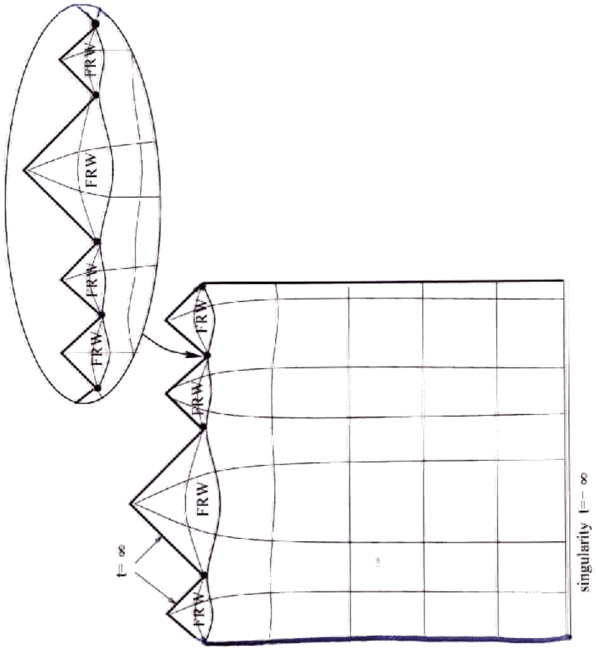
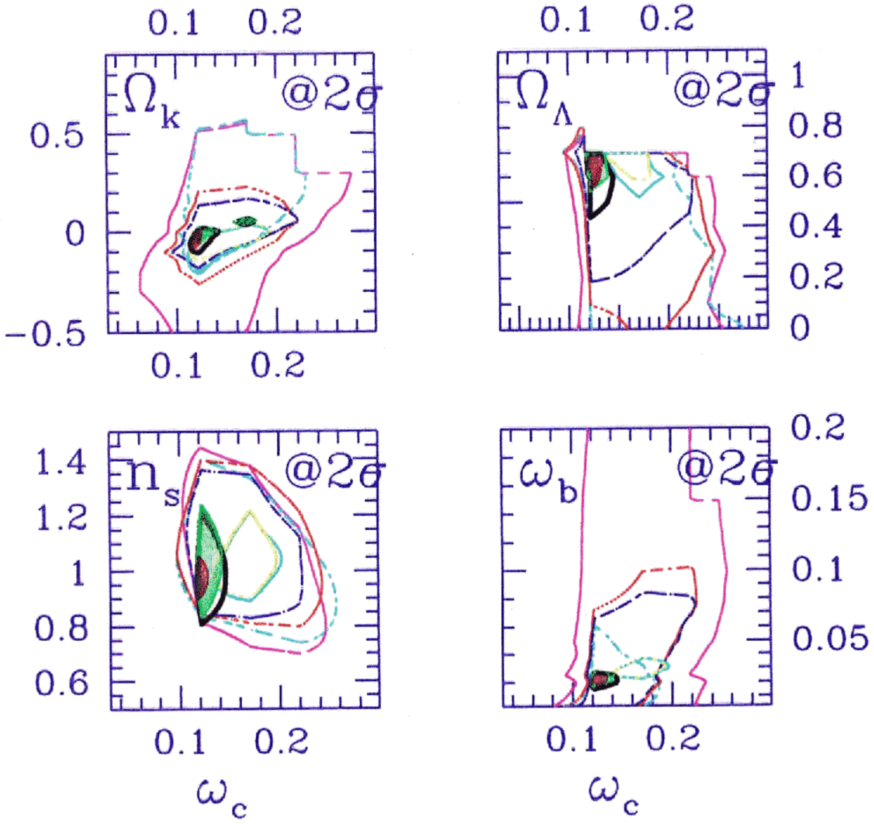




$\omega_a = \Omega_a h^2$

COSMOLOGY
AND
EXTRA DIMENSIONS

COSMOLOGICAL OBSERVABLES

- Bad News So far no models which make distinct predictions
- Good News Many of those models

# COSMOLOGICAL PERTURBATIONS
D = 3 + 1

$$ds^2 = dt^2 - a^2(t)\, d\vec{x}^2$$

$\delta g_{\mu\nu} =$ ϕ, ψ scalar (adiabatical) curvature

E_μ vector

$h_{\mu\nu}$ tensor

$\rho_x << \rho_{tot}$

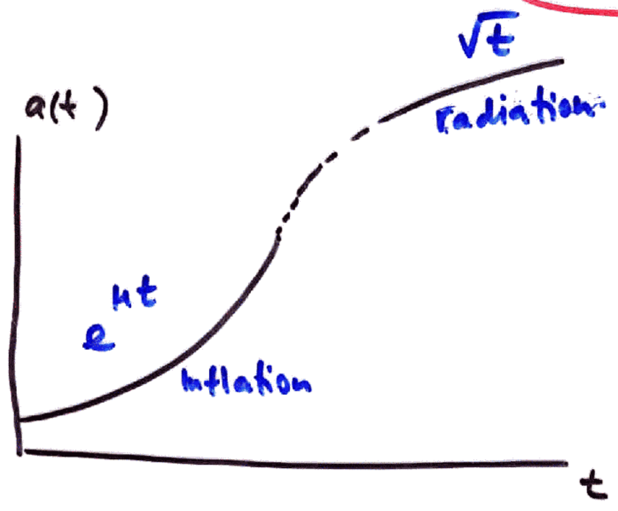
$\frac{\delta\rho_x}{\rho_x}$ isocurvature (entropy)

Inflation predicts

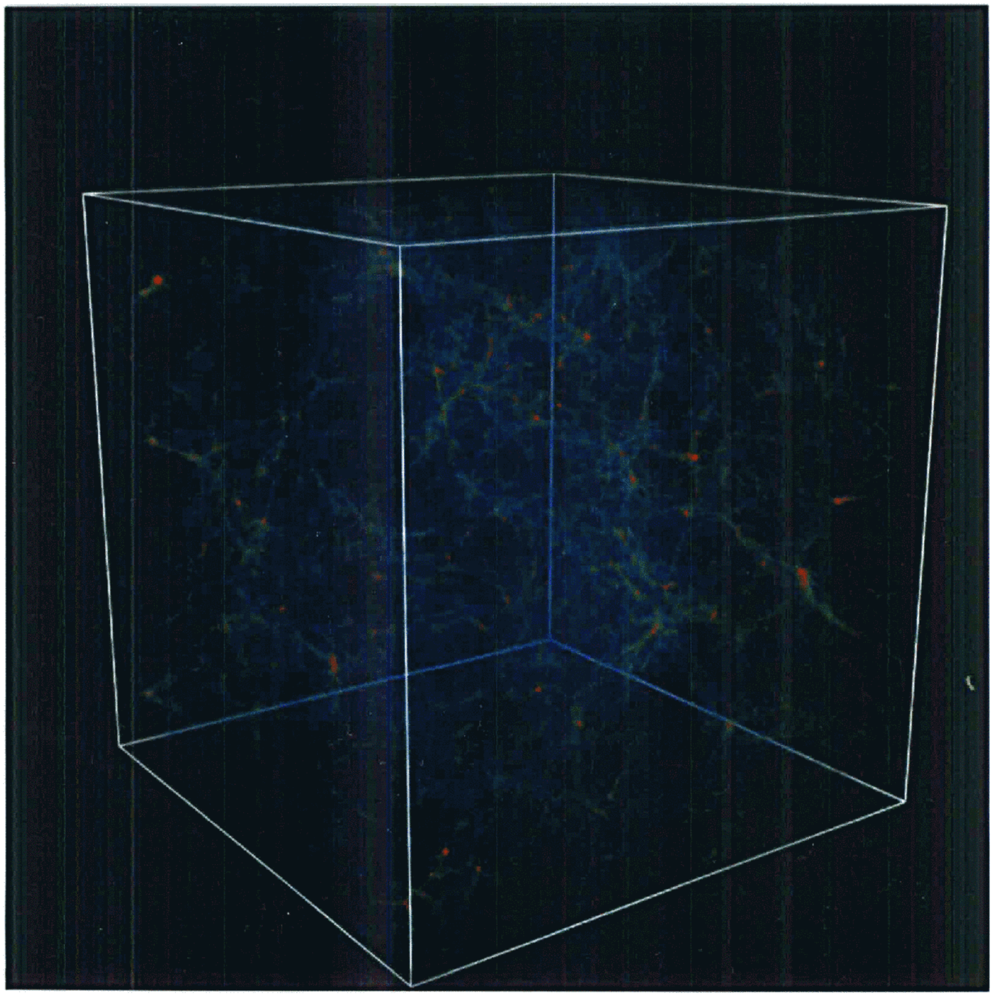
- $\Omega_{tot} = 1$
- no vector perturbations $E_\mu = 0$
- scale-free $n \neq 1$ gaussian scalar fluctuations ϕ
- scale-free gaussian gravitational waves $h_{\mu\nu}$
- fluctuations of all light scalars χ_a
- creation of all particles in process of preheating + thermalization

