Heat transport of quantum excitations: magnons, spinons, and Majorana fermions

Spintronics Meets Topology in Quantum Materials
KITP Workshop
November 12-15, 2019

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Leibniz
Leibniz Association

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Heat transport

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R. Hentrich	
M. Gillig	
B. Büchner	

Cuprate crystals

R. Saint-Martin, A. Revcolevschi	Université Paris-Sud

α-$RuCl_3$ crystals

D. Nowak, A. Isaeva, and T. Doert	TU Dresden
J. Sears, Y.-J. Kim	University of Toronto
P.J. Kelley, S. Nagler	Oak Ridge National Lab

Discussion & Theory

W. Brenig	TU Braunschweig
X. Zotos	University of Crete

€€€

NOVMAG
LOTHERM

Deutsche
Forschungsgemeinschaft
DFG

SFB 1143 Correlated Magnetism:
from frustration to topology

Heat transport: Experiment

Fourier's law: $\boldsymbol{j}_Q = -\kappa \nabla T$

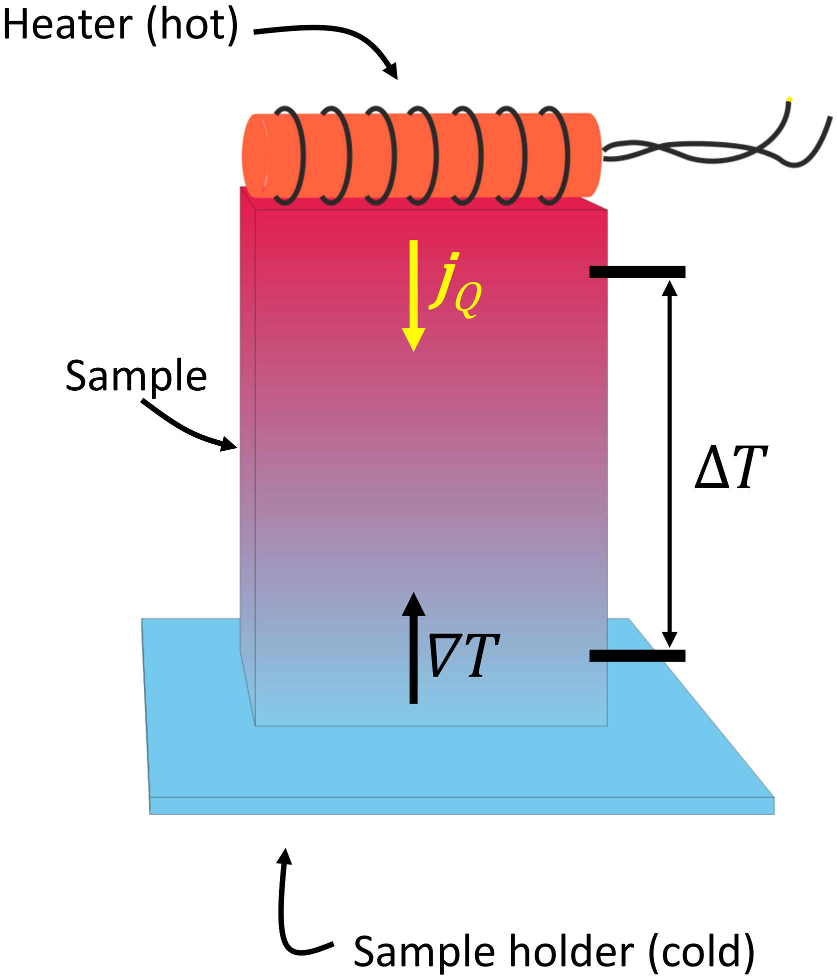
Estimate:

$$\kappa \sim c \cdot v \cdot l$$

c = specific heat

v = velocity

l = mean free path



Heater (hot)

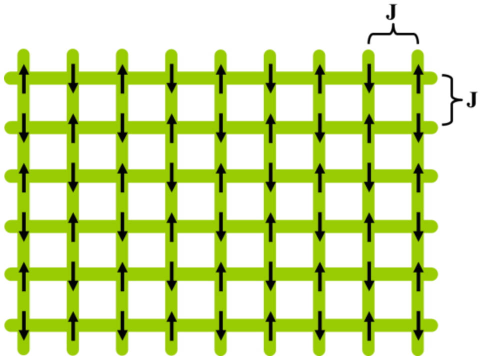
Sample

Sample holder (cold)

Outline

2D-AFM Heisenberg

Magnon heat transport



Material: La_2CuO_4

1D-AFM Heisenberg

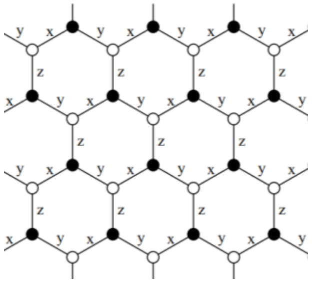
Spinon heat transport

Material: $SrCuO_2$

2D Kitaev

Longitudinal heat transport

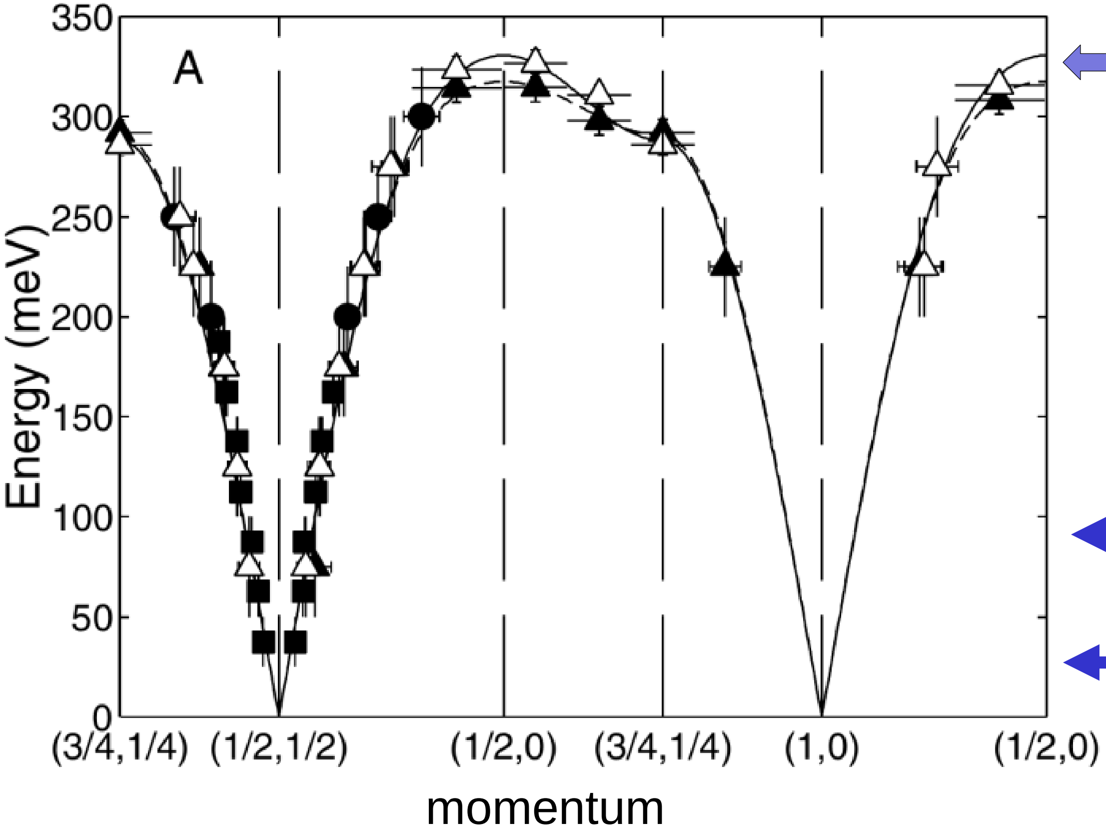
Thermal Hall effect



Material: α-$RuCl_3$

2D antiferromagnetic magnons in La_2CuO_4

s=1/2 Heisenberg AFM on square lattice



325 meV ~ 3800 K

Magnons are fast!

$v_0 \approx 1.3\cdot10^5$ m/s

optical phonons

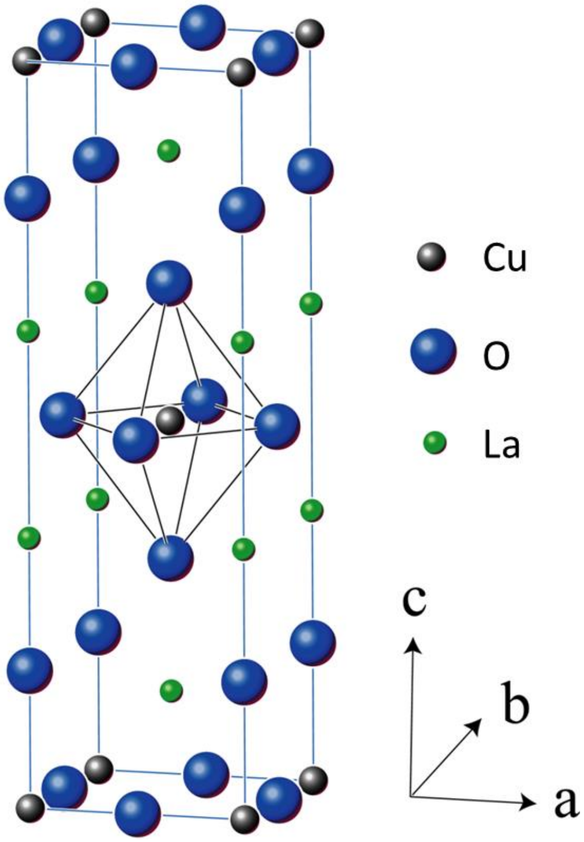
acoustical phonons

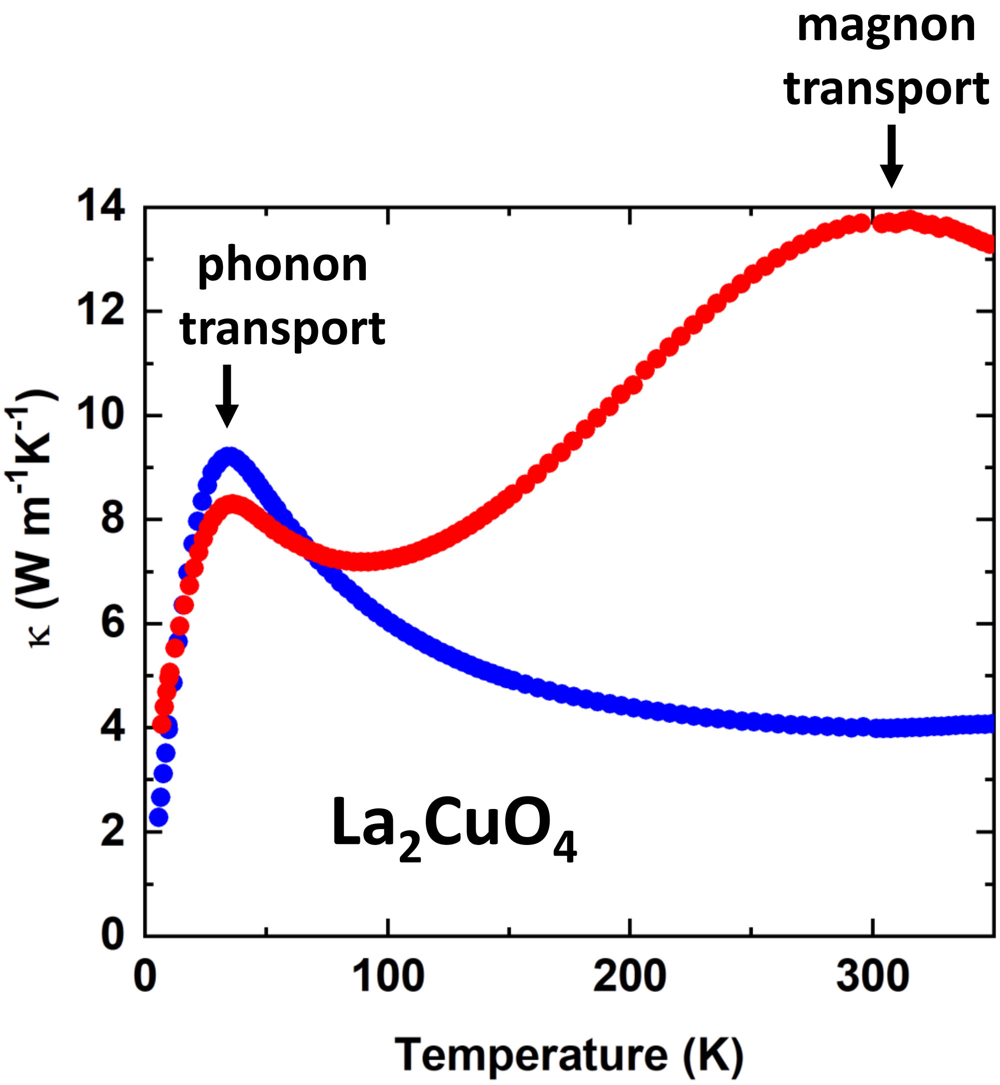
Coldea et al., Phys. Rev. Lett. 2001

Heat conductivity: $\kappa \sim c\, v_0\, l$

Magnon heat transport in La_2CuO_4: evidence

La_2CuO_4





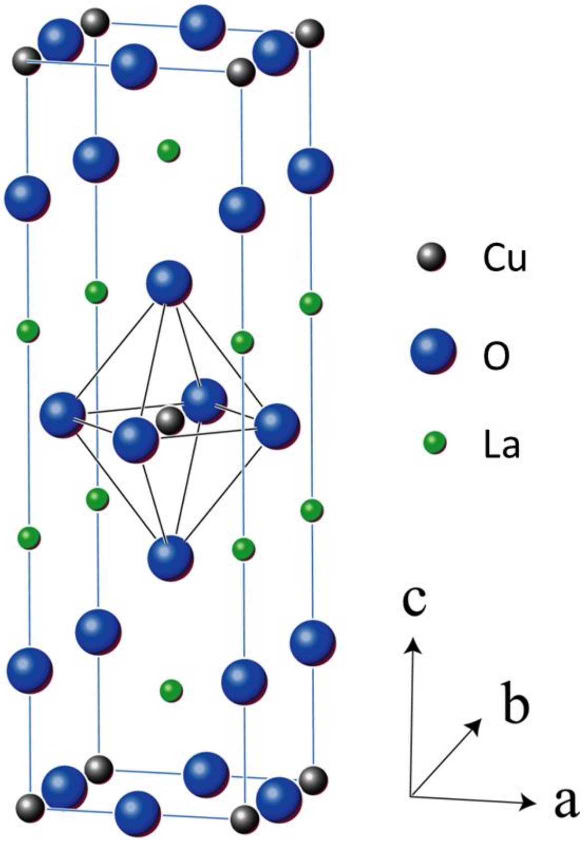
$J_{||} \approx 1500K$

$J_{\perp} \approx 10^{-5} J_{||}$

Hess et al., PRL 2003
Hess, Physics Reports 2019

Magnon heat transport in La_2CuO_4: evidence

La_2CuO_4



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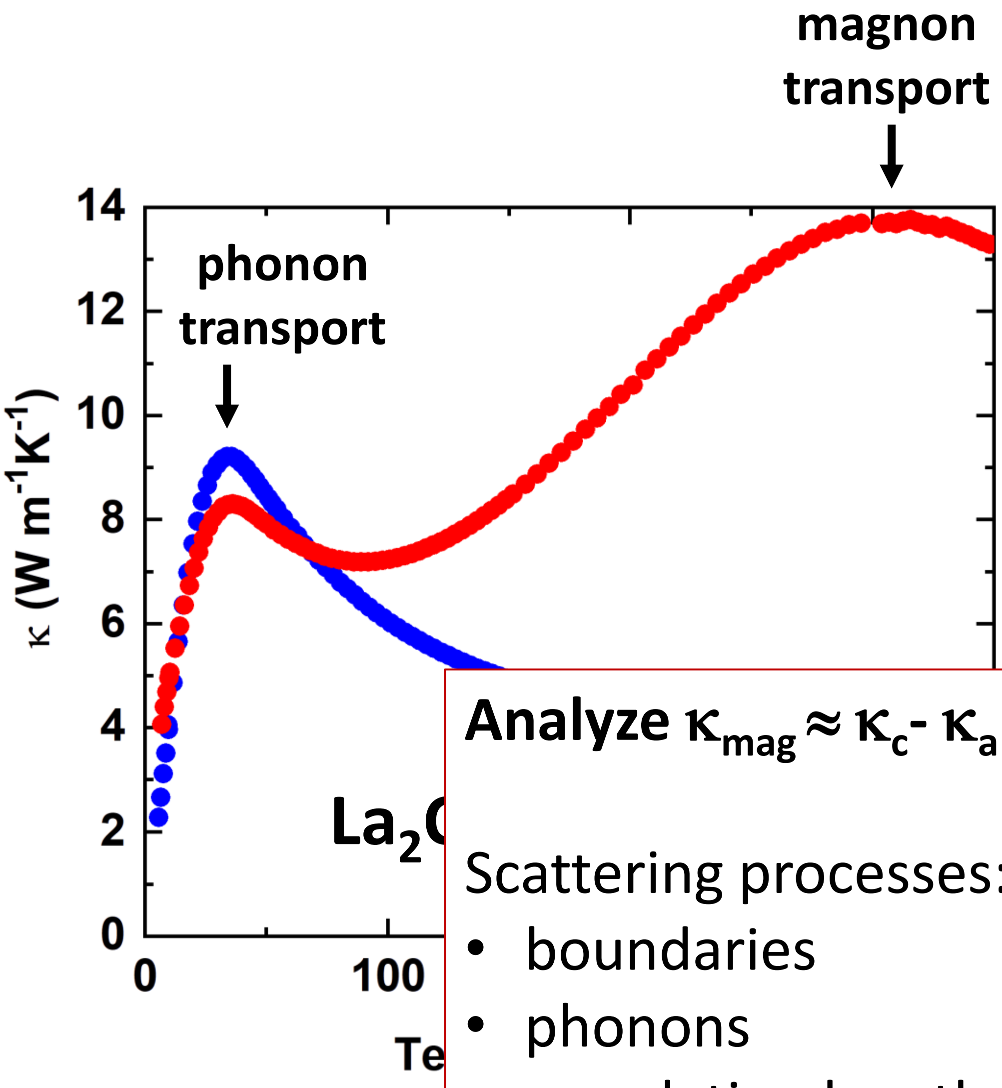

