



# Effects of Young Clusters on Forming Solar Systems

*Fred C. Adams*

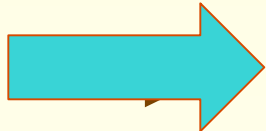
*University of Michigan*

**Star Formation: Then and Now  
KITP/UCSB, August 2007**

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David Hollenbach (NASA Ames), Greg Laughlin (UCSC)*



**Most stars form in clusters:**

 **How does the cluster environment affect process of star/planet formation?**

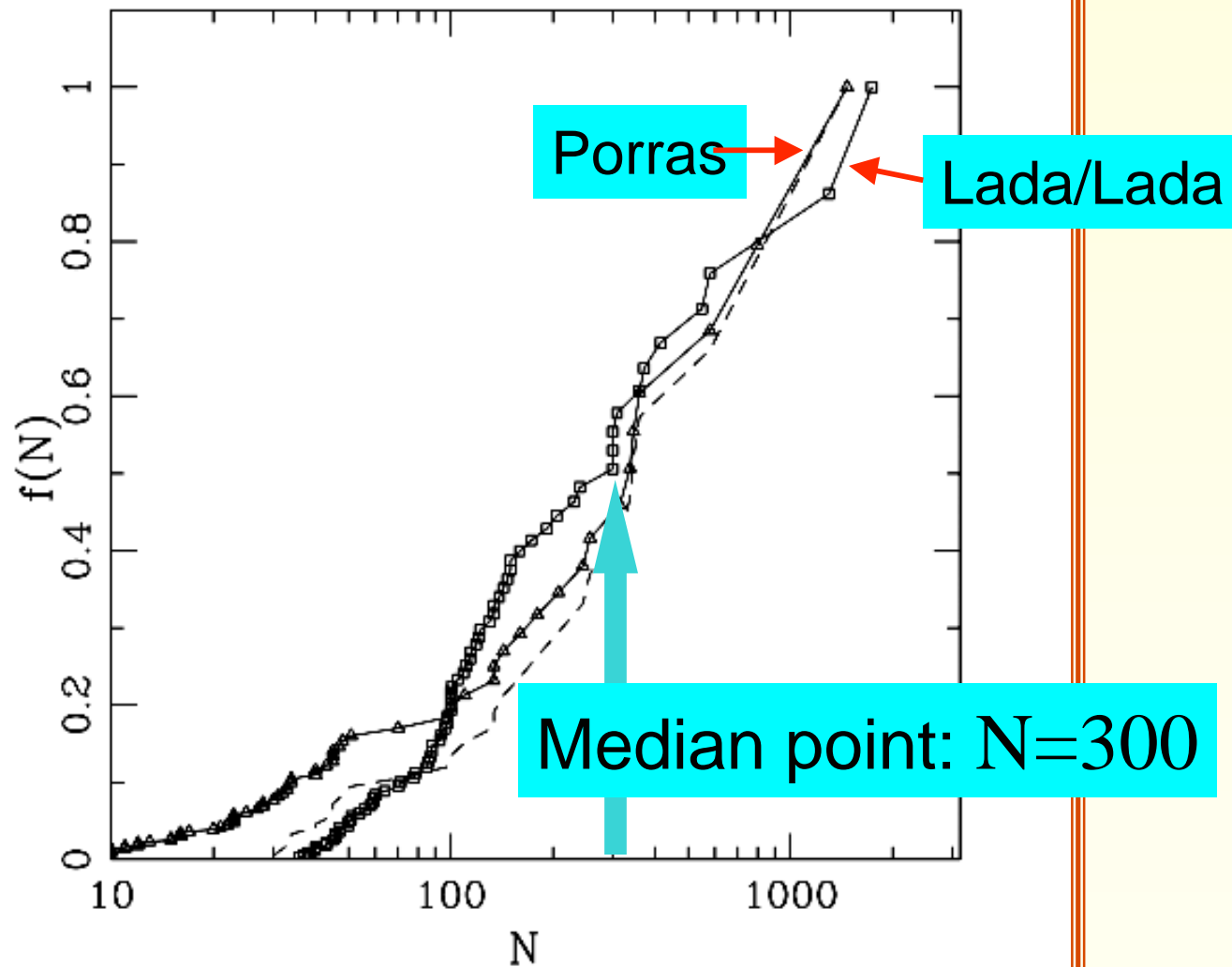
# Outline

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- Distribution of Clusters
- N-body Simulations of Clusters
- UV Radiation Fields in Clusters
- Disk Photoevaporation Model
- Scattering Encounters

Cumulative Distribution: Fraction of stars that form in stellar aggregates with  $N < N$  as function of  $N$



# Simulations of Embedded Clusters

- Modified NBODY2(and 6) Codes (*S. Aarseth*)
- Simulate evolution from embedded stage out to ages of 10 Myr
- Cluster evolution depends on the following:
  - cluster size
  - initial stellar and gas profiles
  - gas disruption history
  - star formation history
  - primordial mass segregation
  - initial dynamical assumptions
- 100 realizations are needed to provide robust statistics for output measures

# Simulation Parameters

Cluster Membership

$$N = 100, 300, 1000$$

Radius

$$R(N) = 1 \text{ pc} \left( \frac{N}{300} \right)^{1/2}$$

Initial Stellar Density  $\rho_* \propto r^{-1}$

Gas Distribution

$$\rho_{gas} = \frac{\rho_0}{\xi(1+\xi)^3}, \quad \rho_0 = \frac{2M_*}{\pi R^3} \quad \xi = \frac{r}{R}$$

Star Formation Efficiency 0.33

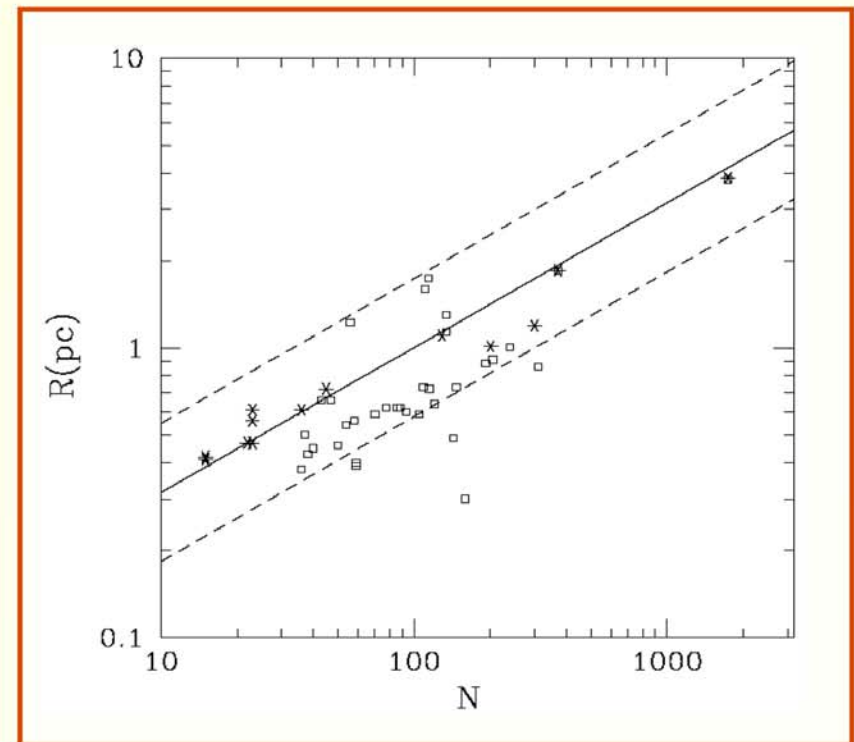
Embedded Epoch  $t = 0-5 \text{ Myr}$

Star Formation  $t = 0-1 \text{ Myr}$

Virial Ratio  $Q = |K/W|$

virial  $Q = 0.5$ ; cold  $Q = 0.04$

Mass Segregation: largest star at center of cluster



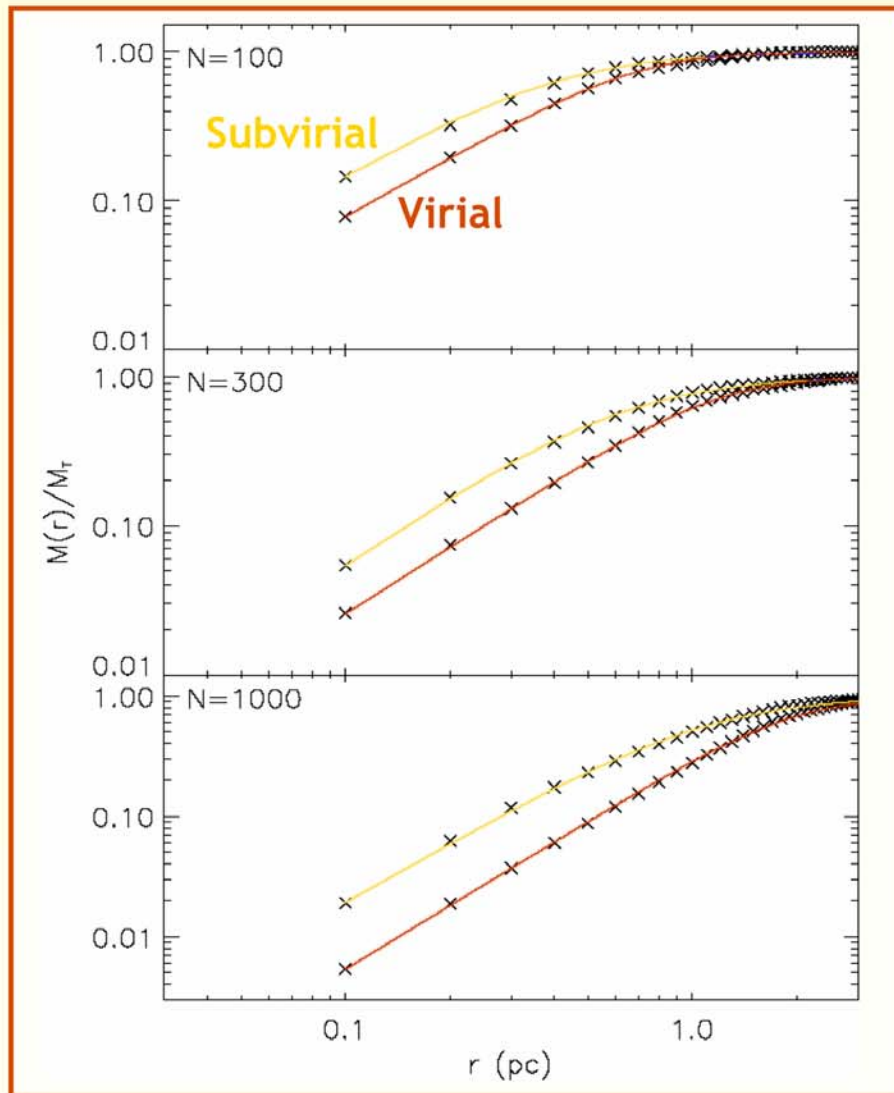
# Dynamical Results

*I. Evolution of clusters as astrophysical objects*

*II. Effects of clusters on forming solar systems*

- Distribution of closest approaches
- Radial position probability distribution  
(given by cluster mass profiles)

# Mass Profiles



$$\frac{M(\xi)}{M_T} = \left( \frac{\xi^a}{1 + \xi^q} \right)^p \quad \xi = r/r_0$$

Simulation	p	$r_0$	a
100 Subvirial	0.69	0.39	2
100 Virial	0.44	0.70	3
300 Subvirial	0.79	0.64	2
300 Virial	0.49	1.19	3
1000 Subvirial	0.82	1.11	2
300 Subvirial	0.59	1.96	3

## Stellar Gravitational Potential

$$\Psi_* = \frac{GM_T}{r_0} \psi_0 \quad \psi_0 = \int_0^\infty \left( \frac{1}{1+u^a} \right)^p du$$

