SPIN TRANSPORT IN METALS AND INSULATORS

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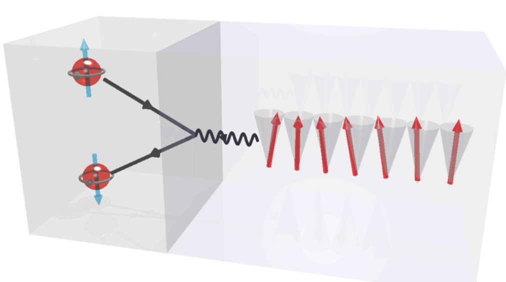
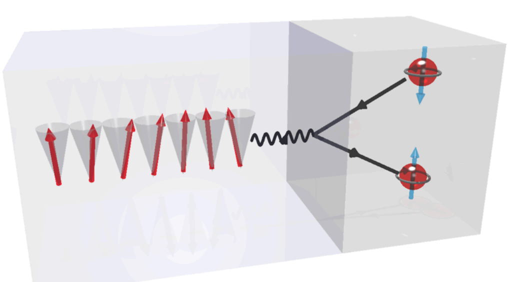
(2013 KITP "tutorial")





OUTLINE

Introduction

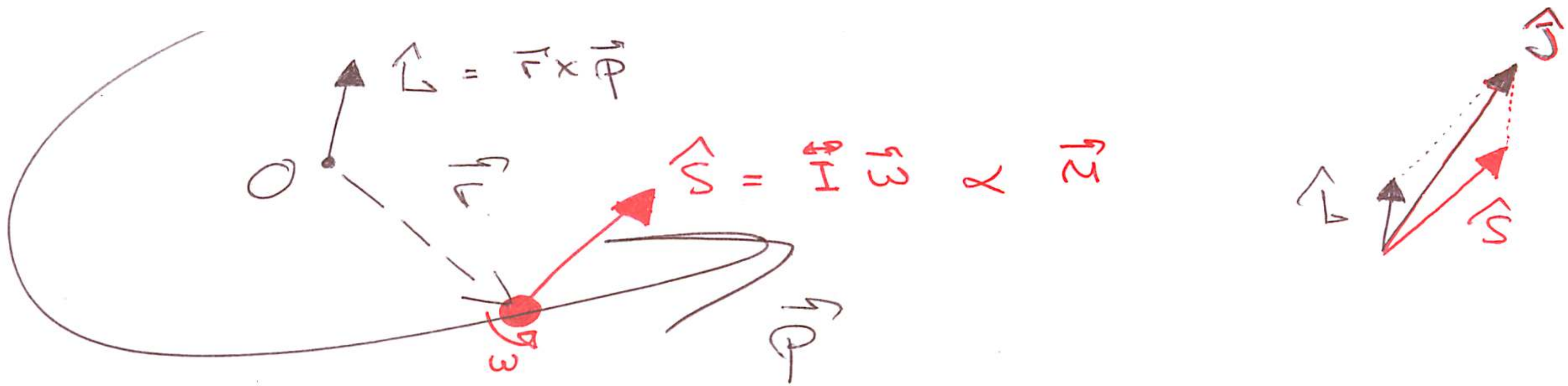
- Landau-Lifshitz theory of magnetic dynamics
- Magnonic and electronic spin-transfer torques
- Fluctuation-dissipation theorem (thermodynamics)
- Onsager reciprocity and spin-motive forces
- Gauge theory perspective

Dynamic phase transitions: swasing/magnon BEC

Spin transfer by Dirac electrons (on TI surface)

Thermal (anomalous) Hall effect due to DMI

ANGULAR MOMENTUM

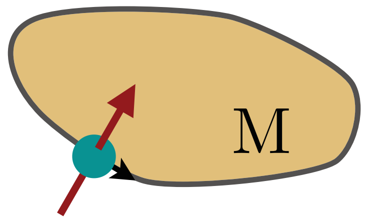


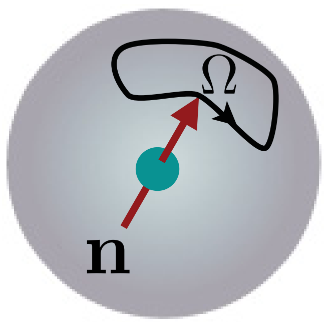
The total angular momentum of a physical system must be conserved due to the *isotropicity of space:*

$$\mathbf{J} = \text{const} \quad \Rightarrow \quad \frac{d\mathbf{L}}{dt} + \frac{d\mathbf{S}}{dt} = 0$$

ACTION FOR A FREE PARTICLE

The classical phase-space action for a point-like particle:





$$\mathbf{S} = S\mathbf{n}$$

$$\mathbf{n} \to (\theta, \phi)$$

$$\mathfrak{S}[\mathbf{p}(t), \mathbf{r}(t), \mathbf{n}(t)] = \int \underbrace{d\mathbf{r}\cdot\mathbf{p}}_{\text{kinetic}} - S\int \underbrace{d\phi(1-\cos\theta)}_{\text{geometric}}$$

The corresponding total angular momentum:

$$\boxed{\mathbf{J} \equiv \frac{\delta\mathfrak{S}}{\delta\varphi} = \mathbf{r}\times\mathbf{p} + S\mathbf{n}}$$

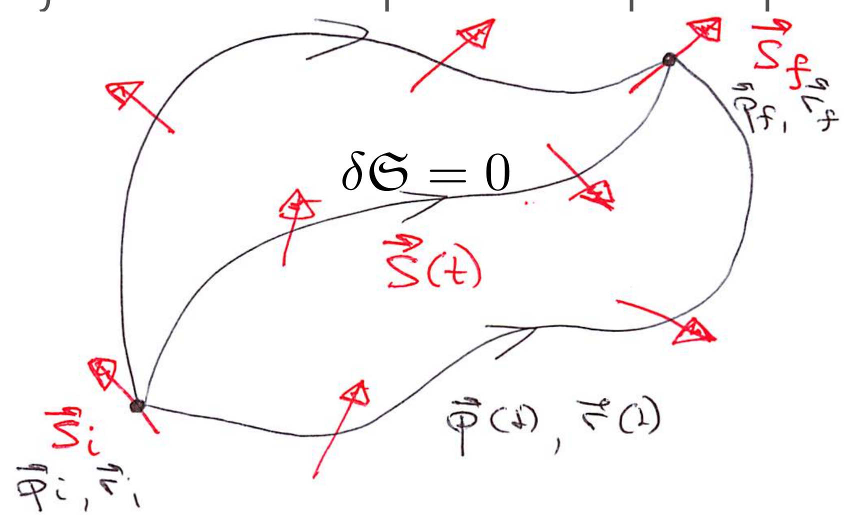
QUANTUM DESCRIPTION

Such Lagrangian formulation can be readily extended to the quantum limit (appropriate for, e.g., molecular magnets or individual quasiparticles that carry a small spin), as follows:

The quantum-mechanical propagator (Green's function) is obtained by an appropriate path integral:

$$G = \int \mathcal{D}[q(t)] e^{\frac{i}{\hbar}\mathfrak{S}[q(t)]}$$

over all possible trajectories in phase+spin space, $q \equiv (\mathbf{p}, \mathbf{r}, \mathbf{n})$



LOCALIZED SPIN DYNAMICS

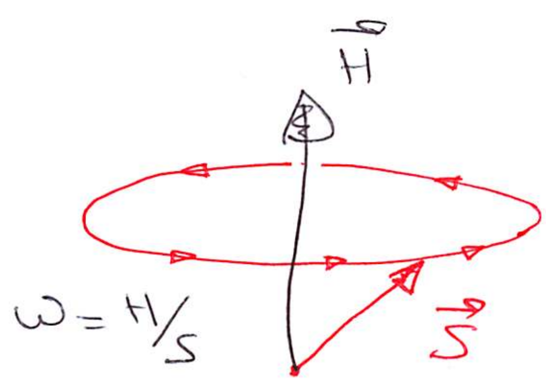
As a starting point (which also reflects historical development of the theory of magnetic dynamics), consider a *localized collective spin variable* $\mathbf{S}(t) = S\mathbf{n}(t)$:

$$\mathfrak{S}[\mathbf{n}(t)] = -S\int \underbrace{d\phi(1-\cos\theta)}_{\text{geometric}} - \int \underbrace{dt\mathcal{H}(\mathbf{n})}_{\text{dynamic}}$$

(Note: Can follow a classical treatment, so long as $S \gg \hbar$)

Minimizing action, $\delta\mathfrak{S} = 0$, we obtain *Larmor precession:*

$$S\frac{d\mathbf{n}}{dt} = \mathbf{H}\times\mathbf{n} \qquad \text{where} \qquad \mathbf{H} \equiv \frac{d\mathcal{H}}{d\mathbf{n}}$$



MAGNETIC CONTINUUM

Well below the *Curie temperature* T_c, the *local spin density* in a ferromagnet can be approximated to be *fully saturated*, i.e.,

$$\mathbf{s}(\mathbf{r},t) = s\mathbf{n}(\mathbf{r},t) \qquad \text{where} \qquad |\mathbf{n}(\mathbf{r},t)| \equiv 1$$

while the spatial structure may exhibit directional inhomogeneity

We will henceforth call it *spin texture:*

Goldstone modes: spin waves (a.k.a. magnons)

