Stringy Origin of non-Abelian Discrete Flavor Symmetries

Stuart Raby

String Phenomenology
KITP
August 29, 2006

THE OHIO STATE UNIVERSITY

Kobayashi, SR, Zhang
hep-ph/0403065 & 0409098

Kobayashi, Nilles, Ploeger, SR, Ratz
- in preparation

Field theory vs. String orbifolds

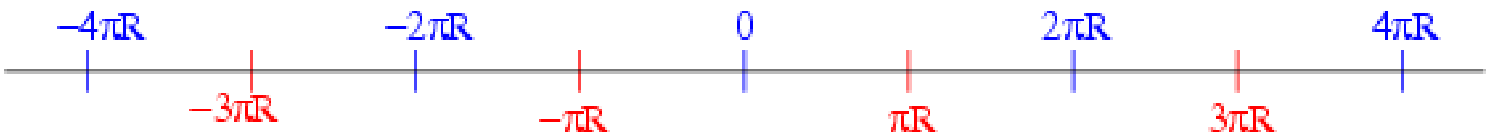
3 Family Orbifold GUT on $M_4 \times S_1/(Z_2 \times Z_2')$

↕

Heterotic string compactified on $[T_2]^3/Z_6$ + Wilson lines

Consider SO(10) example

5D orbifold field theory



- $S^1/\mathbb{Z}_2 = \mathbb{R}/\Omega$

  orbifold space group $\Omega \supset \mathcal{P}, \mathcal{T}$

  $\mathcal{P}: \quad y \to -y$ space reversal

  $\mathcal{T}: \quad y \to y + 2\pi R$ translation

  $\mathcal{P}' = \mathcal{P}\mathcal{T}$ is also a $\mathbb{Z}_2$ action.

- Two conjugacy classes of fixed points

  $$y = 2n\pi R, \quad (2n+1)\pi R, \qquad n \in \mathbb{Z}$$

- Fundamental domain $[0, \pi R]$

  $y = 0$ – fixed point of $\mathcal{P}$;
  $y = \pi R$ – fixed point of $\mathcal{P}'$

- A general 5D field $\phi(x, y)$ transforms as

$$\mathcal{P}: \qquad \phi(x,y) \to \phi(x,-y) = P\phi(x,y)$$
$$\mathcal{T}: \qquad \phi(x,y) \to \phi(x,y+2\pi R) = T\phi(x,y)$$

P, T realize $\mathcal{P}$, $\mathcal{T}$. (T – Wilson line.)

Equivalently, ϕ is characterized by orbifold parities P, P'.

- ϕ has a Kaluza-Klein mode expansion

$$\phi(x,y) = \sum_{m \in \mathbb{Z}} \phi_m(x)\psi_m(y)$$

Depending on the orbifold parities P, P', the wave function $\psi_m(y)$ and KK state masses are

P	P'	Wave functions	Masses
$+$	$+$	$\cos\left(\frac{my}{R}\right)$	$\frac{m}{R}$
$+$	$-$	$\cos\frac{(m+1/2)y}{R}$	$\frac{(m+1/2)}{R}$
$-$	$+$	$\sin\frac{(m+1/2)y}{R}$	$\frac{(m+1/2)}{R}$
$-$	$-$	$\sin\frac{(m+1)y}{R}$	$\frac{(m+1)}{R}$

5D $SO(10)$ orbifold GUT

- $SO(10)$ "bulk"

  Vector multiplet $V(\mathbf{45}) + \Sigma(\mathbf{45})$
  Hyper multiplet $H(\mathbf{10}) + H^c(\mathbf{10})$

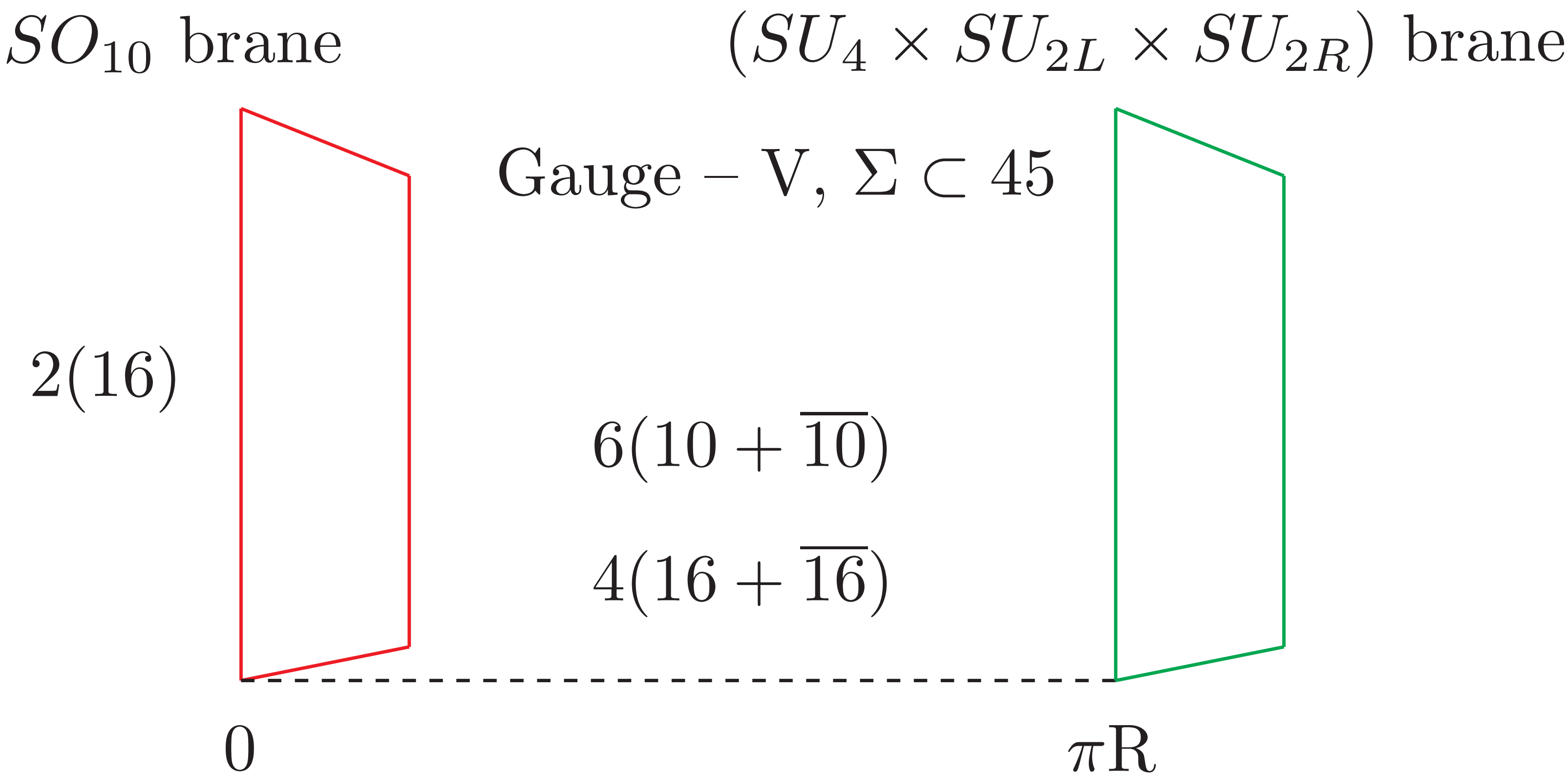
States	P	P'	States	P	P'
$V(\mathbf{15},\mathbf{1},\mathbf{1})$	$+$	$+$	$\Sigma(\mathbf{15},\mathbf{1},\mathbf{1})$	$-$	$-$
$V(\mathbf{1},\mathbf{3},\mathbf{1})$	$+$	$+$	$\Sigma(\mathbf{1},\mathbf{3},\mathbf{1})$	$-$	$-$
$V(\mathbf{1},\mathbf{1},\mathbf{3})$	$+$	$+$	$\Sigma(\mathbf{1},\mathbf{1},\mathbf{3})$	$-$	$-$
$V(\mathbf{6},\mathbf{2},\mathbf{2})$	$+$	$-$	$\Sigma(\mathbf{6},\mathbf{2},\mathbf{2})$	$-$	$+$
$H(\mathbf{6},\mathbf{1},\mathbf{1})$	$+$	$-$	$H^c(\mathbf{6},\mathbf{1},\mathbf{1})$	$-$	$+$
$H(\mathbf{1},\mathbf{2},\mathbf{2})$	$+$	$+$	$H^c(\mathbf{1},\mathbf{2},\mathbf{2})$	$-$	$-$

