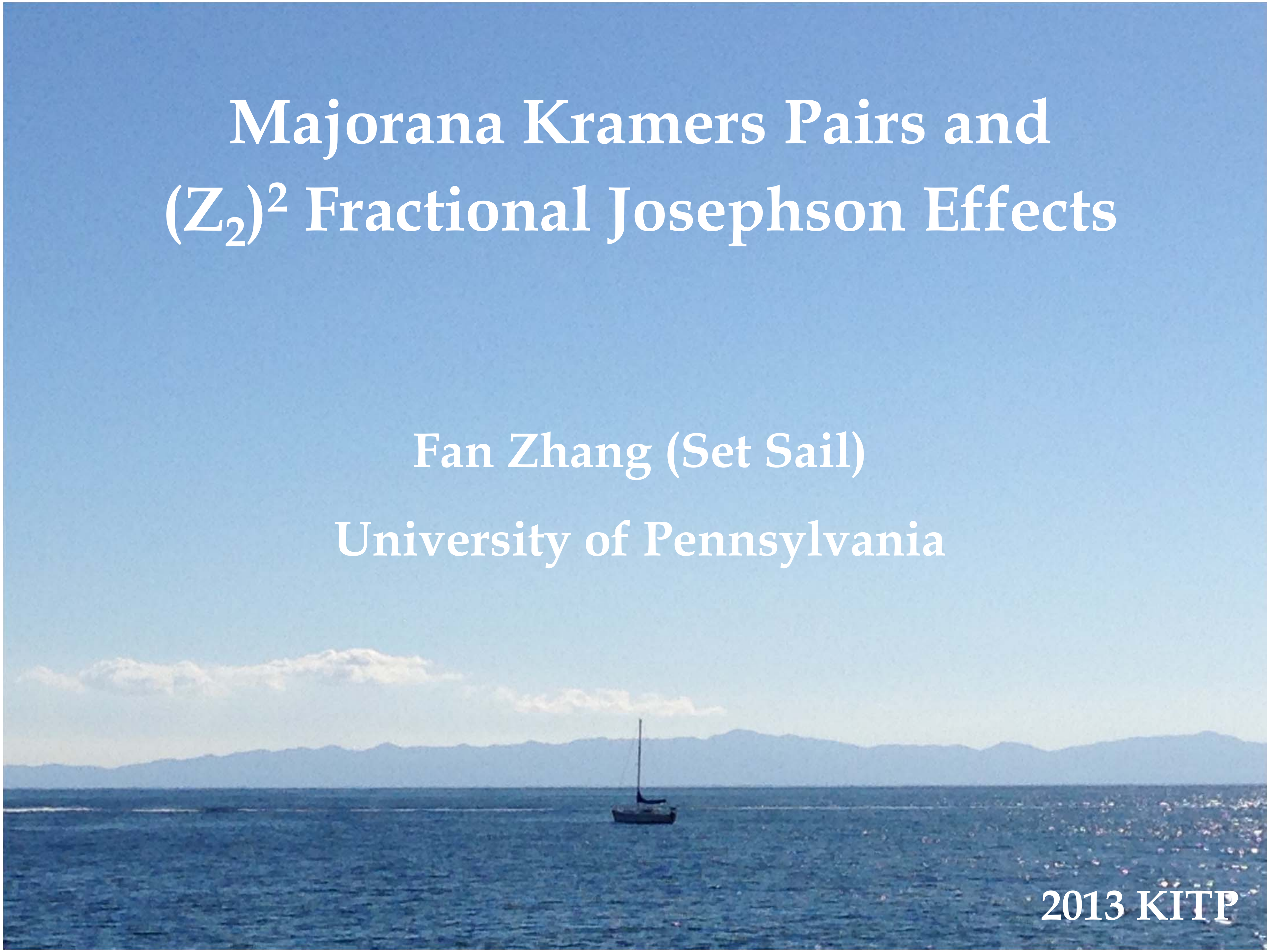


Majorana Kramers Pairs and $(Z_2)^2$ Fractional Josephson Effects

Fan Zhang (Set Sail)

University of Pennsylvania

2013 KITP

My Captains


Charles Kane


Eugene Mele

(Since Oct. 2011)


Allan MacDonald

(Since Jan. 2008)

Conversations with Ken Palley (I)

(**Ken Palley is my landlord at KITP.**)

Ken: Do you really agree with each other?

Fan: Only when there is one "expert".

Ken: What about if there are more than one?

Fan: Only when there is "one" electron.

(**to understand single-particle physics,**

no interaction, no disorder;

but with "spin", as $\theta^2 = -1$.)

Conversations with Ken Palley (II)

Ken: Is there any unsolved physics problem?

Fan: Yes, e.g., do you want to improve your computer?

Ken: No. I am completed satisfied with my current one.
Do you?

Fan: Maybe only when I need to write a proposal.

Ken: Poor boy, let's go surfing.

(**conceptual novelty & experimental realization hopefully,**
instead of making any device or quantum computing)

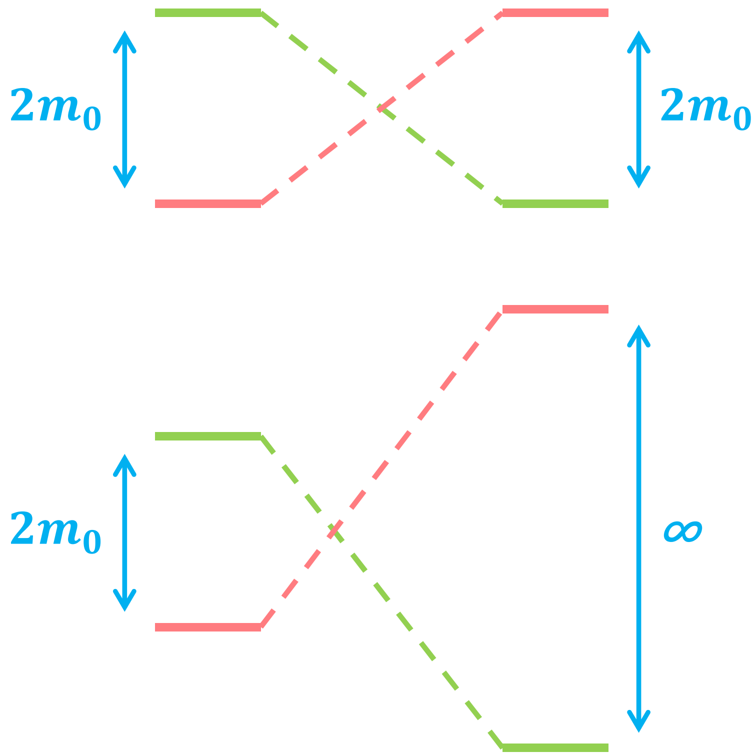
Jackiw-Rebbi Model (1976)

