Nonequilibrium skyrmion dynamics : Driving the magnon reservoir with time periodic fields

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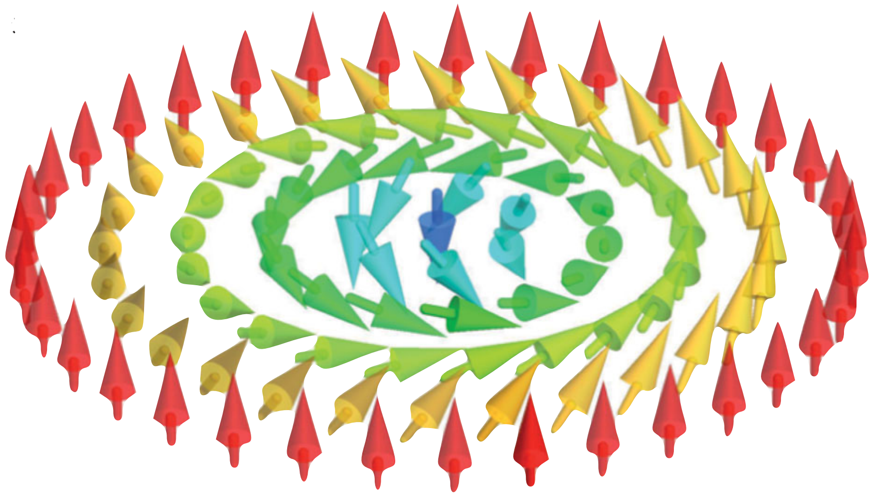
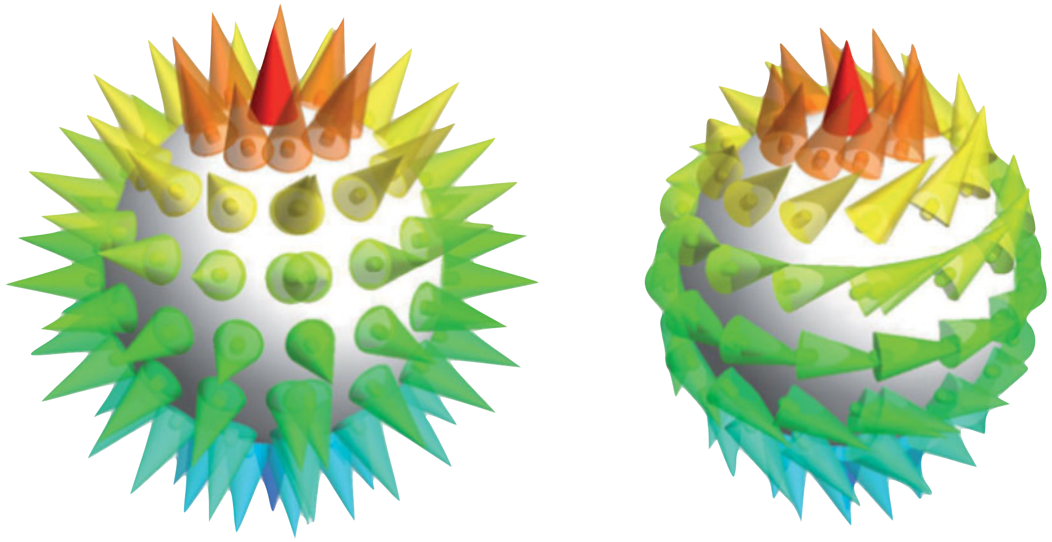
Spin and Heat Transport in Quantum and Topological Materials



Real Space



Spin Space

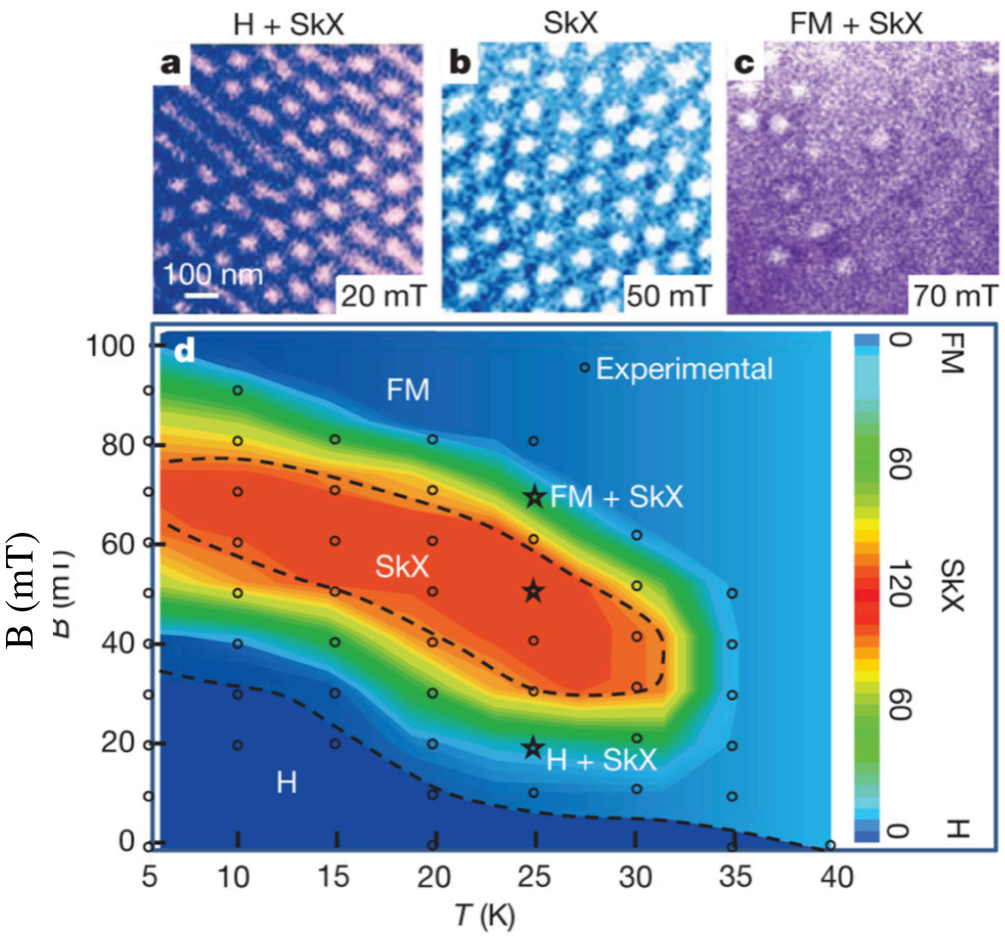
Figure from: Christian Pfleiderer Nature Physics **7***, 673–674 (2011)*

Topological number: $$Q = \frac{1}{4\pi}\int d\mathbf{r}\ \mathbf{m}\cdot(\partial_x\mathbf{m}\times\partial_y\mathbf{m})$$

- Theoretical prediction — N. Bogdanov and A. Hubert, J. Magn. Magn. Mater **138**, 255 (1994)
- First Experimental Detection — S. Mühlbauer, et al., Science **323**, 915 (2009)

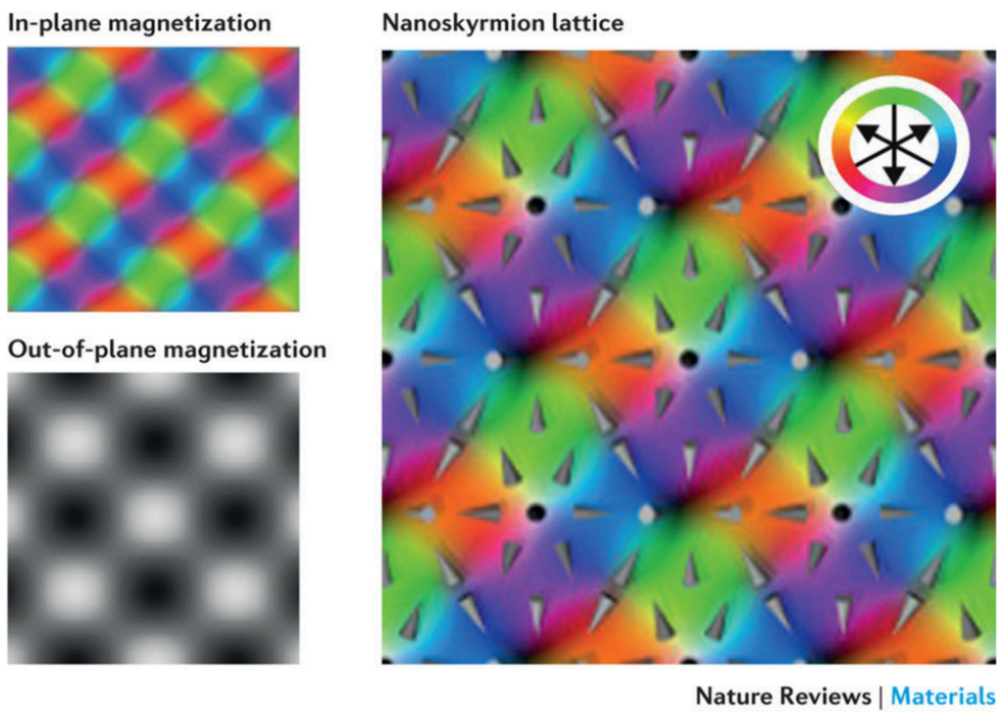
Skyrmion Lattices

$Fe_{0.5}Co_{0.5}Si$ with lattice spacing 90 nm



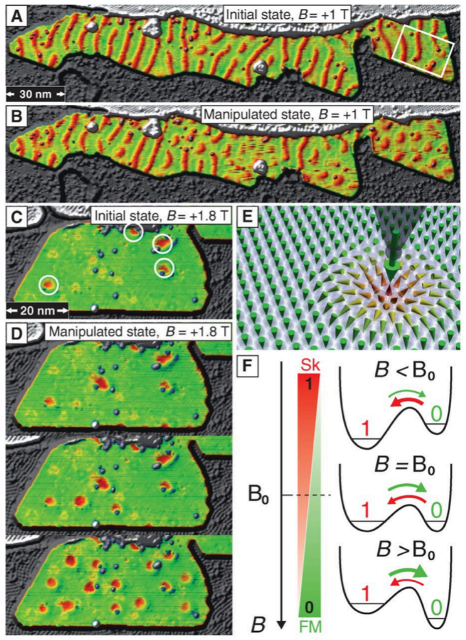
X. Z. Yu, *et al.*, Nature 465, 901 (2010).

Fe ML on Ir(111) with lattice spacing 1 nm

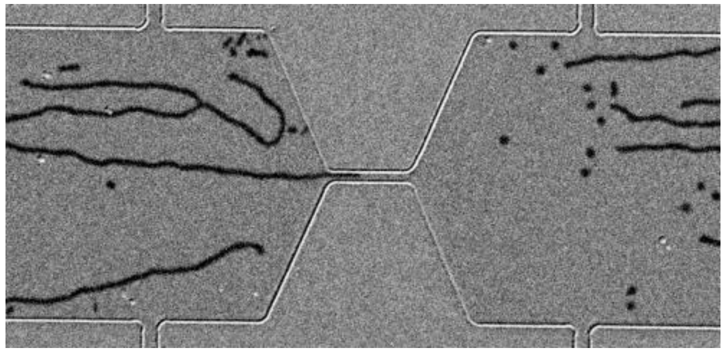


S. Heinze, *et al.*, Nature Physics **7**, 713 (2011).

Individual Skyrmions



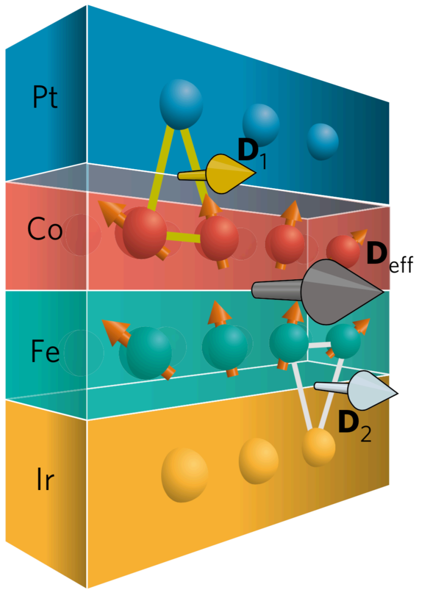
N. Romming, et al.,
Science **341**, 636 (2013).



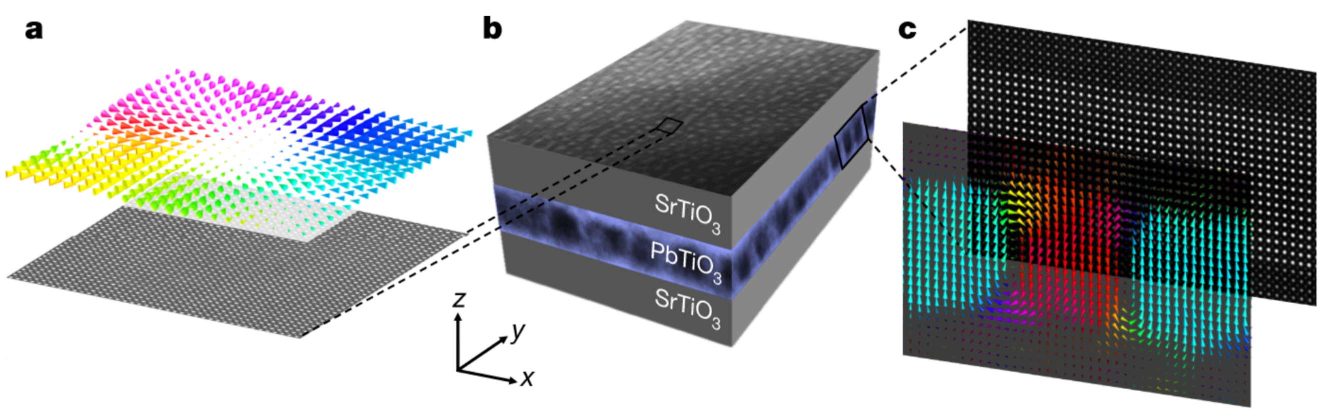
W. Jiang, *et al.*,
Science **349**, 283 (2015).

Stability at room temperature

Skyrmion lattice in multilayers



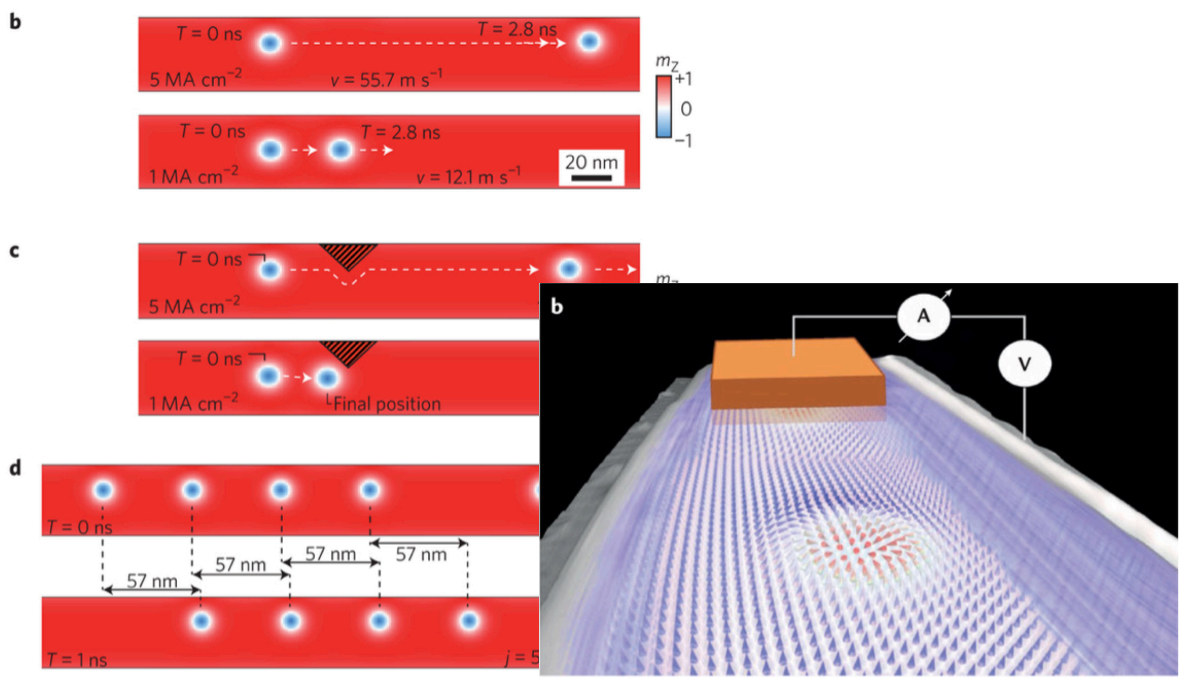
A. Soumyanarayanan, *et al.*, Nature Mater. **16**, 898–904 (2017).



