LIGHT-CONE INTERACTION-POINT OPERATORS AND COVARIANT QUANTIZATION

Nathan Berkovits
(IFT-UNESP, São Paulo)

Stanleyfest, KITP, 13 Feb 2009

Two approaches to covariant quantization:

1) Start with classical covariant action and then quantize;

OR

2) Start with quantum light-cone action and then covariantize.

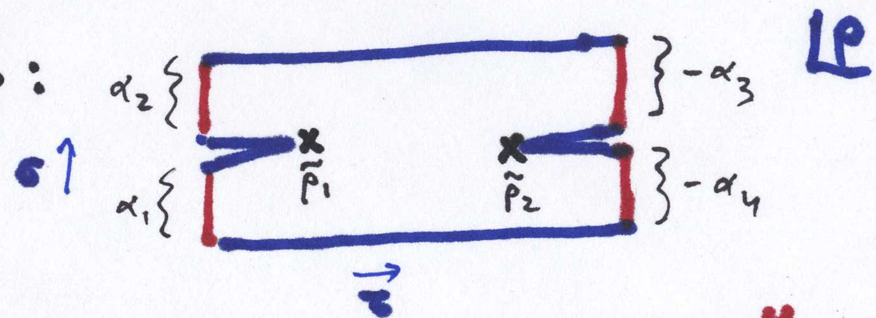
Approach (2) is sometimes easier.

I. Bosonic string (Mandelstam, 1973)

$$S_{LC} = \int d\tau\, d\sigma \;\partial_\rho X^j \,\bar{\partial}_{\bar{\rho}} X^j \qquad j = 1 \ldots D-2\,, \quad \rho = \tau + i\sigma$$

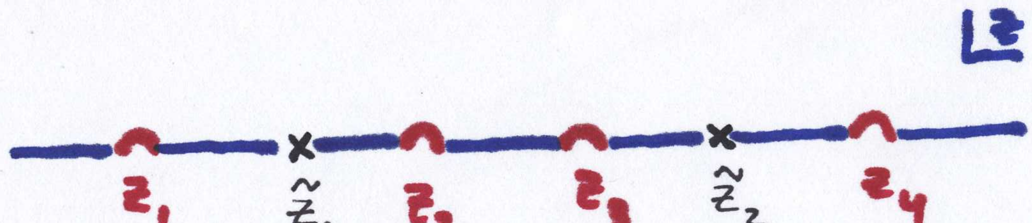
To compute amplitudes, use "interacting string picture".

4-pt. tree amp:



$$\boxed{\alpha_r \equiv P_r^+ = P_r^0 + P_r^{D-1}}$$

Conf. map to semiplane $\downarrow$ $\rho(z) = \sum_{r=1}^{4} \alpha_r \log(z - z_r)$



$$\frac{\partial \rho}{\partial z} = \sum_{r=1}^{4} \frac{\alpha_r}{z - z_r} = 0 \quad \text{when } z = \tilde{z}_I \;\Rightarrow\; \tilde{\rho}_I \equiv \rho(\tilde{z}_I) \text{ are}$$

"interaction points"

To compute 4-pt tree amplitude, perform functional integration using light-cone variables $X^j(\tau,\sigma)$ and use conf. inv. of worldsheet action to map to plane.

$$a = \int d(\tilde{\tau}_2 - \tilde{\tau}_1)\, e^{i\sum_{r=1}^{4} E_r \tau_r} \langle V_1(\rho_1) \ldots V_4(\rho_4) \rangle$$

$$= \int dz_3 \left(\frac{d(\tilde{\tau}_2 - \tilde{\tau}_1)}{dz_3} \right) \langle V_1(z_1) \ldots V_4(z_4) \rangle \, \Delta^{-\frac{D-2}{2}}$$

$$V_r(z_r) = e^{i k_r^j X^j(z_r)} e^{i E_r \tau_r} \quad \text{for tachyon } (k^j k^j = \alpha_r E_r + 2)$$

When $D = 26$, $\Delta^{-\frac{D-2}{2}}$ cancels $\left(\frac{d(\tilde{\tau}_2 - \tilde{\tau}_1)}{dz_3} \right)$

$$\Rightarrow \quad a = \int dz_3 \prod_{r,s} |z_r - z_s|^{-k_r^\mu k_{s\mu}} (z_1 - z_2)(z_2 - z_4)(z_4 - z_1)$$