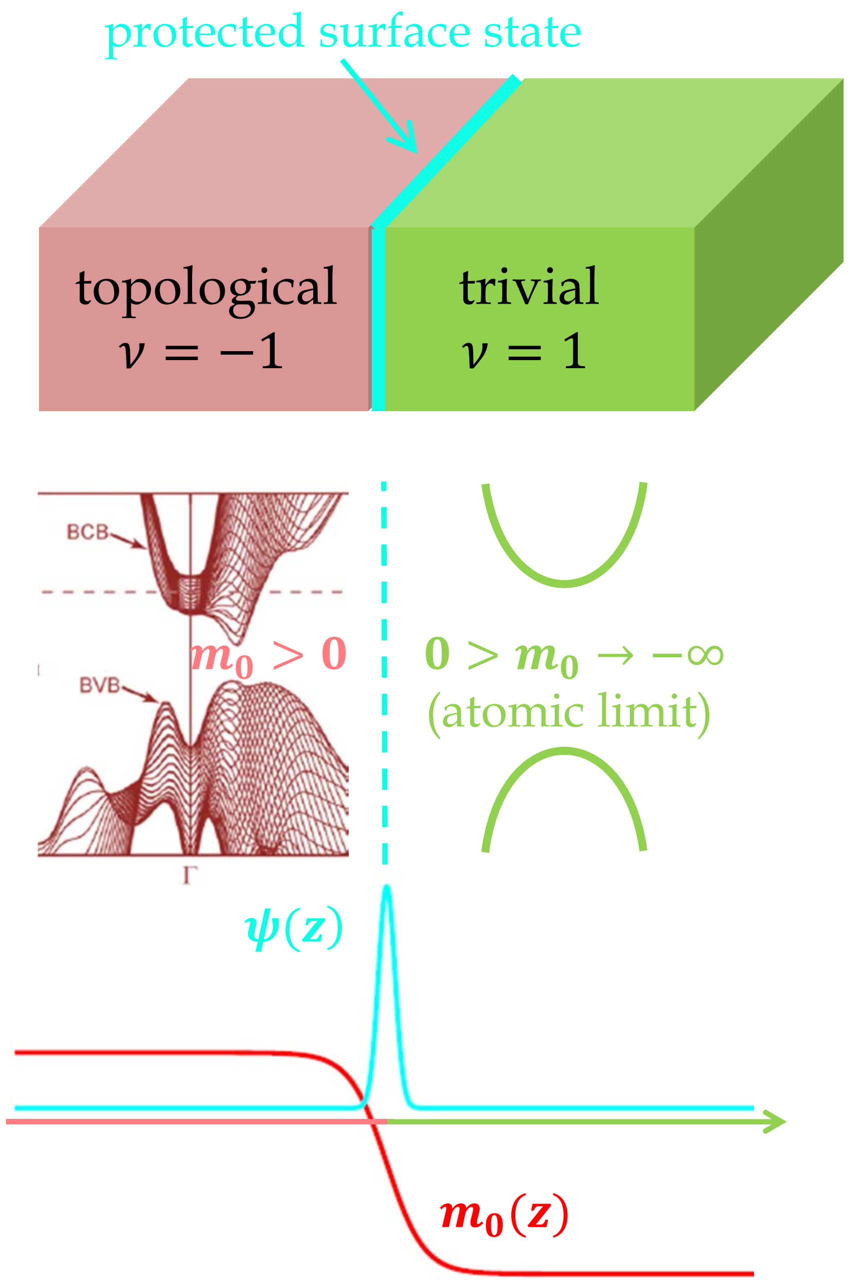
1D Massive Dirac Fermion:

$$H = v k_z \sigma_y + m_0 \mathrm{sgn}(z) \sigma_z$$

a special solution: $E = 0,\ \sigma_x = 1,\ z = 0$

(gapless, fractionalized, localized)





Topological Insulators and Superconductors

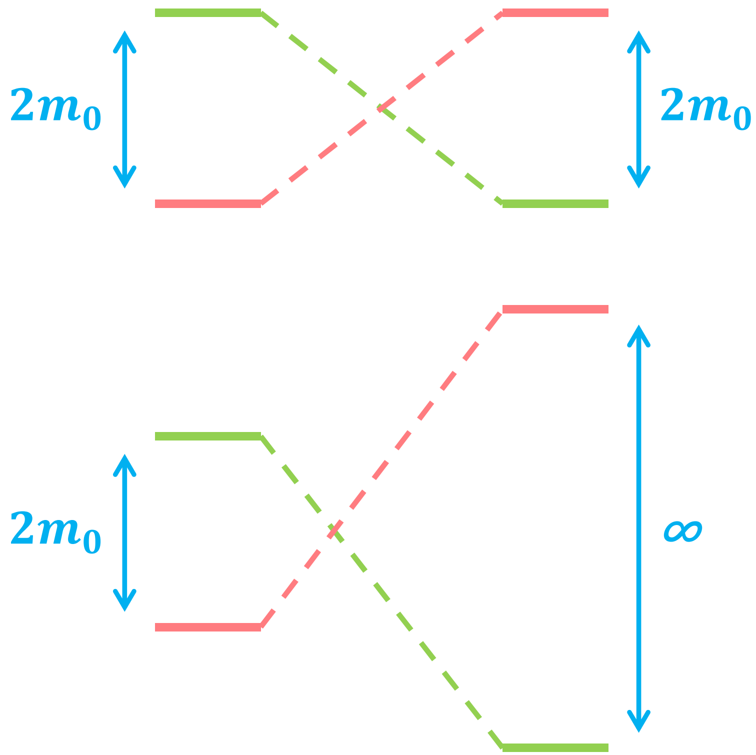
1D Massive Dirac Fermion:

$$H = vk_z\sigma_y + m_0\text{sgn}(z)\sigma_z$$

a special solution: $E = 0,\ \sigma_x = 1,\ z = 0$

(gapless, fractionalized, localized)

- Band inversions (broadly defined)
- (Topological) boundary condition
- Not topological yet !!!



To make a TI or TSC:

- Add dimensions + impose symmetries
- Are gapless modes protected? (topological or trivial)
- If protected, are they still protected if there are two copies? (Z or Z_2)

Jackiw-Rebbi, PRD 13, 3398 (1976);
FZ-Kane-Mele, 2012PRB(R), 2013PRL.

Periodic Table (free fermions)

		Symmetry			Dimension ($\boldsymbol{k}$)							
s	AZ	Θ^2	Ξ^2	Π^2	0	1	2	3	4	5	6	7
0	A	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0
1	AIII	0	0	1	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$
0	AI	1	0	0	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$
1	BDI	1	1	1	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$
2	D	0	1	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$	0
3	DIII	-1	1	1	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$
4	AII	-1	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0
5	CII	-1	-1	1	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0
6	C	0	-1	0	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0
7	CI	1	-1	1	0	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$

(Kitaev; Schnyder-Ryu-Furusaki-Ludwig, Teo-Kane; …)

Outline of My Talk

- ✓ Time-reversal-invariant topological SC [in class DIII]
- × Topological mirror SC [two copies of class AIII insulators]
- ✓ New $Z_2 \times Z_2$ Fractional Josephson effects [beyond the table]

Ref: FZ-Kane-Mele, PRL 111, 056402 & 056403 (2013); FZ-Kane arXiv:1310.5281.