S. Das, *et al.*, Nature **568**, 368–372 (2019).

Applications

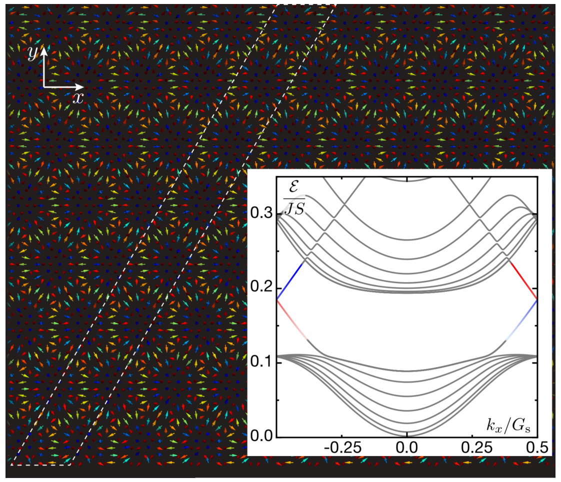
Skyrmion racetrack memory



A. Fert, et al., Nature Nan. **8**, 152 (2013).

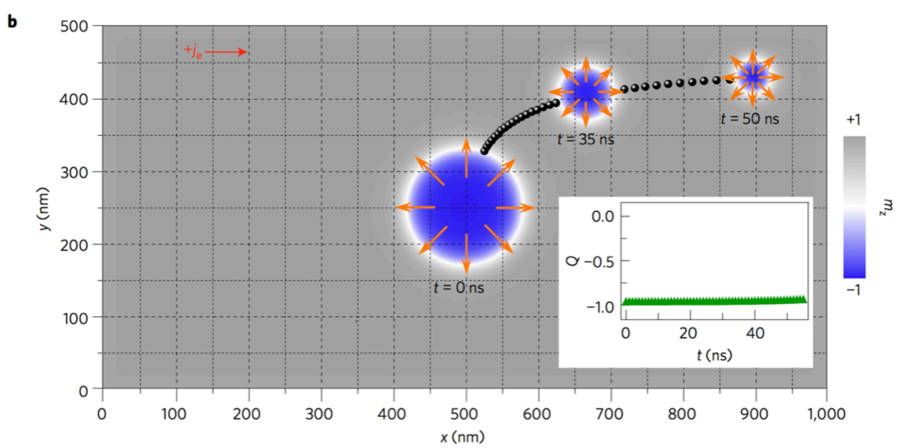
S. Parkin, *et al.*, Science **320**, 197202 (2009).

Topological Magnons



Sebastian A. Diaz, et al., Phys. Rev. Lett. **122**, 187203 (2019)

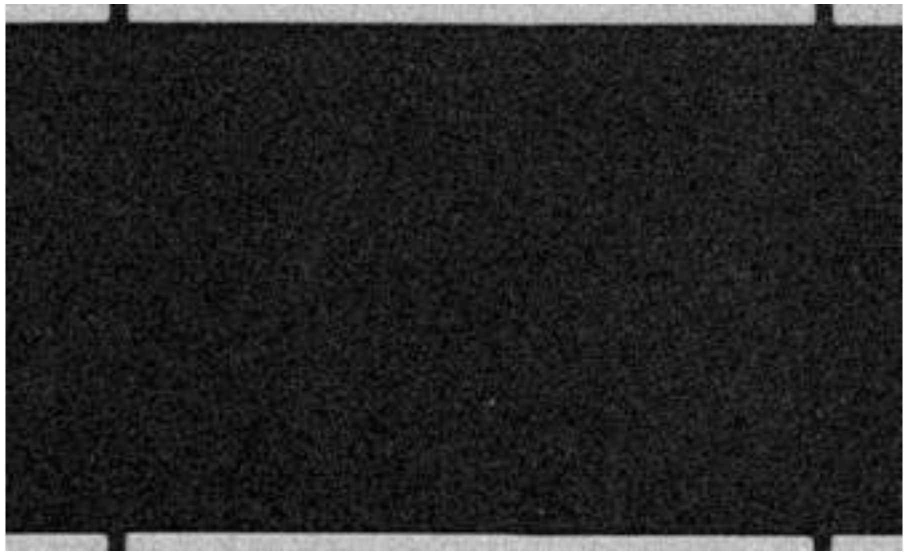
Skyrmion Hall effect

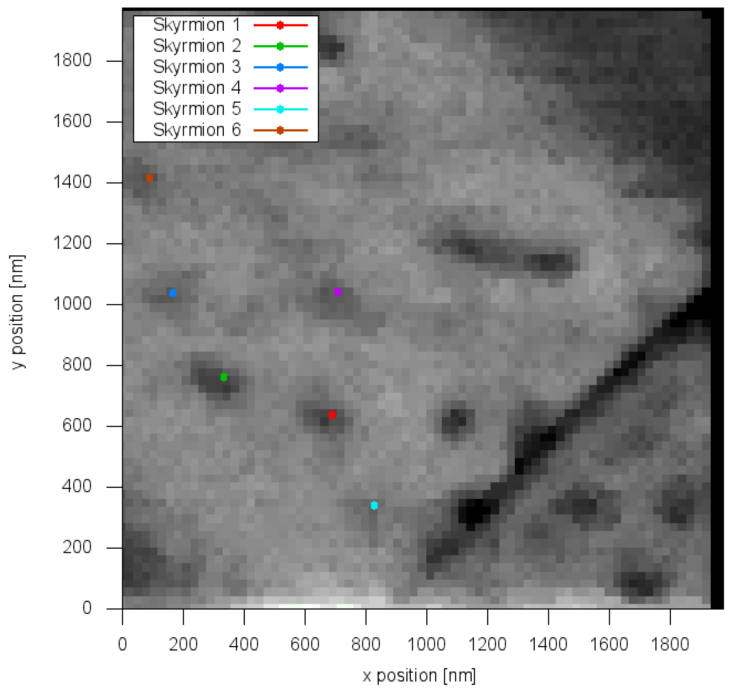


W. Jiang, et al., Nature Physics **13**, 162 (2017)

K. Litzius, et al., Nature Physics **13**, 170 (2017)

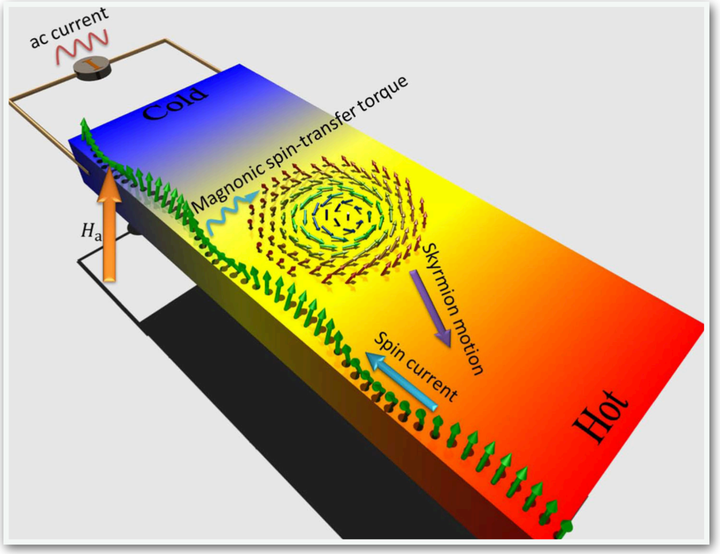
Metals

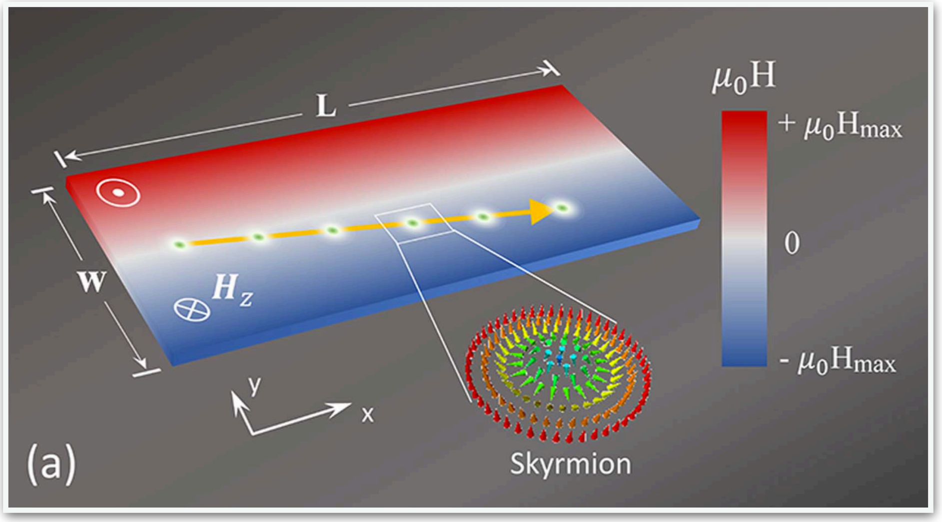




W. Jiang, *et al.*, Nature Physics **13**, 162 (2017). K. Litzius, *et al.*, Nature Physics **13**, 170 (2017).

Insulators





S.-Z. Lin, *et al.*, *Phys. Rev. Lett.* **112**, 187203 (2014) C. Wang, *et al.*, New J. Phys. **19** 083008 (2017)

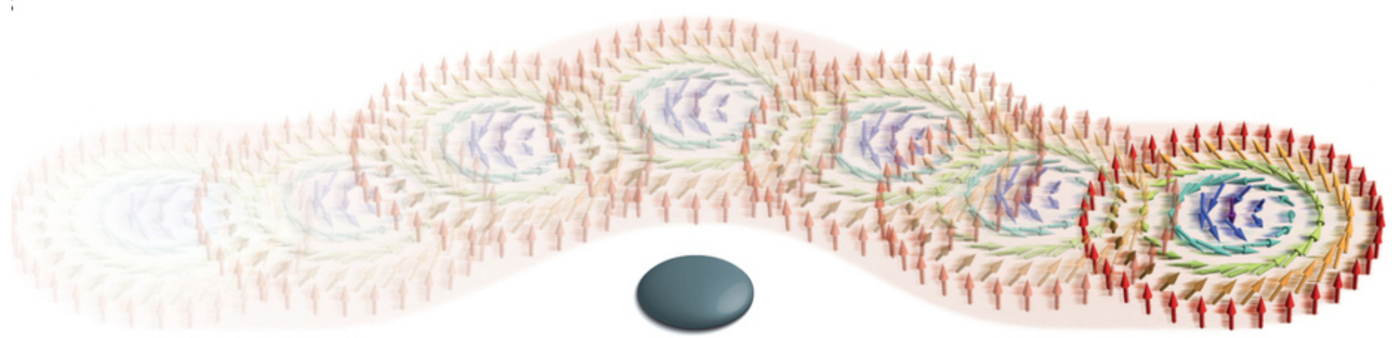
Phenomenological Dissipation

- Landau- Lifshitz-Gilbert equation

$$\frac{d\mathbf{m}}{dt} = -\frac{a^2}{S^2}\mathbf{m} \times \frac{\delta \mathcal{F}}{\delta \mathbf{m}} + \underbrace{\alpha \mathbf{m} \times (\mathbf{m} \times \frac{\delta \mathcal{F}}{\delta \mathbf{m}})}_{\textbf{Damping}}$$

T. L. Gilbert, IEEE Trans. Magn., **40**, 3443 (2004)

- Skyrmion center of mass $\mathbf{R}(t)$



Skyrmion moving around obstacle

Figure from: Achim Rosch. Nature nanotechnology, 8(3):160–161, 2013.

- Thiele's equation of motion

A. A. Thiele, Phys. Rev. Lett. **30**, 230 (1973).

$$\mathbf{G} \times \dot{\mathbf{R}} + \alpha \mathcal{D} \dot{\mathbf{R}} = \mathbf{F}_{\mathrm{ext}}$$

Magnus Force — $\mathbf{G} \parallel z \ , |\mathbf{G}| \sim Q$

Damping → electrons, magnons, phonons

External Forces

Microscopic Dissipation

- Langevin Equation (*1D massive*)

$$M\ddot{q}(t) + M\int_{-\infty}^{t}\gamma(t-t')\dot{q}(t') + V'(q) = \xi(t)$$

U. Weiss, *Quantum Dissipative Systems*, World Scientific (2008)

Dissipation kernel $\gamma(\omega) \propto \omega^{s-1}$

$0 < s < 1$	sub-Ohmic
$s = 1$	Ohmic
$s > 1$	super-Ohmic

Microscopic Dissipation

- Langevin Equation (*1D massive*)

$$M\ddot{q}(t) + M\int_{-\infty}^{t}\gamma(t-t')\dot{q}(t') + V'(q) = \xi(t)$$

Dissipation kernel

$$\gamma(\omega) \propto \omega^{s-1}$$

$0 < s < 1$ sub-Ohmic

$s = 1$ Ohmic

$s > 1$ super-Ohmic

Magnetization textures

1D Domain wall

$$M\ddot{X}(t) + \int_{-\infty}^{t}\eta(t-t')\dot{X}(t') = V'(X) + \xi(t)$$

H.-B. Braun and D. Loss, Phys. Rev. B **53**, 3237 (1996).

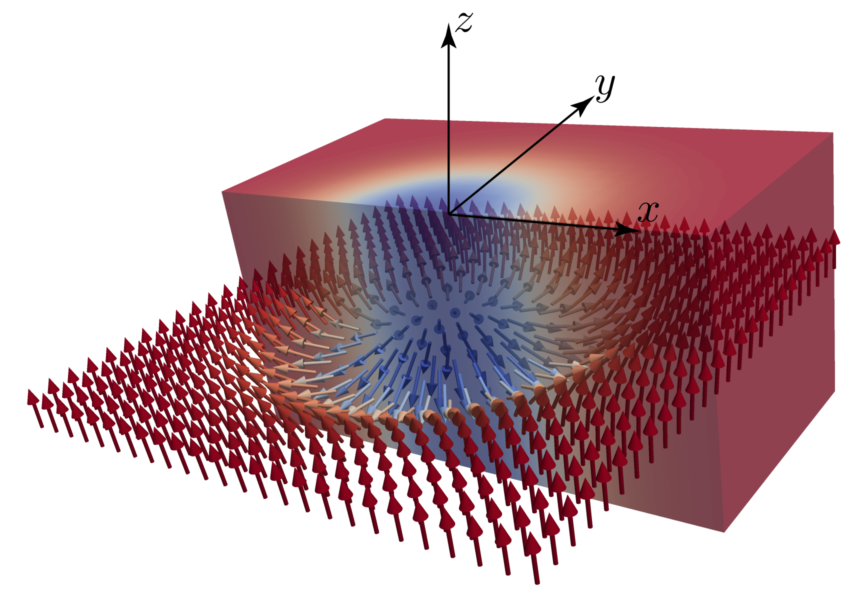
S. K. Kim, O.Tchernyshyov, V. Galitski, and Y. Tserkovnyak, Phys. Rev. B **97**, 174433 (2018).