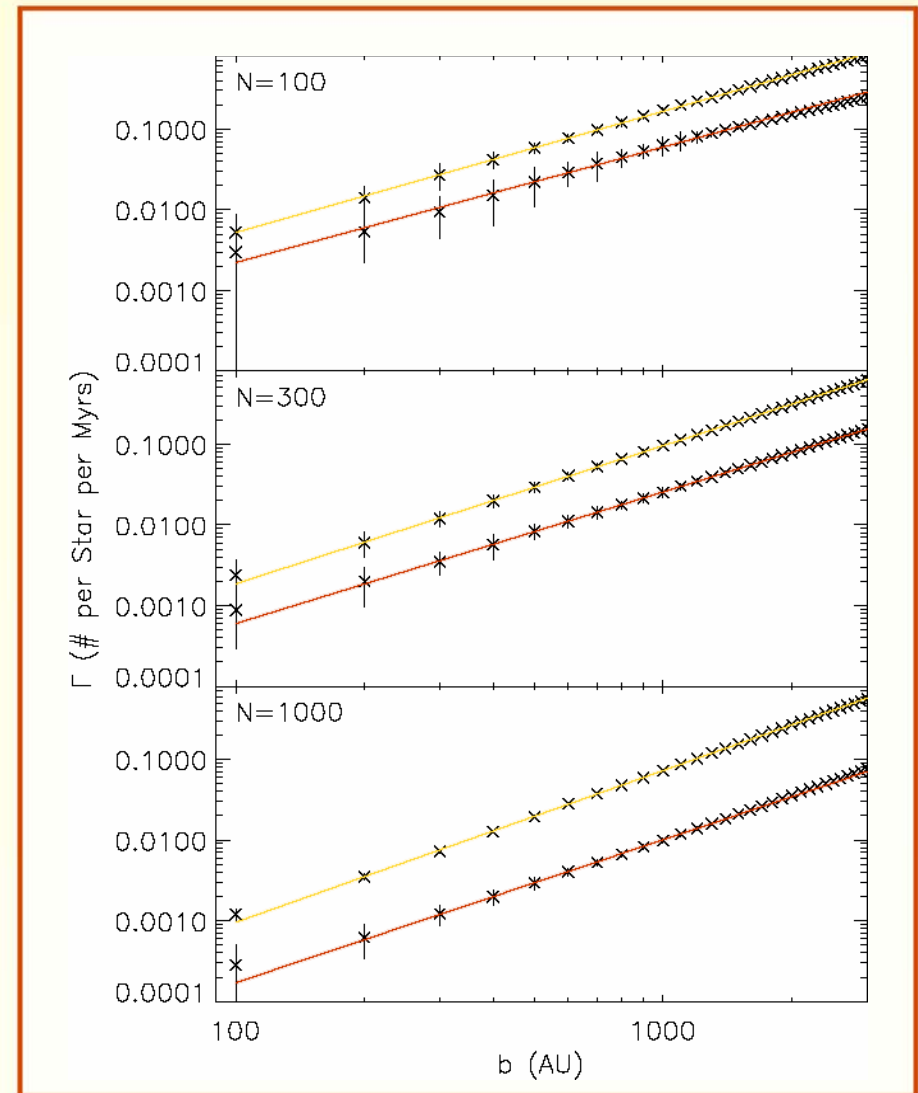


# Closest Approach Distributions

$$\Gamma = \Gamma_0 \left[ \frac{b}{1000 \text{ AU}} \right]^\gamma$$

Simulation	$\Gamma_0$	$\gamma$	$b_c$ (AU)
100 Subvirial	0.166	1.50	713
100 Virial	0.0598	1.43	1430
300 Subvirial	0.0957	1.71	1030
300 Virial	0.0256	1.63	2310
1000 Subvirial	0.0724	1.88	1190
1000 Virial	0.0101	1.77	3650

Typical star experiences one close encounter with impact parameter  $b_c$  during 10 Myr time span



# Effects of Cluster Radiation on Forming/Young Solar Systems

- Photoevaporation of a circumstellar disk
- Radiation from the background cluster often dominates radiation from the parent star (*Johnstone et al. 1998; Adams & Myers 2001*)
- FUV radiation ( $6 \text{ eV} < E < 13.6 \text{ eV}$ ) is more important in this process than EUV radiation
- FUV flux of  $G_0 = 3000$  will truncate a circumstellar disk to  $r_d$  over 10 Myr, where

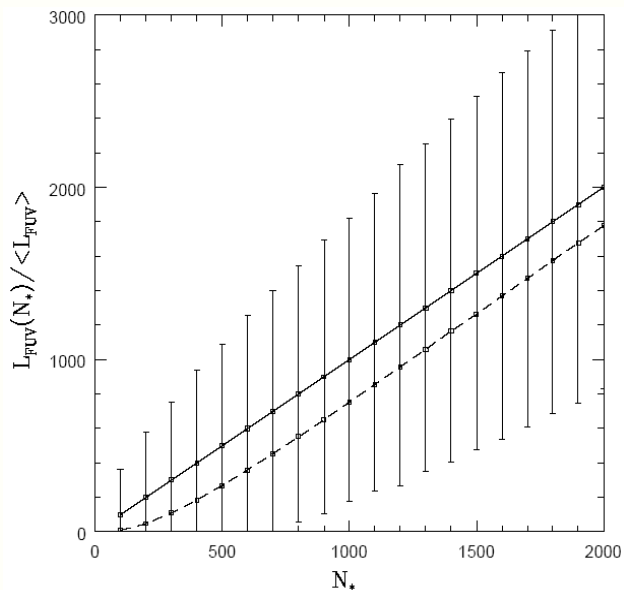
$$r_d = 36 \text{ AU} \left[ \frac{M_*}{M_{sun}} \right]$$

# Calculation of the Radiation Field

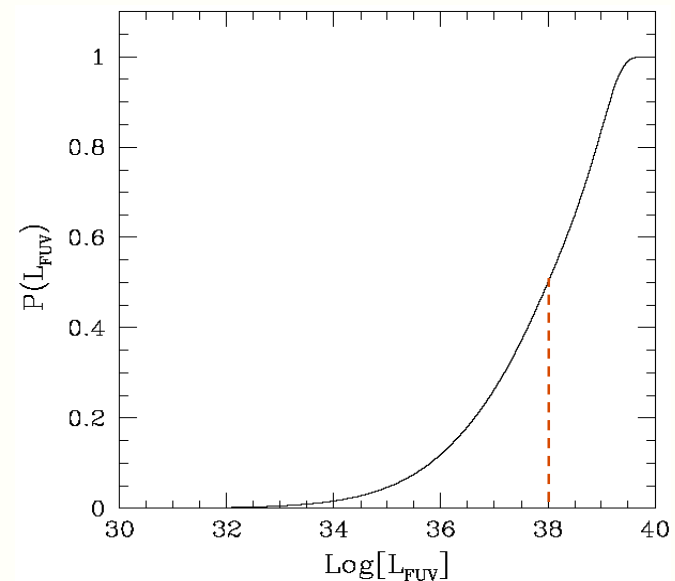
## Fundamental Assumptions

- Cluster size  $N = N$  primaries (ignore binary companions)
- No gas or dust attenuation of FUV radiation
- Stellar FUV luminosity is only a function of mass
- Meader's models for stellar luminosity and temperature

Sample IMF  $\rightarrow L_{\text{FUV}}(N)$



Sample Cluster Sizes  $\rightarrow$  Expected FUV Luminosity in SF Cluster



# Photoevaporation of Circumstellar Disks

FUV Flux depends on:

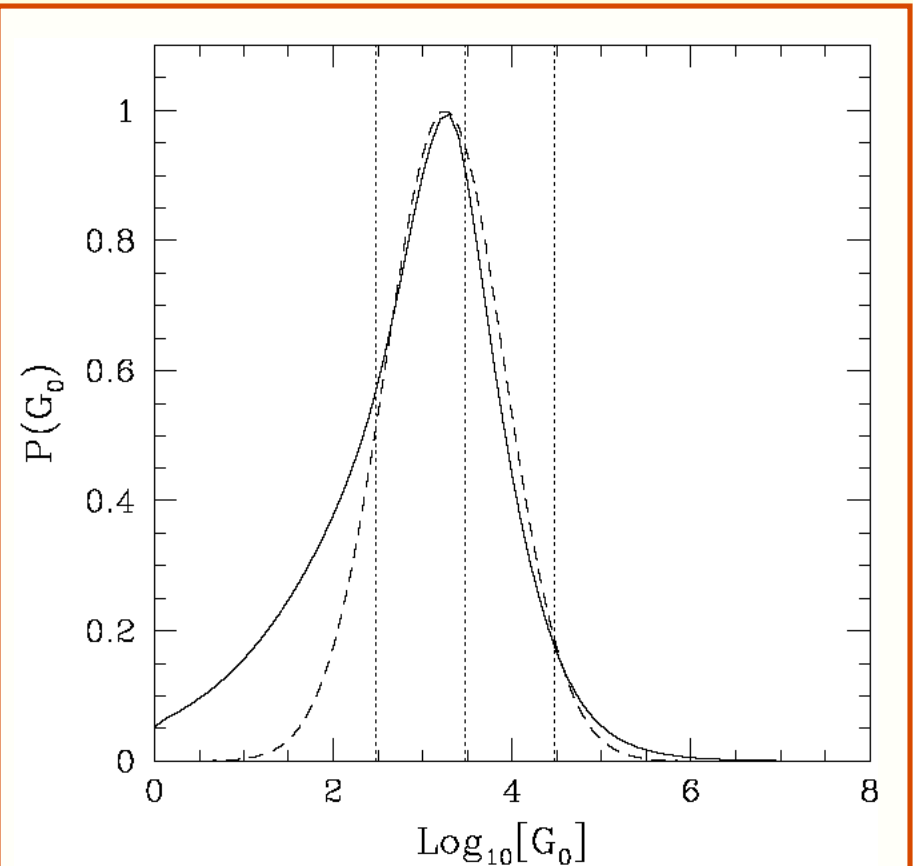
- Cluster FUV luminosity
- Location of disk within cluster

Assume:

- FUV point source located at center of cluster
- Stellar density  $\rho \sim 1/r$