Review: Class D Topological SC

Dimensions	0	1	2
Class D	Z_2	Z_2	Z

$$H_{BDG} = H_N + H_\Delta$$

- In 2D, to goal is to generate the Berry phase π
- Replacing the winding number of chiral p wave pairing by the wind number of a single helical band. [Fu-Kane 2008]
- Time reversal symmetry must be broken.
- Dimension reduction: from 2D to 1D (1D TSC)
- 0D Invariant (implies a fractional Josephson effect)

Q: Is it possible to engineer a topological SC that respects time reversal symmetry using proximity effect **?**

Class DIII (TRI) Superconductors

Dimensions	0	1	2	3
Class DIII	0	Z_2	Z_2	Z

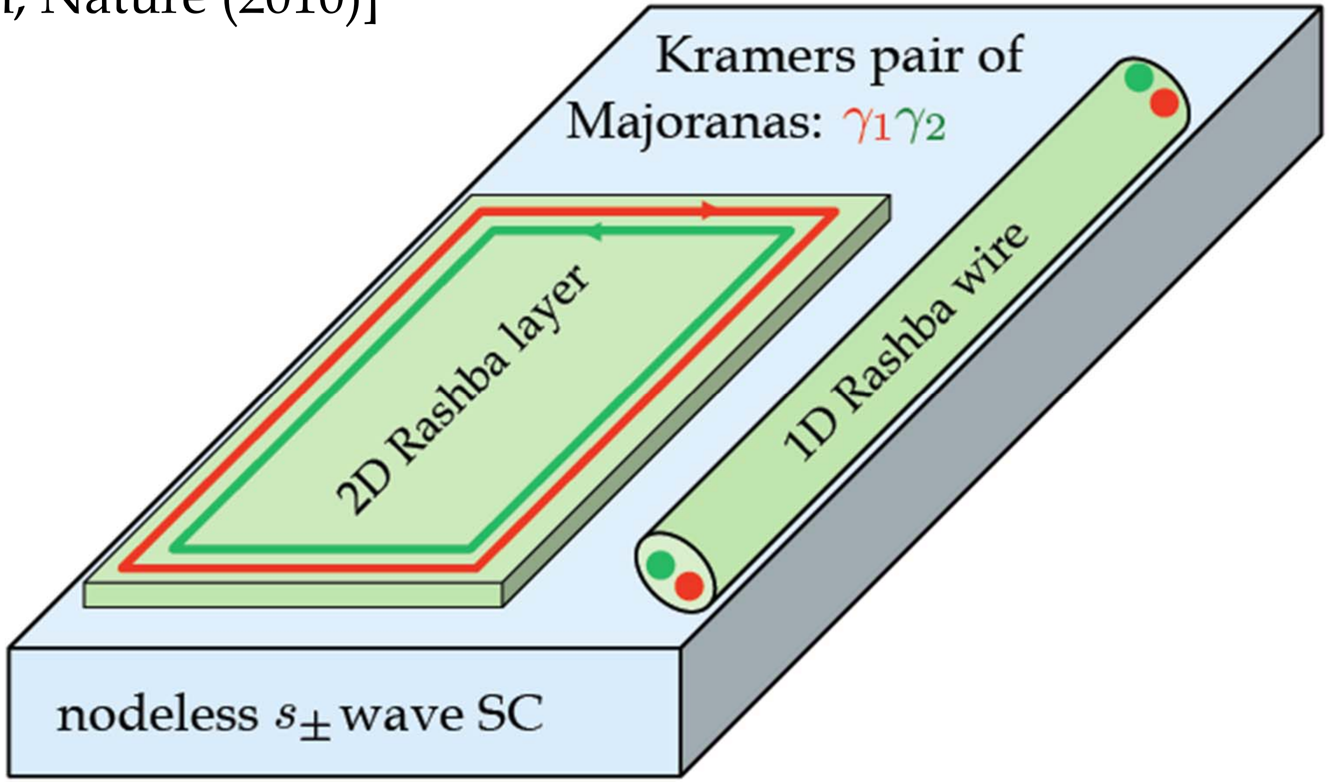
- Without using any interaction or Josephson effect, pure s-wave pairing is impossible to induce TRI TSC. [FZ-Kane-Mele, PRL (2013)]

- My favorite criterion: [There are other equivalent ones.]
  The DIII Z_2 invariant is determined by whether the pairing has a negative sign on odd number of Fermi surfaces, each of which encloses a TRI momentum. [Qi-Hughes-Zhang, PRB (2010).]

Extended S-wave + Rashba semiconductor

- Time-reversal symmetry: no magnetic perturbation
- No interactions: using proximity effect
- nodeless Iron pnictide SC + extended s-wave

[I. I. Mazin, Nature (2010)]



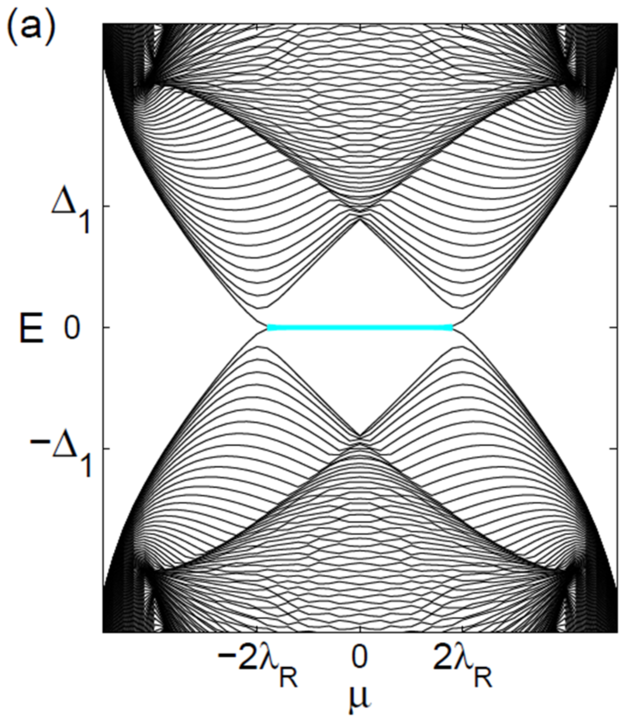
FZ-Kane-Mele, PRL 111, 056402 (2013)