COSMIC INFLATION

Equation of State $\boxed{p \approx -\varepsilon}$



$p \approx 0$



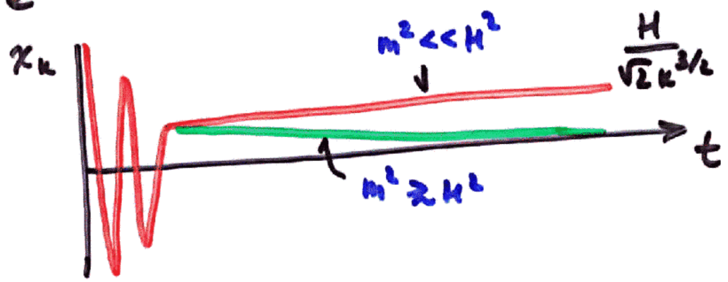
Light field at Inflation

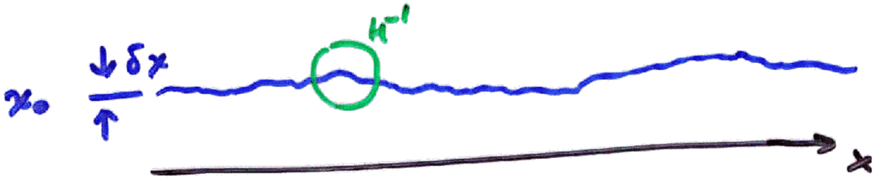
$$\nabla_\mu \nabla^\mu \chi + m^2 \chi = 0$$

$$\chi = \int d^3k\, \chi_k(t) e^{i\vec{k}\vec{x}} + c.c.$$

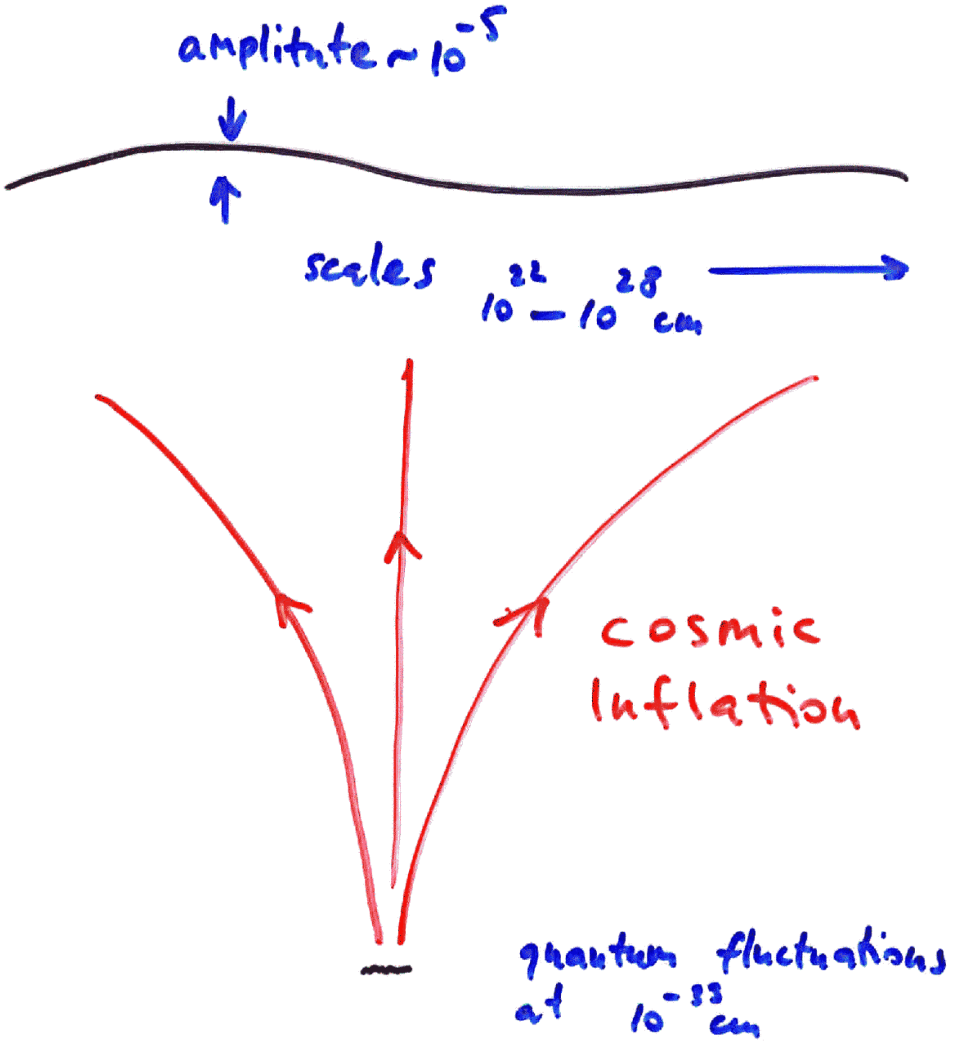
$$\boxed{\ddot{\chi}_k + 3\frac{\dot{a}}{a}\dot{\chi}_k + \left(\frac{k^2}{a^2} + m^2\right)\chi_k = 0}$$

$a(t) = e^{Ht}$





UNIVERSAL AMPLIFIER & STRETCHER



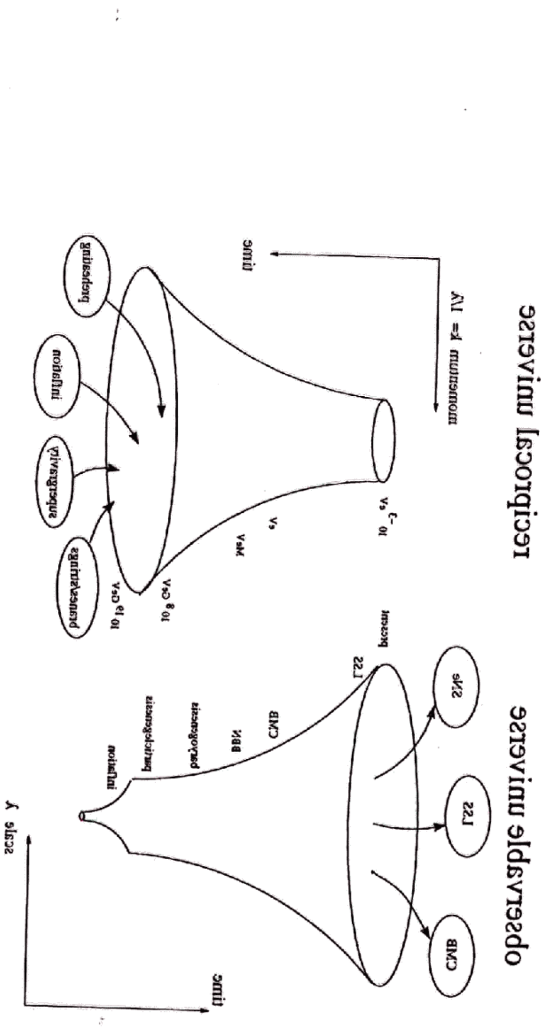
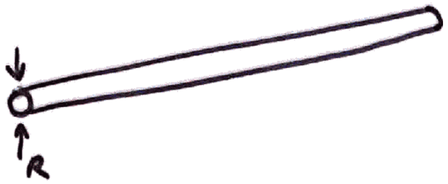
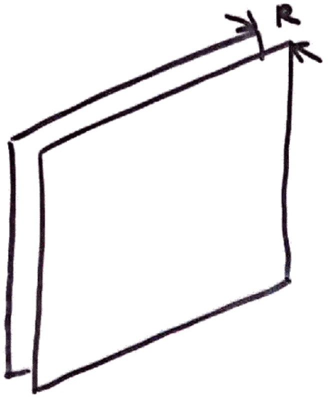


Fig. 1. Sketch of an expanding universe where the wavelengths λ(t) are proportional to a scale factor a(t) and "reciprocal" universe where the momenta k(t) ∼ 1/λ(t) are inversely proportional to a(t).