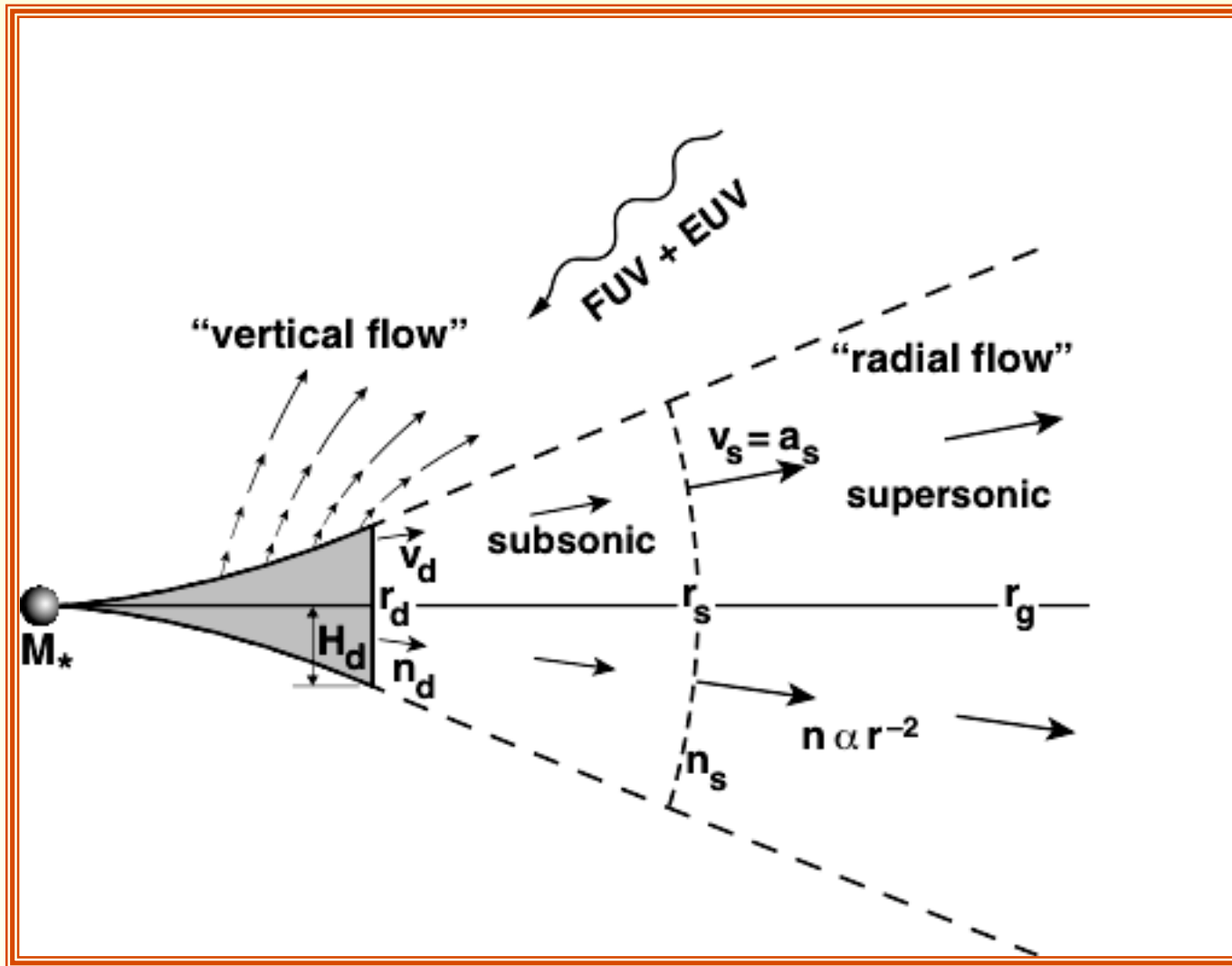
$G_0$  Distribution

Median	900
Peak	1800
Mean	16,500



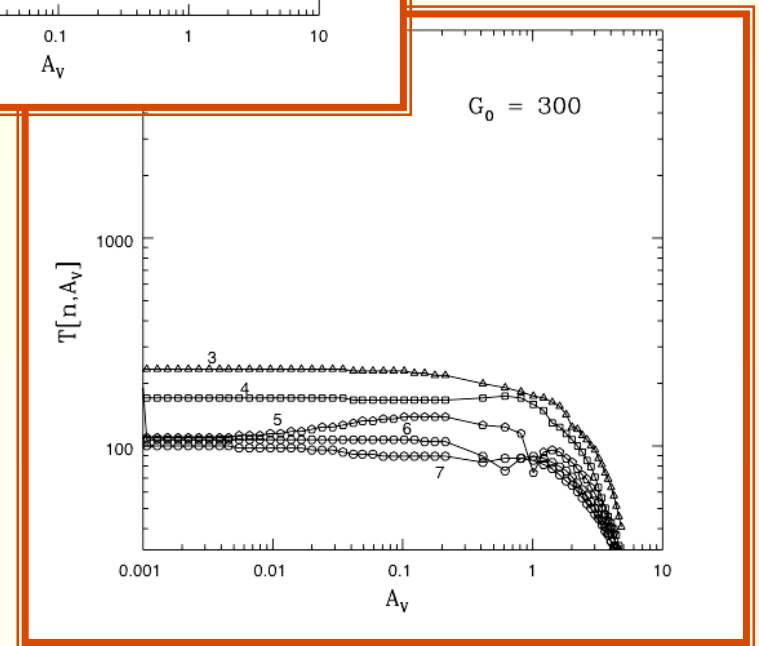
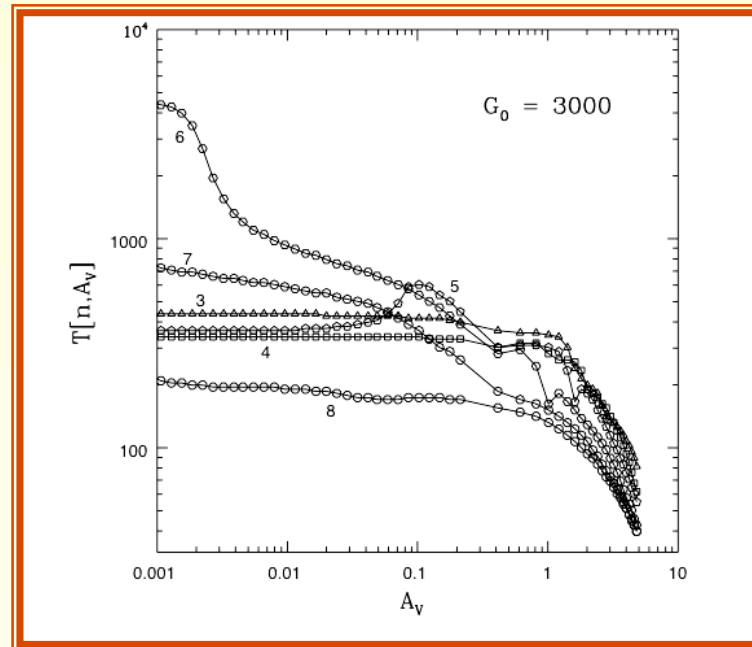
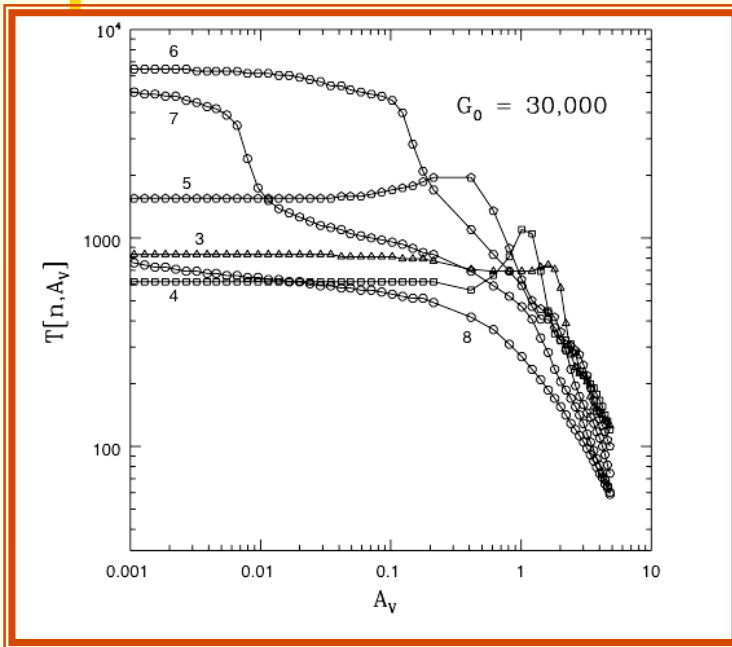
$G_0 = 1$  corresponds to FUV flux  
 $1.6 \times 10^{-3} \text{ erg s}^{-1} \text{ cm}^{-2}$

# Photoevaporation Model



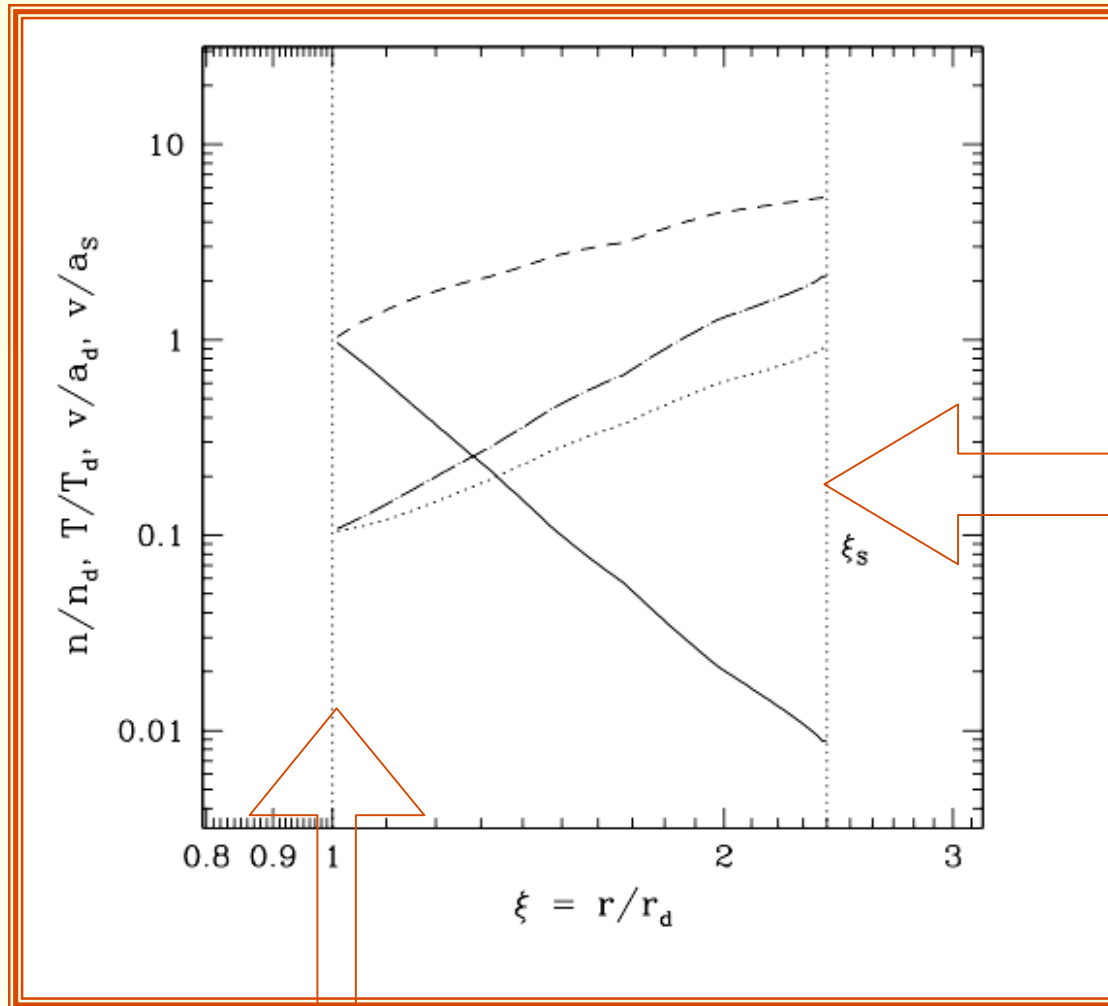
(Adams et al. 2004)

# Results from PDR Code



*Lots of chemistry and many heating/cooling lines determine the temperature as a function of  $G$ ,  $n$ ,  $A$*

