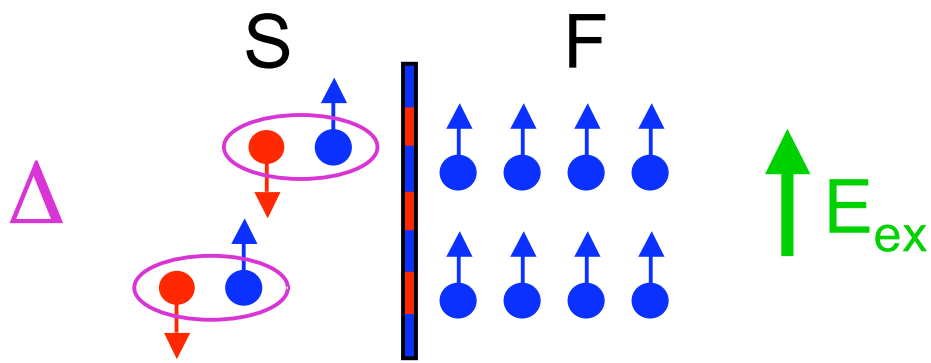
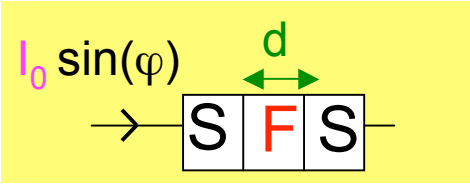
Superconducting proximity effect in a diffusive ferromagnet with spin-active interfaces

A. Cottet and W. Belzig
University of Basel
(now: Orsay and Konstanz)

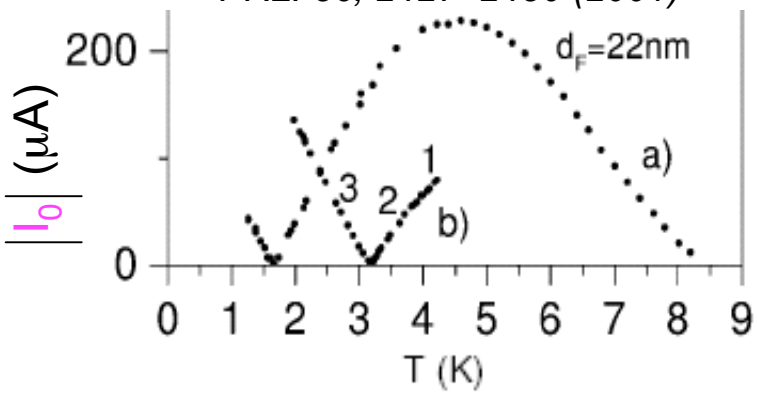
Phys. Rev. B **72**, 180503R (2005)

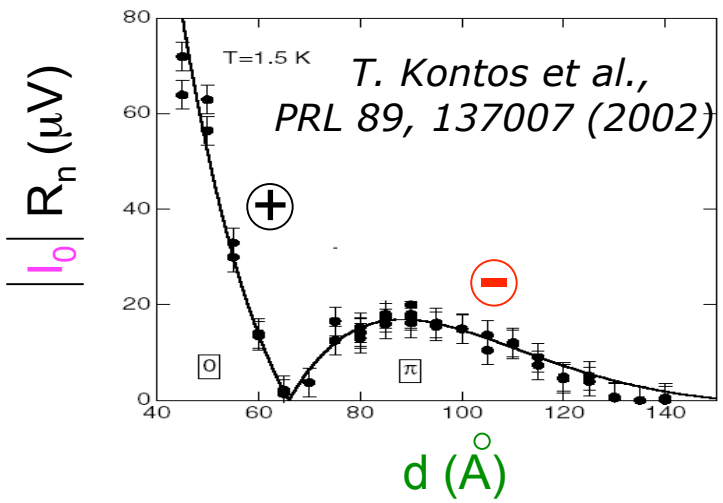


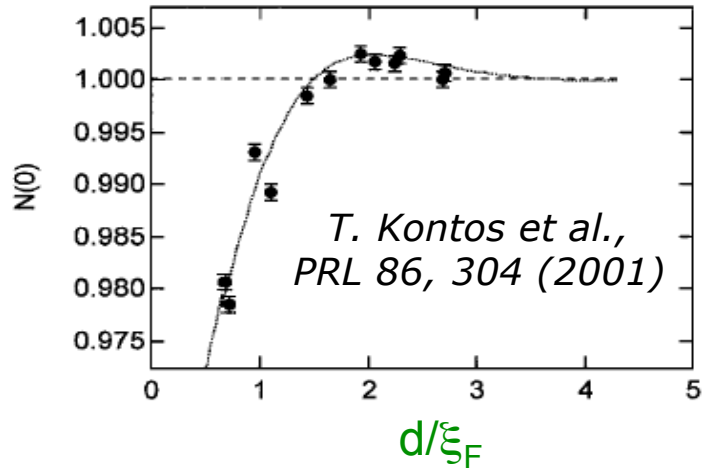
Manifestations of the oscillations of the order parameter

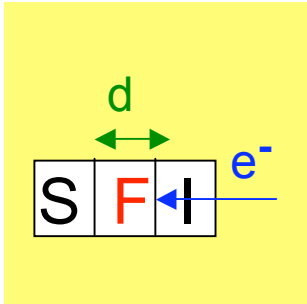


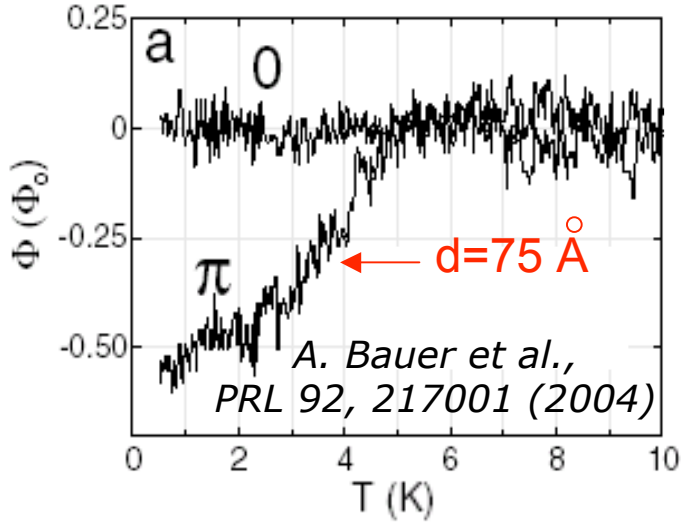
V. V. Ryazanov et al., PRL. 86, 2427–2430 (2001)

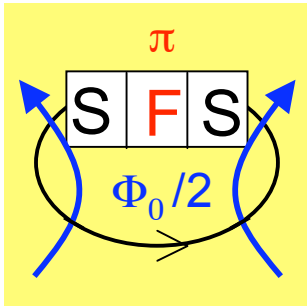












Boundary conditions for a diffusive S/F interface

Following D. Huertas-Hernando, Y. Nazarov and W. Belzig (2002)

V. V. Kuprianov et Lukichev (1988) D. Huertas-Hernando et al.

$$2g_F \hat{G}_F \frac{\partial \hat{G}_F}{\partial x} = G_T [\hat{G}_S, \hat{G}_F] + iG_\phi \hat{D}_F^+ + \frac{G_{MR}}{2} [\hat{D}^+, \hat{G}_F]$$

$$+ iG_\chi [\hat{G}_S \hat{D}^-, \hat{G}_F] + iG_\xi [\hat{D}^- \hat{G}_F, \hat{G}_F]$$

$$\hat{D}^- = [\vec{m}.\vec{\sigma}\hat{\tau}_3, \hat{G}_S]$$

$$\hat{D}^+ = \{\vec{m}.\vec{\sigma}\hat{\tau}_3, \hat{G}_S\}$$

- $G_T = G_Q \sum_n T_n$ Tunnel conductance
- $G_\phi = 2G_Q \,\mathrm{Im}\left(\sum_n r_{n,\uparrow}^F r_{n,\downarrow}^{F\,*} - 4(t_{n,\uparrow}^S t_{n,\downarrow}^{S\,*})/T_n \right)$ Phase-shifting conductance
- $G_{MR} = G_Q \sum_n \left(\left| t_{n,\uparrow}^F \right|^2 - \left| t_{n,\downarrow}^F \right|^2 \right)$ Magnetoresistance term
- $G_\chi = -G_Q \,\mathrm{Im}\left(\sum_n t_{n,\uparrow}^S t_{n,\downarrow}^{S\,*} \right)$
- $G_\xi = G_Q \,\mathrm{Im}\left(\sum_n T_n (r_{n,\uparrow}^F r_{n,\downarrow}^{F\,*} + t_{n,\uparrow}^S t_{n,\downarrow}^{S\,*})/4 \right)$