Analyze $\kappa_{mag} \approx \kappa_c - \kappa_a$

Scattering processes:
- boundaries
- phonons
- correlation length

Mohan, Hess et al., unpublished

Antiferromagnetic S = 1/2 Heisenberg chain: spinons

Heisenberg Model, D=1 : $\mathbf{H} = J \sum_{\langle i,j \rangle} \mathbf{S}_i \cdot \mathbf{S}_j$ $J > 0, S = ½$

Antiferromagnetic Heisenberg chain

Ground state: correlation length $\xi \propto {}^{1}/_{n}$

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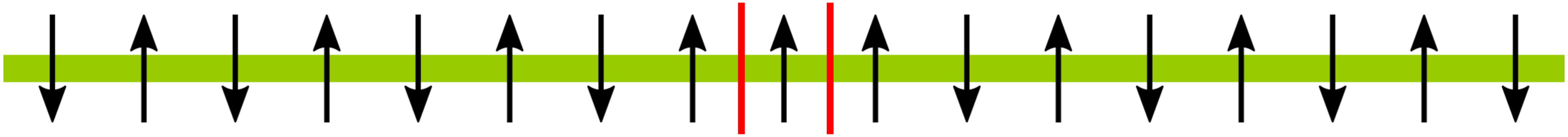

ΔS=1

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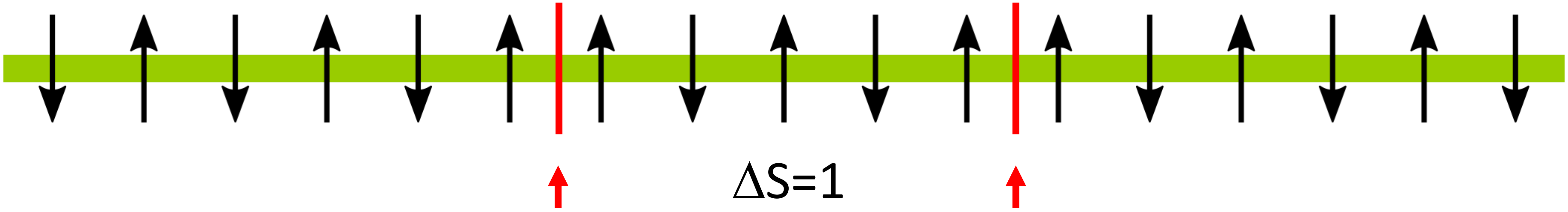

ΔS=1

S=1/2
excitation

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excitation

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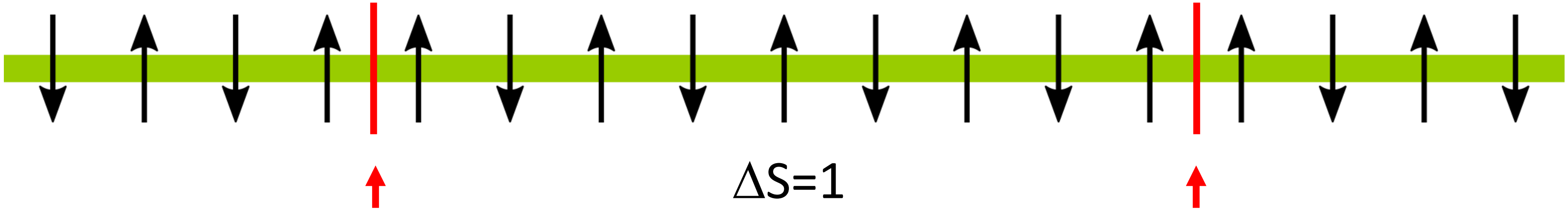

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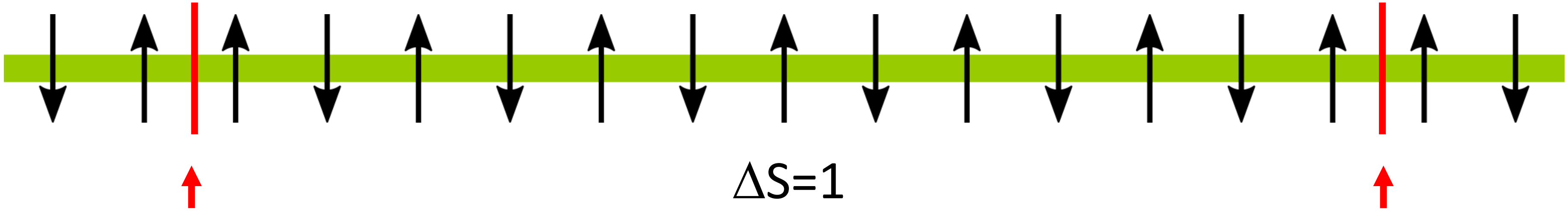

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„Spinons“

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Antiferromagnetic Heisenberg chain

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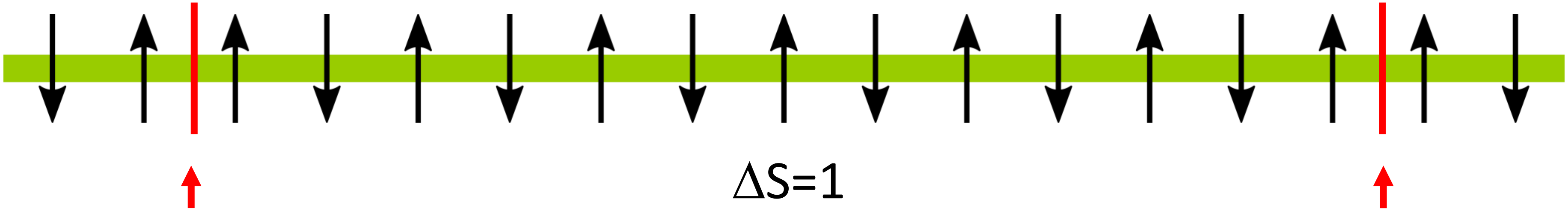


ΔS=1

S=1/2
excitation

„Spinons"

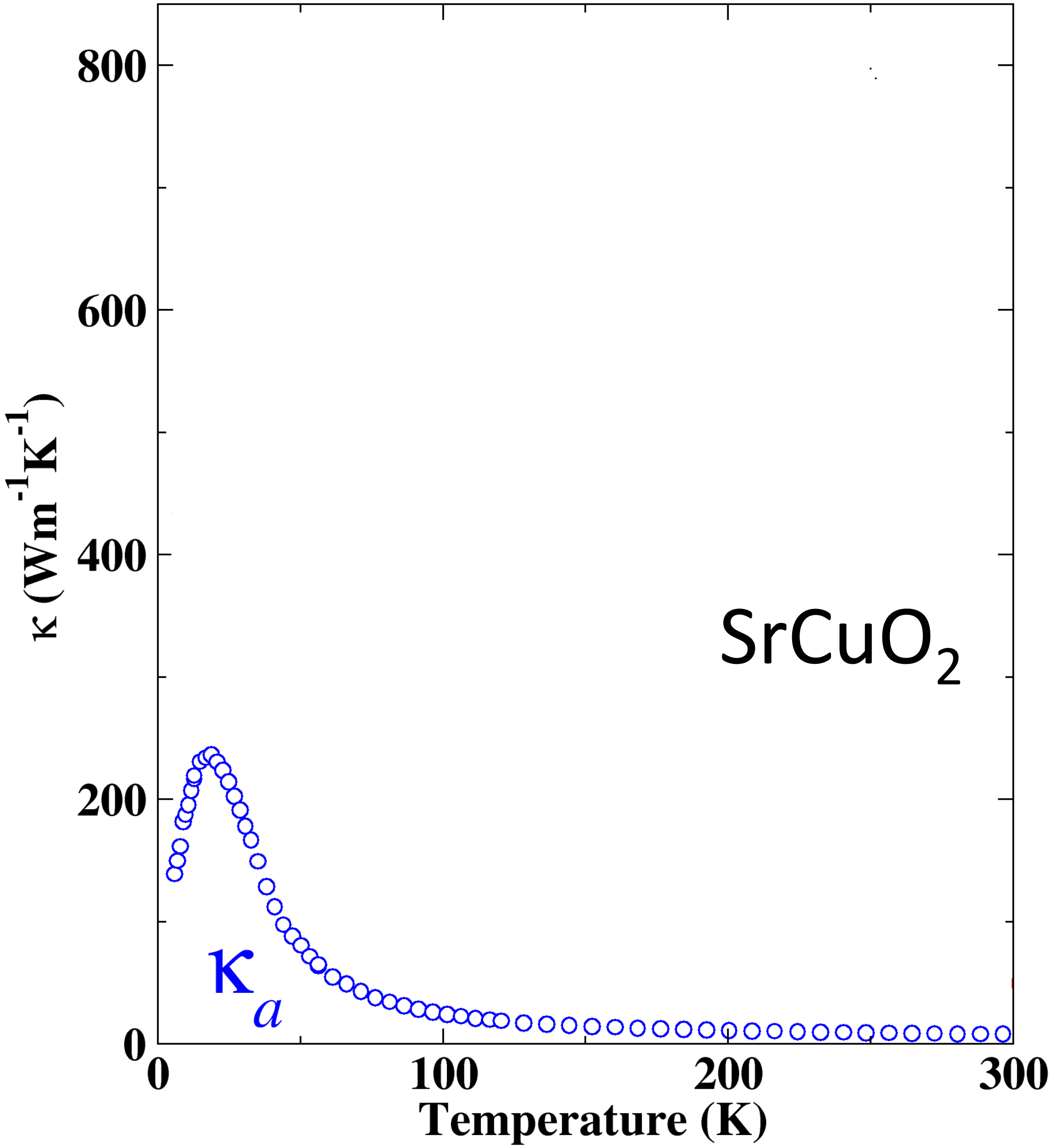
S=1/2
excitation

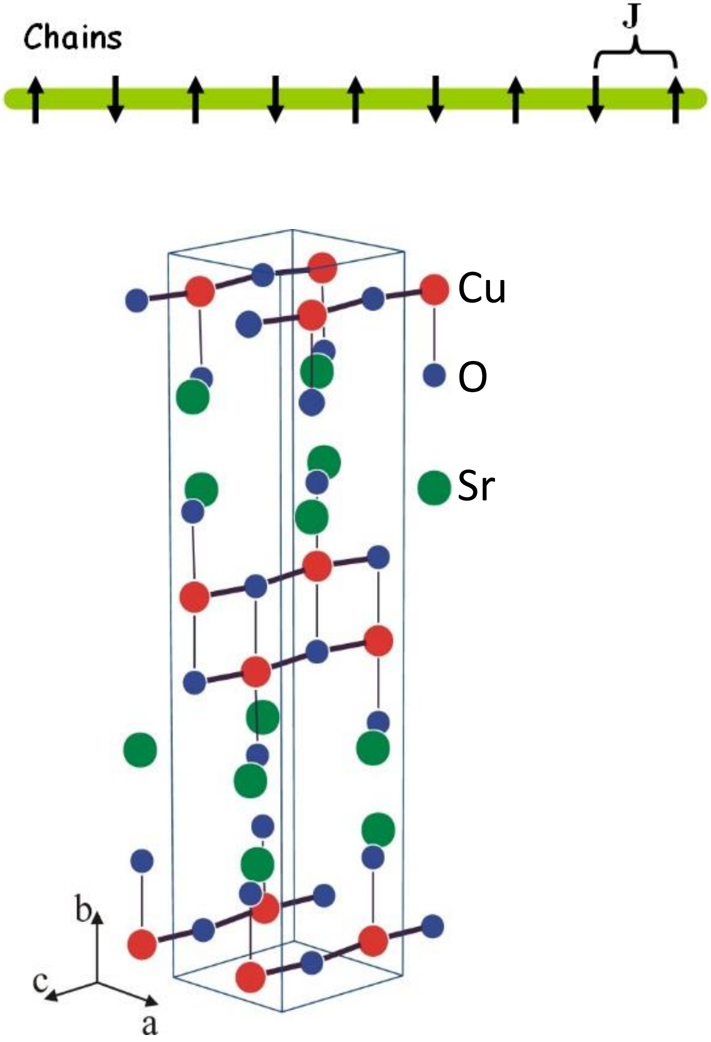
Heat transport: $[j_Q,H]=0 \Rightarrow$ „$\kappa=\infty$" ➡ **Ballistic!**[1]

Heat current operator

[1] Zotos, Naef, Prelovšek, PRB 1997

Large spinon heat transport in $SrCuO_2$





$J/k_B \approx 2000K$

N. Hlubek et al., PRB 2010; Hess, Physics Reports 2019

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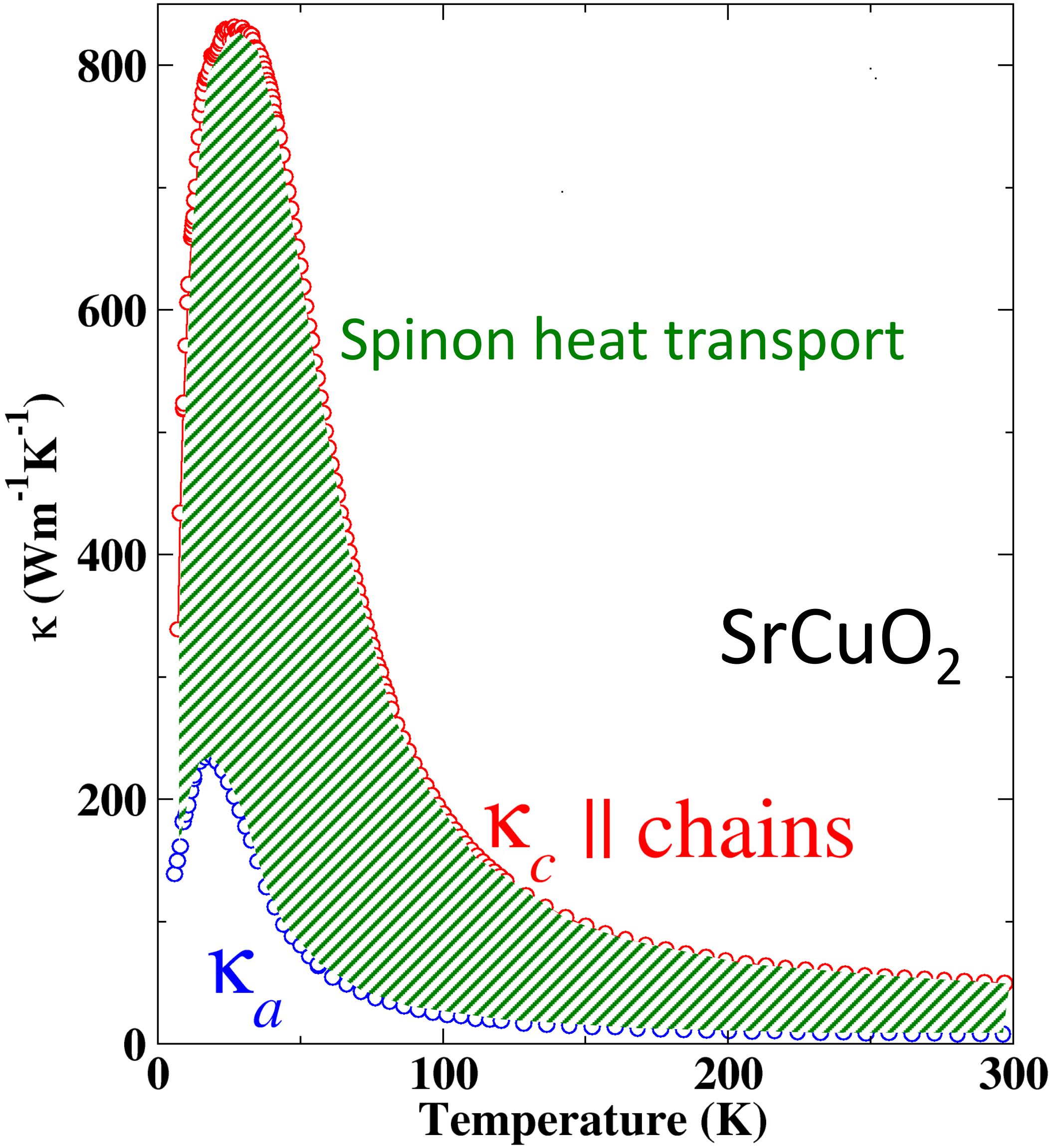