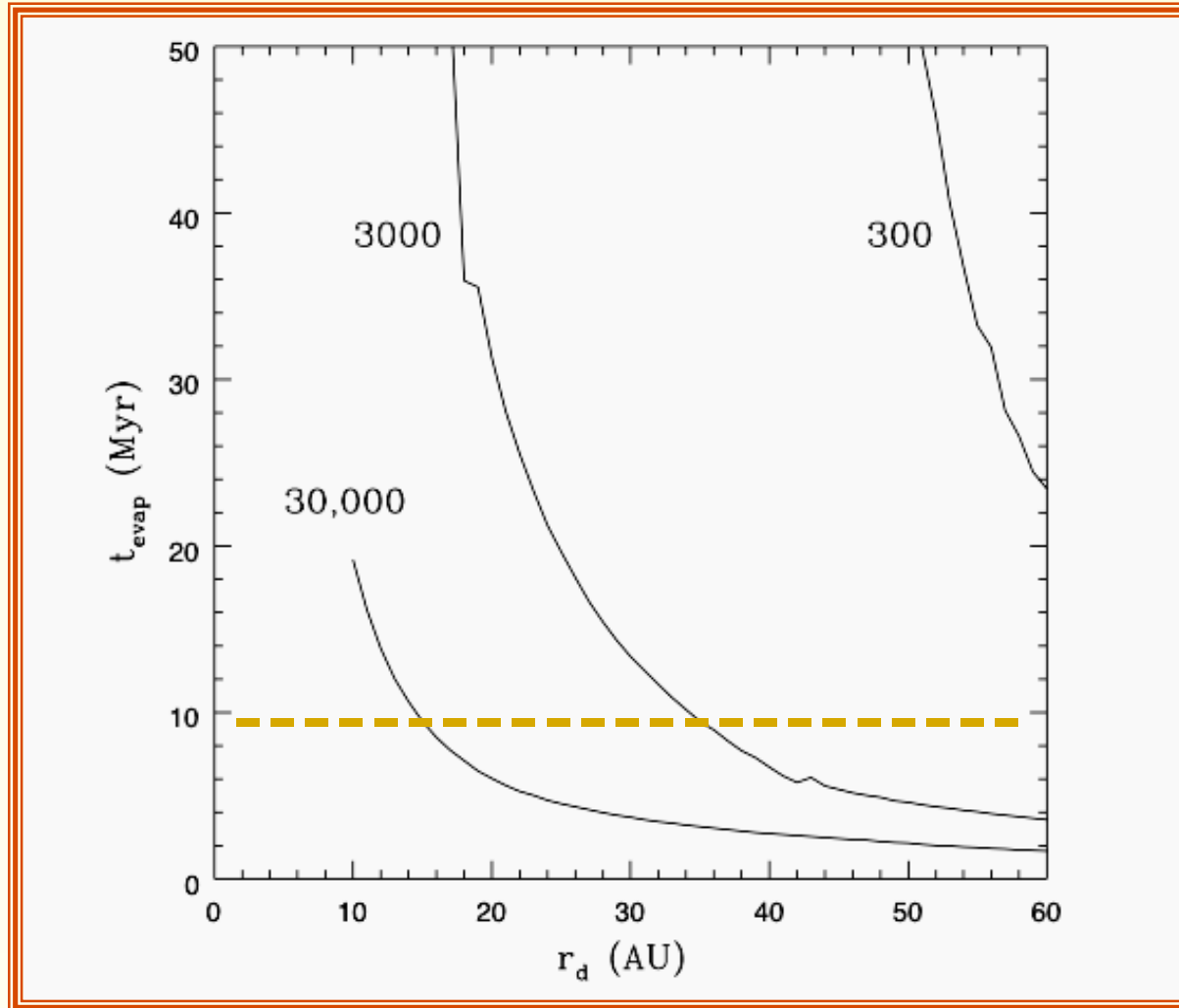
# Solution for Fluid Fields



*sonic surface*

*outer disk edge*

# Evaporation Time vs FUV Field



*(for disks around solar mass stars)*



# Photoevaporation in Simulated Clusters

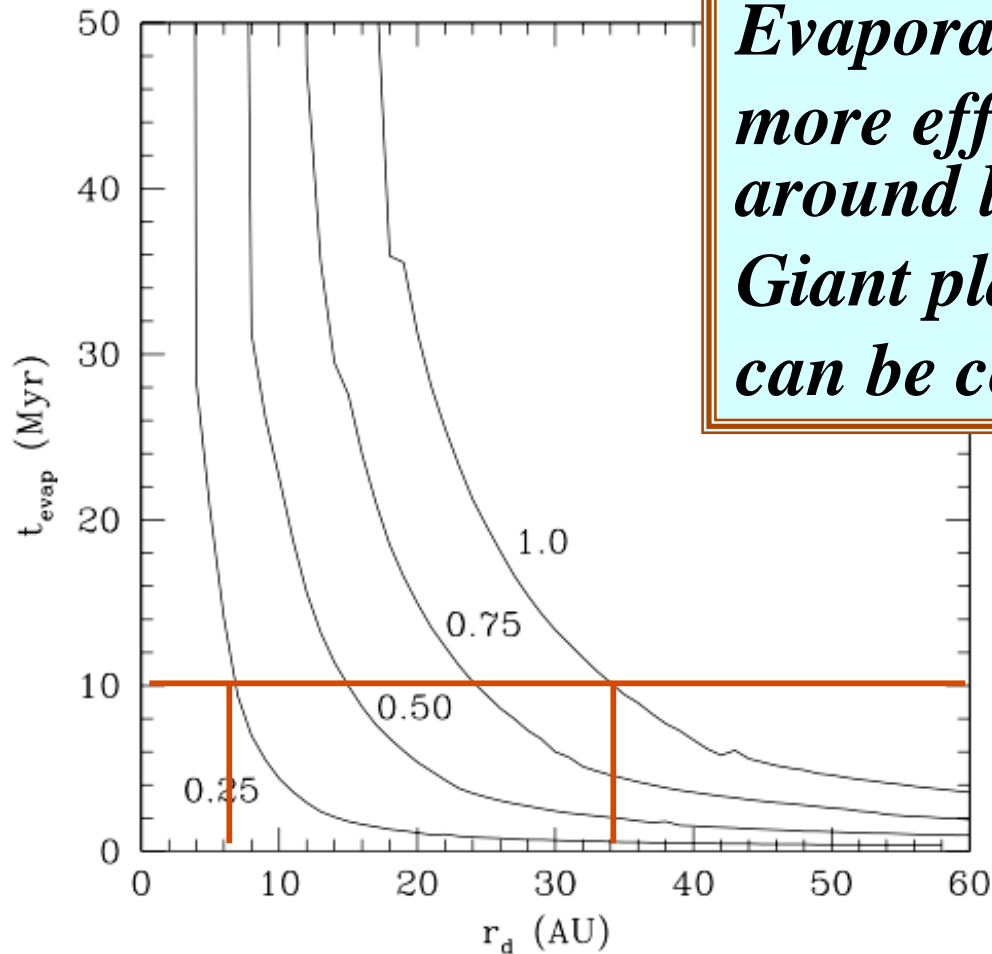
Radial Probability Distributions

$$\frac{N(r)}{N_T} = \left[ \frac{\xi^a}{1 + \xi^a} \right]^p \quad \text{where} \quad \xi = \frac{r}{r_0}$$

Simulation	$r_{eff}$ (pc)	$G_0$ mean	$r_{med}$ (pc)	$G_0$ median
100 Subvirial	0.080	66,500	0.323	359
100 Virial	0.112	34,300	0.387	250
300 Subvirial	0.126	81,000	0.549	1,550
300 Virial	0.181	39,000	0.687	992
1000 Subvirial	0.197	109,600	0.955	3,600
1000 Virial	0.348	35,200	1.25	2,060

FUV radiation does not evaporate enough disk gas to prevent giant planet formation for Solar-type stars

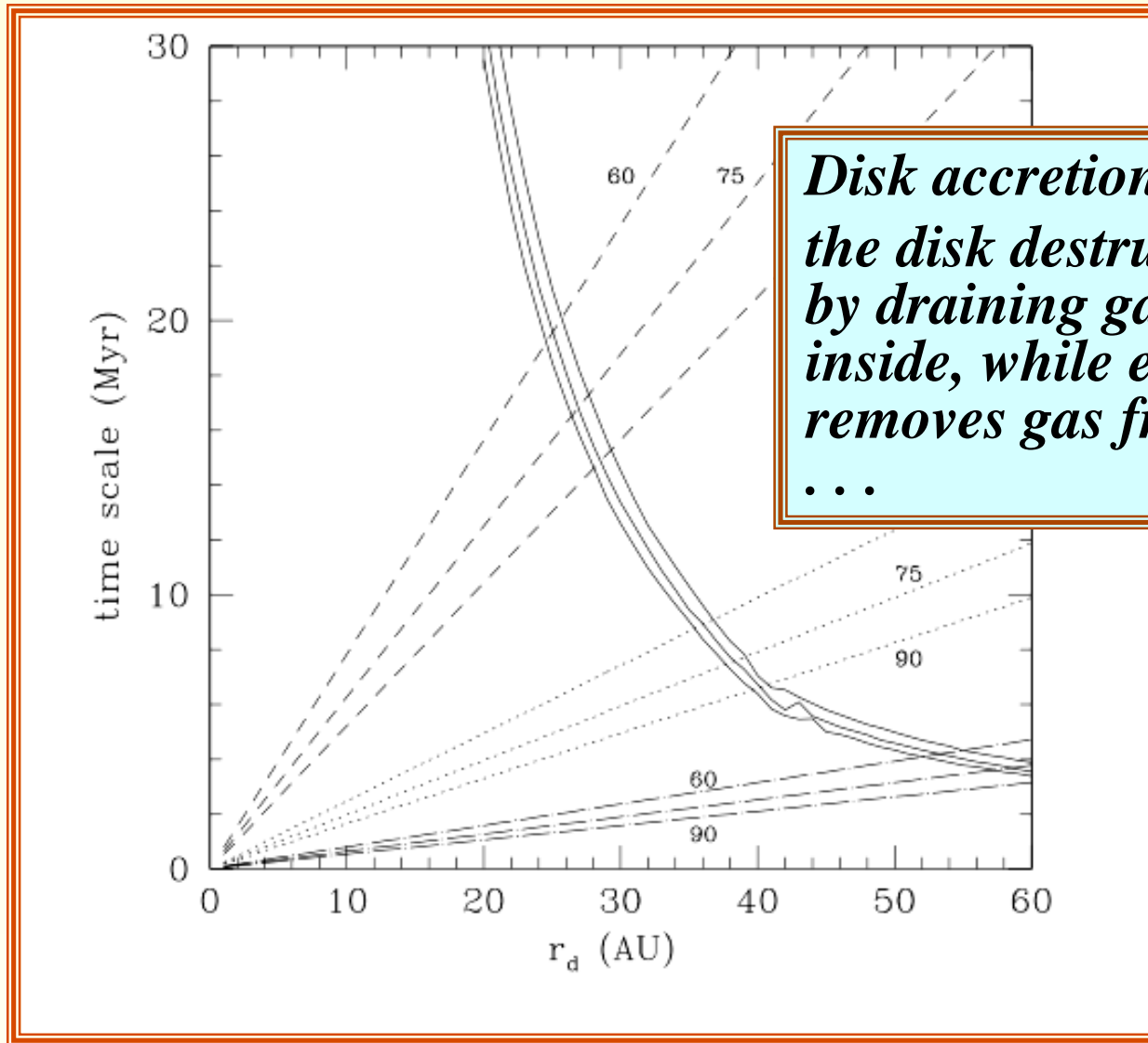
# Evaporation Time vs Stellar Mass



*Evaporation is much more effective for disks around low-mass stars: Giant planet formation can be compromised*

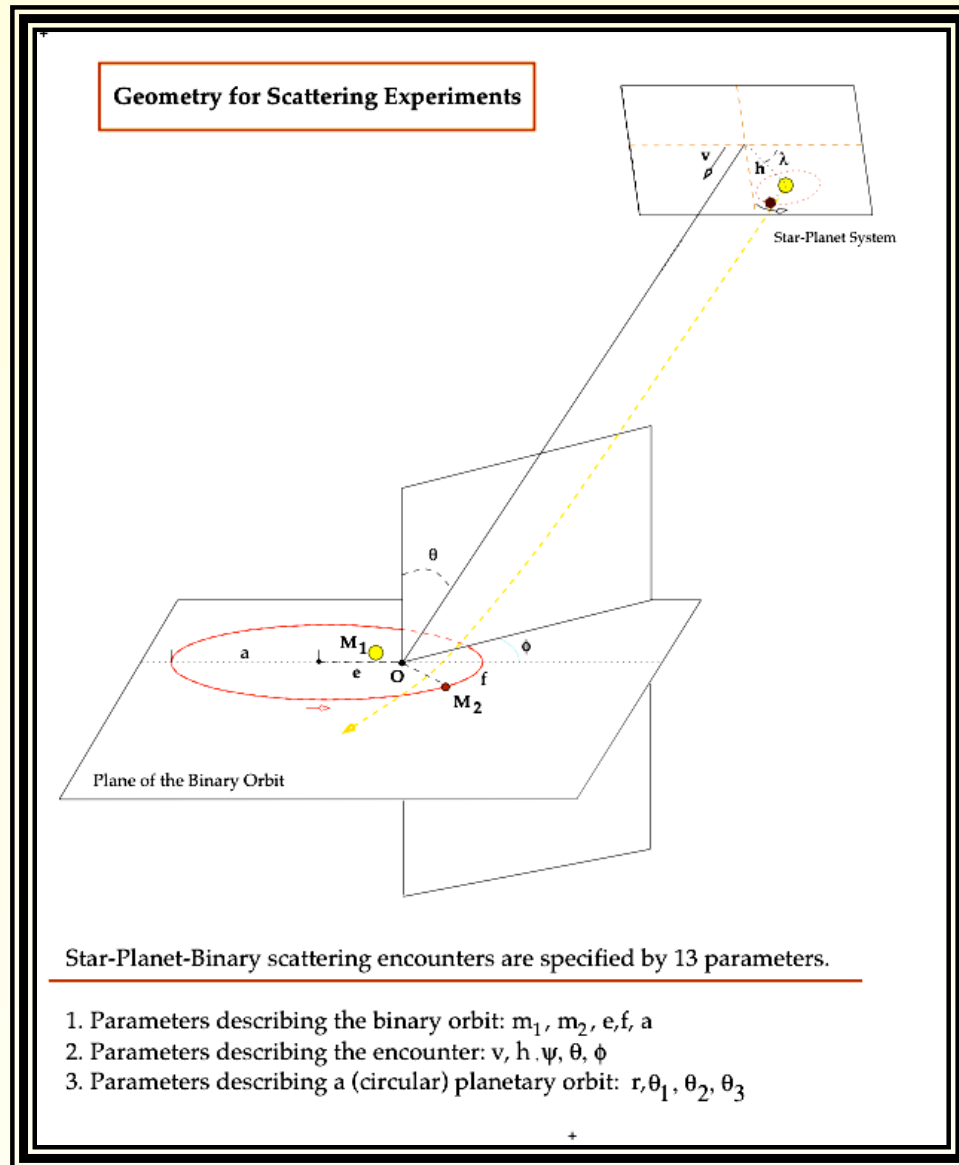
**G=3000**

# Evaporation vs Accretion



*Disk accretion aids and abets the disk destruction process by draining gas from the inside, while evaporation removes gas from the outside*  
•••