2D Skyrmion

$$\tilde{Q}_0\epsilon_{ij}\dot{R}_j + \int_{-\infty}^{t}\dot{R}_j(t')\gamma_{ji}(t-t') = F_i(t) + \xi(t)$$

C. Psaroudaki, S. Hoffman, J. Klinovaja, and D. Loss, Phys. Rev. X **7**, 041045 (2017).

Microscopic Dissipation



Magnetization

$$\mathbf{m} = [\sin\Theta\cos\Phi, \sin\Theta\sin\Phi, \cos\Theta]$$

Action

$$\mathcal{S} = \int dt d\mathbf{r} [\frac{SN_A}{\alpha^2}\dot{\Phi}(\Pi - 1) - N_A \mathcal{F}(\Phi, \Pi)]$$

Partition function

$$Z = \int \mathcal{D}\Phi \mathcal{D}\cos\Theta e^{i\mathcal{S}}$$

$$\mathbf{m}(\mathbf{r},t) = \mathbf{m}_0[\mathbf{r} - \mathbf{R}(t)] + \chi[\mathbf{r} - \mathbf{R}(t), t]$$

→

QFT/Keldysh toolbox

Partition function

$$Z = \int \mathcal{D}\mathbf{R} e^{i\mathcal{S}_{\mathrm{eff}}(\mathbf{R})}$$

$$\tilde{Q}_0 \epsilon_{ij} \dot{R}_j + \int_{-\infty}^{t} \dot{R}_j(t')\gamma_{ji}(t - t') = F_i(t) + \xi(t)$$

Local in time → $\propto \mathcal{M}\omega^2 R_i(\omega)$

$\omega \ll \epsilon_{\mathrm{gap}}$

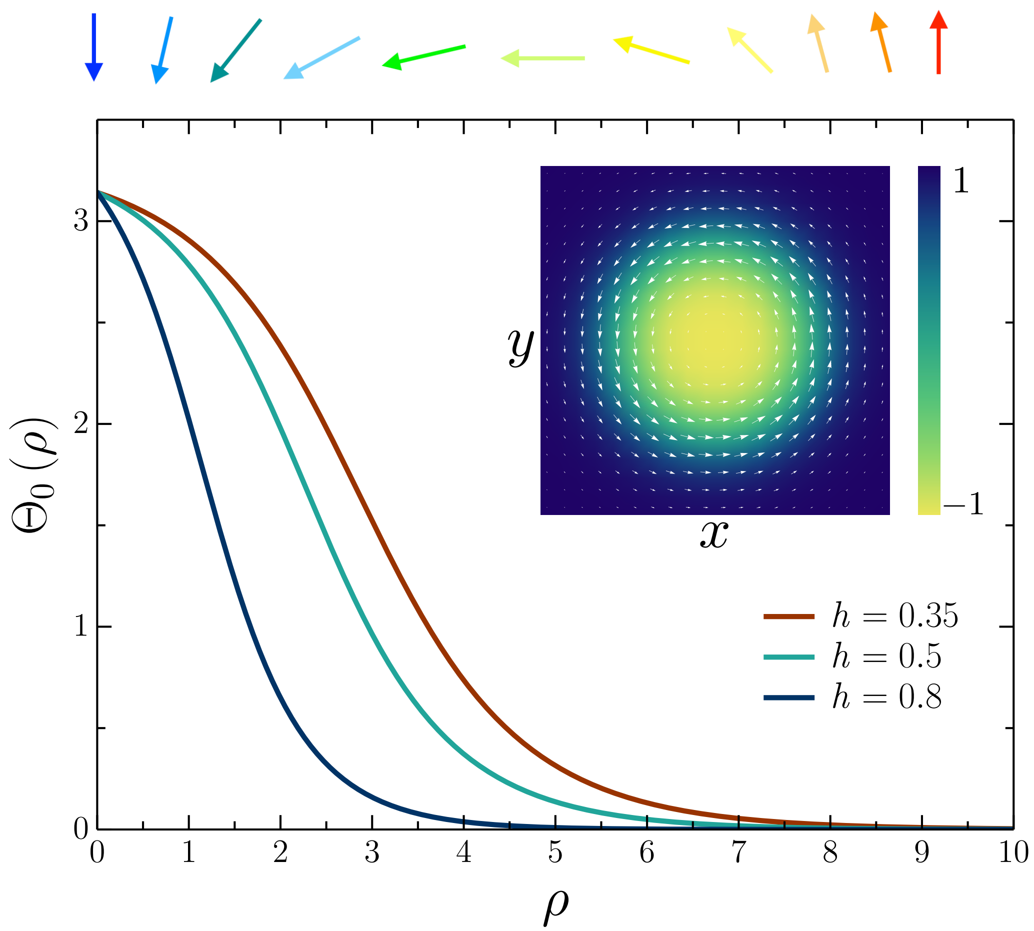
$T \ll \epsilon_{\mathrm{gap}}$

C. Psaroudaki, et al., Phys. Rev. X, **7** 041045 (2017)

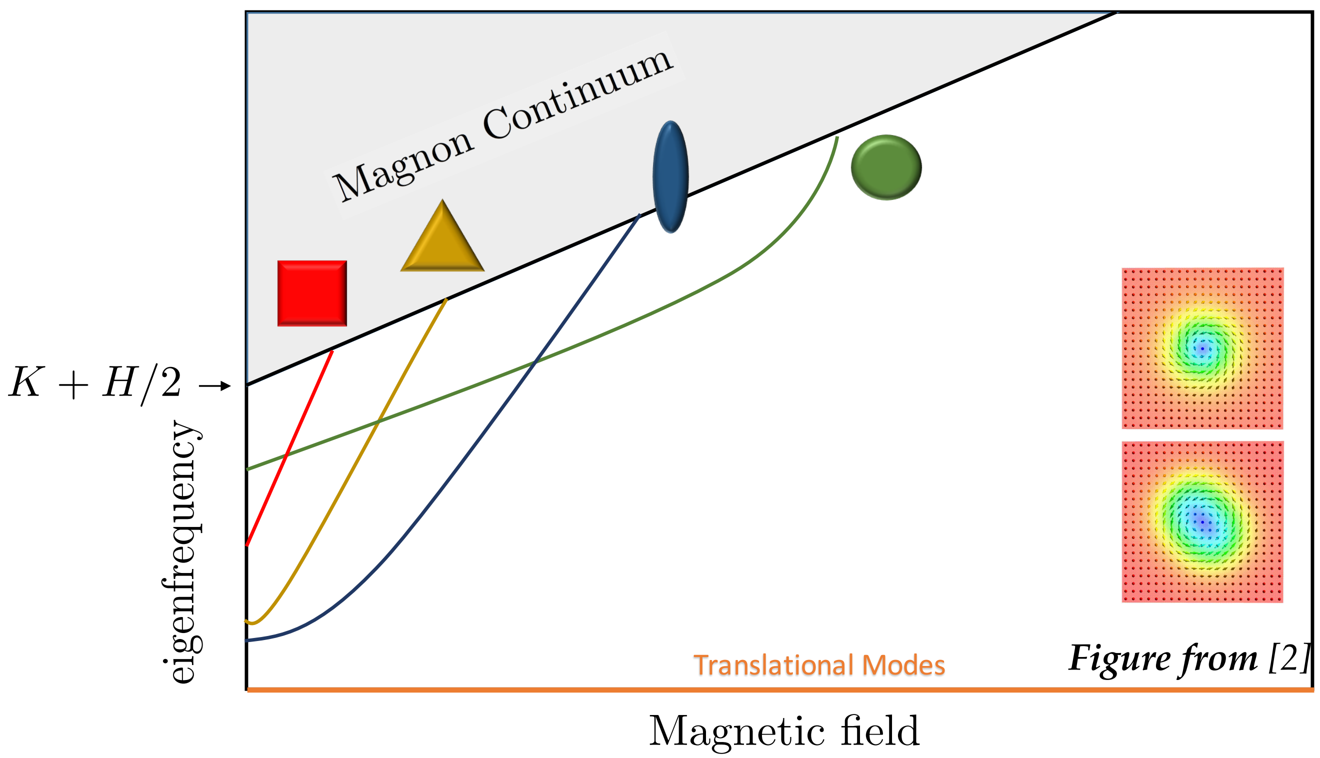
Chiral Magnets

Free Energy

$$\mathcal{F}(\mathbf{m}) = J \sum_{i=x,y} \left(\frac{\partial \mathbf{m}}{\partial \tilde{r}_i} \right)^2 + D\mathbf{m} \cdot \nabla \times \mathbf{m} - K m_z^2 - H m_z$$



$$H\Psi_n = \varepsilon_n \sigma_z \Psi_n$$



$$\Phi_0(\rho, \phi) = \phi + \pi/2$$

$$\Theta_0(\rho) = 2\tan^{-1}\left(\frac{\lambda}{\rho} e^{\frac{\rho-\lambda}{\rho_0}}\right)$$

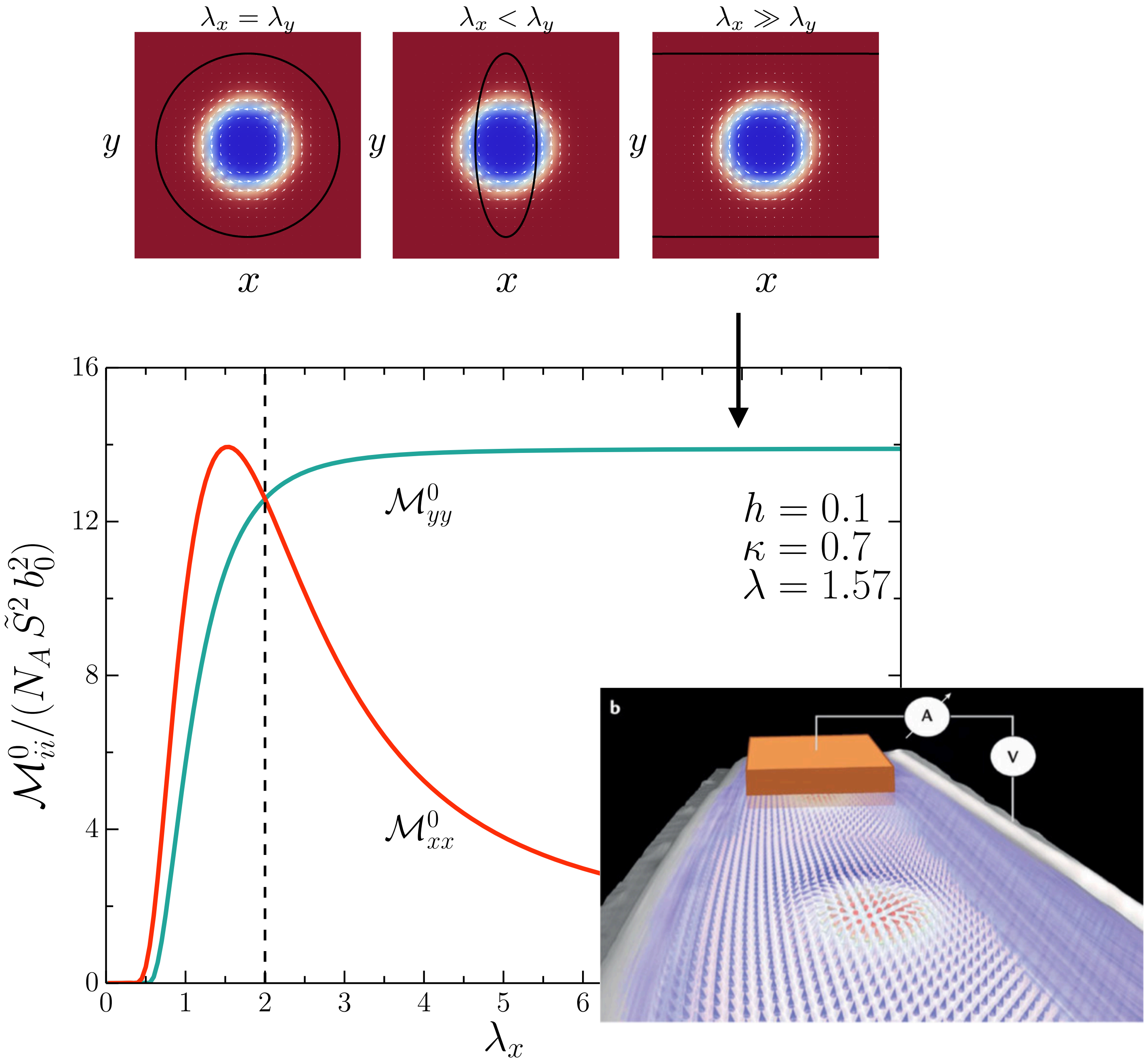
[1] S. Lin, *et al.*, Phys. Rev. B **89,** 024415 (2014)

[2] C. Schütte, and M. Garst, Phys. Rev. B **90** 094423 (2014).

Skyrmion Mass

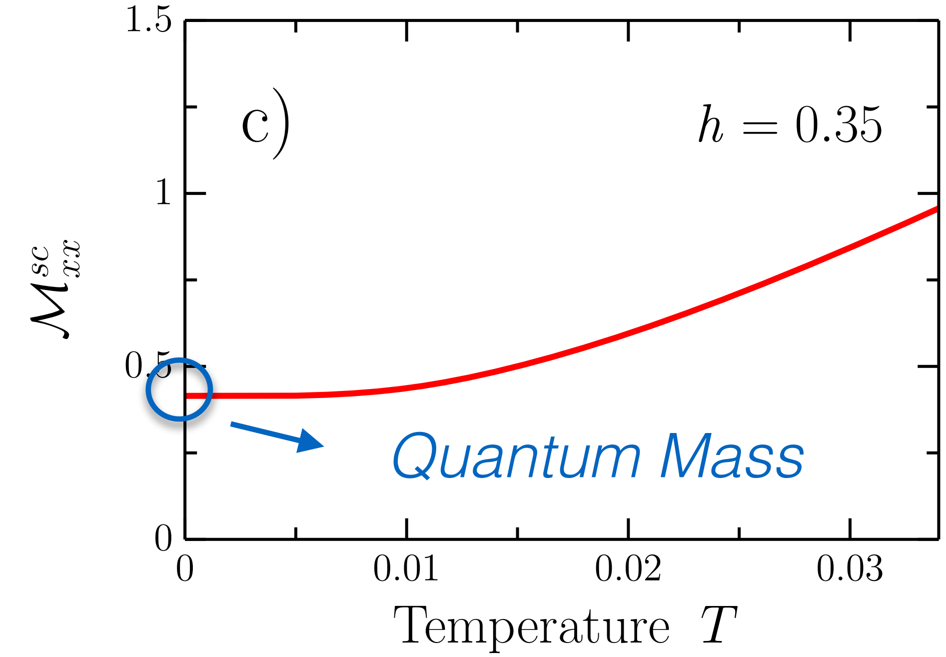
Semiclassical Mass

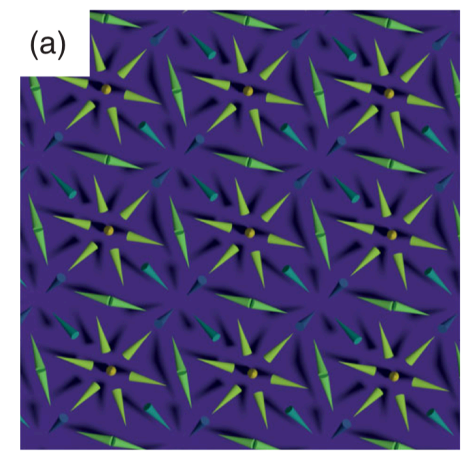
$$\mathcal{M}^0_{ij} = N_A \bar{S} \sum_n{}' \frac{\mathcal{A}^n_{ij}}{\bar{\varepsilon}_n}$$



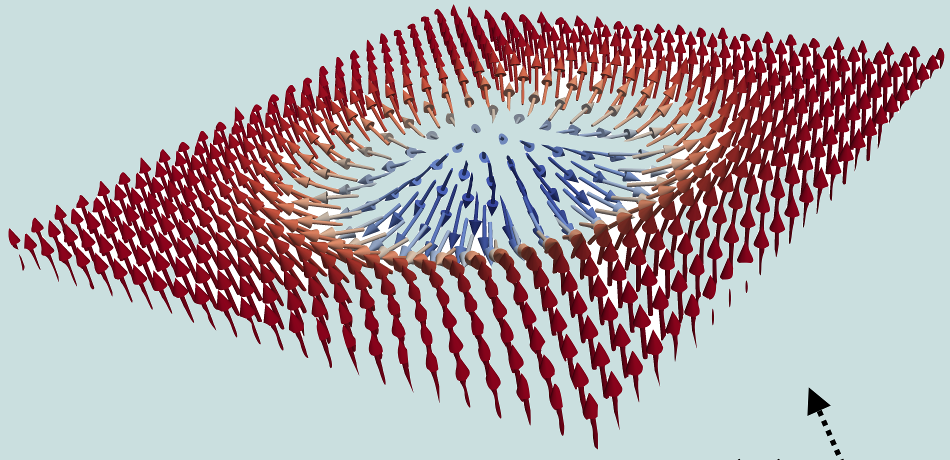
Quantum Mass

$$\mathcal{M}^T_{ij} = \sum_{m,n}{}' \mathcal{B}^{n,m}_{ij} \frac{\coth(\frac{\beta\bar{\varepsilon}_m}{2}) - \coth(\frac{\beta\bar{\varepsilon}_n}{2})}{\bar{\varepsilon}_n - \bar{\varepsilon}_m}$$



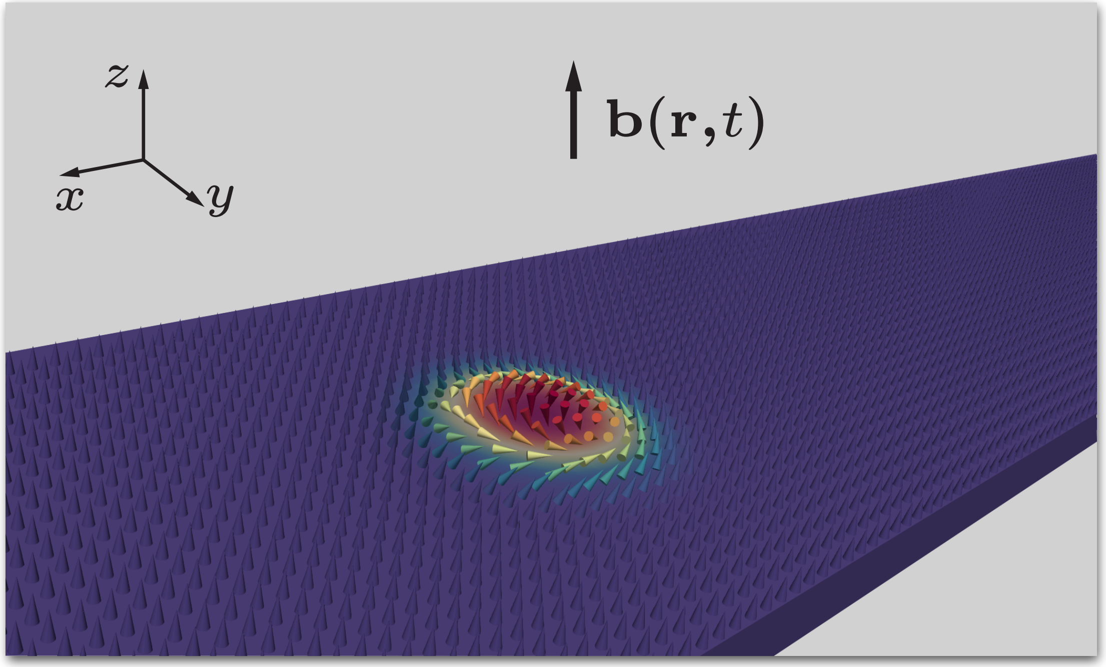


J. Grenz et al., Phys. Rev. Lett. **119**, 047205 (2017).



$\mathbf{F}_{\text{ext}}(t)$ + Magnon Bath ?