At a finite (but still low, compared to T_c) temperature, the free-energy density is given by a *nonlinear* $\boldsymbol{\sigma}$ *model:*

$$\mathcal{F}[\mathbf{n}(\mathbf{r})] = \mathbf{H}\cdot\mathbf{n} + \frac{A}{2}(\boldsymbol{\nabla}\mathbf{n})^2 + (\text{relativistic corrections})$$

dipolar interactions, crystalline anisotropies, DMI

LANDAU-LIFSHITZ EQUATION

The Larmor precession is generalized as follows:

$$s\frac{d\mathbf{n}}{dt} = \mathbf{H}_{\text{eff}} \times \mathbf{n}$$

where

$$\mathbf{H}_{\text{eff}} \equiv \frac{\delta F}{\delta \mathbf{n}} = \mathbf{H} - A\nabla^2\mathbf{n}$$

in the *exchange approximation* (i.e., neglecting relativistic effects)

We can put it together as:

$$s\frac{d\mathbf{n}}{dt} = \mathbf{H} \times \mathbf{n} - \nabla_i \mathbf{j}_i \qquad \text{where} \qquad \mathbf{j}_i = A\nabla_i \mathbf{n} \times \mathbf{n}$$

spin continuity equation with local precession

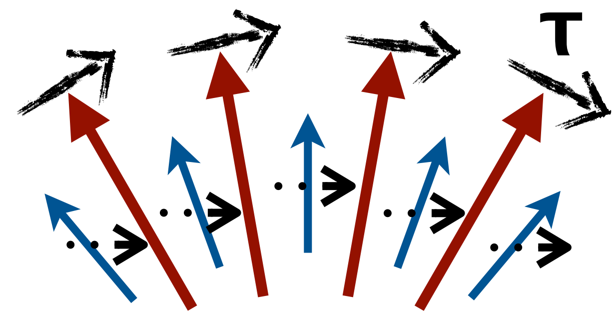
we interpret this as the *spin current* carried by magnons

SPIN TRANSFER TORQUE

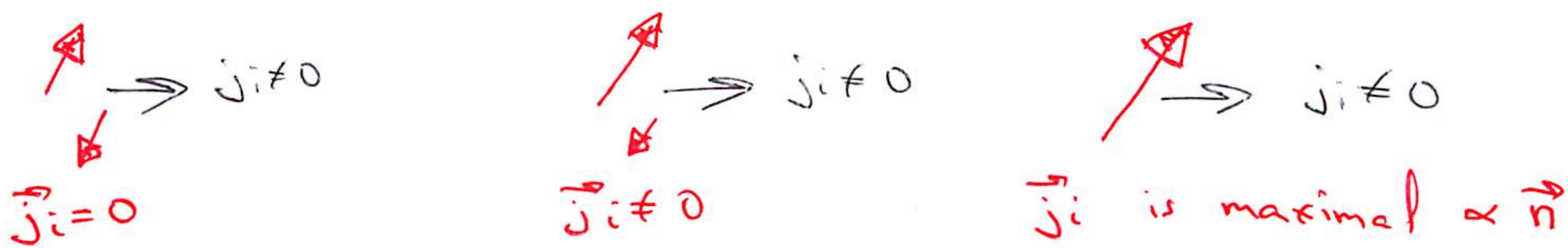
In metals, furthermore, there is a *torque* associated with the spin current carried by itinerant electrons:

$$\mathbf{j}_i = A\nabla_i \mathbf{n} \times \mathbf{n} - P\frac{\hbar}{2e}\mathbf{n} j_i$$

magnon spin current — electron spin current



$P = 0$	$0 < P < 1$	$P = 1$
normal metal	ferromagnet	halfmetal



This produces a new term in the equation of motion:

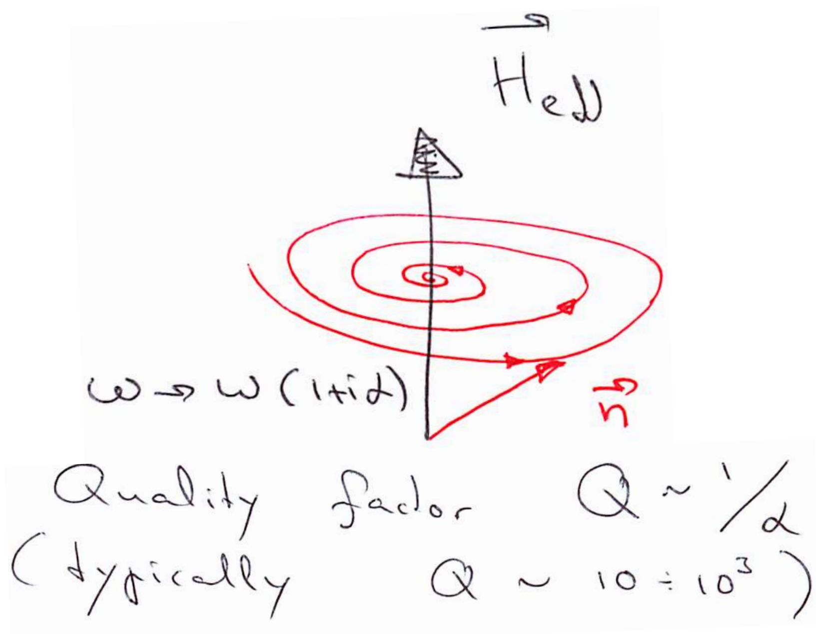
$$s\frac{d\mathbf{n}}{dt} = (\text{LL terms}) + P\frac{\hbar}{2e}(\mathbf{j}\cdot\boldsymbol{\nabla})\mathbf{n}$$

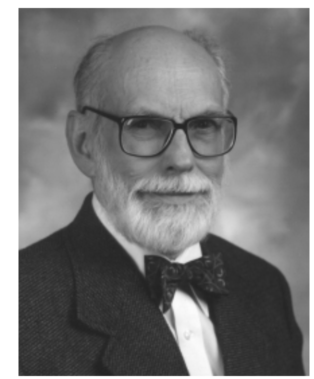
Volovik, *JPC* (1987)

GILBERT DAMPING

The ferromagnetic damping is most naturally introduced as an exponential spiraling down of magnetic precession:

$$s(1 + \alpha \mathbf{n} \times) \frac{d\mathbf{n}}{dt} = \mathbf{H}_{\text{eff}} \times \mathbf{n}$$





Gilbert, *IEEEM* (2004)

Typically α is isotropic (i.e., is a scalar) and independent of $\mathbf{H}_{\text{eff}}$

contrast this to the Bloch phenomenology, where the quality factor of transverse precession, ~HT_2, increases with the applied field H

ENERGY DISSIPATION

The rate of energy dissipation according to the LLG equation:

$$P \equiv -\frac{dF}{dt} = -\int dV \frac{\delta F}{\delta \mathbf{n}} \cdot \frac{d\mathbf{n}}{dt} = -\int dV\, \mathbf{H}_{\text{eff}} \cdot \frac{d\mathbf{n}}{dt}$$

$$= \alpha s \int dV \frac{d\mathbf{n}}{dt} \cdot \frac{d\mathbf{n}}{dt} \rightarrow s \int dV \frac{d\mathbf{n}}{dt} \cdot \hat{\alpha} \cdot \frac{d\mathbf{n}}{dt}$$

scalar damping tensorial damping

$\hat{\alpha}$ must thus generally be a positive-definite matrix

This expression is an instance of the *Rayleigh dissipation functional*

According to a fundamental principle of thermodynamics, the dissipation in a driven system suggests the presence of fluctuations in thermal equilibrium

FLUCTUATION-DISSIPATION THEOREM

In general, inverting the LLG equation as

$$\mathbf{H}_{\text{eff}} \left(\equiv \frac{\delta F}{\delta \mathbf{n}} \right) = -\hat{\gamma} \frac{d\mathbf{n}}{dt}$$

we have for the stochastic *Langevin field*

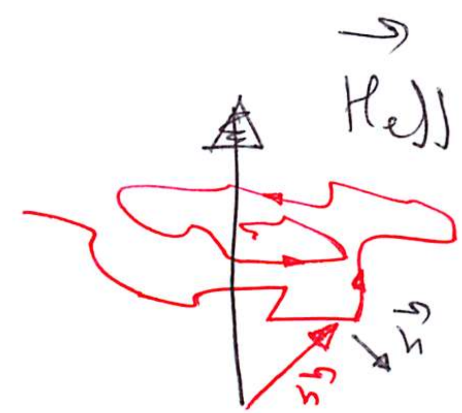
$$\langle h_i(t) h_j(t') \rangle = k_B T (\gamma_{ij} + \gamma_{ji}) \delta(t - t')$$

Landau & Lifshitz, *vol. 5*

In case of isotropic Gilbert damping, the full finite-temperature LLG equation thus reads