COMPACTIFICATION OF EXRA DIMENSIONS





$$\ell_D^{D-2} = V_{D-4}\,\ell_4^2$$

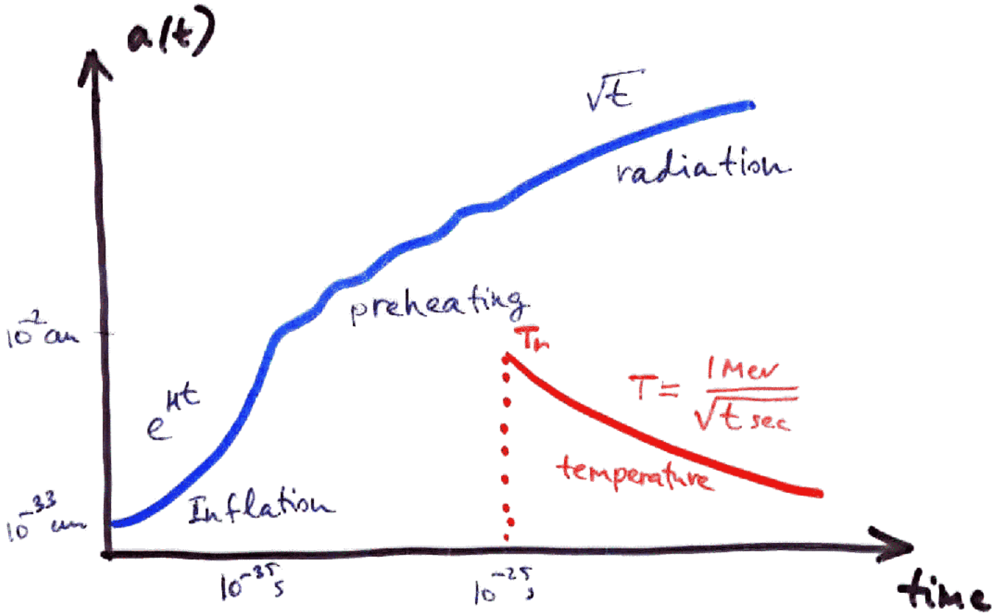
$$M_p^2 = M_s^{D-2}\,V_{D-4}$$

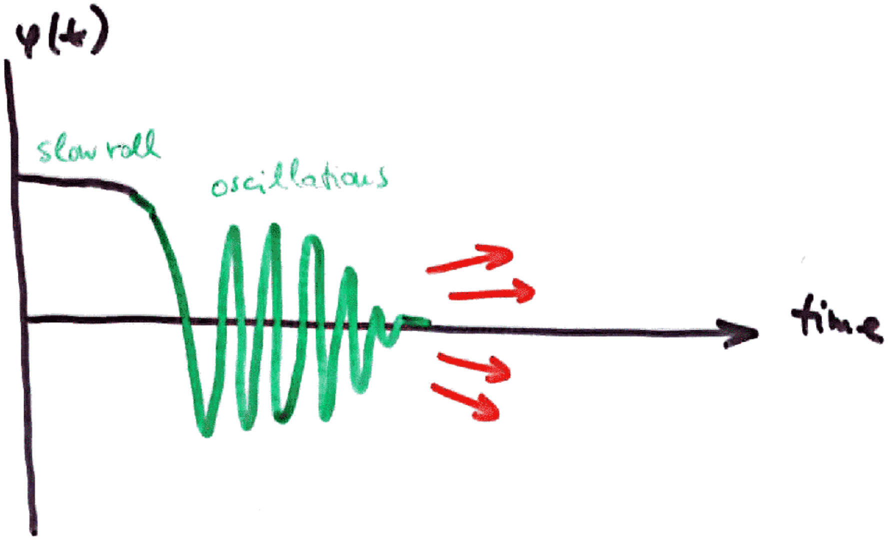
$$\ell_4^2 = G_4$$

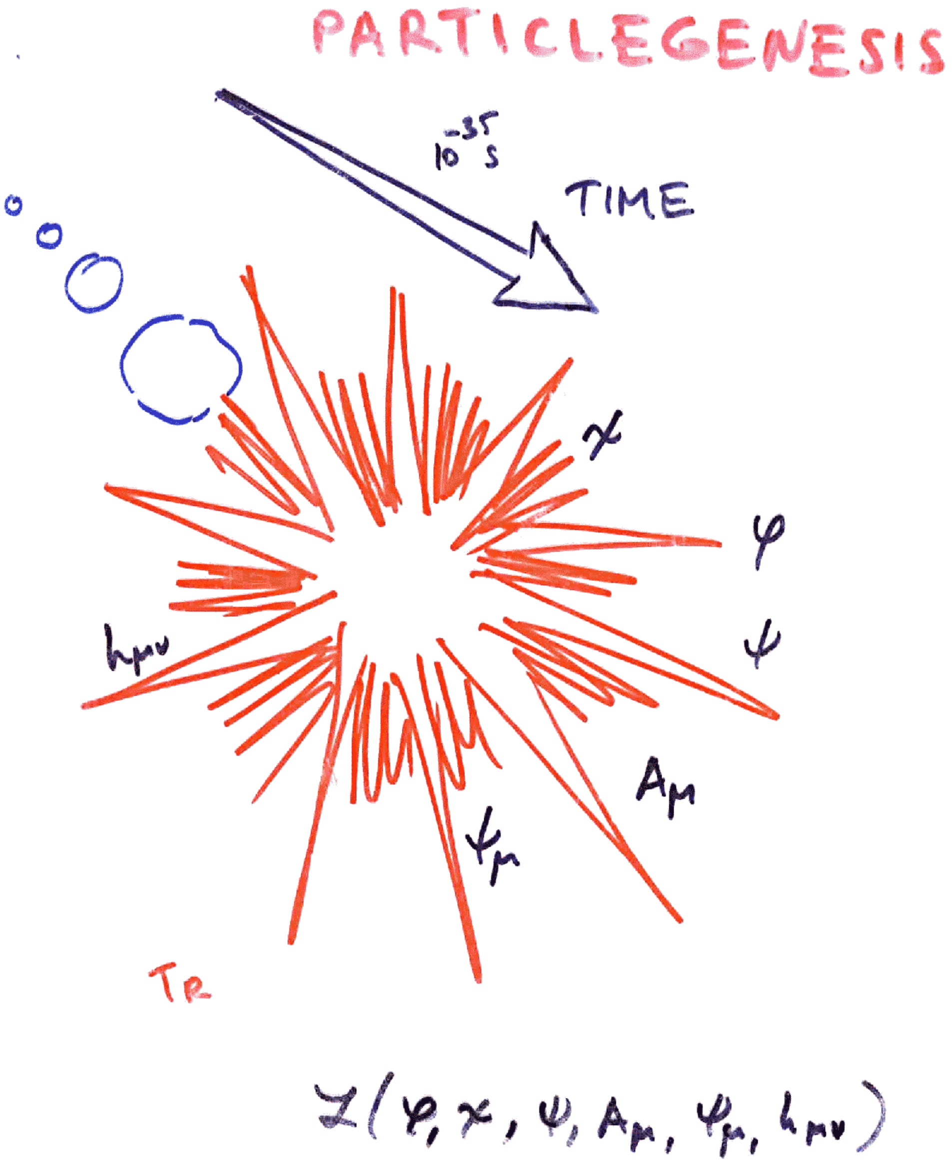
- Dimensional reduction gives us scalars (if light) candidates to be produced from inflation
- Coupling constants are moduli dependent

$$\alpha = \alpha(\chi_a)$$

$$\delta\alpha(\vec x) = \frac{\partial\alpha}{\partial\chi}\,\delta\chi(\vec x)$$


a(t)
$\sqrt{t}$
radiation
preheating
T_r
$T = \frac{1\,\mathrm{Mev}}{\sqrt{t\,\mathrm{sec}}}$
temperature
e^{Ht}
Inflation
10^{-2} cm
10^{-33} cm
10^{-35} s
10^{-25} s
time


φ(t)
slow roll
oscillations
time


PARTICLEGENESIS
10^{-35} s
TIME
χ
φ
ψ
$h_{\mu\nu}$
A_μ
ψ_μ
T_R
$\mathcal{L}(\varphi, \chi, \psi, A_\mu, \psi_\mu, h_{\mu\nu})$

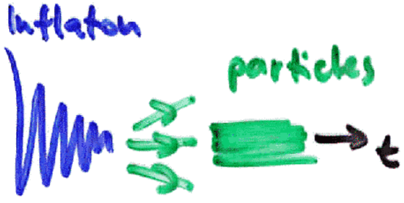
General YM-matter-supergravity theory with
$N=1$ local SUSY

