Introduction & context

General fmwk: description of sp-time instantons in ST

* Effects of D-inst b.c. (prior to recognition of Dp-branes as solitons..)
"combinatorics of boundaries in ST" 9407031 Polchinski

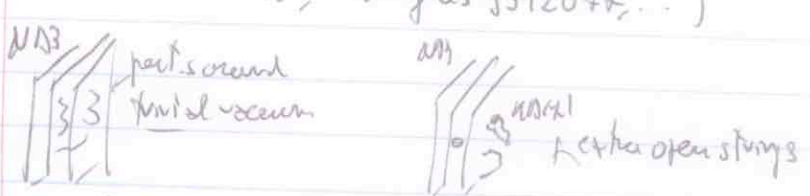
$\sim \frac{2\pi}{g_s}$ ($\to 2\pi i\tau$) , exp needed $1 + \ldots + \frac{1}{2} \ldots$
axio-dilaton

$\sim e^{2\pi i \tau}$ effect

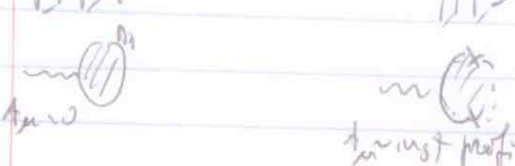
* (After D-branes) generalization to Ep-branes (p+1-like in sp-time, wrapped on int dirs) as possible sources of n.p. corrections
Early work, e.g., BBS 9507158, Witten 9604030, OJ Ganor 9612077
(rôle of (mixed) 0-modes)

* Rôle of D(-1)'s for SL(2,Z) of Type IIB eff action (R^4 terms) GG 9701093, ...
- related comp: D-instanton partition function (→ moduli integration, see later)
$\sim$ 0-Dim SYM integral → approach based on what's now known as "localization" by MNS 9803265; math techniques we will also use
- work on rôle of inst branes for other string dualities, in particular het/type I dualities (see later)

* D-instantons as gauge instantons. D-1/D3 (in general Dp-4/Dp) correspond to inst. sectors of D3 gauge theory, reproduce ADHM constr. of moduli space
(Witten 9510135, Douglas 9512077, ...)



$S_{D3} \supset \int C_0 \, \underbrace{\mathrm{tr} F \wedge F}_{\text{inst } \#}$
$\sim k C_0$ as $k D(-1)$



(Billo et al 0211250)

- D(-1)/D(-1) and D(-1)/D3 zero momentum ("moduli") ⟷ (pseudo-) moduli of susy ADHM construction

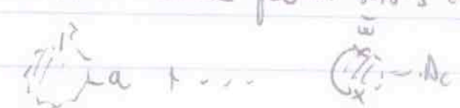
e.g. D(-1)/D(-1) NS a^μ "posits" along D3 $\xrightarrow{\text{after}}$ x^μ: centre (goldstone boson of broken transl inv)

R $M_{\alpha A}$ superpartner ⟶ θ^α

...

D3/D(-1) NS w_α ~ size [of the ADHM constraints..]

- the moduli action comes from disks (partly) attached to the D(-1)

$S_{mod} =$ 

[N.B. contains also Lag mult × (ADHM constraints)]
(aux fields)

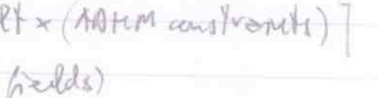
• Cvetic - Richter - Weigand 0609191

* Rules to get the n.p. contribs to eff action for gauge fields (from E/D inst)
[Among others GG 0002011, Billo et al 2002, [in N=2 context] • Blumenhagen - Cvetic - Weigand
Ibanez Uranga 0609213, Florea Kachru McGreevy Saulina 0610003
Akerblom Blumenhagen Lüst Plauschinn MSS 0612132, Blumenhagen MSS 0803.1562

Roughly: chiral multiplet

$$S_{eff}(\Phi) \sim \sum_k \int d^4x\, d^2\theta\, e^{2\pi i \tau k} \underbrace{\int d\hat{M}_k\, e^{-S_{mod}(M_k,\, \Phi(x,\theta))}}_{Z_k}$$

centred moduli — incorporates int.s of moduli with gauge/matter fields

(more later) ~  ~ $\mathrm{Tr}(\Phi(x,\theta)\mu\bar{\mu}) \subset S_{mod}$

* Integration over moduli (crucial technical point)