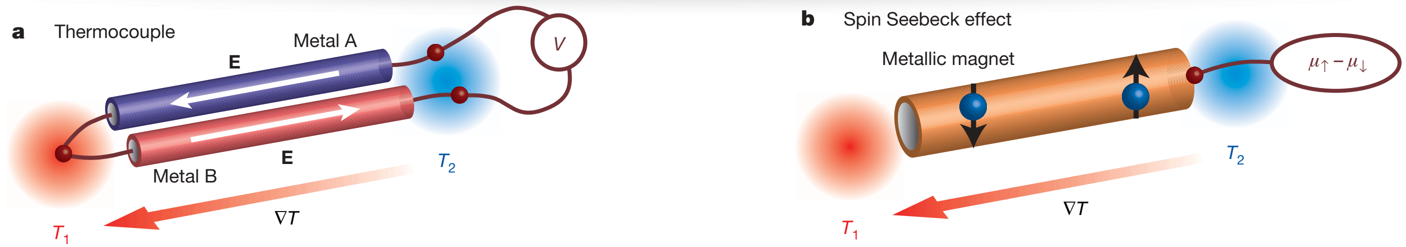
$$s(1 + \alpha \mathbf{n} \times) \frac{d\mathbf{n}}{dt} = (\mathbf{H}_{\text{eff}} + \mathbf{h}) \times \mathbf{n}$$

$$\langle h_i(t) h_j(t') \rangle = 2 \alpha s k_B T \delta(t - t')$$

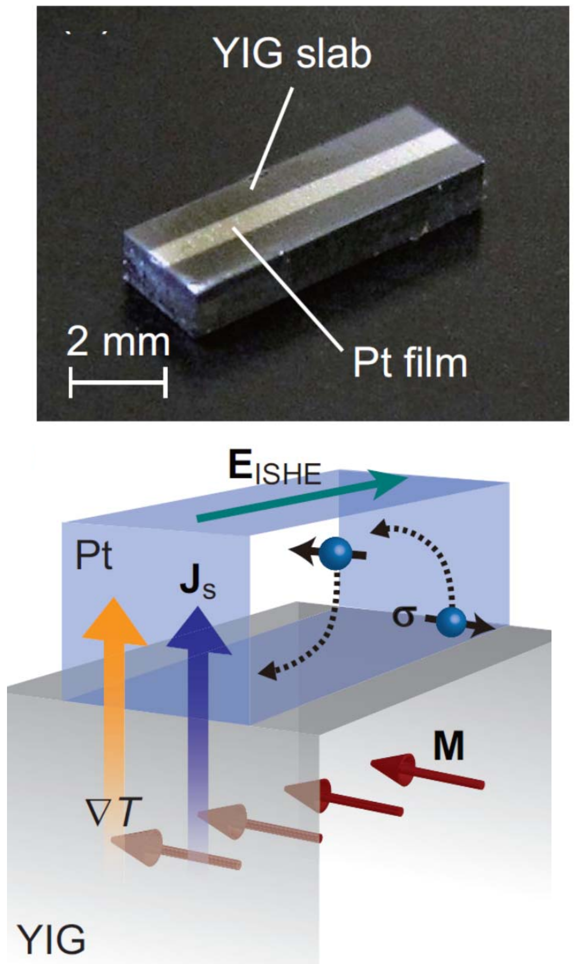


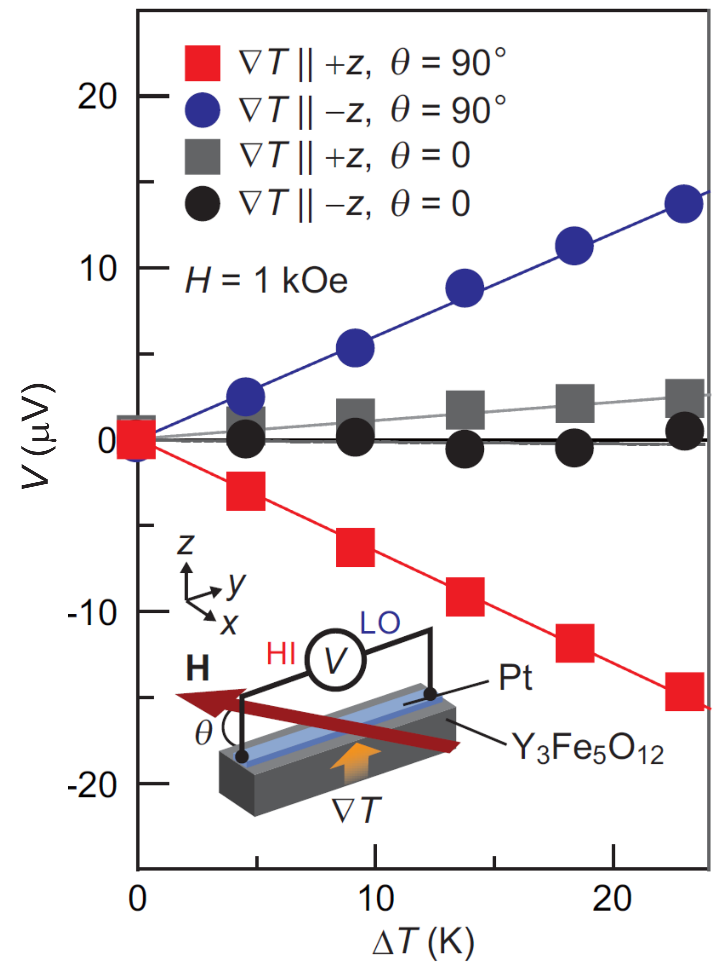
Brown, *PR* (1963)

SPIN SEEBECK EFFECT



Uchida, Saitoh *et al.*, *Nature* (2008)



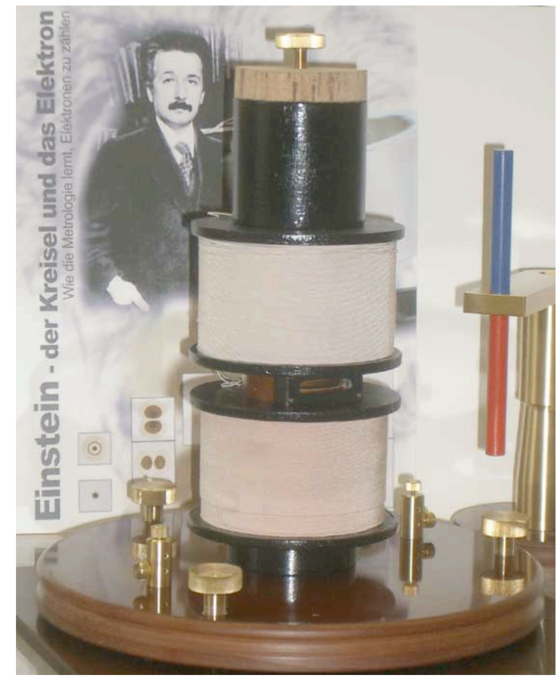


Uchida, Saitoh *et al.*, *APL* (2010)

DISSIPATIVE SPIN TORQUE

Gilbert damping transfers angular momentum from the magnetic subsystem to the crystal that supports it

as is the case in the *Einstein-de Haas effect:*



This means that the spin continuity equation also needs to be revised, as the angular momentum is now leaking away from the magnetic degrees of freedom

We will do it in a fashion formally analogous to the introduction of Gilbert damping:

$$s(1 + \alpha \mathbf{n}\times)\frac{d\mathbf{n}}{dt} = \mathbf{H}_{\text{eff}} \times \mathbf{n} + P\frac{\hbar}{2e}(1 + \beta \mathbf{n}\times)(\mathbf{j} \cdot \boldsymbol{\nabla})\mathbf{n}$$

The *Galilean-invariant* limit of the *Stoner model* is established by setting $\alpha = \beta$

ONSAGER RECIPROCITY

One more element is necessary to complete the thermodynamic picture: The *Onsager*-reciprocal *motive force* exerted on the electrons by magnetic dynamics

We proceed by casting the coarse-grained equations in the form of a *quasistationary* relaxation toward thermodynamic equilibrium:

$$F[\rho(\mathbf{r}), \mathbf{p}(\mathbf{r}), \mathbf{n}(\mathbf{r})]$$

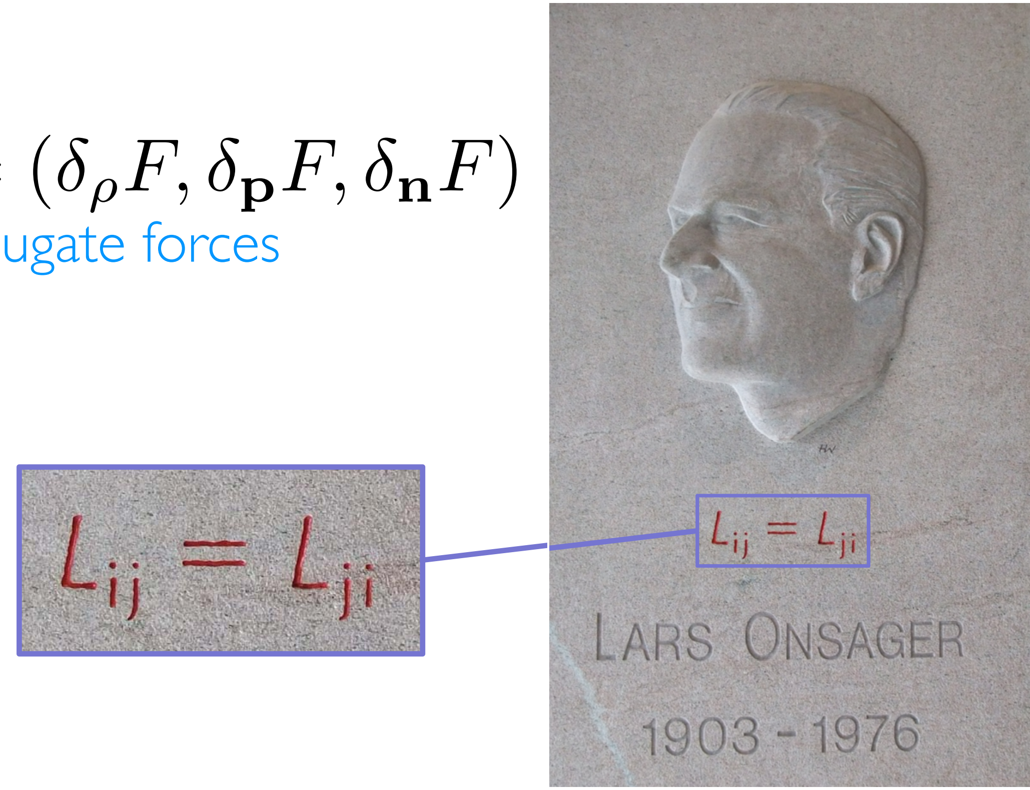
free energy

$$(\mu, \mathbf{j}, \mathbf{H}) = (\delta_\rho F, \delta_\mathbf{p} F, \delta_\mathbf{n} F)$$

conjugate forces

$$\partial_t \begin{pmatrix} \rho \\ \mathbf{p} \\ \mathbf{n} \end{pmatrix} = \widehat{L}[\mathbf{n}(\mathbf{r})] \begin{pmatrix} \mu \\ \mathbf{j} \\ \mathbf{H} \end{pmatrix}$$