Driving Field

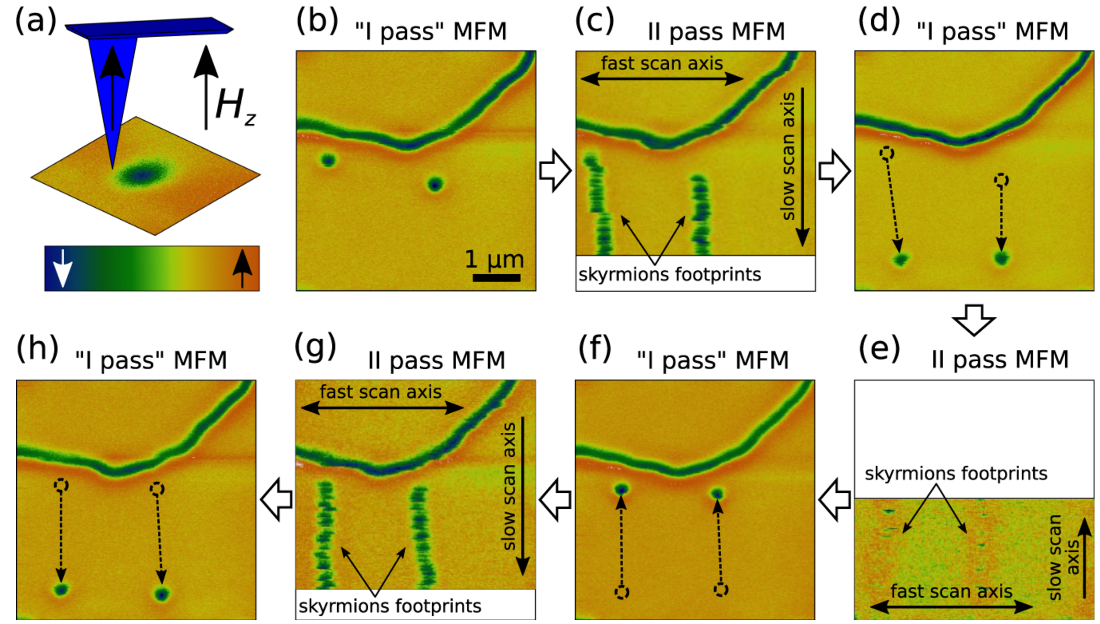


Time-periodic magnetic field gradient

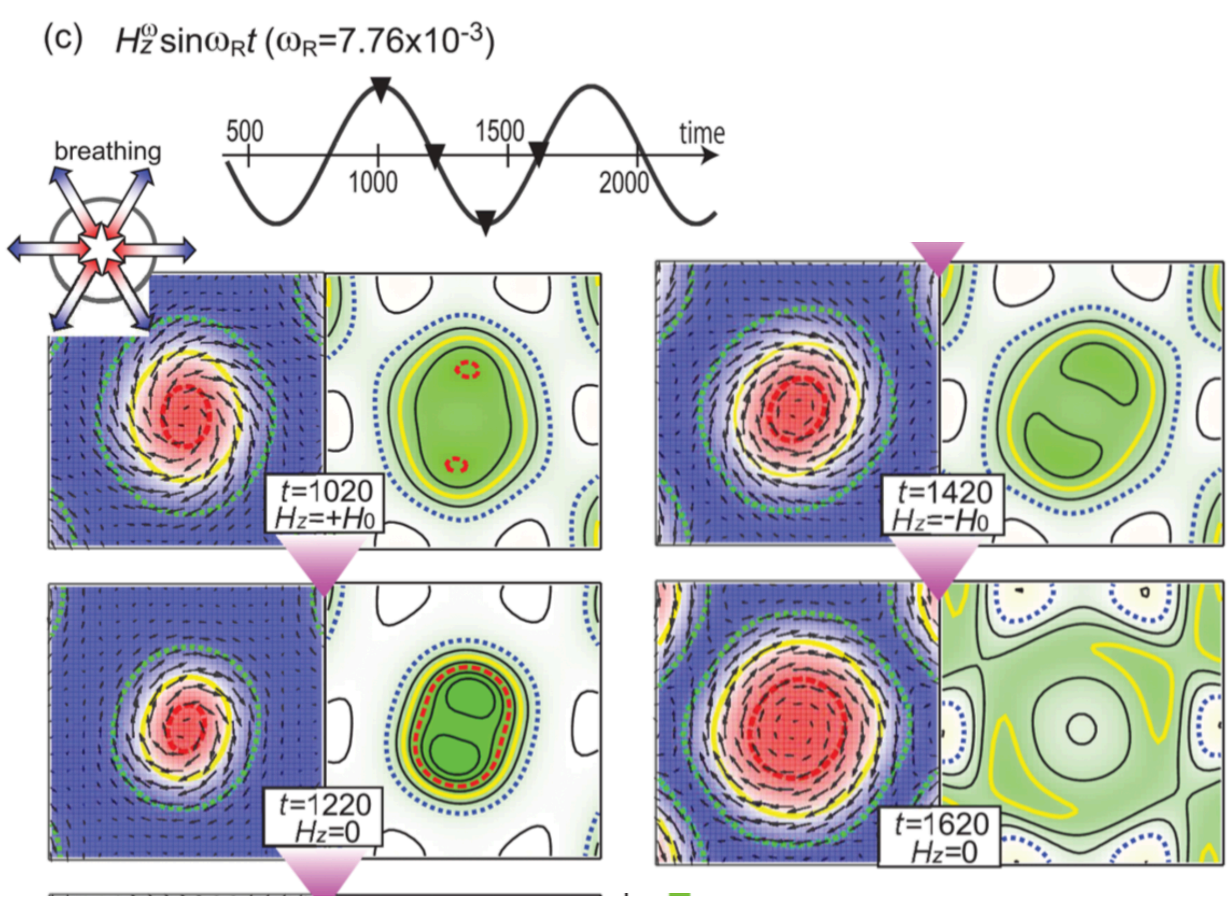
$$\mathbf{b}(\mathbf{r},t) = \Theta(t-t_0) h_z x \cos(\omega_{\text{ext}} t)\hat{z}$$

C. Psaroudaki and D. Loss, Phys. Rev. Lett. **120**, 237203 (2018)

Individual skyrmion manipulation by local magnetic field gradients



A. Casiraghi, *et al*. Commun. Phys. **2,** 145 (2019).



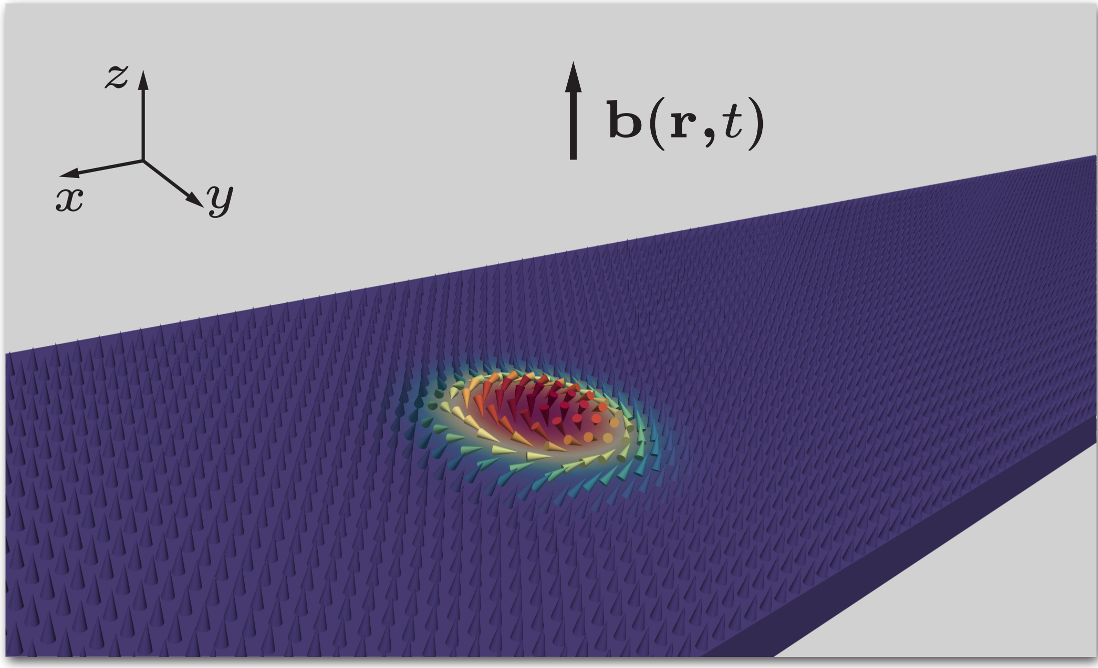
Breathing mode activation by out of plane microwave fields

M. Mochizuki, Phys. Rev. Lett. **108**, 017601 (2012)

Experimental observation in insulator Cu_2OSeO_3

Y. Onose, *et al*., Phys. Rev. Lett. **109**, 037603 (2012)

Skyrmions Driven by Intrinsic Magnons



Time-periodic magnetic field gradient

$$\mathbf{b}(\mathbf{r},t) = \Theta(t-t_0)h_z x\cos(\omega_{\text{ext}}t)\hat{z}$$

$$\mathcal{S}_B = \mathcal{S}_0 + N_A \int dt d\mathbf{r}\ \mathbf{b}(\mathbf{r},t)\cdot\mathbf{m}(\mathbf{r},t)$$

$$\tilde{Q}_0\epsilon_{ij}\dot{R}^j(t) + \int_{t_0}^{t} dt'\dot{R}^j(t')\gamma_{ji}(t,t') = F^i_{\text{ext}}(t) + \mathcal{K}_{ij}R^j(t)$$

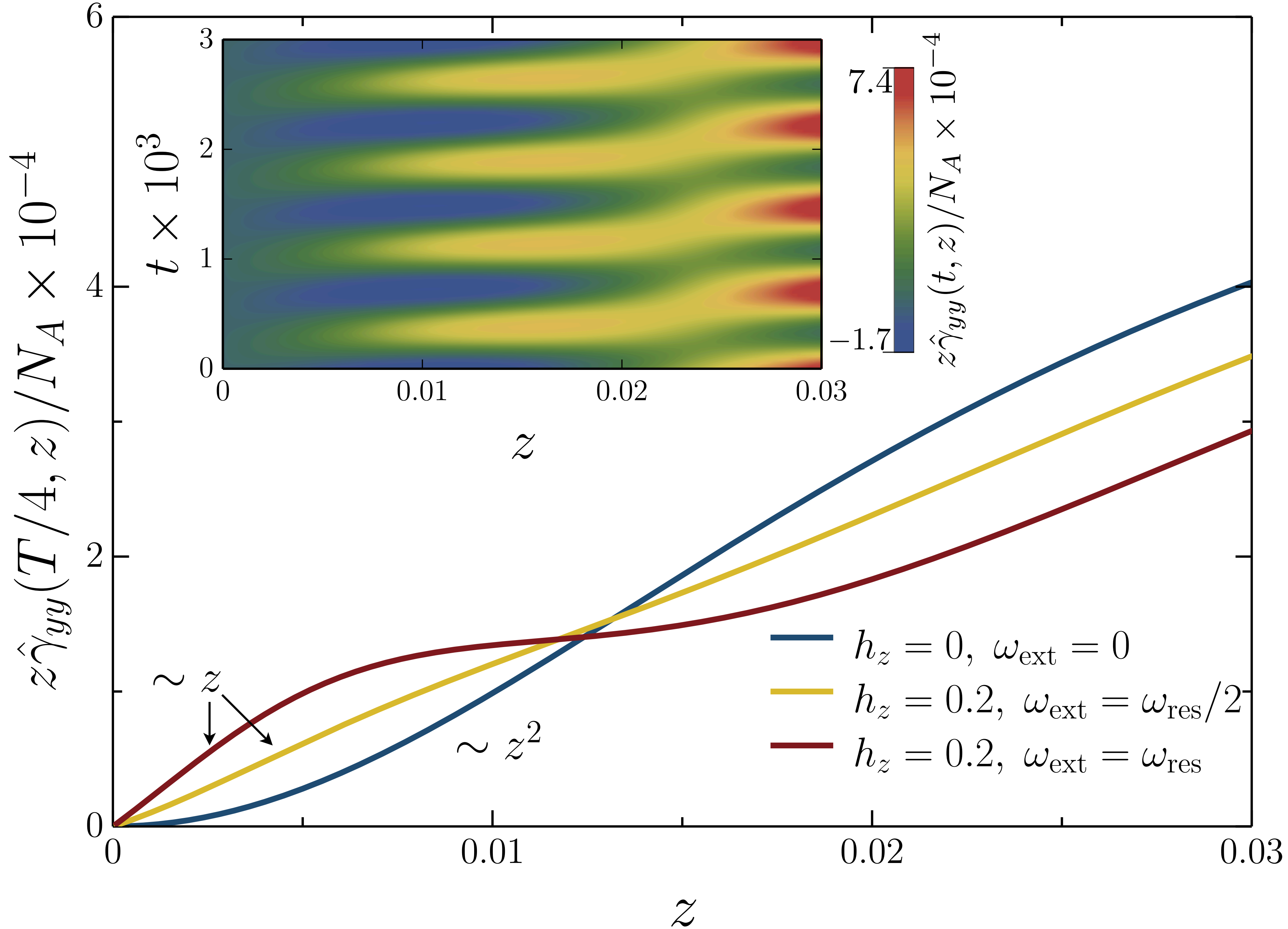
Driven Magnon Bath

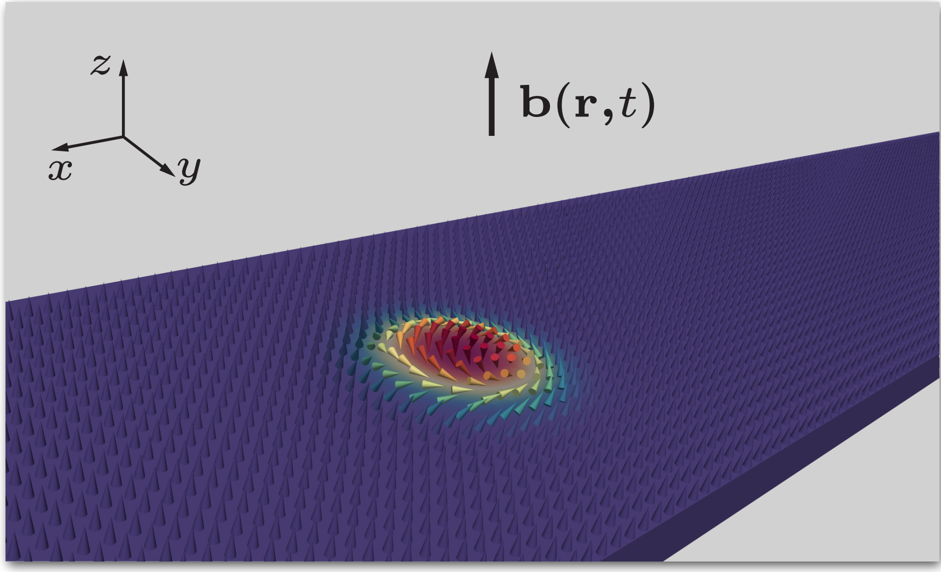
$$\gamma_{ji}(t,t') = \gamma^0_{ji}(t-t') + \Delta\gamma_{ji}(t,t-t')$$

C. Psaroudaki and D. Loss, Phys. Rev. Lett. **120**, 237203 (2018)

Super-Ohmic to Ohmic transition

$$\hat{\gamma}_{ij}(t,z) = \gamma_{ij}(t, \omega = iz)$$





$$\tilde{Q}_0 \epsilon_{ij} \dot{R}^j(t) + \int_{t_0}^{t} dt' \dot{R}^j(t') \gamma_{ji}(t,t') = F^i_{\text{ext}}(t) + \mathcal{K}_{ij} R^j(t)$$