- $P = +$: complete multiplet of $SO(10)$
  $\rightarrow y = 0$ – $SO(10)$ "brane"

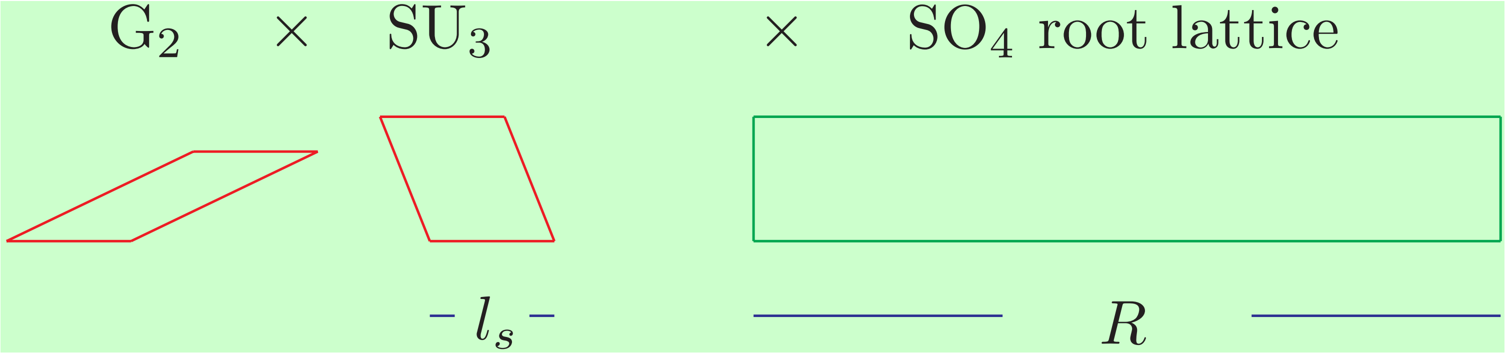
  $P' = +$: complete multiplet of $SU(4) \times SU(2)_L \times SU(2)_R$
  $\rightarrow y = \pi R$ – PS "brane"

- Solve the doublet-triplet splitting problem.

SO(10) Orbifold GUT on $M_4 \times S_1/(Z_2 \times Z_2')$



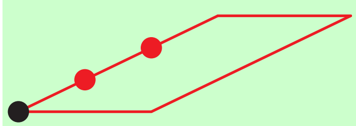
Compactify 6D on $(T^2)^3$

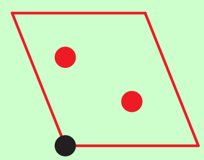


Then mod by $Z_6 = (Z_3 \times Z_2)$
and
Add Wilson lines

consistent with mod. inv. !

G_2 SU_3 SO_4

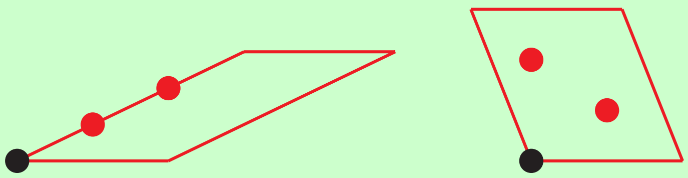




$$V,\ \Sigma\ \in SO(10)$$

$$6(\mathbf{10} + \overline{\mathbf{10}})$$

$$4(\mathbf{16} + \overline{\mathbf{16}})$$



$$V \in PS, \quad f_3 \in \mathbf{16}[U_1]$$

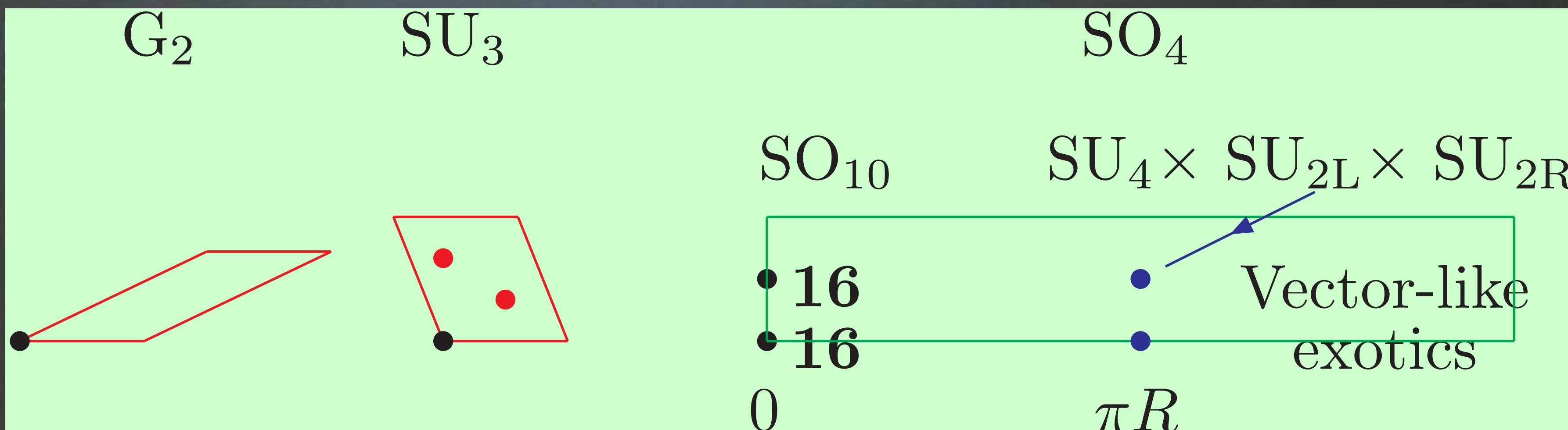
$$3(h + C) \in 6(\mathbf{10} + \overline{\mathbf{10}})$$

$$f_3^c + (\chi^c + \bar{\chi}^c) \in 3(\mathbf{16} + \overline{\mathbf{16}})$$

SO(10) → SO(10) → $SU(4) \times SU(2)_L \times SU(2)_R$

$f_3 = (4,2,1)$, $f_3^c = (4^c,1,2)$, $h = (1,2,2)$

Strings & Exotics
No chiral exotics !!