Landau & Lifshitz, *vol. 5*



SPIN-MOTIVE FORCE

Starting with the charge continuity and LLG equations:

$$\dot{\rho} = -\boldsymbol{\nabla} \cdot \mathbf{j}$$

$$s(1 + \alpha \mathbf{n}\times)\dot{\mathbf{n}} = \mathbf{H} \times \mathbf{n} + q^*(1 + \beta \mathbf{n}\times)(\mathbf{j} \cdot \boldsymbol{\nabla})\mathbf{n}$$

we obtain the Onsager-reciprocal *motive force*

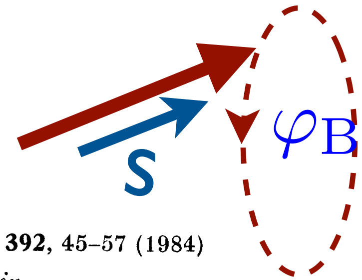
$$\mathbf{F} = -\boldsymbol{\nabla}\mu - q^*(\mathbf{n} \times \dot{\mathbf{n}} - \beta\dot{\mathbf{n}})_i \boldsymbol{\nabla} n_i$$

YT and Mecklenburg, *PRB* (2008)

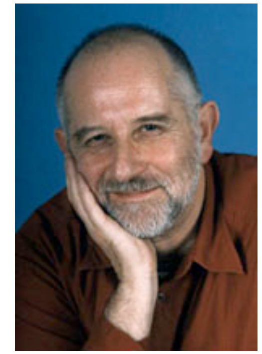
The coupling constant in this theory of *spin magnetohydrodynamics* is given by

$$q^* = P\frac{\hbar}{2e}$$

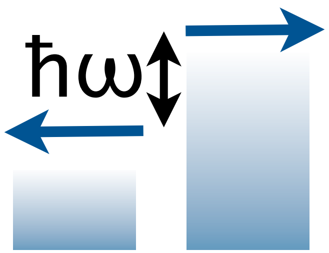
GEOMETRIC NATURE



$$\varphi_{\text{Berry}} = s\Omega$$

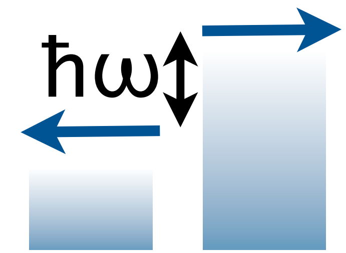


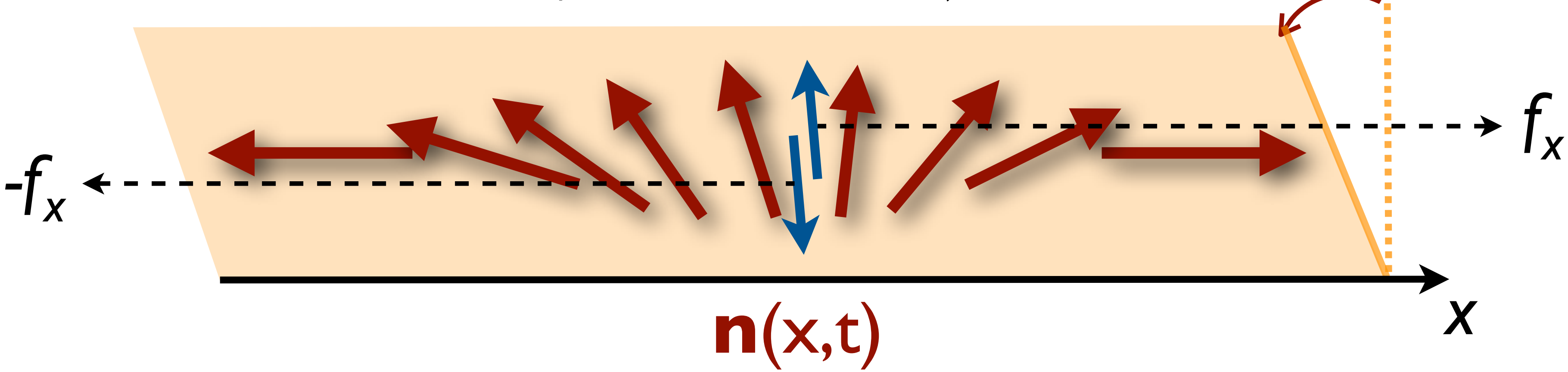
Proc. R. Soc. Lond. A **392**, 45–57 (1984)
Printed in Great Britain



Quantal phase factors accompanying adiabatic changes

By M. V. Berry, F.R.S.
H. H. Wills Physics Laboratory, University of Bristol, Tyndall Avenue, Bristol BS8 1TL, U.K.



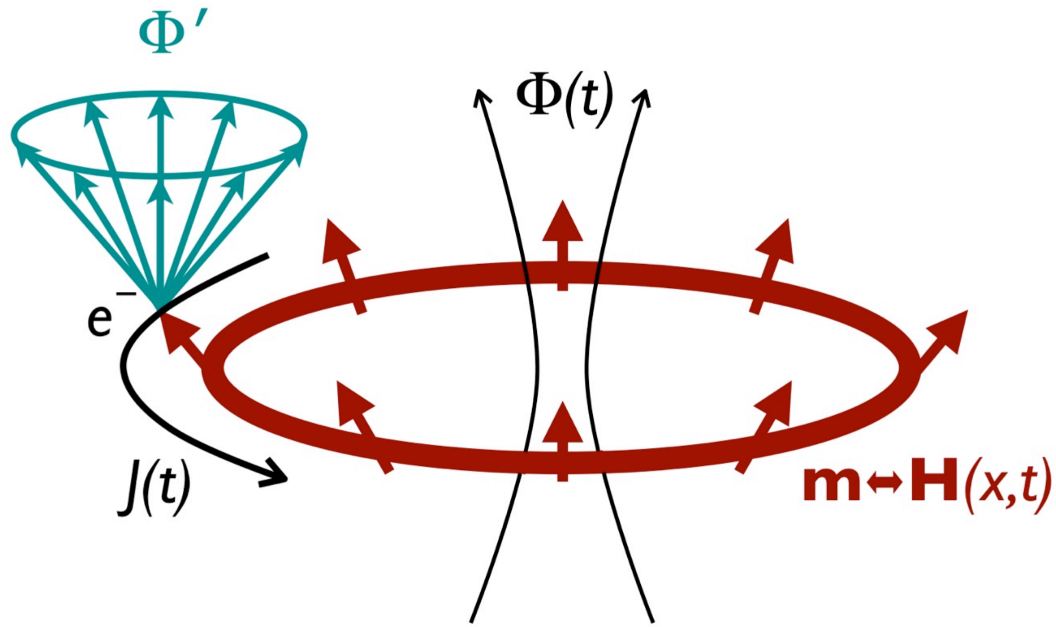


Barnes and Maekawa, *PRL* (2007)

Note that in the Onsager reciprocal of the spin continuity equation we return to the familiar geometric action!

SPIN MAGNETOHYDRODYNAMICS

The collective spin magnetohydrodynamics can be recast as a gauge theory



with the free energy

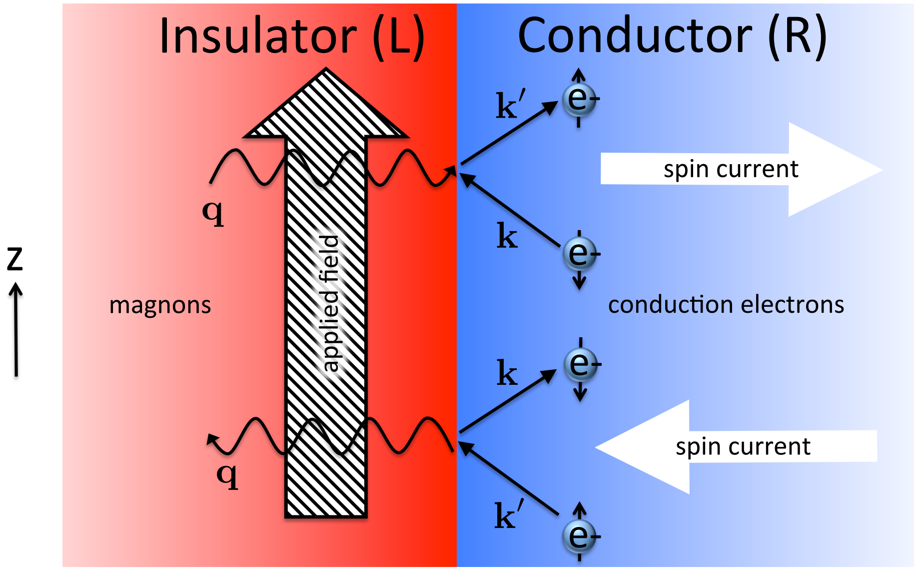
$$\mathcal{F}(\mathcal{J}, \Phi, \Phi'[\mathbf{m}]) = \frac{[\mathcal{J} - (\Phi + \Phi')/c]^2}{2L}$$

which contains the essential information about the structure of the underlying spin-transfer torques and reciprocal pumping