- fermionic moduli must be saturated (absorb all possible int.s)

- Very complicated matrix integrals. Occur in instanton calculus in SYM theories also from purely field-theoretic perspective

- much work on this (english group Dorey Hollowood... results up to $k=2$)
- Breakthrough by Nekrasov (2002) in $\mathcal{N}=2$ SYM instanton calculus

$$\mathcal{F}(\Phi) = \text{pert} + \sum_k c_k \left(\frac{\Lambda}{\Phi}\right)^{4k}$$

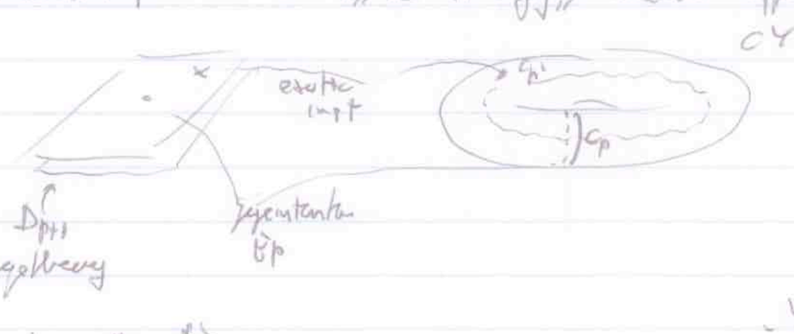
explicitly evaluated by int. over moduli space of $\# k$
using "localization" techniques [see later] (proposed by other groups e.g. Rome group...)

N.B. these techniques due to MNS
are mentioned for $O(2)$ part functions

- Much activity in the phenomenologically interesting context of $\mathcal{N}=1$ orientifold comps with D-branes, in particular considering n.p. cons to the superpotential [we heard some of it in last week's talks by Richter, (Rivas)...]

- effects in the closed sector (e.g. used for moduli stab. in KKLT, 0301240)
- and in the open sector, e.g. reproducing the ADS superpotential in SQCD [Akerblom et al, Argurio - Bert Fer-Lerda-Pet 0704.0202]

- Renewed interest: possible 'exotic' or "stringy" instanton effects



- stringy by construction
- appear with exp. factor not tied to gt scale: gauge theory gauge inst $e^{-\frac{V(C_p)}{g_s}} \sim e^{-\frac{1}{g^2_{YM}}}$; ex $e^{-\frac{V(C_p')}{g_s}}$

- may give rise to phenomen. important terms perturbatively forbidden in these string models: neutrino Majorana mass terms, μ-term, Yukawa couplings, susy-breaking terms

[Some ref.s: Blumenhagen-Cvetic-Weigand 0609191
Ibanez Uranga 0609213 (Ibanez U Hellerman 0704.1079)
Florea-Kachru McGreevy Saulina 0610003
Blum Cvetic Kast Richter Weigand 0707.1871
Argurio Bertolini Franco Kachru 0703236
Aharony Kachru Silverstein 0708.0493
Sen... Hollowell...: 0710.0780 (9th)]

* the work I'm giving not in this line, but phenom. implications ~~give~~ motivation to pursue the technical aspects of stringy instanton calculus
[other big motivation: rôle in dualities]

* ws characterization of exotic inst.-s
(for simplicity: toroidal comp., different b.c.'s on int cycles → twisted eqn.s)
D_{p+3}/E_p strings

$$\text{ws} \quad L_0 - \frac{1}{2} = N_x + N_\psi + \sum_{i>1}^{3} \frac{\theta_i}{2} + \frac{1}{2} - \frac{1}{2}$$

(int. cycles; sp. time (ND))

- gauge inst.s: $\theta_i = 0 \Rightarrow \exists$ 0-modes w_α (related to size)
- exotic inst.s: some $\theta_i \neq 0$ no such 0-modes

OR mode varies already present. Certain modes λ in gauge case ~ Lag. mult. for ferm ADHM constraints. In exotic case, abelian part of f ADHM constraints vanish ($\propto w_\alpha$)
⇒ ab part of λ dangerous (true ferm 0-modes): kills all contrib.s if not lifted!

