

The physical properties of most theories of particle physics depend intimately on their vacuum structures, and string theory is no exception.

How is SUSY broken, if it's broken at all?

What other symmetries are manifest at low energies?

Do metastable vacua exist in addition to the ground state?

If so, what are their lifetimes?

What about vacuum energy?

The point is this: the vacuum structure of any given model plays a significant, and often dominant, role in its phenomenology.

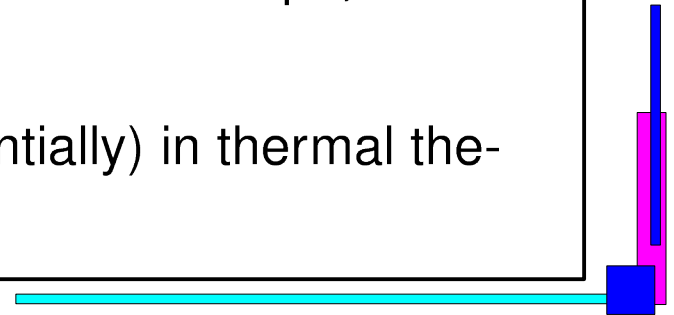


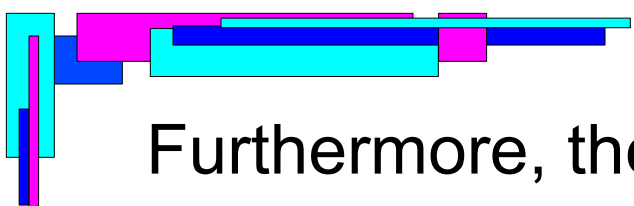


If fact, in studies of the string landscape, the stakes may be **even higher**:

As we shall demonstrate, many structures expected to occur generically on the string landscape give rise to vast numbers of **metastable minima**.

Such minima are important for a variety of reasons:

- They have markedly different phenomenologies than the true ground states hence, theories whose true ground states are not viable may actually be viable after all.
 - They can significantly affect vacuum counting in statistical studies of the string landscape, and even **dominate** the landscape, if the number of metastable vacua is large.
 - They may be populated (perhaps even preferentially) in thermal theories, in the early universe.
- 



Furthermore, they also have a host of phenomenological implications, including:

 New scenarios for **metastable SUSY-breaking** in SUSY theories.

- Metastable vacua arise at tree level, and all of the relevant dynamics is perturbative. Dienes, BT [arXiv:0806.3364]
- No cumbersome nonperturbative dynamics: lifetimes and transition rates can be calculated reliably.

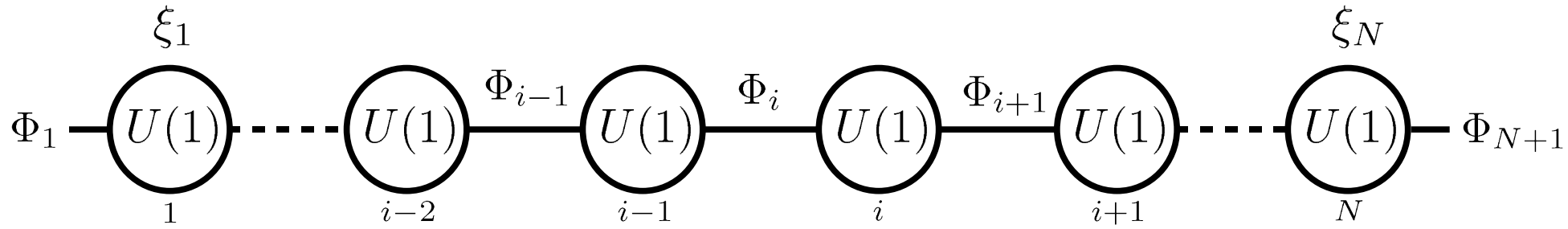
 Transitions between these minima that give rise to a highly nontrivial system of vacuum dynamics. Dienes, BT [arXiv:08113335]

- Possible implications for cosmology: (multiple, rapid phase transitions; topological defects, etc.).

 A host of **new particle states** which could give rise to an exciting Z' phenomenology at the LHC.



A Simple Abelian Scenario

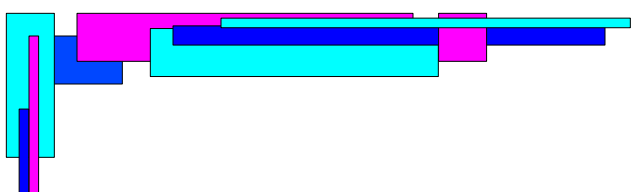


$$Q_{ai} = \begin{pmatrix} -1 & +1 & 0 & \dots & 0 & 0 \\ 0 & -1 & +1 & & 0 & 0 \\ 0 & 0 & -1 & & 0 & 0 \\ \vdots & & & \ddots & +1 & 0 \\ 0 & 0 & 0 & \dots & -1 & +1 \end{pmatrix}$$

- Consider an orbifold moose diagram with N $U(1)$ sites and $N + 1$ chiral superfields.
- For simplicity, we take all gauge couplings to be equal: $g_a = g$ for all a .
- Field charges can be written in terms of an $N \times N + 1$ matrix Q_{ai} .
- Nonzero **FI terms** $\xi_1 = \xi_N \equiv \xi$ are included for the endpoint gauge groups.
- **Wilson line operator** superpotential.

$$W = \lambda \prod_{i=1}^{N+1} \Phi_i$$

Non-renormalizable for $N > 2$



These structures are nothing special,
complicated, or esoteric.

Indeed, they are nothing but Abelian orbifold
moose constructions.

Structures of this sort arise generically in flux compactifications,
which reduce at low-energies to “deconstructed” supersymmetric
Abelian gauge theories.

Hence, these structures are a feature of the **real string
landscape**.



One More Ingredient: Kinetic Mixing

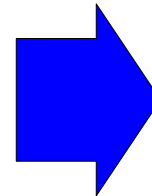
- The field-strength tensor for an Abelian gauge field is gauge invariant by itself.
- Our theory contains N Abelian gauge factors, and no symmetry prevents kinetic mixing terms from appearing in the field Lagrangian.

Supersymmetrized Kinetic Mixing

$$\mathcal{L} \ni \frac{1}{32} \int d^2\theta W_{a\alpha} X_{ab} W_b^\alpha$$

General

$$X_{ab} \equiv \begin{pmatrix} 1 & -\chi_{12} & -\chi_{13} & \dots & -\chi_{1N} \\ -\chi_{21} & 1 & -\chi_{23} & \dots & -\chi_{2N} \\ -\chi_{31} & -\chi_{32} & 1 & \dots & -\chi_{3N} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -\chi_{N1} & -\chi_{N2} & -\chi_{N3} & \dots & 1 \end{pmatrix}$$



Nearest Neighbors

$$\begin{pmatrix} 1 & -\chi & 0 & \dots & 0 & 0 \\ -\chi & 1 & -\chi & \dots & 0 & 0 \\ 0 & -\chi & 1 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & 1 & -\chi \\ 0 & 0 & 0 & \dots & -\chi & 1 \end{pmatrix}$$

- To simplify matters, let us restrict ourselves to the case where mixing occurs only between nearest-neighbor sites on the moose, with a common mixing angle χ .