F is weakly polarized

$T_n << 1$

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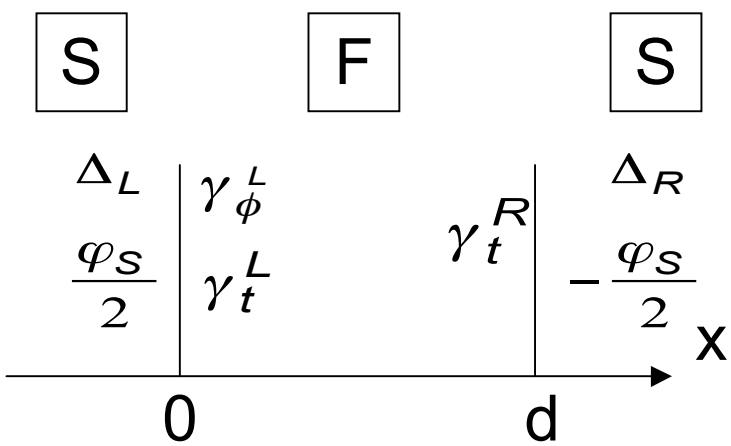
F is weakly polarized

$T_n << 1$

Josephson current in a SFS junction

$$I_S = \pi\, g_F\, k_B T \sum_{n,\sigma} Q_\sigma(\omega_n)/e$$

Asymmetric case : $\gamma_t^R \overline{>>} \gamma_t^L$



$$\theta_{SFS}^\sigma(x) = \theta_{SFI}^\sigma(x) + \delta\theta^\sigma(x)$$

$$\varphi_{SFS}^\sigma(x) = \varphi_{SFI}^\sigma(x) + \delta\varphi^\sigma(x)$$

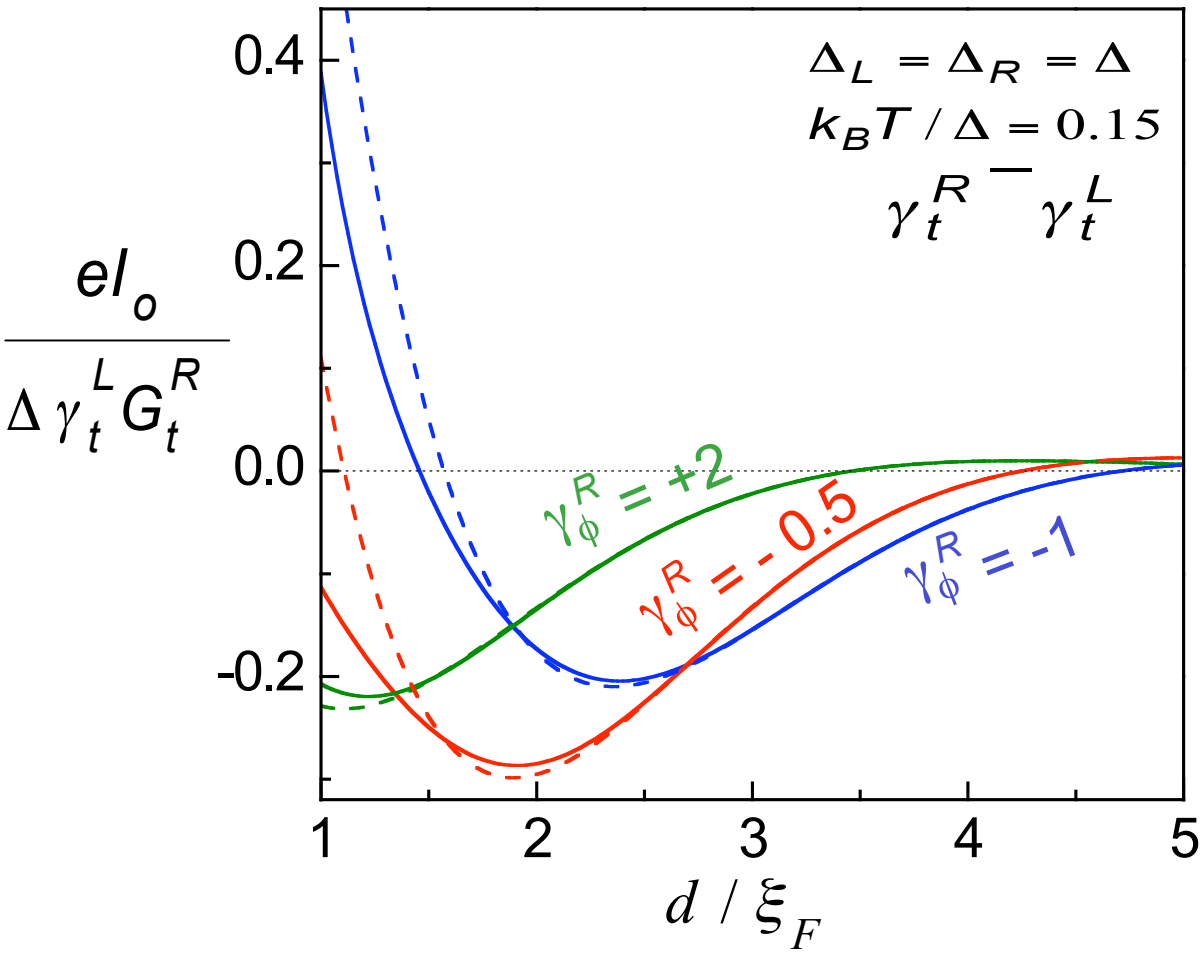
$d \ll \xi_F$, T=0 $\Rightarrow$

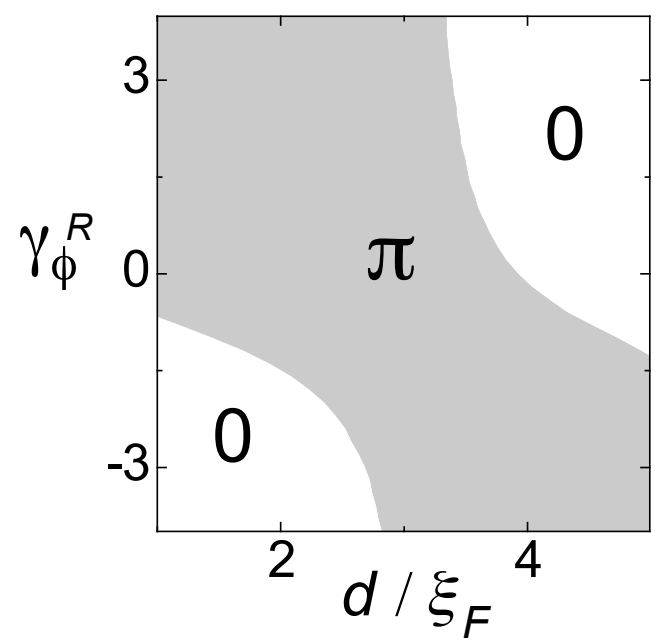
$$\frac{e I_0}{\gamma_t^L G_t^R \Delta} = \pi \frac{\sin\left(\frac{d}{\xi_F} + \lambda(\gamma_\phi)\right)}{\sqrt{1+(1+\gamma_\phi)^2}} e^{-\frac{d}{\xi_F}}$$

$$\lambda(\gamma_\phi) = \arg(i - (1+\gamma_\phi))$$

$$\gamma_\phi = G_\phi / G_t$$

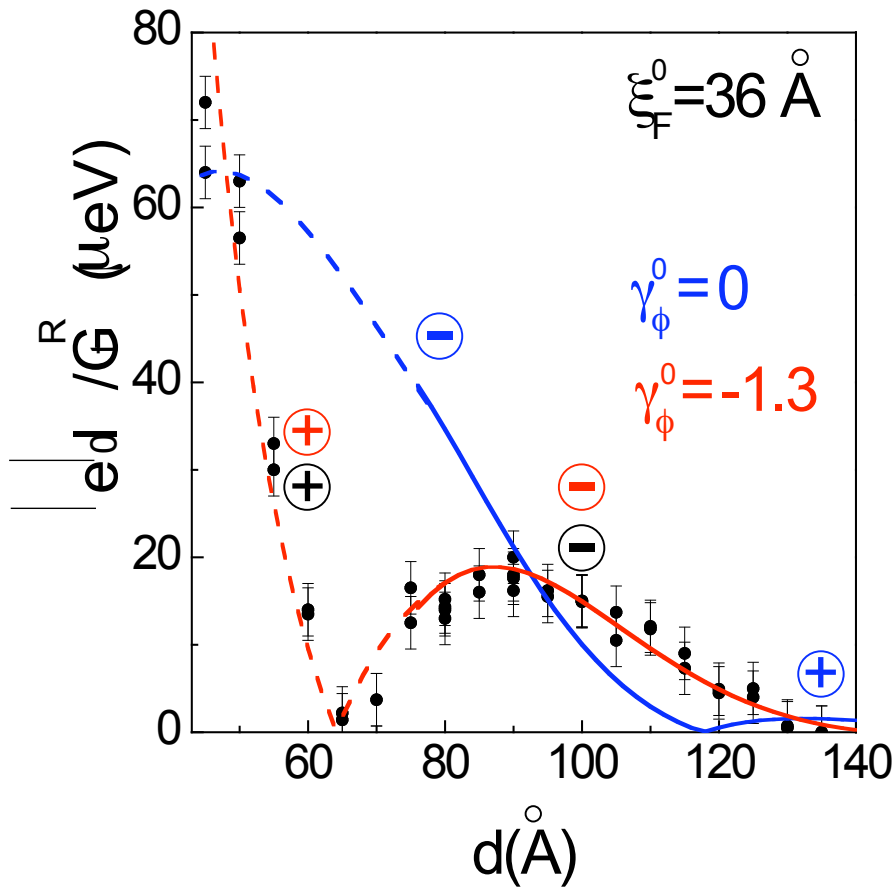
Josephson current in a SFS junction
very asymmetric case