Proposed mechanisms to lift them:
- orientifold projection [Argurio Bertolini Franco Kachru 0703.236
Argurio " Ferretti Petersson 0704.0262
...]

- closed string fluxes [Blum, Cvetic, Richter, Weigand 0708.0403
...]

- other mechanisms [Petersson, 0711.1837]

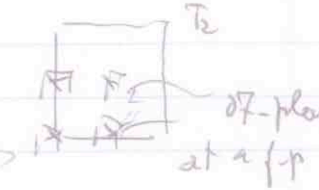
Billo, Ferro, Frau, Gallot, Lerda, Pesando

An 8d example of stringy inst calculus (0905.4586)

Concentrate on a simple example: D7/D(-1) in Type I' on T_2

Type I' ~ IIB on T_2 projected by $\Omega = \omega (-1)^{F_L} I_2$ (w.s. parity; inv. on T_2)

- (local) tadpole cancellation ⇒ 4 D7 at ∀ f.p



- Ω projection → in ∀ f.p the D7 support a SO(8) gauge theory

d.o.f. arranged into $\Phi(x,\theta) = \phi(x) + \theta^\alpha \lambda_\alpha + \theta \gamma^{\mu\nu} \theta F_{\mu\nu}$ (similar to $N=2$ $d=4$)

- In this 8d theory, we add D(-1)'s and study their effect (focusing on one f.p SO(8) gauge theory; on top of D7 → generalization of O(4) insts of $N=2$ cases)

$$\text{D7} \quad \overset{T_2}{\ldots} \qquad L_0^{ns} - \tfrac{1}{2} = N_x + N_\psi + \tfrac{1}{2} + \cancel{\tfrac{1}{2}} - \tfrac{1}{2} \Rightarrow \text{no } w_\alpha\text{-moduli}$$

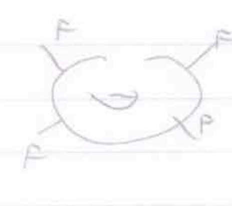
$\theta_1 = \frac{1}{2}$ $\theta_2 = \frac{1}{2}$ ⇒ exotic from the ws point of view!

- the "dangerous" 0-modes (as part of λ_i) projected out by Ω (see later)
⇒ D(-1) can contribute

- we'll see that they contribute (at all # k) to the quantization for the D7 gauge fields
⇒ technically interesting (exotic calculus at to all orders)

- Other (main) interest: duality to Het $SO(8)^4$ theory (obtained from Het SO(32) on T_2 with suitable Wilson lines)

F^4 amplitudes saturated at 1-loop exact?



- Computed by several groups (Lerche-Stieberger, Gutperle, Kiritsis-Obers-Pioline, Gava-Narain-Sarmadi)
(earlier results by Lerche-Nilsson-Schellekens)

$$\rightarrow \quad t_8 \frac{\mathrm{Tr} F^4}{4} \log \left| \frac{\eta(4T)}{\eta(2T)} \right| + t_8 \frac{(\mathrm{Tr} F^2)^2}{16} \log \left(\overset{\text{Kahler}}{\mathrm{Im}T} \, \overset{\text{cmplx str of } T_2}{\mathrm{Im}U} \, \frac{|\eta(2T)|^8 \, |\eta(U)|^4}{|\eta(4T)|^4} \right)$$

$$+ \, 2 t_8 \, \mathrm{Pf}(F) \log \left| \frac{\eta(T+1/2)}{\eta(T)} \right|$$

- Duality map: $T \leftrightarrow \tau$ (axiodilaton)
- Contains terms that map to the tree level + one-loop result in type I' (if correctly evaluated!) plus a series of corrections in $e^{2\pi i T \cdot k} \rightarrow e^{2\pi i \tau k} \equiv q^k$:

$$-\frac{1}{2} t_8 \mathrm{Tr} F^4 \left\{ \sum_{k=1}^{\infty} \left(d_k q^{4k} - d_k q^{2k} \right) + c.c. \right\} + \frac{1}{8} t_8 (\mathrm{Tr} F^2)^2 \left\{ \sum_k \left(d_k q^{4k} - 2 d_k q^{2k} \right) + c.c. \right\}$$

$$+ 8 \, t_8 \, \mathrm{Pf} F \left\{ \sum_{k=1}^{\infty} d_{2k-1} q^{2k-1} + c.c. \right\} \qquad \left(d_k = \sum_{\ell | k} \frac{1}{\ell} \right)$$