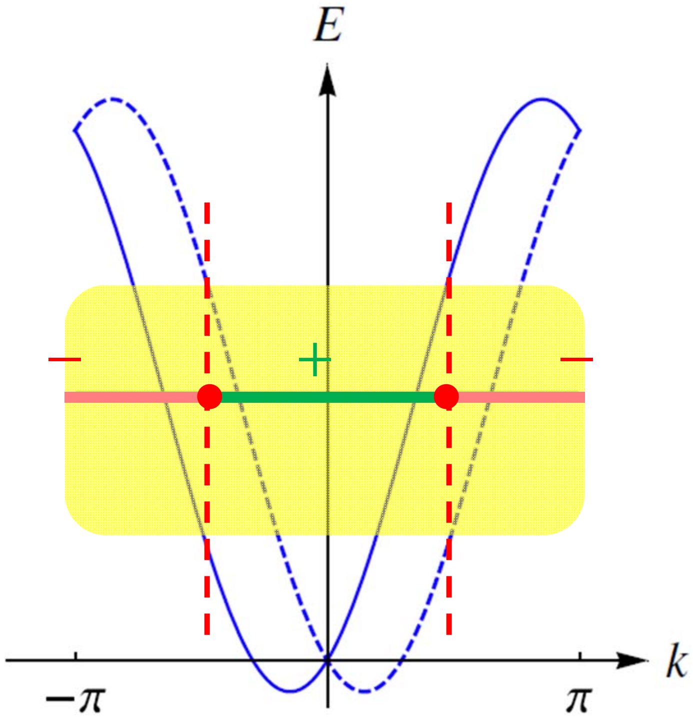
Time-Reversal-Invariant Topological Superconductivity

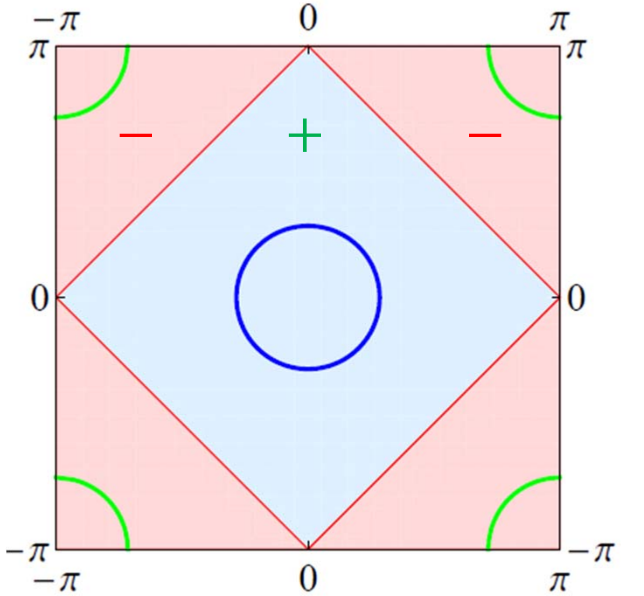
$$\mathcal{H} = (-2t\cos k_x + 2\lambda_R \sin k_x\,\sigma_z - \mu)\,\tau_z + (\Delta_0 + 2\Delta_1 \cos k_x)\,\tau_x\,,$$

For simplicity we consider: $\Delta_0 = 0$

Topological criterion: $|\mu| < 2\lambda_R$



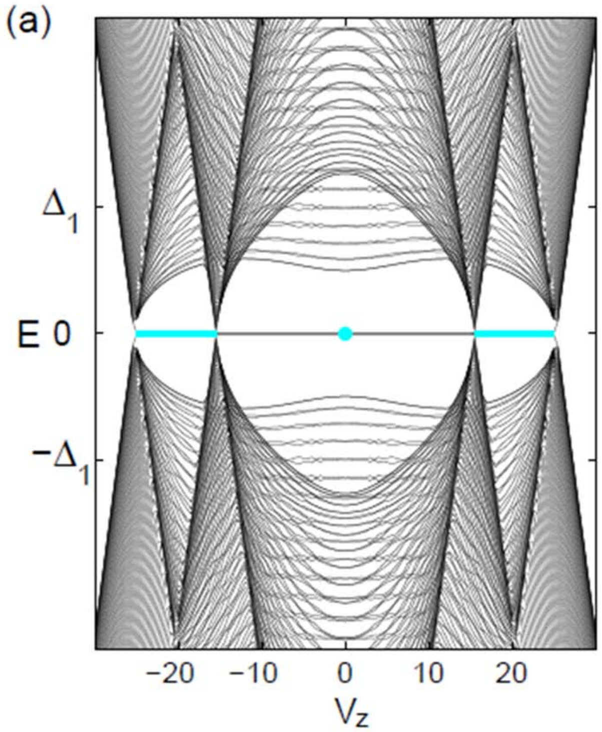




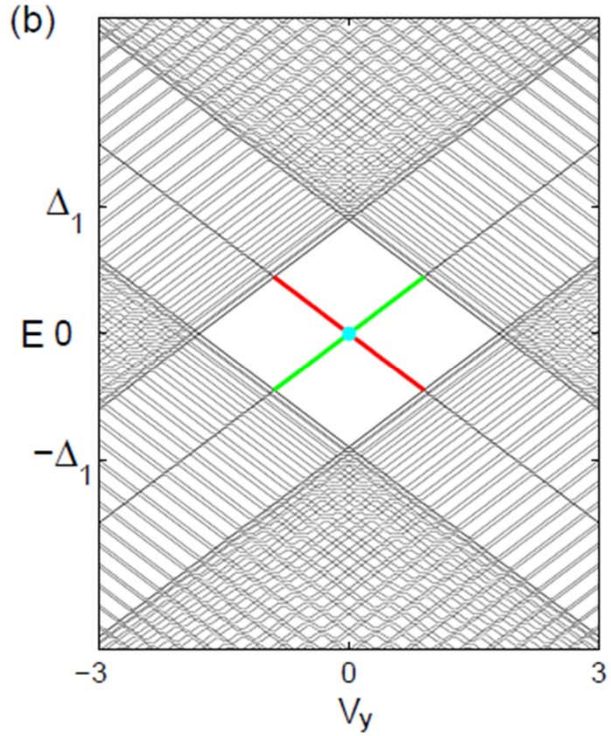
FZ-Kane-Mele, PRL 111, 056402&056403 (2013)

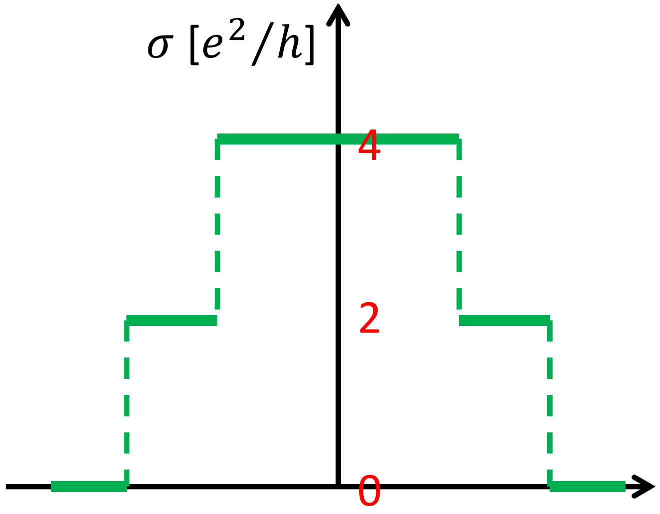
Evolution of a Majorana Kramers Pair in Zeeman Fields