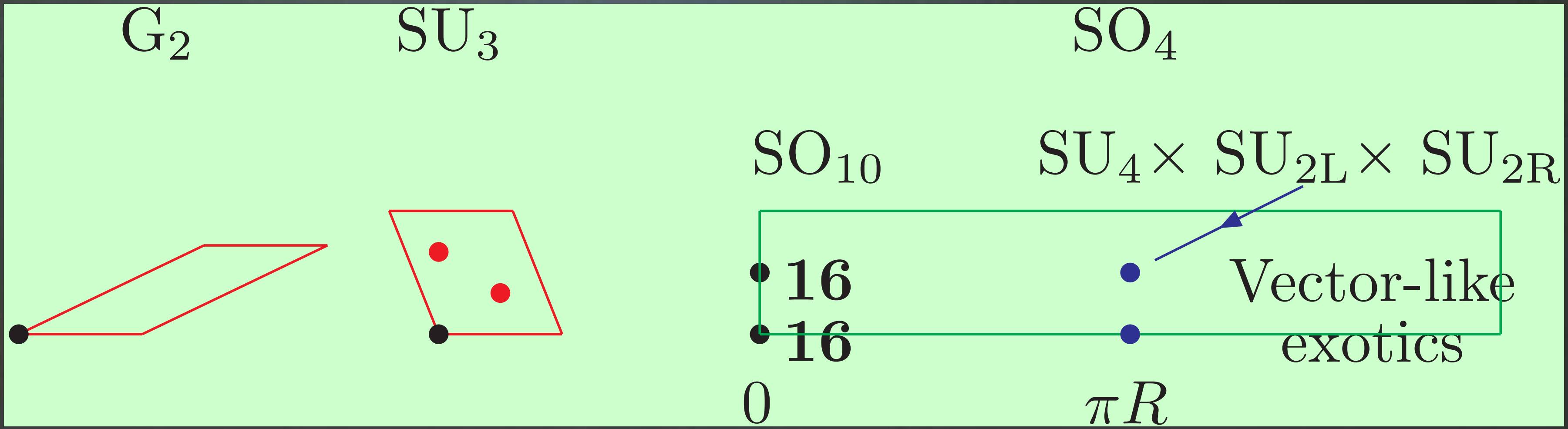


Vector-like exotics

$$[(4,1,1)+(\bar{4},1,1)]+(1,2,1)+(1,1,2)$$
$$+(1,2,1;1,2)+(1,1,2;1,2)$$

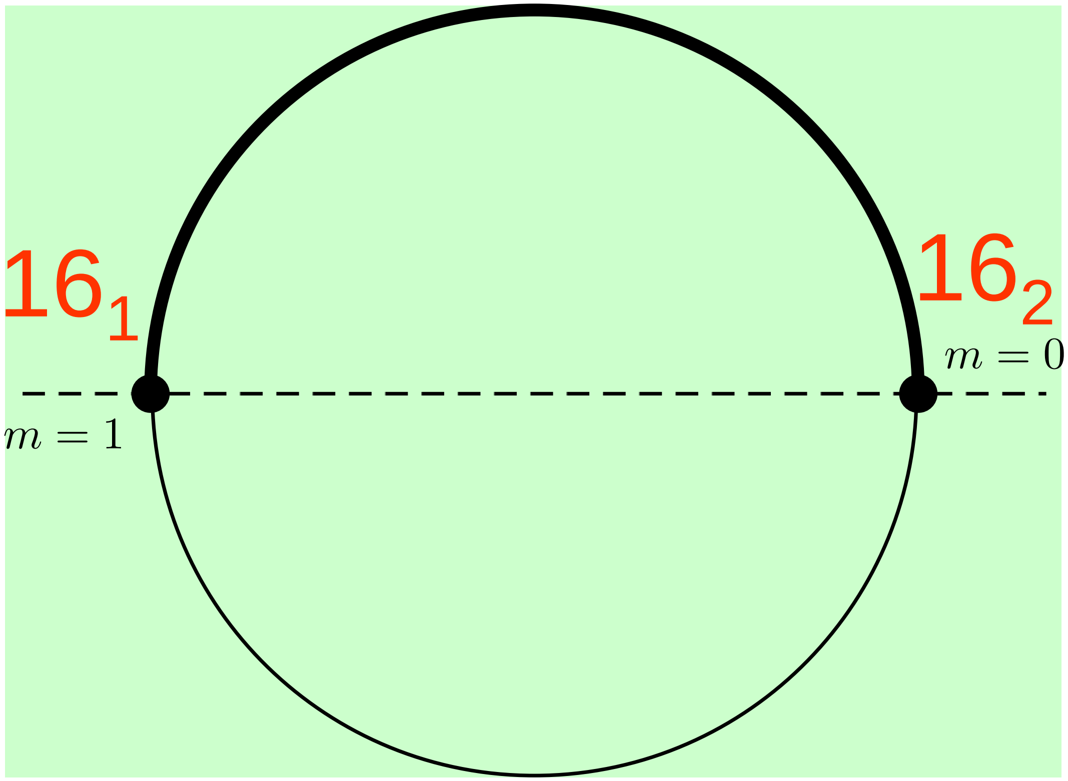
Massive &
Decouple !!

Strings
& Discrete N.A. family symmetries



2 Light fam's D_4

S_1/Z_2



D_4 family symmetry

$$D_4 = \{\pm 1, \pm\sigma_1, \pm\sigma_3, \mp i\sigma_2\}$$

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} : f_1 \leftrightarrow f_2 \qquad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} : f_2 \leftrightarrow -f_2$$

geometry space group

$$\begin{pmatrix} f_1 \\ f_2 \end{pmatrix} \text{ doublet} \qquad f_3 \text{ singlet}$$

Fermion mass hierarchy

PS breaking VEVs

$$O_i = \langle \chi^c_\alpha \; \bar{\chi}^c_i \rangle, \quad i = 1, 2$$

- Fermion mass matrix [simple form]

