

# Strongly interacting Rydberg slow light polaritons

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## Collaboration

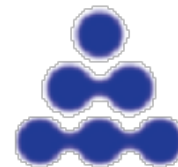
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Strongly interacting  
Rydberg slow light  
polaritons



SFB TRR21:  
Tailored quantum matter

# Interacting Rydberg slow light polaritons

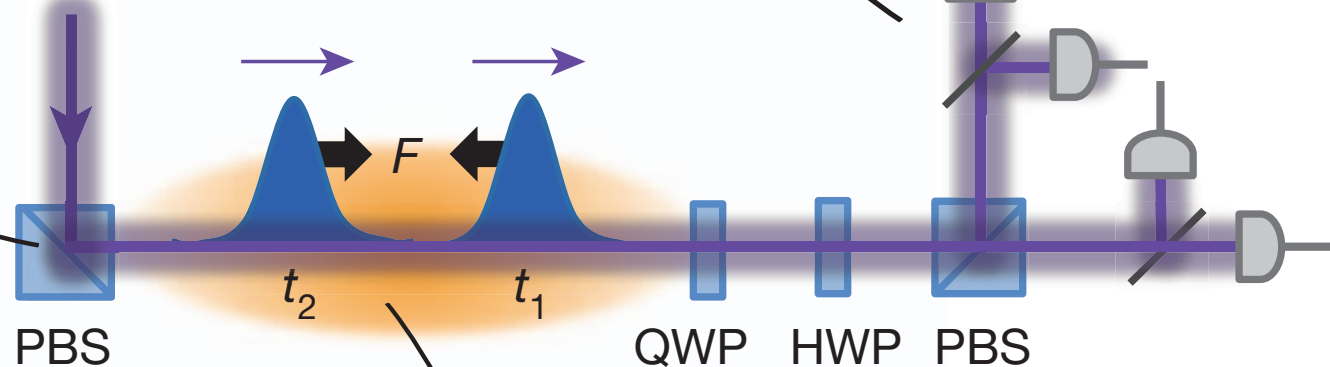
## Input:

- photons in a single transverse channel
- no-backscattering as atomic cloud is smooth
- chiral-one-dimensional photons

## Readout:

- detection transmitted photons
- photon correlations

$$|V\rangle = |\sigma^+\rangle + |\sigma^-\rangle$$



Firstenberg et al 2013

## Media:

- cold atomic gases
- strongly interacting slow light polaritons in one-dimension

# Slow light polaritons

## Electromagnetic induced transparency (EIT)

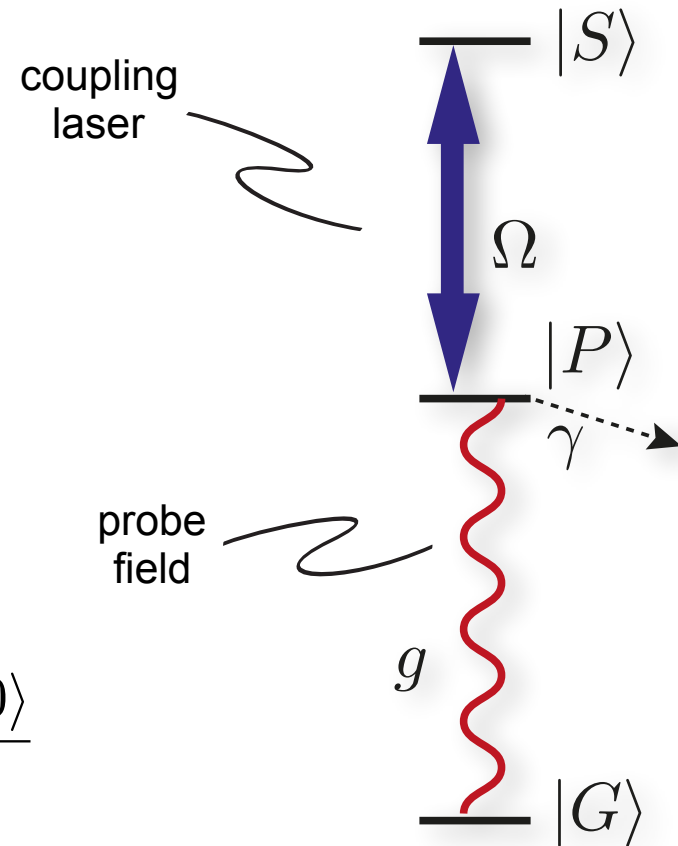
- photons in an atomic media
- three level setup for the atoms

- coupling laser
- probe field
- losses only from intermediate p-level

- dark state

- dissipation free state
- polariton: superposition of photon and excited state

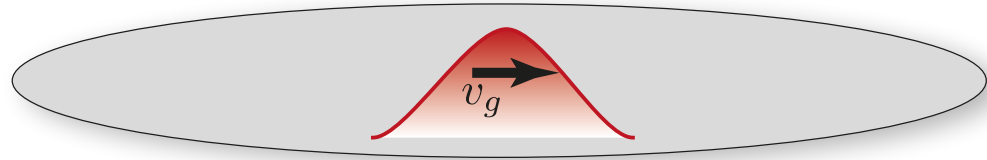
$$\frac{\Omega|G, 1\rangle - g|S, 0\rangle}{\sqrt{\Omega^2 + g^2}}$$