Breaking mirror symmetry:



Respecting mirror symmetry:



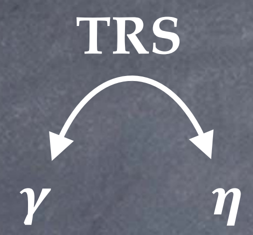


- a MKP forms two ABS
- opposite fermion parity
- Zeeman splitting

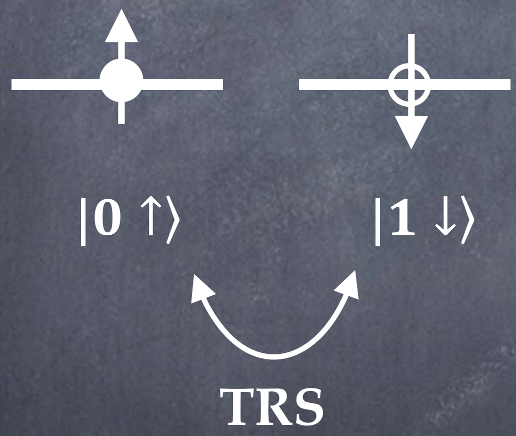
signatures in tunneling spectroscopy !

FZ-Kane-Mele, PRL 111, 056402 (2013)

Majorana Kramers Pair



MKP forms a fermion level



MF's: $\gamma^\dagger = \gamma,\ \eta^\dagger = \eta$

TRS: $\gamma \to \eta \to -\gamma$

Define: $c_\uparrow = \gamma + i\eta$

TRS:
$$c_\downarrow = \eta + i\gamma = i(\gamma - i\eta) = ic_\uparrow^\dagger$$

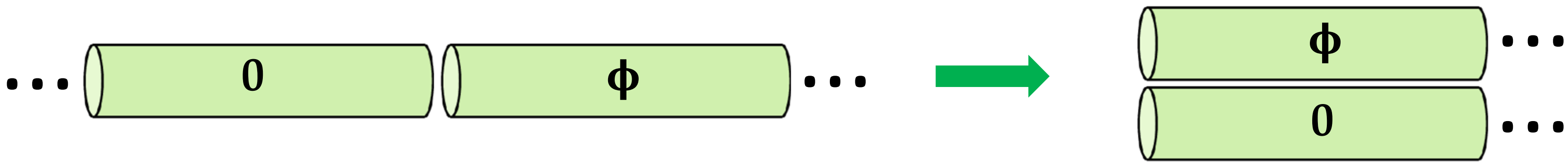
TRS = Super Symmetry

$$c_\sigma = ic_{\bar{\sigma}}^\dagger$$

FZ-Kane-Mele, PRL 111, 056402&056403 (2013)

Puzzle: 4π Josephson Effects? Non-Abelian Statistics?

- Tunneling Cooper pairs or electrons? [Yes, 4π]
- One minus sign for each Majorana? [No, 2π]



- When folded into each other, the Josephson effects can thus be interpreted as the boundary consequences of the bulk invariant of $H(k,\phi)$;
- $H(k,\phi)$ inherits PH, TR, and chiral symmetry constraints.
- How many topological inequivalent $H(k,\phi)$?

Anomalous Pumps $H(k, r, \phi)$

- **Class D can be understood by the original table;**
- **Class AIII, DIII, … cannot !!!**

	PHS	*TRS*
k	$-$	$-$
r	$+$	$+$
ϕ	$+$	$-$
θ	$-$	$+$

Class AIII:

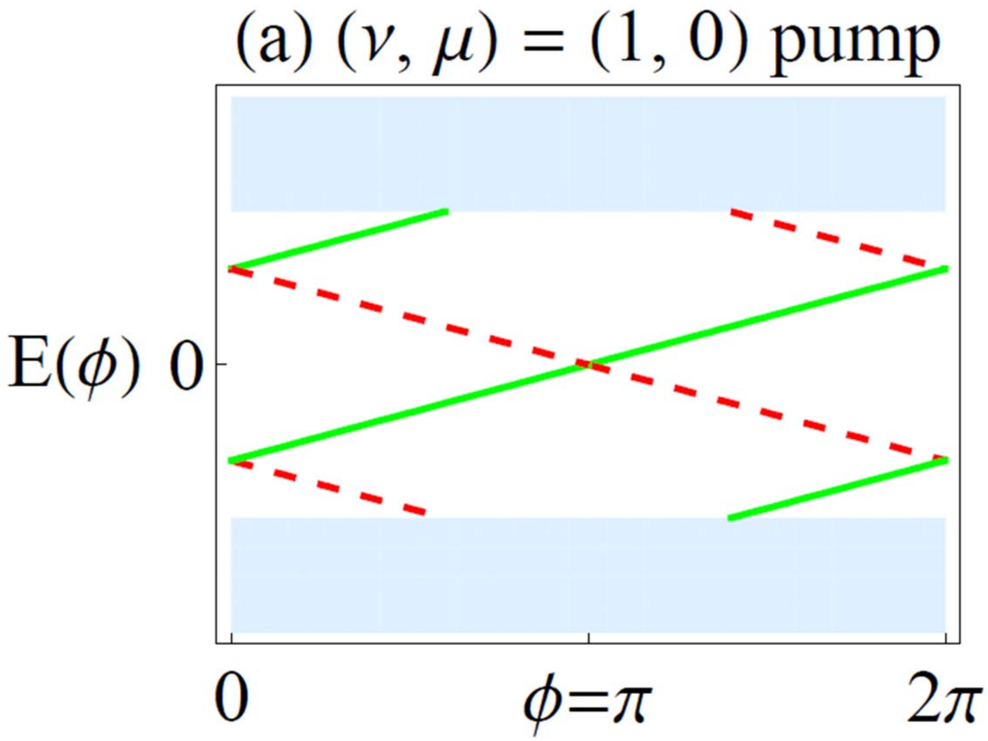
$d_k - d_r$		even	odd
$d_\phi - d_\theta$	even	0	$\mathbb{Z}$
	odd	0	$\mathbb{Z} \times \mathbb{Z}$