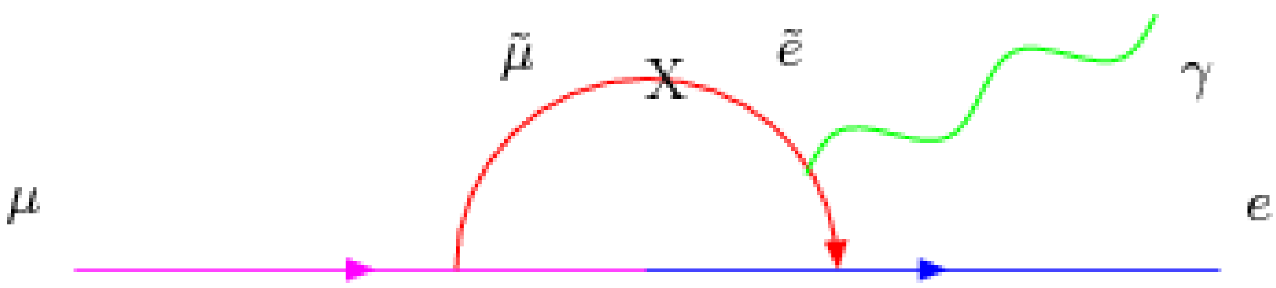
$$(f_1 \; f_2 \; f_3) \; h \; \mathcal{M} \begin{pmatrix} f^c_1 \\ f^c_2 \\ f^c_3 \end{pmatrix}$$

$$\mathcal{M} = \begin{pmatrix} (O_2 \; \tilde{S}_e + S_e) & (O_2 \; \tilde{S}_o + S_o) & (O_1 \; O_2 \; \phi_e + \tilde{\phi}_e) \\ (O_2 \; \tilde{S}_o + S_o) & (O_2 \; \tilde{S}_e + S_e) & (O_1 \; O_2 \; \phi_o + \tilde{\phi}_o) \\ \phi'_e & \phi'_o & 1 \end{pmatrix}$$

Flavor Violation

Non-abelian family symmetry

Fermion hierarchy $\longleftrightarrow$ scalar flavor viola



Suppresses flavor violation

Kobayashi, SR, Zhang

Kobayashi, Nilles, Ploeger, SR & Ratz

Orbifolds preserving N=1 SUSY

(a) $\mathbb{Z}_N$

orbifold	twist
$\mathbb{Z}_3$	$(1,1,-2)/3$
$\mathbb{Z}_4$	$(1,1,-2)/4$
$\mathbb{Z}_6$-I	$(1,1,-2)/6$
$\mathbb{Z}_6$-II	$(1,2,-3)/6$
$\mathbb{Z}_7$	$(1,2,-3)/7$
$\mathbb{Z}_8$-I	$(1,2,-3)/8$
$\mathbb{Z}_8$-II	$(1,3,-4)/8$
$\mathbb{Z}_{12}$-I	$(1,4,-5)/12$
$\mathbb{Z}_{12}$-II	$(1,5,-6)/12$

(b) $\mathbb{Z}_N \times \mathbb{Z}_M$

orbifold	v^1	v^2
$\mathbb{Z}_2 \times \mathbb{Z}_2$	$(1,0,-1)/2$	$(0,1,-1)/2$
$\mathbb{Z}_2 \times \mathbb{Z}_3$	$(1,0,-1)/2$	$(0,1,-1)/3$
$\mathbb{Z}_2 \times \mathbb{Z}_4$	$(1,0,-1)/2$	$(0,1,-1)/4$
$\mathbb{Z}_2 \times \mathbb{Z}_6$	$(1,0,-1)/2$	$(0,1,-1)/6$
$\mathbb{Z}_2 \times \mathbb{Z}'_6$	$(1,0,-1)/2$	$(1,1,-2)/6$
$\mathbb{Z}_3 \times \mathbb{Z}_3$	$(1,0,-1)/3$	$(0,1,-1)/3$
$\mathbb{Z}_3 \times \mathbb{Z}_6$	$(1,0,-1)/3$	$(0,1,-1)/6$
$\mathbb{Z}_4 \times \mathbb{Z}_4$	$(1,0,-1)/4$	$(0,1,-1)/4$
$\mathbb{Z}_6 \times \mathbb{Z}_6$	$(1,0,-1)/6$	$(0,1,-1)/6$

Table 1: (a) $\mathbb{Z}_N$ and (b) $\mathbb{Z}_N \times \mathbb{Z}_M$ orbifold twists for 6D $\mathbb{Z}_N$ orbifolds leading to N=1 SUSY.

Orbifold + Twisted Sectors

$$x^i = x^i + n_a e_a^i, \quad (i = 1,..,d)$$

$$n_a e_a^i \subset \Lambda \quad \text{lattice}$$

$$T^d = R^d \big/ \Lambda$$ torus

$$\theta\Lambda = \Lambda \qquad \theta^N = 1$$

$$T^d \big/ \mathbf{Z}_N$$ orbifold

$$f = \left(\theta^k f\right) + \Lambda$$

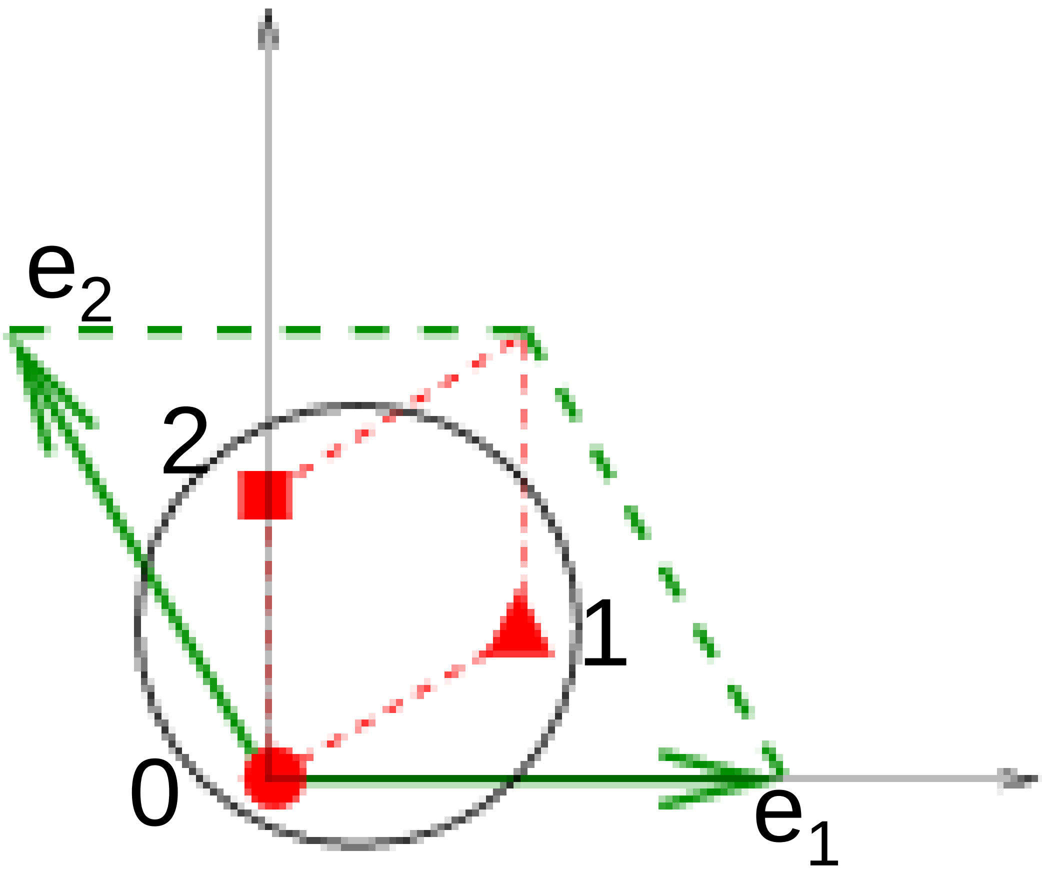
Fixed points, k^{th} twisted sector

$$\left(1-\theta^k\right) f = \left(1-\theta^k\right)\Lambda \equiv \Lambda_k$$

Conj. classes

T^2/Z_3 orbifold

$(1-\theta)\Lambda$ spanned by $3e_1$, e_2-e_1



fixed points

Stringy Selection Rules

Eg. : Yukawa couplings of n states in first twisted sector

$$\prod_i^n \left(\theta,\ m_1^{(i)} e_1\right) = \left(\theta^n,\ \sum_i m_1^{(i)} e_1\right) = \left(1,\ \left(1-\theta\right)\Lambda\right)$$

$$\text{(I)} \quad n = 3\mathbf{Z}, \qquad \text{(II)} \quad \sum_i m_1^{(i)} = 0 \ \left(\operatorname{mod} 3\right)$$