Volovik, *JPC* (1987); YT and Wong, *PRB* (2009)

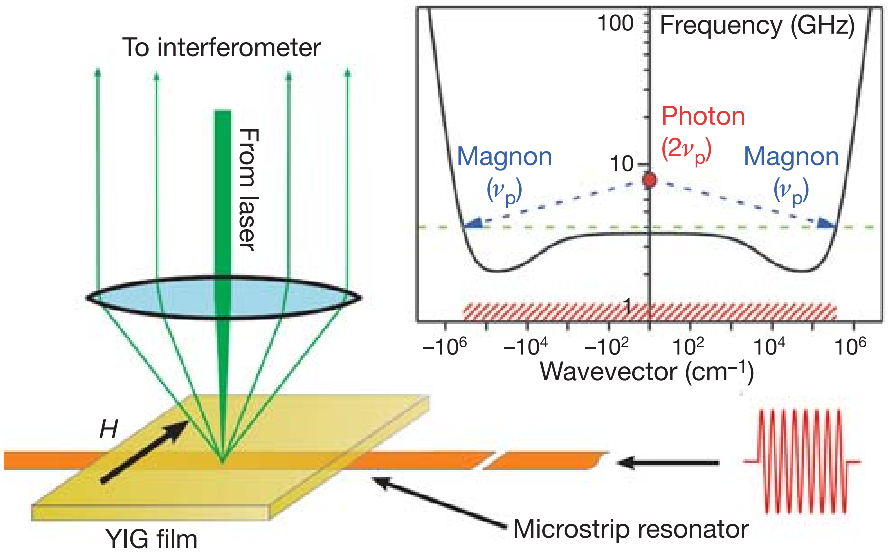
ELECTRONIC PUMPING OF BEC

We want to develop a dc-transport route to inducing BEC of magnons in magnetic thin-film heterostructures



Bender, Duine, and YT, *PRL* (2012)

VS.

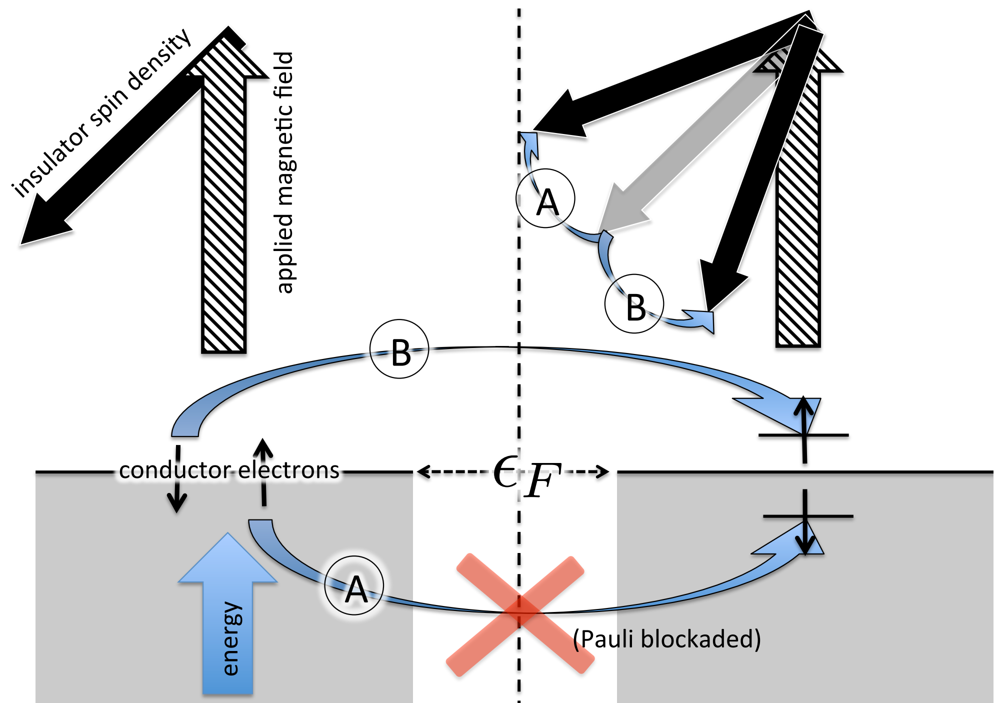


Demokritov, *Nature* (2006)

Microwave agitation (resonant or parametric) of the ferromagnet is replaced by electronic spin transfer

SWASING

$$\hat{V}_{\text{int}} = \sum_{\mathbf{qkk'}} V_{\mathbf{qkk'}} \hat{c}_{\mathbf{q}} \hat{a}^{\dagger}_{\mathbf{k'}\uparrow} \hat{a}_{\mathbf{k}\downarrow} + \text{H.c.}$$

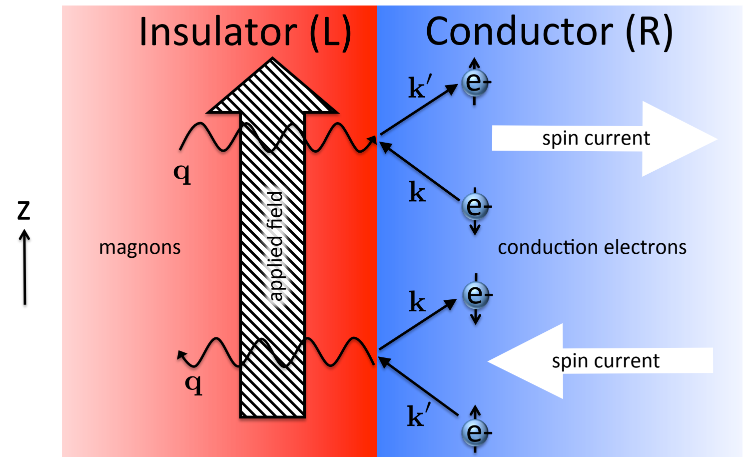


$$\Delta S = \frac{\mu' - \epsilon_0}{T} \Delta N$$

Berger, *PRB* (1996)

SPIN-TRANSFER RATES

The total (z axis) spin current (toward ferromagnet):



$$j_s = j_0 + j_x$$

consists of the ground-state (condensed) magnon contribution

$$j_0 = \frac{\hbar g^{\uparrow\downarrow}}{2\pi s}\left(\mu' - \epsilon_0\right) n_0$$

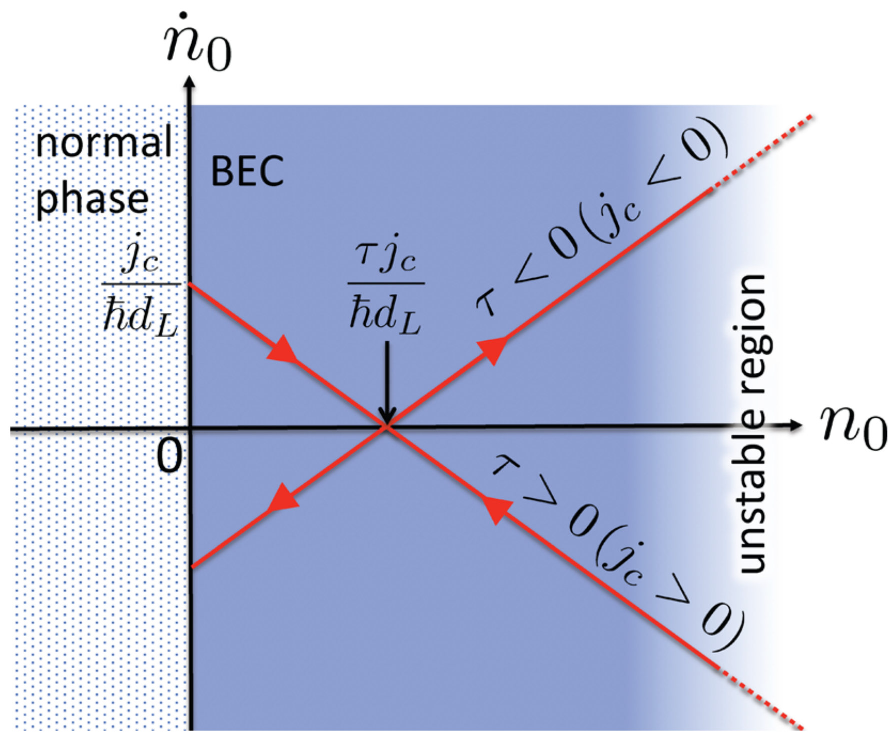
as well as the thermal magnon contribution

$$j_x = \frac{\hbar g^{\uparrow\downarrow}}{\pi s}\int_{\epsilon_0}^{\infty} d\epsilon D(\epsilon)(\epsilon - \mu')\left[n_{\mathrm{BE}}\left(\beta(\epsilon-\mu)\right) - n_{\mathrm{BE}}\left(\beta'(\epsilon-\mu')\right)\right]$$