# Slow light polaritons



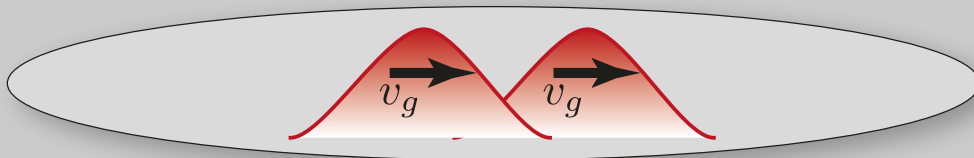
photon wave packet



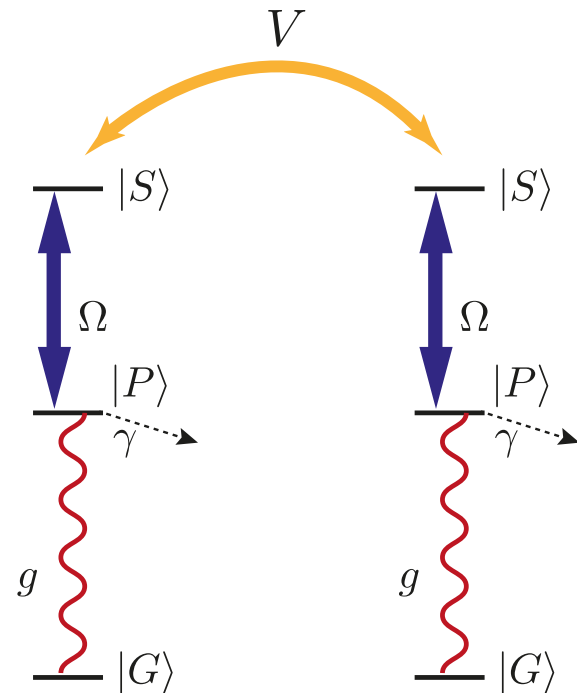
slowed down and compressed within the media

- slow light ( $\sim 1$  m/s)
- single photon storage (a few seconds)
- light pulse to width of  $20\mu\text{m}$

Is it possible to induce a strong interaction between the photons?



- yes, if we use Rydberg states as an excited state
- Rydberg slow light polaritons



# Rydberg excitations

## Rydberg-Rydberg interaction

- strong van der Waals interactions for s-wave states

- depending on  $n$  attractive or repulsive

- $C_6 \sim n^{11}$

- dipole-dipole interactions in presence of an electric field

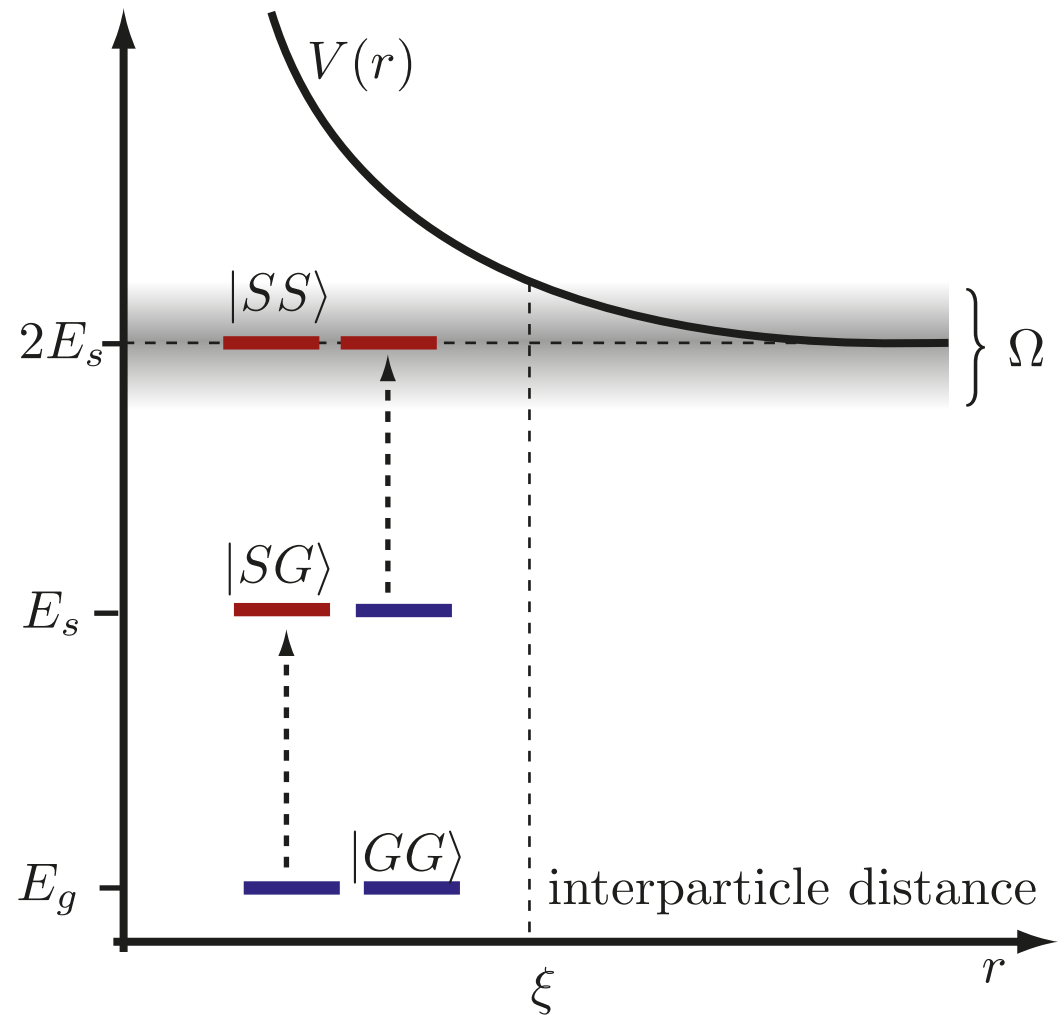
$$d \sim n^2 e a_0$$

## Blockade phenomena

- once a Rydberg atom is excited, further excitations are shifted out of resonance