# Solar System Scattering



Many Parameters

+

Chaotic Behavior



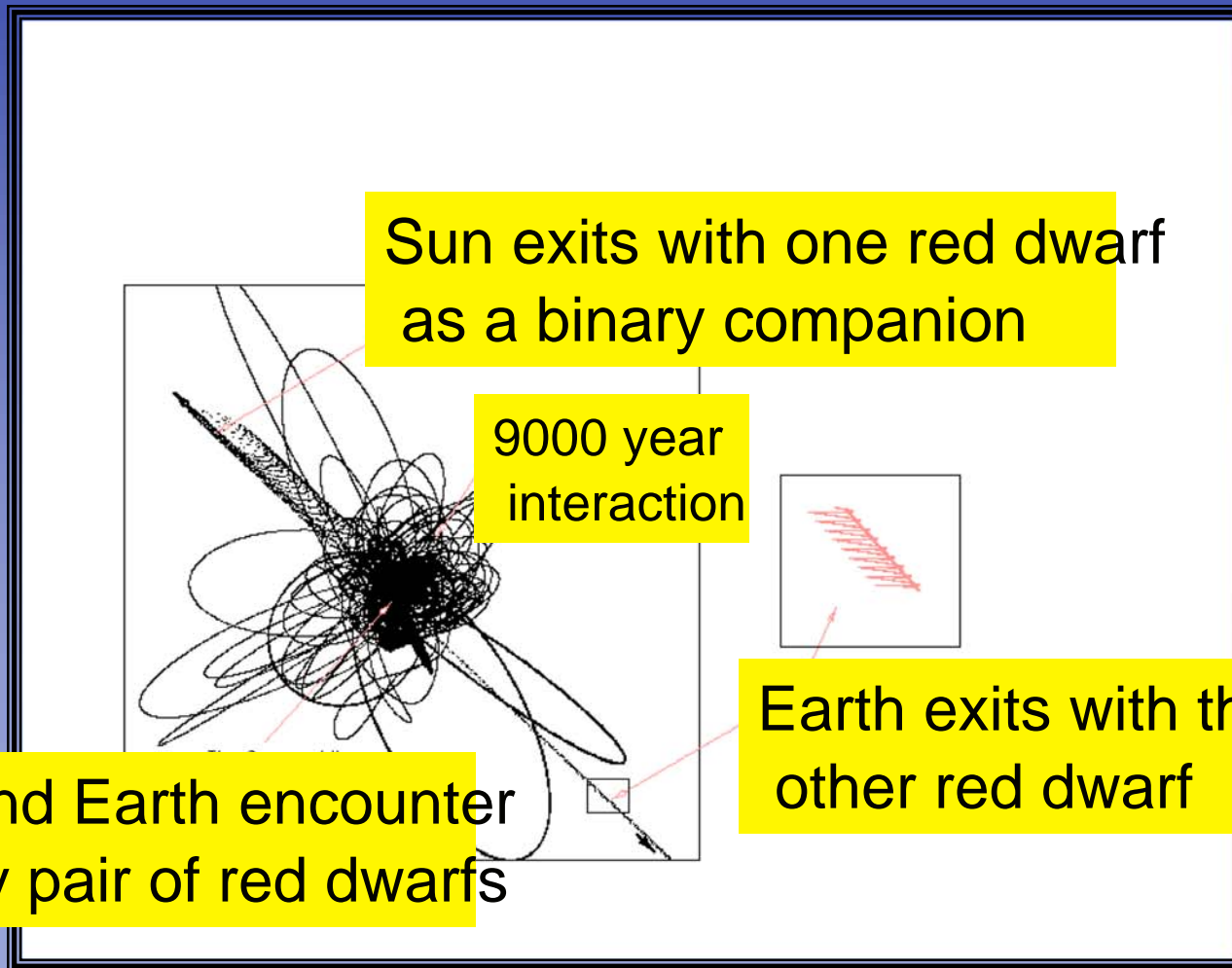
Many Simulations

Monte Carlo

# Monte Carlo Experiments

- **Jupiter only,  $v = 1$  km/s,  $N=40,000$  realizations**
- **4 giant planets,  $v = 1$  km/s,  $N=50,000$  realizations**
- **KB Objects,  $v = 1$  km/s,  $N=30,000$  realizations**
- **Earth only,  $v = 40$  km/s,  $N=100,000$  realizations**
- **4 giant planets,  $v = 40$  km/s, Solar mass,  $N=100,000$  realizations**
- **4 giant planets,  $v = 1$  km/s, varying stellar mass,  $N=100,000$  realizations**