In general, the kinetic mixing matrix can be diagonalized by a rotation combined with a diagonal rescaling:

$$[(M^{-1})^T]_{ab} X_{bc} M_{cd} = I_{N \times N}$$

with:

$$M_{ab} = S_{ac} O_{cb}$$

Scaling

Rotation

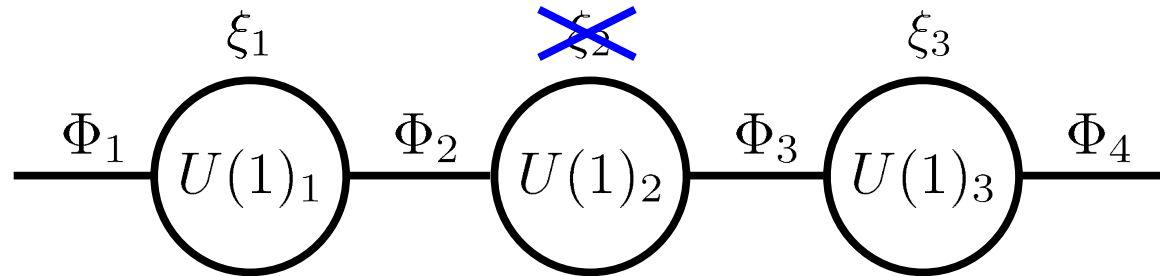
The gauge fields and D-terms in the new basis are:

$$\hat{\xi}_a = [(M^{-1})^T]_{ab} \xi_b \quad \hat{Q}_{ai} = [(M^{-1})^T]_{ab} Q_{bi} \quad \hat{A}_a^\mu = M_{ab} A_b^\mu$$

This is a **nontrivial modification** of the D-term potential, and it has **physical consequences**, as we shall soon see.

Now let's turn to some examples...

$N = 3:$



- Three $U(1)$ gauge groups, four chiral superfields

$$\mathcal{L} \ni \frac{1}{32\pi} \int d^2\theta (W_\alpha^1 W_\alpha^2 W_\alpha^3) \begin{pmatrix} 1 & -\chi & 0 \\ -\chi & 1 & -\chi \\ 0 & -\chi & 1 \end{pmatrix} \begin{pmatrix} W_\alpha^1 \\ W_\alpha^2 \\ W_\alpha^3 \end{pmatrix} \boxed{W = \lambda \Phi_1 \Phi_2 \Phi_3 \Phi_4}$$

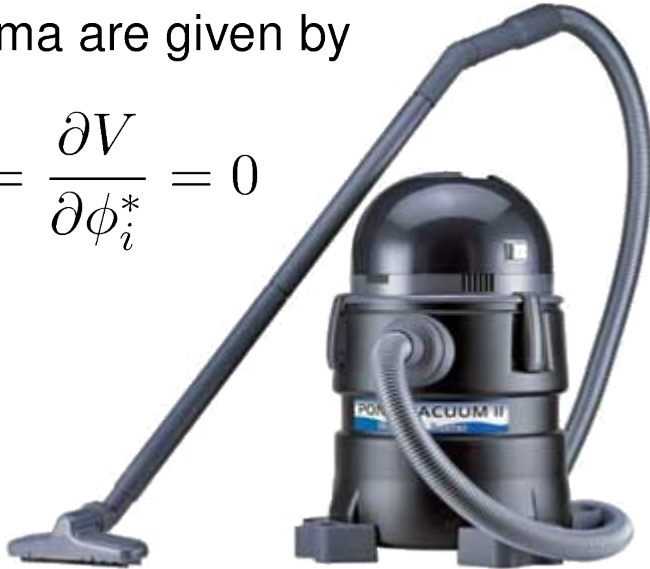
$\dim[\lambda] = -1$

- We will restrict ourselves to the case where $\xi_1 = \xi_3 \equiv \xi > 0$ for simplicity.
- The same qualitative behavior occurs over large regions of parameter space where $\xi_1 \neq \xi_3$.

Testing for Stability

- Extrema are given by

$$\frac{\partial V}{\partial \phi_i} = \frac{\partial V}{\partial \phi_i^*} = 0$$



Scalar Potential

$$V(\phi) = \frac{1}{2} \sum_{a=1}^N g_a^2 D_a^2 + \sum_{i=1}^n |F_i|^2$$

where

$$D_a = \xi_a + g \sum_{i=1}^n Q_{ai} |\phi_i|^2, \quad F_1 = \frac{\partial W}{\partial \phi_i}$$

- Stability of the extrema is governed by a $2(N+1) \times 2(N+1)$ mass matrix.

Diagonalize...

$$\mathcal{M}^2 = \begin{pmatrix} \frac{\partial^2 V}{\partial \phi_i \partial \phi_j} & \frac{\partial^2 V}{\partial \phi_i^* \partial \phi_j} \\ \frac{\partial^2 V}{\partial \phi_i \partial \phi_j^*} & \frac{\partial^2 V}{\partial \phi_i^* \partial \phi_j^*} \end{pmatrix} \rightarrow$$

One massless Nambu-Goldstone boson per broken $U(1)$, and a vacuum that's...

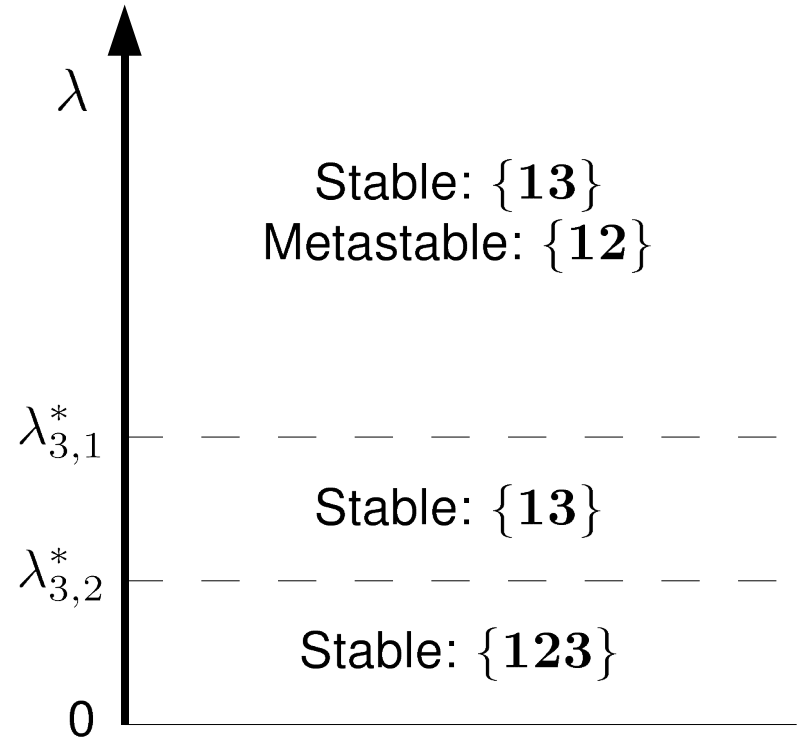
Stable if all other $m^2 > 0$.

Unstable if $\exists m^2 < 0$.