- Blockade radius

$$\xi = (C_6 / \hbar \Omega)^{1/6} \sim 5 \mu m$$



Exp: T. F. Gallagher, Charlottesville; M. Weidemüller, Freiburg; P. Pillet, Orsay; Rolston, JQI; van den Heuvel, Amsterdam; P. Gould, Storrs; T. Pfau Stuttgart, A. Browaeys, P. Grangier, Orsay; M. Saffman.

Th: Robicheaux and Hernández, Ates, Pohl, Pattard, Rost, Stanojevic, Côté, Lukin, Fleischhauer, Cirac, Zoller,

# References

## Theory:

Friedler, Petrosyan, Fleischhauer, Kurizki, (2005);  
Gorshkov, Otterbach, Fleischhauer, Pohl, Lukin (2011);  
Otterbach, Muth, Moos, Fleischhauer (2013);  
Peytrosoyan, Otterbach, Fleischhauer (2011);  
He, Sharypov, Shen, Simon, Xiao (2014);  
Bienias, Choi, Firstenberg, Maghrebi, Gullan, Lukin, Gorshkov, Büchler (2014);

## Exp:

Jones, Adams, Durham  
Rempe, Dürr, Munich  
Firstenberg, Peyronel, Liang, Gorshkov, Lukin, Vuletic (2013);  
Peyronel, Firstenberg, Liang, Hofferberth, Gorshkov, Pohl, Lukin, Vuletic (2012);  
Parigi, Bimbard, Stanojevic, Hilliard, Nogrette, Tualle-Brouri, Ourjoumtsev, Grangier (2012);  
Dudin and Kuzmich, (2012);  
Pritchard, Maxwell, Gauguet, Weatherill, Jones, Adams (2010);

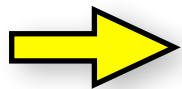
S. Hofferberth (Stuttgart)  
O. Firstenberg (Weizmann)  
J. Simon (Chicago)

# Interacting Rydberg slow light polaritons

(Firstenberg et al 2013)

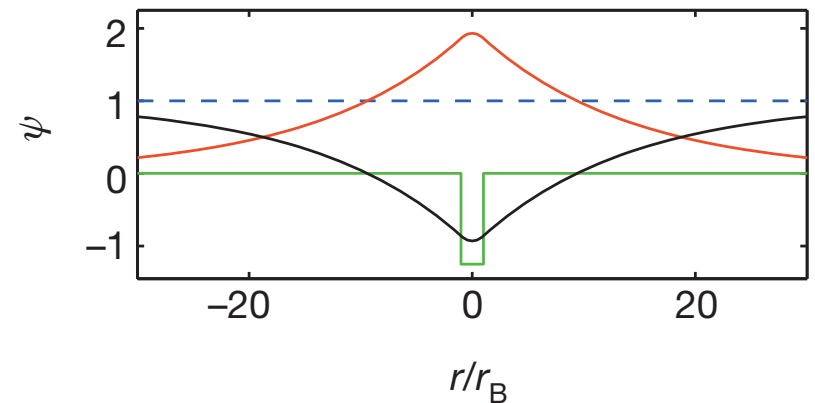
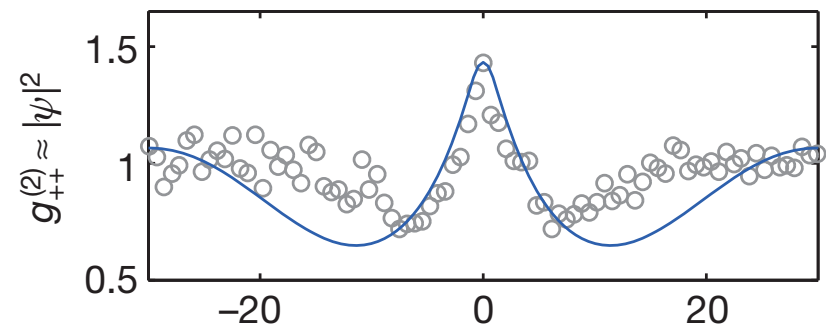
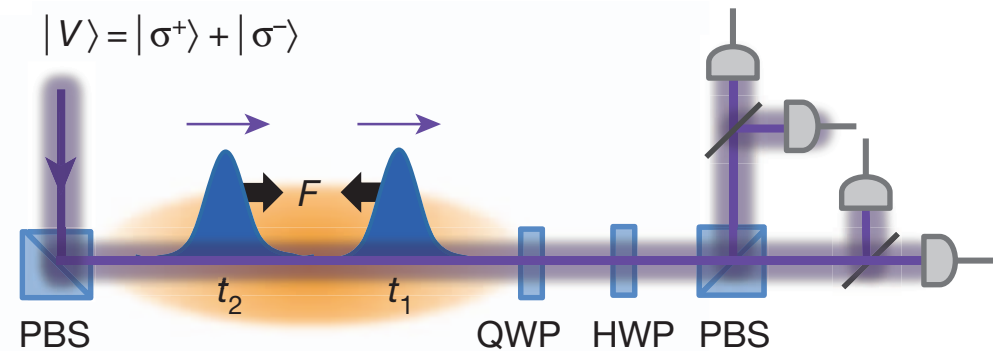
## Photonic bound state