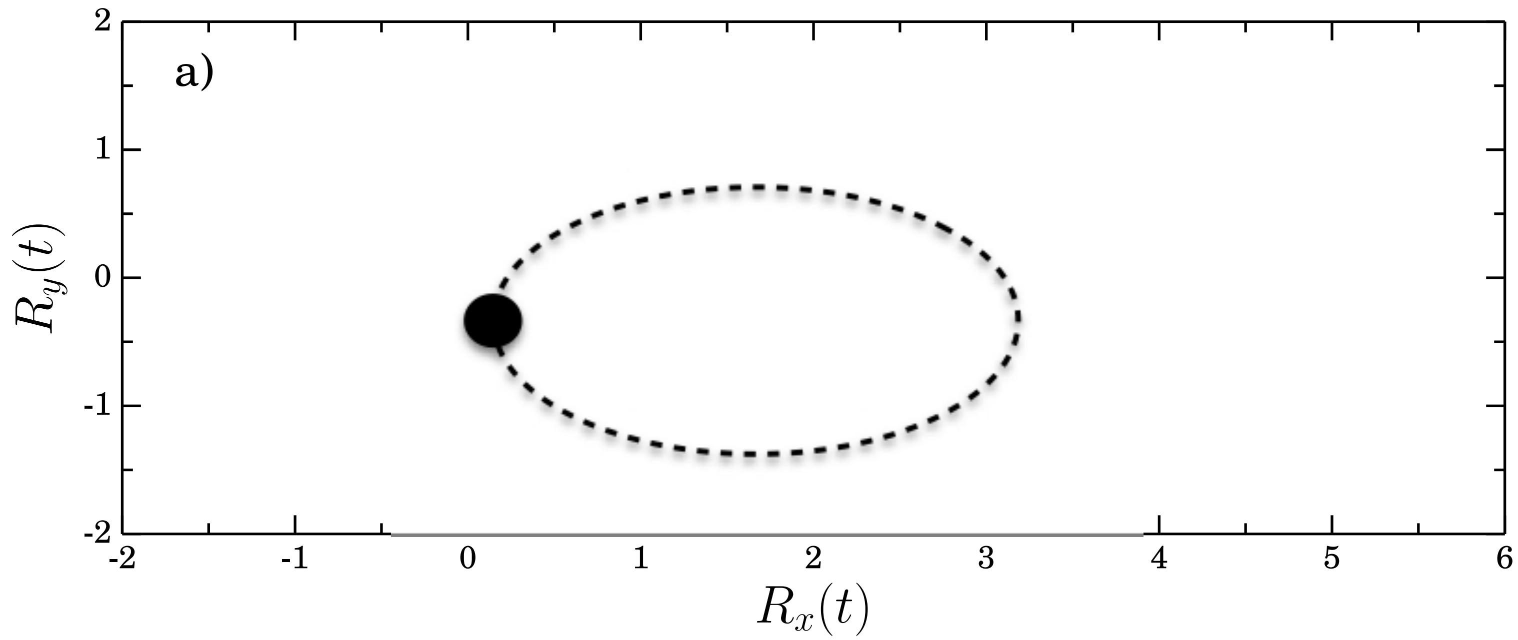
Local equation of motion

$\propto \sin(\omega_{\text{ext}} t)$ $\propto \cos(\omega_{\text{ext}} t)$

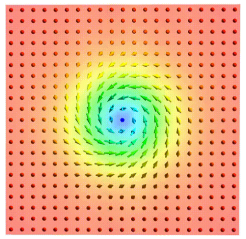
$$\begin{bmatrix} D_x(t) + M_x(t)\partial_t & Q(t) + G(t)\partial_t \\ -Q(t) - G(t)\partial_t & D_y(t) + M_y(t)\partial_t \end{bmatrix} \begin{pmatrix} \dot{R}_x \\ \dot{R}_y \end{pmatrix} = \begin{pmatrix} F^x_{\text{ext}}(t) \\ K^y R_y \end{pmatrix}$$

Time-dependent dissipation

Skyrmions Driven by Intrinsic Magnons



$\omega_{\rm ext} = 2.05$ GHz



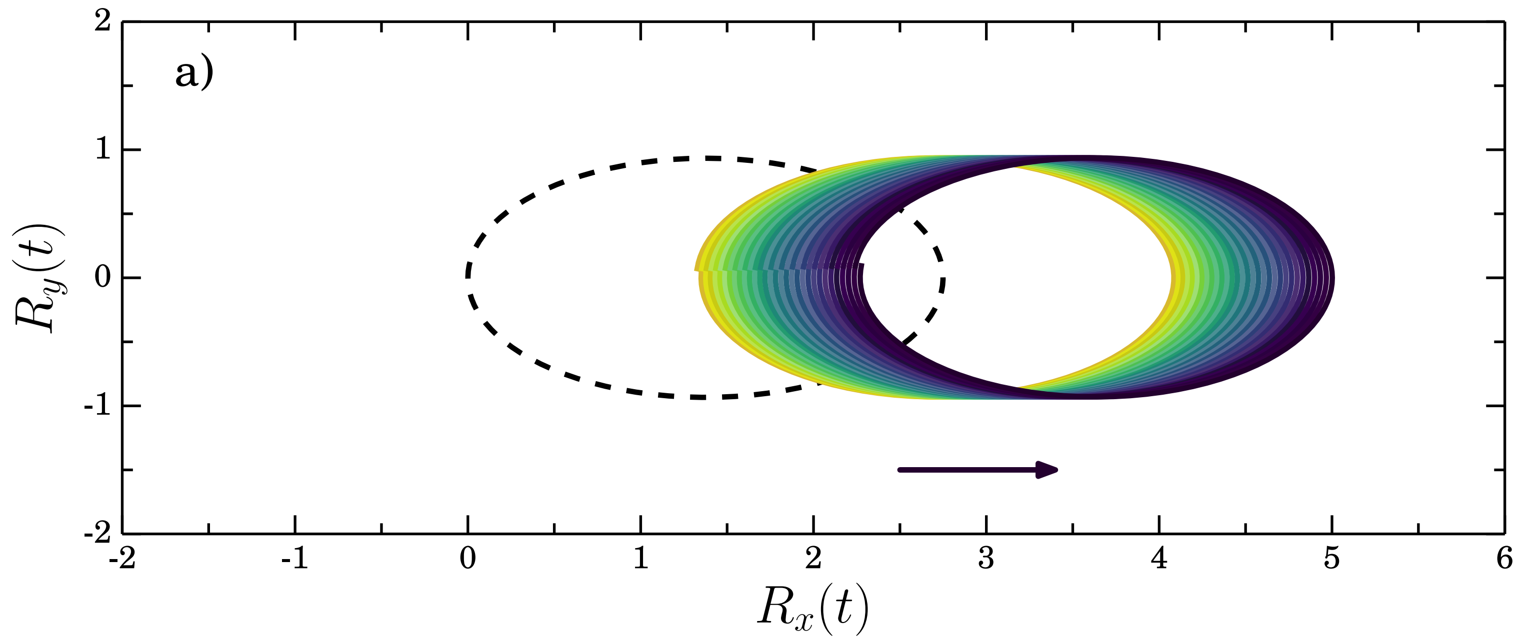
~ resonant with breathing mode

$h_z = 5.4$ mT nm^{-1}

$$\begin{bmatrix} M_x\partial_t & \tilde{Q}_0 \\ -\tilde{Q}_0 & M_y\partial_t \end{bmatrix} \begin{pmatrix} \dot{R}_x \\ \dot{R}_y \end{pmatrix} = \begin{pmatrix} F^x_{ext}(t) \\ K^y R_y \end{pmatrix}$$

Bounded Periodic Motion

Skyrmions Driven by Intrinsic Magnons



$\omega_{\text{ext}} = 2.05$ GHz

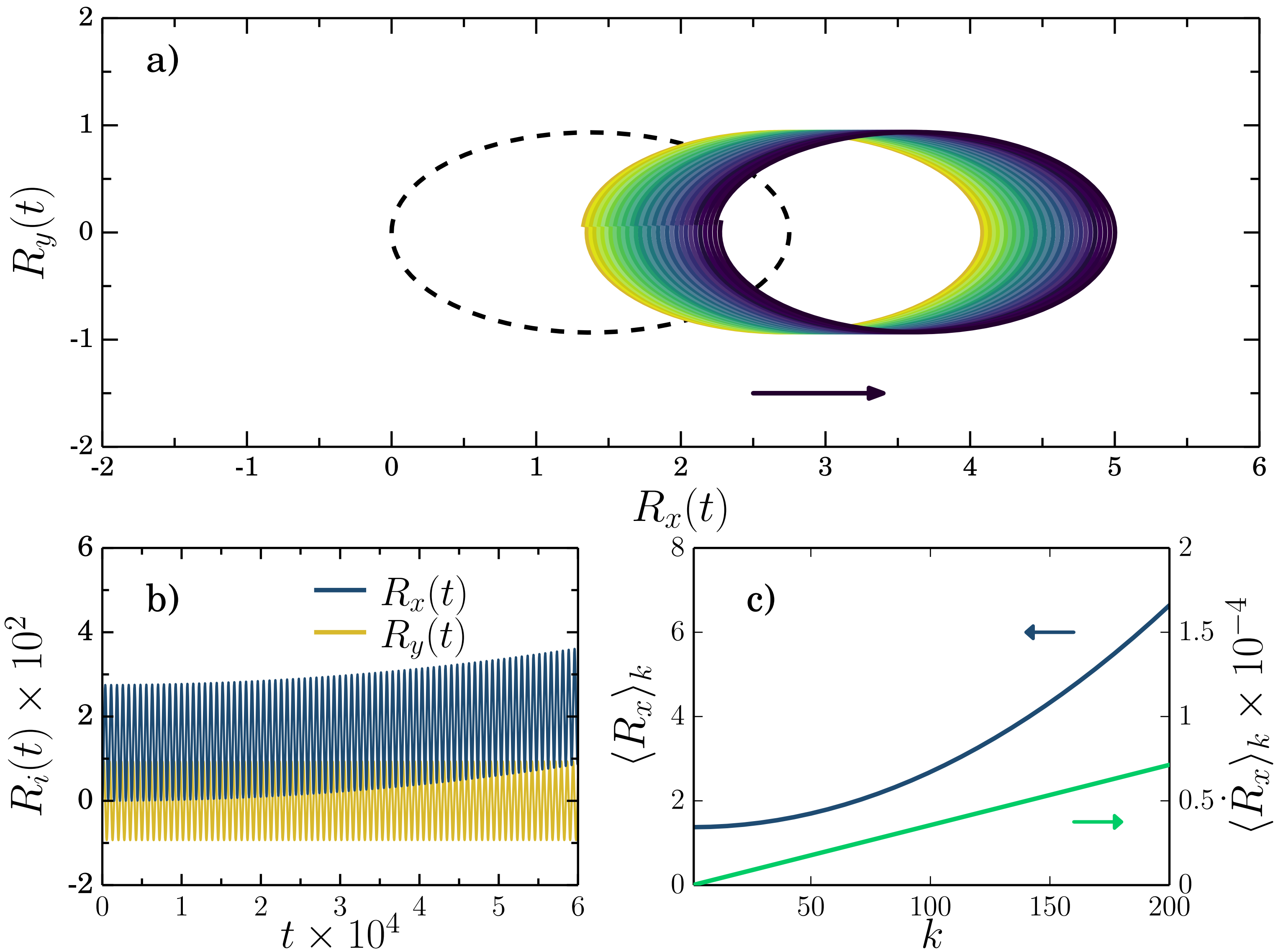
~ resonant with breathing mode

$h_z = 5.4$ mT nm^{-1}

$$\begin{bmatrix} D_x(t) + M_x(t)\partial_t & Q(t) + G(t)\partial_t \\ -Q(t) - G(t)\partial_t & D_y(t) + M_y(t)\partial_t \end{bmatrix} \begin{pmatrix} \dot{R}_x \\ \dot{R}_y \end{pmatrix} = \begin{pmatrix} F^x_{\text{ext}}(t) \\ K^y R_y \end{pmatrix}$$

Unidirectional helical propagation of the skyrmion

Unidirectional helical propagation of the skyrmion



$$\omega_{\text{ext}} = 2.05 \text{ GHz}$$

~ resonant with breathing mode

$$h_z = 5.4 \text{ mT nm}^{-1}$$

$$\langle R_x \rangle_k = 1.37 + 1.32 \times 10^{-4} k^2$$

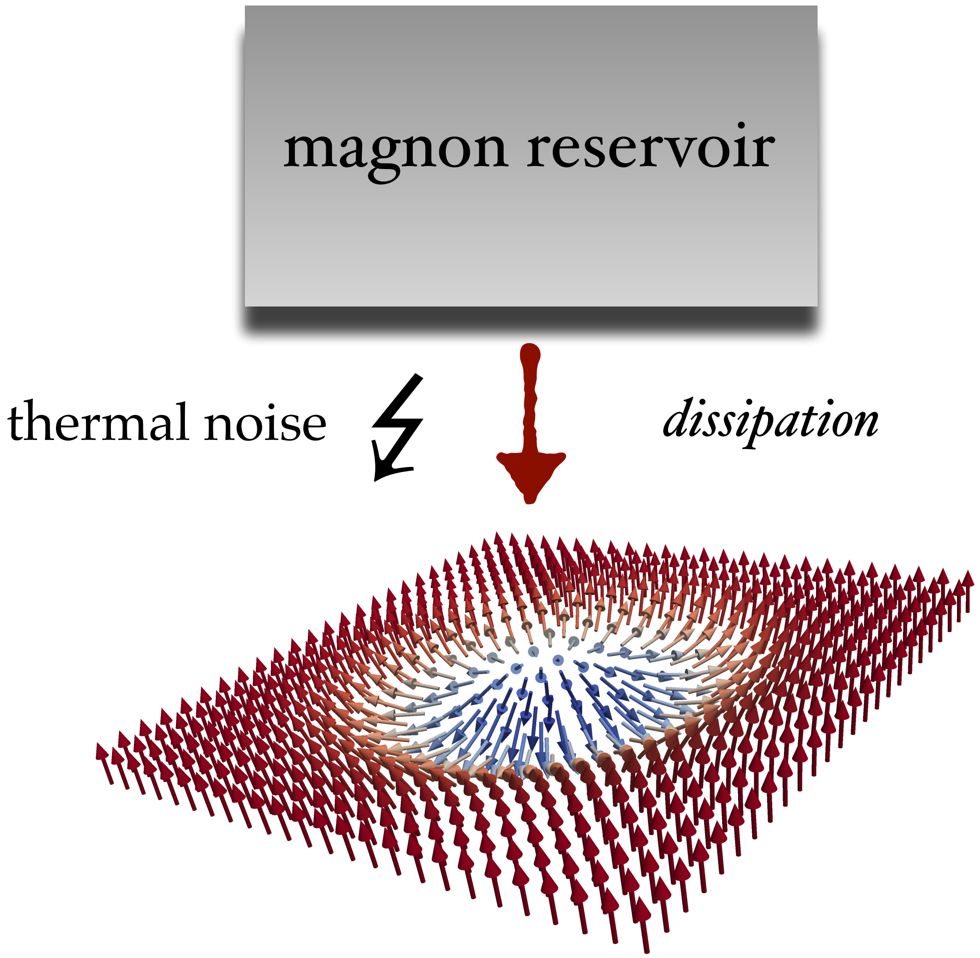
$$\langle \dot{R}_x \rangle_k = 3.57 \times 10^{-7} k$$

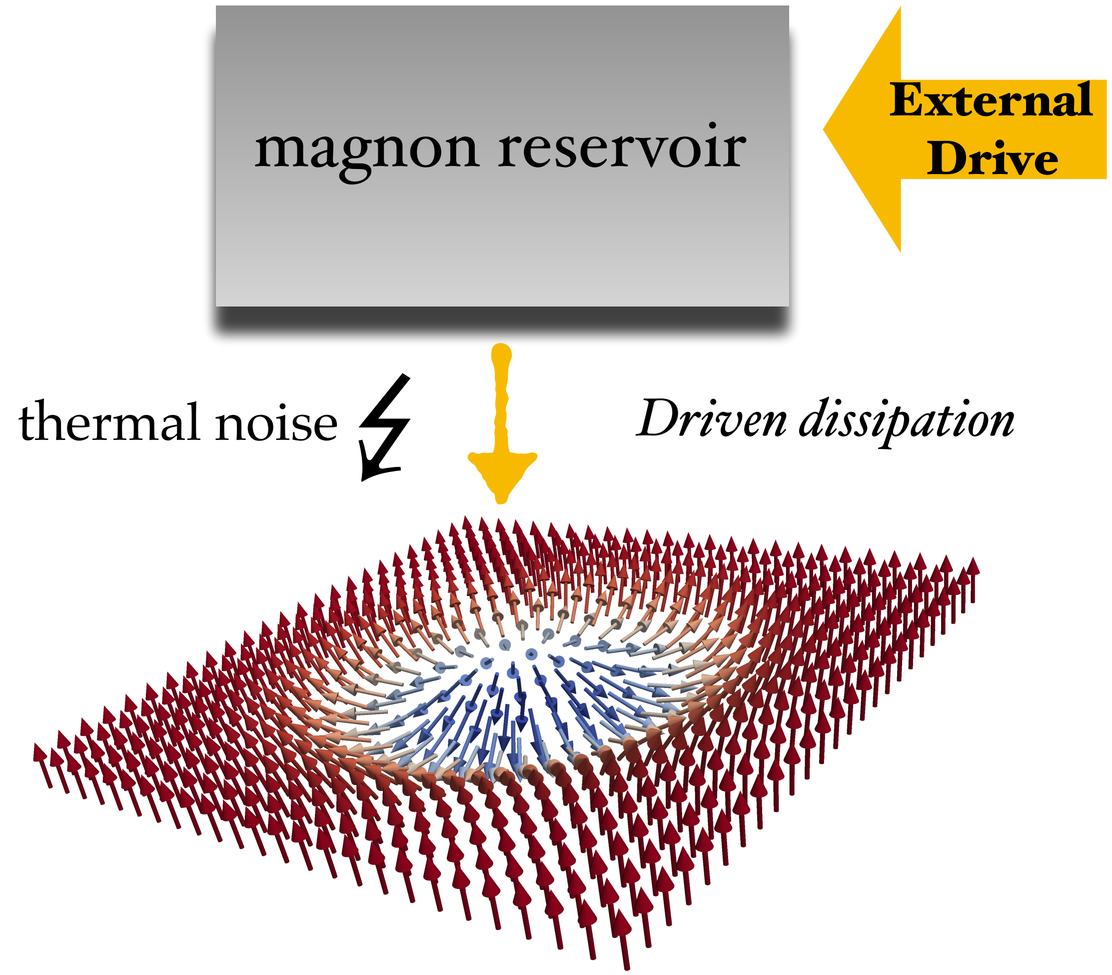
$$\langle R_i \rangle_k = 1/T \int_{kT}^{(k+1)T} R_i(t) dt$$