. Gutperle (1999) showed that the moduli space integral over $k=2$ D-instantons had the right structure, but this was before localis. techniques were commonly available

* From now on: sketch the comp. that allows to obtain the structure and the exact coeffs c_k by moduli space integration in type I' [we analyzed $k=5$ (I said later) but that's just a technical limitation]

* First step: identify the moduli and their transf. properties (determined by the Ω projection) under $SO(4)$ and $SO(8)$

$\uparrow$ $\eta(-T)$ c.p. $\qquad \uparrow$ $Q4$ c.p.

		moduli	SO(4)	× SO(8)	
−1/−1	NS	a_μ	$\boxplus$	1	positions in $\mathbb{R}^8$ (ab. part → x^μ)
		$\chi, \bar\chi$	$\boxminus$	1	" $\mathbb{R}^2$
		D_m	$\boxminus$	1	aux to decouple $[a^\mu,a^\nu][a_\mu,a_\nu]$ just 7 needed
	R	M_α	$\square\square$	1	(ab. part : θ_α)
		$\lambda_{\dot\alpha}$	$\boxminus$	1	(no ab. part)
−1/2	R	μ	$\square$	$\square$	
(NS		w	$\square$	$\square$)	aux (picture 0) no special role

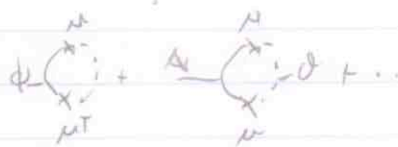
* Get the moduli action through the comp of disk diagrams

$$S(\mathcal{M},\Phi) = \mathrm{Tr}\Big\{ \lambda_{\dot\alpha}\, \gamma_\mu^{\dot\alpha\beta} [a^\mu, M_\beta] + \lambda_{\dot\alpha}[\chi, \lambda^{\dot\alpha}] + M^\alpha [\bar\chi, M_\alpha]$$

$$+ D_m D^m - D_m (\bar\tau^m)_{\mu\nu} [a^\mu, a^\nu] + [a_\mu, \bar\chi][a^\mu, \chi] + [\bar\chi, \chi]^2 \Big\}$$

$$+ \mathrm{Tr}\big\{ \mu \mu^T \chi + \mu^T\, \Phi(x,\theta)\, \mu + w^T w \big\}$$

x, θ appear only here



* Supersymmetric w.r.t. transf. determined by the action of the supercharge $Q^{\dot\alpha}$ preserved by D-1/D7 on the emission vertices of the moduli:

$$Q^{\dot\alpha} a^\mu = (\gamma^\mu)^{\dot\alpha\beta} M_\beta \;; \; \ldots$$

* Single out a given component (say $Q \equiv Q^{\dot 8}$), use it as a BRS-charge organizing the action as Q-exact

- Q seen as a scalar → SO(8) "Lorentz" symmetry reduced to SO(7) subgroup leaving this particular spinor invariant. The decomps. is such that

$8_v \to 8_s$ of SO(7)   index μ = spinor of SO(7)

$8_s \to 8_s$ $M_\alpha \to M_\mu = (M_m, -M_8)$

$8_c \to 7+1$ $\lambda_{\dot\alpha} \to (\lambda_m, \eta \equiv \lambda_8)$

$28 \to 21 + 7$ $f_{\mu\nu} \to f_{mn} (\bar\tau^{mn})_{\mu\nu} + h_m (\bar\tau^m)_{\mu\nu}$

. The BRS action is then written as

$$Q a^\mu = M^\mu \qquad Q M^\mu = [\chi, a^\mu]$$

$$Q \lambda_m = D_m \qquad Q D_m = [\chi, \lambda_m]$$

$$Q \bar{\chi} = \eta \qquad Q \eta = [\chi, \bar{\chi}]$$

$$Q \mu = w \qquad Q w = \chi_\mu + \mu \phi$$

$\phi = \langle \Phi(x,0) \rangle$ we restrict to constant vevs sufficient to do the comp. (see later) reinstate $\Phi(x,0)$ at the end

↳ equivariant BRS cohomology wrt $SO(K) \times SO(8)$:

$$Q^2 \cdot = T_{SO(K)}(\chi) \cdot + T_{SO(8)}(\phi) \cdot$$

↑ params of the transf.