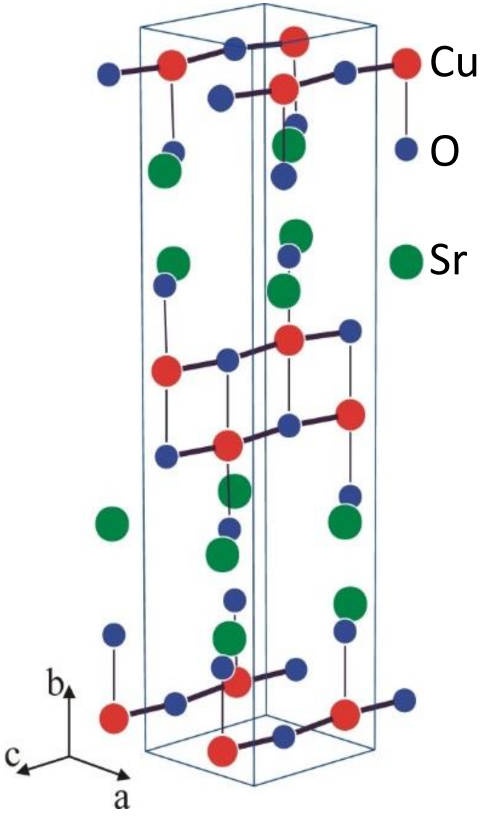

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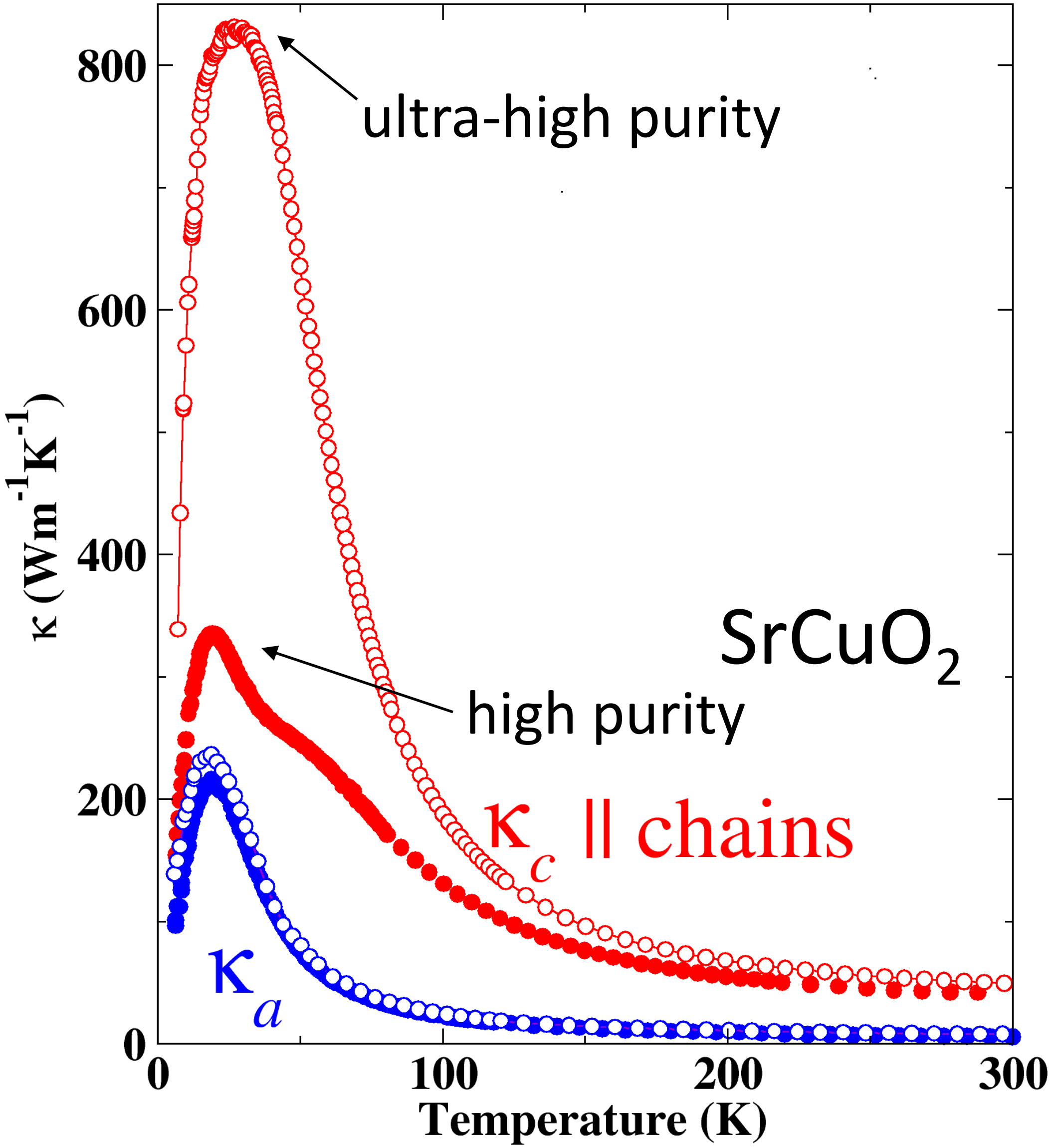

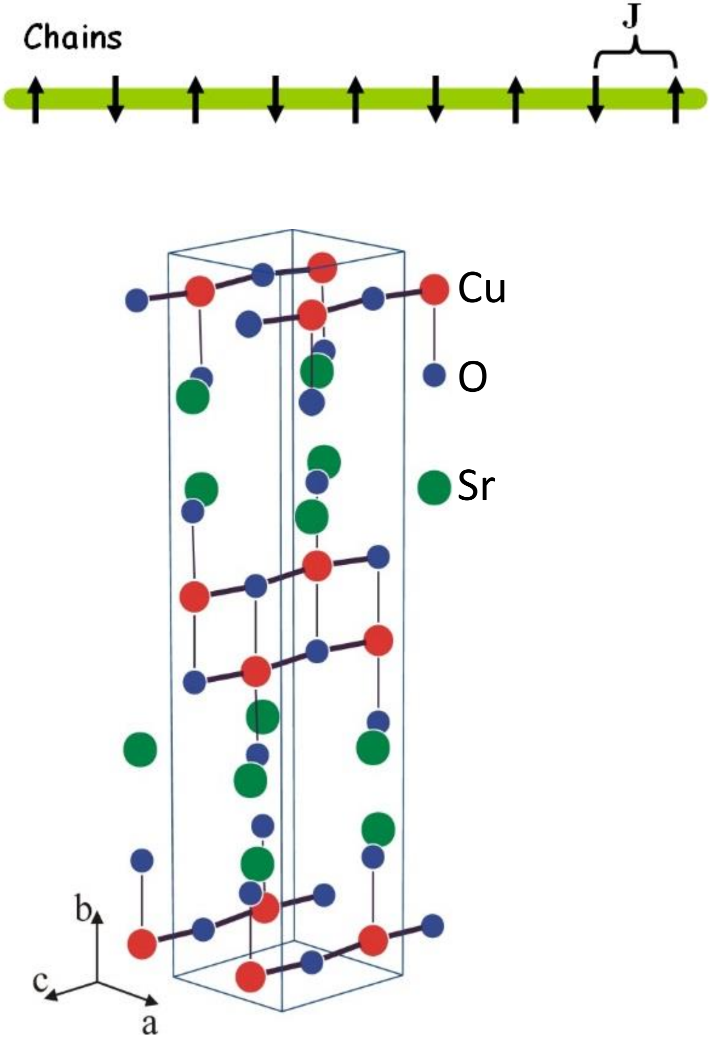

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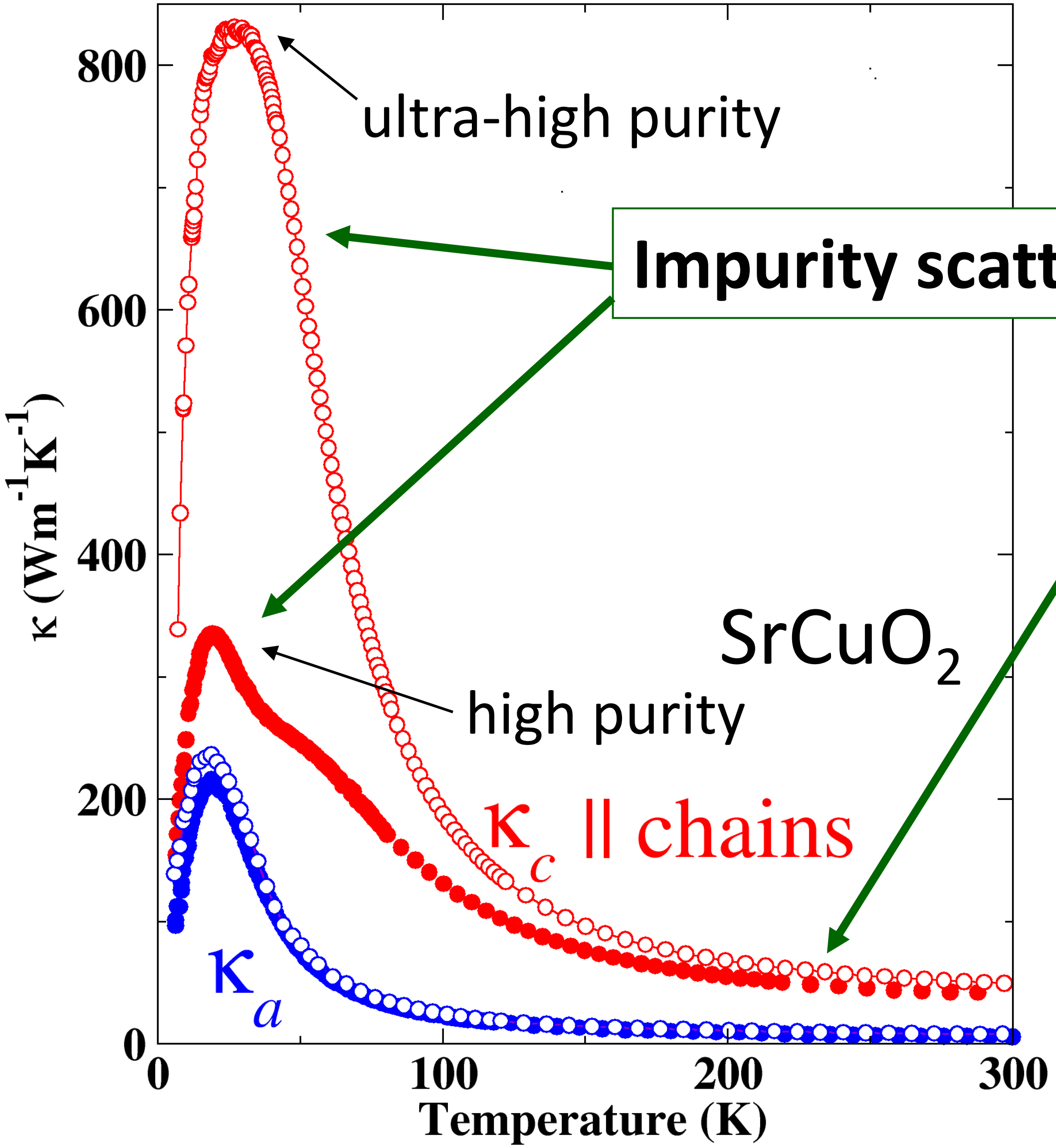


Impurity scattering!

Phonon scattering!

Analyze $\kappa_{mag} \approx \kappa_c - \kappa_a$

Scattering processes:

- boundaries
- Phonons

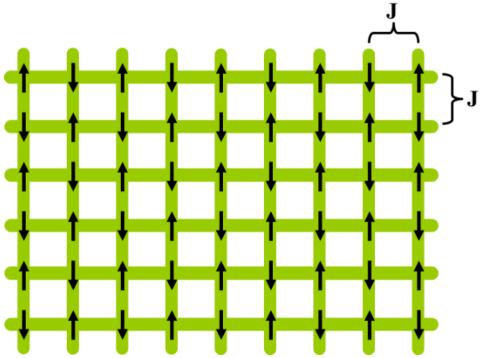
→ Ballistic!

N. Hlubek et al., PRB 2010; Hess, Physics Reports 2019

Outline

2D-AFM Heisenberg

Magnon heat transport



Material: La_2CuO_4

1D-AFM Heisenberg

Spinon heat transport

Material: $SrCuO_2$

2D Kitaev

Longitudinal heat transport

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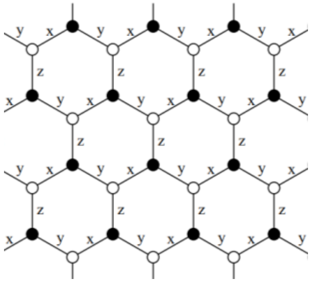
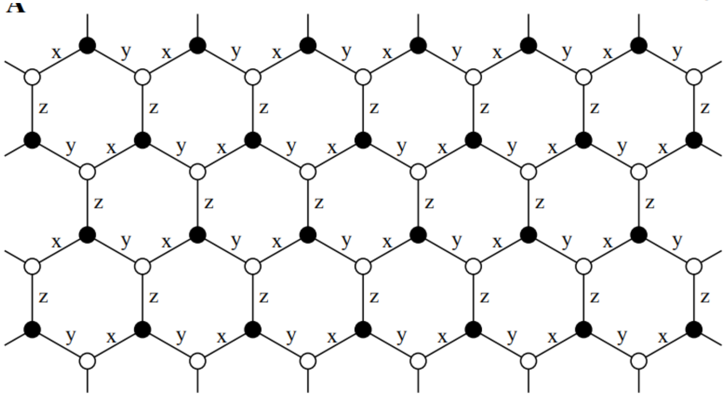
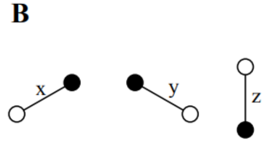


Material: α-$RuCl_3$

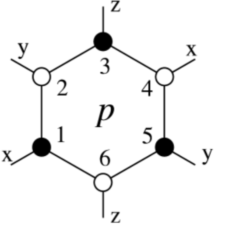
Kitaev model

$$H = -J_x \sum_{x\text{-links}} \sigma_j^x \sigma_k^x - J_y \sum_{y\text{-links}} \sigma_j^y \sigma_k^y - J_z \sum_{z\text{-links}} \sigma_j^z \sigma_k^z$$





A. Kitaev,
Ann. Phys. **321**, 2 (2006)



Flux loops

$$[H, W_p] = 0$$

$$W_p = \sigma_1^x \sigma_2^y \sigma_3^z \sigma_4^x \sigma_5^y \sigma_6^z$$

Immobile -> **no transport**

Spins are mapped to 4 **Majorana fermions**

$c^\alpha, \alpha = 0, x, y, z$ with $\{c^\alpha, c^\beta\} = 2\delta_{\alpha\beta}$

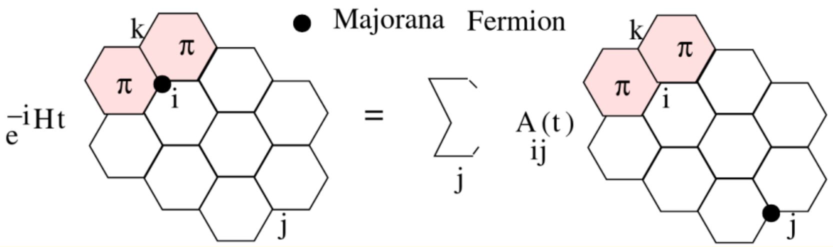
$\sigma_i^a = i c_i c_i^a, \qquad a = x, y, z$

$\hat{u}_{\langle ij \rangle_a} \equiv i c_i^a c_j^a$

$$H = -\sum_{a=x,y,z} J_a \sum_{\langle ij \rangle_a} i c_i \hat{u}_{\langle ij \rangle_a} c_j \qquad [H, \hat{u}_{\langle ij \rangle_a}] = 0$$