- low intensity coherent light on EIT resonance
- two-photon correlations after the media



characteristic correlations as expected for a bound state

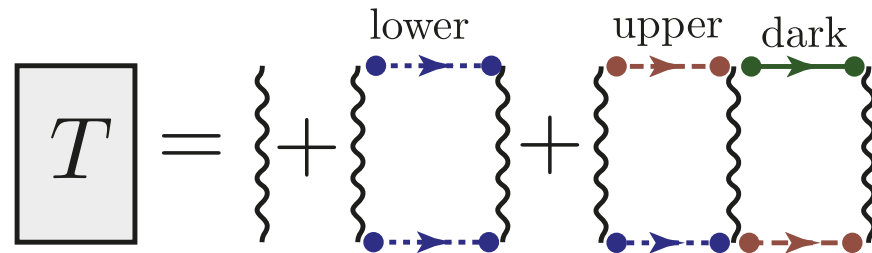
- Theorie:
- full numerical simulations for two photons
  - approximative equation derived by comparison with full solution



# Outline

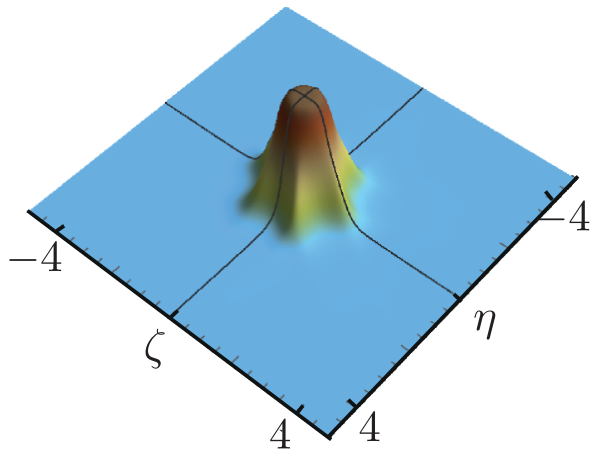
## Many-body theory

- microscopic derivation
- two-photon: bound states and scattering states



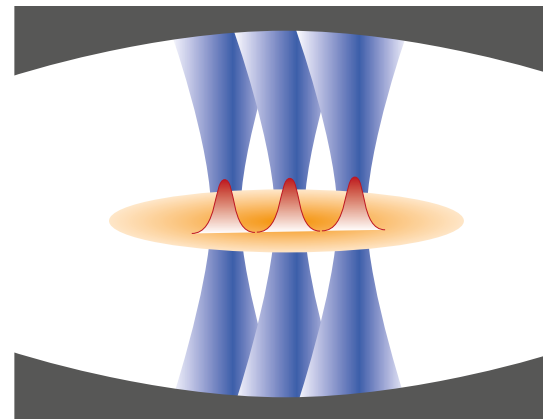
## Three-body interactions

- demonstration of strong influence



## Applications

- extension into two-dimensions
- multi-mode cavity
- phase gates



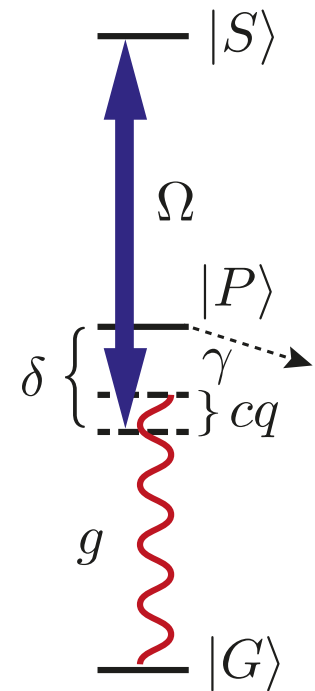


# Microscopic Hamiltonian

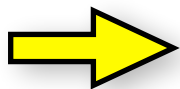
- restriction to a single transverse mode
- continuous distribution of atoms
- photonic density much smaller than atomic density

$$H_0 = \hbar \int dz \begin{pmatrix} \psi_e^\dagger \\ \psi_p^\dagger \\ \psi_s^\dagger \end{pmatrix} \begin{pmatrix} -ic\partial_z & g & 0 \\ g & \Delta & \Omega \\ 0 & \Omega & 0 \end{pmatrix} \begin{pmatrix} \psi_e \\ \psi_p \\ \psi_s \end{pmatrix}$$

kinetic energy for photons



- rotating frame and rotating wave approximation
- chiral photonic field



no back scattering as spatial variations are smooth on the photonic wave length

- collective coupling:  $g = g_0 \sqrt{n}$  atomic density

- detuning:  $\Delta = \delta - i\gamma$  decay from p-level

$\psi_e^\dagger(z)$  : photon creation operator with transverse mode

$$u_\perp(\mathbf{x}) e^{ik_c z}$$

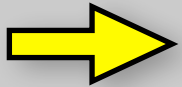
$\psi_p^\dagger(z)$  : creation operator for the excitation into p-level