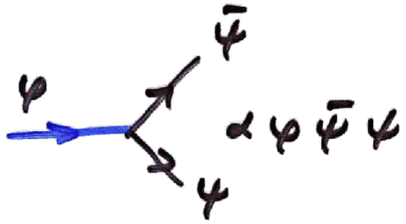
$$\begin{aligned}
e^{-1}\mathcal{L} \;=\;& -\tfrac{1}{2}M_P^2\left[R+\bar{\psi}_\mu R^\mu+\mathcal{L}_{SG,torsion}\right]-M_P^2 g_i{}^j\left[(\hat{\partial}_\mu z^i)(\hat{\partial}^\mu z_j)+\bar{\chi}_j \not{\mathcal{D}}\chi^i+\bar{\chi}^i \not{\mathcal{D}}\chi_j\right]\\
&+ (\mathrm{Re}\ f_{\alpha\beta})\left[-\tfrac{1}{4}F_{\mu\nu}^{\alpha}F^{\mu\nu\,\beta}-\tfrac{1}{2}\bar{\lambda}^\alpha \hat{\not{\mathcal{D}}}\lambda^\beta\right]+\tfrac{1}{4}\mathrm{i}(\mathrm{Im}\ f_{\alpha\beta})\left[F_{\mu\nu}^{\alpha}\tilde{F}^{\mu\nu\,\beta}-\hat{\partial}_\mu\left(\bar{\lambda}^\alpha\gamma_5\gamma^\mu\lambda^\beta\right)\right]\\
&- M_P^{-2}e^K\left[-3WW^*+(\mathcal{D}^iW)g^{-1}{}_i{}^j(\mathcal{D}_jW)\right]-\tfrac{1}{2}(\mathrm{Re}\ f)^{-1\,\alpha\beta}\mathcal{P}_\alpha\mathcal{P}_\beta\\
&+ \tfrac{1}{8}(\mathrm{Re}\ f_{\alpha\beta})\bar{\psi}_\mu\gamma^{\nu\rho}\left(F_{\nu\rho}^{\alpha}+\hat{F}_{\nu\rho}^{\alpha}\right)\gamma^\mu\lambda^\beta\\
&+ \Big\{M_P^2 g_j{}^i\bar{\psi}_{\mu L}(\hat{\not{\partial}}z^j)\gamma^\mu\chi_i-\tfrac{1}{4}f_{\alpha\beta}^i\bar{\chi}_i\gamma^{\mu\nu}\hat{F}_{\mu\nu}^{-\alpha}\lambda_L^\beta\\
&\quad +\tfrac{1}{2}e^{K/2}W\bar{\psi}_{\mu R}\gamma^{\mu\nu}\psi_{\nu R}+\bar{\psi}_R\cdot\gamma\left[\tfrac{1}{2}\mathrm{i}\lambda_L^\alpha\mathcal{P}_\alpha+\chi_i e^{K/2}\mathcal{D}^iW\right]\\
&\quad -e^{K/2}(\mathcal{D}^i\mathcal{D}^jW)\bar{\chi}_i\chi_j+\tfrac{1}{2}\mathrm{i}(\mathrm{Re}\ f)^{-1\,\alpha\beta}\mathcal{P}_\alpha f_{\beta\gamma}^i\bar{\chi}_i\lambda^\gamma-2M_P^2\xi_\alpha{}^i g_i{}^j\bar{\lambda}^\alpha\chi_j\\
&\quad +\tfrac{1}{4}M_P^{-2}e^{K/2}(\mathcal{D}^jW)g^{-1}{}_j{}^i f_{\alpha\beta i}\bar{\lambda}_R^\alpha\lambda_R^\beta\\
&\quad -\tfrac{1}{4}f_{\alpha\beta}^i\bar{\psi}_R\cdot\gamma\chi_i\bar{\lambda}_L^\alpha\lambda_L^\beta+\tfrac{1}{4}(\mathcal{D}^i\partial^j f_{\alpha\beta})\bar{\chi}_i\chi_j\bar{\lambda}_L^\alpha\lambda_L^\beta+h.c.\Big\}\\
&+ M_P^2 g_j{}^i\left(\tfrac{1}{8}e^{-1}\varepsilon^{\mu\nu\rho\sigma}\bar{\psi}_\mu\gamma_\nu\psi_\rho\bar{\chi}^j\gamma_\sigma\chi_i-\bar{\psi}_\mu\chi^j\bar{\psi}^\mu\chi_i\right)\\
&+ M_P^2\left(R_{ij}^{k\ell}-\tfrac{1}{2}g_i{}^k g_j{}^\ell\right)\bar{\chi}^i\chi^j\bar{\chi}_k\chi_\ell\\
&+ \tfrac{3}{64}M_P^{-2}\left((\mathrm{Re}\ f_{\alpha\beta})\bar{\lambda}^\alpha\gamma_\mu\gamma_5\lambda^\beta\right)^2-\tfrac{1}{16}M_P^{-2}f_{\alpha\beta}^i\bar{\lambda}_L^\alpha\lambda_L^\beta g^{-1}{}_i{}^j f_{\gamma\delta j}\bar{\lambda}_R^\gamma\lambda_R^\delta\\
&+ \tfrac{1}{8}(\mathrm{Re}\ f)^{-1\,\alpha\beta}\left(f_{\alpha\gamma}^i\bar{\chi}_i\lambda^\gamma-f_{\alpha\gamma i}\bar{\chi}^i\lambda^\gamma\right)\left(f_{\beta\delta}^j\bar{\chi}_j\lambda^\delta-f_{\beta\delta j}\bar{\chi}^j\lambda^\delta\right).
\end{aligned}$$

MODULATED COSMOLOGICAL FLUCTUATIONS

L.K. astro-ph/0303614
cosmo02

Dvali et al astro-ph/0303591





$$\Gamma(\varphi\to\bar{\psi}\psi)\simeq\frac{\alpha^2 m_\varphi}{8\pi}$$

$$T_R\simeq\sqrt{\Gamma M_p}$$

$$\rho_r\simeq\frac{M_p^2}{t^2}\simeq\Gamma^2M_p^2$$

$\alpha=\dfrac{\chi}{M}$ coupling constant spatial variation

$$\frac{\delta\rho_r}{\rho_r}=\frac{\delta\chi_k}{M}\simeq\frac{H}{M}k^{-3/2}$$

Modulated fluctuations $\delta\Gamma$ generate scalar metric perturbations

Simple model

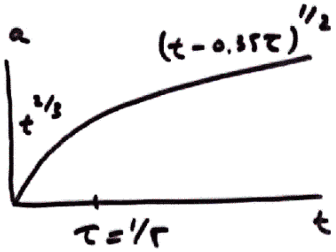
inflaton oscillations - pressureless fluid

$$\varepsilon_m = \frac{\varepsilon_{m0}}{a^3} e^{-\Gamma t}$$

products of decay - relativistic fluid

$$\varepsilon_r = \frac{\Gamma \varepsilon_{r0}}{a^4} \int^t dt' a e^{-\Gamma t'}$$

$$\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a}\right)^2 = \frac{4\pi G}{3} \frac{\varepsilon_{m0}}{a^3} e^{-\Gamma t}$$



Perturbations of metric ϕ, δ_r, δ_m, $\delta\Gamma$

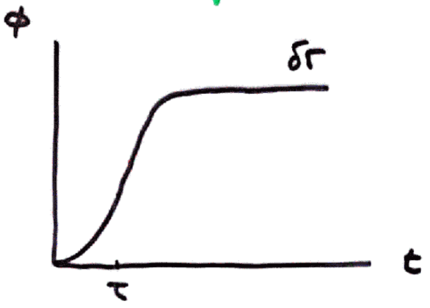
$$T_m{}^\mu{}_{\nu;\mu} = -\Gamma \varepsilon_m u_{m;\nu} \qquad T_r{}^\mu{}_{\nu;\mu} = \Gamma \varepsilon_m u_{m;\nu}$$

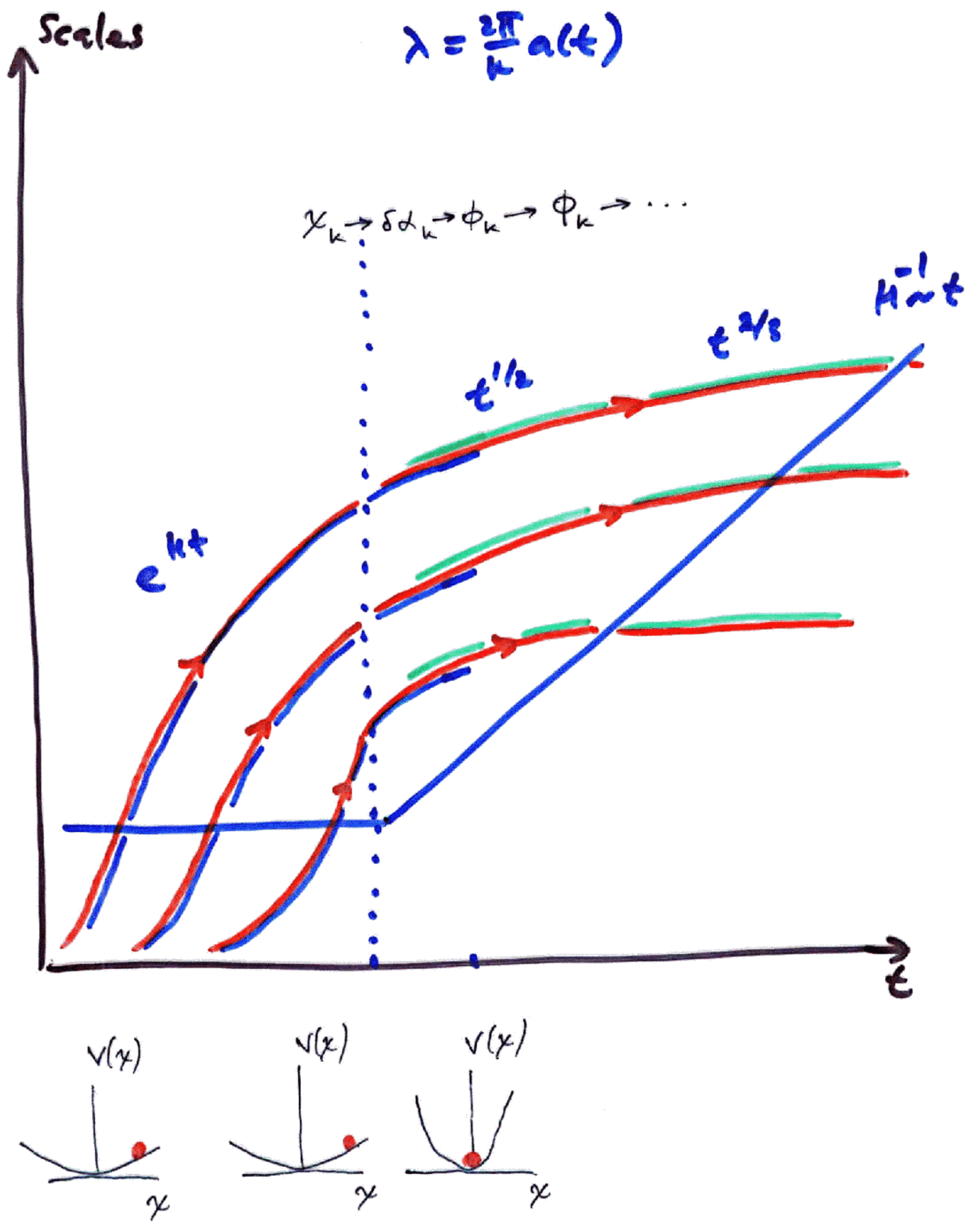
$k \to 0$

$$H\dot{\phi} + H^2\phi = -\frac{4\pi G}{3}(\varepsilon_m \delta_m + \varepsilon_r \delta_r)$$

$$\dot{\delta}_m = 3\dot{\phi} - \delta\Gamma - \Gamma\phi$$

$$\dot{\delta}_r = 4\dot{\phi} + \frac{\varepsilon_m}{\varepsilon_r}(\delta\Gamma + \Gamma\phi + \Gamma\delta_m - \Gamma\delta_r)$$