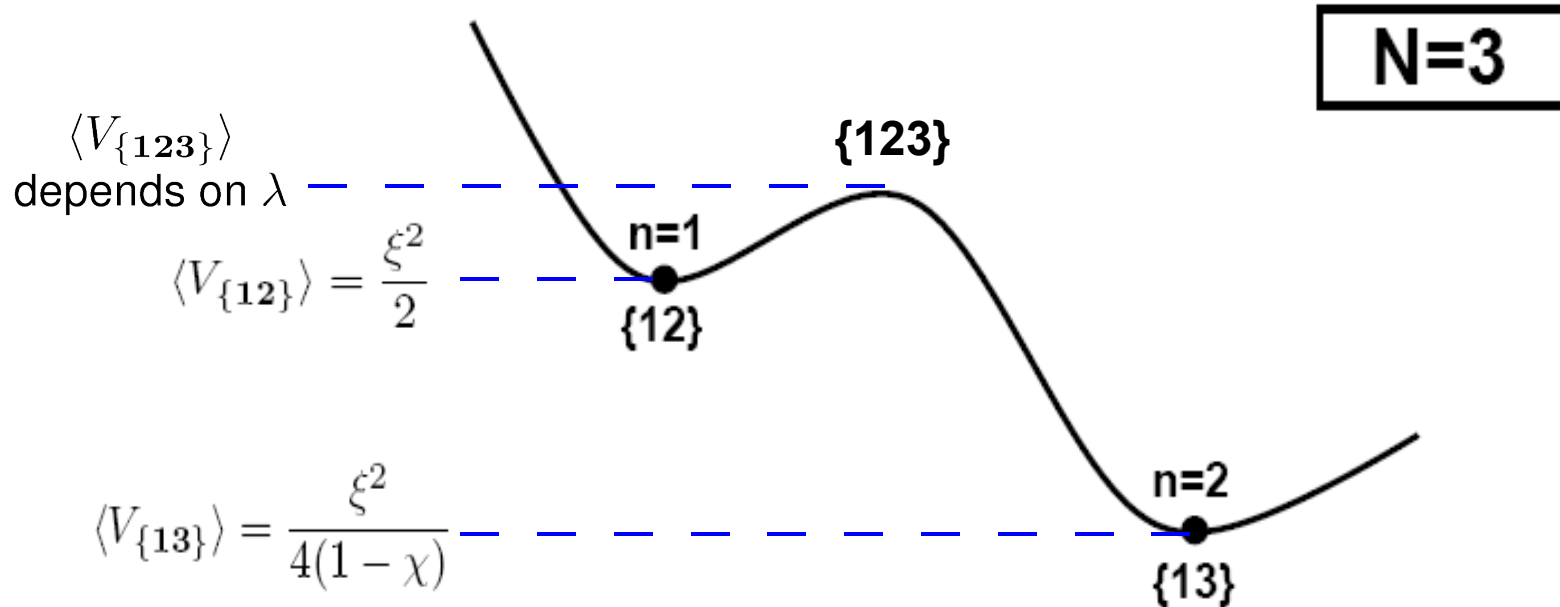
A **flat direction** for each additional $m^2 = 0$.

Vacuum Structure:

- For small λ , the ground state of the theory is the $\{123\}$ vacuum.
- Above some critical $\lambda_{3,2}^*$, the $\{13\}$ vacuum becomes the ground state.
- Further increasing λ beyond a second critical $\lambda_{3,1}^*$ results in the addition of a second, **metastable** minimum: $\{12\}$.

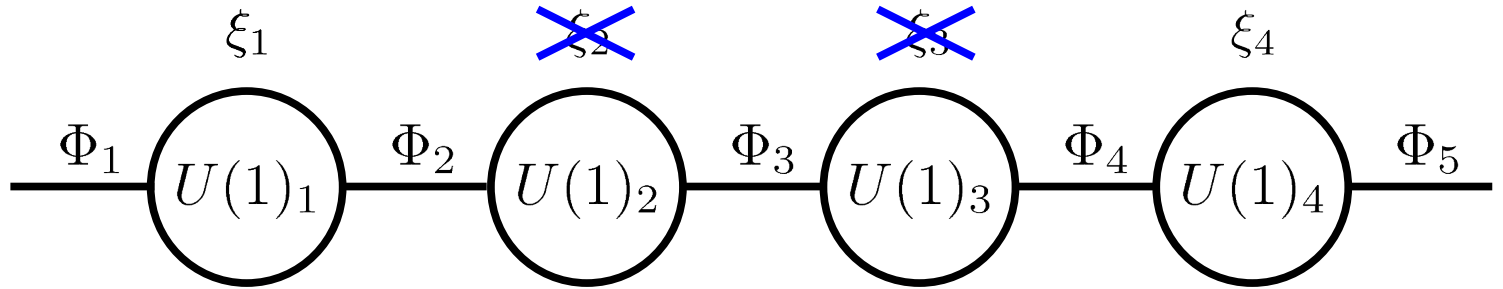


A Sketch of the Vacuum Structure:



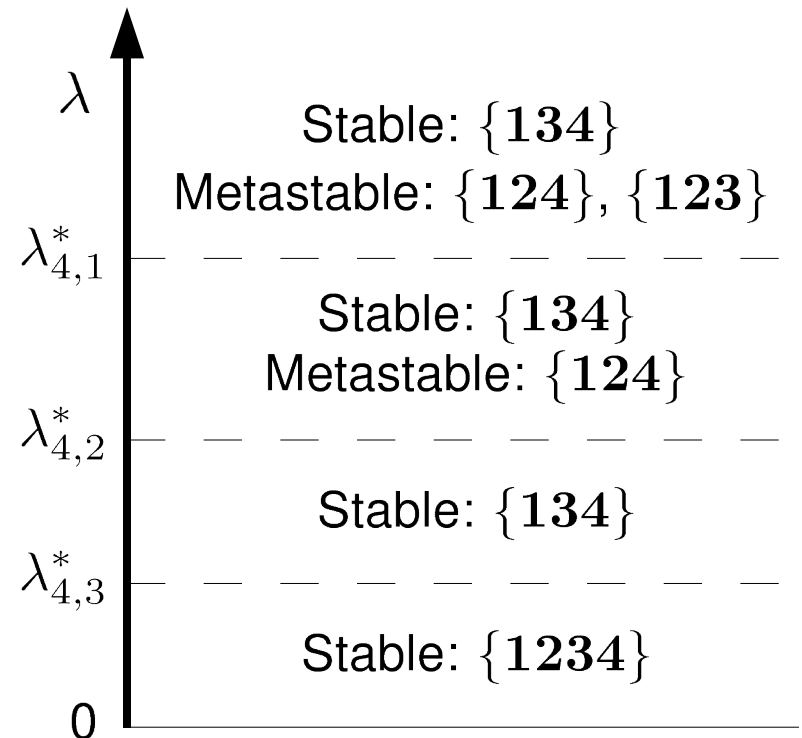
- The vacuum energies (and field configurations) of the vacua are λ -independent.
- The heights of the potential barriers separating these vacua (and hence stability) are controlled by λ .