$\psi_s^\dagger(z)$  : creation operator for the excitation into s-level

# Hamiltonian

## Non-interacting polaritons

$$H_0 = \hbar \int dz \begin{pmatrix} \psi_e^\dagger \\ \psi_p^\dagger \\ \psi_s^\dagger \end{pmatrix} \begin{pmatrix} -ic\partial_z & g & 0 \\ g & \Delta & \Omega \\ 0 & \Omega & 0 \end{pmatrix} \begin{pmatrix} \psi_e \\ \psi_p \\ \psi_s \end{pmatrix}.$$



$$H_0 = \sum_{q, \alpha \in \{0, \pm 1\}} \epsilon_{\alpha q} \bar{\psi}_{\alpha q}^\dagger \tilde{\psi}_{\alpha q}$$

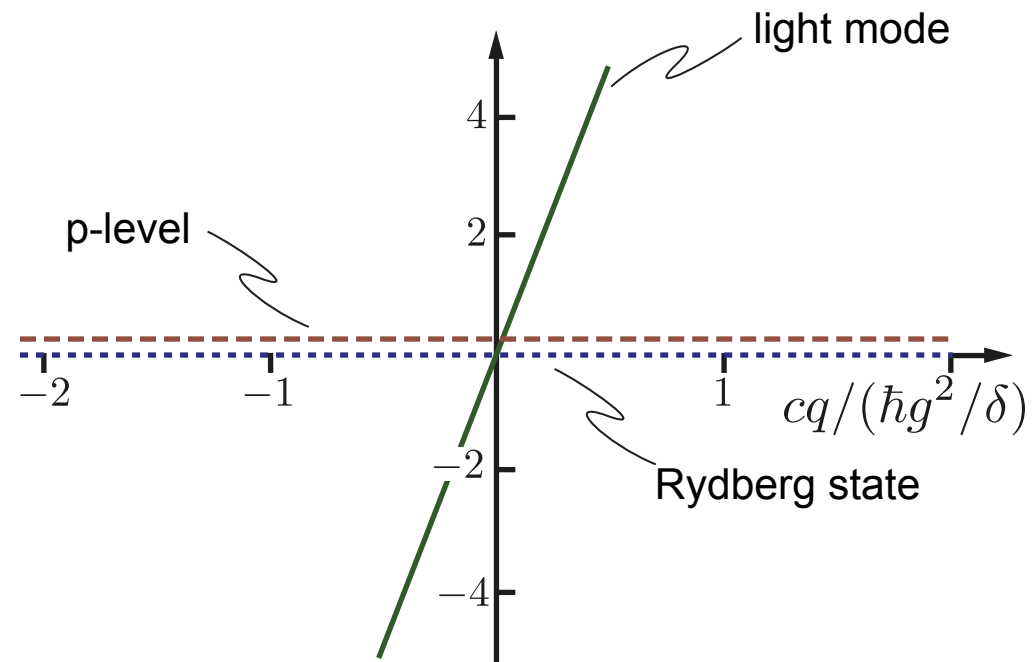
$$\tilde{\psi}_{\alpha q} = \sum_{\beta \in \{e, p, s\}} U_\alpha^\beta(q) \psi_{\beta q}$$

- dispersion for slow light polariton

$$\epsilon_0 = v_g q + \frac{\hbar^2}{2m} q^2 + \dots$$

slow light  
velocity:  $v_g = \frac{\Omega^2}{g^2 + \Omega^2} c$

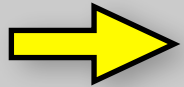
effective  
mass:  $m = \hbar \frac{(g^2 + \Omega^2)^3}{2c^2 g^2 \Delta \Omega^2}$



# Hamiltonian

## Non-interacting polaritons

$$H_0 = \hbar \int dz \begin{pmatrix} \psi_e^\dagger \\ \psi_p^\dagger \\ \psi_s^\dagger \end{pmatrix} \begin{pmatrix} -ic\partial_z & g & 0 \\ g & \Delta & \Omega \\ 0 & \Omega & 0 \end{pmatrix} \begin{pmatrix} \psi_e \\ \psi_p \\ \psi_s \end{pmatrix}.$$



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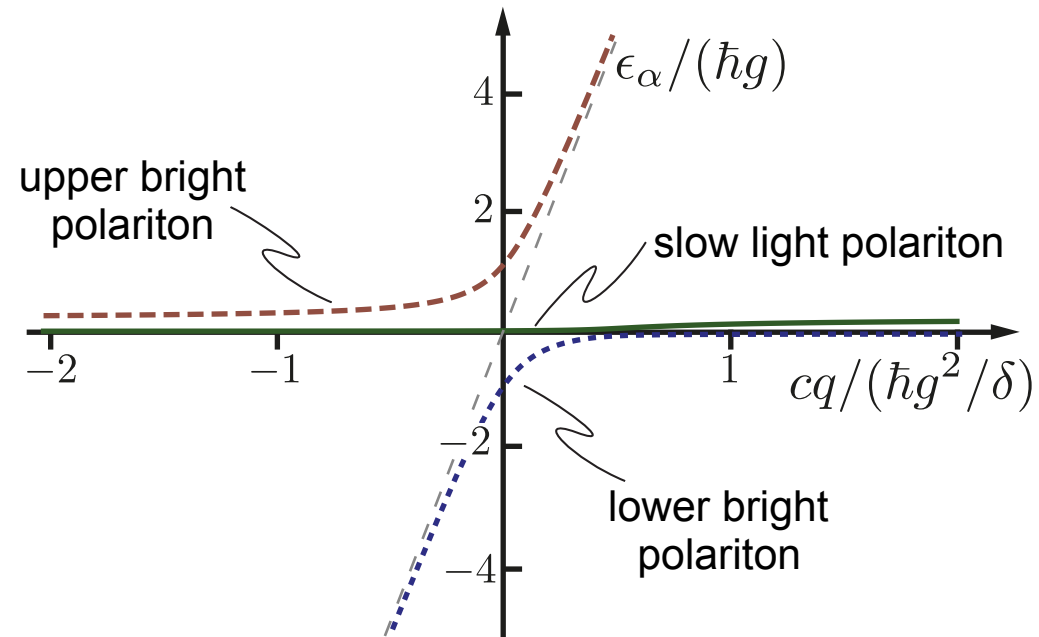
$$\tilde{\psi}_{\alpha q} = \sum_{\beta \in \{e, p, s\}} U_\alpha^\beta(q) \psi_{\beta q}$$