For $t = 73$ ns, $\langle R_x \rangle = 5.94$ nm

$$\langle \dot{R}_x \rangle = 16.3 \text{ cm/s}$$

Stochastic dynamics

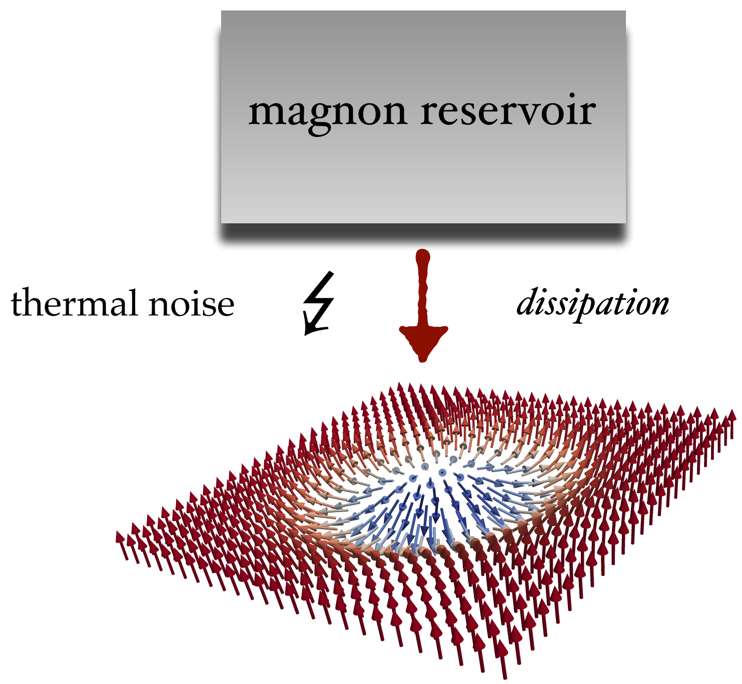


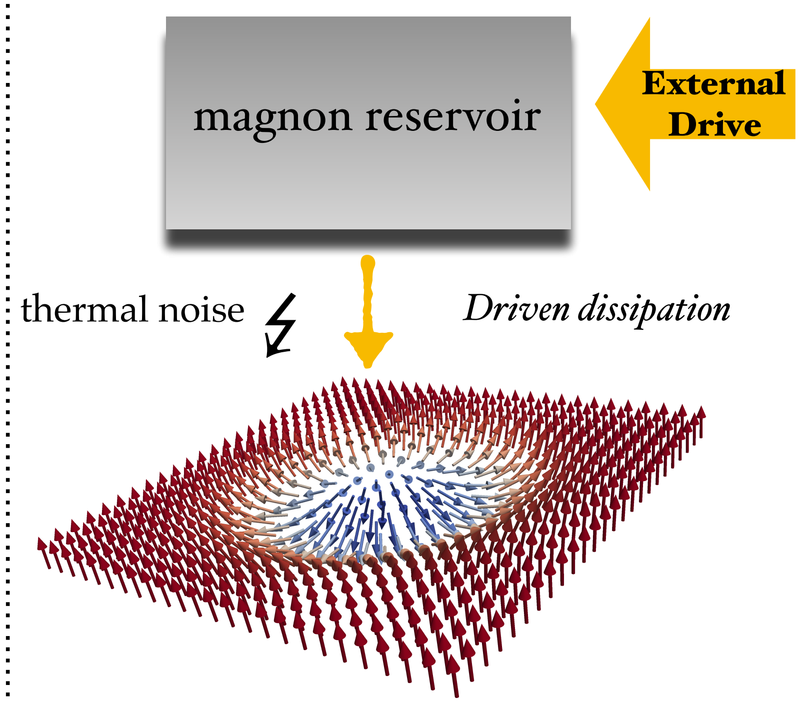


$$\tilde{Q}_0 \epsilon_{ij} \dot{R}_c^j(t) + \int_{t_0}^{t} dt' \; \dot{R}_c^j(t') \gamma_{ji}(t,t') = \xi_i(t)$$

C. Psaroudaki, P. Aseev, and Daniel Loss,
Phys. Rev. B **100**, 134404 (2019)

Quantum Brownian Motion





$$\tilde{Q}_0 \epsilon_{ij} \dot{R}_c^j(t) + \int_{t_0}^{t} dt' \; \dot{R}_c^j(t') \gamma_{ji}(t,t') = \xi_i(t)$$

$$\langle \xi_i(t) \rangle = 0$$

$$\langle \xi_i(t) \xi_j(t') \rangle = C_{ij}(t-t')$$

magnon reservoir — external drive

$$\Delta\gamma_{ji}(t,t') = \partial_t W_{ji}(t-t') G_{\mathrm{ext}}(t,t')$$

$$\Delta C_{ji}(t,t') = \partial_t \partial_{t'} U_{ji}(t-t') G_{\mathrm{ext}}(t,t')$$

Equilibrium Fluctuation dissipation

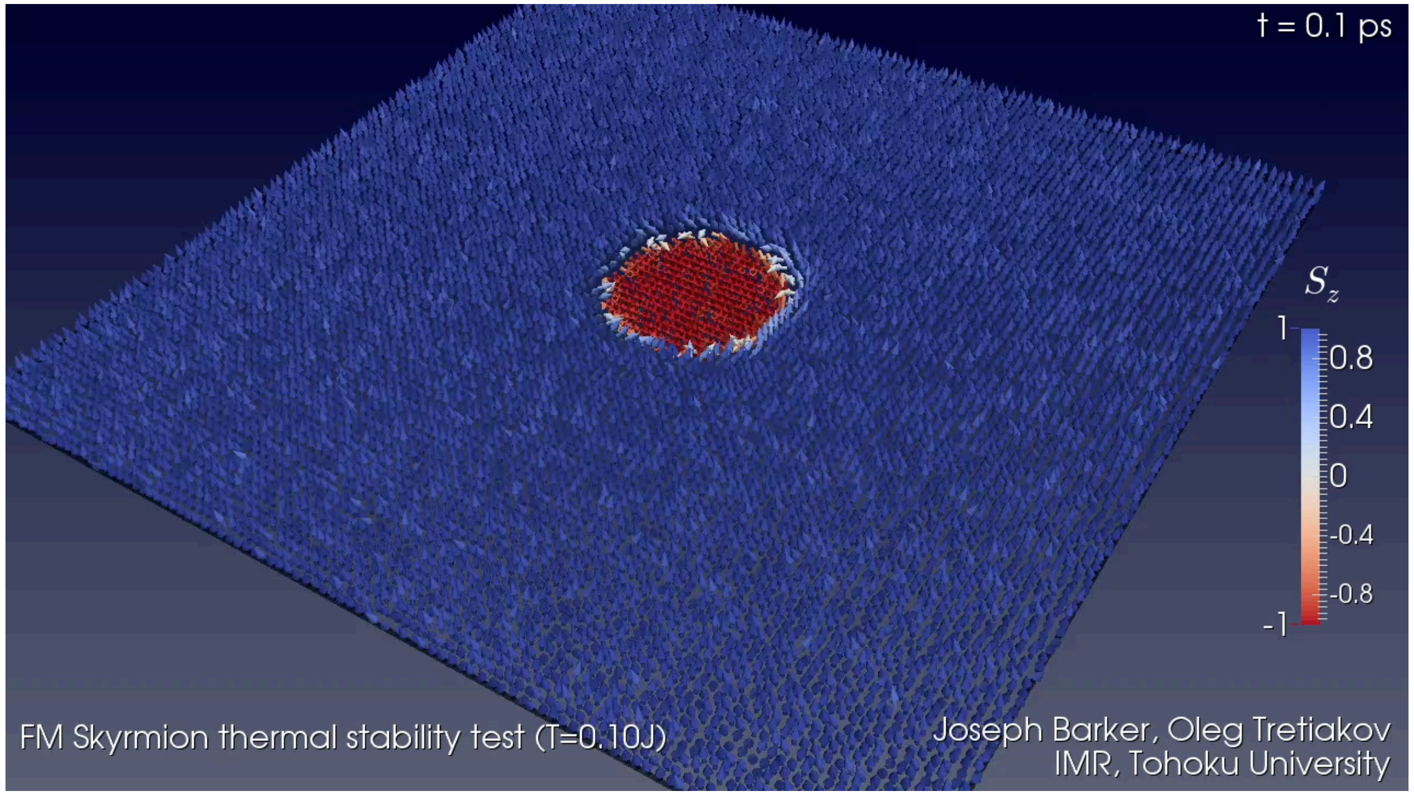
$$C_{ij}(\omega) + C_{ij}(-\omega) = \omega \coth(\frac{\beta\omega}{2}) \left[\gamma_{ij}(\omega) + \gamma_{ji}(-\omega)\right]$$

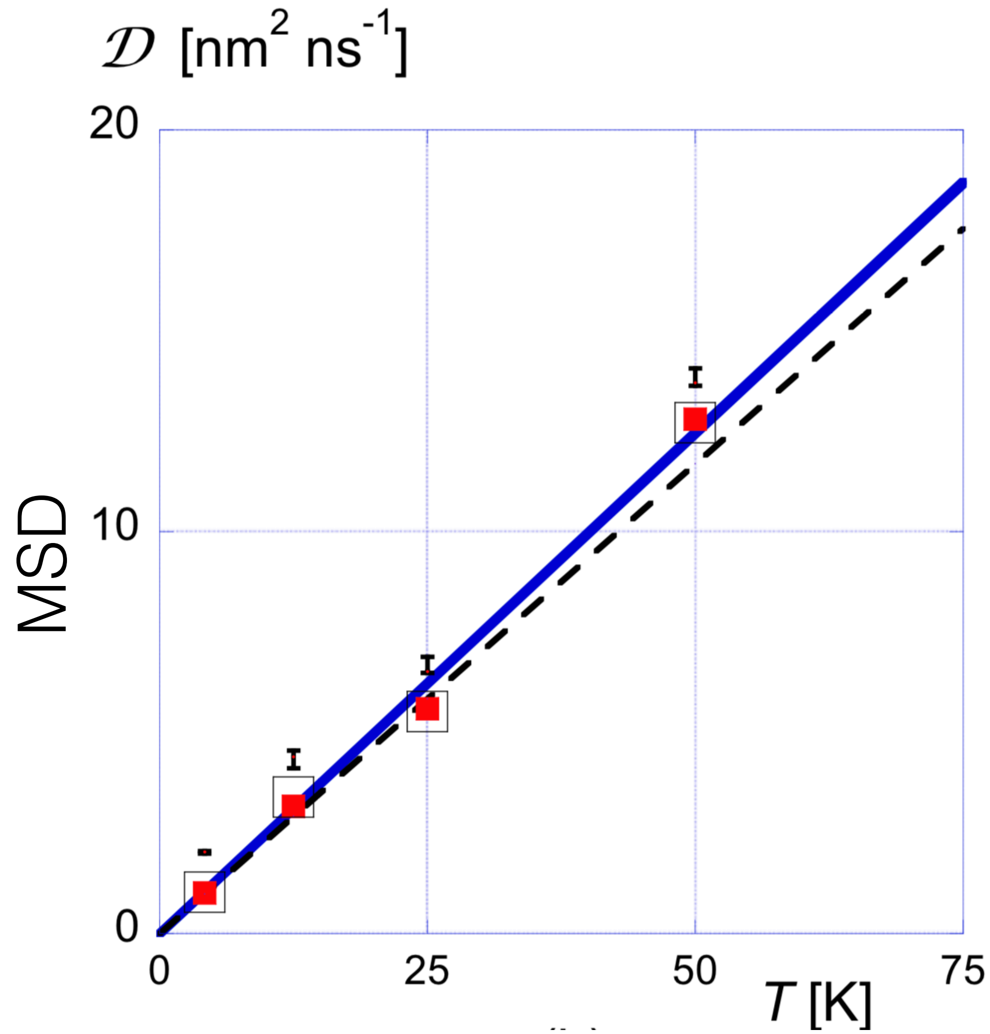
Nonequilibrium Fluctuation dissipation

$$U_{ji}(\omega) + U_{ij}(-\omega) = \coth(\frac{\beta\omega}{2})[W_{ji}(\omega) + W_{ij}(-\omega)]$$

Classical Brownian Motion

$$\langle \xi_i(t)\xi_j(t')\rangle_{\mathrm{cl}} = \alpha \mathcal{D} k_B T \delta_{ij}\delta(t-t')$$





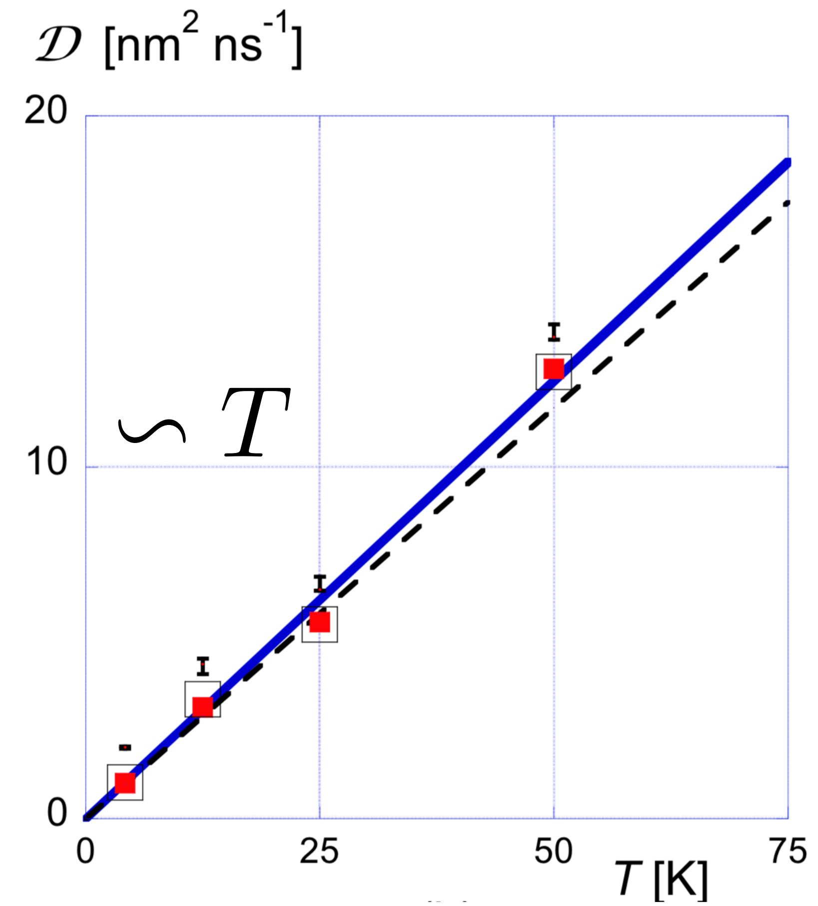
J. Miltat, S. Rohart, and A. Thiaville, Phys. Rev. B **97**, 214426 (2018)

$$\mathbf{G} \times \dot{\mathbf{R}}(t) + \alpha \mathcal{D} \dot{\mathbf{R}}(t) = \mathbf{F}_{\mathrm{ext}}(t) + \boldsymbol{\xi}(t)$$

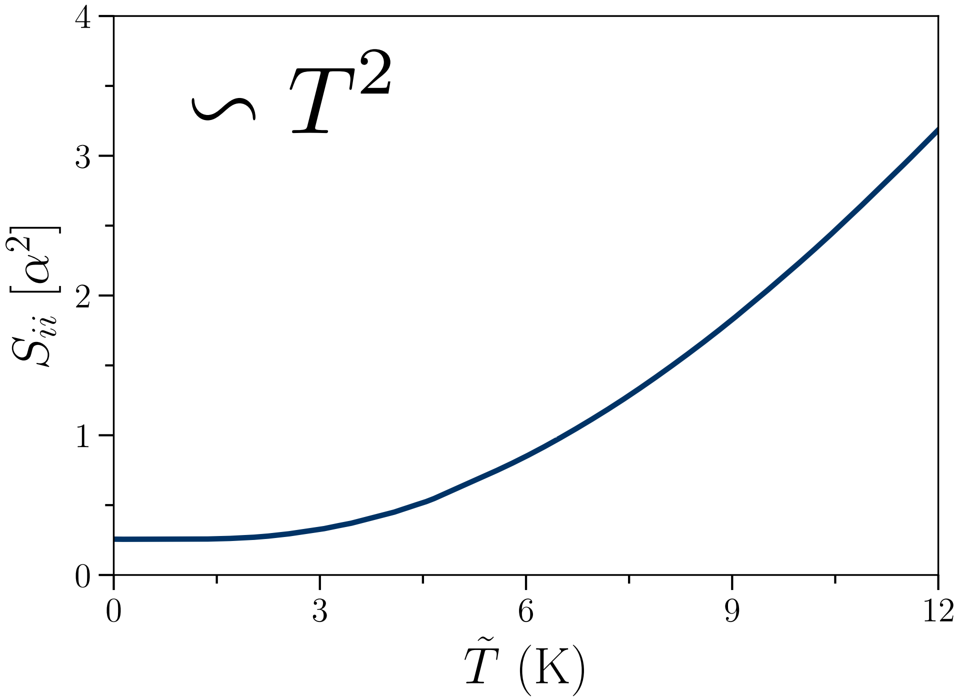
R. E. Troncoso and A. S. Núñez, Phys. Rev. B **89** , 224403 (2014).
C. Schütte, J. Iwasaki, A. Rosch, and N. Nagaosa, Phys. Rev. B **90**, 174434 (2014).
J. Barker and O. A. Tretiakov, Phys. Rev. Lett. **116**, 147203 (2016).