-> free propagation -> **heat transport possible**

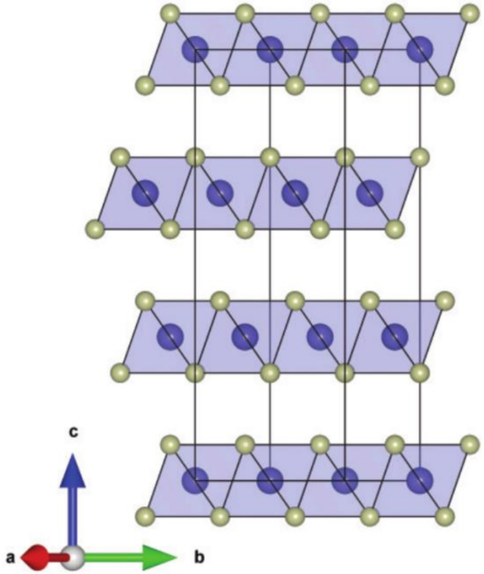
Spin flips fractionalize into
flux pair + Majorana fermion

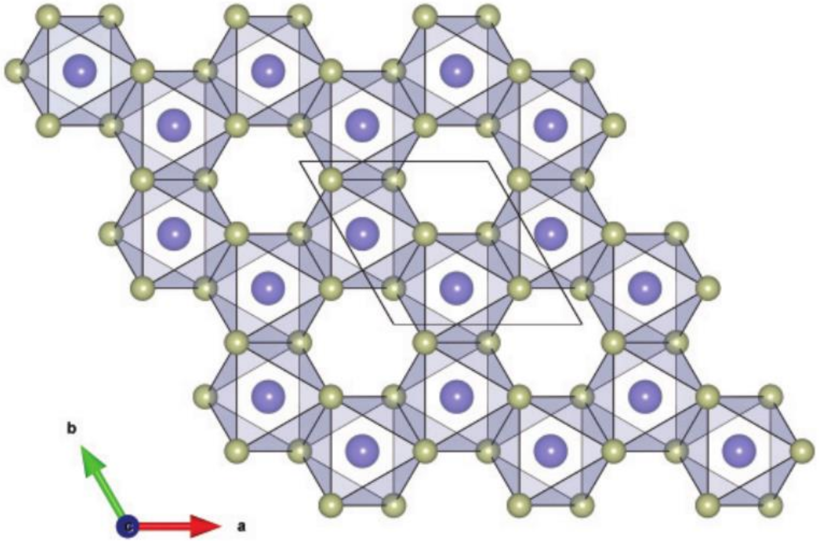


Baskaran, Mandal, Shankar,
PRL **98**, 247201 (2007)

α-RuCl$_3$: a candidate Kitaev-honeycomb system

Plumb et al., PRB **90**, 041112(R) (2014)



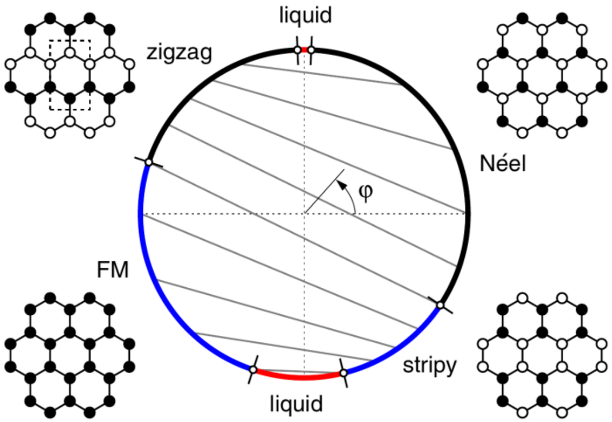


AFM order at $T_N \approx 7$K

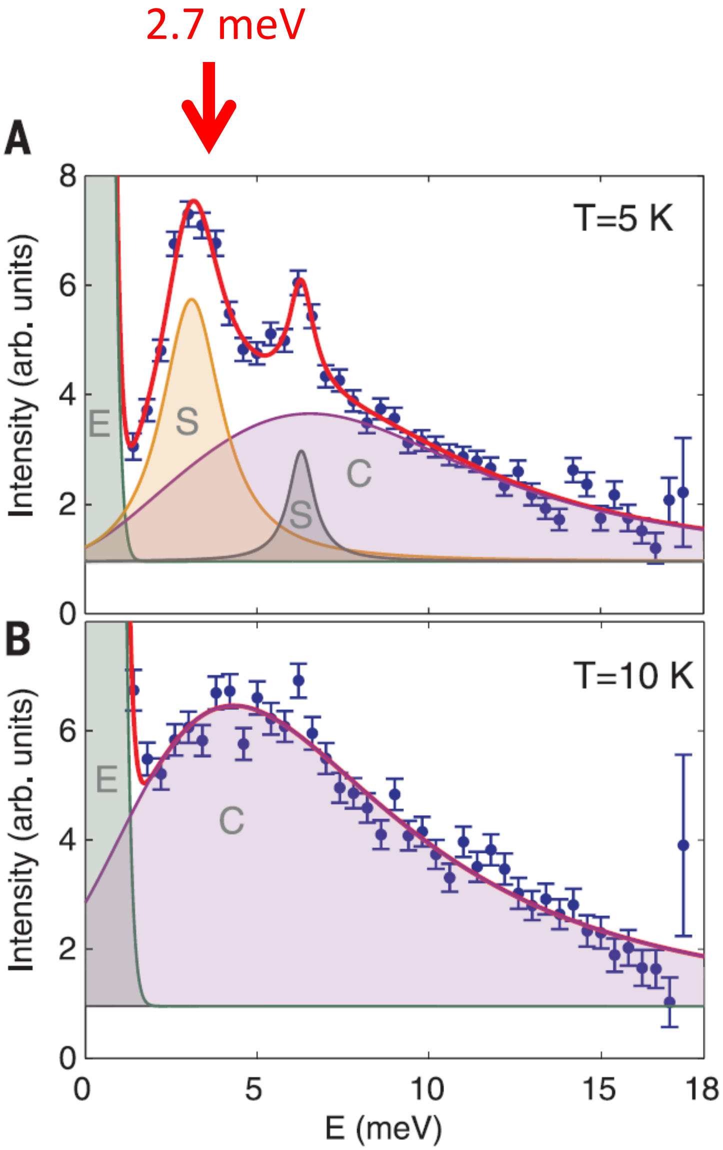
$$H = K \sum_{\langle ij \rangle_\gamma} S_i^\gamma S_j^\gamma + J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j + ...$$

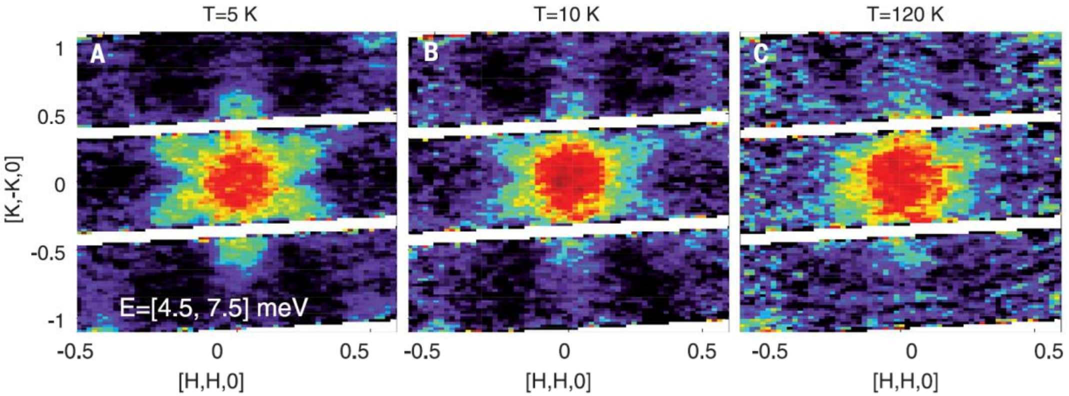
G. Jackeli and G. Khaliullin, PRL **102**, 017205 (2009)

J. Chaloupka , G. Jackeli and G. Khaliullin, PRL **110**, 097204 (2013)



Magnetic excitations of α-$RuCl_3$





A. Banerjee et al., Science **356**, 1055 (2017)

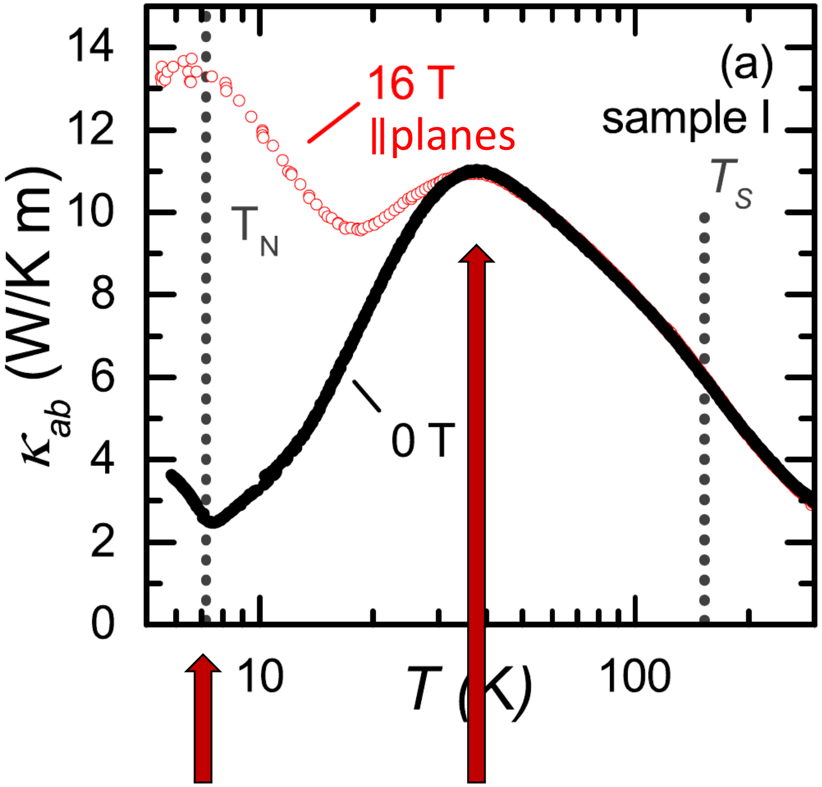
Inelastic neutron scattering at q ≈ 0:

- Sharp and gapped spin waves at $T < T_N$
- Broad excitation continuum up to T ~ 100 K at higher energies
- |K|≈5 meV

Longitudinal thermal conductivity $\boldsymbol{\kappa}_{xx}$
with B‖ planes

Heat transport of α-$RuCl_3$: Results

In-plane data:



magnetic ordering

phononic peak?

R. Hentrich et al., PRL **120**, 117204 (2018)

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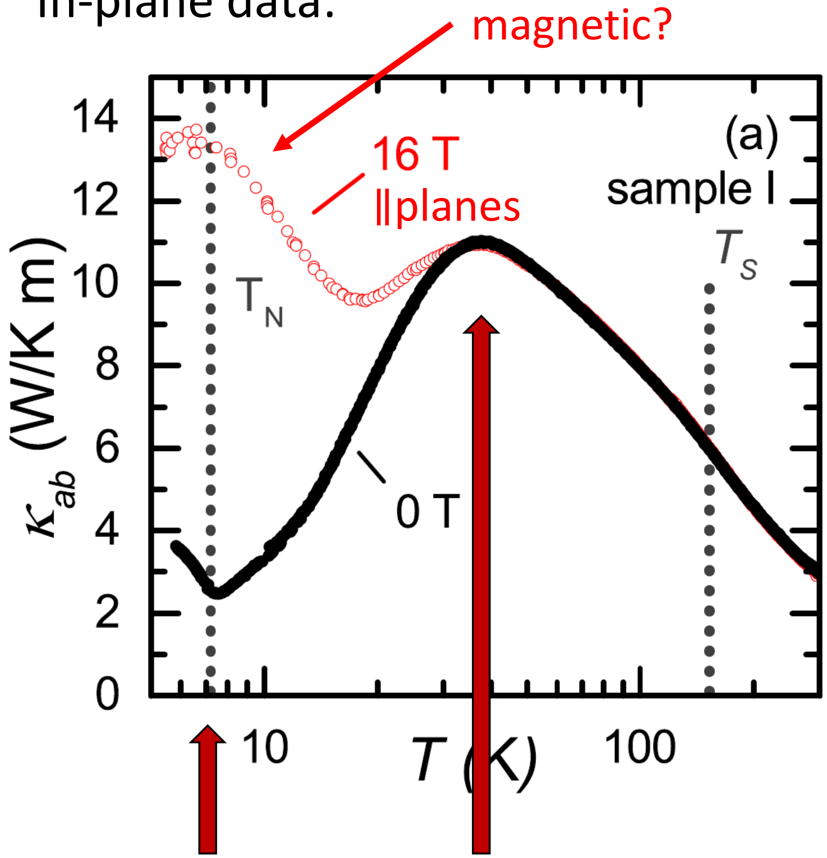

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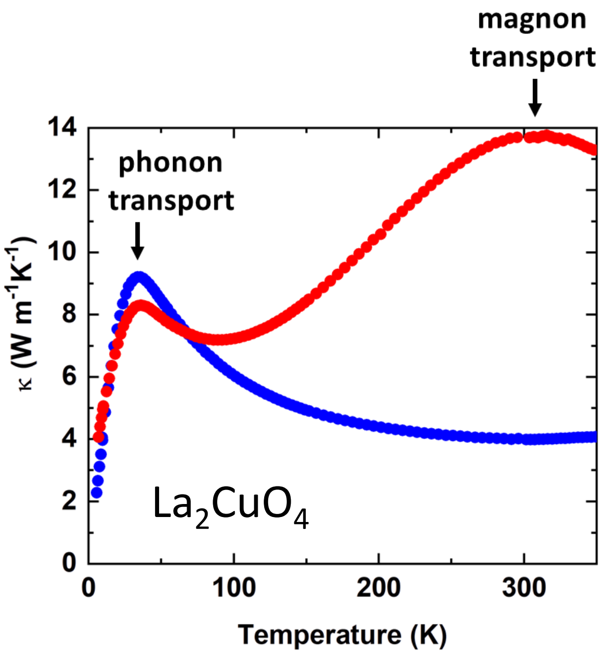
Reminder:
phononic + magnetic heat transport



Conjecture
1. Zero field data purely phononic
2. High-field (16T) peak: Majorana excitations

Measure transport ⊥ planes

R. Hentrich et al., PRL **120**, 117204 (2018)

Heat transport of α-$RuCl_3$: Results

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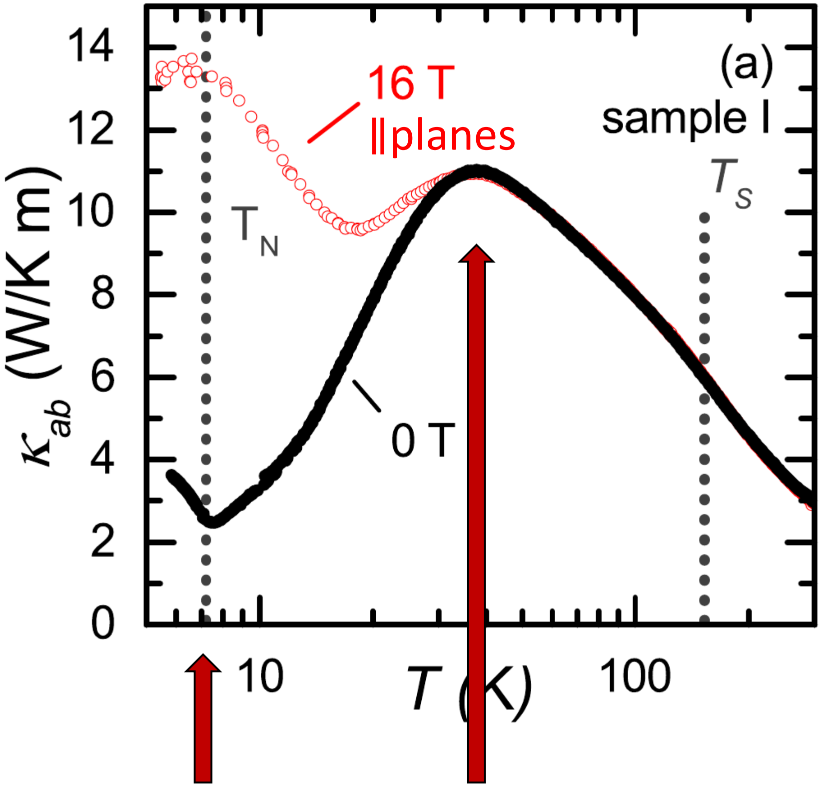