- dispersion for slow light polariton

$$\epsilon_0 = v_g q + \frac{\hbar^2}{2m} q^2 + \dots$$

slow light velocity:  $v_g = \frac{\Omega^2}{g^2 + \Omega^2} c$

effective mass:  $m = \hbar \frac{(g^2 + \Omega^2)^3}{2c^2 g^2 \Delta \Omega^2}$



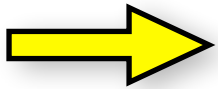
# Microscopic Hamiltonian

## Interaction Hamiltonian

- strong van der Waals interaction between Rydberg atoms

$$V(z) = \frac{C_6}{z^6} \quad : \text{attractive as well as repulsive interactions are possible}$$

$$H_{\text{int}} = \frac{1}{2} \int dz dz' V(z - z') \psi_s^\dagger(z) \psi_s^\dagger(z') \psi_s(z') \psi_s(z)$$



$$H = H_0 + H_{\text{int}}$$

: three bosonic fields  
with quartic interaction

: energy and momentum conservation

: broken Galilei/Lorentz invariance

# Goal

Is there a many-body theory for slow light polaritons alone?

## Two-polariton problem

- scattering properties
- two-photon bound states

- effective interaction potential
- pseudo-potential for slow light polaritons
- many-body theory in dilute regime
- three-body interactions as small correction

In analogy:

- interactions in cold atoms are determined by s-wave scattering length



$$V_{\text{eff}}(r) = \frac{4\pi\hbar^2 a_s}{m} \delta(r) \partial_r r$$

# Two-body problem

## Two-body problem

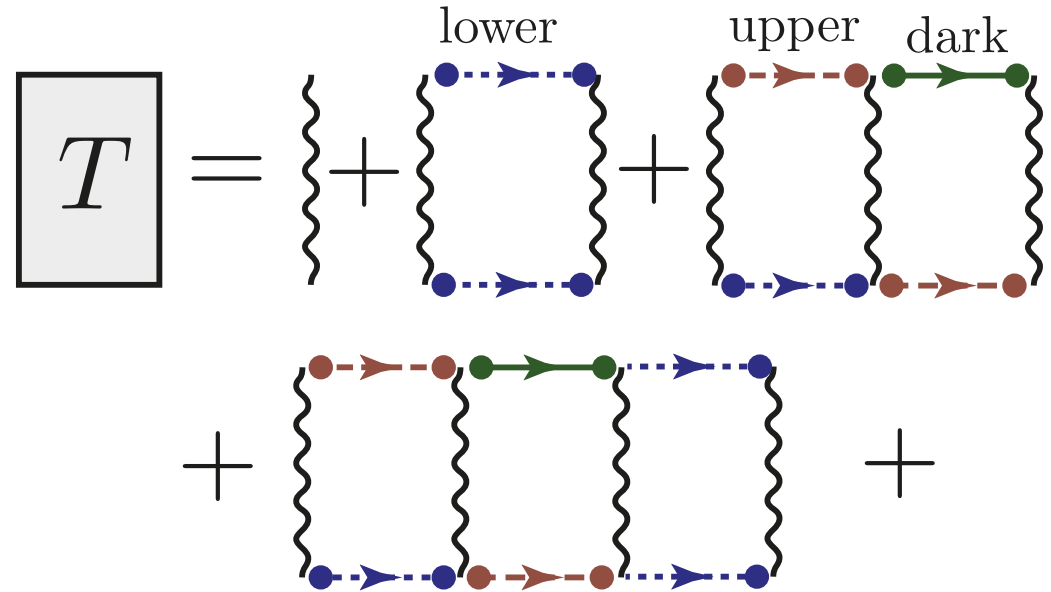
- general equation involves 9 wave functions

$$\psi_{\alpha\beta}(z, z') \quad \alpha, \beta \in \{e, p, s\}$$

- fixed energy and center of mass momentum:

$$\hbar\omega \quad \hbar K$$

- T-matrix for the Rydberg part: resummation of all ladder diagrams



$$T_{kk'}(K, \omega) = V_{k-k'} + \int \frac{dq}{2\pi} V_{k-q} \chi_q(K, \omega) T_{qk'}(K, \omega)$$

- two-particle propagator

$$\chi_q(K, \omega) = \sum_{\alpha, \beta \in \{0, \pm 1\}} \frac{\bar{U}_s^\alpha(p) U_\alpha^s(p) \bar{U}_s^\beta(p') U_\beta^s(p')}{\hbar\omega - \epsilon_\alpha(p) - \epsilon_\beta(p') + i\eta}, \quad \begin{aligned} p &= K/2 + q \\ p' &= K/2 - q \end{aligned}$$