. the moduli organize in BRS doublets:

	$SO(K)$	$SO(8)$	$SO(7)$
(a^μ, M^μ)	$\square\!\square$	1	8_s
(λ_m, D_m)	$\boxminus$	1	7
$(\bar{\chi}, \eta)$	$\boxminus$	1	1
(μ, w)	$\square$	$\square$	1
vector χ	$\boxminus$	1	1

N.B. $\chi, \bar{\chi}$ treated as indep. customary in all instanton calculus

[. Completes that $[d\hat{\mathcal{M}}_K] = L^{-4}$ $\forall K$

$\Rightarrow \int d\hat{\mathcal{M}}_K \, e^{-S[\mathcal{M}_K, \Phi(x,0)]}$ = quartic inv. in $\Phi(x,0)$ $\forall K$]

* The moduli action is BRS-~~inv~~ exact: $S_{mod} = Q \square$ (apart from deg. not matter here)

* This allows to reduce moduli integration to the evaluation of fluct. determinants around the fixed points of Q (practical way to proceed: see later)

- However, fixed points must be isolated. This requires to deform the action, making Q equivariant also wrt $SO(7)$
- This is the perfect analogue of ε-defs in Nekrasov's comp of $N=2$ $d=4$ instantons
- Just as in that case (Billo et al, ...) these def.s correspond to bkg field strengths from the RR sector. $\mathcal{F}_{\mu\nu} = F_{\mu\nu z}$, $\bar{\mathcal{F}}_{\mu\nu} = F_{\mu\nu \bar{z}}$
  $\uparrow T_2$

Moduli action modified by interactions such as 

- the $\underline{21}$ part of $\mathcal{F}_{\mu\nu}$ (fmn) $\to$ ~~accounted~~ defs accounted for by changing Q-action:

  e.g. $Q a^{\mu} = M^{\mu}$, $Q M^{\mu} = [\chi, a^{\mu}] + \bar{\mathcal{F}}_{(21)}^{\mu\nu} a_{\nu}$

  In general: $Q^2 \cdot = T_{SO(K)}(\chi)\cdot + T_{SO(8)}(\phi)\cdot + T_{SO(7)}(\mathcal{F})\cdot$

  $\uparrow$ equiv. wrt $SO(7)$ as well

- the 7 part of $\mathcal{F}_{\mu\nu}$ and $\bar{\mathcal{F}}_{\mu\nu}$ $\to$ def. the $\mathbb{C}$ ~~~~ fermion orig $\Rightarrow$ final result should not depend on them

* We are now ready to carry out the integration over the moduli

- Int. over $-7/7$ moduli is quadratic:

  $$\int d\mu \, dw \; e^{-\mathrm{tr}(w^{\dagger} w + \mu^{T} Q^2 \mu)}_{\substack{\uparrow \\ \chi\mu + \mu\phi}} = \mathrm{Pf}_{\mu}(Q^2)$$ $\leftarrow$ depends just on the transf. properties of the μ's

- it's suff. to take $\phi \in$ Cartan of $SO(8)$ $\quad \phi = (\phi_1, \phi_2, \phi_3, \phi_4, -\phi_1, -\phi_2, -\phi_3, -\phi_4)$
- bring χ $\in$ Cartan of $SO(K)$ ($\Rightarrow$ Vandermonde!!)

  to get
  $K = 2N$: $\prod_{u=1}^{4} \prod_{i=1}^{N} (\phi_u^2 - \chi_i^2)$

  $K = 2N+1$: $\mathrm{Pf}\,\Phi \cdot \prod \,''$

- The integrals over other moduli are not gaussian but we're free to rescale BRS doublets arbitrarily (bos/ferm Jacobians cancel) and also we can rescale $\bar{\mathcal{J}}_{\mu\nu}$ (since the gauge fermion only)

