



value for gamma going down!

Model Predictions

- Neutrino Sector (3 parameters)
- with $\eta_0'' = 0$

Georgi-Jarlskog relations ⇒ $V_{d,L} \neq I$

SU(5) ⇒ $M_d = (M_e)^T$

⇒ corrections to TBM related to θ_c

$$\theta_c \simeq \left|\sqrt{m_d/m_s} - e^{i\alpha}\sqrt{m_u/m_c}\right| \sim \sqrt{m_d/m_s},$$

⇒

$$\theta_{12}^e \simeq \sqrt{\frac{m_e}{m_\mu}} \simeq \frac{1}{3}\sqrt{\frac{m_d}{m_s}} \sim \frac{1}{3}\theta_c$$

$$U_{\text{MNS}} = V_{e,L}^\dagger U_{\text{TBM}} = \begin{pmatrix} 1 & -\theta_c/3 & * \\ \theta_c/3 & 1 & * \\ * & * & 1 \end{pmatrix} \begin{pmatrix} \sqrt{2/3} & 1/\sqrt{3} & 0 \\ -\sqrt{1/6} & 1/\sqrt{3} & -1/\sqrt{2} \\ -\sqrt{1/6} & 1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix}$$

$$\theta_{13} \simeq \theta_c/3\sqrt{2}$$

← CGs of SU(5) & T′

$$\tan^2\theta_\odot \simeq \tan^2\theta_{\odot,TBM} + \frac{1}{2}\theta_c\cos\delta$$

→ CG: leptonic Dirac CPV

neutrino mixing angle ($\tan^2\theta_\odot$) | 1/2 | quark mixing angle (θ_c)

$$\xi_0 = -0.0791\,, \quad \eta_0 = 0.1707\,, \quad S_0 = 10^{12}\text{ GeV}$$

$$|m_1| = 0.00134\text{ eV},\; |m_2| = 0.00882\text{ eV},\; |m_3| = 0.0504\text{ eV}$$

Model Predictions

M.-C. Chen, K.T. M., J. Huang, A. Wijangco, under preparation

- Neutrino Sector (3 parameters) $\eta_0'' \neq 0$
- with $\xi_0 = -0.051,\ \eta_0 = 0.23,\ \eta_0'' = -0.055$ $\quad S_0 = 10^{12}$ GeV

$$U_{\text{MNS}} = V_{e,L}^{\dagger}\, \mathrm{U_V} = \begin{pmatrix} 1 & -\theta_c/3 & * \\ \theta_c/3 & 1 & * \\ * & * & 1 \end{pmatrix} \begin{pmatrix} 0.808875 & -0.57735 & 0.111303 \\ -0.308046 & -0.57735 & -0.756158 \\ -0.500829 & -0.57735 & 0.644854 \end{pmatrix}$$

new contribution does not change the eigenvector corresponds to m2

- sum rules that exist in $\eta_0'' = 0$ case are modified

$$\sin\theta_{13}^{\text{MNS}} \simeq \frac{\theta_c}{3\sqrt{2}} + \theta_{13}^{\nu} + \kappa\frac{\theta_c}{3}$$

$$\tan^2\theta_{\odot} \simeq \frac{1}{2} + \left(\frac{1}{2} + \kappa'\right)\theta_c \cos\delta$$

θ_{13}^{ν} , κ , κ' : contributions from $\eta_0'' \neq 0$

κ : related to deviation of θ_{23} from $\pi/4$

Numerical Results: Neutrino Sector

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- Diagonalization matrix for charged leptons: $\begin{pmatrix} 0.997e^{i177^o} & 0.0823e^{i131^o} & 1.31\times10^{-5}e^{-i45^o} \\ 0.0823e^{i41.8^o} & 0.997e^{i176^o} & 0.000149e^{-i3.58^o} \\ 1.14\times10^{-6} & 0.000149 & 1 \end{pmatrix}$
- MNS Matrix

$$|U_{MNS}| = \begin{pmatrix} 0.824259 & 0.542816 & 0.161084 \\ 0.264063 & 0.609846 & 0.747234 \\ 0.500867 & 0.577441 & 0.644743 \end{pmatrix}$$

$$\begin{aligned} \sin^2\theta_{12} &= 0.30 \\ \sin^2\theta_{23} &= 0.43 \\ \sin^2\theta_{13} &= 0.026 \end{aligned}$$

- Neutrino Masses:

$$\begin{aligned} m_1 &= 0.0036 \text{ eV} \\ m_2 &= 0.0093 \text{ eV} \\ m_3 &= 0.051 \text{ eV} \end{aligned}$$

3 independent parameters in neutrino sector

predicted 3 masses and 3 angles:
all agree with exp within 1σ

- Leptonic CP violation from CG coefficients:

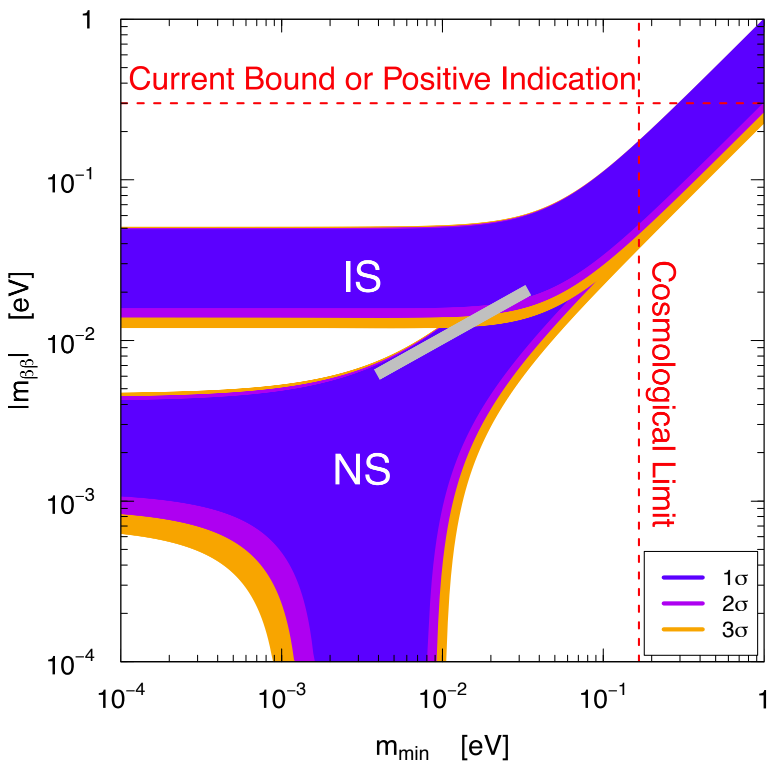
prediction for Dirac CP phase: δ = 197 degrees (in standard parametrization)

Two Majorana CPV measures:

$$S_1 \equiv \text{Im}\left\{U_{\text{MNS, e1}} U^*_{\text{MNS, e3}}\right\} = 0.034 \qquad S_2 \equiv \text{Im}\left\{U_{\text{MNS, e2}} U^*_{\text{MNS, e3}}\right\} = -0.029$$

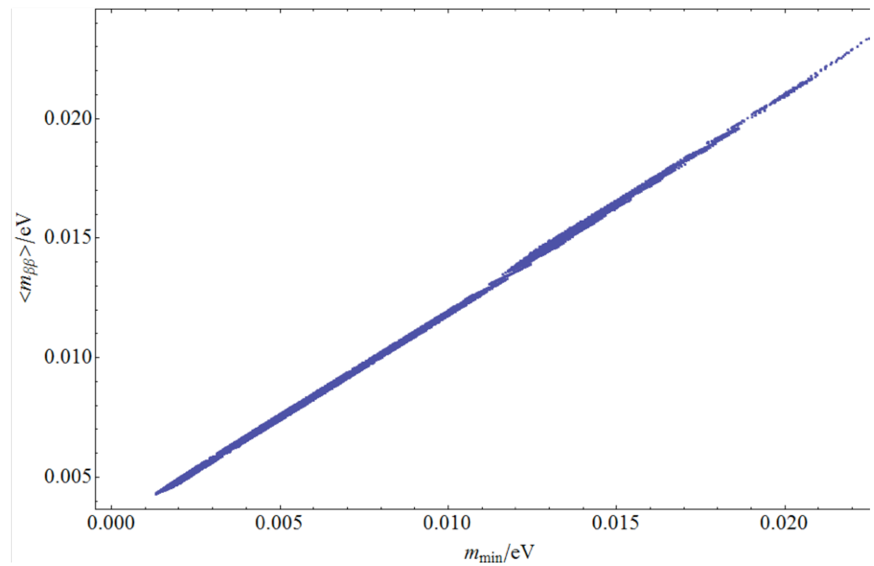
Neutrinoless Double Beta Decay

- neutrino-less double beta decay



[Plot taken from C. Giunti, LIONeutrino2012]

our model prediction



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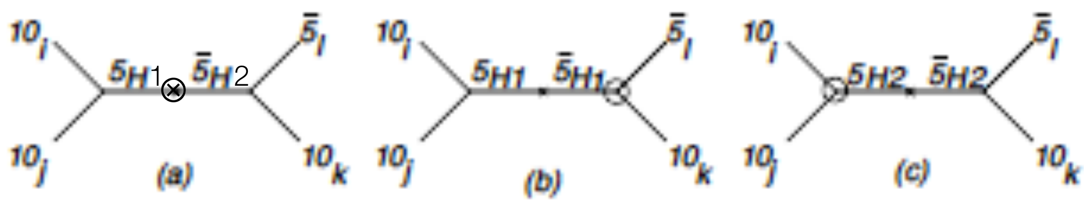
Proton Decay in SUSY SU(5) x T´ Model