$N = 4$

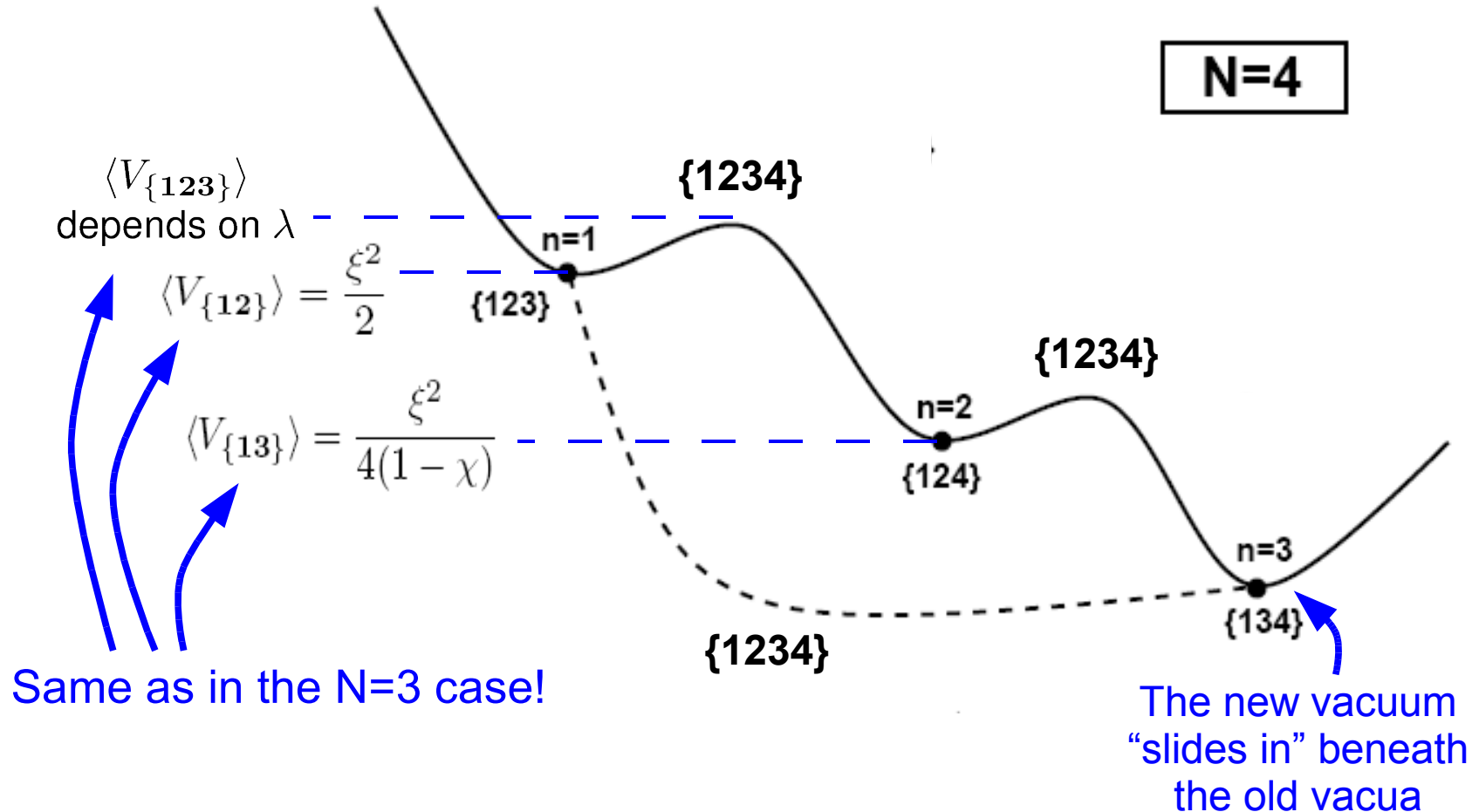


- Four $U(1)$ gauge groups
- Five chiral superfields
- $\dim[\lambda] = -2$
- As λ increases beyond a series of critical values, $\lambda_{N,n}^*$, new vacua become stabilized.

| | |
|---------|---------|
| $n = 1$ | : {123} |
| $n = 2$ | : {124} |
| $n = 3$ | : {134} |



- Here we see a tower of $N - 1$ vacua, with $(N - 1)$ choose 2 saddle point barriers separating them.



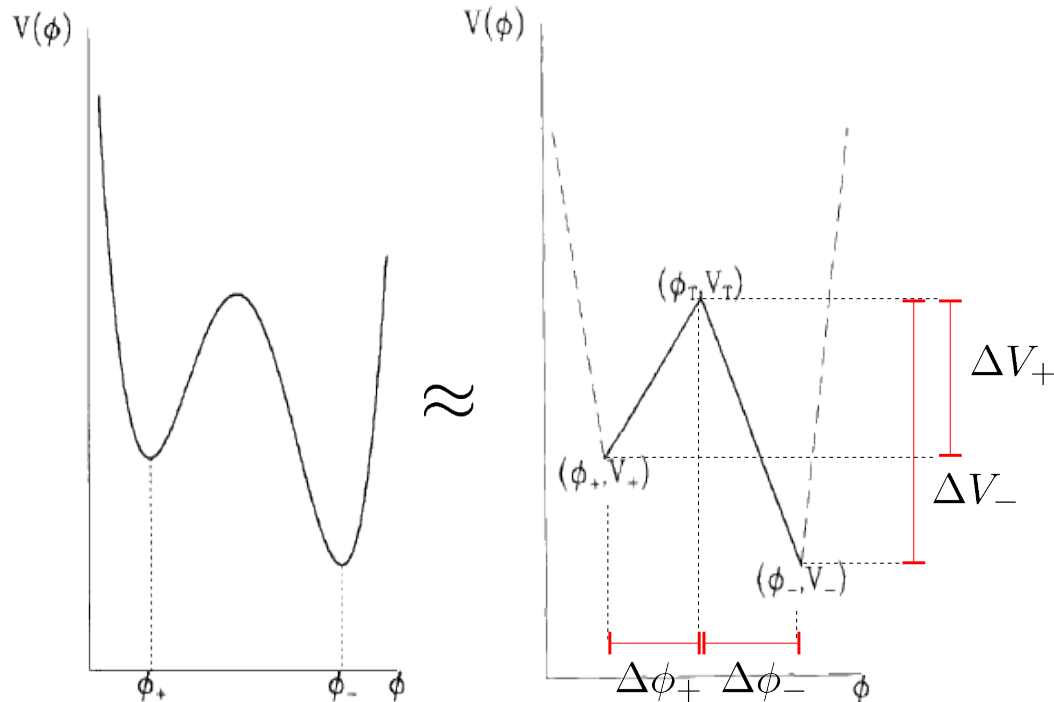
The Dynamics of Vacuum Transitions

- Our theory includes multiple metastable minima. So what are the lifetimes of these minima? To what other states do they decay?

$$\frac{\Gamma}{[\text{Vol.}]} = Ae^{-B}$$

$$B = S_E(\phi) - S_E(\phi_+)$$

↑
Bounce Action



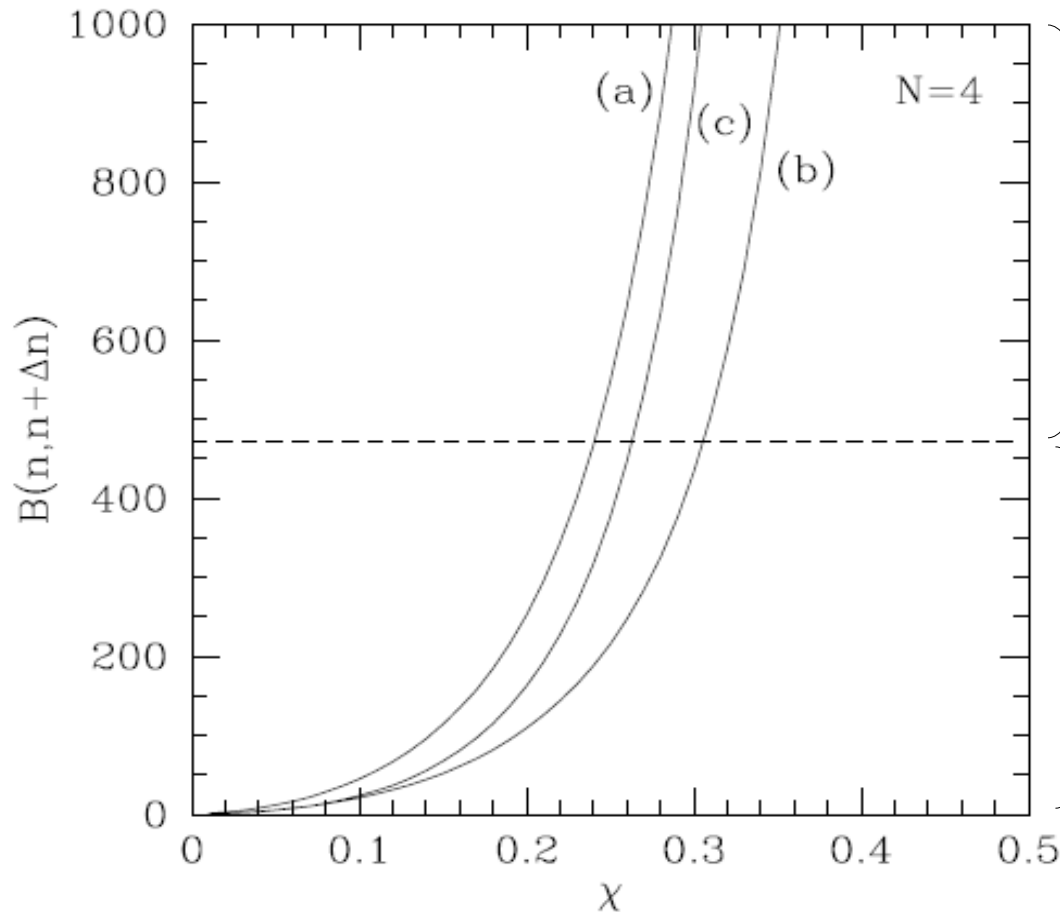
- The decay rate of a metastable vacuum can be calculated by standard, semiclassical instanton methods [Coleman & De Luccia; Hawking & Moss, et al.].
- Approximate the potential in the region between stable and metastable vacua as triangular [Duncan & Jensen].
- The bounce action can be determined explicitly in this case.