Space group selection rules

$$(\mathrm{I})\quad \begin{pmatrix} |(\theta,0)\rangle \\ |(\theta,\mathrm{e}_1)\rangle \\ |(\theta,2\mathrm{e}_1)\rangle \end{pmatrix} \Rightarrow \begin{pmatrix} \omega & 0 & 0 \\ 0 & \omega & 0 \\ 0 & 0 & \omega \end{pmatrix} \begin{pmatrix} |(\theta,0)\rangle \\ |(\theta,\mathrm{e}_1)\rangle \\ |(\theta,2\mathrm{e}_1)\rangle \end{pmatrix} \qquad \omega = e^{2\pi i/3}$$

$$(\mathrm{II})\quad \begin{pmatrix} |(\theta,0)\rangle \\ |(\theta,\mathrm{e}_1)\rangle \\ |(\theta,2\mathrm{e}_1)\rangle \end{pmatrix} \Rightarrow \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega & 0 \\ 0 & 0 & \omega^2 \end{pmatrix} \begin{pmatrix} |(\theta,0)\rangle \\ |(\theta,\mathrm{e}_1)\rangle \\ |(\theta,2\mathrm{e}_1)\rangle \end{pmatrix}$$

$\Delta(54) =$
$S_3 \rtimes (Z_3 x Z_3')$

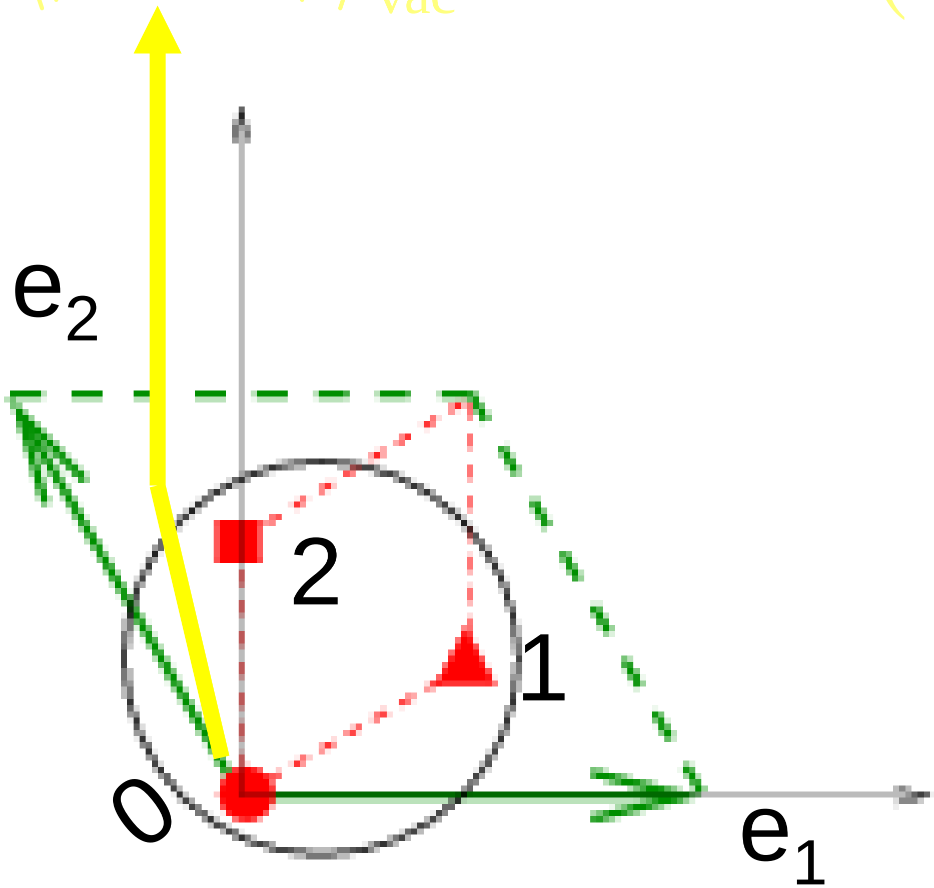
$$(\mathrm{III})\quad \begin{pmatrix} |(\theta,0)\rangle \\ |(\theta,\mathrm{e}_1)\rangle \\ |(\theta,2\mathrm{e}_1)\rangle \end{pmatrix} \Rightarrow \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} |(\theta,0)\rangle \\ |(\theta,\mathrm{e}_1)\rangle \\ |(\theta,2\mathrm{e}_1)\rangle \end{pmatrix}$$

S_3 perm's.

geometry

Flavor Symmetry Breaking

$$\left\langle \left| (\theta, 0) \right\rangle \right\rangle_{\text{vac}} \neq 0 \qquad \Delta(54) \to D_3 = S_2 \cup \mathbf{Z}_3$$



$$\begin{pmatrix} \left| (\theta, e_1) \right\rangle \\ \left| (\theta, 2e_1) \right\rangle \end{pmatrix}$$