- proton decay mediated by color triplet Higgsinos (dim-5 operators)
    - generally gives too fast decay rate
    - Z_{12} x Z_{12} forbid (vertices in circles)



- no Higgsino mediated proton decay
- Planck induced operators: Yukawa suppressed
- proton decay mediated by gauge boson (dim-6 operators)
    - non-minimal Higgs content, model prediction is within current experimental limits

Summary

- SUSY SU(5) x T′ : near tri-bimaximal lepton mixing & realistic CKM matrix
- complex CG coefficients in T′: origin of CPV both in quark and lepton sectors
- Z_{12} x Z_{12}': only 10 parameters in Yukawa sector
  - dynamical origin of mass hierarchy (including m_b vs m_t)
  - forbid Higgsino-mediated proton decay
- realistic theta13: generated by 1′′ flavon in neutrino sector

$$\sin\theta_{13}^{\mathrm{MNS}} \simeq \frac{\theta_c}{3\sqrt{2}} + \theta_{13}^{\nu} + \kappa\frac{\theta_c}{3}$$

- CP phases from CG:

quark CP phase: γ = 45.6 degrees

leptonic Dirac CP phase: δ = 197 degrees
(global fit: ~180 degrees)

Vacuum Alignment

- Z_{12} x Z_{12}' symmetry: too restrictive
    - resort to extra dimensions (5D)
    - in the bulk: Z_{12} x Z_{12}' symmetric
    - on the boundary branes: Z_{12} x Z_{12}' explicitly broken
- Neutrino sector:
    - invariants: $B_1^\nu = \xi^2, \;\; B_2^\nu = \eta^2, \;\; T_1^\nu = \xi^3, \;\; T_2^\nu = \xi^2\eta, \;\; T_3^\nu = \eta^3 \qquad B_3^\nu \; = \; S^2 \, ,$
    - superpotential: $T_4^\nu \; = \; S^3 \, , \quad T_5^\nu \; = \; \xi^2 S \, , \; T_6^\nu \; = \; \eta^2 S \, , \quad T_7^\nu \; = \; \eta S^2 \, , \quad T_8 = \eta''\xi^2$

$$\mathcal{W}_\nu^{flavon} = \sum_i m_i' B_i + \sum_j p_j' T^j$$

- supersymmetric minima:

$$F_{\xi_1} = F_{\xi_2} = F_{\xi_3} = 2(m_1' + p_5 s_0 + p_2 \eta_0 + p_8 \eta_0'') = 0$$

$$F_\eta \;\; = \;\; p_7 s_0^2 + 2m_2' \eta_0 + 2p_6 \eta_0 s_0 + 3p_3 \eta_0^2 + 3p_2 \xi_0^2 = 0$$

$$F_s \;\; = \;\; 3p_4 s_0^2 + 2p_7 \eta_0 s_0 + p_6 \eta_0^2 + 3p_5 \xi_0^2 = 0$$

$$\langle \xi \rangle = \xi_0 \Lambda \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \qquad \langle \eta'' \rangle = \eta_0'' \Lambda$$

$$\langle \eta \rangle = \eta_0 \Lambda \qquad \langle S \rangle = S_0$$

Vacuum Alignment

- charged fermion sector:
  - invariants

$$B_1 = \phi^2, \quad B_2 = \phi'^2, \quad B_3 = \phi\phi', \quad B_4 = \zeta N$$

$$T_1 = \phi^3, \ T_2 = \phi'^3, \ T_3 = \phi^2\phi', \ T_4 = \phi'^2\phi, \ T_5 = N^3, \ T_6 = \zeta^3, \ T_7 = \phi^2\zeta$$
$$T_8 = \phi'^2\zeta, \ T_9 = \phi\phi'\zeta, \ T_{10} = \phi^2 N, \ T_{11} = \phi'^2 N, \ T_{12} = \phi\phi' N, \ T_{13} = \psi'^2\phi$$
$$T_{14} = \psi'^2\phi', \ T_{15} = \psi^2\phi, \ T_{16} = \psi^2\phi', \ T_{17} = \psi\psi'\phi, \ T_{18} = \psi\psi'\phi', \ T_{19} = \psi\psi'\zeta$$

  - superpotential

$$\mathcal{W}_c^{flavon} = \sum_i m_i'' B_i + \sum_i \mu_j'' T^j$$

  - Supersymmetric minima: envision parameter space that satisfy minimization conditions (F=0)