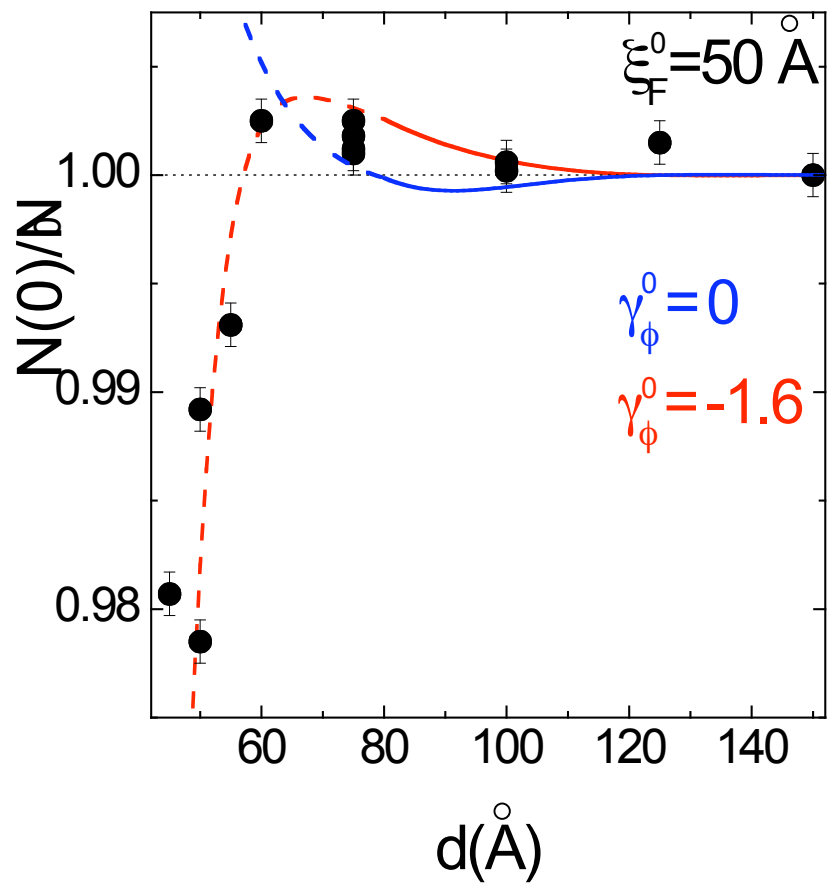




Theoretical interpretation of Proximity effect measurements in PdNi

a_t^0=0.4



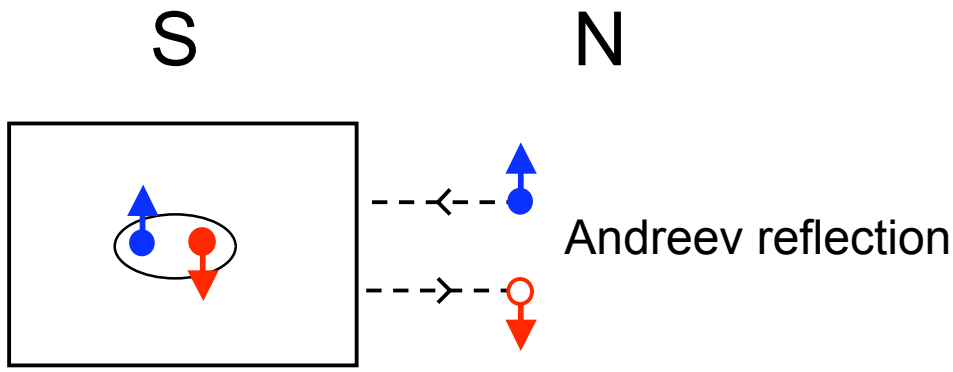


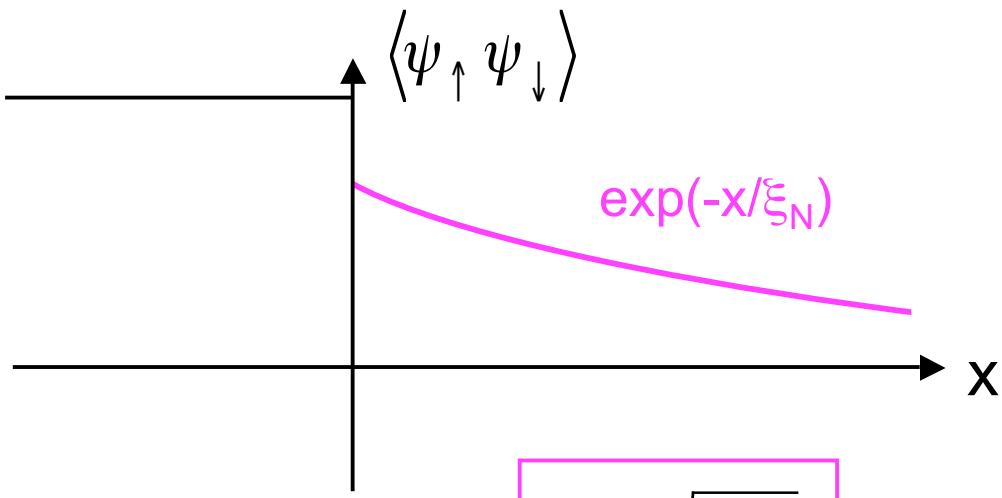
CONCLUSIONS

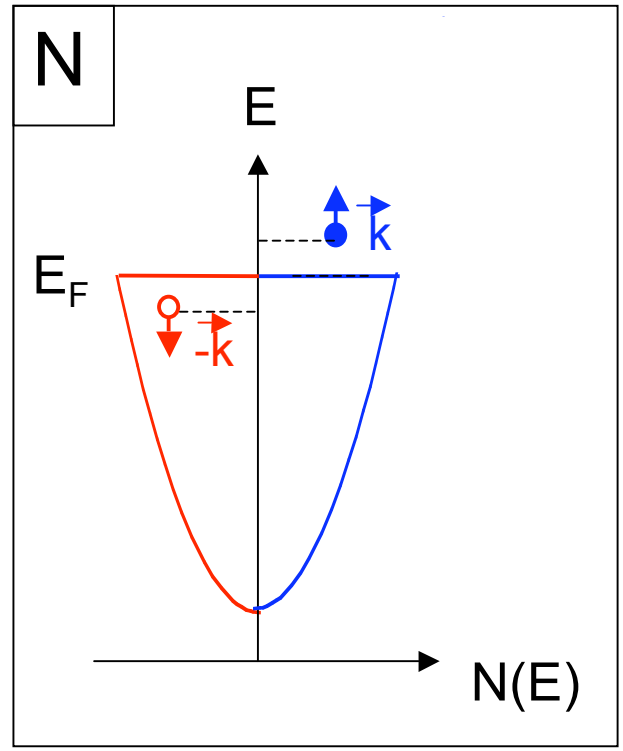
A new framework for describing the superconducting proximity effect in diffusive ferromagnets:

- The interfaces are characterized by G_T and G_ϕ
- G_ϕ modifies the phase and amplitude of the spatial oscillations of physical signals
- New interpretation of proximity effect measurements in PdNi
- Identification of experimental signatures of spin-dependent interfacial phase shifting

Superconducting proximity effect at a superconducting/normal interface (S/N)







$$\xi_N = \sqrt{\frac{\hbar D}{k_B T}}$$

D diffusion constant in N

PROGRAM:

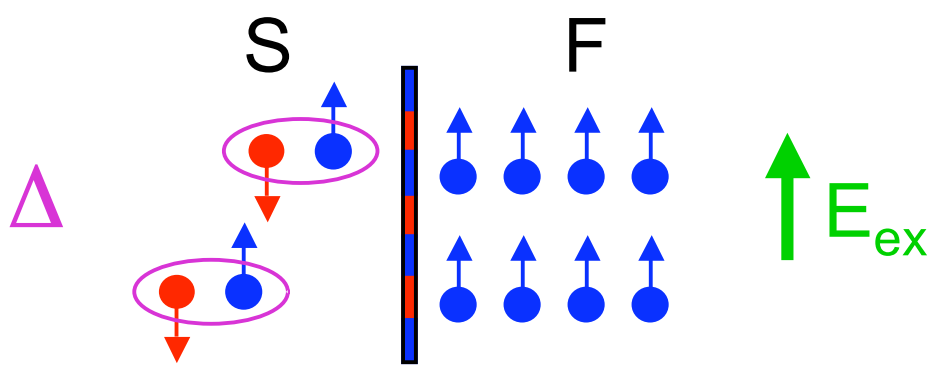
1) Proximity effect in diffusive S/F circuits

2) New boundary conditions for describing S/F interfaces

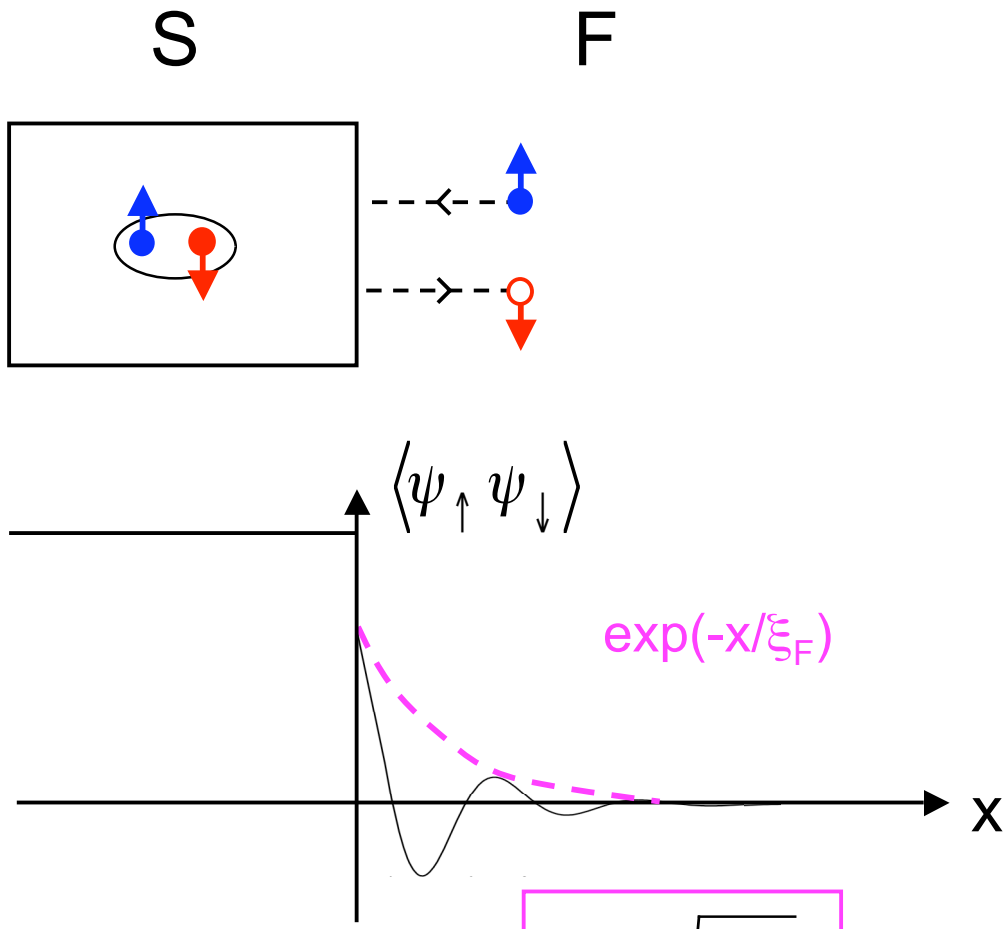
3) Predictions for S/F, S/F/I and S/F/S geometries

4) Interpretation for the proximity effect measurements in Nb/PdNi

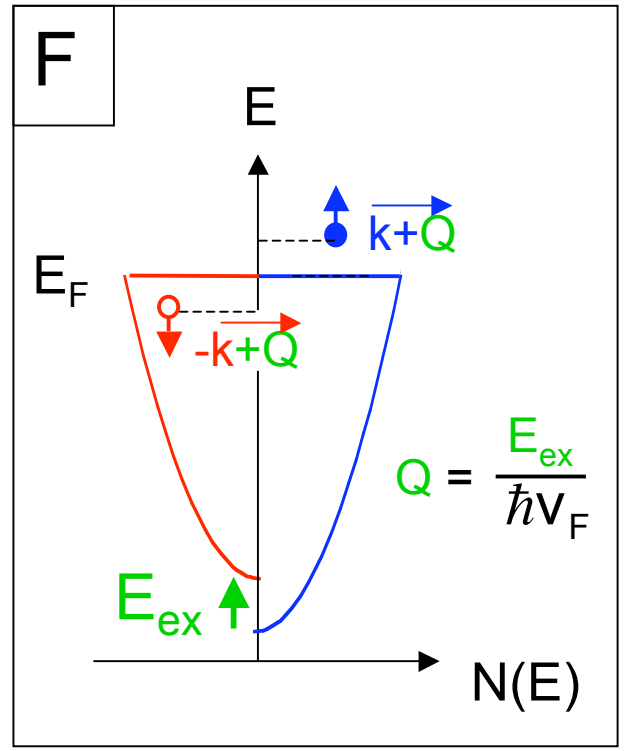
5) Conclusions



Superconducting proximity effect at a superconducting/ferromagnetic interface (S/F)



$$\xi_F = \sqrt{\frac{\hbar D}{E_{ex}}}$$



D diffusion constant in F

Theoretical description of proximity effect from Gorkov to Usadel equations

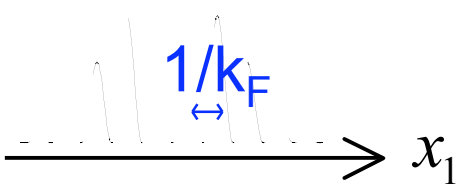
BCS description

$\hat{\Gamma}$

$(\vec{P},\vec{r},E)\langle \quad \rangle_{|\vec{P}|}$

$\vec{r} = (\vec{r}_1 + \vec{r}_2)/2$

$\vec{R} = (\vec{r}_1 - \vec{r}_2)/2$

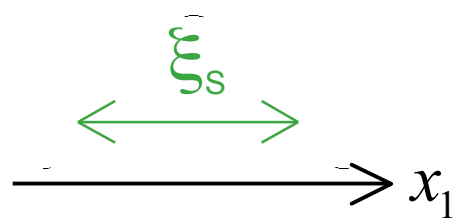


Gorkov Eqs.

Quasi-classical limit

$\vec{v}_F$

$\hat{g}(\frac{v_F}{\ }, \vec{r}, E)$



Eilenberger Eqs.

Diffusive limit

$\hat{G}(\vec{r},\omega_n)$

$\langle \quad \rangle_{\vec{v}_F/v_F}$

$D = v_F l/3$

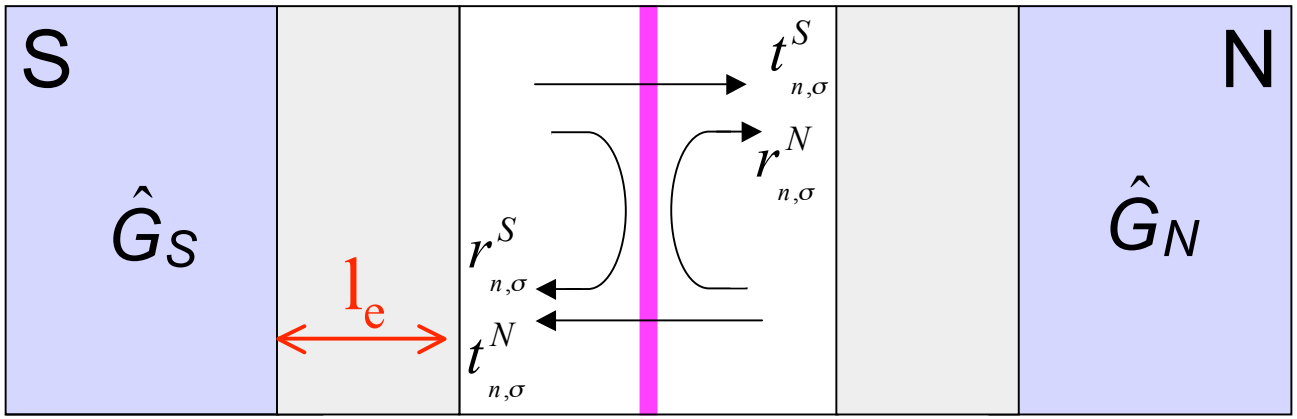
$\omega_n = \pi(2n+1)k_B T$

Usadel Eqs.