which is enhanced when $T < T'$

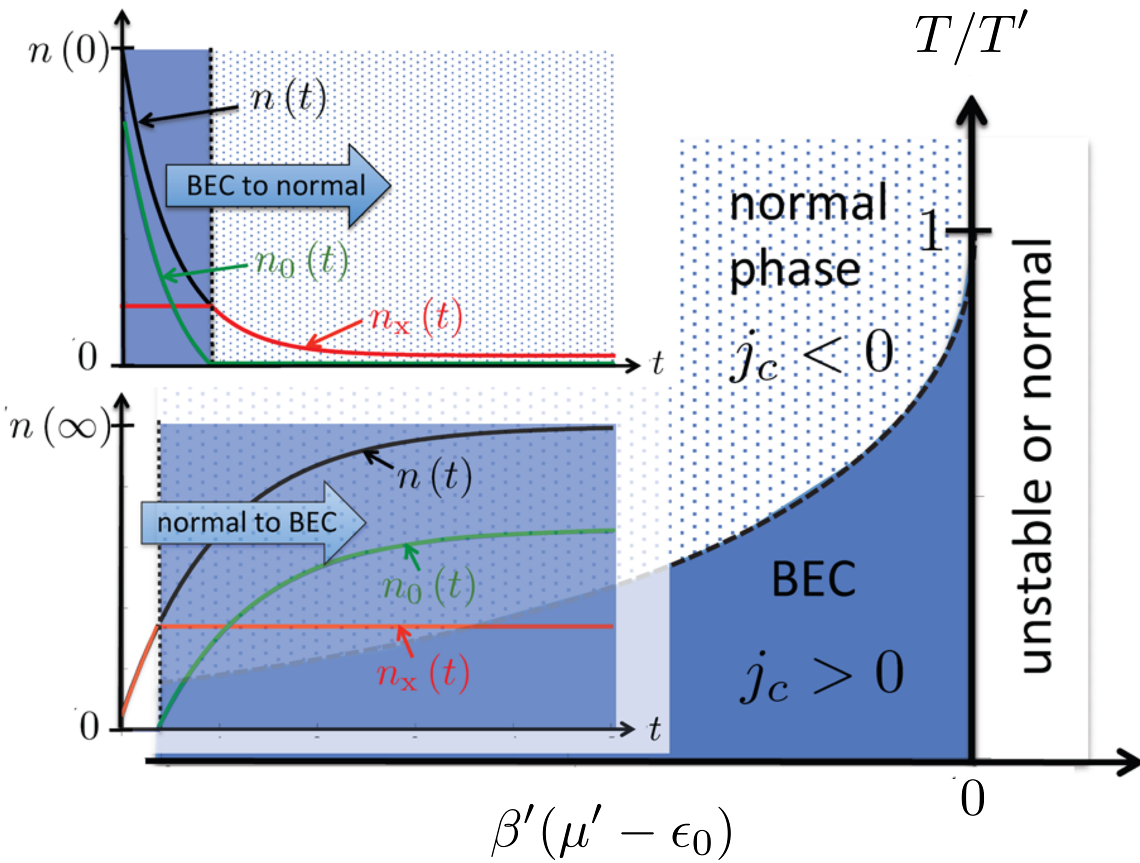
BEC RATE EQUATION

$$\dot{n}_0 = \frac{j_s}{\hbar d} = \frac{j_c}{\hbar d} - \frac{n_0}{\tau} \qquad \frac{\hbar}{\tau} \equiv \frac{\hbar g^{\uparrow\downarrow}}{2\pi s d}(\epsilon_0 - \mu')$$

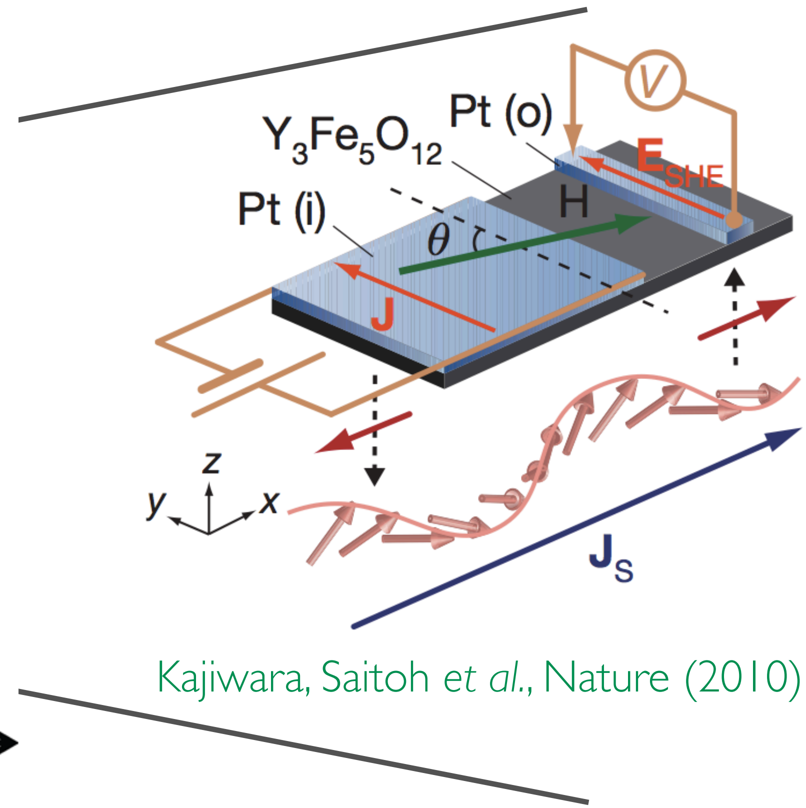


$$n_0(t) = \frac{\tau j_c}{\hbar d} + \left[n_0(0) - \frac{\tau j_c}{\hbar d} \right] e^{-t/\tau}$$

DYNAMIC PHASE DIAGRAM



Bender, Duine, and YT, *PRL* (2012)



Kajiwara, Saitoh *et al.*, Nature (2010)

CONDENSATE INTERACTIONS

Total condensate/cloud spin density:

$$\mathbf{s} = \left(\sqrt{2}\mathrm{Re}\psi, \sqrt{2}\mathrm{Im}\psi, n\right)$$

$$\psi = \sqrt{n_0} e^{i\phi} \qquad n = n_0 + n_x$$

condensate dynamics

cloud damping

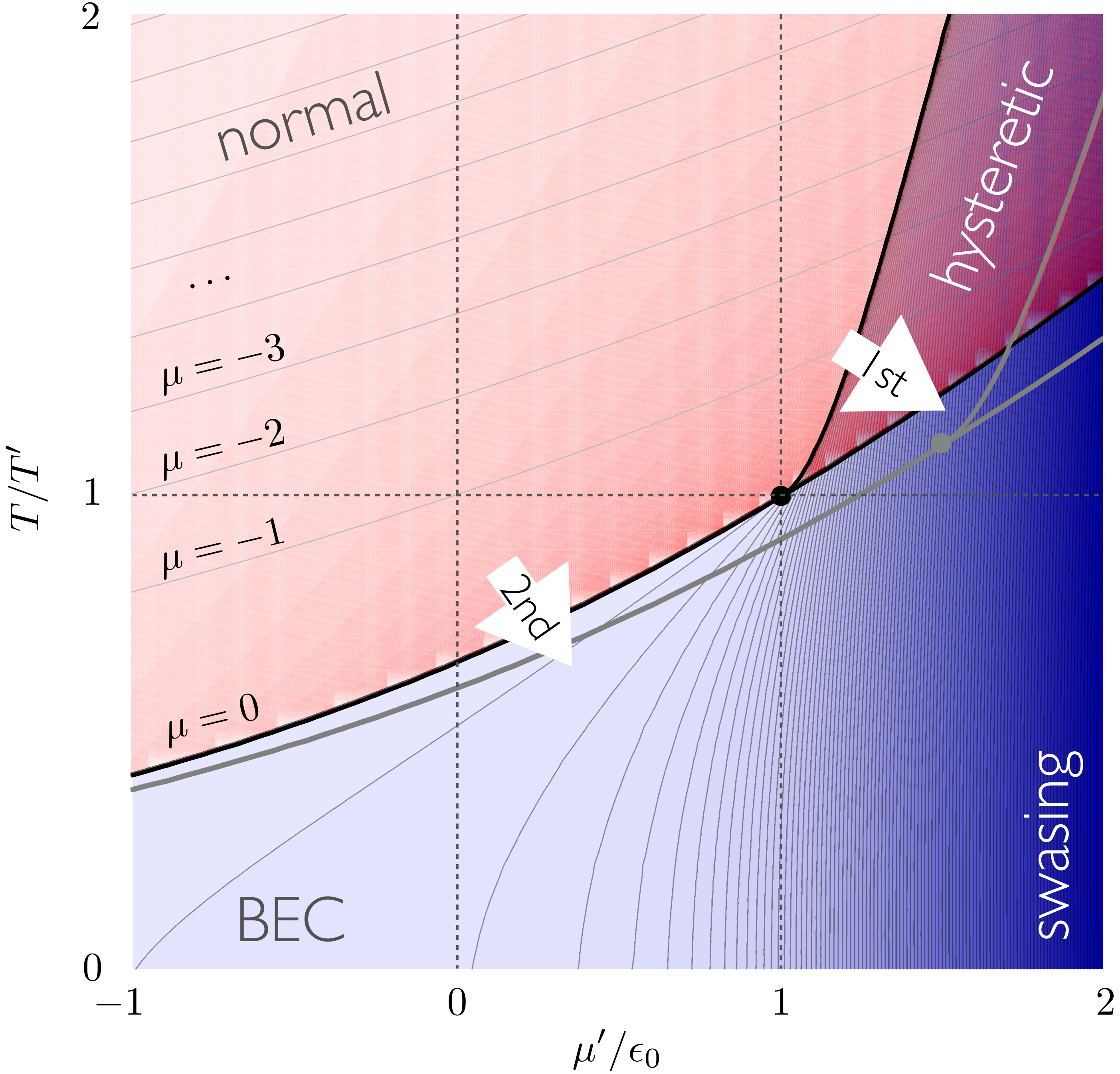
cloud pumping

$$\begin{aligned}\frac{dn}{dt} =& -\left[\omega + \alpha'(\omega - \mu')\right] n_0 \\ & - \int_0^\infty d\epsilon D(\epsilon) \left\{(\epsilon + \omega)\left[n_{\mathrm{BE}}(\beta(\epsilon - \mu)) - n_{\mathrm{BE}}(\beta(\epsilon + \omega))\right]\right\} \\ & - 2\alpha' \int_0^\infty d\epsilon D(\epsilon) \left\{(\epsilon + \omega - \mu')\left[n_{\mathrm{BE}}(\beta(\epsilon - \mu)) - n_{\mathrm{BE}}(\beta'(\epsilon + \omega - \mu'))\right]\right\}\end{aligned}$$