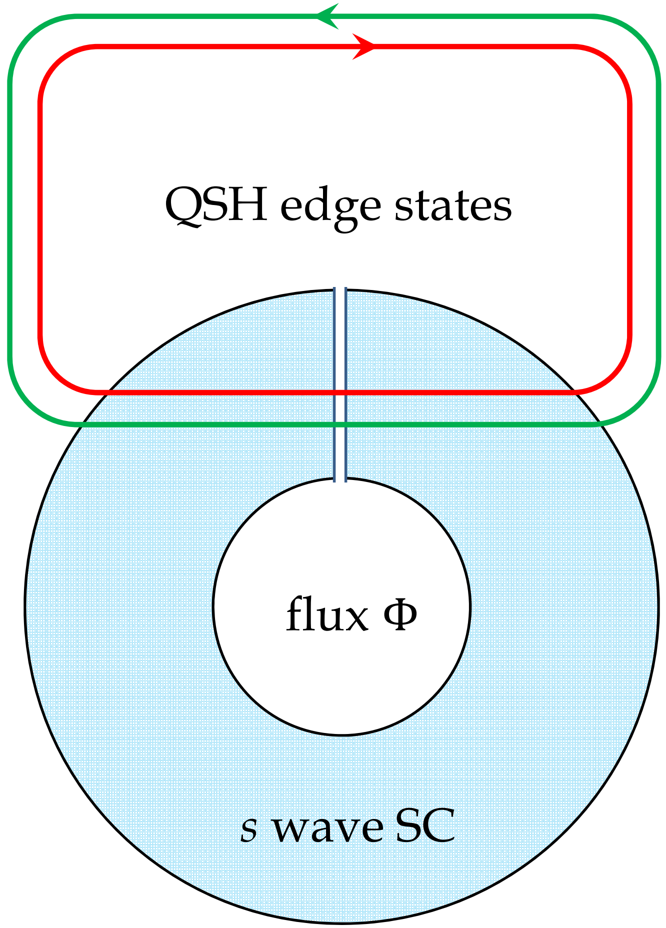
Class DIII:

$(d_k - d_r) \bmod 8$		$0, 4, 5, 6$	1	2	3	7
$d_\phi - d_\theta$	0	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	$2\mathbb{Z}$
	1	0	$\mathbb{Z}_2 \times \mathbb{Z}_2$	$\mathbb{Z}_2 \times \mathbb{Z}_2$	$\mathbb{Z} \times \mathbb{Z}$	$2\mathbb{Z} \times 2\mathbb{Z}$

Homotopy Argument: FZ-Kane arXiv:1310.5281.

The 1st Z_2 Invariant



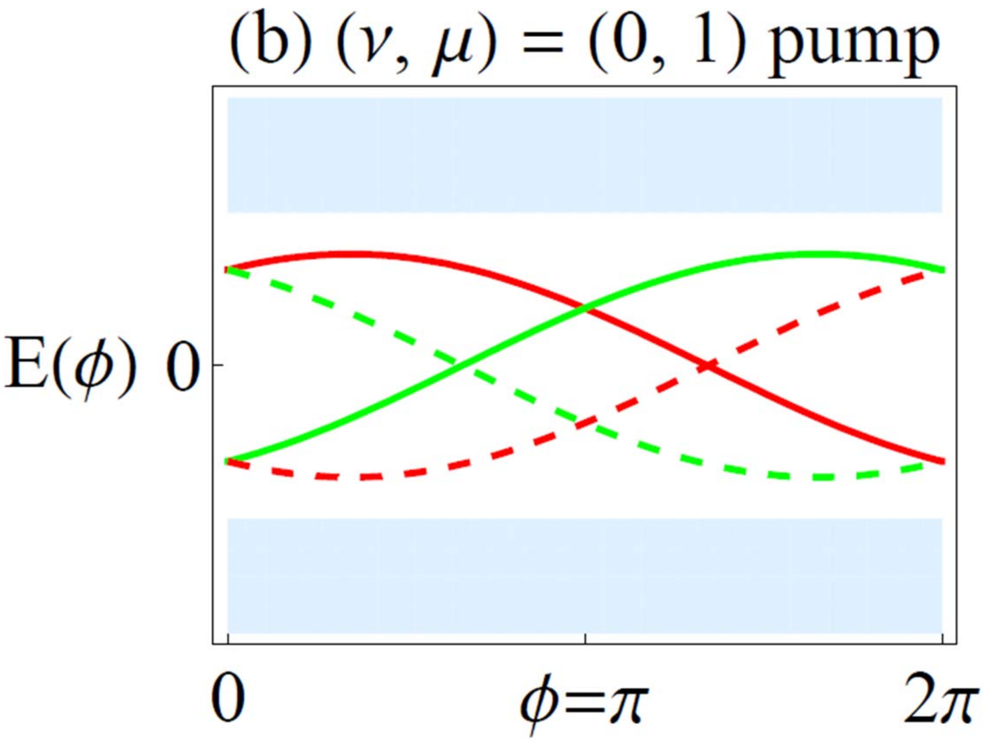


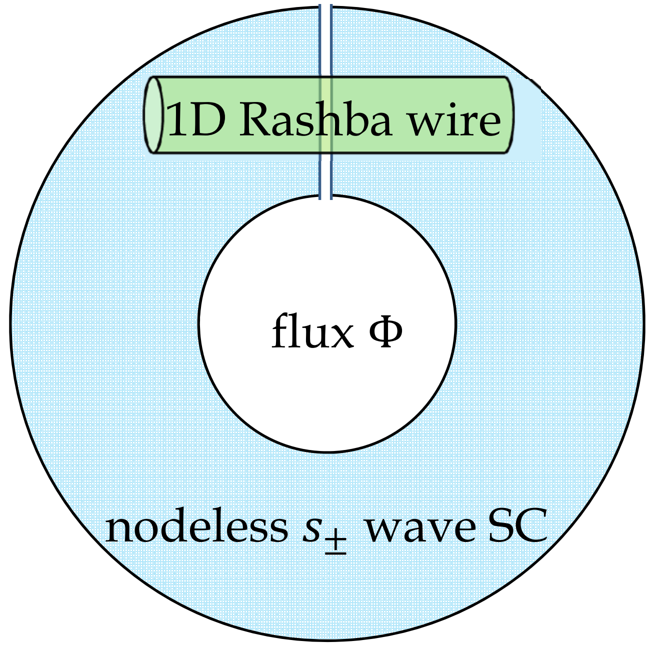
- **4π periodic Josephson effect**
  (non-Abelian statistics of MFs)

- **Robust to TRS breaking**

(L. Molenkamp, A. Yacoby, R. Du, ...)