Quantum Brownian Motion

Mean Square Displacement $\frac{1}{2}\langle[R_i(t) - R_j(t')]^2\rangle$



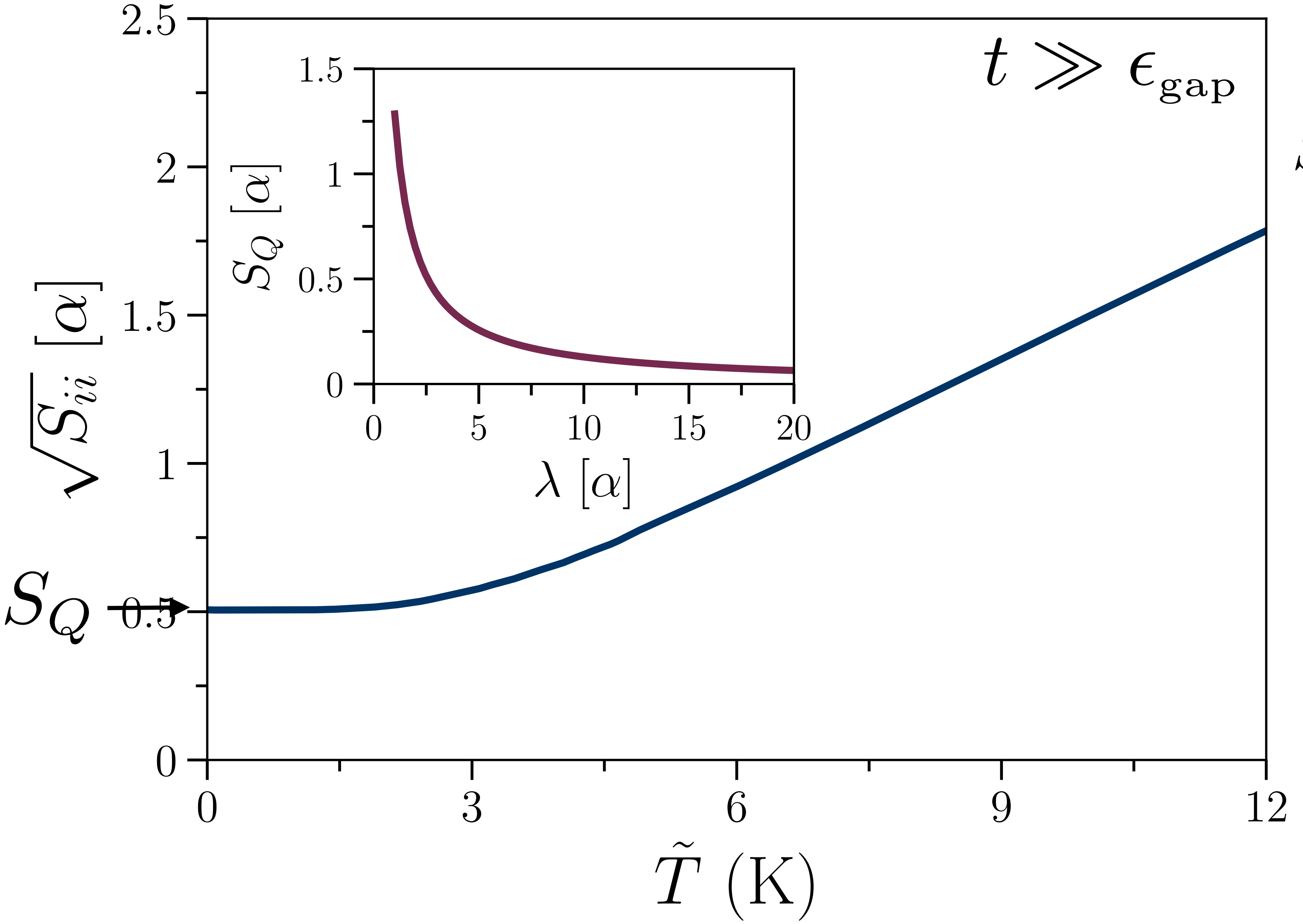
$$\langle\xi_i(\omega)\xi_j(\omega')\rangle = \alpha\mathcal{D}k_B T\delta_{ij}\delta_{\omega,\omega'}$$



$$\langle\xi_i(t)\xi_j(t')\rangle = C_{ij}(t - t')$$

$$C_{ij}(\omega) + C_{ij}(-\omega) = \omega\coth(\frac{\beta\omega}{2})\,[\gamma_{ij}(\omega) + \gamma_{ji}(-\omega)]$$

Quantum RMSD



Mean square displacement

$$S_{ij}(t,t') = \frac{1}{2}\langle[R_i(t) - R_j(t')]^2\rangle$$

Response function

$$R_i(t) = \int_{-\infty}^{t} dt' \chi_{ij}(t-t')\xi_j(t')$$

Symmetrized autocorrelation function

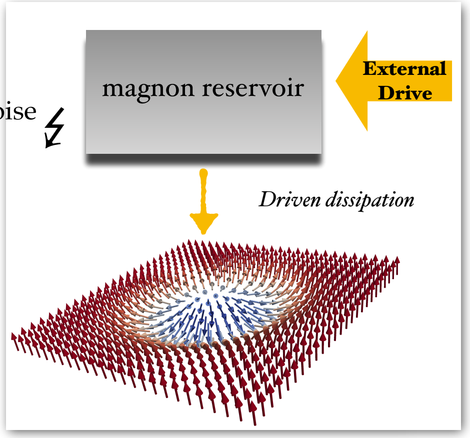
$$\mathcal{X}_{ij}(\omega) = C_{ij}(\omega) + C_{ji}(-\omega)$$

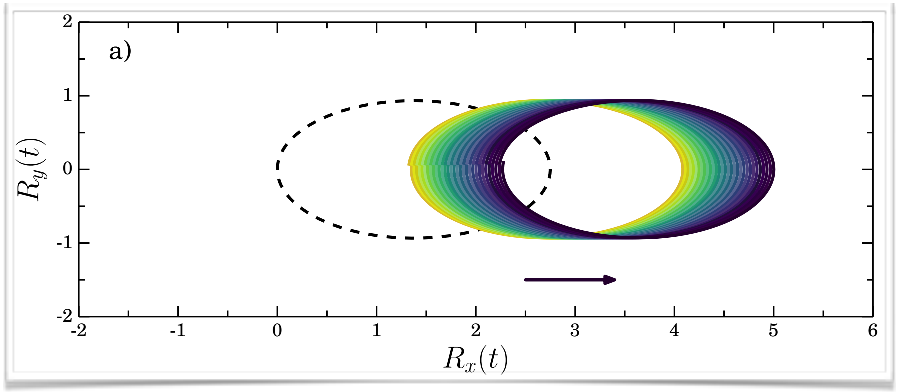
$$\langle \xi_i(t)\xi_j(t')\rangle = C_{ij}(t-t')$$

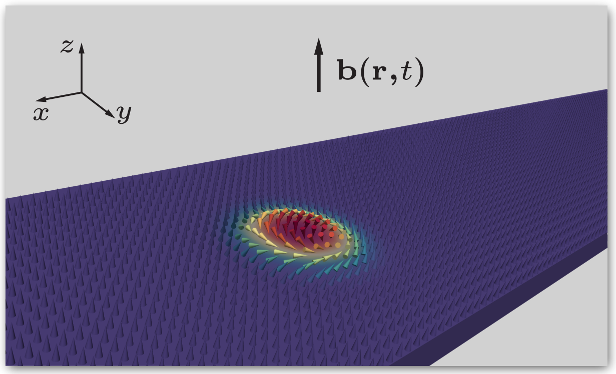
Diagonal MSD

$$S_{ii}(\bar{t}) = \int \frac{d\omega}{2\pi}(e^{-i\omega\bar{t}} - 1)\chi_{il}(\omega)\mathcal{X}_{lk}(\omega)\chi_{ik}(-\omega)$$

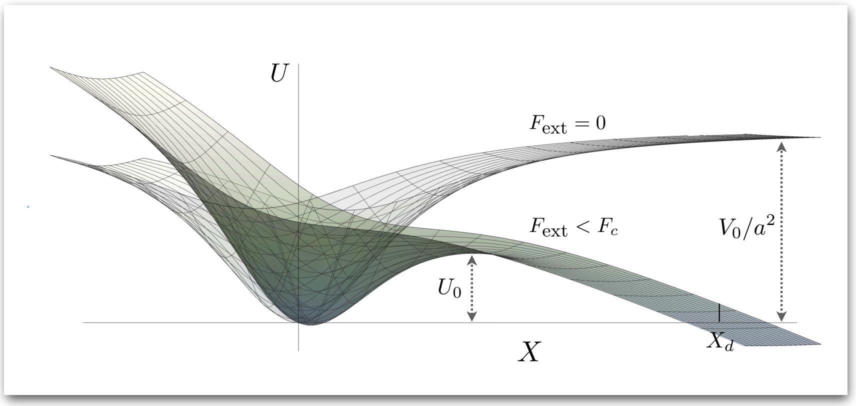
NonEquilibrium and Stochastic dynamics of skyrmions under time periodic driving fields

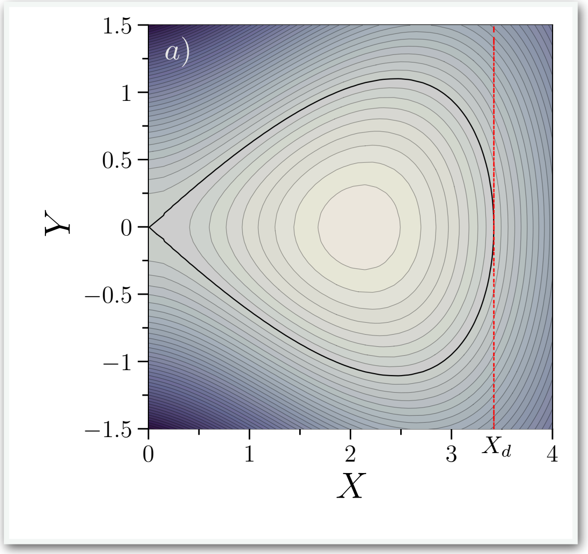






Quantum Depinning of a Magnetic Skyrmion

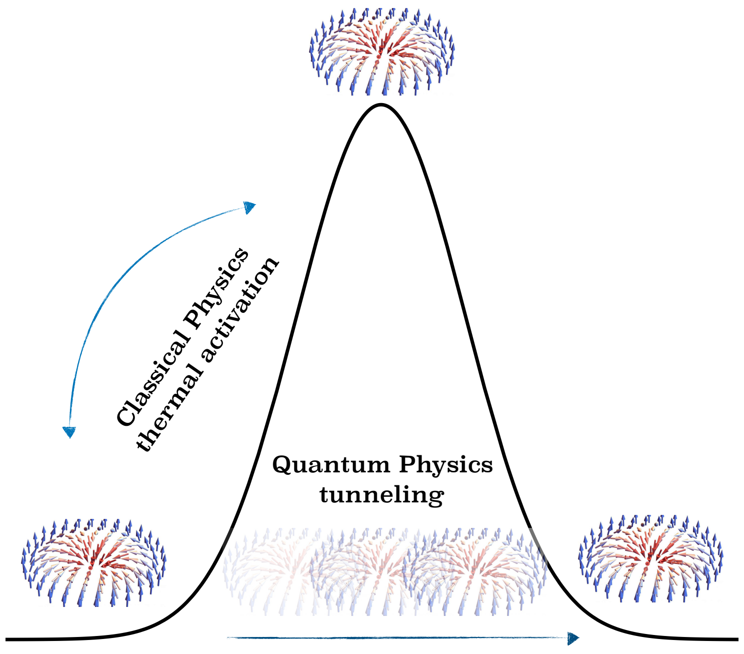


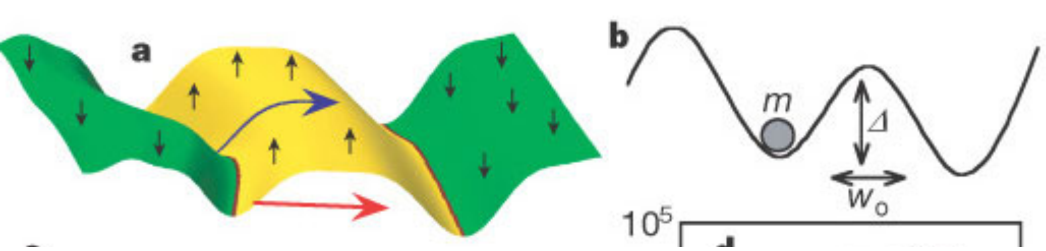


C. P, et al., Phys. Rev. Lett. **120**, 237203 (2018)
C. P, et al., Phys. Rev. B **100**, 134404 (2019)
C. P, et al., arXiv:1910.09585

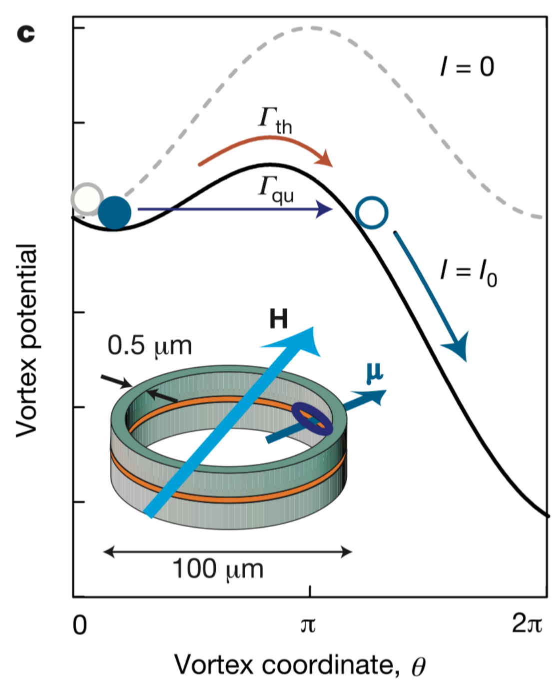
Quantum Tunneling

- Quantum tunnelling of a magnetic skyrmion

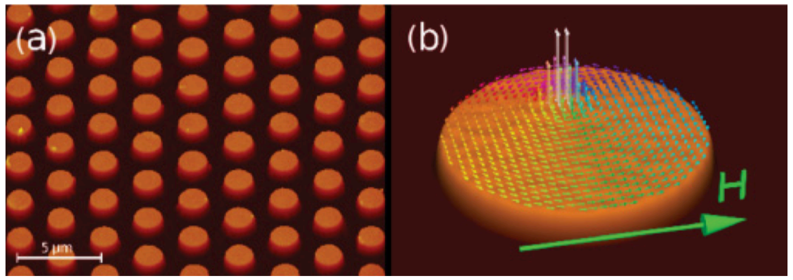




J. Brooke, *et al.*, Nature **413**, 610 (2001).



A. Wallraff , *et al.*, Nature **425**, 155 (2003).



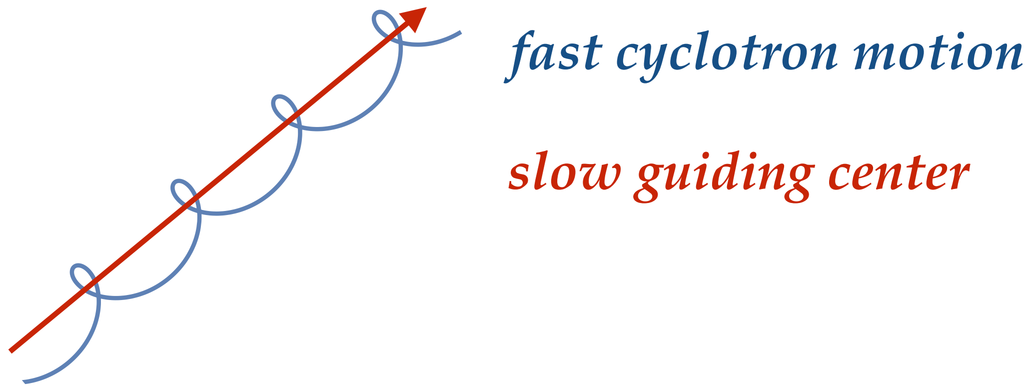
R. Zarzuela, et al.,
Phys. Rev. B **85**, 180401(R) (2012).