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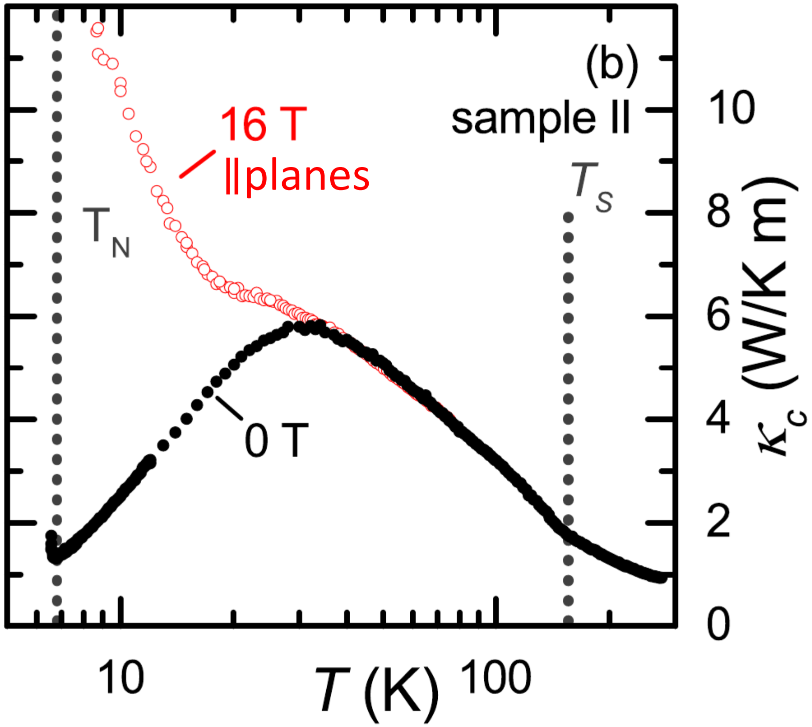
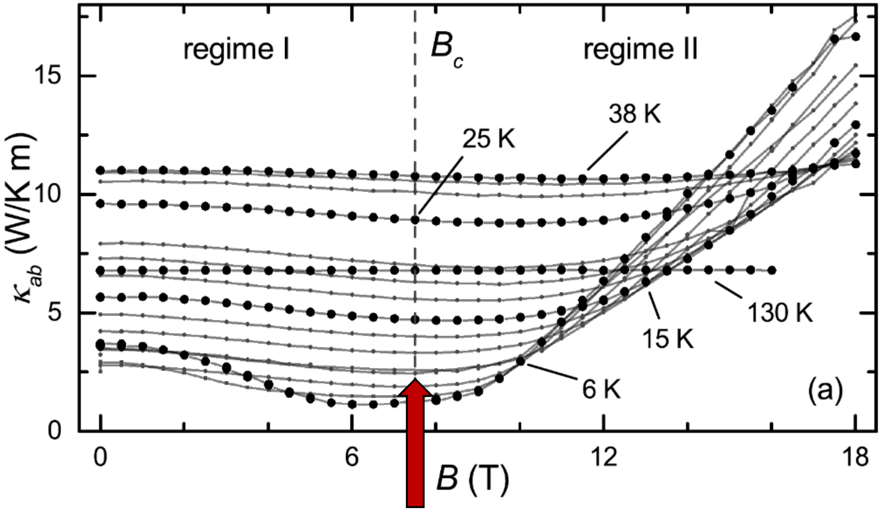


Low-T enhancement: phononic!

R. Hentrich et al., PRL **120**, 117204 (2018)

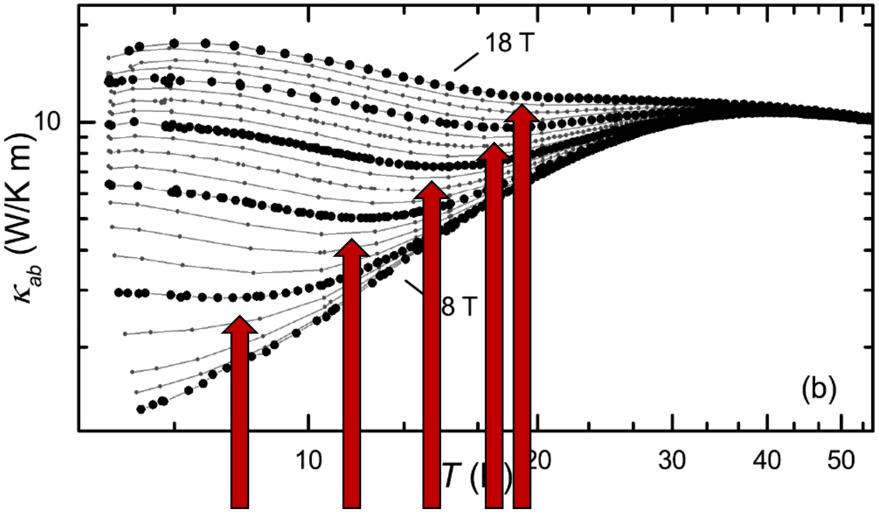
Heat transport of α-$RuCl_3$: Results

Field dependence



critical field B_c

Temperature dependence @ $B>B_c$



Minima at T^*

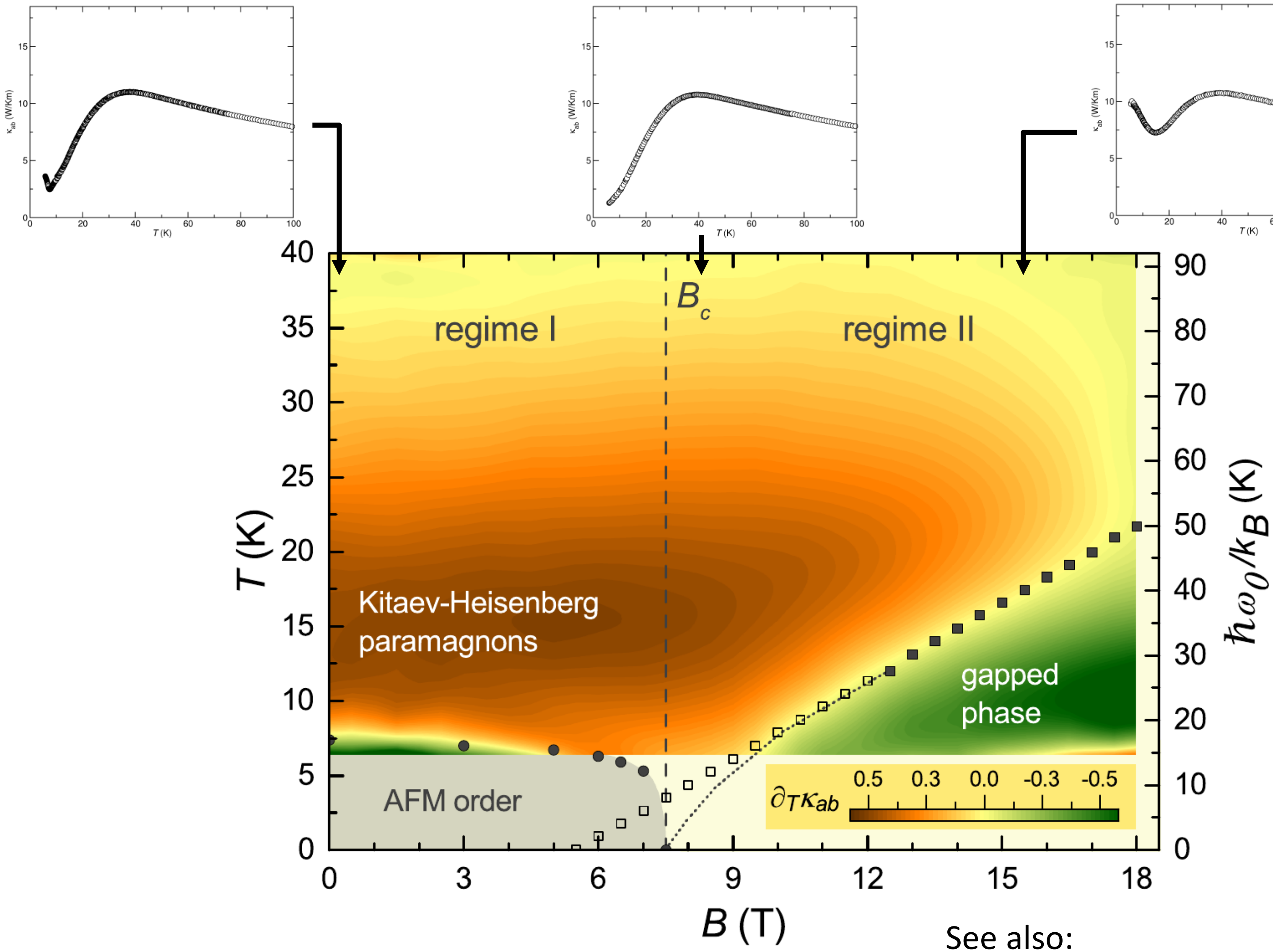
Poor man‘s approach: minimum at T^*

→ “resonant” scattering process at $\omega_0 \sim 4k_B T^*$

(in 3D)

R. Hentrich et al., PRL **120**, 117204 (2018)

Heat transport of α-$RuCl_3$: Results



R. Hentrich et al., PRL **120**, 117204 (2018)

See also:
Baek et al., PRL 2017 (NMR)
Banerjee et al. NPJ Quant. Mat. 2018 (INS)

Heat transport of α-RuCl$_3$: Analysis

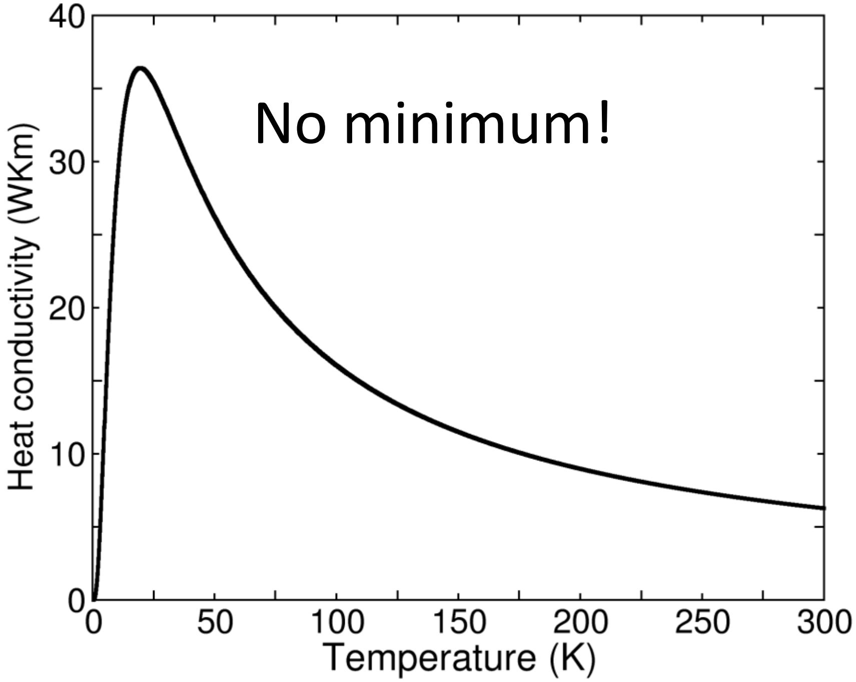
$$\kappa(T) = \frac{k_B}{2\pi^2 v_s}\left(\frac{k_B T}{\hbar}\right)^3 \int_0^{\Theta_D/T} \frac{x^4 e^x}{(e^x-1)^2}\tau_c(x)\; dx$$

$$\tau_c^{-1} = \tau_P^{-1} + \tau_D^{-1} + \tau_B^{-1}$$

umklapp

point defects

sample boundary



No minimum!

R. Hentrich et al., PRL **120**, 117204 (2018)

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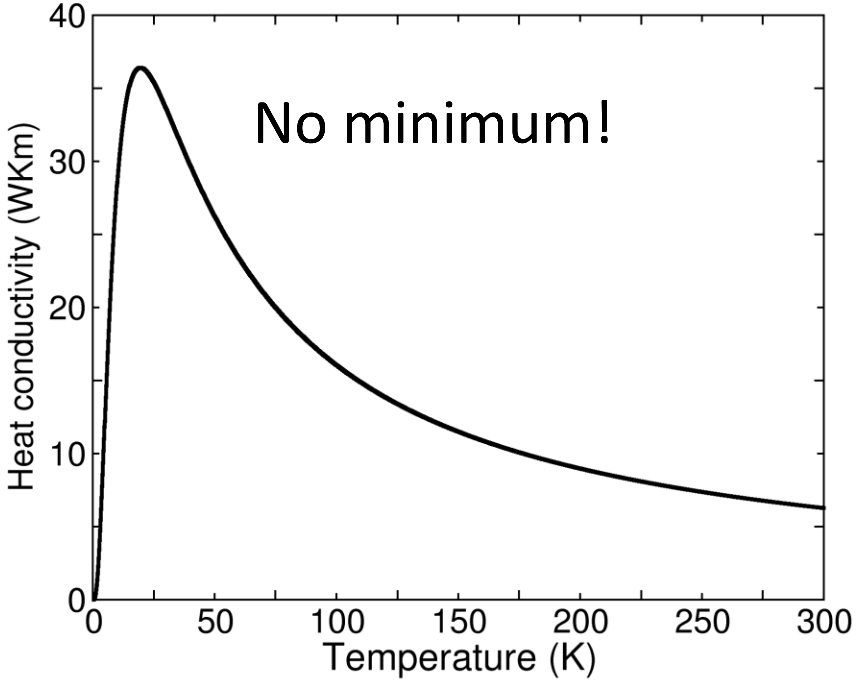
$$\tau_c^{-1} = \tau_P^{-1} + \tau_D^{-1} + \tau_B^{-1} + {\tau_{mag}}^{-1}$$

↑ umklapp

↑ point defects

↑ sample boundary



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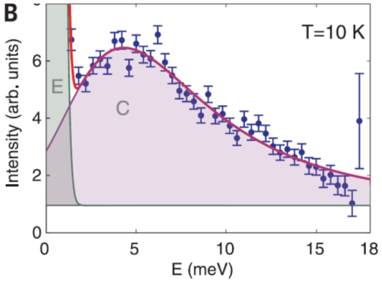
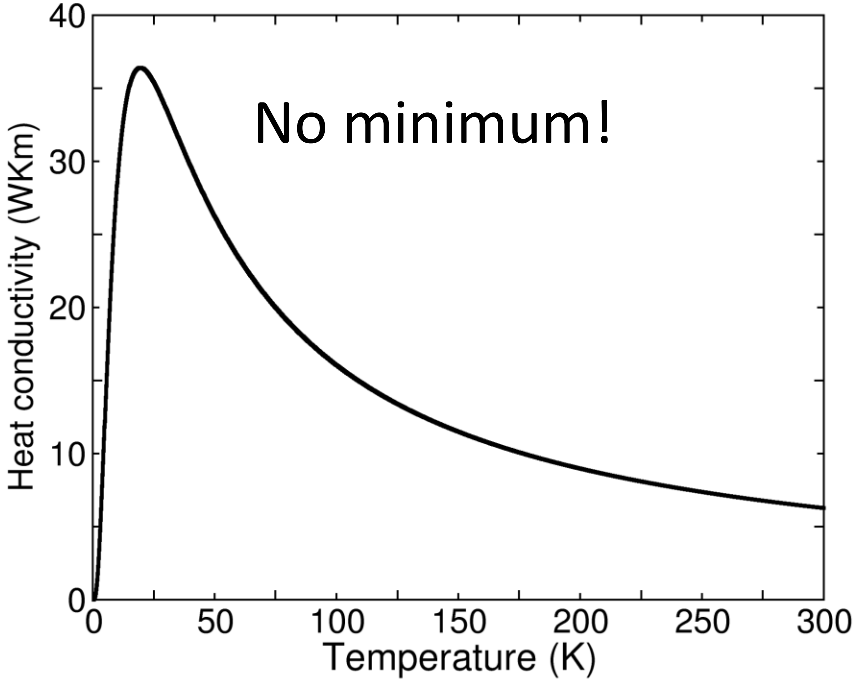
$$\tau_c^{-1} = \tau_P^{-1} + \tau_D^{-1} + \tau_B^{-1} + \tau_{mag}^{-1}$$

$$\boxed{\tau_{\rm mag}^{-1} = C\theta(K - \hbar\omega)\frac{e^{\frac{-\hbar\omega_0}{k_B T}}}{1 + 3e^{\frac{-\hbar\omega_0}{k_B T}}}}$$