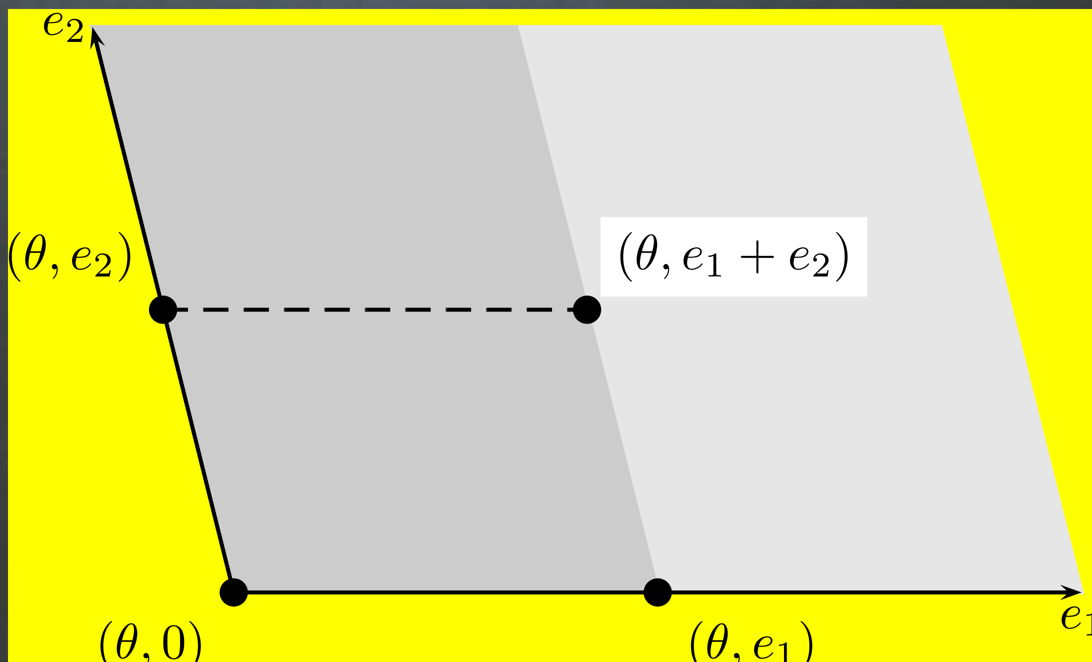
D_3

doublet

T^2/Z_2 orbifold

$(D_4 \times D_4)/Z_2$

4 - plet

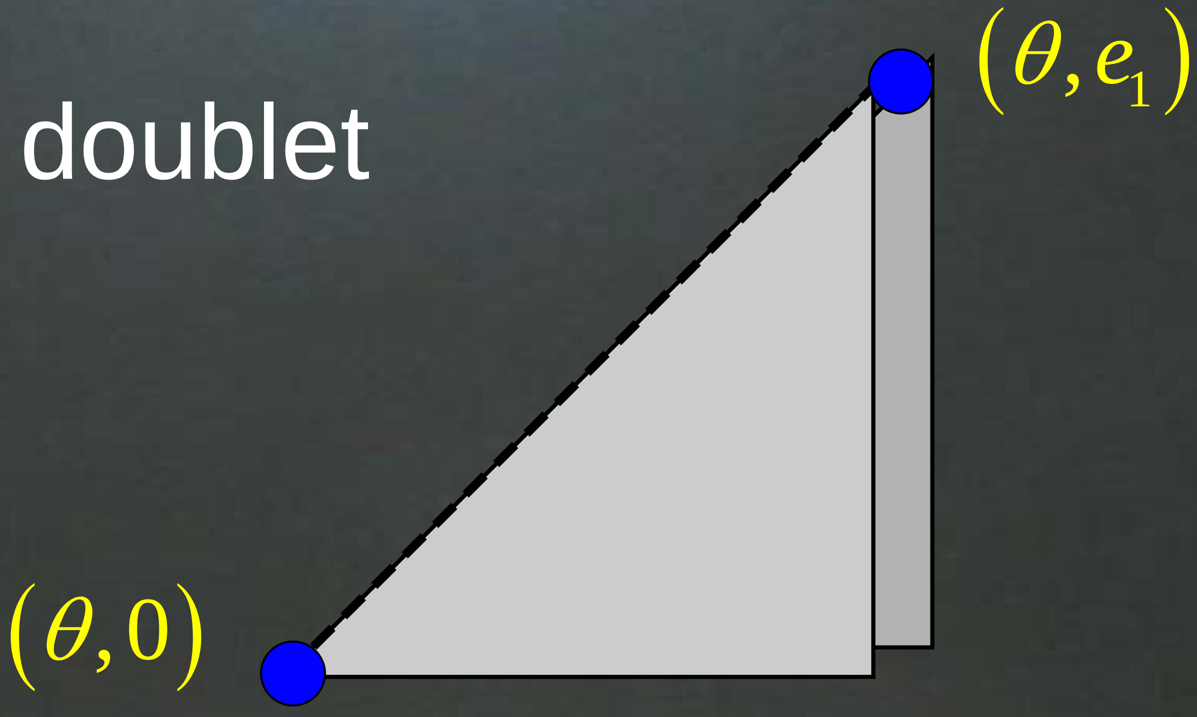


T^2/Z_4 orbifold (θ- twisted sector)

$$\left\{ \left| (\theta, 0) \right\rangle, \left| (\theta, e_1) \right\rangle \right\}$$

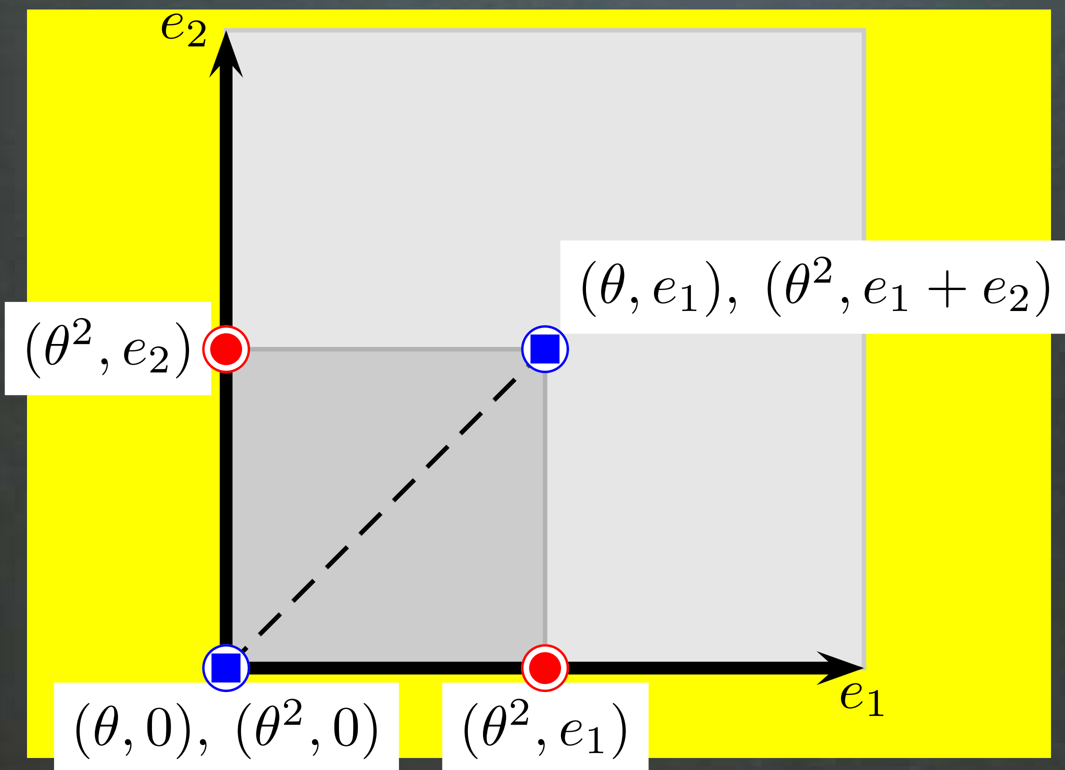
$(D_4 \times Z_4)/Z_2$

doublet



T^2/Z_4 orbifold (θ^2 – twisted sector)

$$\left\{ \left|\left(\theta^2, 0\right)\right\rangle, \ \left|\left(\theta^2, e_1 + e_2\right)\right\rangle, \ \left|\left(\theta^2, e_1\right)\right\rangle, \ \left|\left(\theta^2, e_2\right)\right\rangle \right\}$$