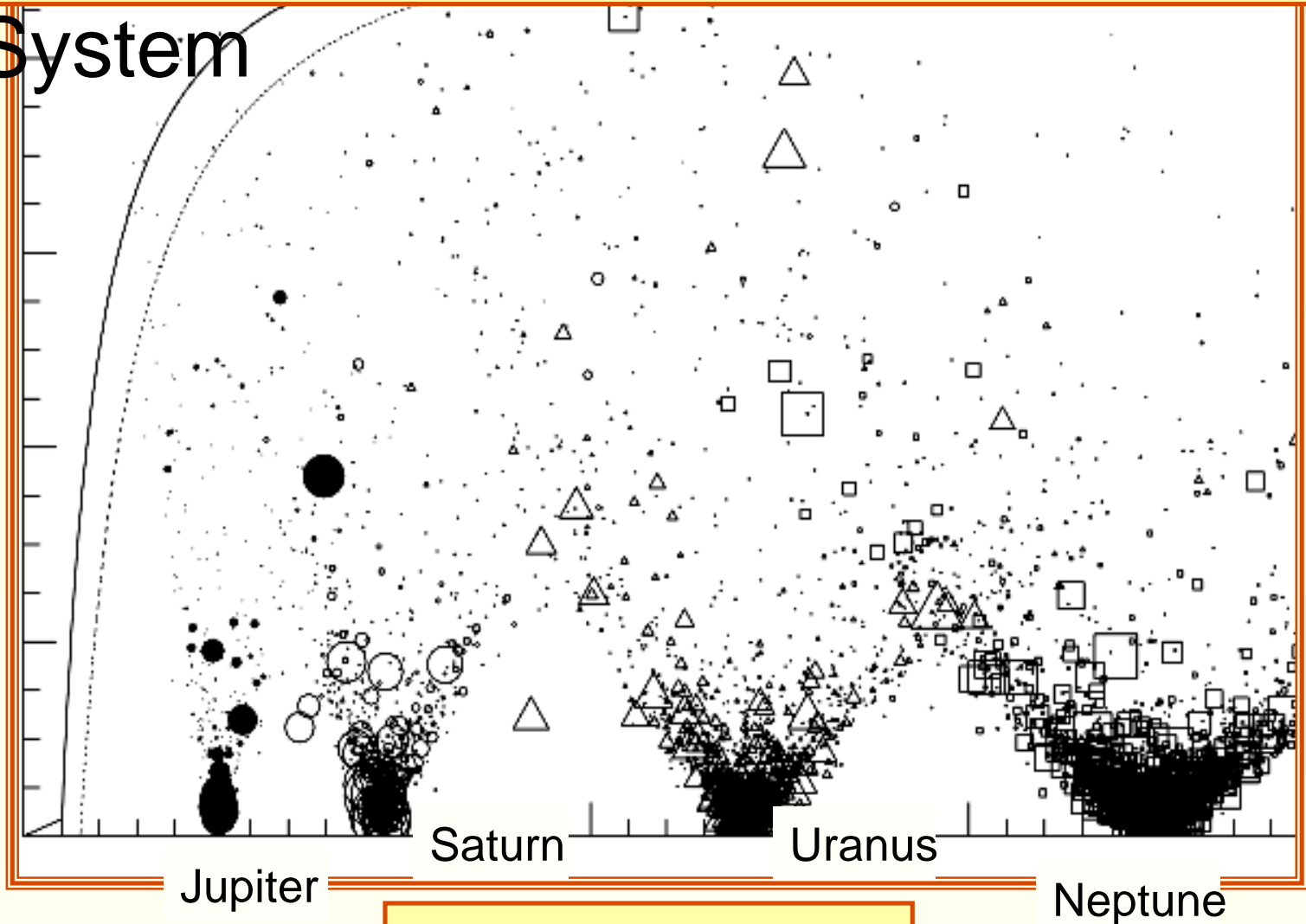
# Red Dwarf captures the Earth



# Scattering Results for our Solar System

System

Eccentricity  $e$



Jupiter

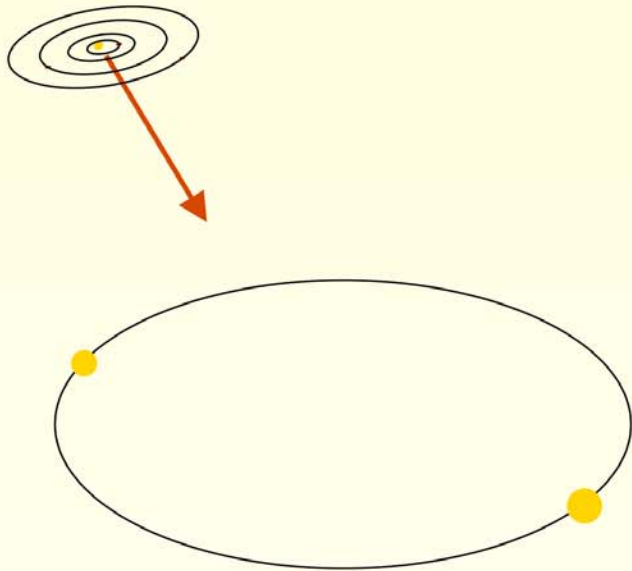
Saturn

Uranus

Neptune

Semi-major axis  $a$

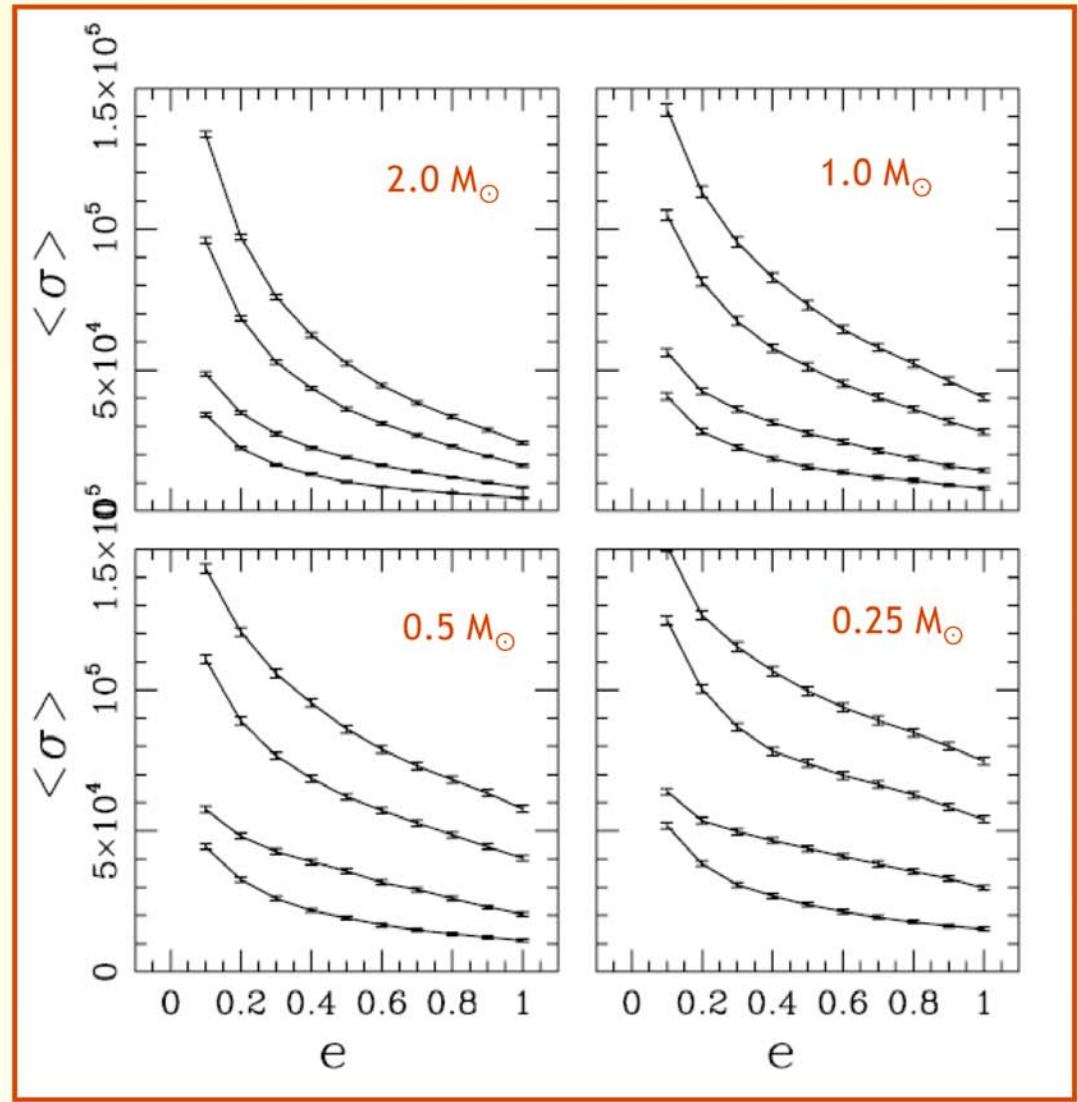
# Cross Sections



$$\langle \sigma \rangle_{ej} = C_0 \left( \frac{a_p}{AU} \right) \left( \frac{M_*}{M_{sun}} \right)^{-1/2}$$

where

$$C_0 = 1350 \pm 160 (AU)^2$$





# Solar System Scattering in Clusters

Ejection Rate per Star  
(for a given mass)

$$\Gamma_{\text{eject}} = \Gamma_0 \left( \frac{C_0 (a_p / \text{AU})}{\pi (1000 \text{ AU})} \right)^{\gamma/2} \left( \frac{M_*}{M_{\odot}} \right)^{-\gamma/4}$$



Integrate over IMF  
(normalized to cluster size)

$$\int dm \left( \frac{dN}{dm} \right) m^{-\gamma/4} \text{ where } N = \int dm \left( \frac{dN}{dm} \right)$$

Subvirial N=300 Cluster

$$\Gamma_0 = 0.096, \gamma = 1.7$$

$$\Gamma_J = 0.15 \text{ per Myr}$$

1-2 Jupiters are  
ejected in 10 Myr

Less than number of  
ejections from internal  
solar system scattering

*(Moorhead & Adams 2005)*

# Conclusions

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- Clusters have moderate effects on star formation:
    - FUV fluxes significantly shorten total disk lifetime (but still allow for Jovian planet formation)
    - Disruption of planetary systems rare,  $b_c \sim 700\text{-}4000$  AU
    - Planet ejection rates via scattering encounters are low
    - All modes of destruction more important for M stars
- 

- Photoevaporation model for external FUV radiation
- Distributions of FUV flux and luminosity
- Distributions of radial positions and closest approaches
- Cross sections for solar system disruption
- [Orbit solutions, triaxial effects, spirographic approx.]



# Bibliography

- *Adams et al. 2007, ApJ, in press*
- *Adams & Bloch 2007, SIAM J. Ap. Math*
- *Adams, Proszkow, Fatuzzo, Myers 2006, ApJ, 641, 504*
- *Adams & Bloch 2005, ApJ, 629, 204*
- *Adams, Hollenbach, Laughlin, Gorti 2004, ApJ, 611, 360*
- *Adams & Myers 2001, ApJ, 553, 744*
- *Adams & Laughlin 2001, Icarus, 150, 151*

# Orbits in Cluster Potentials

$$\rho = \frac{\rho_0}{\xi(1+\xi)^3} \Rightarrow \Psi = \frac{\Psi_0}{1+\xi}$$

$$\varepsilon \equiv |E|/\Psi_0 \quad \text{and} \quad q \equiv j^2/2\Psi_0 r_s^2$$

$$\varepsilon = \frac{\xi_1 + \xi_2 + \xi_1 \xi_2}{(\xi_1 + \xi_2)(1 + \xi_1 + \xi_2 + \xi_1 \xi_2)}$$

$$q = \frac{(\xi_1 \xi_2)^2}{(\xi_1 + \xi_2)(1 + \xi_1 + \xi_2 + \xi_1 \xi_2)}$$


# Orbits (continued)

$$q_{\max} = \frac{1}{8\varepsilon} \frac{(1 + \sqrt{1 + 8\varepsilon} - 4\varepsilon)^3}{(1 + \sqrt{1 + 8\varepsilon})^2} \quad (\text{angular momentum of the circular orbit})$$

$$\xi_* = \frac{1 - 4\varepsilon + \sqrt{1 + 8\varepsilon}}{4\varepsilon} \quad (\text{effective semi-major axis})$$

$$\frac{\Delta\theta}{\pi} = \frac{1}{2} + \left[ (1 + 4\varepsilon)^{-1/4} - \frac{1}{2} \right] \left[ 1 + \frac{\log(q/q_{\max})}{6\log 10} \right]^{3.6}$$

$$\lim_{q \rightarrow q_{\max}} \Delta\theta = \pi(1 + 8\varepsilon)^{-1/4} \quad (\text{circular orbits do not close})$$

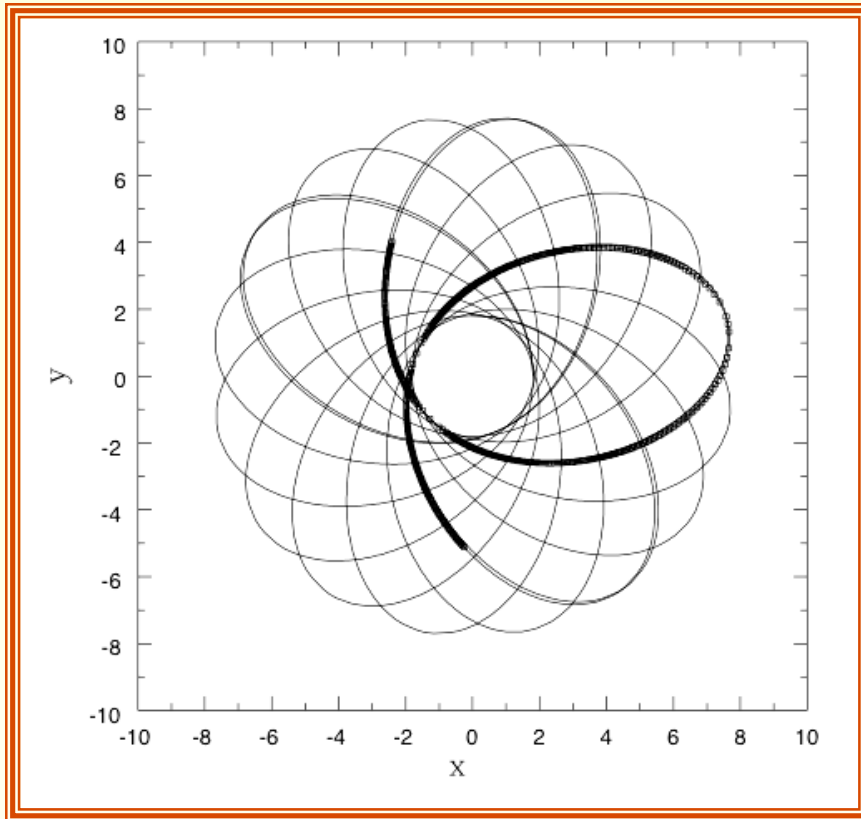


*These results can be used to determine the radiation exposure of a star averaged over its orbit, as a function of energy, where the result is nearly independent of angular momentum:*

$$\langle F_{fuv} \rangle \approx \frac{L_{fuv}}{8r_s^2} \frac{A\epsilon^{3/2}}{\cos^{-1} \sqrt{\epsilon} + \sqrt{\epsilon}\sqrt{1-\epsilon}}$$

*where*  $1 \leq A \leq \sqrt{2}$

# Spirographic Orbits!



## Orbital Elements

$$(\varepsilon, q)$$

$$(\xi_1, \xi_2)$$

$$(\alpha, \beta, \gamma)$$

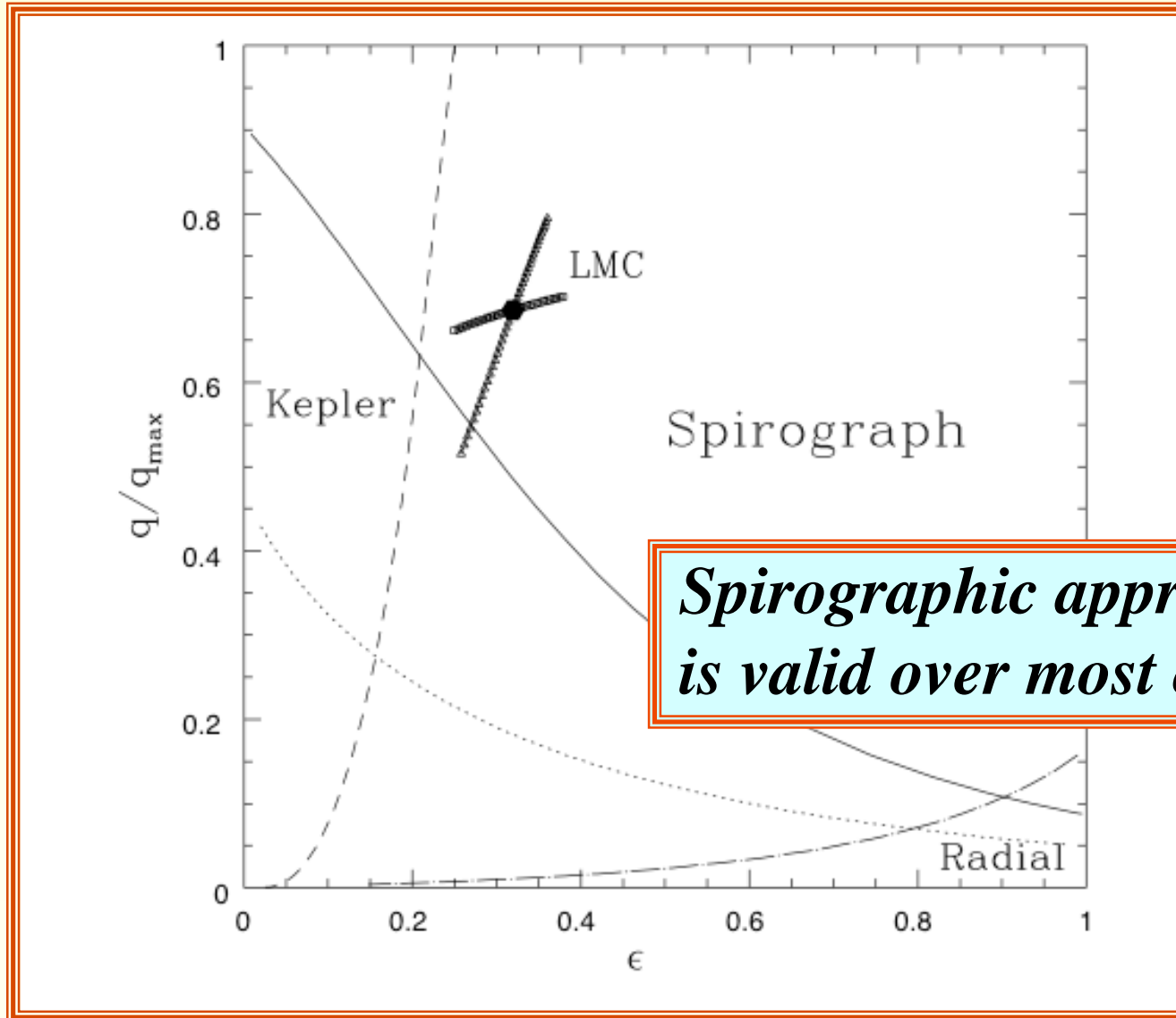
$$x(t_p) = (\alpha - \beta) \cos t_p + \gamma \cos[(\alpha - \beta)t_p / \beta]$$

$$y(t_p) = -(\alpha - \beta) \sin t_p + \gamma \sin[(\alpha - \beta)t_p / \beta]$$