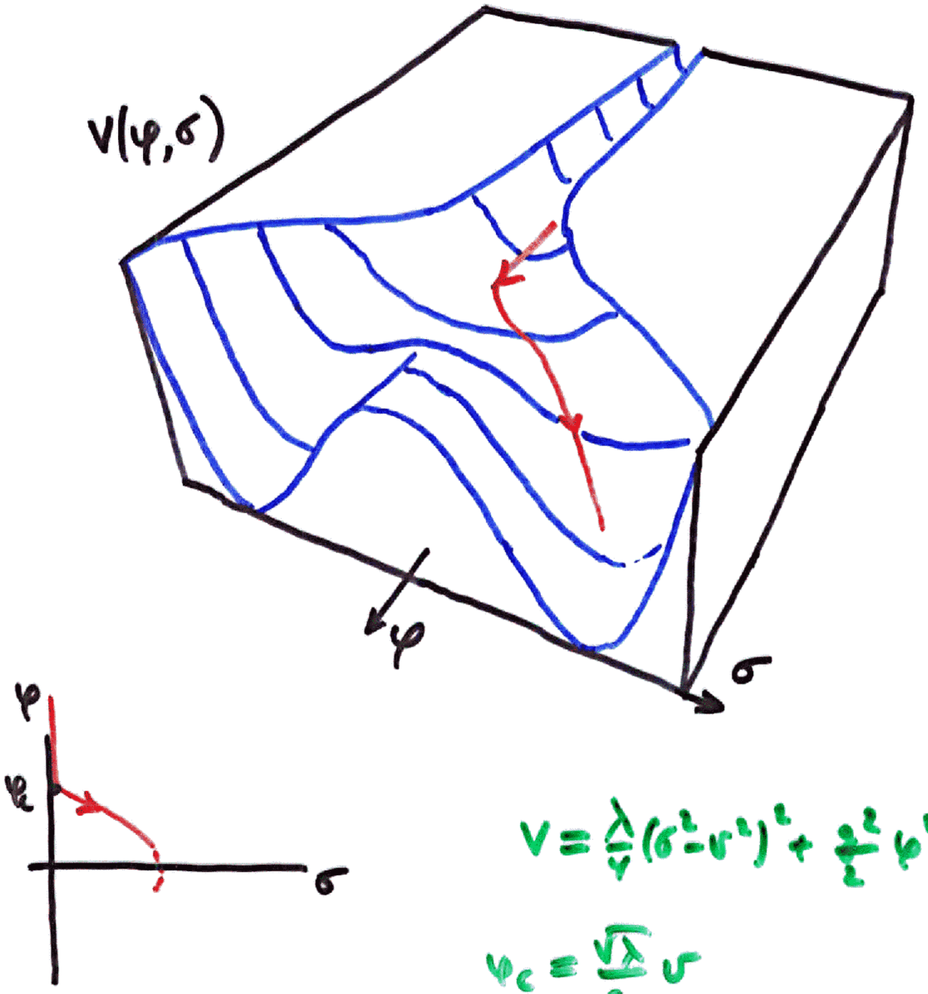


TACHYONIC PREHEATING and Dynamics of Symmetry breaking

hep-ph/0012142

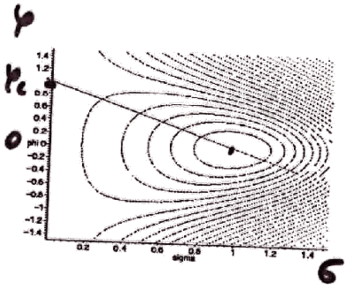
hep-ph/0106179

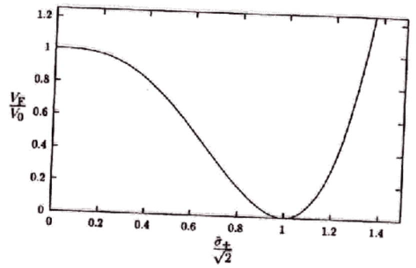
Hybrid Inflation



$$V = \frac{\lambda}{4}(\sigma^2 - v^2)^2 + \frac{g^2}{2}\varphi^2\sigma^2$$

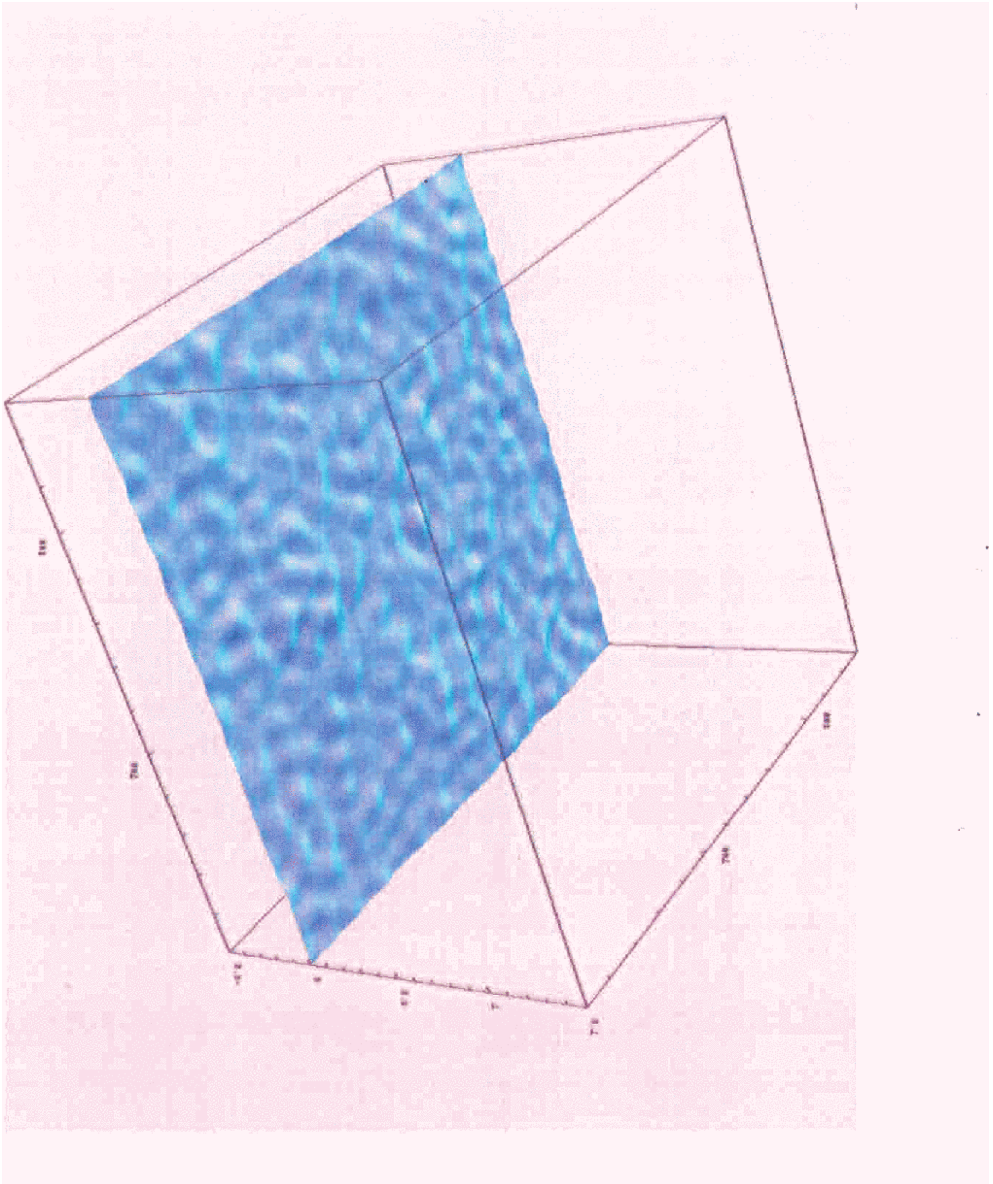
$$\varphi_c = \frac{\sqrt{\lambda}}{g} v$$