Decay sequence:

First, (b): $\{123\} \rightarrow \{134\}$ ($n_i = 1, n_f = 3$)

Then, (c): $\{124\} \rightarrow \{134\}$ ($n_i = 2, n_f = 3$)

Then, (a): $\{123\} \rightarrow \{124\}$ ($n_i = 1, n_f = 2$)



The Result: a “collapse” to the ground state.

- Multiple vacuum transitions can occur for $N = 4$, and it is necessary to ask which decay channel dominates for a given choice of parameters.
- The results shown here are for the large λ regime.

Observations:

- Lifetimes increase with χ .
- Decays to the “bottom” (ground state) generally favored.

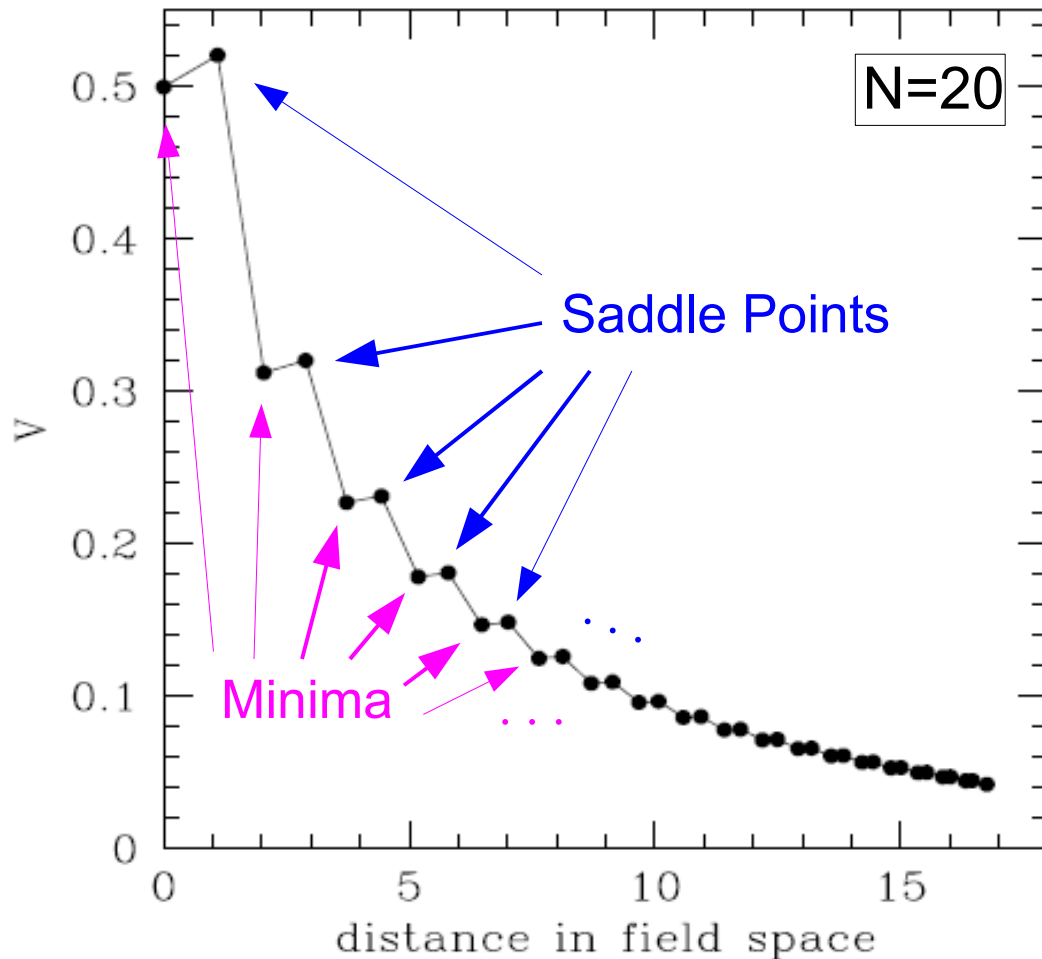
Minima on the moose

Now, we move to
the general case of
an N-site Moose.



And we find...

...a Tower of Metastable Vacua!



- An N -site Abelian moose gives rise to a **tower of $N - 1$ vacua**, each with a vacuum energy and field VEV configuration.
- For large N , many of these vacua have small vacuum energies, relative to the fundamental scale in the theory.
- When λ is not too small, each vacuum is separated from any other vacuum by a unique saddle point of type $\{1, 2, \dots, N - 1, N\}$.

- The vacuum energies and field VEV configurations of each vacuum in the tower are independent of λ , and the vacuum energies themselves are also independent of N .

VEV Configurations

$$v_j^2 = \xi \left(1 + \frac{1}{R_n(\chi)} \right) \quad j = 1$$

$$v_j^2 = \xi \left(\frac{1}{R_n(\chi)} \right) \quad j = 2, \dots, N - n$$

$$v_j^2 = 0 \quad j = N - n + 1$$

$$v_j^2 = \xi \left(\frac{R_{j-N+n-1}(\chi) - 1}{R_n(\chi)} \right) \quad j = N - n + 2, \dots, N$$