Here, $\omega \equiv \dot{\phi} = \frac{\partial F_{\mathrm{GL}}}{\partial n_0}$ in terms of the free energy $F_{\mathrm{GL}} = Hn + \frac{K}{2} n^2$

Bender, Brataas, Duine, and YT, in preparation

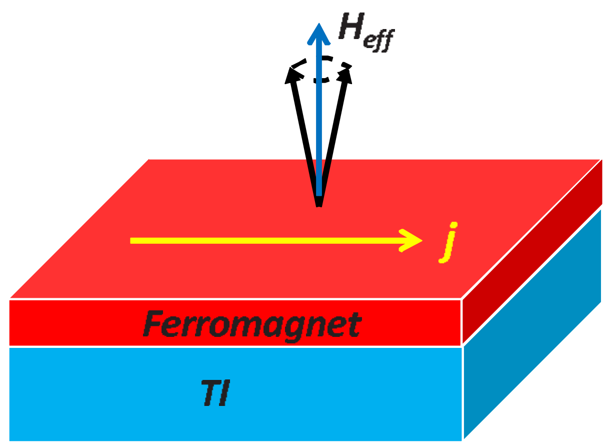
PHASE DIAGRAM

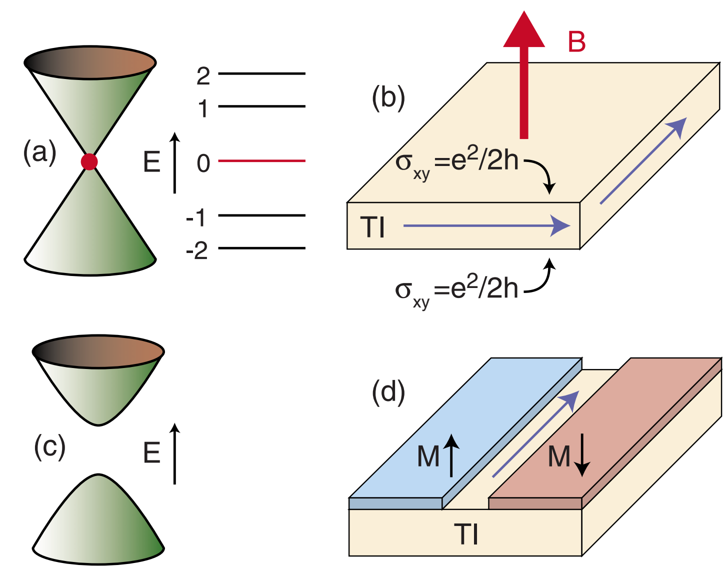


MI/TI EXCHANGE INTERACTION

$$H_0 = v\mathbf{p}\cdot\mathbf{z}\times\hat{\boldsymbol{\sigma}} + \Delta\hat{\sigma}_z \quad \rightarrow \quad \mathbf{v} \equiv \partial_{\mathbf{p}} H_0 = v\mathbf{z}\times\hat{\boldsymbol{\sigma}}$$

Redlich, Semenoff, Jackiw *et al.* (1983)





$$H' = J(m_x\hat{\sigma}_x + m_y\hat{\sigma}_y) + J_\perp m_z\hat{\sigma}_z$$

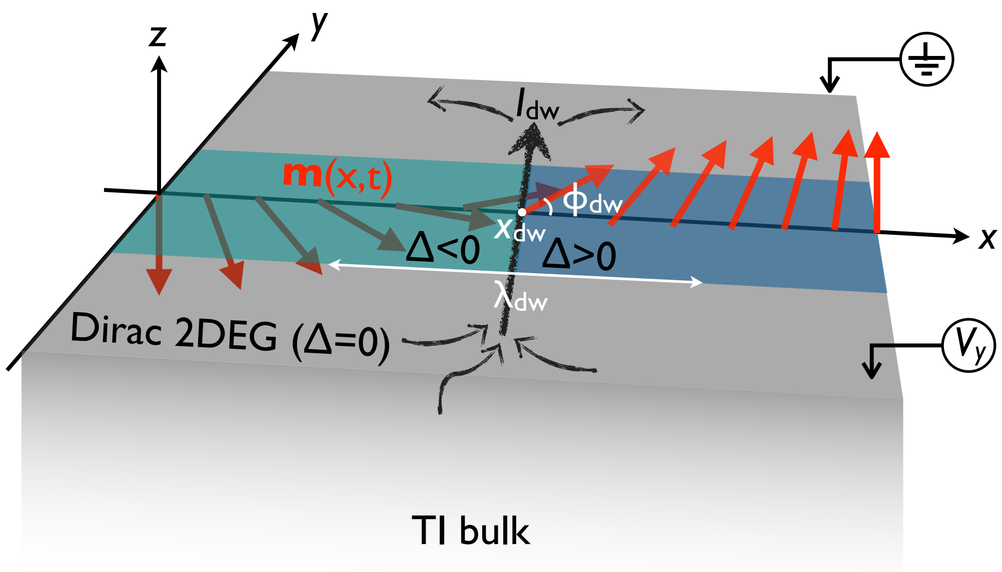
Hasan and Kane, *RMP* (2010)

$$\rightarrow \quad \mathbf{a} = \frac{J}{ev}\mathbf{m}\times\mathbf{z} \quad \text{and} \quad \Delta = J_\perp m_z$$

$$\rightarrow \quad \rho = \text{sgn}(\Delta)\frac{eJ}{4\pi\hbar v}\boldsymbol{\nabla}\cdot\mathbf{m}\,, \quad \mathbf{j} = -\text{sgn}(\Delta)\frac{eJ}{4\pi\hbar v}\mathbf{z}\times\partial_t\mathbf{m}\times\mathbf{z}$$

Garate, Franz, Yokoyama, Nomura, Nagaosa *et al.* (2010)

MI/TI DOMAIN-WALL DYNAMICS



$$\mathbf{j} = \frac{eJ_\perp}{2\pi\hbar}\mathbf{z} \times \boldsymbol{\nabla} m_z$$

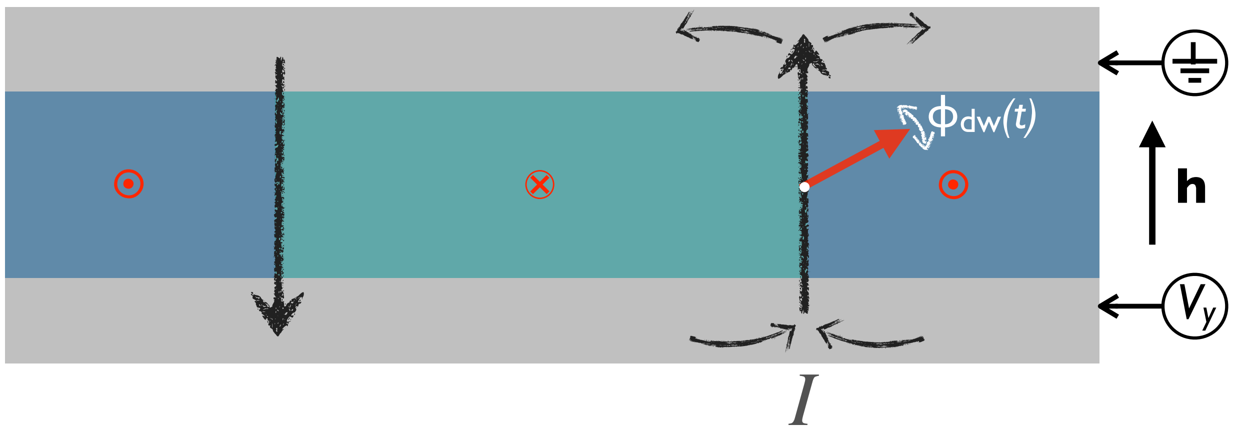
persistent current

$$\mathcal{F} = \frac{JJ_\perp}{2\pi\hbar v}\mathbf{m} \cdot \boldsymbol{\nabla} m_z$$

DMI-type free energy

YT and Loss, *PRL* (2012)

DW/CHIRAL MODE COUPLING



$$\dot{x}_{\rm dw} = -\frac{f_\phi + (J/ev) I \sin\phi_{\rm dw}}{(4+\alpha^2)s}\,, \qquad \dot{\phi}_{\rm dw} = -\frac{\alpha}{2\lambda_{\rm dw}}\dot{x}_{\rm dw}$$