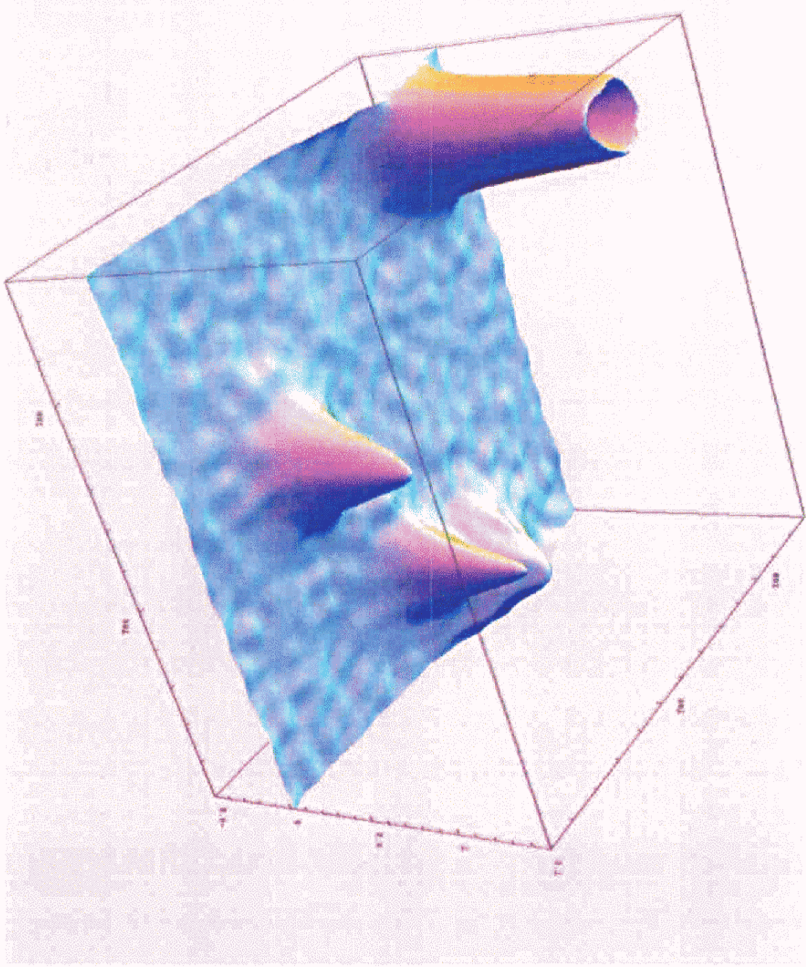


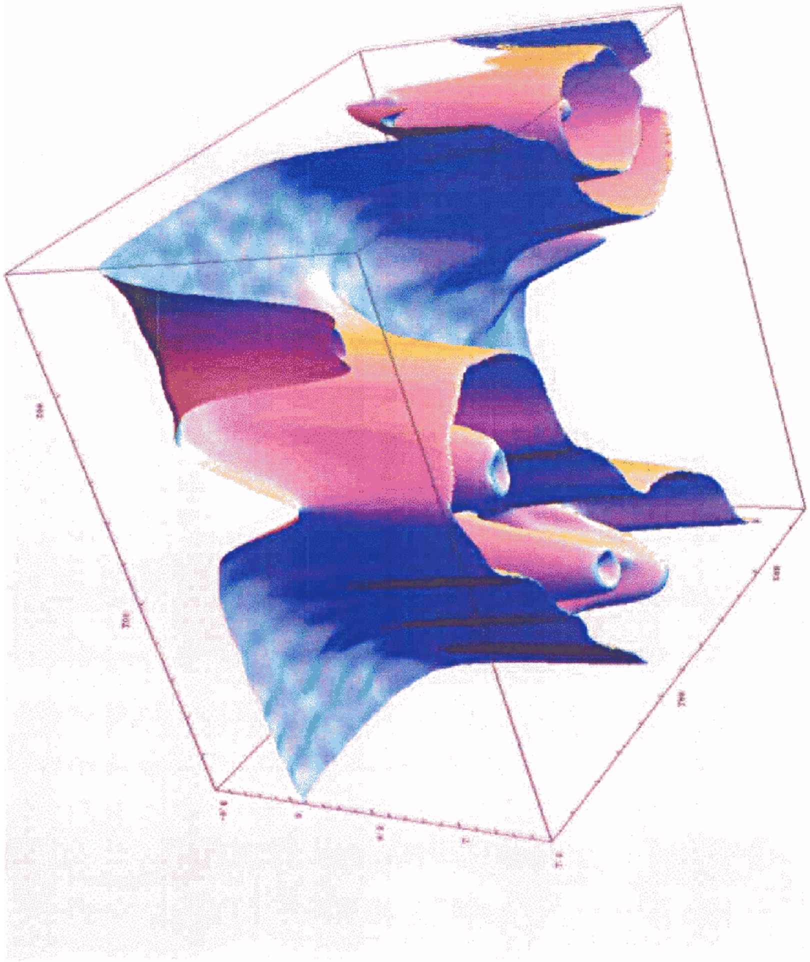


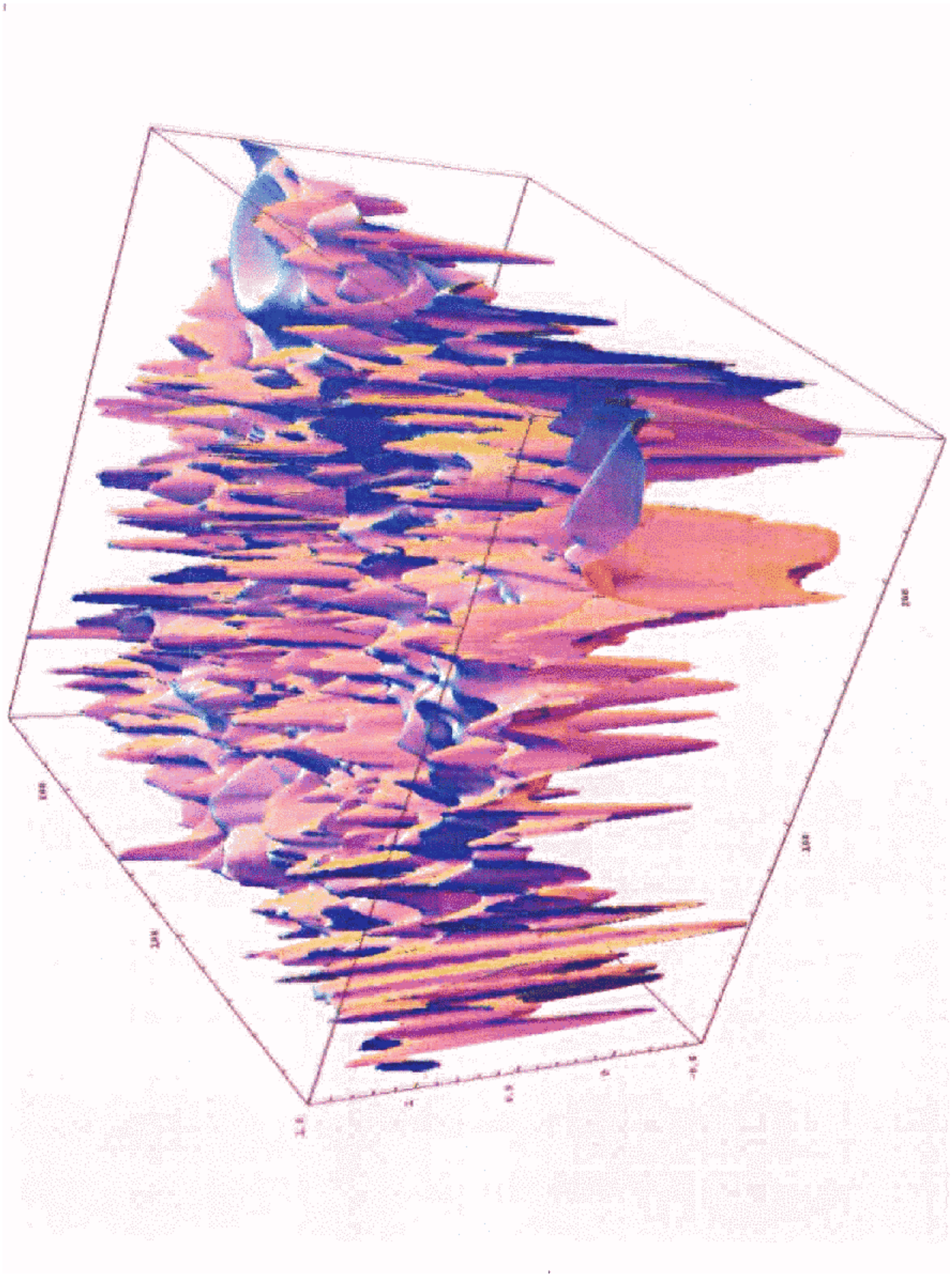
$$\varphi(t) + \frac{1}{\sqrt{2}}\sigma_+(t) = \varphi_c$$

$$V_F = \frac{\lambda v^4}{4} - \frac{\lambda}{4}\sigma_+^3 + \frac{\lambda}{4}\sigma_+^4 + \lambda\sigma_+^2$$