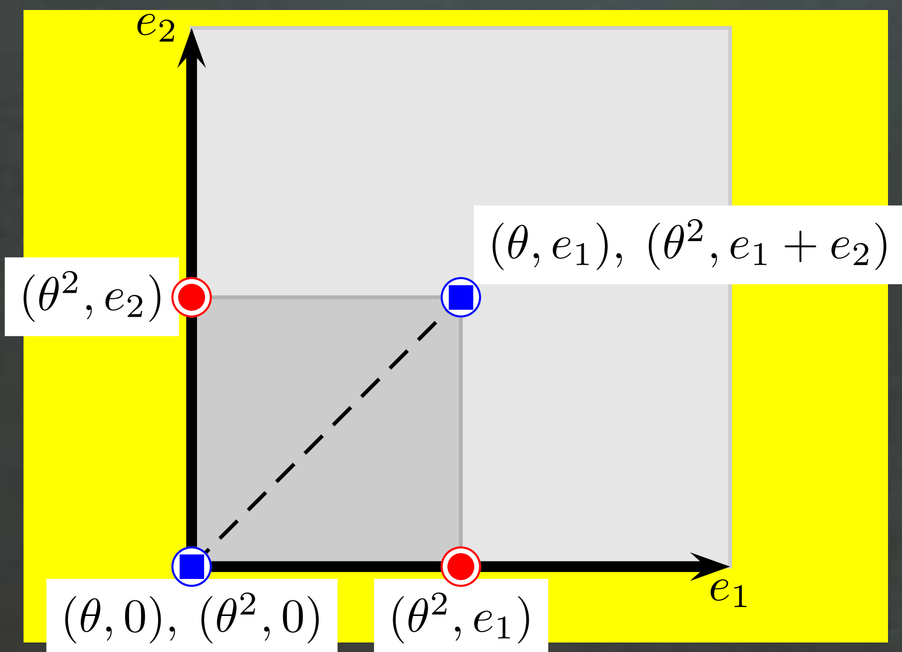
$(D_4 \times D_4)/Z_2$

4-plet

T^2/Z_4 orbifold ($\theta+\theta^2$ – twisted sectors)

$(D_4 \times Z_4)/Z_2$ combined ⇒ smaller sym.

$$\left\{ \left|(\theta^2,0)\right\rangle \pm \left|(\theta^2,e_1+e_2)\right\rangle,\ \left|(\theta^2,e_1)\right\rangle \pm \left|(\theta^2,e_2)\right\rangle \right\}$$



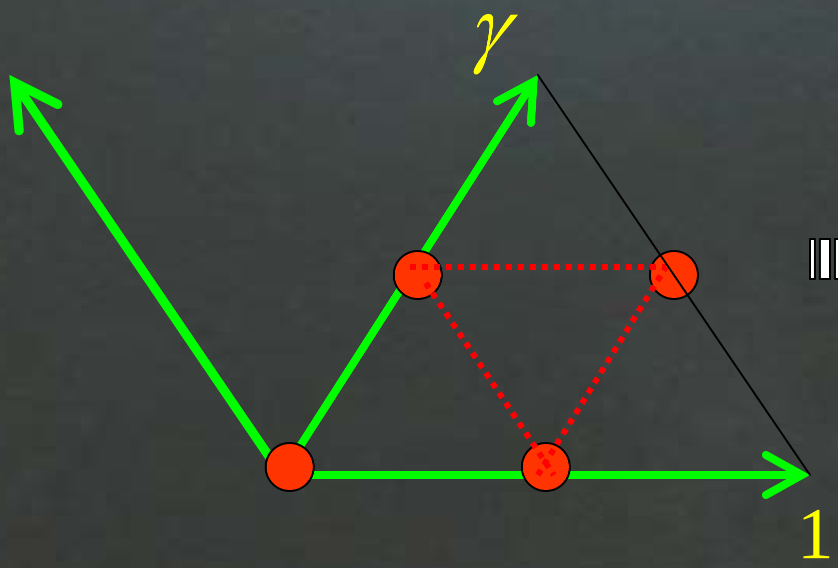
$$\left\{ \left|(\theta,0)\right\rangle,\ \left|(\theta,e_1)\right\rangle \right\}$$

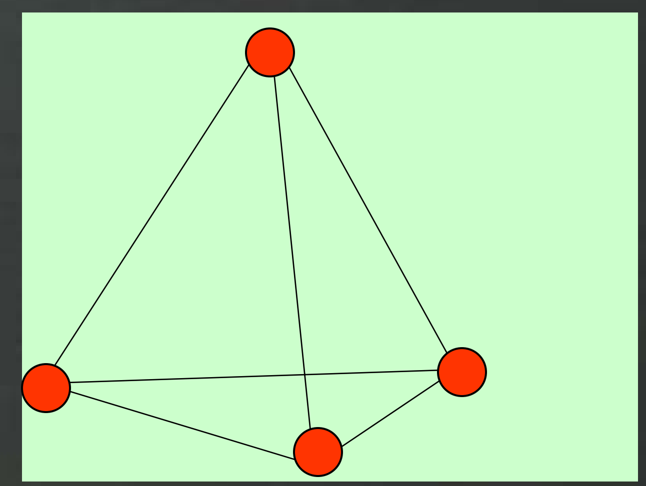
4- singlets !
+ doublet

T^2/Z_2 orbifold (SU(3) lattice)

$$z \to z + \gamma, \;\; \gamma = e^{i\pi/3}$$

$$z \to z + 1, \qquad z \to -z$$





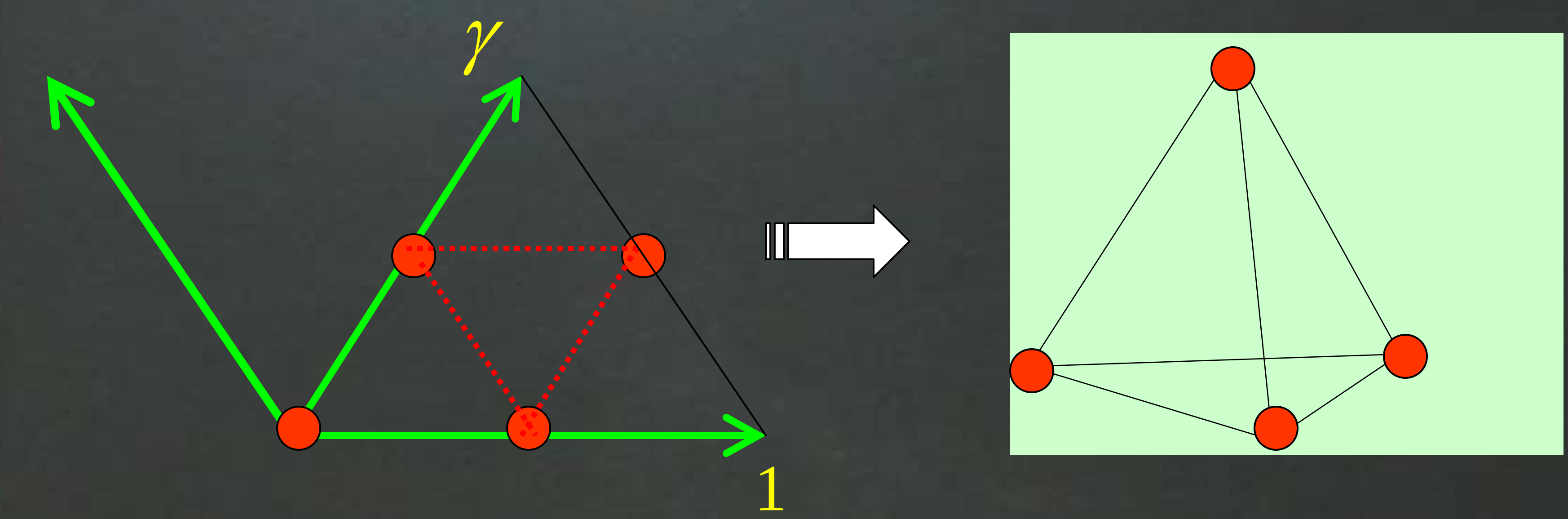
T^2/Z_2 orbifold (SU(3) lattice)