# Two-particle propagator

General behavior

$$\chi_q = \bar{\chi} + \frac{\alpha}{\hbar\bar{\omega} - \hbar^2 q^2/m + i\eta} + \frac{\alpha_B}{\hbar\bar{\omega}_B - \hbar^2 q^2/m + i\eta}$$

- saturation for large relative momenta

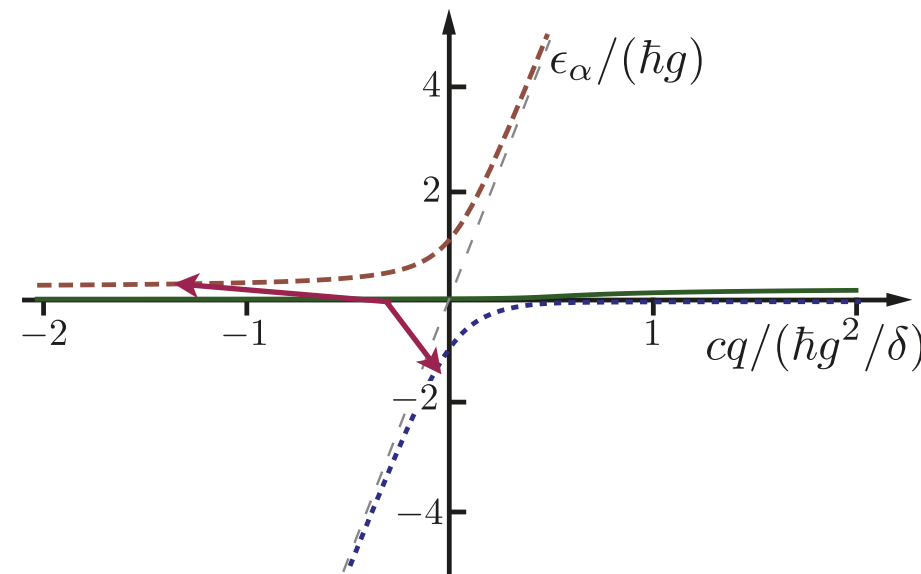
$$\bar{\chi}(\omega) = \frac{1}{\hbar\omega} \frac{\Delta - \frac{\omega}{2} - \frac{\Omega^2}{\Delta - \omega}}{+ 2\Omega^2}$$

- pole for propagation of slow light polariton:

$$\bar{\omega}(K, \omega) \quad \alpha(K, \omega)$$

- resonant excitation into two bright polaritons

$$\bar{\omega}_B(K, \omega) \quad \alpha_B(K, \omega)$$



# Two-particle propagator

$$\chi_q = \bar{\chi} + \frac{\alpha}{\hbar\bar{\omega} - \hbar^2 q^2/m + i\eta} + \frac{\alpha_B}{\hbar\bar{\omega}_B - \hbar^2 q^2/m + i\eta}$$

Effective interaction potential

- T-matrix equation

$$\begin{aligned} T_{kk'}(K, \omega) &= V_{k-k'} + \int \frac{dq}{2\pi} V_{k-q} \chi_q(K, \omega) T_{qk'}(K, \omega) \\ &= V_{k-k'}^{\text{eff}} + \int \frac{dq}{2\pi} V_{k-q}^{\text{eff}} [\chi_q(K, \omega) - \bar{\chi}(\omega)] T_{qk'}(K, \omega) \end{aligned}$$



# Two-particle propagator

$$\chi_q = \bar{\chi} + \frac{\alpha}{\hbar\bar{\omega} - \hbar^2q^2/m + i\eta} + \frac{\alpha_B}{\hbar\bar{\omega}_B - \hbar^2q^2/m + i\eta}$$