$$v_j^2 = 0 \quad j = N + 1$$

where $R_n(\chi) \equiv (1/\chi - 2)n + 2$

Vacuum Energies

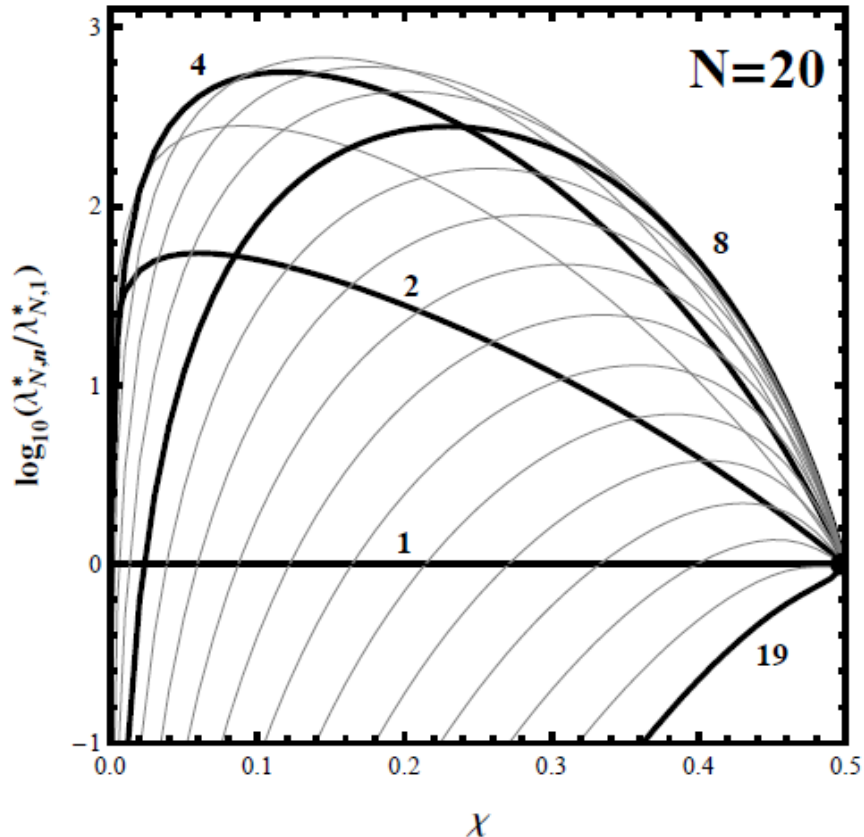
$$V_n = \frac{\xi^2}{2} \left(\frac{1}{\chi R_n(\chi)} \right)$$

Independent of λ

Really a spiral staircase



- As before, the stability of the vacua in the tower is primarily controlled by λ :
- There critical λ above which the $n = 1$ vacuum (the highest state in the tower for $0 < \chi < 1/2$) becomes stable is given by the general expression:



$$\tilde{\lambda}_{N,1}^{*2} = g^2 \frac{1}{1 + \chi} (\xi \chi)^{(2-N)}$$

- For an arbitrary value of n :

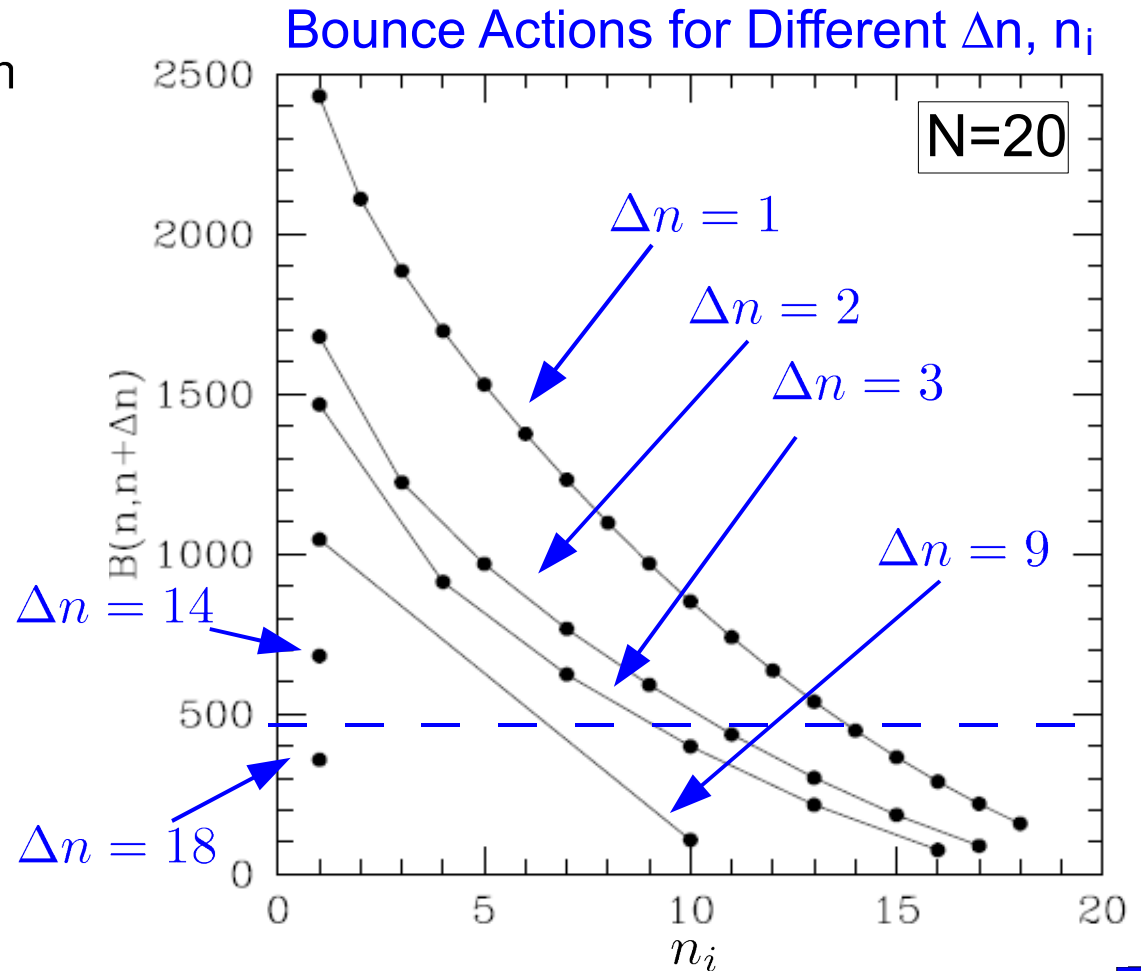
$$\lambda_{N,n}^{*2} = y^n \frac{\Gamma(y)}{\Gamma(n + y)} \frac{R_n^{N-2}}{\chi(1 + R_n)}$$

(where $y \equiv \chi/(1 - 2\chi)$)

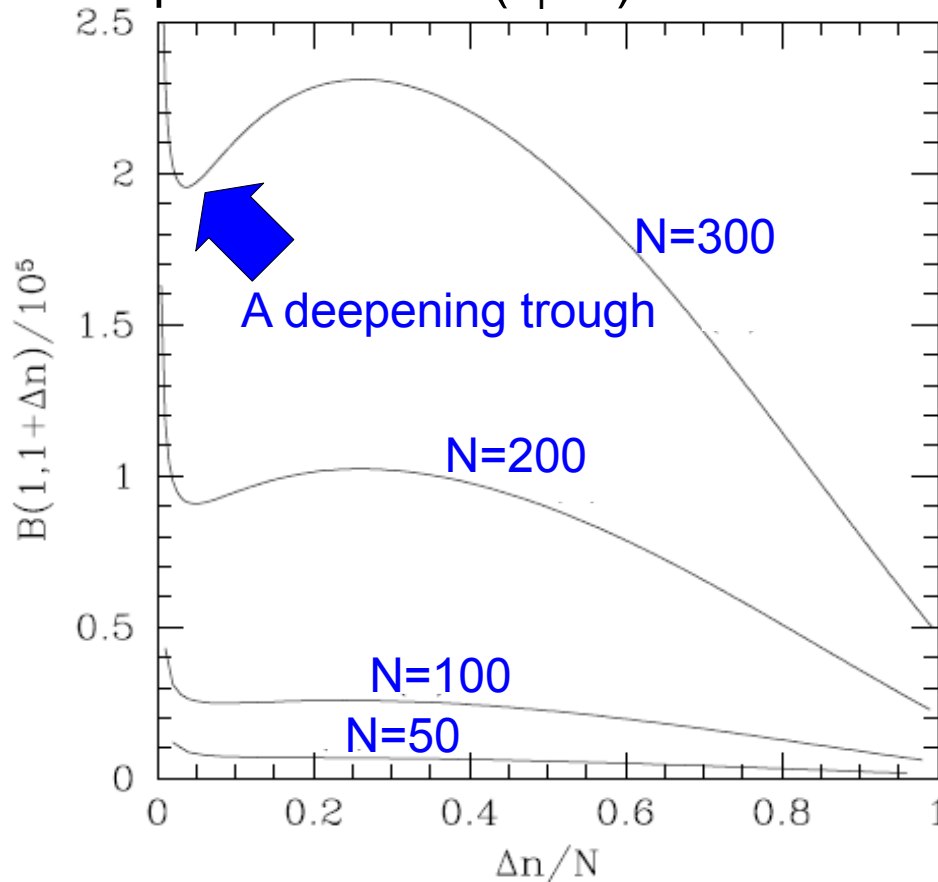
- All vacua can easily be stabilized while λ remains perturbative (note that $\lambda_{N,n}^{*2} \propto \xi^{1-N/2}$).