$$D\,\partial_r\left(\hat{G}\partial_r\hat{G}\right) = [(\omega_n + iE_{ex}\hat{\sigma})\hat{\tau}_3 + \hat{\Delta}(\vec{r}), \hat{G}]$$

Spin degenerate boundary conditions for a diffusive S/N interface

V. V. Kuprianov et Lukichev (1988)



Ballistic zone

Isotropisation zone

Diffusive zone

$l_e \sim \xi_{S(N)}$

Barrier described in a scattering approach

$$S_\sigma = \begin{bmatrix} r^S_{n,\sigma} & t^N_{n,\sigma} \\ t^S_{n,\sigma} & r^N_{n,\sigma} \end{bmatrix} \qquad S_\uparrow = S_\downarrow$$

$$G_T = G_Q \sum_n T_n$$

$$G_Q = e^2/\hbar$$

g_N : Conductance per unit lenght of N

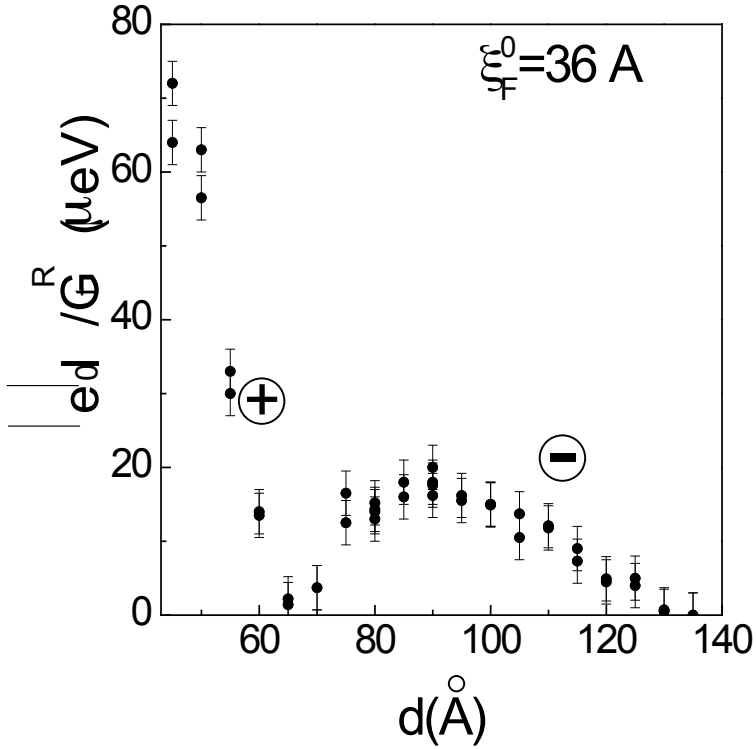
Tunnel limit:

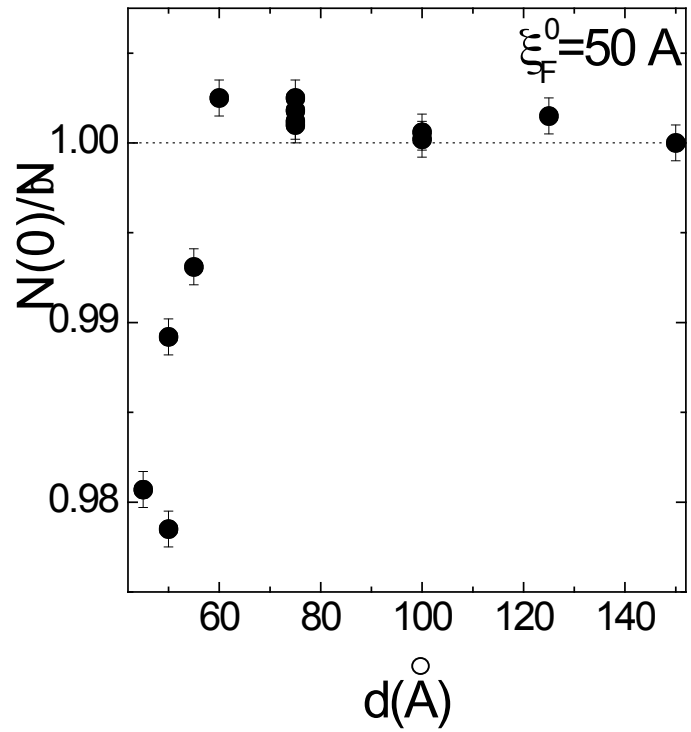
$$T_n = \sum_\sigma \left| t_{n,\sigma} \right|^2 << 1 \Rightarrow \boxed{2 g_N \hat{G}_N \frac{\partial \hat{G}_N}{\partial x} = G_T [\hat{G}_S, \hat{G}_N]}$$

Theoretical interpretation of Proximity effect measurements in PdNi

$$\xi_F = (\hbar D / E_{ex})^{1/2}$$

$$a_t^0 = \left.\frac{G_t^L d}{g_F}\right|_{d\ \mathrm{small}} = 0.4$$





Data of T. Kontos et al. (2001, 2002)

Theoretical interpretation of Proximity effect measurements in PdNi

Usadel Equations

+ spin degenerate boundary conditions

of *V. V. Kuprianov et Lukichev (1988)*

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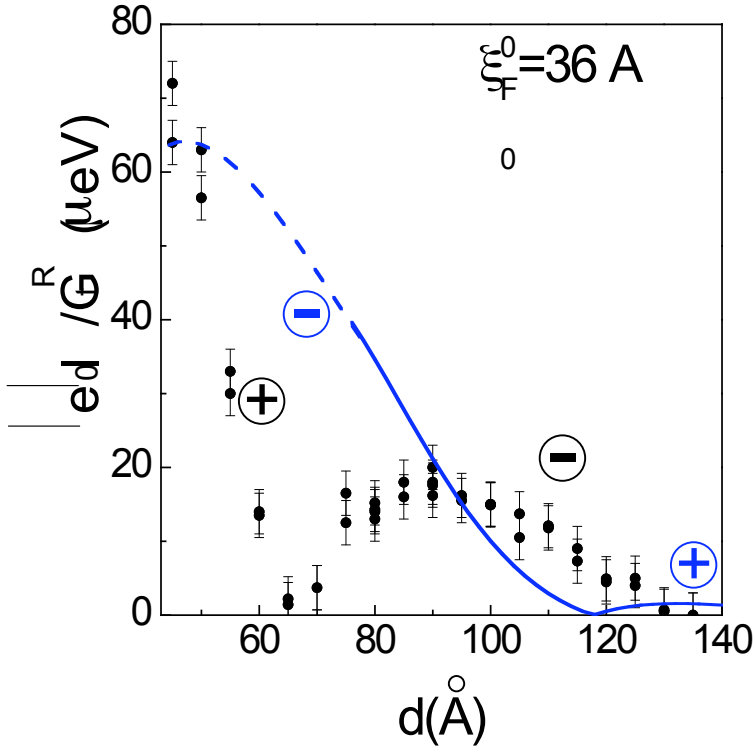
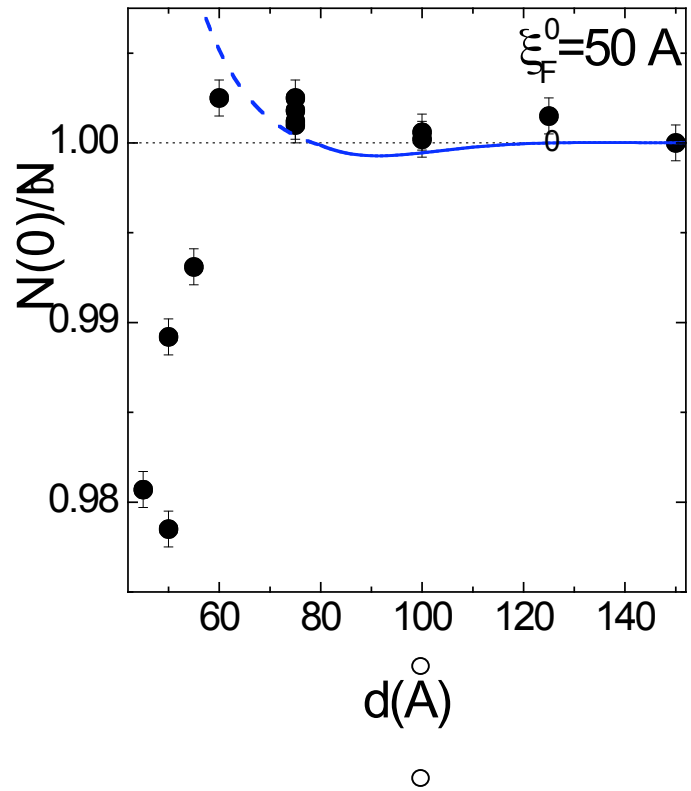
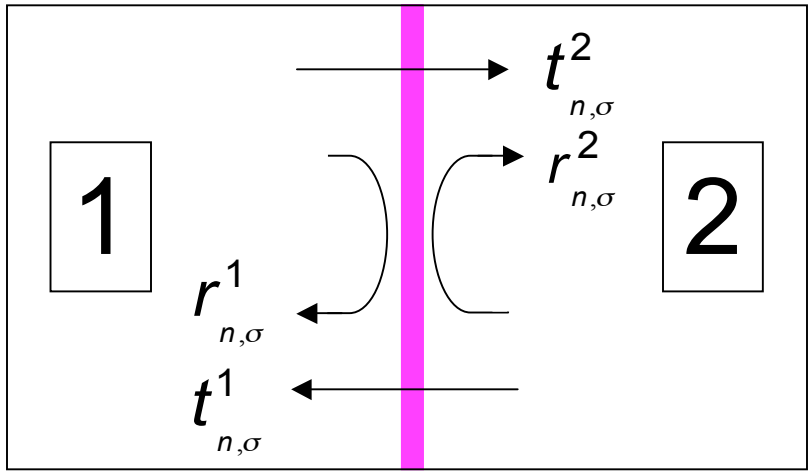