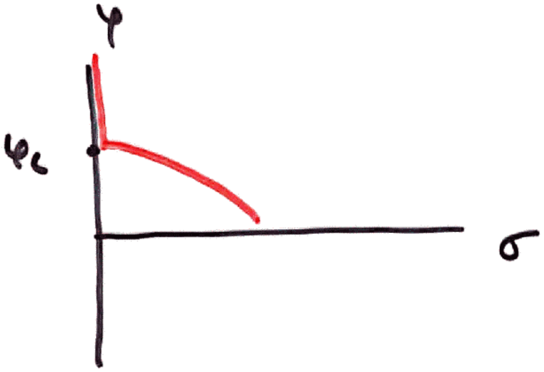








Modulated fluctuations in hybrid inflation



$$\varphi_c = \frac{\sqrt{\lambda}}{g}\, v$$

$$\lambda = \lambda(\chi_a)$$
$$S = g(\chi_a)$$

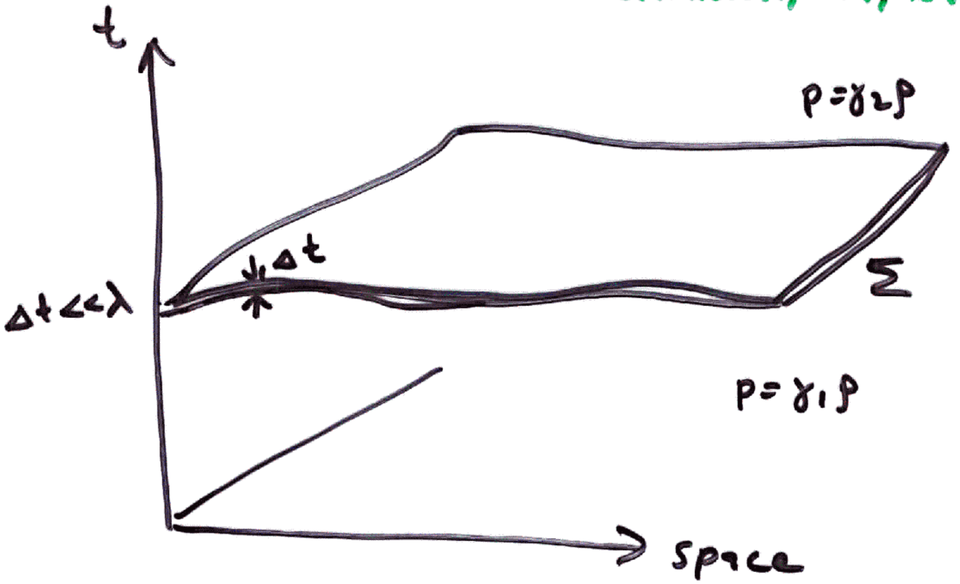
$$\varphi_c = \varphi_c(\chi_a)$$

$$\boxed{\delta\varphi_c(\vec{x}) = \frac{\partial\varphi}{\partial\chi}\,\delta\chi_a(\vec{x})}$$

Bernardeau, L.K., Uzan 03

SIMPLE WAY TO TREAT MODULATED PERTURBATIONS

Bernardeau, L.K., Uzan



Junction Conditions

Deruelle Mukhanov 1995

$[a]_\pm=0 \quad [H]_\pm=0$

$[\Phi]_\pm=0 \quad \left[\dot\Phi+\dot H\frac{\delta q}{q}\right]_\pm=0 \quad \left[\frac{\delta q}{\dot q}\right]_\pm=0$

$\Sigma: \quad q(t,\vec{x})=\text{const}$

Hybrid inflation $q=\varphi_c$

FEATURES OF MODULATED FLUCTUATIONS

- $$\frac{\delta\rho}{\rho}=\frac{V^{3/2}}{M_P^3V'}+\frac{\delta\alpha}{\alpha}$$

  inflaton ↑ ↑ interaction

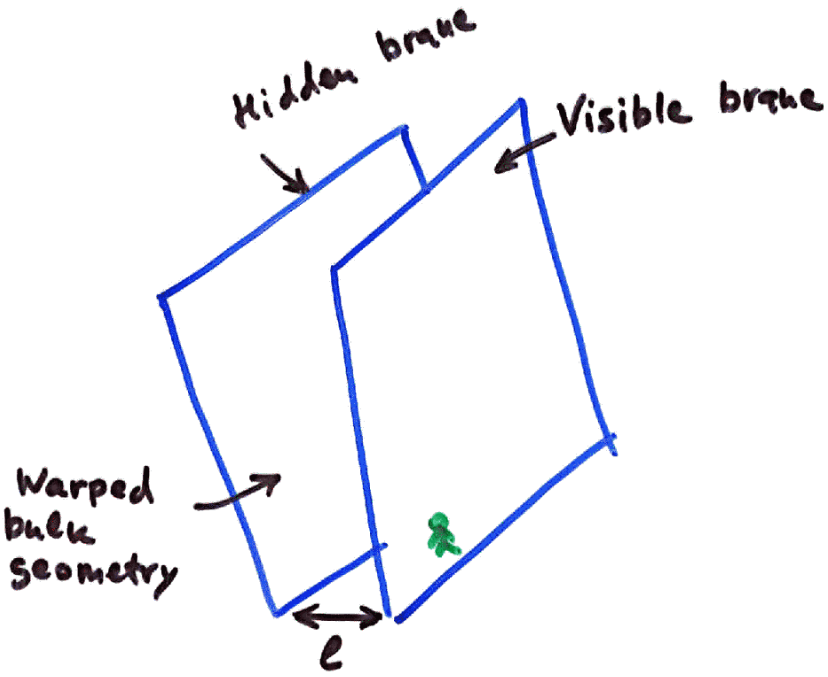
  which mass is light

- no consistency relation $\frac{T}{S}\sim(n_s-1)$
- isocurvature $\left(\frac{\delta\rho}{\rho}\right)_{CDM}, \frac{\delta\rho_B}{\rho_B}$
- different (small) non-gaussian signal

  Dvali et al 03
- χ - moduli, sneutrino, MSSM flat directions

Allahverdi, L.K. Peloso

BRANE WORLDS



$$ds^2 = dy^2 + A^2(y)\, ds_4^2$$

$$M_p^2 = M_s^{D-2} V_{D-4}$$

HW theory
ADD model
RS models

$$S = \int d^5x \sqrt{-g} \left(R + \frac{1}{2} \varphi_{,A} \varphi^{,A} - V(\varphi) \right)$$

$$- \sum_{a=1,2} \int d^4x \sqrt{-\gamma} \left([K] + U_a(\varphi) \right)$$

Junction conditions

$$[K_{\mu\nu} - K g_{\mu\nu}] = g_{\mu\nu} U$$

$$[n \cdot \nabla \varphi] = U_{,\varphi}$$

Bulk eqs

$$\varphi'' + 4 \frac{A'}{A} \varphi' - V_{,\varphi} = 0$$

$$6 \left(\frac{A'}{A} \right)^2 = \frac{\varphi'^2}{2} - V + \frac{H^2}{6A^2}$$

FLUCTUATIONS FROM INFLATION AND EXTRA DIMENSIONS

- Fluctuations of scalars
- Gravitational waves h_{AB}

$D=3+1$

two TT tensor components

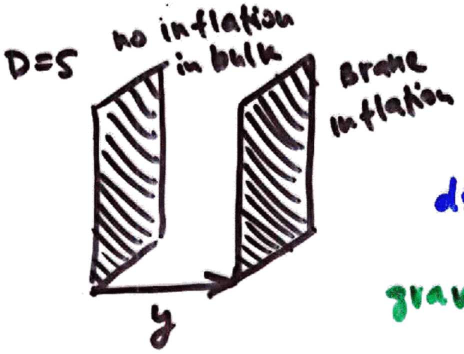
$$h_k = \frac{H}{M_p} \cdot \frac{1}{k^{3/2}}$$

- in general $\frac{D(D-3)}{2}$ components
- 4D appearance as 2 tensor + V + S modes
- more scales (M_s, M_p, R, H) to amplitude ?
- KK modes

Inner space is not inflating

Inner space is inflating

Braneworld INFLATION



Frolov, L.K. hep-th/0209133

$$ds^2 = dy^2 + A^2(y)(-dt^2 + e^{2Ht} d\vec{x}^2)$$

grav. waves $h_{AB}(y, \vec{x}, t)$

$$h_{\mu\nu} = A^{1/2} h_m(y) Q^{(m)}_{\mu\nu}(t, \vec{x})$$

$h_{yA} = h_{yy} = 0$

$$\Box_{ds} Q^{(m)}_{\mu\nu} = m^2 Q^{(m)}_{\mu\nu}$$

$$h_m'' + (m^2 - V_{eff}) h_m = 0$$

$$V_{eff} = \frac{3}{2}\frac{A''}{A} + \frac{A'^2}{2A}$$

$$x = \int \frac{dy}{A(y)}$$

NO-GO RESULTS

for arbitrary $A(y)$, $V(\varphi)$

- massive KK graviton modes are not generated from Inflation due to the gap

$$\Delta m \geq \sqrt{\frac{3}{2}}\, H$$

- massless scalar and vector projections of the bulk gravitons are absent

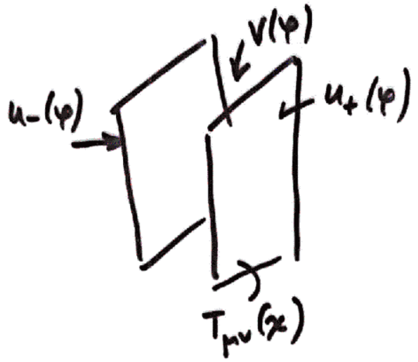
- amplitude

$$k^{3/2} h_k = \frac{H}{\tilde{M}_p}$$

$\tilde{M}_p$ not necessarily $= M_p$ today

Scalar fluctuations from braneworld inflation

Frolov, L.K.
hep-th/0309002



Background

$$ds^2 = dy^2 + A^2(y)\, ds_4^2$$

$$V(\varphi),\ u_\pm(\varphi),\ T^{\pm}_{\mu\nu}(\chi)$$

Perturbations

$$\delta g_{AB},\ \zeta(x^\mu),\ \delta\varphi,\ \delta\chi$$

Many works about $\delta\chi$ back to usual slow roll inflation

Neglect $\delta\chi$

Consider only $\delta\varphi(y, x^\mu)$

$$ds_4^2 = -dt^2 + e^{2Ht} d\vec{x}^2$$

$d=3+1$ FRW geometry

$$ds^2 = a^2(\tau)\left[(1+2\phi)\,d\tau^2 - (1+2\psi)\,\gamma_{ij}\,dx^i dx^j\right]$$