Can be obtained covariantly using $V_r(z_r) = c\, e^{i k_r^\mu X_\mu(z_r)}$

$\mu = 0$ to 25

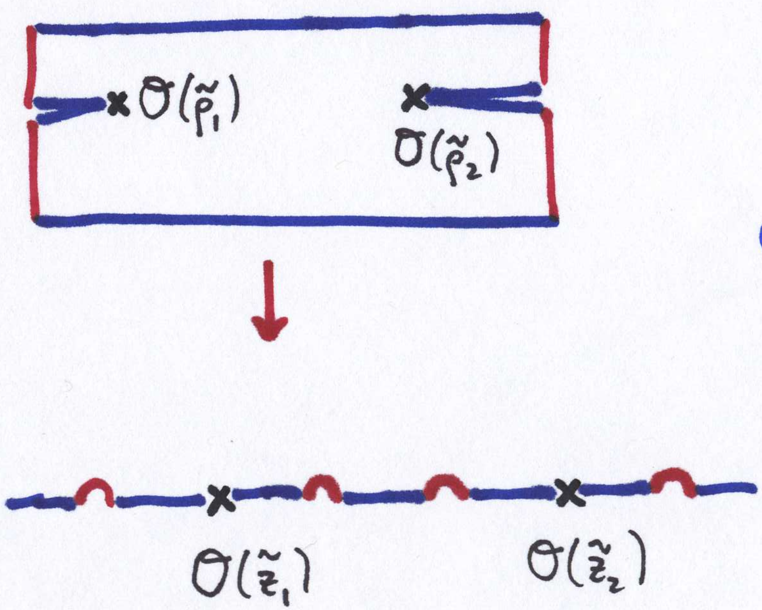
II. Neveu-Schwarz string (Mandelstam, 1974)

$$S_{LC} = \int d\tau\, d\sigma \left(\partial_\rho X^j \bar{\partial}_{\bar{\rho}} X^j + \psi^j \bar{\partial}_{\bar{\rho}} \psi^j \right) \qquad j=1\ldots 8$$

ψ^j has $\frac{1}{2}$ conf. wt $\Rightarrow$ $\psi^j(z) = \sqrt{\frac{\partial\rho}{\partial z}}\, \psi^j(\rho)$

NS $\Rightarrow$ $\psi^j(z)$ has no branch cuts $\Rightarrow$ $\psi^j(\rho)$ has branch cuts at $\begin{cases} \rho = \rho_r \\ \rho = \tilde{\rho}_I \end{cases}$

4 pt. tree:



$$\sigma(\tilde{\rho}) = \lim_{\rho\to\tilde{\rho}} (\rho-\tilde{\rho})^{3/4}\, \partial_\rho X^j(\rho)\, \psi^j(\rho)$$

σ is "interaction-point operator"

$$\sigma(\tilde{z}) = \left(\frac{\partial^2\rho}{\partial z^2}\right)\Bigg|_{z=\tilde{z}}^{-3/4} \partial_z X^j(\tilde{z})\, \psi^j(\tilde{z})$$

Insertions of $\sigma(\tilde{\rho})$ are needed for Lorentz invariance.

To compute amplitude, need to include both vertex op's V_r at ρ_r and interaction-point operators σ at $\tilde{\rho}_I$.

$$a = \int d(\tilde{\tau}_2 - \tilde{\tau}_1)\, e^{i\sum_{r=1}^{4} E_r \tau_r} \langle V_1(\rho_1) \ldots V_4(\rho_4)\, \sigma(\tilde{\rho}_1)\, \sigma(\tilde{\rho}_2) \rangle$$

$$= \int dz_3 \left(\frac{d(\tilde{\tau}_2 - \tilde{\tau}_1)}{dz_3} \right) \Delta^{-\frac{3(D-2)}{4}} \langle V_1(z_1) \ldots V_4(z_4)\, \sigma(\tilde{z}_1)\, \sigma(\tilde{z}_2) \rangle$$

$V_r(z_r) = e^{i k_r^j X^j(z_r)} e^{i E_r \tau_r}$ for tachyon $(k_r^j k_r^j = \alpha_r E_r + 1)$, $\sigma(\tilde{z}) = \left(\frac{\partial^2 \rho}{\partial z^2}\right)^{-3/4} \partial X^j \psi^j$

Can be obtained covariantly using Friedan, Martinec, Shenker 1985

$$V^{cov} = c e^{-\varphi} e^{i k^\mu X_\mu}, \quad \sigma^{cov} = \{Q, \xi\} = e^{\varphi} \partial X^\mu \psi_\mu + \ldots$$

To relate to FMS, use $N=1$ super-worldsheets where

$$\mathcal{X}^j = X^j + \kappa \psi^j \rightarrow \mathcal{X}^\mu = X^\mu + \kappa \psi^\mu$$

(NB 1987; Sin 1988; Aoki, D'Hoker, Phong 1990)

III. Green-Schwarz superstring (Green, Schwarz 1984; Mandelstam 1985)

$$S_{lc} = \int d\tau\, d\sigma \left(\partial_\rho X^j \bar{\partial}_{\bar\rho} X^j + S^a \bar{\partial}_{\bar\rho} S^a \right) \qquad j = 1 \dots 8,\ a = 1 \dots 8$$

$S^a(\rho)$ has no branch cuts in ρ-plane because of spacetime susy

If S^a has $+\frac{1}{2}$ conf. wt $\Rightarrow S^a(z)$ has branch cuts at $\begin{cases} z = z_r \\ z = \tilde{z}_I \end{cases}$

Convenient to break $SO(8) \to U(4)$ and split

$$S^a \begin{cases} \nearrow S^A & \text{0 conf. wt} \\ \searrow S_A & \text{+1 conf. wt} \end{cases}$$

$A = 1 \dots 4$ $\qquad S_{lc} = \int d\tau\, d\sigma \left(\partial_\rho X^j \bar{\partial}_{\bar\rho} X^j + S_A \bar{\partial}_{\bar\rho} S^A \right)$

$\Rightarrow$ $S^A(z) = S^A(\rho)$

$S_A(z) = \left(\frac{\partial\rho}{\partial z} \right) S_A(\rho)$ $\qquad$ have no branch cuts in z plane

After breaking $SO(8) \to U(4)$ where $S^a \to (S^A, S_A)$, there are two choices for how the $SO(8)$ vector splits.