Data of T. Kontos et al. (2001, 2002)

Spin dependent interfacial scattering

Following D. Huertas-Hernando, Y. Nazarov and W. Belzig (2002)

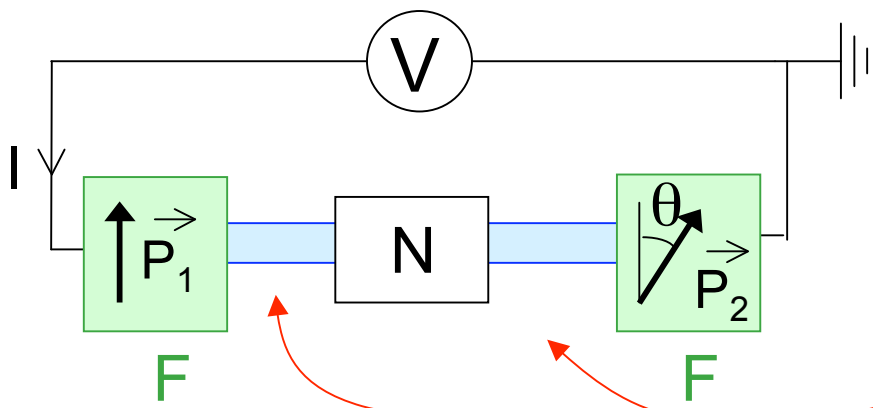


$$t^{1(2)}_{n,\sigma} = \left| t^{1(2)}_{n,\sigma} \right| e^{i\varphi^{1(2)}_{t,\sigma}}$$

$$r^{1(2)}_{n,\sigma} = \left| r^{1(2)}_{n,\sigma} \right| e^{i\varphi^{1(2)}_{r,\sigma}}$$

Spin-dependence of the phases !

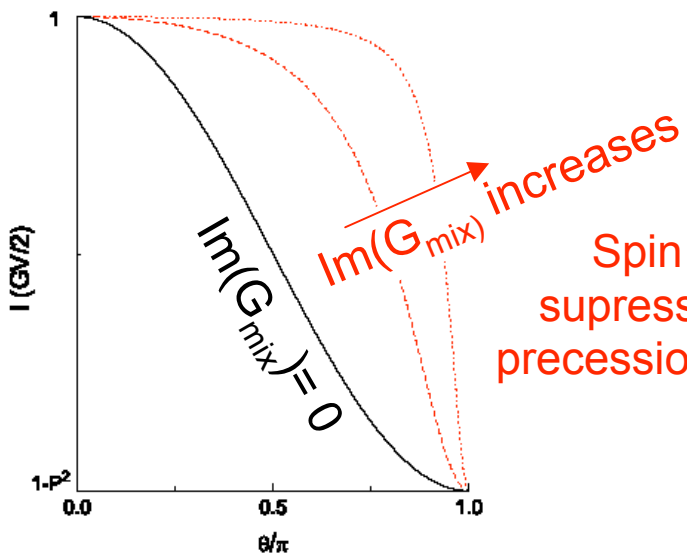
Spin dependence of the interfacial diffusion phase shifts
Effects in the F/N/F diffusive case



A. Braatas et al., (2001)

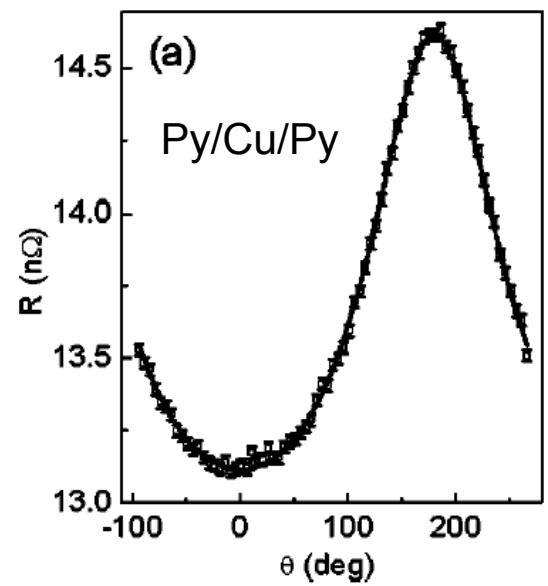
$$G_{mix} = G_Q \sum_n \left(1 - r^N_{n,\uparrow} r^{N\,*}_{n,\downarrow}\right)$$

$$\mathrm{Im}(G_{mix}) = 0 \text{ if } \varphi^N_{r,\uparrow} = \varphi^N_{r,\downarrow}$$

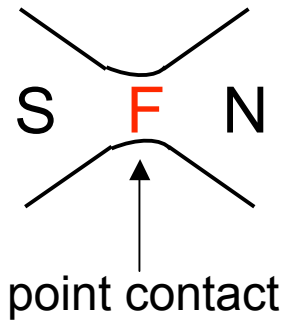


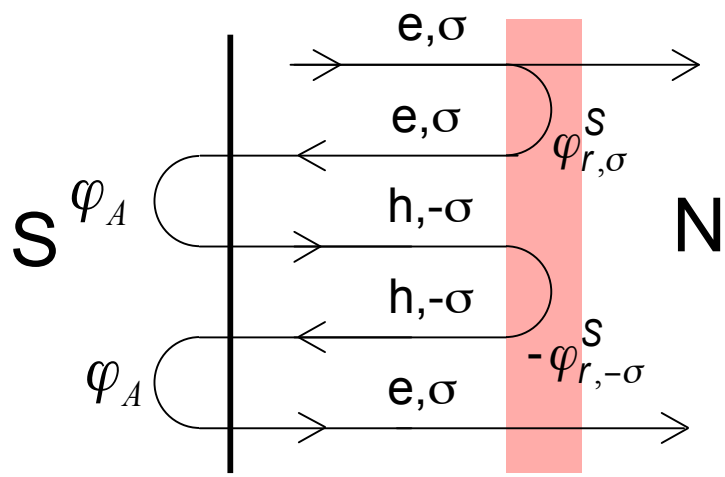
Spin accumulation supressed by interfacial precession effects for $\theta \neq 0, \pi$

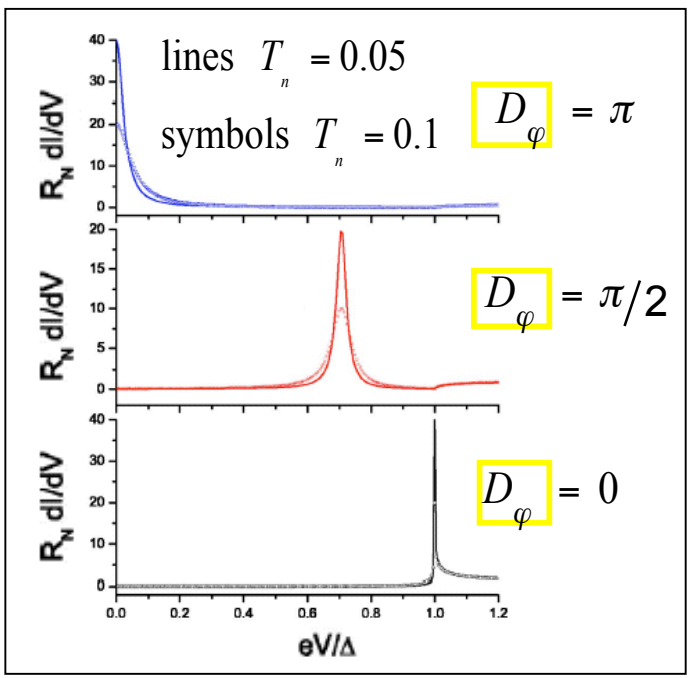
*S. Urazdin et al., Phys. Rev. B **71**, 100401 (2005)*