$\uparrow K$

$$\psi = -\phi$$

Solutions

$$\phi(\tau,\vec{x}) = \tilde{\phi}(\tau)\,Q_\lambda(\vec{x})$$

$$\delta\tilde{\varphi}(\tau)$$
$$\tilde{\phi}(\tau)$$

$$\nabla^2 Q_\lambda = \lambda Q_\lambda$$

$$\left(\frac{a\,\delta\varphi}{\dot{\varphi}}\right)^{\cdot} = \left(1 - 2\,\frac{\lambda+3K}{\dot{\varphi}^2}\right) a\phi$$

$$(a\phi)^{\cdot} = \frac{1}{2}\,a^2\dot{\varphi}\,\delta\varphi$$

Scalar perturbations in ~~3~~4+1 warp geometry

$$ds^2 = A^2(y)\left[(1+2\phi)\,dy^2 + (1+2\psi)\,ds_4^2\right]$$

$$T^A{}_B = \varphi^{,A}\varphi_{,B} - \frac{1}{2}\,\delta^A_B V(\varphi)$$

$$\delta G^A{}_B = \delta T^A{}_B \qquad 8\pi G = 1$$

$$\psi = -\frac{1}{2}\phi$$

Solutions

$$\phi(y,x^\mu) = \tilde{\phi}_m(y)\,Q_m(t,\vec{x})$$

$$\delta\tilde{\varphi}(y)$$
$$\tilde{\phi}(y)$$

$$(\Box + m^2)\,Q_m = 0$$

$$\left(\frac{A\,\delta\varphi}{\varphi'}\right)' = \left(1 - \frac{3}{2}\,\frac{m^2+4H^2}{\varphi'^2}\right) A\phi$$

$$(A^2\phi)' = \frac{2}{3}\,A^2\varphi'\,\delta\varphi$$

$$u = \sqrt{\frac{3}{2}}\,\frac{A^{3/2}}{\varphi'}\,\Phi$$

$$\boxed{u_m'' + [m^2 + 4H^2 - V_{eff}]\,u_m = 0}$$

$$V_{eff} = \frac{z''}{z} + \frac{2}{3}\varphi'^2 \qquad z = \frac{1}{\sqrt{A}\,\varphi'}$$

Boundary condition

$$(\delta\varphi' - \varphi'\Phi)|_\pm = \pm\frac{1}{2}u''a\,\delta\varphi|_\pm$$

Lower eigenvalue

$$\boxed{m_n^2 = -4H^2 + m_{0n}^2}$$

$$m_0 \simeq \frac{2}{3}\,\frac{\int\frac{dy}{A}}{\int\frac{dy}{A\varphi'^2}}$$

eigenvalue problem

Rayleigh's method

At 4d slice

$$Q_m(t,\vec{x}) = e^{i\vec{k}\vec{x}}\,Q_m(t)$$

$$\ddot{Q}_m + 3H Q_m + \left(\frac{k^2}{a^2} + m^2\right)Q_m = 0$$

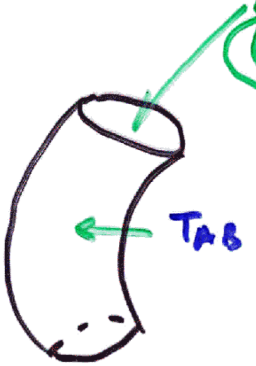
$$m^2 = -4H^2 + m_0^2 < 0$$

$$Q_m(t) \sim \exp\left(\sqrt{\frac{9}{4} + \frac{|m^2|}{H^2}} - \frac{3}{2}\right)Ht$$

tachyonic instability

Conjecture for general case

$$ds^2 = \underset{(1+2\phi)}{g_{AB}}\, dy^A dy^B + \underset{(1+2\psi)}{A(y)}\, ds_4^2$$



Scalar fluctuations

$$\delta G_{AB} = 8\pi G\, \delta T_{AB}$$

$$\boxed{({}^4\Box - 4H^2)\phi + \cdots}$$

$$\phi(y, x^\mu) = \tilde{\Phi}_n(y) Q_n(x^\mu)$$

$$({}^4\Box + m^2) Q_n = 0$$

$$\boxed{(m^2 + 4H^2)\tilde{\Phi}(y) + \cdots}$$

$$\boxed{m_n^2 = -4H^2 + m_{0n}^2}$$

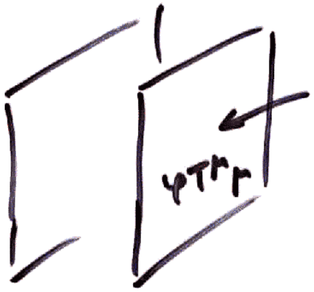
with bulk gravity

without bulk gravity

$$\boxed{m_n^2 = -\cancel{4H^2} + \tilde{m}_{0n}^2}$$

Implementations of $m^2 = -4H^2 + m_0^2$

- Inflation possible for $H < m_0$
- For $|m^2| << H^2$ we have induced metric fluctuations with scale free spectrum without slow roll
- Branes are stabilized for 4d cosmological constant smaller than m_0^4



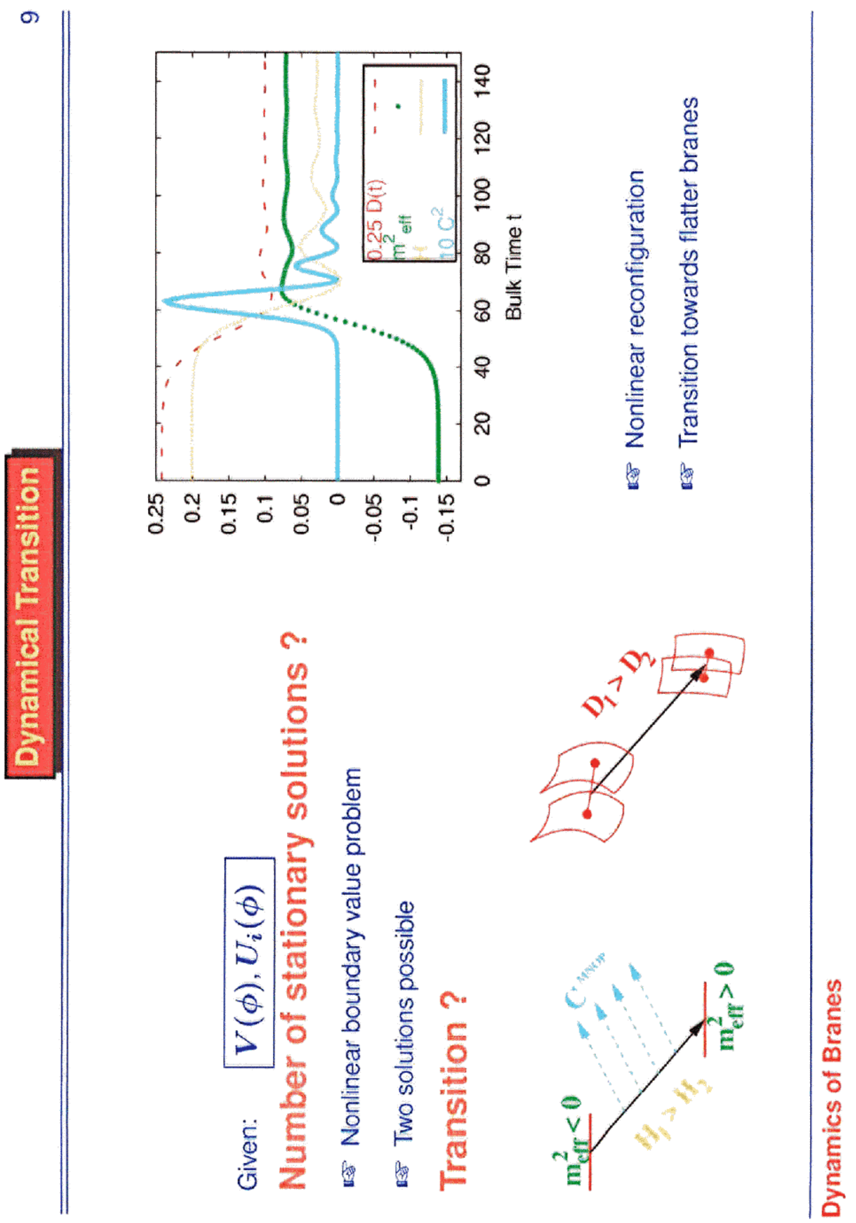
at brane

$m^2 = m_0^2 + \Delta m^2$

from interaction

Classical
Cosmological
Constant
Problem

C C C P



Dynamical Transition

Given: $V(\phi), U_i(\phi)$

Number of stationary solutions ?

- Nonlinear boundary value problem
- Two solutions possible

Transition ?

- Nonlinear reconfiguration
- Transition towards flatter branes