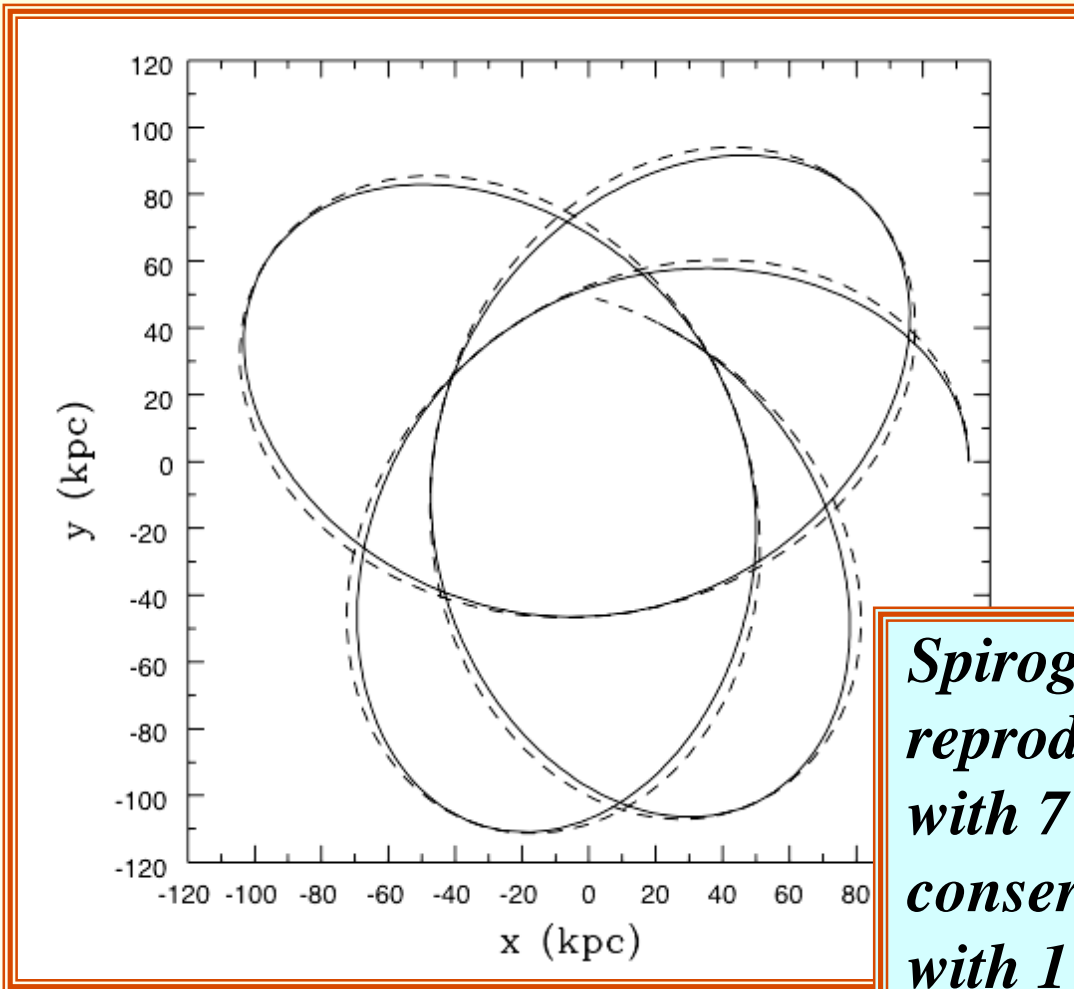
*(Adams & Bloch 2005)*



# Allowed Parameter Space



# Application to LMC Orbit



*Spirographic approximation reproduces the orbital shape with 7 percent accuracy & conserves angular momentum with 1 percent accuracy. Compare with observational uncertainties of 10-20 percent.*

# Triaxial Potential

$$\Phi = \int_0^{\infty} du \frac{\psi(m)}{\sqrt{(u+a^2)(u+b^2)(u+c^2)}} \quad \psi(m) = \int_{\infty}^{m^2} \rho(m) dm^2$$

- In the inner limit the above integral can be simplified to

$$\Phi = -I_1 + I_2$$

where  $I_1$  is the depth of the potential well and the effective potential is given by

$$I_2 = 2 \int_0^{\infty} du \frac{\sqrt{\xi^2 u^2 + \Lambda u + \Gamma}}{(u+a^2)(u+b^2)(u+c^2)}$$

$\xi, \Lambda, \Gamma$  are polynomial functions of  $x, y, z, a, b, c$

# Triaxial Forces

$$F_x = \frac{-2 \operatorname{sgn}(x)}{\sqrt{(a^2 - b^2)(a^2 - c^2)}} \ln \left( \frac{2G(a)\sqrt{\Gamma} + 2\Gamma - a^2\Lambda}{2a^2\xi G(a) + \Lambda a^2 - 2a^4\xi^2} \right)$$

$$F_y = \frac{-2 \operatorname{sgn}(y)}{\sqrt{(a^2 - b^2)(b^2 - c^2)}} \left[ \sin^{-1} \left( \frac{\Lambda - 2b^2\xi^2}{\sqrt{\Lambda^2 - 4\Gamma\xi^2}} \right) - \sin^{-1} \left( \frac{2\Gamma/b^2 - \Lambda}{\sqrt{\Lambda^2 - 4\xi^2\Gamma}} \right) \right]$$

$$F_z = \frac{-2 \operatorname{sgn}(z)}{\sqrt{(a^2 - c^2)(b^2 - c^2)}} \ln \left( \frac{2G(c)\sqrt{\Gamma} + 2\Gamma - c^2\Lambda}{2c^2\xi G(c) + \Lambda c^2 - 2c^4\xi^2} \right)$$

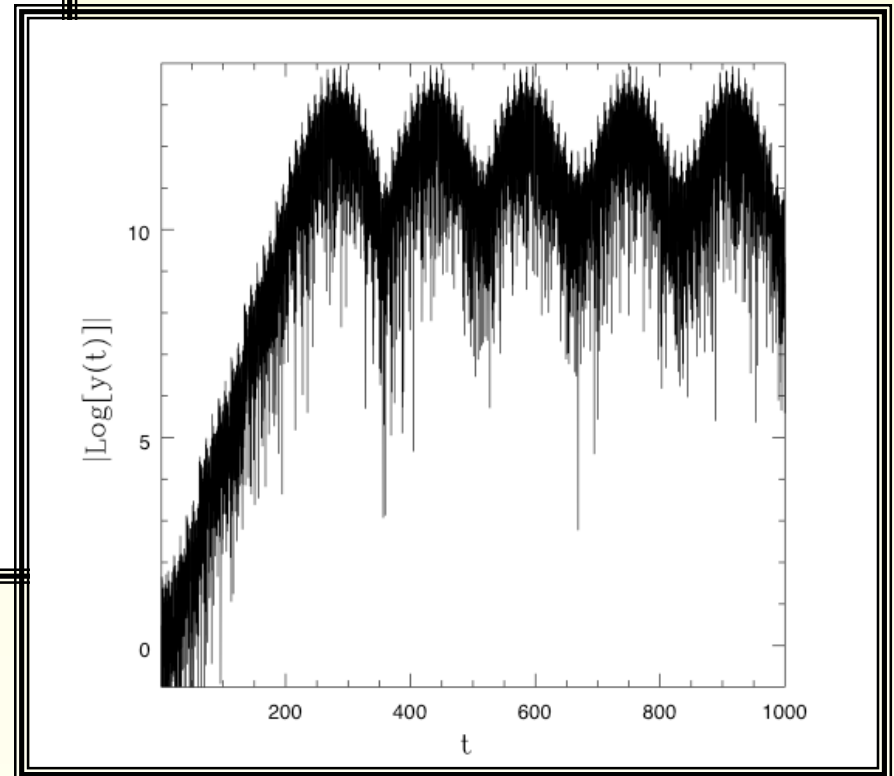
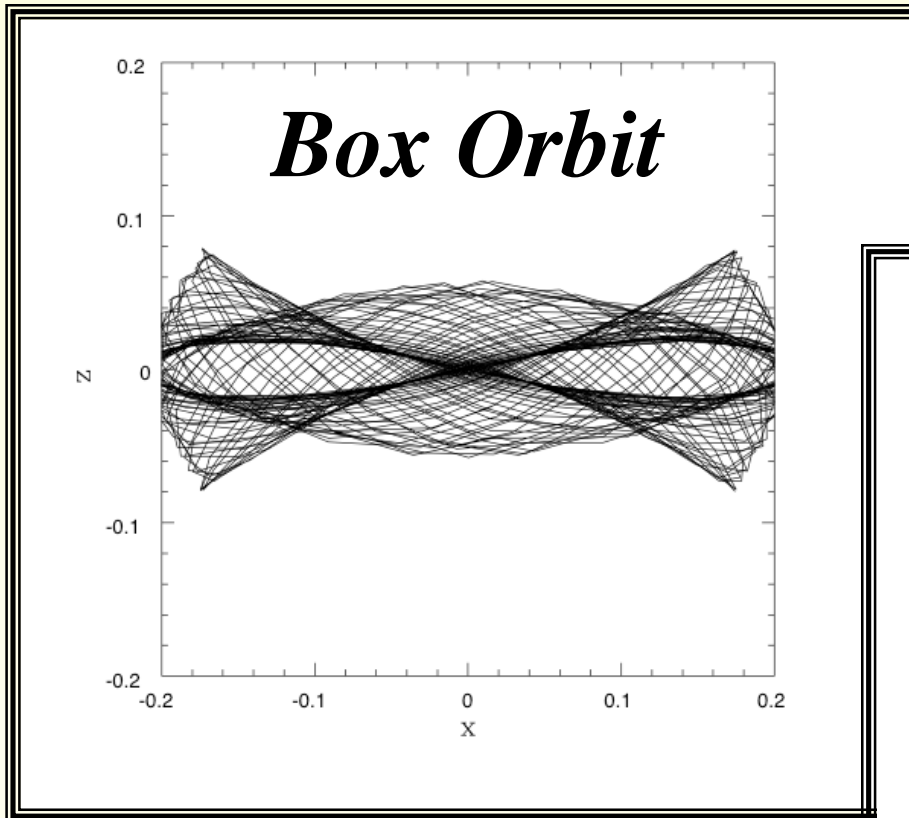
$$G(u) = \xi^2 u^4 - \Lambda u^2 + \Gamma$$