The 2nd Z_2 Invariant



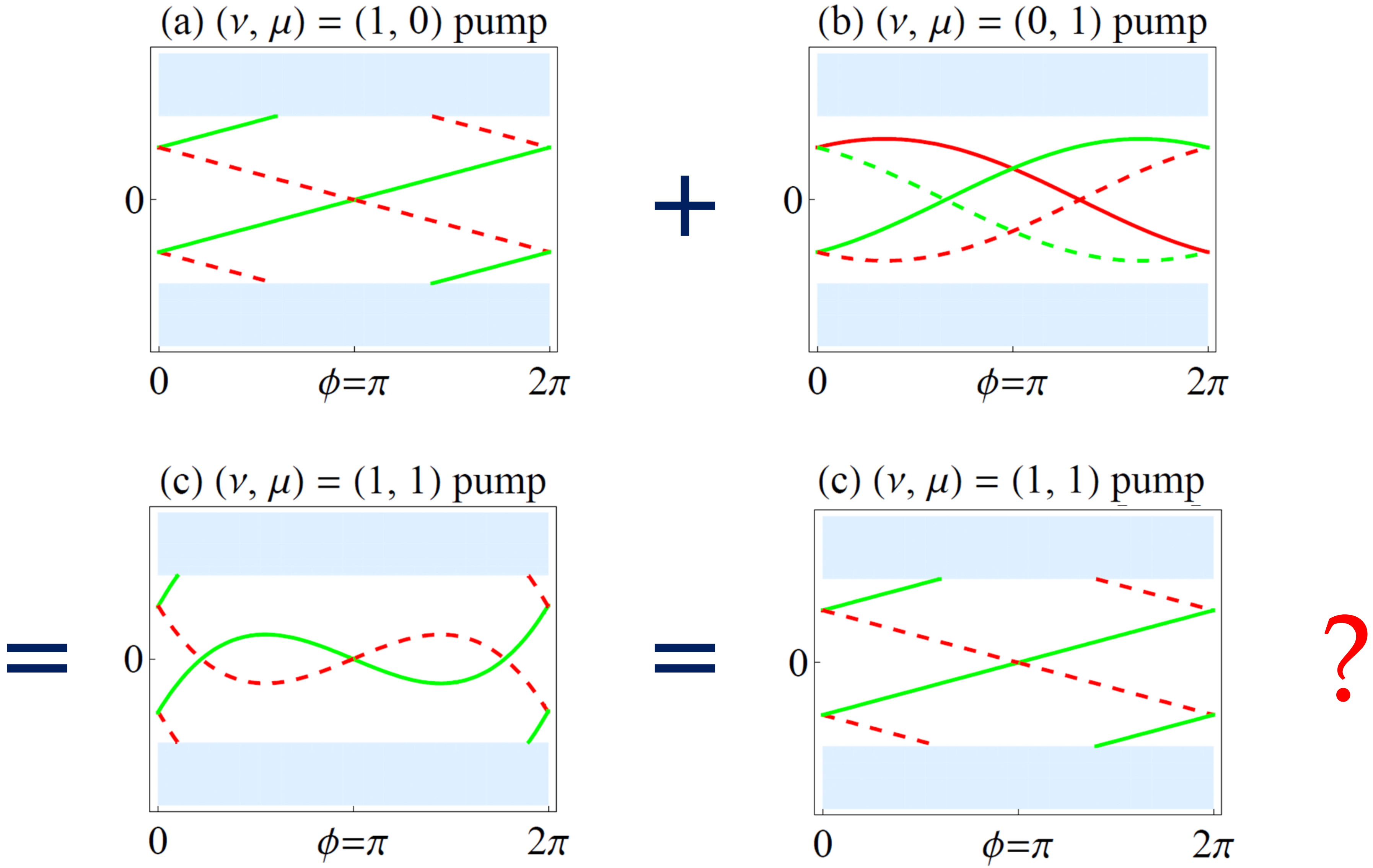


- 4π periodic Josephson effect (non-Abelian statistics of Majorana Kramers pairs)
- In the fermion parity (FP: 0 or 1) basis of each "spin", the adiabatic pumping of FP and "spin" follows: (ϕ advances by π in each step)

$$|0_\uparrow 0_\downarrow\rangle \rightarrow |1_\uparrow 0_\downarrow\rangle \rightarrow |1_\uparrow 1_\downarrow\rangle \rightarrow |1_\uparrow 0_\downarrow\rangle \rightarrow |0_\uparrow 0_\downarrow\rangle$$

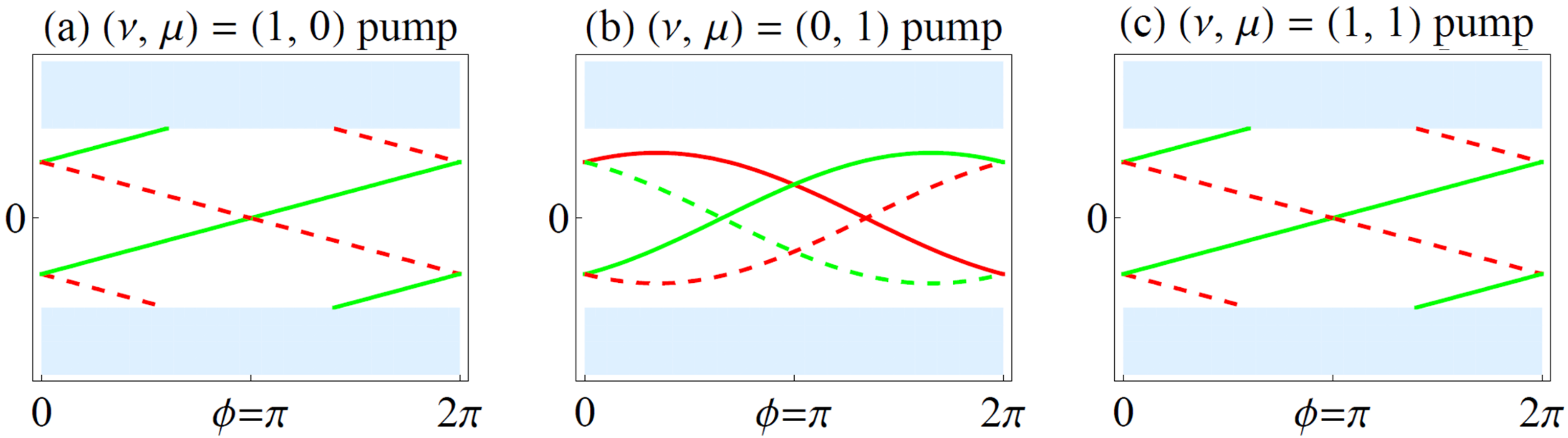
FZ-Kane arXiv:1310.5281; FZ-Kane-Mele, PRL 111, 056402 (2013).

The 3rd Z_2 Invariant



FZ-Kane arXiv:1310.5281.

Models for Andreev Bound States



$H_a = \delta\phi\sigma_y$ $\quad$ $H_b = \delta\phi\sigma_y\tau_z$ $\quad$ $H_c = -\delta\phi\sigma_y$

PHS: $\Xi H(k,\phi)\Xi^{-1} = -H(-k,\phi)$; $\quad$ TRS: $\theta H(k,\phi)\theta^{-1} = H(-k,-\phi)$