S/F/N ballistic point contact







*E. Zhao et al. PRB **70**, 134510 (2004)*

Andreev resonances:

$$2\varphi_A - \sigma D_\varphi = 0 \; [2\pi]$$

$$D_\varphi = \varphi^S_{r,\uparrow} - \varphi^S_{r,\downarrow}$$

$$\varphi_A = 2\arccos\,(eV/\Delta)$$

Angular parametrisation of the problem

$\theta_\sigma(\omega_n)$: Pairing angle

$\varphi_\sigma(\omega_n)$: Superconducting phase

Bulk solutions

In S : $\theta_\sigma(\omega_n) = \pi / 2$

In F : $\theta_\sigma(\omega_n) = 0$

- Usadel equations in the limit of a weak proximity effect :

$$\begin{cases} \dfrac{\partial^2 \theta_\sigma}{\partial x^2} - \dfrac{k_\sigma^2}{\xi_F^2}\theta_\sigma - \dfrac{Q_\sigma^2}{\theta_\sigma^3} = 0 \\ Q_\sigma = \dfrac{\partial \varphi_\sigma}{\partial x}\theta^2{}_\sigma \end{cases}$$

with

$$k_\sigma = \sqrt{2\left(i\sigma \,\mathrm{sgn}(\omega_n) + \frac{|\omega_n|}{E_{ex}} \right)}$$

$$\xi_F = (\hbar D / E_{ex})^{1/2}$$

Description of S/F diffusive hybrid circuits in the limit of weak proximity effect

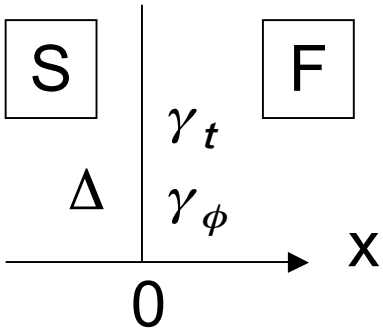
Boundary conditions in F for a S/F interface:

$$\begin{cases} g_F \dfrac{\partial \theta_\sigma}{\partial x} = G_T\left(\cos(\theta_S)\theta_\sigma - \sin(\theta_S)\cos(\varphi_\sigma - \varphi_S)\right) + \boxed{iG_\phi \sigma \theta_\sigma} \\ g_F \dfrac{\partial \varphi_\sigma}{\partial x}\theta_\sigma = G_T \sin(\theta_S)\sin(\varphi_\sigma - \varphi_S) \end{cases}$$

In this seminar:

- Rigid boundary conditions: $\theta_S = \arctan\left(\dfrac{\Delta}{|\omega_n|}\right)$
- Δ — E_{ex} $\Longrightarrow k_\sigma = 1 + i\sigma\,\mathrm{sgn}(\omega_n)$

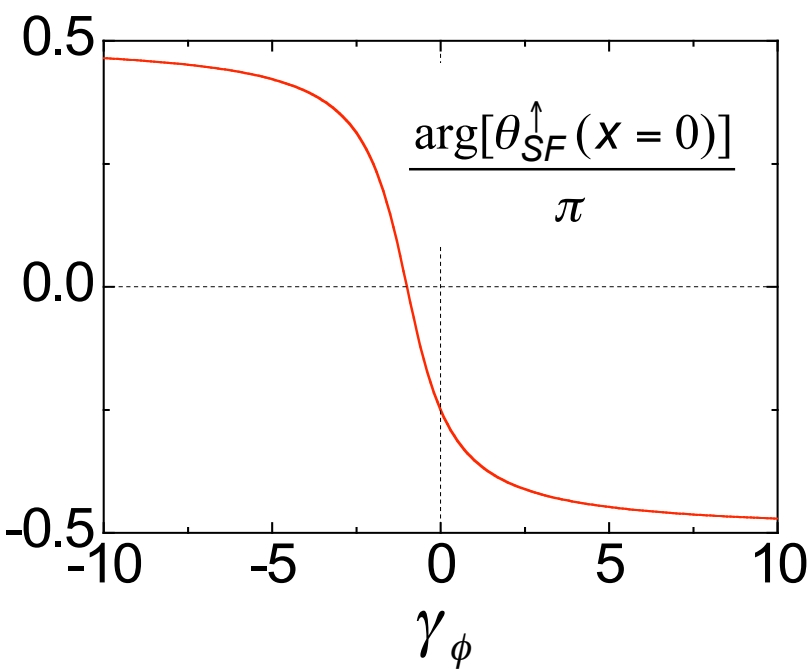
Semi-infinite S/F structure

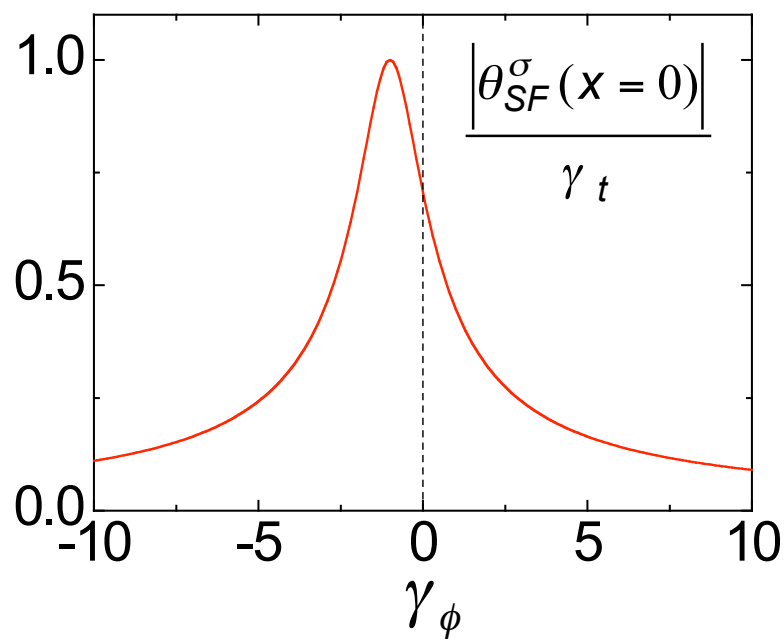


$$\gamma_{t(\phi)} = \frac{G_{t(\phi)}\xi_F}{g_F}$$

$$Q_\sigma = 0$$

$$\theta_{SF}^{\sigma}(x) = \frac{\gamma_t \sin(\theta_S)}{k_\sigma + i\gamma_\phi \sigma \,\mathrm{sgn}(\omega_n)} e^{-k_\sigma \frac{x}{\xi_F}}$$





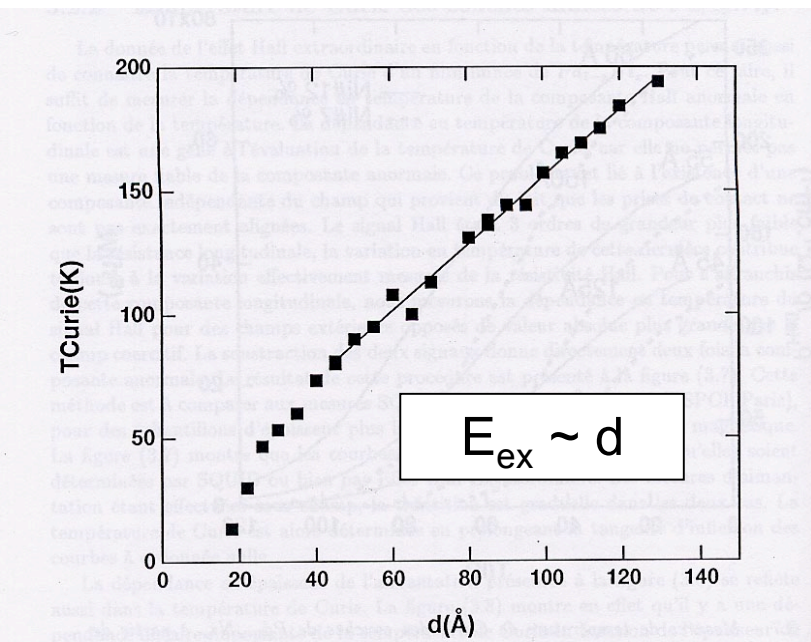
Experimental parameters for Nb/PdNi structures