$$\xi^2 = x^2 + y^2 + z^2$$

$$\Lambda = (b^2 + c^2)x^2 + (a^2 + c^2)y^2 + (a^2 + b^2)z^2$$

$$\Gamma = b^2c^2x^2 + a^2c^2y^2 + a^2b^2z^2$$

# Triaxial Potentials in Clusters



*Growth of perpendicular coordinate*

# Where did we come from?



# Solar Birth Aggregate

Supernova  
enrichment

requires large  $N$

$$M_* > 25 M_\odot$$

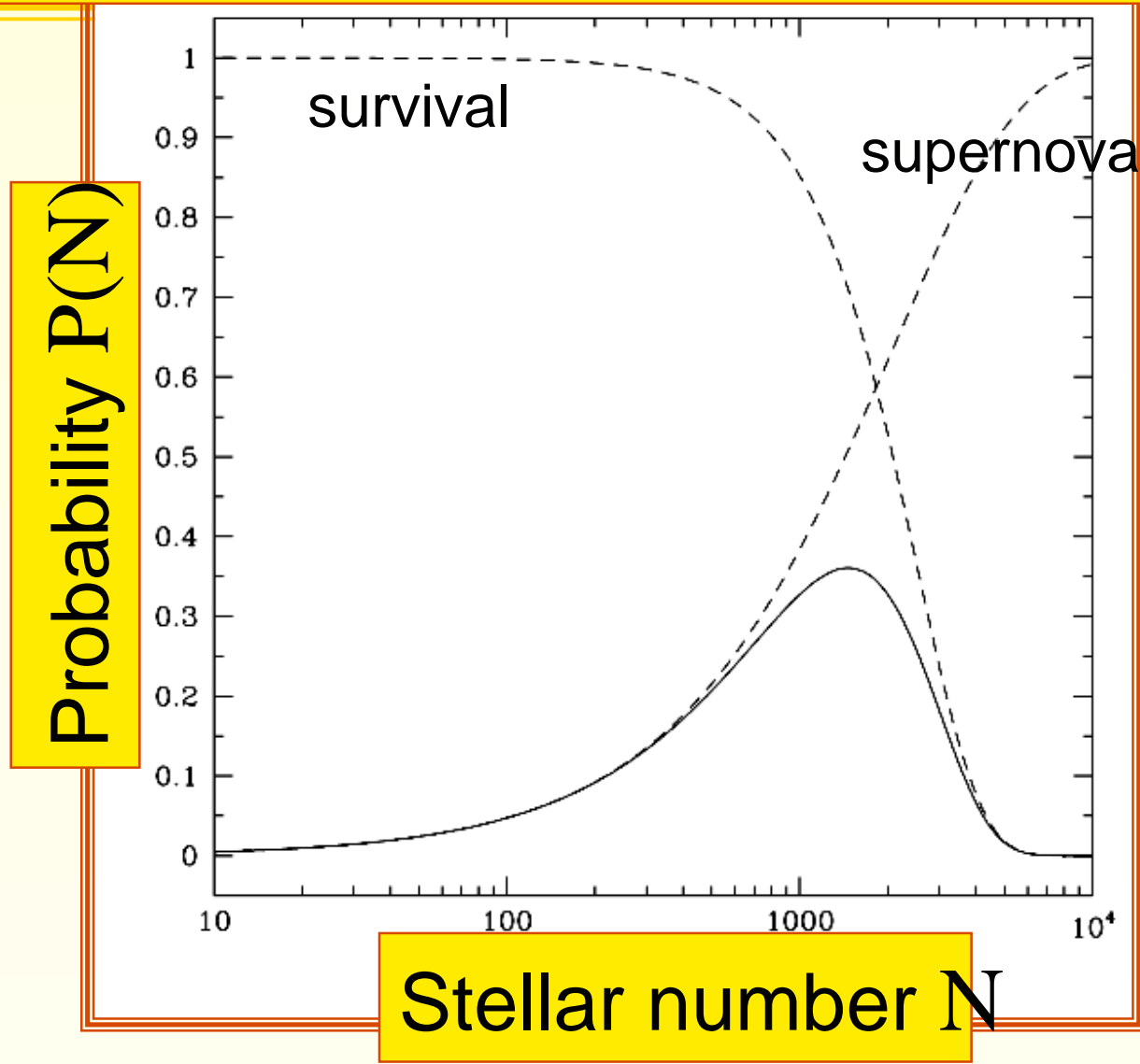
$$F_{SN} = 0.000485$$

Well ordered solar system  
requires small  $N$

$$\varepsilon(\textit{Neptune}) < 0.1$$

$$\Delta\Theta_j < 3.5^\circ$$

# Expected Size of the Stellar Birth Aggregate





# Constraints on the Solar Birth Aggregate

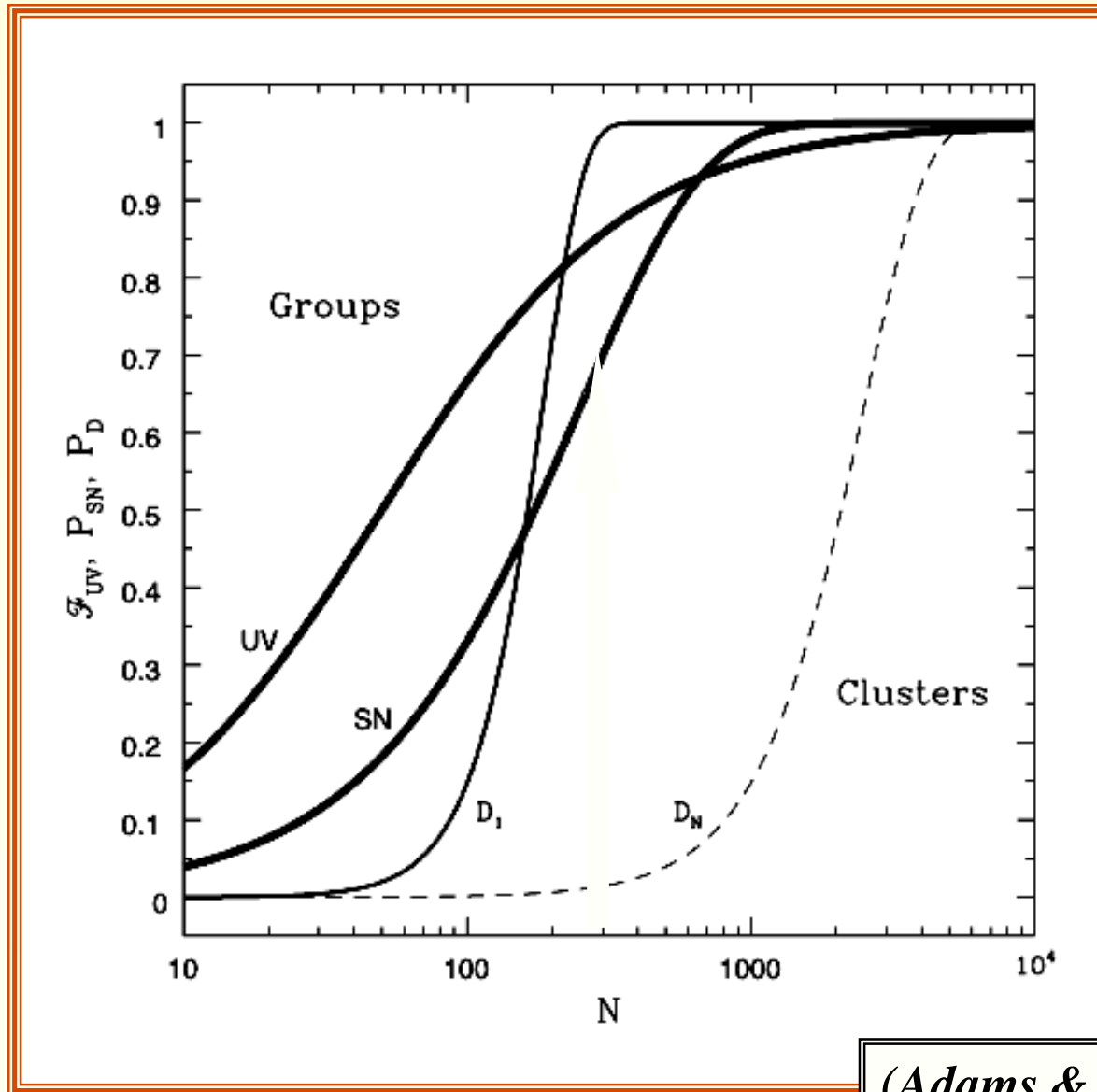
$$\langle N \rangle \approx 2000 \pm 1100$$

$$P \approx 0.017 \quad (1 \text{ out of } 60)$$

*(Adams & Laughlin 2001 - updated)*

# Probability as function of system size

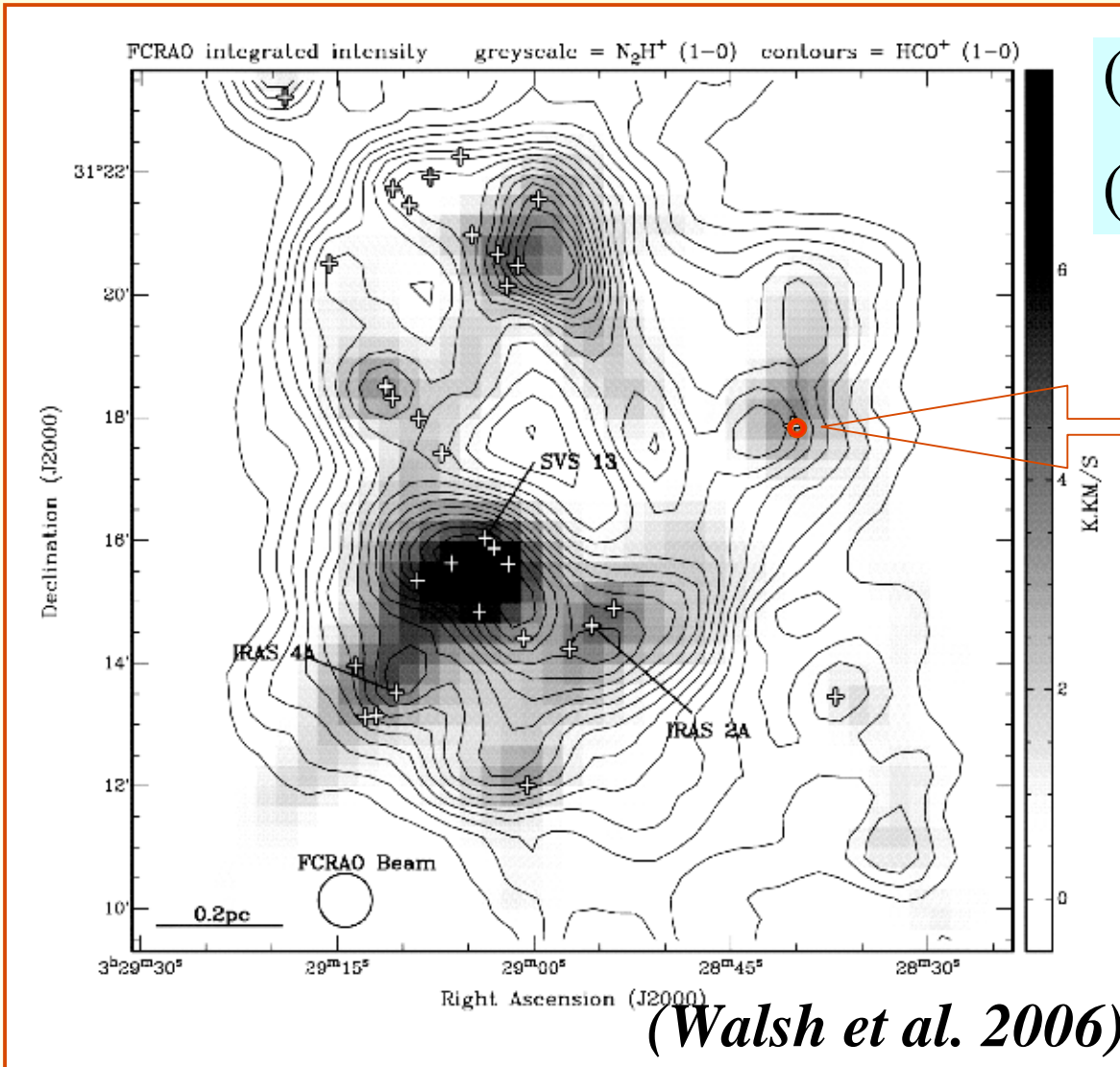
N



(Adams & Myers

2001)

# NGC 1333 - cold start



$$(\Delta v)_Z \approx 0.1 \text{ km/s}$$

$$(\Delta v)_T (\Delta t)_{SF} \approx 0.02 \text{ pc}$$