$$T: \ z \rightarrow \gamma^2 z$$

$$S: \ z \rightarrow z + \tfrac{1}{2}$$

A_4 Altarelli, Feruglio, Lin

$$S^2 = T^3 = (ST)^3 = 1$$



T^2/Z_2 orbifold (SU(3) lattice)

S_4

S_4 ✝ $(Z_2 \times Z_2 \times Z_2)$

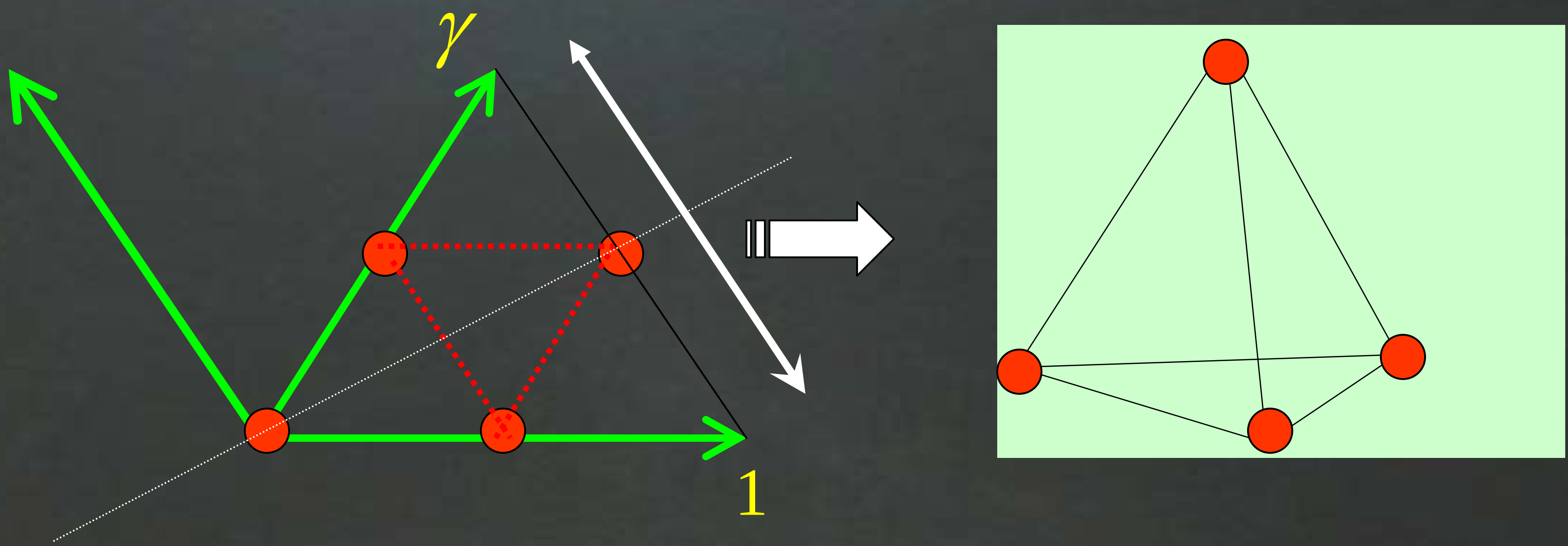


Table of Orbifold Symmetries

orbifold	flavor symmetry	twisted sector	string fundamental states
$\mathbb{S}^1/\mathbb{Z}_2$	$D_4 = S_2 \ltimes (\mathbb{Z}_2 \times \mathbb{Z}_2)$	**untwisted sector**	$\mathbf{1}$
		θ-twisted sector	$\mathbf{2}$
$\mathbb{T}^2/\mathbb{Z}_2$	$(D_4 \times D_4)/\mathbb{Z}_2 = (S_2 \times S_2) \ltimes \mathbb{Z}_2^3$	**untwisted sector**	$\mathbf{1}$
		θ-twisted sector	$\mathbf{4}$
$\mathbb{T}^2/\mathbb{Z}_3$	$\Delta(54) = S_3 \ltimes (\mathbb{Z}_3 \times \mathbb{Z}_3)$	**untwisted sector**	$\mathbf{1}$
		θ-twisted sector	$\mathbf{3}$
		θ^2-twisted sector	$\bar{\mathbf{3}}$
$\mathbb{T}^2/\mathbb{Z}_4$		**untwisted sector**	$\mathbf{1}$
	$(D_4 \times \mathbb{Z}_4)/\mathbb{Z}_2$	**θ-twisted sector**	$\mathbf{2}$
	$(D_4 \times D_4)/\mathbb{Z}_2$	**θ^2-twisted sector**	$\mathbf{4}$
	or $(D_4 \times \mathbb{Z}_4)/\mathbb{Z}_2$	**$\theta + \theta^2$-twisted sectors**	**or** $\mathbf{1}_{A_1} + \mathbf{1}_{B_1} + \mathbf{1}_{B_2} + \mathbf{1}_{A_2}$
$\mathbb{T}^2/\mathbb{Z}_6$	**trivial**		
$\mathbb{T}^4/\mathbb{Z}_8$		**untwisted sector**	$\mathbf{1}$
	$(D_4 \times \mathbb{Z}_8)/\mathbb{Z}_2$	**θ-twisted sector**	$\mathbf{2}$
	combined	**θ^2-twisted sector**	$\mathbf{1}_{A_1} + \mathbf{1}_{B_1} + \mathbf{1}_{B_2} + \mathbf{1}_{A_2}$
		θ^3-twisted sector	$\mathbf{2}$
		θ^4-twisted sector	$4 \times (\mathbf{1}_{A_1} + \mathbf{1}_{B_1} + \mathbf{1}_{B_2} + \mathbf{1}_{A_2})$
$\mathbb{T}^4/\mathbb{Z}_{12}$	**trivial**		
$\mathbb{T}^6/\mathbb{Z}_7$		**untwisted sector**	$\mathbf{1}$
	$S_7 \ltimes (\mathbb{Z}_7)^6$	**θ^k-twisted sector**	$\mathbf{7}$
		θ^{7-k}-twisted sector	$\bar{\mathbf{7}}$

Conclusions

- non-Abelian flavor symmetries
  - spont. breaking ➡ fermion mass h
  - suppress flavor violating processes
- Orbifolds ➡ discrete n-A. flavor. sy
  - point group selection rules
  - geometry
- GOAL: Find realistic models using these tools.