High-energy cutoff

triplet excitations





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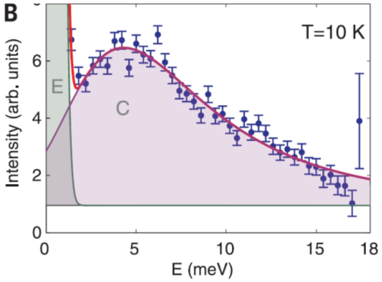
$$\kappa(T) = \frac{k_B}{2\pi^2 v_s}\left(\frac{k_B T}{\hbar}\right)^3 \int_0^{\Theta_D/T} \frac{x^4 e^x}{(e^x - 1)^2}\tau_c(x)\ dx$$

$$\tau_c^{-1} = \tau_P^{-1} + \tau_D^{-1} + \tau_B^{-1} + \tau_{mag}^{-1}$$

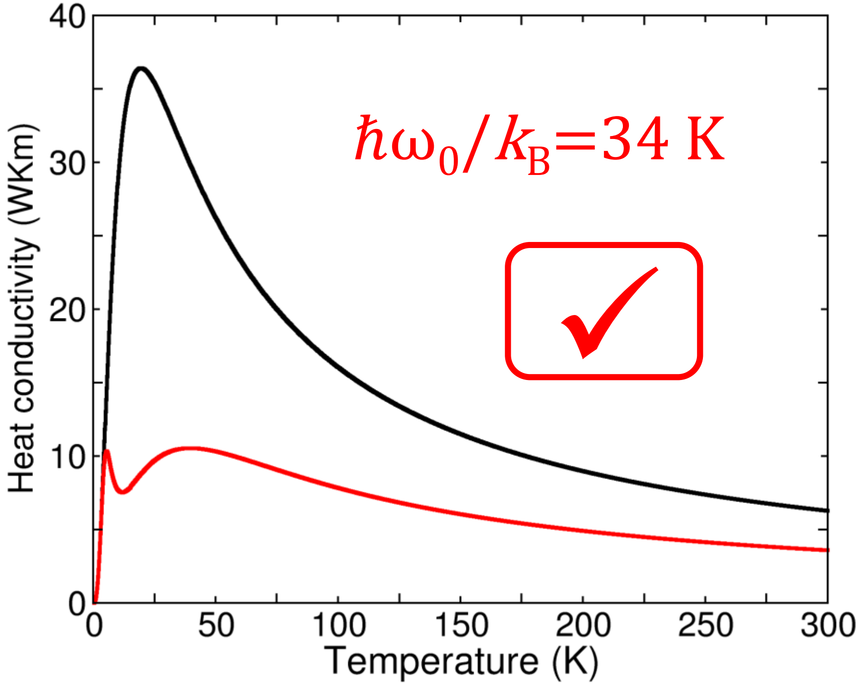
$$\boxed{\tau_{\mathrm{mag}}^{-1} = C\theta(K - \hbar\omega)\frac{e^{\frac{-\hbar\omega_0}{k_B T}}}{1 + 3e^{\frac{-\hbar\omega_0}{k_B T}}}}$$

High-energy cutoff

triplet excitations



R. Hentrich et al., PRL **120**, 117204 (2018)

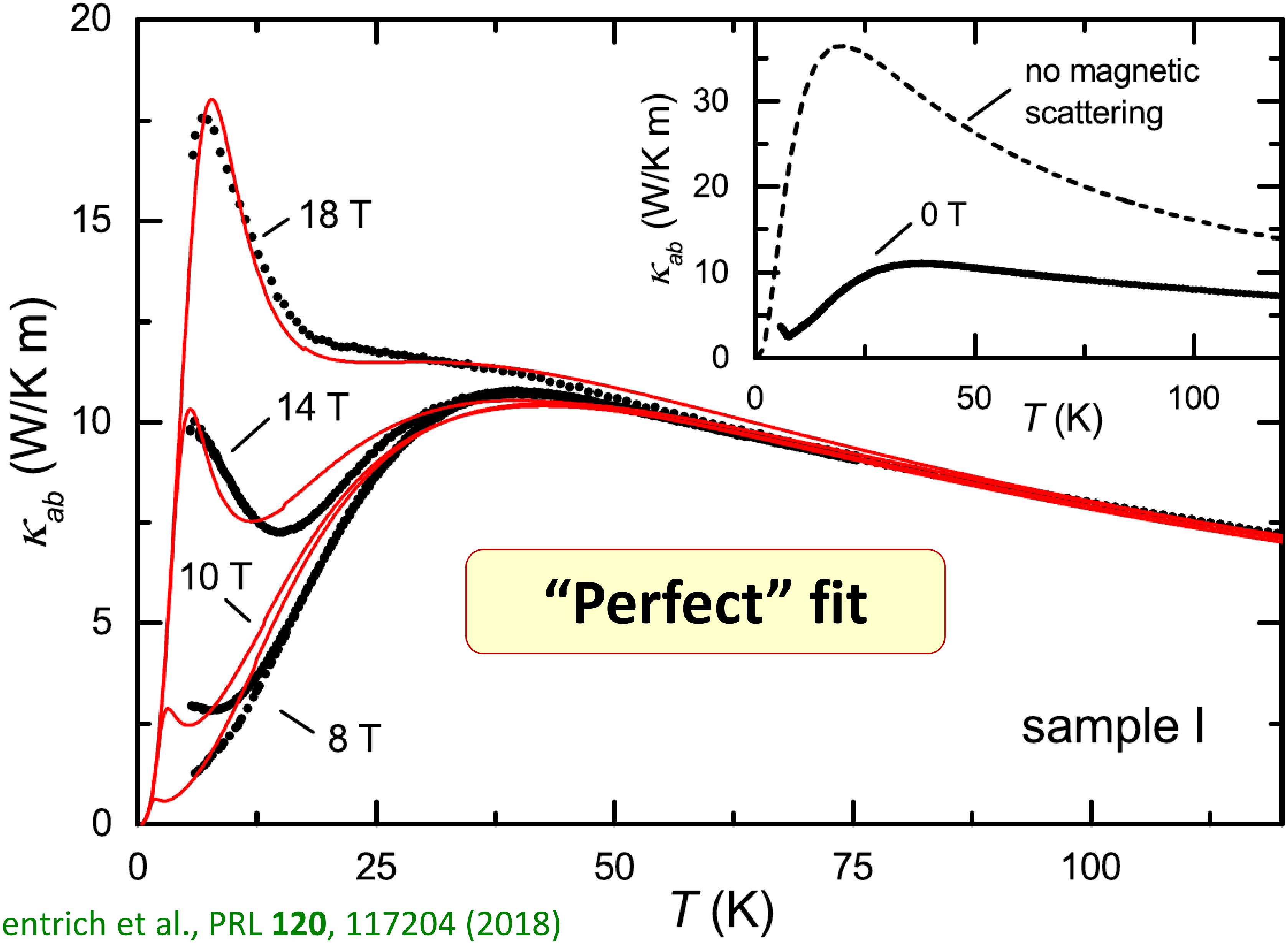


$\hbar\omega_0/k_B$=34 K

✓

→ fit all data, extract $\omega_0(B)$

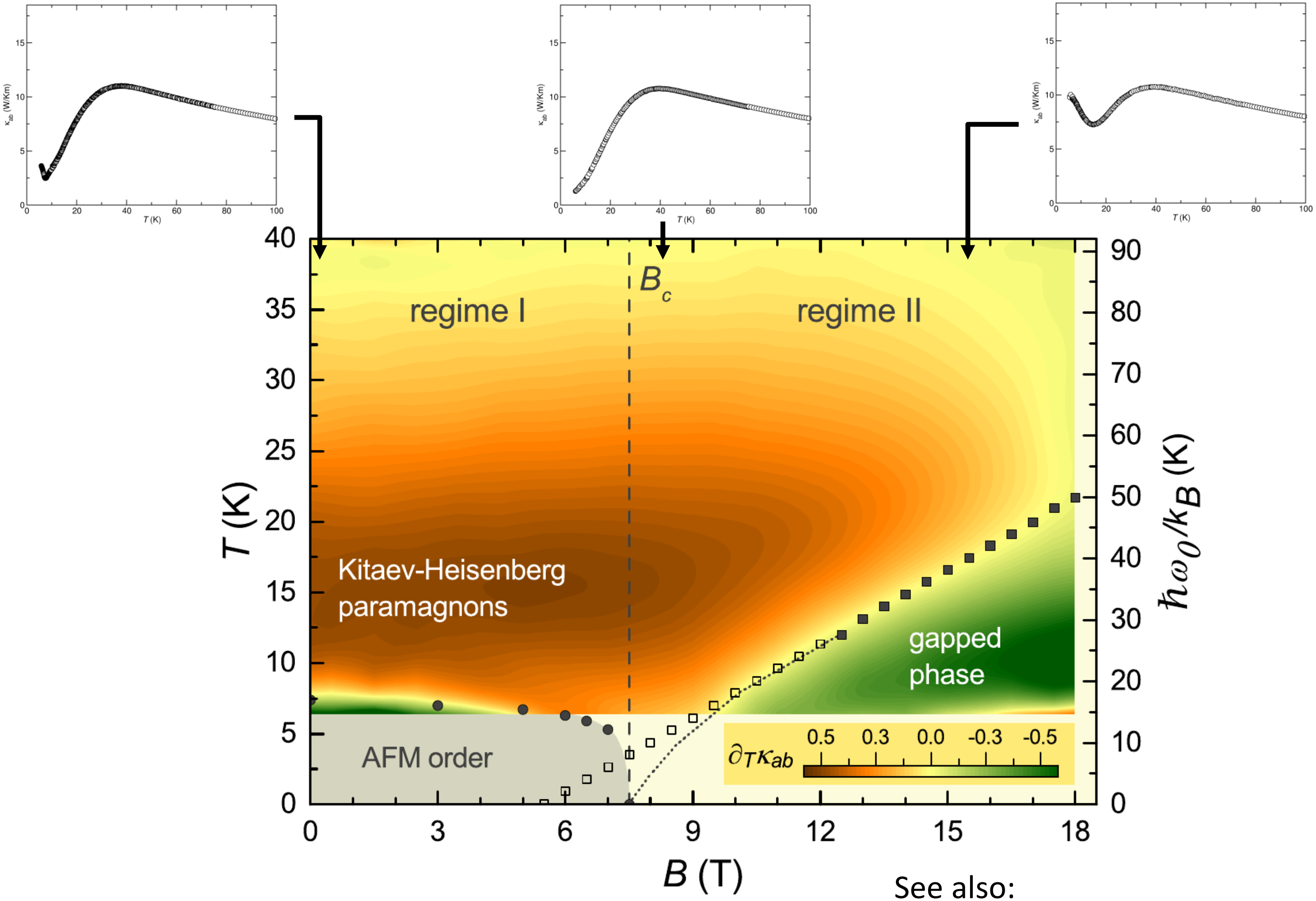
Heat transport of α-$RuCl_3$: Analysis



"Perfect" fit

R. Hentrich et al., PRL **120**, 117204 (2018)

Heat transport of α-$RuCl_3$: Results

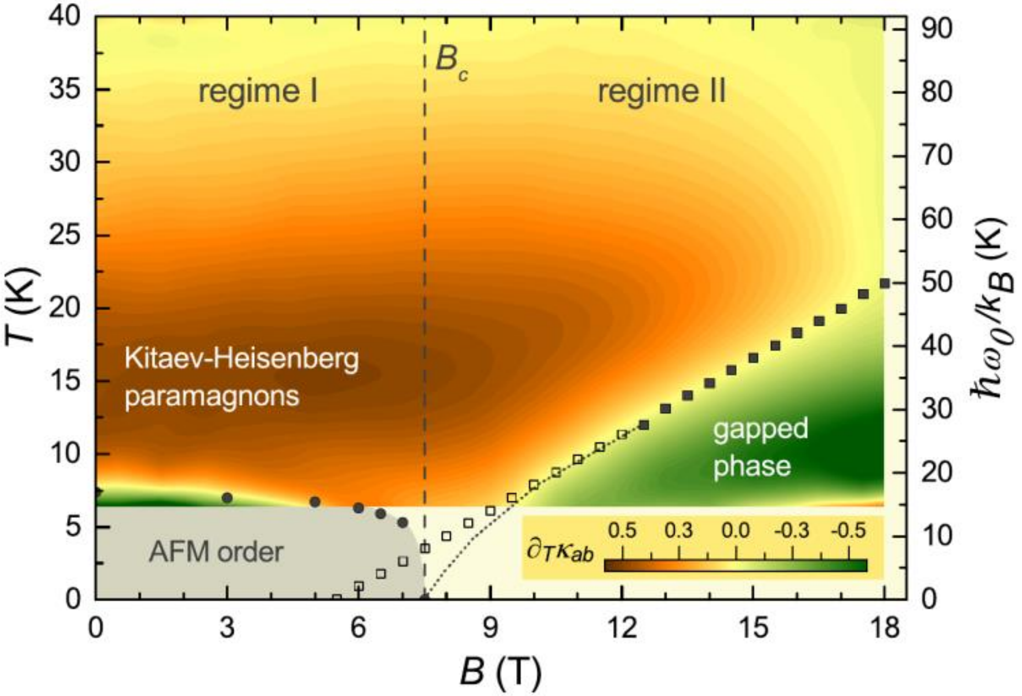


See also:
Baek et al., PRL 2017 (NMR)
Banerjee et al. NPJ Quant. Mat. 2018 (INS)

R. Hentrich et al., PRL **120**, 117204 (2018)

α-$RuCl_3$: Conclusions I

- **Longitudinal thermal transport of α-$RuCl_3$: primarily phononic**
- **Phonons scatter off spin fluctuations**
- **Gapped spectrum for $B_{\|} > B_{cr} \sim 7.5$ T, putative QSL at B_{cr}**



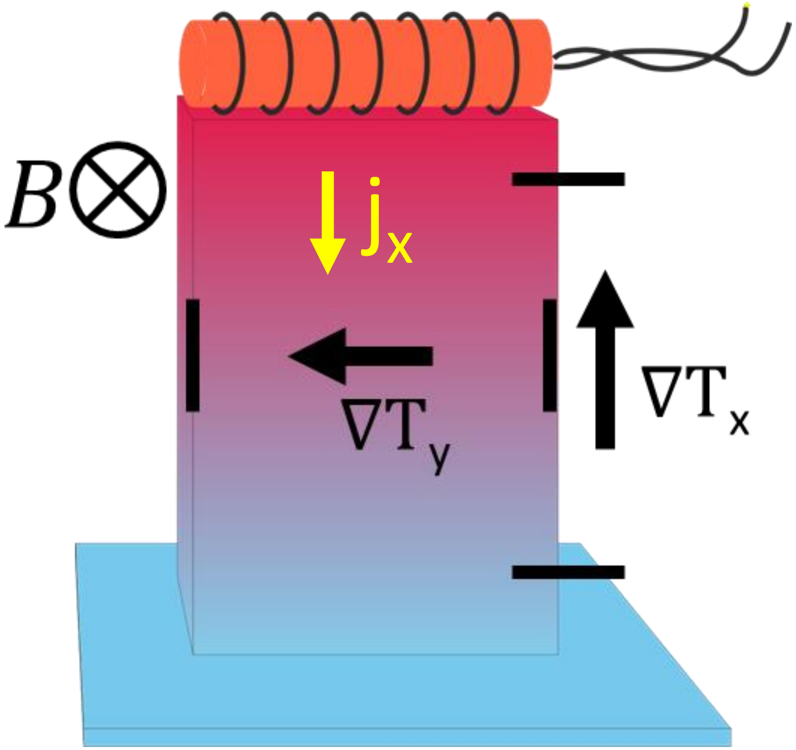
To clarify:

- True nature of spin model
- Spin-phonon coupling
- Is there Majorana fermion transport?