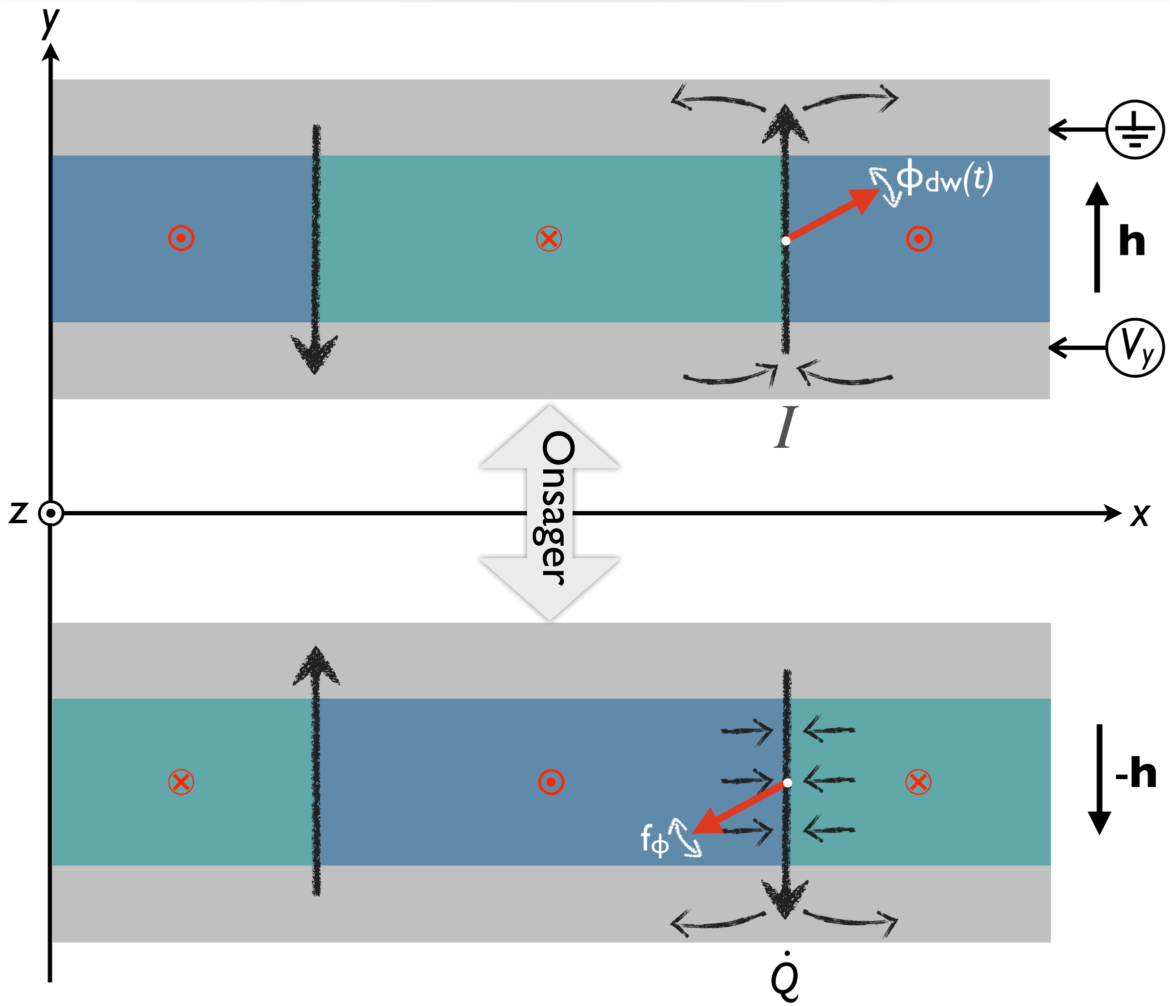
Josephson-type relations

$$f_\phi \equiv -\frac{1}{L}\frac{\partial F}{\partial \phi_{\rm dw}} = \frac{JJ_\perp}{4\hbar v}\sin\phi_{\rm dw} + \pi h \lambda_{\rm dw}\cos\phi_{\rm dw}$$

torque on the DW structure

YT and Loss, *PRL* (2012)

RECIPROCAL CHARGE PUMPING



$$\dot{Q} = \frac{eJ}{4\pi\hbar v}\frac{\alpha \sin\phi_{\rm dw}}{\lambda_{\rm dw}(4+\alpha^2)s}\frac{\partial F}{\partial \phi_{\rm dw}} \rightarrow \frac{e}{2\pi\hbar}L\frac{J}{v}\partial_t m_x(x_{\rm dw})$$

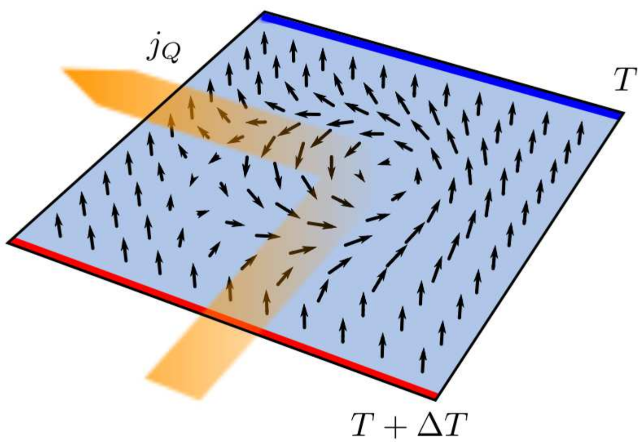
conductance

force

THERMAL HALL EFFECT

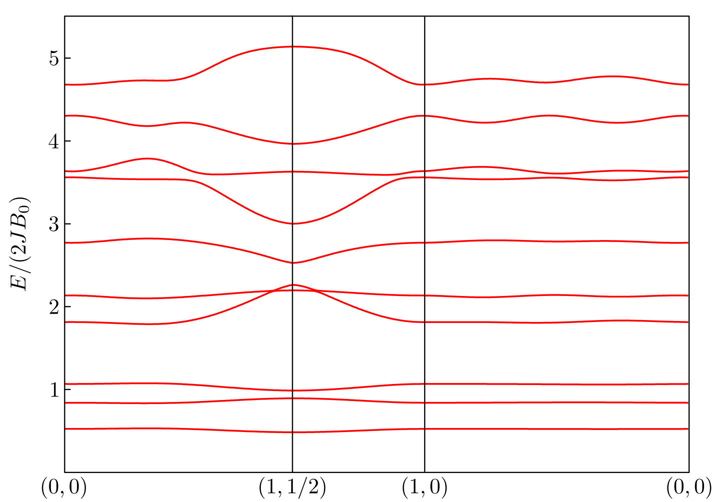
A magnetic texture fomented by a DM-type spin-orbit interaction:

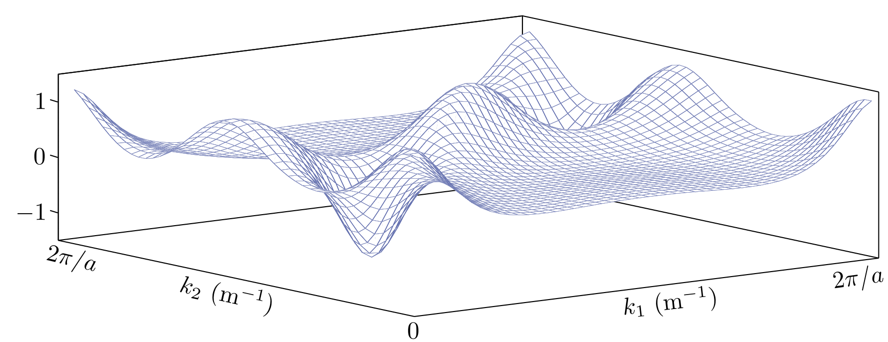
$$\mathcal{F}_{\rm so}(\mathbf{m}, \partial_j \mathbf{m}) = \Gamma_{\rm R} m_z \boldsymbol{\nabla} \cdot \mathbf{m} + \Gamma_{\rm DM} \mathbf{m} \cdot \boldsymbol{\nabla} \times \mathbf{m}$$

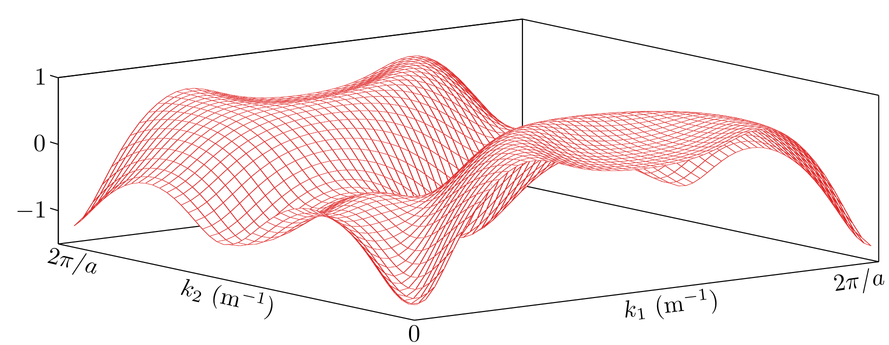


$$i\hbar \partial_t m_+ = \left[J(\boldsymbol{\nabla}/\boldsymbol{i} + \mathbf{A})^2 + \varphi \right] m_+$$

$$\mathbf{A} = \hat{R}^{-1} \left(\boldsymbol{\nabla} - \frac{\Gamma}{J} \hat{\mathbf{I}} \right) \hat{R} \Big|_{12}$$







Katsura, Nagaosa, and Lee, *PRL* (2010); Onose *et al., Science* (2010)

Hoogdalem, YT, and Loss, *PRB* (2013)

SUMMARY

Nonequilibrium magnetism is enriched by the interplay between the itinerant (electron or magnon) and the collective (monodomain, domain wall etc.) degrees of freedom

The core physics is based on the Onsager-reciprocal spin torque and pumping phenomena, in practice facilitated by spin Hall effect

Strong pumping generally leads to condensation of magnons

The (*Galilean*) phenomenology can be constructed based on the SU(2) (*Berry phase*) gauge structure of the interaction between *fluxes* and spatiotemporally inhomogeneous magnetic *precession*

Strong spin-orbit interactions enrich the gauge structure and lead to a myriad of Hall-like phenomena for charge and heat transport