Lifetimes on the tower

- In general, transitions between vacua with larger separation in vacuum energy (or large $\Delta n \equiv n_f - n_i$) occur exponentially more quickly.
- The transition from any vacuum to the ground state is usually much faster than all other, competing decays.
- However, this is **NOT always the case**...



Bounce action for decays from the top of the tower ($n_i=1$) to different n_f .

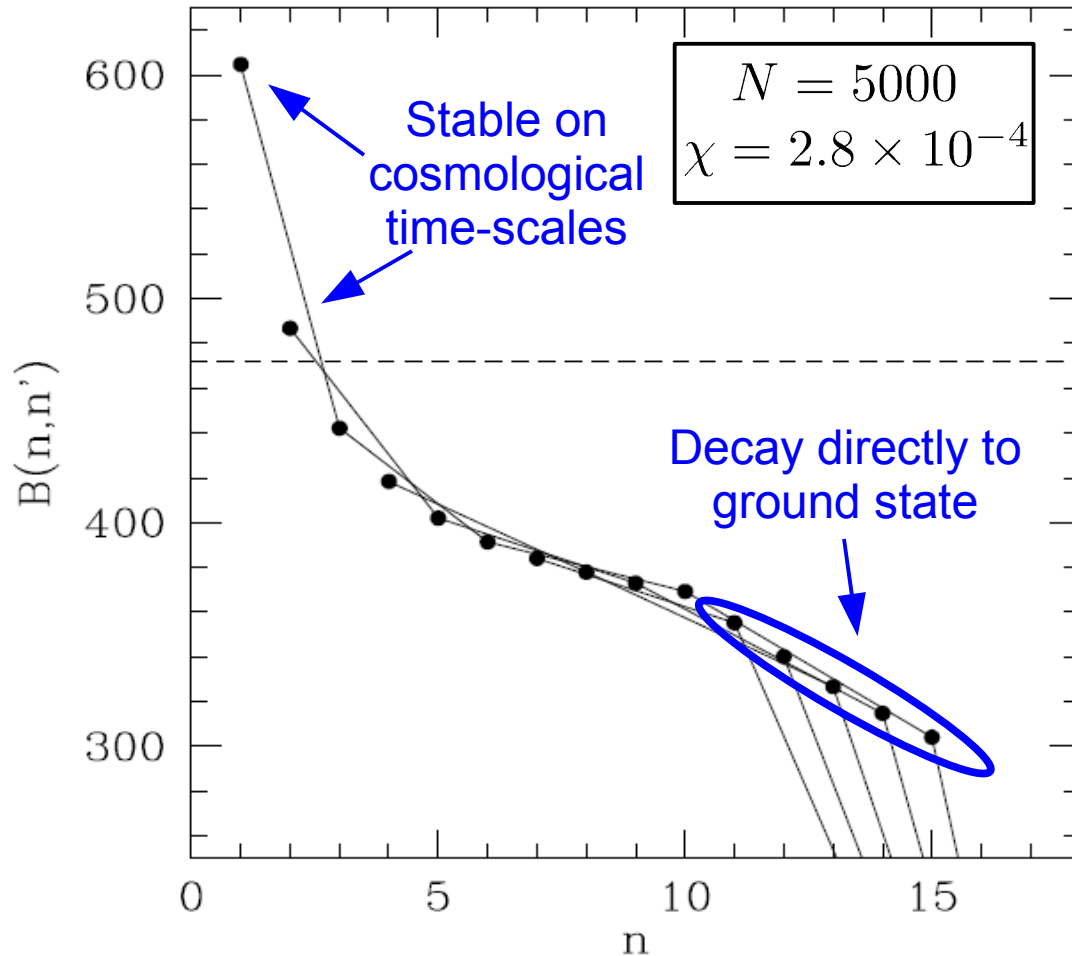


- Decays to the ground state don't always dominate for a given n_i .
- When N becomes large, other behavior is possible: decays to other regions in the tower can dominate over decays to the ground state.
- For the values of N shown, decays to the ground state still dominate, but other decay rates are exponentially increasing.

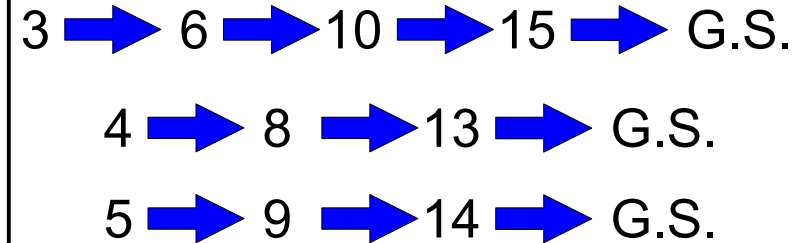
This means that for still larger N , we find...

... a Metastable Vacuum Cascade!

In this case, some vacua in the tower do not decay directly to the ground state.



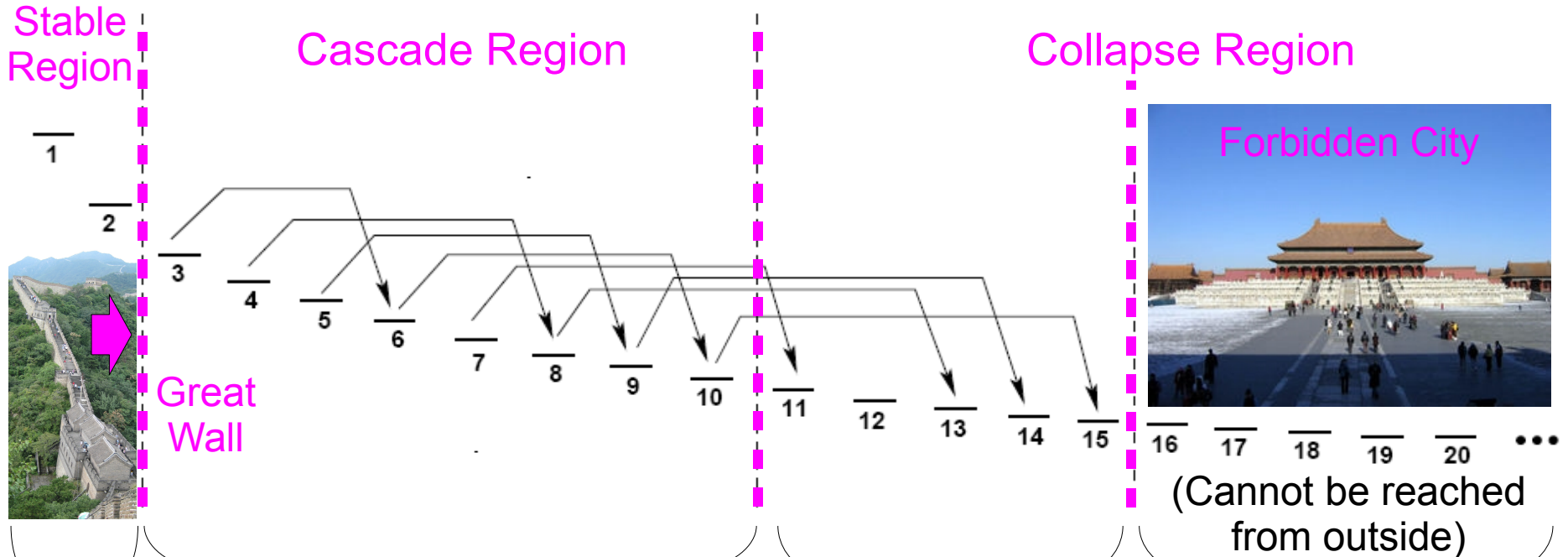
Vacuum decays can exhibit nontrivial cascade behavior:



(G.S. = Ground State)

We see that the tower of vacua can be divided into distinct regions which evolve very differently over time.