R. Hentrich et al., PRL **120**, 117204 (2018)

Transversal thermal conductivity $\boldsymbol{\kappa}_{xy}$ with B⊥ planes

Thermal Hall effect



$$\kappa_{xy} = \frac{\kappa_{xx}^2 \partial_y^a T}{j_x}$$

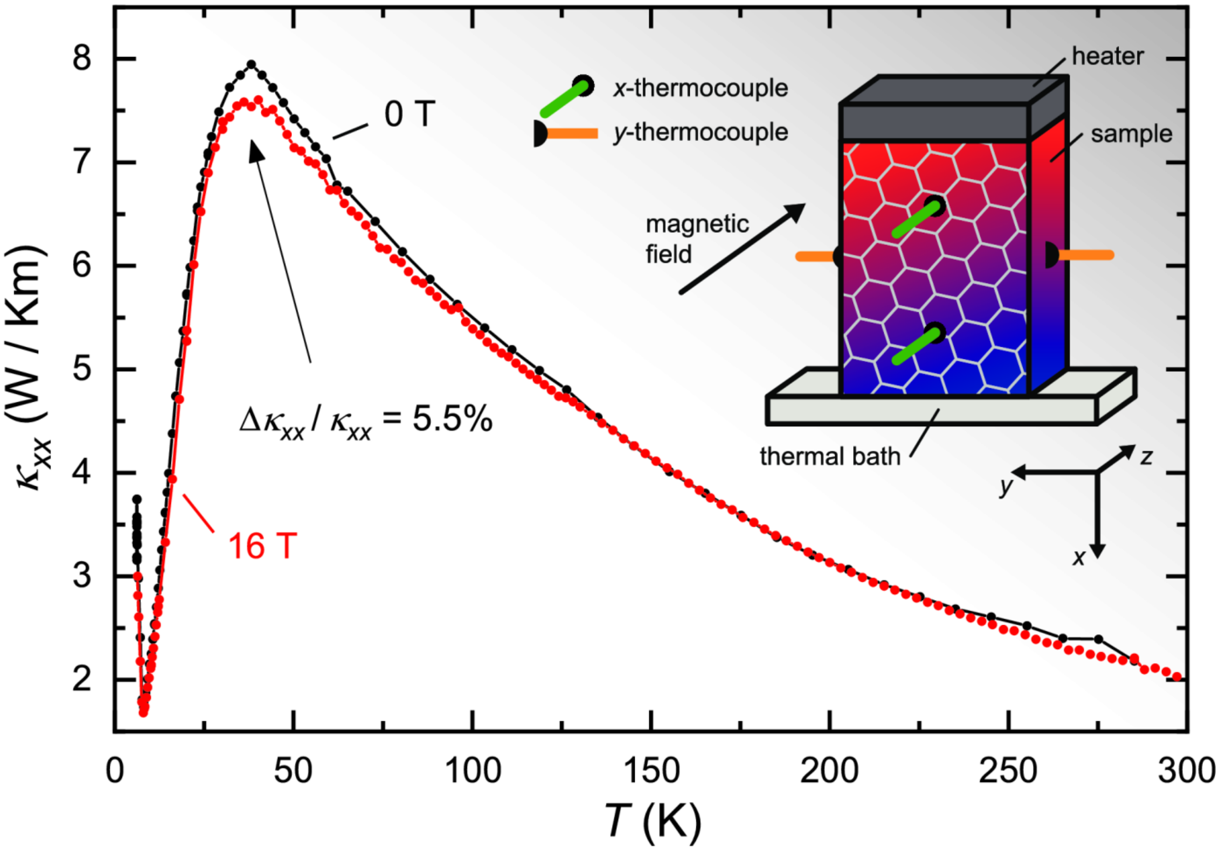
Expectation for spin liquids:
deconfined fermionic excitations experience Lorenz force

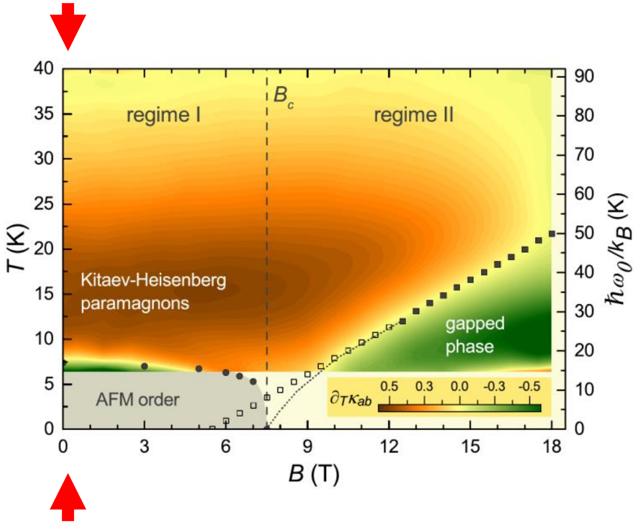
→ Thermal Hall effect (analog of electronic Hall effect)

Katsura, Nagaosa, and Lee, PRL 2010

Results

Longitudinal thermal conductivity κ_{xx} with B⊥ planes



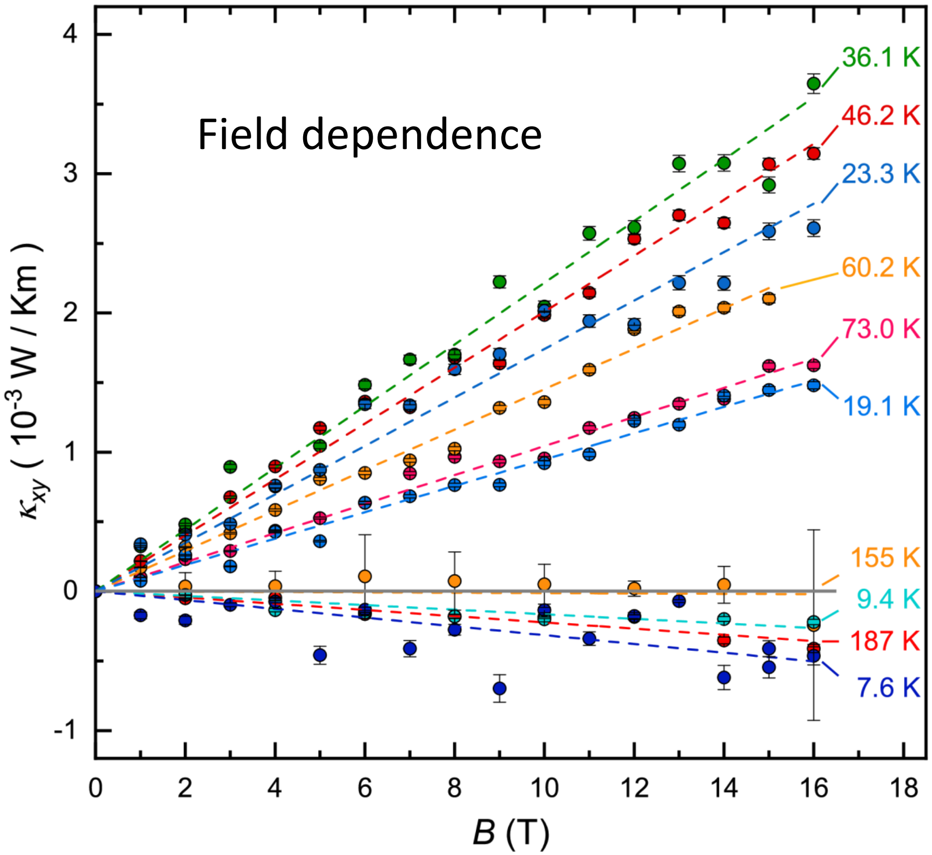


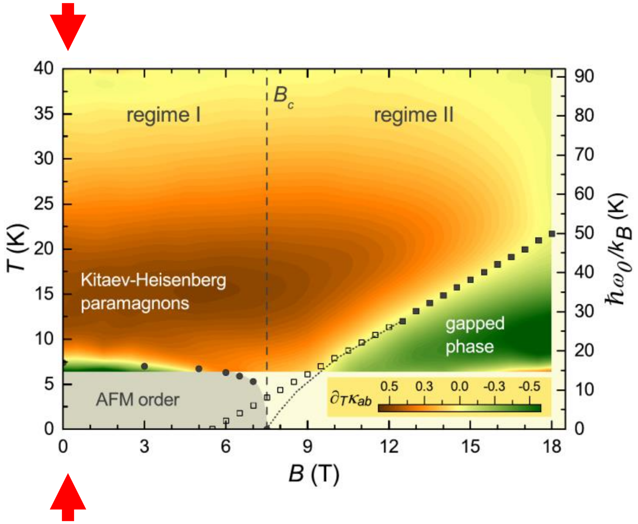
Slight suppression in field

R. Hentrich, et al., PRB **99**, 085136 (2019)

Results

Transversal thermal conductivity **κ_{xy}** with B⊥ planes





Large (!) positive $\kappa_{xy} \propto$ B in paramagnetic phase

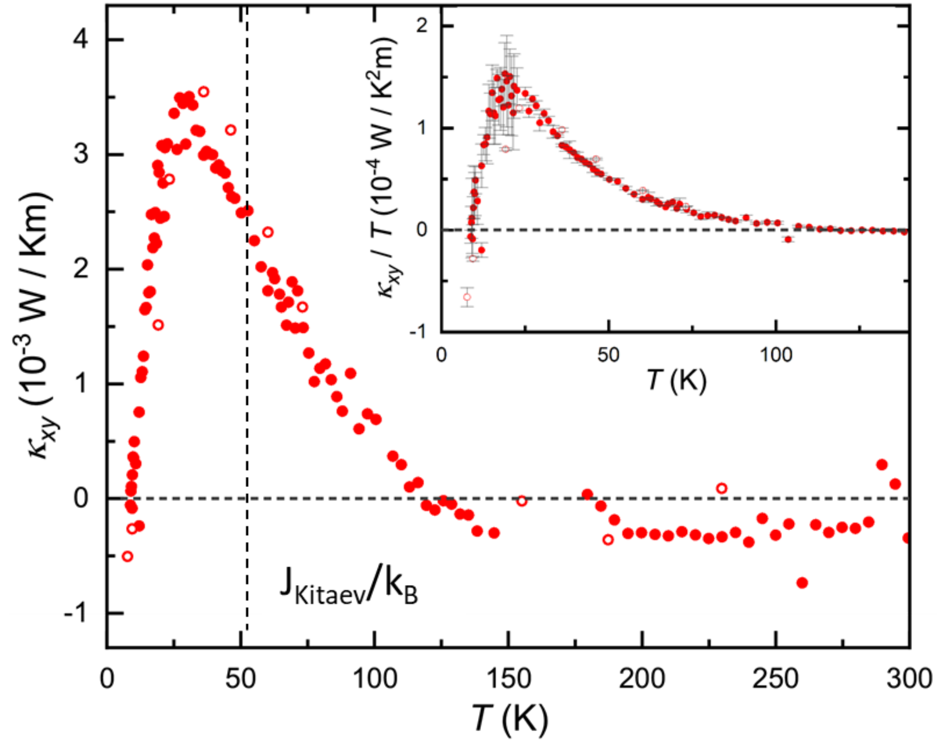
R. Hentrich, et al., PRB **99**, 085136 (2019)

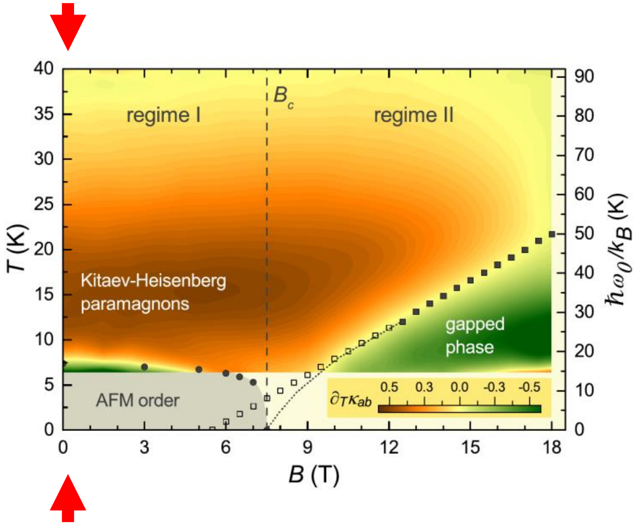
See also: Kasahara et al., PRL 2018

Results

Transversal thermal conductivity κ_{xy} with B⊥ planes

Temperature dependence





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R. Hentrich, et al., PRB **99**, 085136 (2019)

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Results

Transversal thermal conductivity κ_{xy} with B⊥ planes

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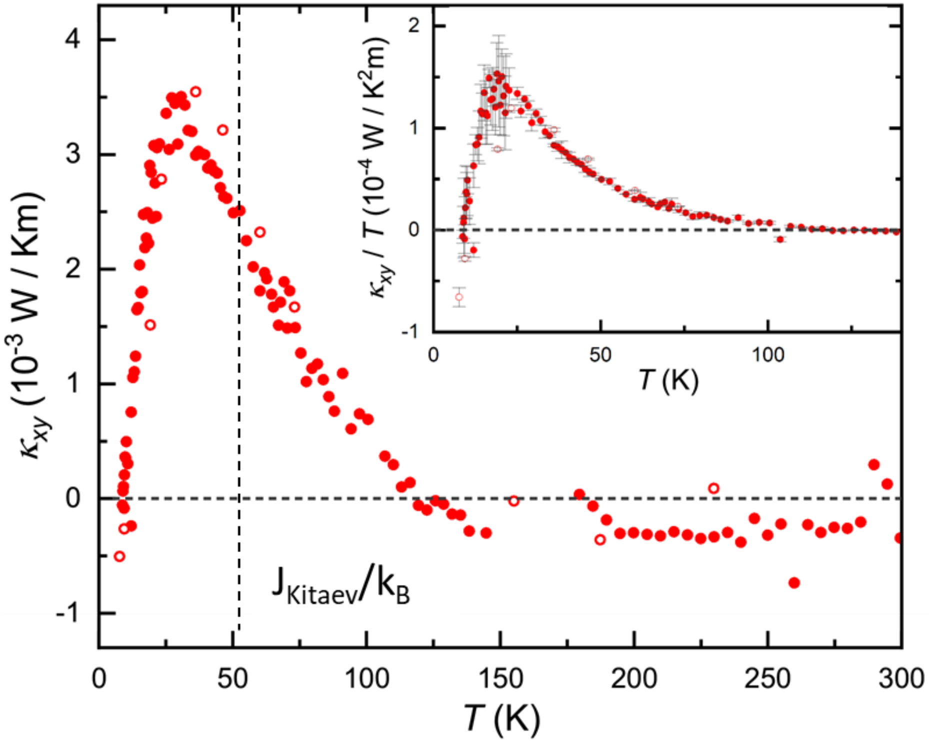
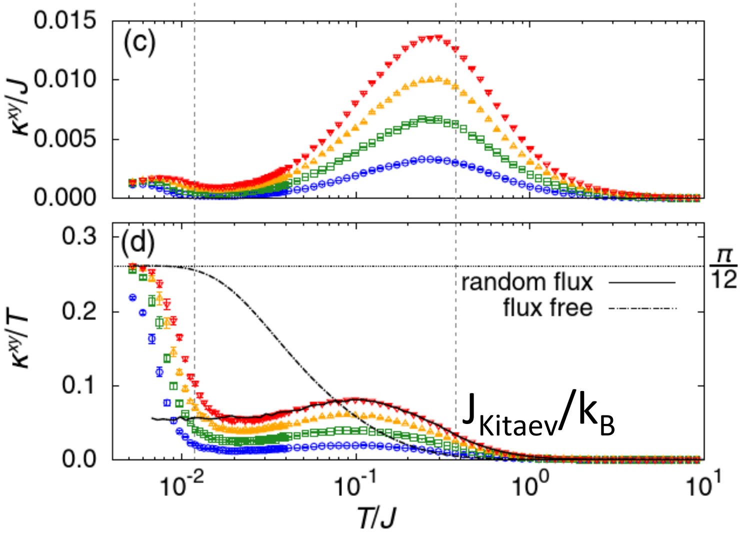


QMC-Results



Nasu, Yoshitake, Motome, PRL 2017

Large (!) positive $\kappa_{xy} \propto B$ in paramagnetic phase

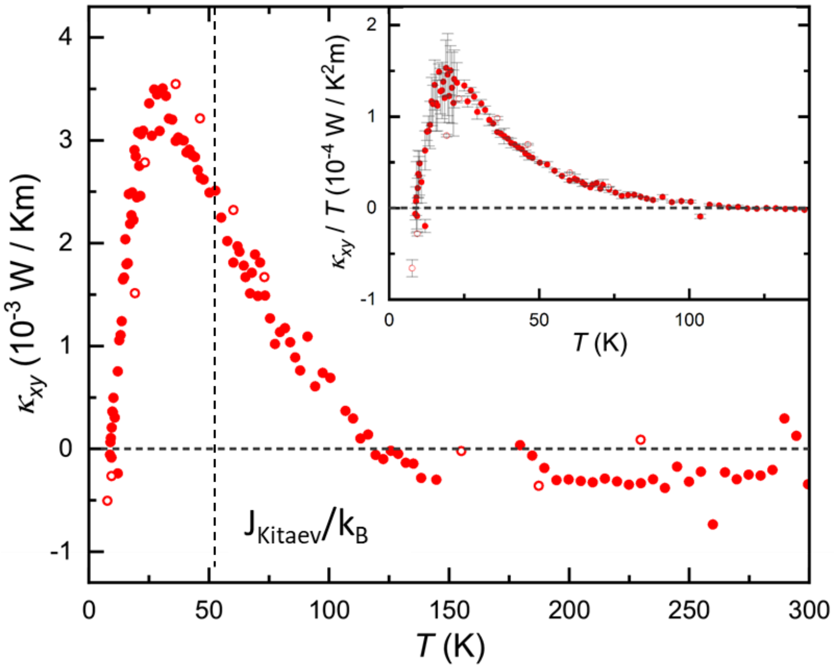
→ **Transport by Kitaev-Heisenberg paramagnons?**