1) $X^j \to (X^L, X^{[AB]}, X^R)$ $\quad [AB] = 1$ to 6, $L = 7+i8$, $R = 7-i8$

OR

2) $X^j \to (X^A, X_A)$ $\quad A = 1$ to 4

As in NS string, GS superstring also requires interaction-pt. op's

$$a = \int d(\tilde{\tau}_2 - \tilde{\tau}_1)\, e^{i\sum_{r=1}^{4} E_r \tau_r} \langle V_1(\rho_1) \ldots V_4(\rho_4)\, \Theta(\tilde{\rho}_1)\, \Theta(\tilde{\rho}_2) \rangle$$

$V_r(z_r) = \Phi(S^A)\, e^{i k_r^j X^j} e^{i E_r \tau_r}$ for super-YM multiplet $(k_r^j k_r^j = \alpha_r E_r)$

Choice 1) $\Theta(\tilde{\rho}) = \lim_{\rho \to \tilde{\rho}} (\rho - \tilde{\rho})^{1/2} \partial X^L + \lim_{\rho \to \tilde{\rho}} (\rho - \tilde{\rho})^{3/2} \partial X^{[AB]} S_A S_B + \lim_{\rho \to \tilde{\rho}} (\rho - \tilde{\rho})^{5/2} \partial X^R \epsilon^{ABCD} S_A S_B S_C S_D$

Choice 2) $\Theta(\tilde{\rho}) = \lim_{\rho \to \tilde{\rho}} (\rho - \tilde{\rho})\, \partial X^A S_A + \lim_{\rho \to \tilde{\rho}} (\rho - \tilde{\rho})^2 \epsilon^{ABCD} \partial X_A S_B S_C S_D$

To covariantize, choose (2) for breaking $SO(8) \to U(4)$

$$\Rightarrow \quad \mathcal{O}(\tilde{\rho}) = \lim_{\rho\to\tilde{\rho}} (\rho-\tilde{\rho})\, \partial X^A S_A + \lim_{\rho\to\tilde{\rho}} (\rho-\tilde{\rho})^3\, \partial X_A S^A \left(\epsilon^{BCDE} S_B S_C S_D S_E\right)$$

$\mathcal{O}(\tilde{\rho})$ comes from twisted $N=2$ superconf. algebra

$\mathcal{X}^A = x^A + \kappa S^A$, $\bar{\mathcal{X}}_A = x_A + \bar{\kappa} S_A$, $\epsilon^{ABCD} S_A S_B S_C S_D$ is "spectral flow" operator

Twisted $N=2$ generators $= (T, G^+, G^-, J)$ of "hybrid" formalism

(NB 1992)

To get $SO(9,1)$ covariance, first extend $U(4) \to U(5)$ (Wick-rotated)

$$(x^A, S^A) \to (x^a, S^a) \quad , \quad (x_A, S_A) \to (x_a, S_a) \quad \text{where } \boxed{a = 1 \ldots 5}$$

Then extend $U(5)$-cov. variables to (Wick-rotated) $SO(9,1)$-cov. variables

$$x^m \nearrow x^a, \searrow x_a \quad , \quad \theta^\alpha \nearrow \theta^+, \to \theta_{(ab)}, \searrow S^a \quad , \quad p_\alpha \nearrow S_a, \to p^{(ab)}, \searrow p_+$$

$m = 0 \ldots 9$

$\alpha = 1 \ldots 16$

Introduce ghosts to cancel additional variables

$(\lambda^\alpha, w_\alpha)$ where λ^α satisfies "pure spinor" constraint

$$\lambda\gamma^m\lambda = 0$$

$\Rightarrow$ λ^α parametrizes ways of breaking $SO(10) \to U(5)$.

$$S_{cov} = \int d\tau\, d\sigma \left(\partial_\rho x^m \bar{\partial}_\rho x_m + p_\alpha \bar{\partial}_\rho \theta^\alpha + w_\alpha \bar{\partial}_\rho \lambda^\alpha\right) \qquad \text{(NB 2000)}$$

In light-cone gauge where $\theta^+ = \theta_{(ab)} = p_+ = p^{(ab)} = 0$ and $\lambda^+ = 1$, $\lambda_{(ab)} = 0$

$$V_{cov} = \lambda^\alpha A_\alpha(x,\theta) \to V_{lc} = \Phi(s^A)\, e^{ik^j x^j} e^{iE\tau}$$

where $A_\alpha(x,\theta)$ is on-shell sYM superfield.

$\mathcal{O}(\tilde{\rho})$ comes from composite operator for "b ghost".

Stanley, thank you for all of the wonderful tricks you have taught me.