Cascades, Great Walls, and a “Forbidden City”



Vacua are stable on cosmological-time scales

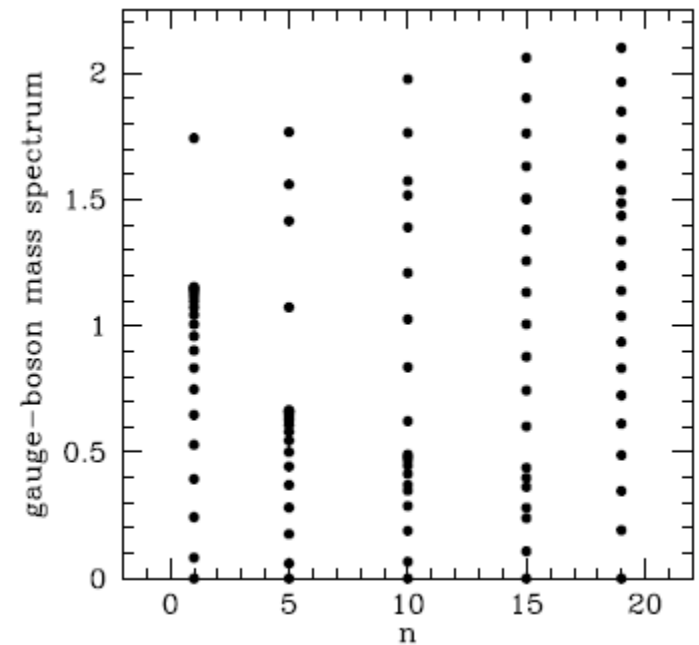
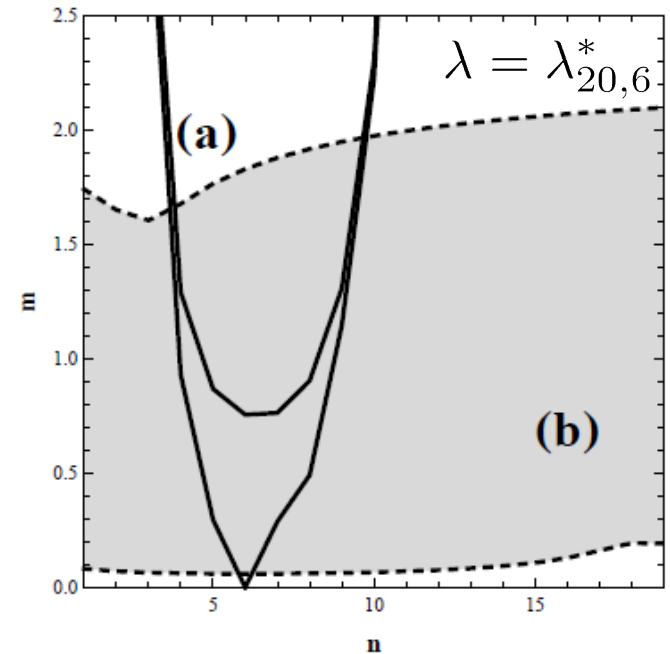
Vacua decay to other metastable vacua in the tower

Vacua decay directly to ground state

Evolution of Particle Spectra

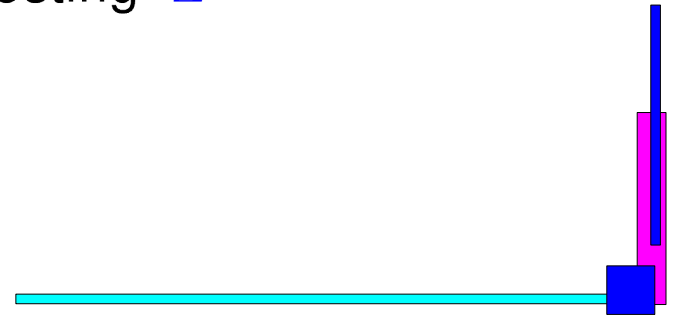
The spectrum of particles in each vacuum includes towers of scalars, fermions, and gauge bosons.

- Each vacuum includes one massless gauge field associated with an unbroken combination of $U(1)$'s.
- This massless gauge field couples only to the fields in the chiral supermultiplets Φ_{N-n+1} and Φ_{N+1} , which acquire masses $m^2 \propto \lambda \xi^{N-1}$.
- The fields in the rest of the Φ_i in W_a^α form “towers” of massive fermions, scalars, and gauge bosons with masses $m^2 \propto \xi$.





This is clearly a **highly nontrivial vacuum structure**, and one with a wealth of possible phenomenological applications:

- For large N , many states in the tower will have vacuum energies $V \ll \xi^2$. This could be used to **address the cosmological constant problem** if such states are stable on cosmological time-scales.
 - One could potentially address the hierarchy between the SUSY-breaking scale and M_P in weak-scale models in the same way.
 - Each vacuum generically contains a tower of massive, kinetically-mixed gauge bosons. This can lead to interesting **Z' phenomenology** at the LHC.
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Summary

- Supersymmetric theories with a large number of $U(1)$ gauge groups can give rise to highly nontrivial vacuum structures involving **large numbers of metastable minima**.
- Such structures arise generically in the full string landscape and therefore must be contended with.
- A highly nontrivial set of vacuum dynamics can arise, involving **cascades** of successive vacuum-transitions.
- A trove of phenomenological possibilities exist in scenario of this sort, including ways of addressing the cosmological constant problem, breaking SUSY, etc.

To wit, these potentials have a lot of potential!

Implications for the landscape

Most discussions of the landscape assume that the low-energy limit of a given string model has a relatively simple field-theory structure, consisting of:

- a single vacuum state (the ground state)
- a tower of excited states built on that vacuum.

As such, the resulting phenomenology associated with that string model is uniquely determined, and each string model corresponds to a unique possible state for the universe:

one string model \leftrightarrow one vacuum so... counting models \sim counting vacua

But if we really want to understand the landscape, all of this is now in question!

As we've seen, the phenomenological properties of a metastable vacuum may be **completely different** than those of true ground state.

The Upshot:

The one-to-one connection between models and vacua **need not apply!**
The full landscape of string theory can be even richer than previously imagined, since **all long-lived metastable vacua** must be included in the analysis.

- The structures I have presented here, which arise generically in flux compactifications, give rise to infinite towers of metastable vacua with higher and higher energies!
- As the number of vacua grows towards infinity, the energy of the highest vacuum remains fixed while the energy of the true ground state tends towards zero.

Thus, even if such models are relatively rare across the landscape, the fact that they give rise to infinitely many vacua means that they could **completely dominate** the properties of the landscape as a whole! But if we really want to understand the landscape, all of this is now in question!