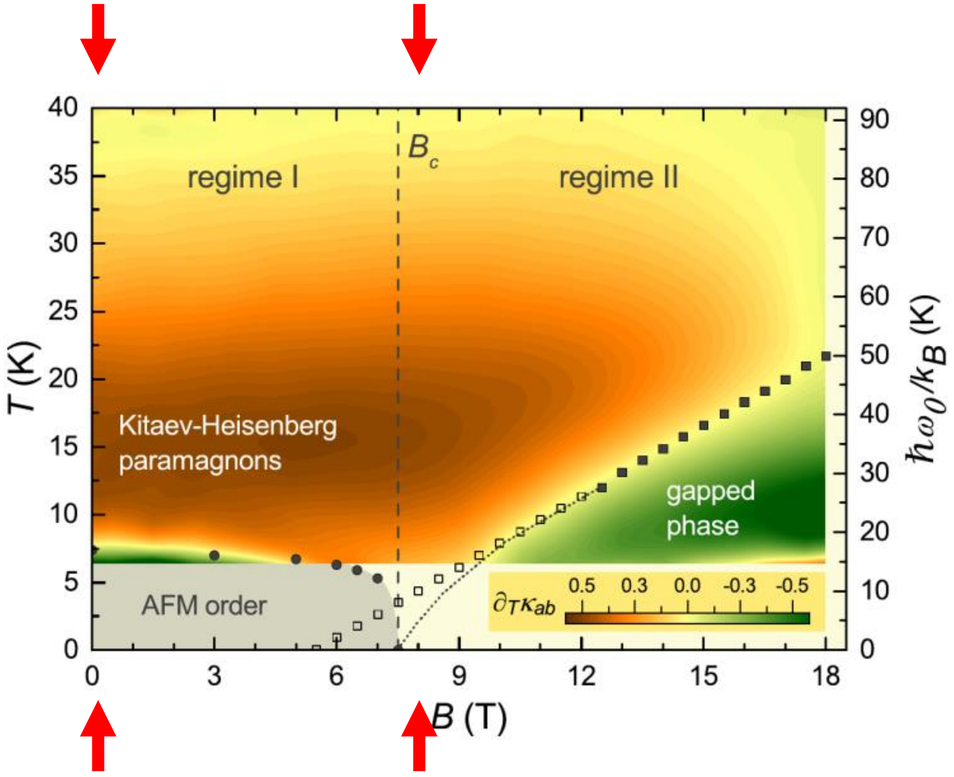
R. Hentrich, et al., PRB **99**, 085136 (2019)

See also: Kasahara et al., PRL 2018

Open questions

Temperature dependence



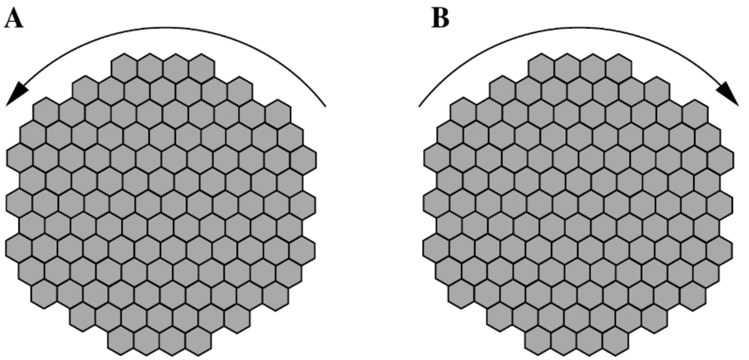


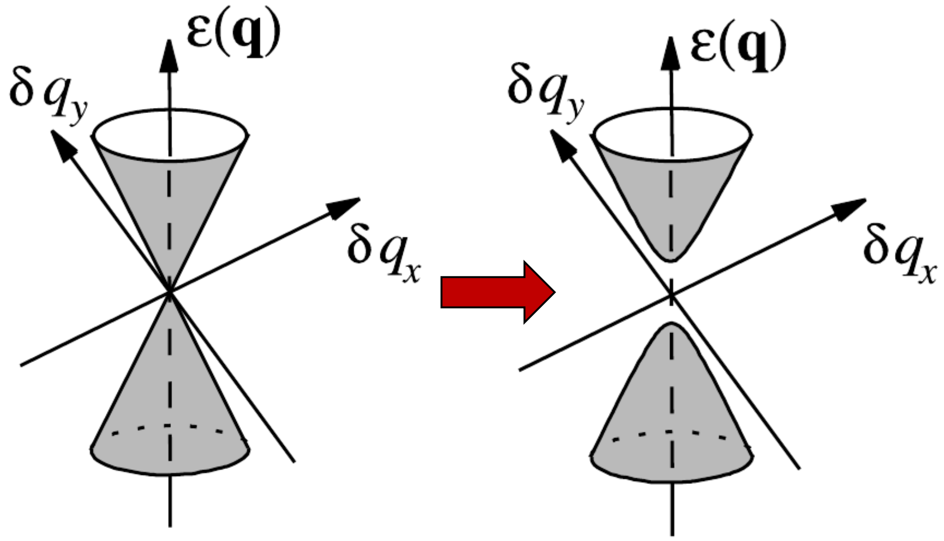
- Kitaev-Heisenberg paramagnons = Majorana Fermions?
- Temperature dependence of $\boldsymbol{\kappa}_{xy}$?
- Phononic origin?
- Evolution upon suppressing long-range order → edge state?

Thermal Transport in the Kitaev Model

Kitaev model in magnetic field:

-> quantized edge currents





-> quantized transversal heat conductivity

$$\kappa_{xy}/T = \frac{\pi}{12}\frac{k_B^2}{\hbar} \text{ for } T \ll \Delta$$

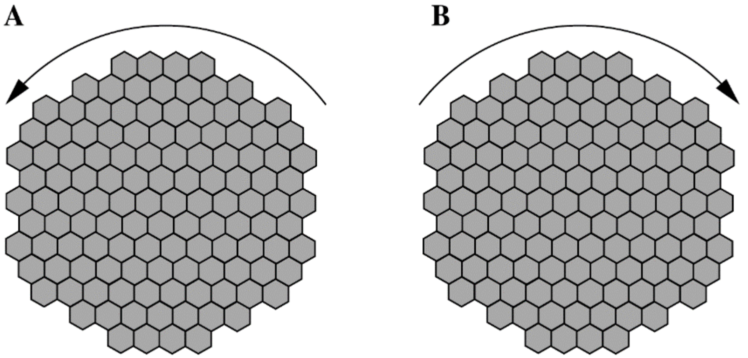
A. Kitaev, Ann. Phys. **321**, 2 (2006)

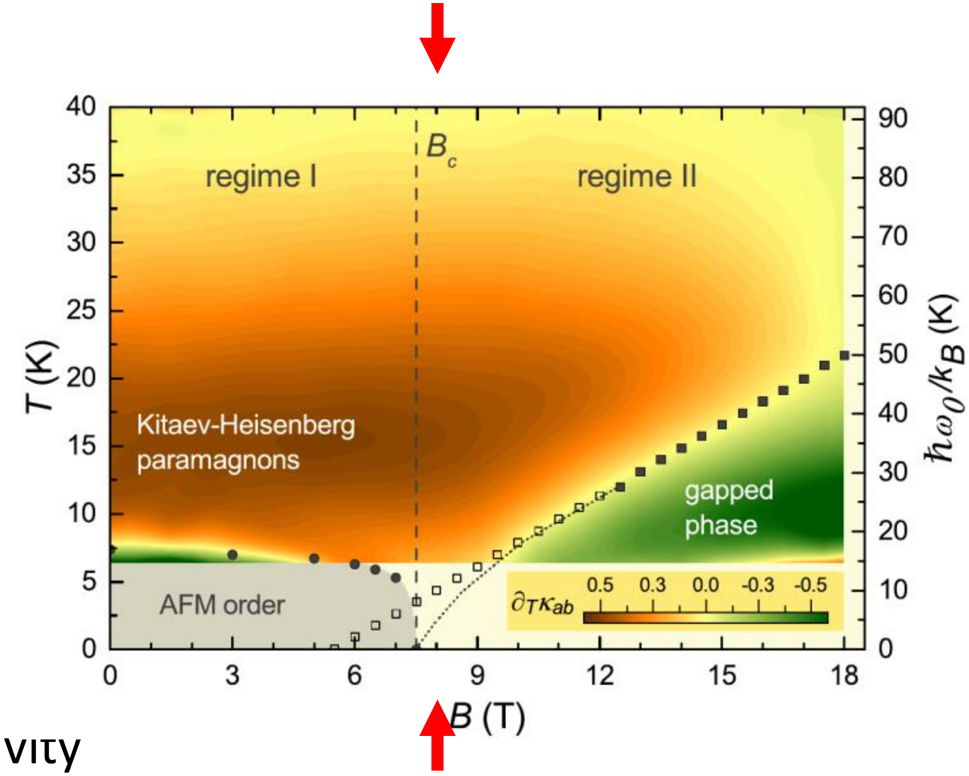
Fractional Thermal Hall effect!?

Thermal Transport in the Kitaev Model

Kitaev model in magnetic field:

-> quantized edge currents





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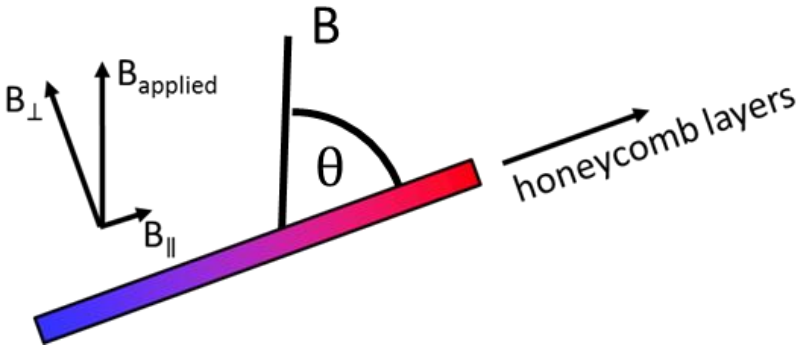
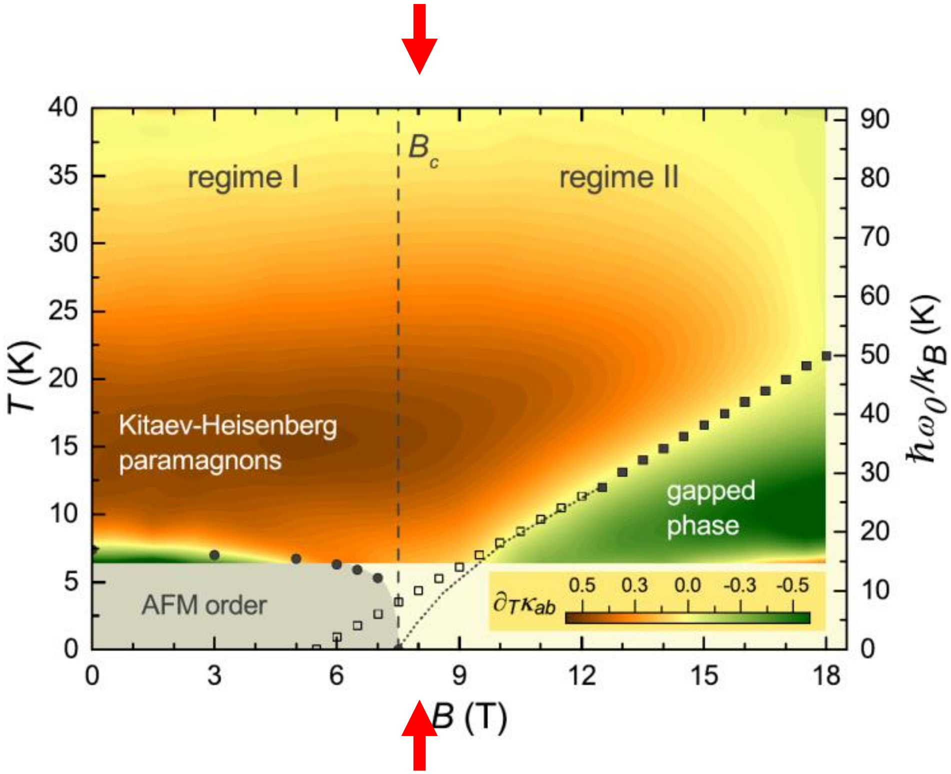
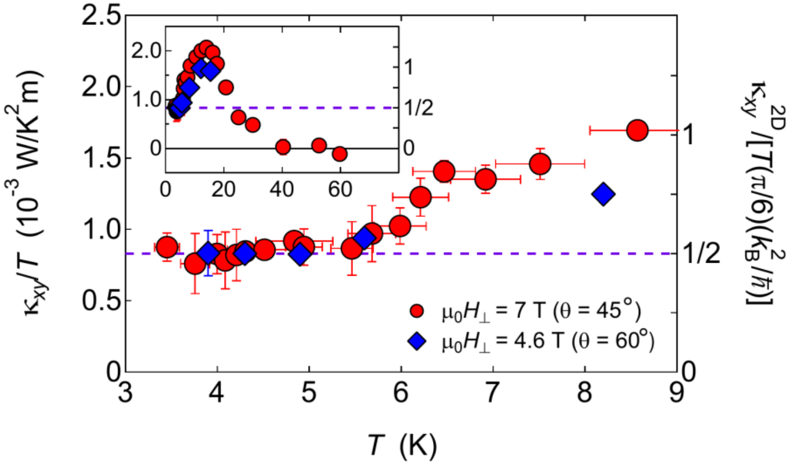
Evidence for Majorana edge states?





Kasahara et al., Nature 2018:



…to be confirmed

Fractional Thermal Hall effect!?

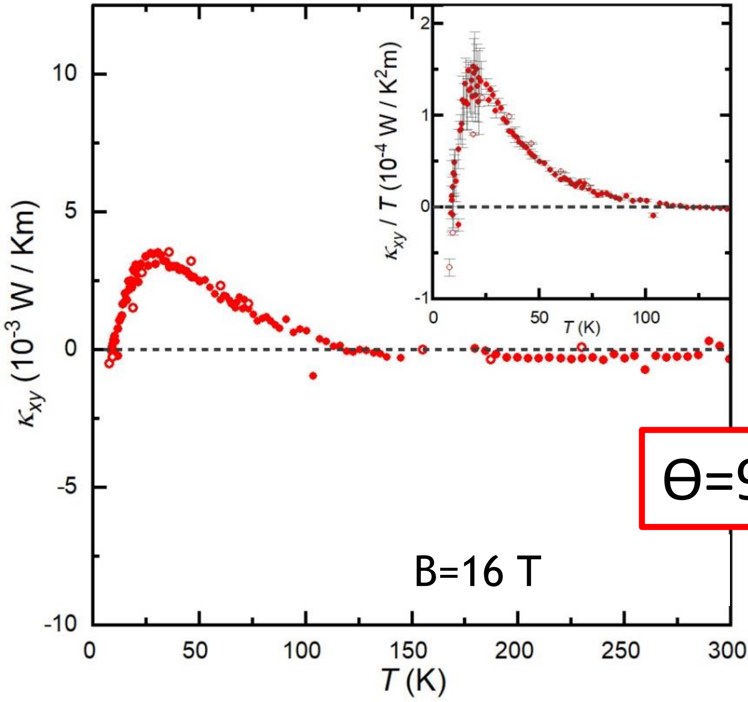
Evidence for Majorana edge states?

To do:

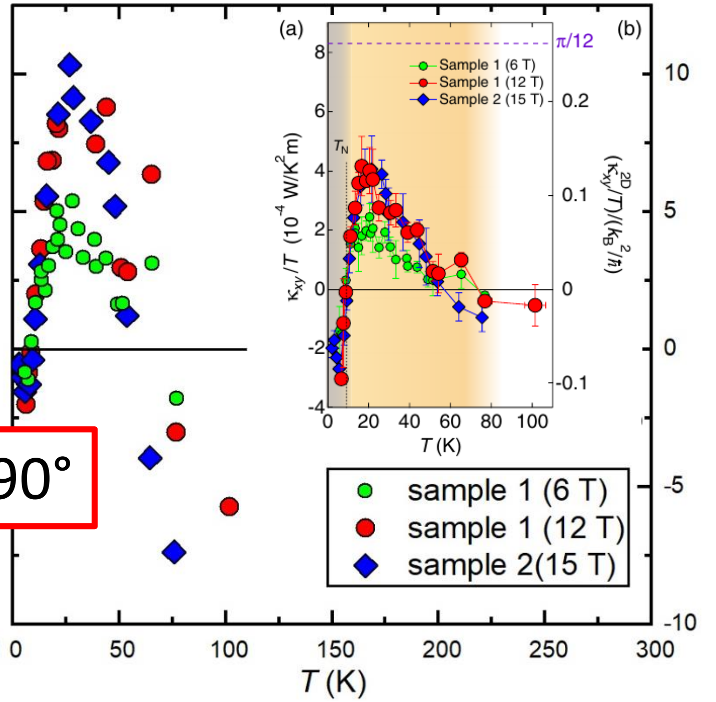
1) Confirm plateau at tilted magnetic field: **<u>difficult</u>**

2) Confirm fractionalized value $\kappa_{xy}/T \xrightarrow[T\ll\Delta]{} \boxed{\frac{1}{2}}\frac{\pi k_B^2}{6\hbar}$: **<u>difficult</u>**

R. Hentrich et al., PRB **99**, 085136 (2019)



Y. Kasahara et al., PRL **120**, 217205 (2018)



Conclusions

Large spin heat transport in S=1/2 Heisenberg magnets

➡ 2D magnon heat transport in La_2CuO_4

➡ Ballistic heat transport of 1D spinons in $SrCuO_2$

Thermal transport in Kitaev material α-$RuCl_3$

➡ Phononic κ_{xx} (mostly), strong phonon-spin scattering

➡ Sizeable thermal Hall effect (!)
→ heat transport by Majorana fermions?