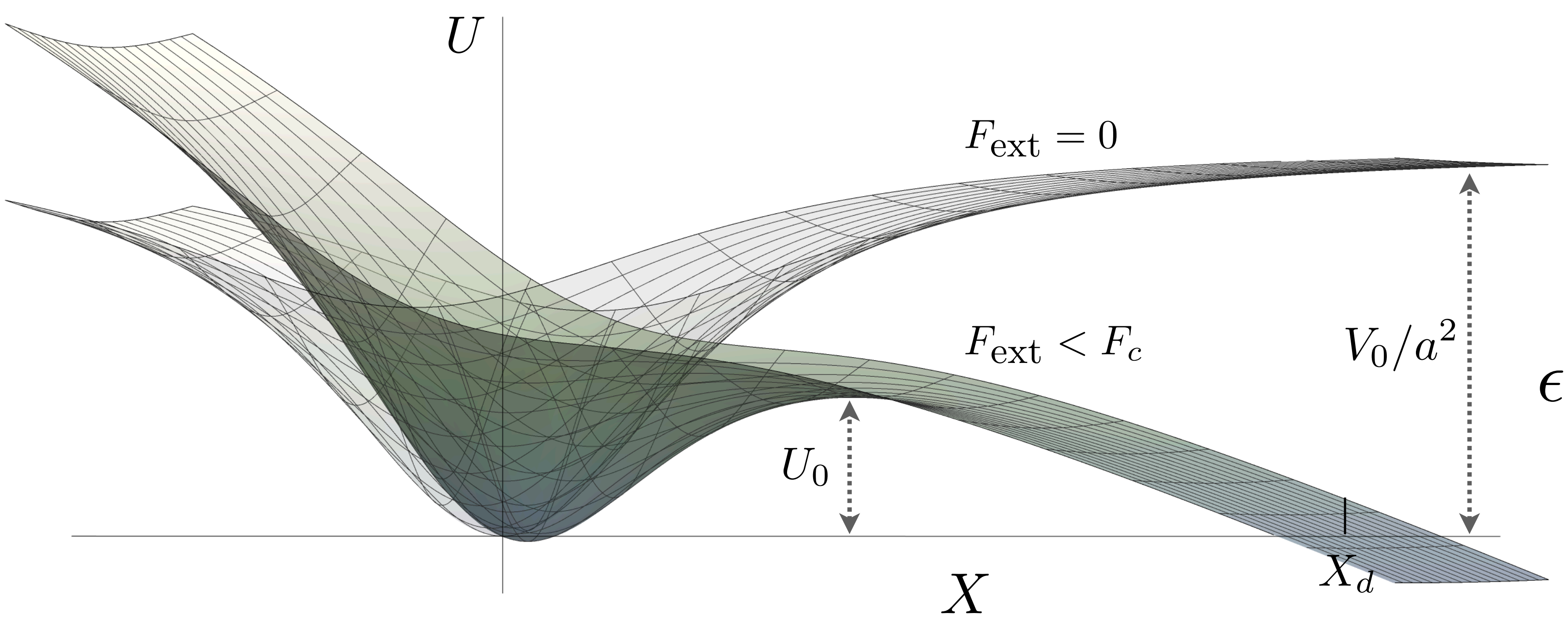
Quantum Depinning

$$\mathcal{S}_E = \int_0^\beta d\tau [-i\tilde{Q}(\dot{\mathcal{X}}\mathcal{Y} - \dot{\mathcal{Y}}\mathcal{X}) + \frac{1}{2}\mathcal{M}\dot{\mathbf{R}}^2 + U(\mathcal{X},\mathcal{Y})]$$

$$U(\mathcal{X},\mathcal{Y}) = V_p(\sqrt{\mathcal{X}^2 + \mathcal{Y}^2}) - F_{ext}\mathcal{X}$$

$$\tilde{Q}/\mathcal{M} \gg 1$$





$$\epsilon = 1 - F_{\text{ext}}/F_c \ll 1$$

Quantum Depinning

$$\mathcal{S}_E = \int_0^\beta d\tau[-i\tilde{Q}(\dot{\mathcal{X}}\mathcal{Y} - \dot{\mathcal{Y}}\mathcal{X}) + \frac{1}{2}\mathcal{M}\dot{\mathbf{R}}^2 + U(\mathcal{X},\mathcal{Y})]$$

$$\downarrow$$

$$\mathcal{S}_E = \int_0^\beta d\tau \left[i\tilde{Q}(\dot{Y}X - \dot{X}Y) + U(X,Y)\right] \qquad [X,Y] = -i/2\tilde{Q}$$

Equations of motion

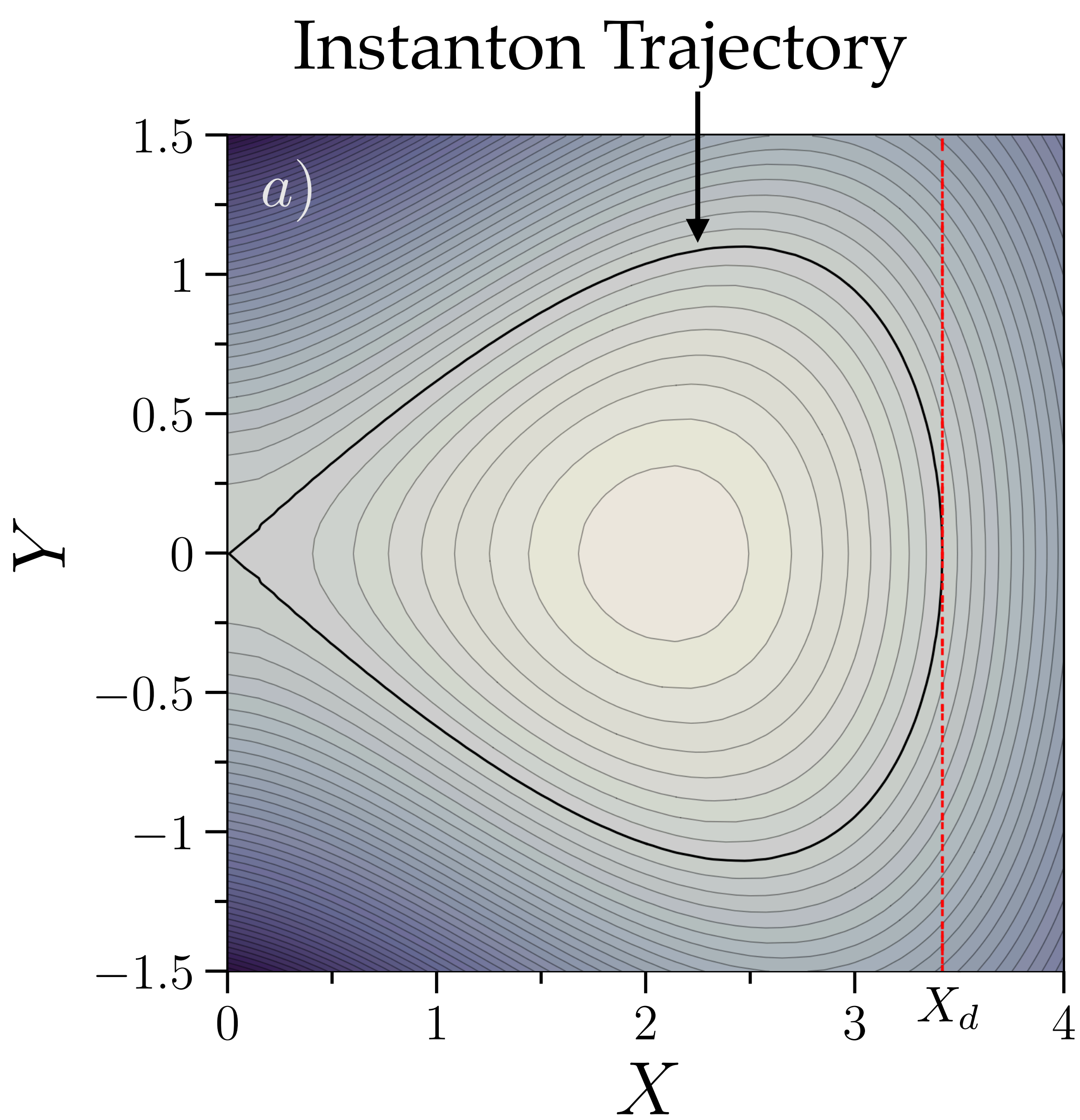
$$2\tilde{Q}\dot{Y} + \frac{\partial U}{\partial X} = 0$$

$$-2\tilde{Q}\dot{X} + \frac{\partial U}{\partial Y} = 0$$

Instanton Trajectories →

$$U(X, iY) = 0$$

Quantum Depinning



Tunnel Frequency

$$\omega_\tau = \frac{9V_0(3\epsilon)^{1/4}(d\alpha)^2}{16\hbar|\tilde{Q}|a^4}$$

WKB exponent $e^{-\mathcal{S}_0/\hbar}$

$$\mathcal{S}_0 = 5.6\hbar|\tilde{Q}|a^2\epsilon^{5/4}$$

Decay Rate

$$\Gamma \simeq \frac{\omega_\tau}{2\pi}e^{-\mathcal{S}_0/\hbar} \simeq \frac{9V_0(3\epsilon)^{1/4}(d\alpha)^2}{32\pi\hbar|\tilde{Q}|a^4}e^{-\mathcal{S}_0/\hbar}$$

Crossover Temperature

$$T_c = \frac{\hbar U_0}{k_B\mathcal{S}_0}$$

Quantum Depinning

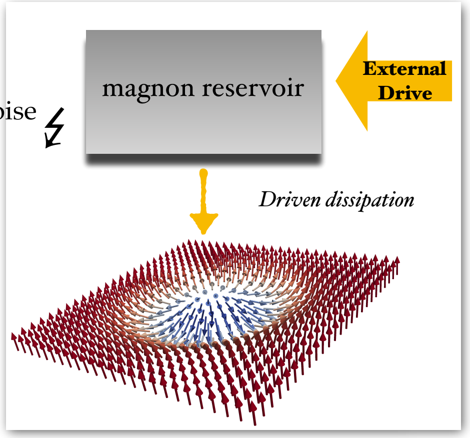
TABLE I. Tunneling quantities for the chiral magnetic insulator Cu_2OSeO_3, with $J_0 = 3.34$ meV, $D = 0.79$ meV, $K = 6.8 \times 10^{-2}$ meV, $M_s = 111.348$ kA m^{-1}, $\alpha = 8.911$ Å, $S = M_s\alpha^3/g\mu_B$ [50], and $Q = 1$, $\lambda_d = \lambda$, $J'/J_0 = 0.3$, $D' = 0$, and $N_A = 30$.

λ	N	ϵ	X_d	ω_τ	$\mathcal{S}_0/\hbar$	Γ^{-1}	T_c
4.3 nm	8.8×10^2	5×10^{-2}	3.02 nm	2.90×10^{10} s^{-1}	288.76	5.51×10^{115} s	30.76 mK
		2×10^{-3}	0.60 nm	1.30×10^{10} s^{-1}	5.16	8.48×10^{-8} s	13.76 mK
		5×10^{-4}	0.30 nm	9.17×10^{9} s^{-1}	0.91	1.71×10^{-9} s	9.73 mK
7.4 nm	2.61×10^3	5×10^{-2}	5.3 nm	3.54×10^{9} s^{-1}	886.59	1.96×10^{376} s	3.76 mK
		2×10^{-3}	1.06 nm	1.58×10^{9} s^{-1}	15.86	0.03 s	1.68 mK
		5×10^{-4}	0.5 nm	1.12×10^{9} s^{-1}	2.80	9.25×10^{-8} s	1.19 mK
10.3 nm	5.05×10^3	5×10^{-2}	7.40 nm	1.04×10^{9} s^{-1}	1731.46	5.56×10^{743} s	1.10 mK
		2×10^{-3}	1.48 nm	4.66×10^{8} s^{-1}	30.97	38.15×10^{4} s	0.49 mK
		5×10^{-4}	0.74 nm	3.29×10^{8} s^{-1}	5.47	4.55×10^{-6} s	0.35 mK

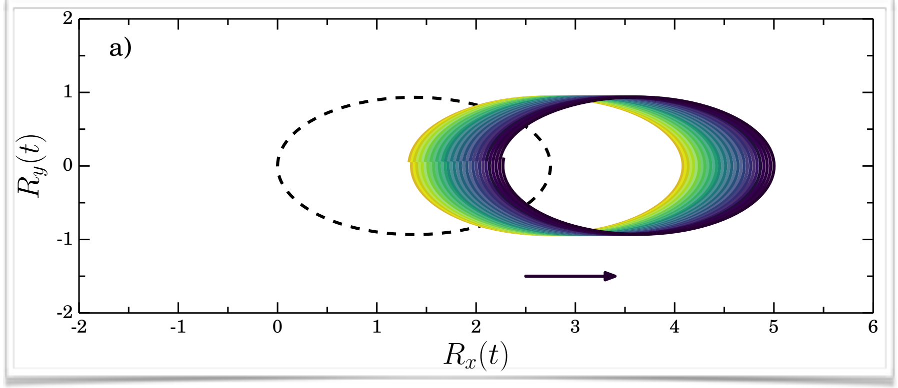
Macroscopic Quantum Tunnelling for a Topological Particle

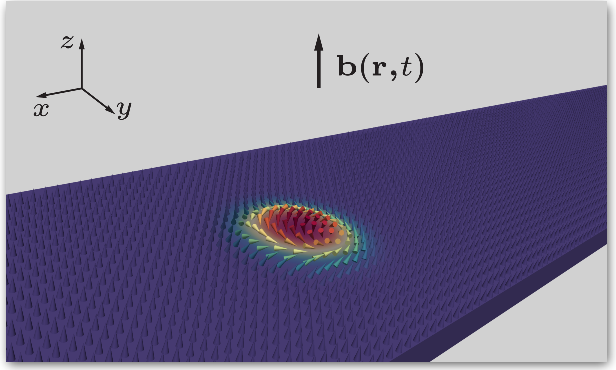
C. Psaroudaki and D. Loss, *Quantum Depinning of a Magnetic Skyrmion*, manuscript in preparation (2019).

Discussion

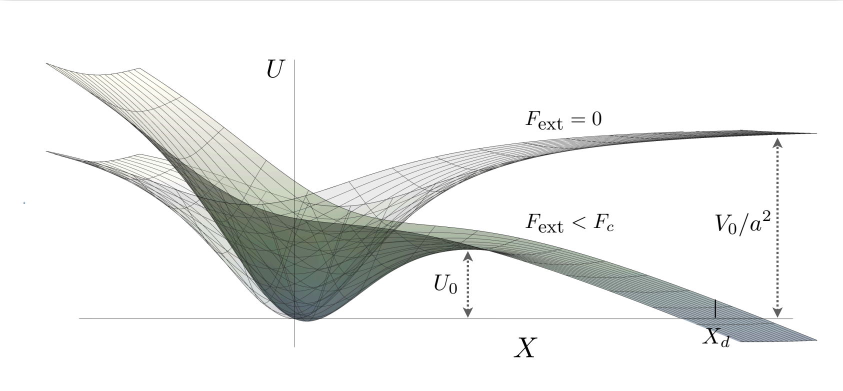


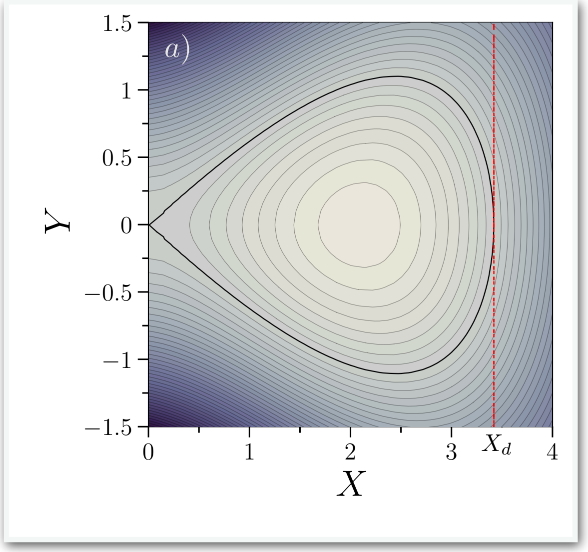
NonEquilibrium and Stochastic dynamics of skyrmions under time periodic driving fields





Quantum Depinning of a Magnetic Skyrmion





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