$\Rightarrow$ it is possible to find a rescaling limit (sending to ∞ some resc. parameters) in such a way that the leading action is quadratic in all moduli (exc. χ)

- The relevant part (the int. over $D_\chi, \lambda_\chi, \bar{\lambda}, \eta \to$ constant) becomes:

$$\int \underset{(\text{except } \chi)}{da_\mu\, dM_\mu\, d\lambda_{\hat{m}}\, dD_{\hat{m}}}\; e^{-\mathrm{tr}\{D_{\hat{m}} D^{\hat{m}} - \lambda_{\hat{m}} Q^2 \lambda^{\hat{m}} + a_\mu \bar{\mathcal{J}}^{\mu\nu} (Q^2 a)_\nu + M_\mu \bar{\mathcal{J}}^{\mu\nu} M_\nu\}}$$

cancels in the ratio of gaussian integrals

- Finally, the int. over the χ's (diagonalised $\to$ ~~Cartan~~ $\to$ Vandermonde) leads to the above:

$$Z_k \sim \int \{d\chi_i\}\; \Delta(\chi)\; \underbrace{\frac{\mathrm{Pf}_\lambda(Q^2)}{\det_D^{1/2}(Q^2)}}\; \mathrm{Pf}_\mu(Q^2)$$

all det. by symmetry prop. under $SU(k) \times SO(6) \times SO(4)$

i.e. by the weights of relevant reps once parameters are in the Cartan:

$\chi \to \chi_i \qquad \phi \to \phi_u, \qquad f_{mn} \to f_i \; (i=1,2,3)$

N.B. Acting on the a_μ: 4 4's spinorial, as diag $(\varepsilon_1, \varepsilon_2, \varepsilon_3, \varepsilon_1 - \varepsilon_2 \ldots)$ with $\sum \varepsilon_i = 0$

$\underline{k=1}$ (no χ) $\quad Z_1 = \dfrac{\mathrm{Pf}\, \Phi}{\varepsilon_1 \varepsilon_2 \varepsilon_3 \varepsilon_4}$ from integration over abelian comp. of $(a_\mu, M_\mu) \sim \underline{x^\mu, \theta^\alpha}$

- undeformed action: appear in the action only through $\Phi(\chi, a)$
$\to$ leave the integral $\int d^4x\, d^8\theta$ apart

- Deformed: they couple to def.s! $\to$ we do the integral, with the result $1/\varepsilon_1 \varepsilon_2 \varepsilon_3 \varepsilon_4$

$\Rightarrow$ in the result we have to replace $\quad \cdot \ \frac{1}{\varepsilon_1\varepsilon_2\varepsilon_3\varepsilon_4} \to \int d^4x\, d^8\theta$

$\cdot \ \phi \to \Phi(x,\theta)$

- $\underset{\sim}{K=2}$

$$Z_2 = \int d\chi \ \frac{f_1 f_2 f_3}{\varepsilon_1\varepsilon_2\varepsilon_3\varepsilon_4} \ \frac{\prod_{u=1}^{4} (\phi_u^2 - \chi_u^2)}{\prod_{A=1}^{4} (2\chi - \varepsilon_A)(2\chi + \varepsilon_A)}$$

- Integration prescription [MNS] : $\int d\chi \to$ contour integral, sing. avoided by small imparts for the ε_A
- seems arbitrary & problematic but works perfectly in all instances it's been applied (YM integrals in D: 10, 6, 4, ... ; various cases of instanton calcules ..)