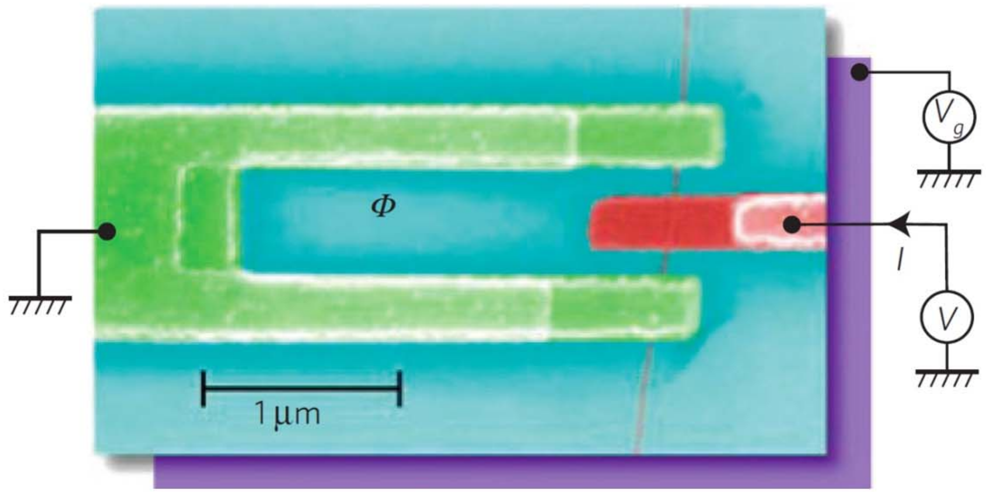
Choose a gauge in which: $\Xi = K$ $\quad$ (with $\Xi^2 = +1$)

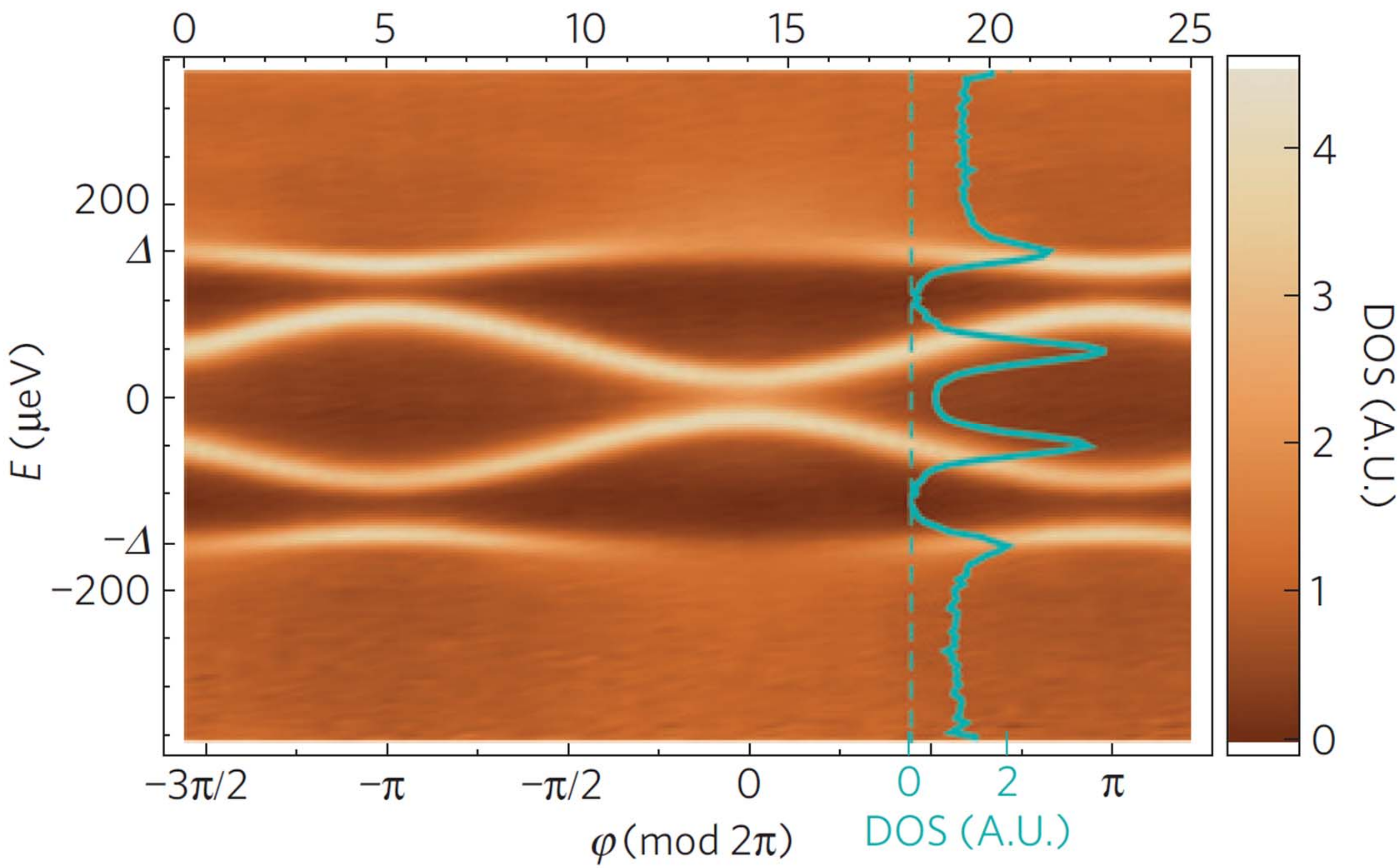
$\theta = \sigma_y K$ $\quad$ (with $\theta^2 = -1$)

FZ-Kane arXiv:1310.5281.

P. Joyez Group
Nature Physics (2010)

- **CNT**
- **STM**

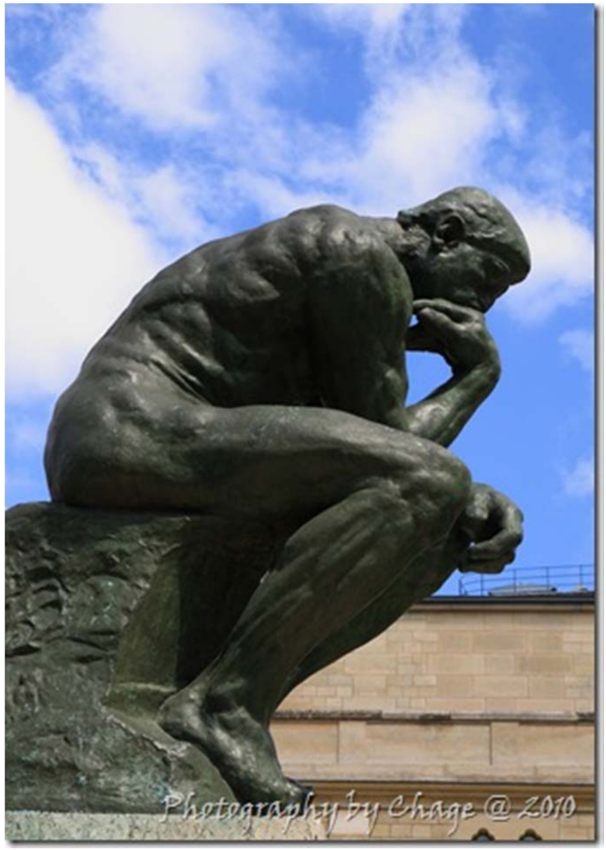




Conclusions

Theory: How many kinds of topological states there are?

Experiment: When will we observe one Majorana fermion?



Majoranas ?

Thank you and have a nice trip !