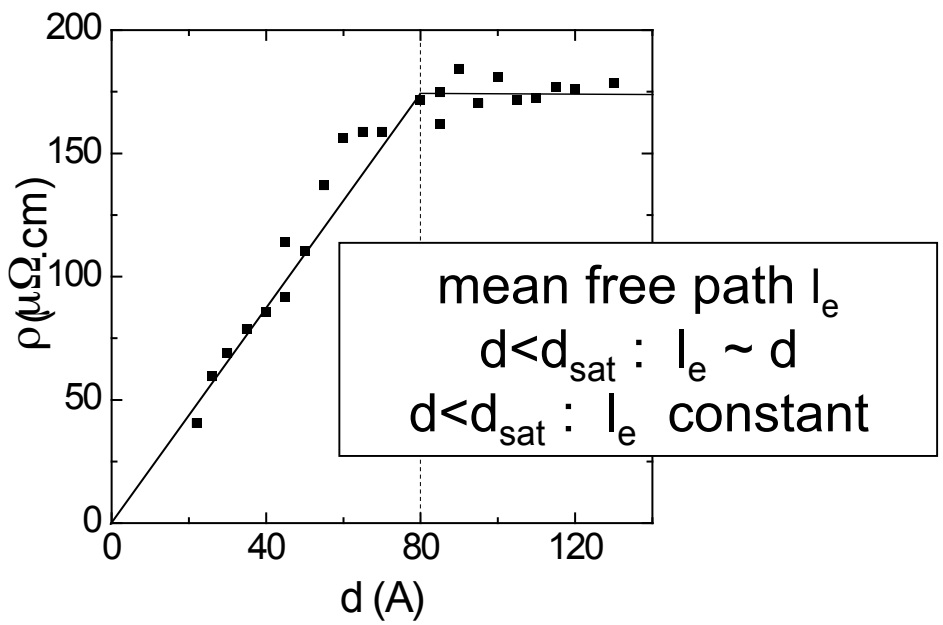
S/F/I = Nb/PdNi/Alox/Al

S/F/S = Nb/PdNi/Alox/Al/Nb $\Rightarrow$ $\gamma_t^R / \gamma_t^L \neq 1$

Δ_{Nb}=1.35 meV, $\Delta_{Al/Nb}$=0.6 meV $\neq$ $E_{ex} \sim 10$ meV $\Rightarrow$ $k_\sigma = 1 + i\sigma\,\mathrm{sgn}(\omega_n)$

T=1.5K $\sim T_c / 6$





Experimental parameters for Nb/PdNi structures

	γ_t^L	γ_ϕ^L	ξ_F
$d<d_{sat}$	$\frac{a_t^0 \xi_F^0}{d}$	γ_ϕ^0	ξ_F^0
$d>d_{sat}$	$\frac{a_t^0 \xi_F^0}{\sqrt{d_{sat} d}}$	$\gamma_\phi^0 \sqrt{\frac{d}{d_{sat}}}$	$\xi_F^0 \sqrt{\frac{d_{sat}}{d}}$

$$a_t^0 = \frac{G_t^L d_{sat}}{g_{F,sat}}$$

$$\sigma_F = 2e^2 N_0 D$$

$$D = v_F l_e / 3$$

$$\xi_F = (\hbar D / E_{ex})^{1/2}$$

$$\gamma_{t(\phi)} = \frac{G_{t(\phi)} \xi_F}{\sigma_F \eta}$$

$$E_{ex} \propto d$$

$$G_\phi^L \propto E_{ex}$$

$E_{ex} - E_F$

3 fitting parameters: $a_t^0, \gamma_\phi^0, \xi_F^0$

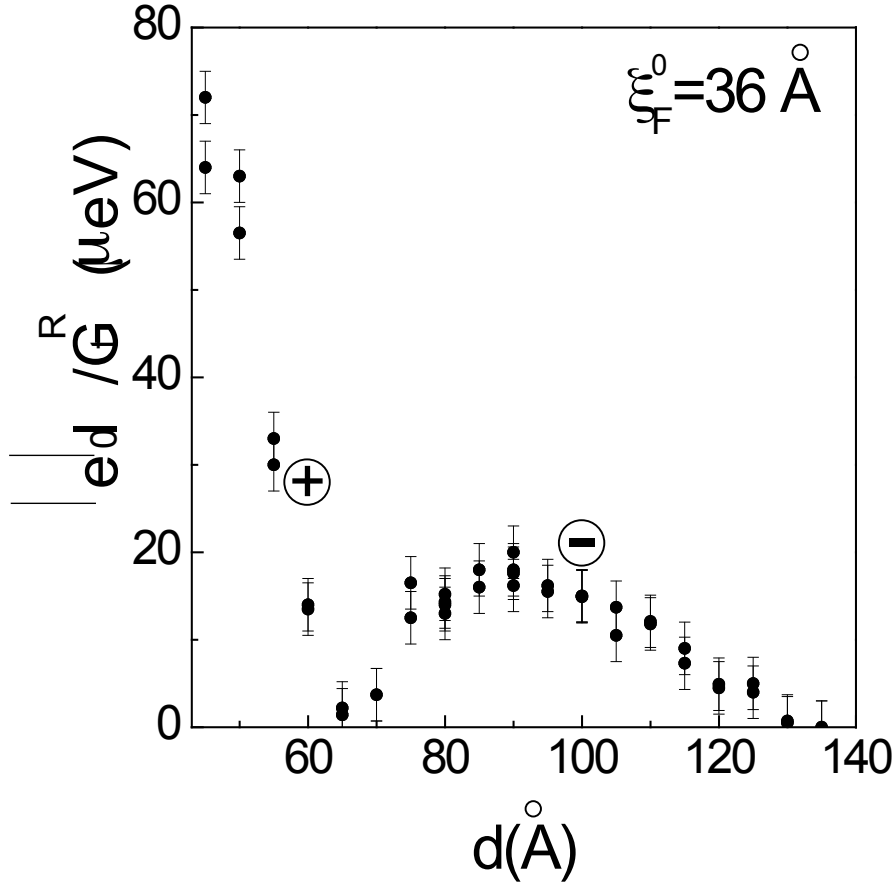
constraints:

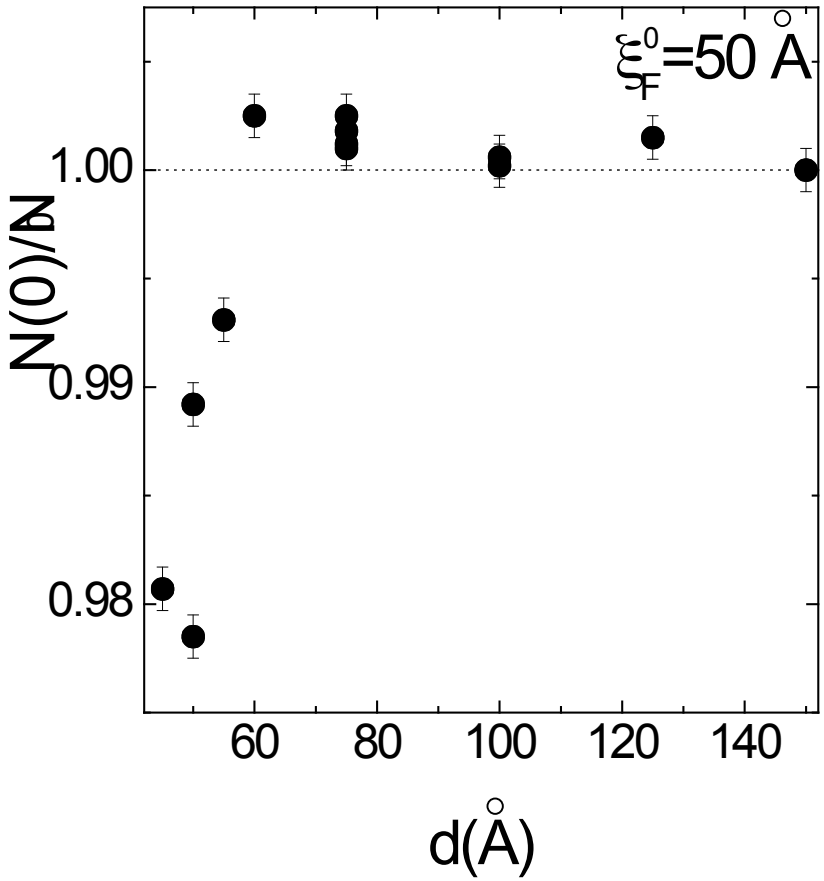
Nb/Pd : $a_t^0 = 0.2$

$\xi_F^0 \sim 50$ Å

Theoretical interpretation of Proximity effect measurements in PdNi

a_t^0=0.4





Theoretical interpretation of Proximity effect measurements in PdNi

a_t^0=0.4