- Thus one remaining with sum over residues. Results:

$$Z_2 = \frac{(\mathrm{Pf}\,\phi)^2}{4(\varepsilon_1\varepsilon_2\varepsilon_3\varepsilon_4)} + \frac{1}{\varepsilon_1\varepsilon_2\varepsilon_3\varepsilon_4} \ \frac{\mathrm{Tr}\,\phi^4 - \frac{1}{2}(\mathrm{tr}\,\phi^2)^2}{256} + \text{terms vanishing with defs} \to 0$$

higher divergence

- Similar structure (more and more divergent) for Z_k with higher k's

- Due to disconnected confs $\{k_i\}$, $\sum k_i = k$ of lower ist contributing to Z_k ~~beyond~~ on top of true instanton #k conf. $\Rightarrow$ must restrict to connected contributions

$$Z = \sum_k Z_k \, e^{2\pi i \tau k} \longrightarrow \text{take } \log Z$$

- Thus the actual formula for n.p. contrib.s is

$$S_{np}[\Phi] = \int d^4x\, d^8\theta \ \underbrace{\lim_{\varepsilon \to 0} \varepsilon_1\varepsilon_2\varepsilon_3\varepsilon_4 \log Z \Big|_{\phi \to \Phi(x,\theta)}}_{F_{np}[\Phi]}$$

Just complicated algebra: identify residues, Kummer-Ollen, take log ...
we were able to do it up to $k=5$ (just technical limitation). Result:

$$F_{n.p} = \frac{1}{2}\,\mathrm{Tr}\,\phi^4\left(\frac{1}{2}q^2 + \frac{1}{4}q^4 + \dots\right) - \left(\mathrm{Tr}\,\phi^2\right)^2\left(\frac{1}{4}q^2 + \frac{1}{4}q^4 + \dots\right)$$
$$+\,8\,\mathrm{Pf}\,\phi\left(q + \frac{4}{3}q^3 + \frac{6}{5}q^5 + \dots\right)$$

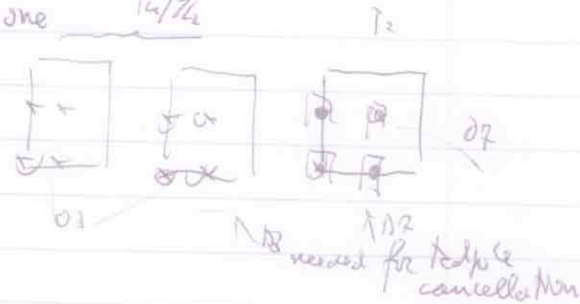
Agrees with the het result upon duality
(N.B. for the terms $\phi^4 \to t_8 F^4$)

- A 4d example 1002.4322 (MB, M. Frau, F. Fucito, A. Lerda, J. Morales, R. Poghossian)

shares certain characteristics of previous 8d model:
- exotic instantons at all orders k contribute to the gauge eff. action. Computed directly via localization techniques
- ∃ a computable het. dual, we computed it → full agreement

Confirms these exotic calculus techniques also apply to 4d examples

• Model = compactif. on $T_4/\mathbb{Z}_2$ of the 8d one



- Can also be seen as T-dual on T_2 of the GP model (9601038)

[r.3 het side + exp.: with D-instantons for GP considered by Camara-Dudas 08063102]

- Brane arrangement can be taken so that (at one O7 f.p.) we get a $U(4)$ conformal $\mathcal{N}=2$ S.YM
- D(-1)'s are exotic w.r.t. the D7, contribute at all orders to the quadratic gauge coupling (comps rather complex, we could go up to $k=3$ (triple integrals over the χ_i)

. the het dual is a U(10) het SO(n) on $T_4/\mathbb{Z}_2$ + Wilson lines on T_2 → $U(4)^4$

. Interesting point: the log Z shows divergences of different types

8d → quadratic prop. in previous case ⟺ untwisted heterotic

4d → quadratic prop ⟺ twisted het

$$F_{(4)} = \lim_{\varepsilon \to 0} \varepsilon_1 \varepsilon_2 \varepsilon_3 \varepsilon_4 \log Z$$

$$F_{(2)} = \lim_{\varepsilon \to 0} \left(\varepsilon_1 \varepsilon_2 \log Z - \frac{1}{\varepsilon_3 \varepsilon_4} F_{(4)} \right)$$

* Perspectives

- phenom. appl.s where such techniques might be needed

- relation to F-th. - ory bkgs

e.g. D7 in type I' → exactly the set of Sen "F-theory & orientifolds", 9605150

↓

noncompact D-instantons → F-theory curve with moduli τ

relation between quantum gauge potential and τ of F-theory?