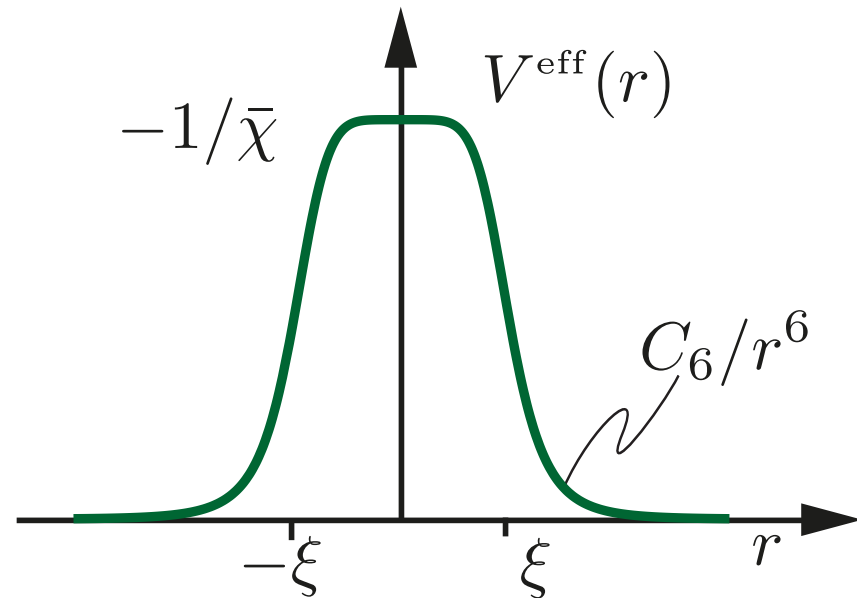
Effective interaction potential

$$V^{\text{eff}}(r) = \frac{V(r)}{1 - \bar{\chi}(\omega)V(r)}$$

saturation on the blockade radius

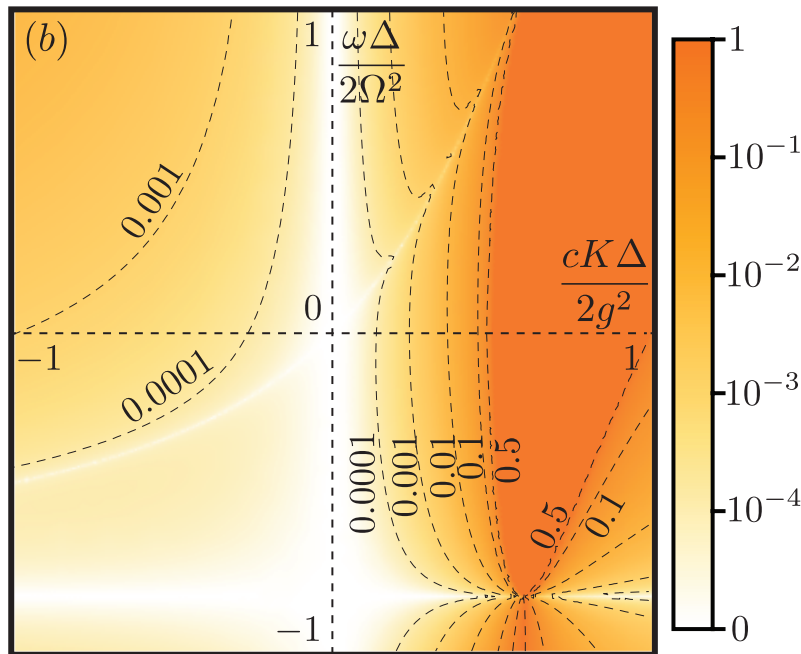
$$\xi = (|C_6|\bar{\chi})^{1/6}$$

resonance feature for two-Rydberg excitations possible



# Two-particle propagator

$$\chi_q = \bar{\chi} + \frac{\alpha}{\hbar\bar{\omega} - \hbar^2q^2/m + i\eta} + \frac{\alpha_B}{\hbar\bar{\omega}_B - \hbar^2q^2/m + i\eta}$$



Influence of resonant excitation into bright polaritons

- second pole vanishes for

- low momenta and energy regime

- far detuning  $\Omega \ll |\Delta|$

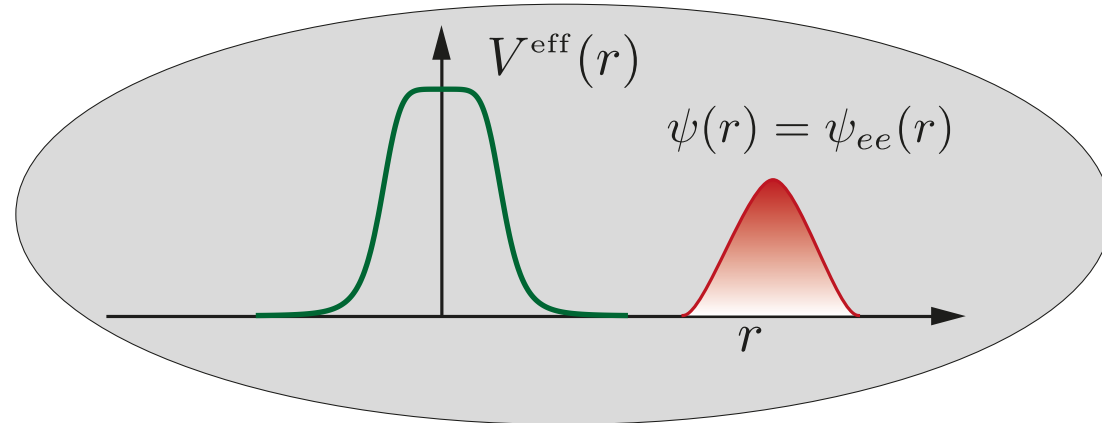
- extremely small for many experimentally relevant regimes

$$\zeta(K, \omega) = \sqrt{|(\bar{\omega}\alpha_B^2)/(\bar{\omega}_B\alpha^2)|}$$

# Effective Schrödinger equation

## Effective Schrödinger equation

- wave function two polaritons
- Schrödinger equation for two-polariton wave function



$$\hbar\bar{\omega}\psi(r) = \left[ -\frac{\hbar^2}{m}\partial_r^2 + \alpha V_{\text{eff}}(r) \right] \psi(r)$$

on-shell  
condition

massive  
particles

overlap of polaritons  
into Rydberg state

effective  
interaction

- parameters depend on total energy and center of mass

$$\bar{\omega}(K, \omega) \quad \alpha(K, \omega)$$

# Low energy and momentum regime

Interaction strength:  $\xi/\lambda$

$$\lambda = \sqrt{|\hbar^2 \bar{\chi} / (\alpha m)|} \quad \text{de-Broglie wavelength}$$

$$\xi = (|C_6 \bar{\chi}|)^{1/6} \quad \text{blockade radius}$$

Scattering properties

- weak interactions

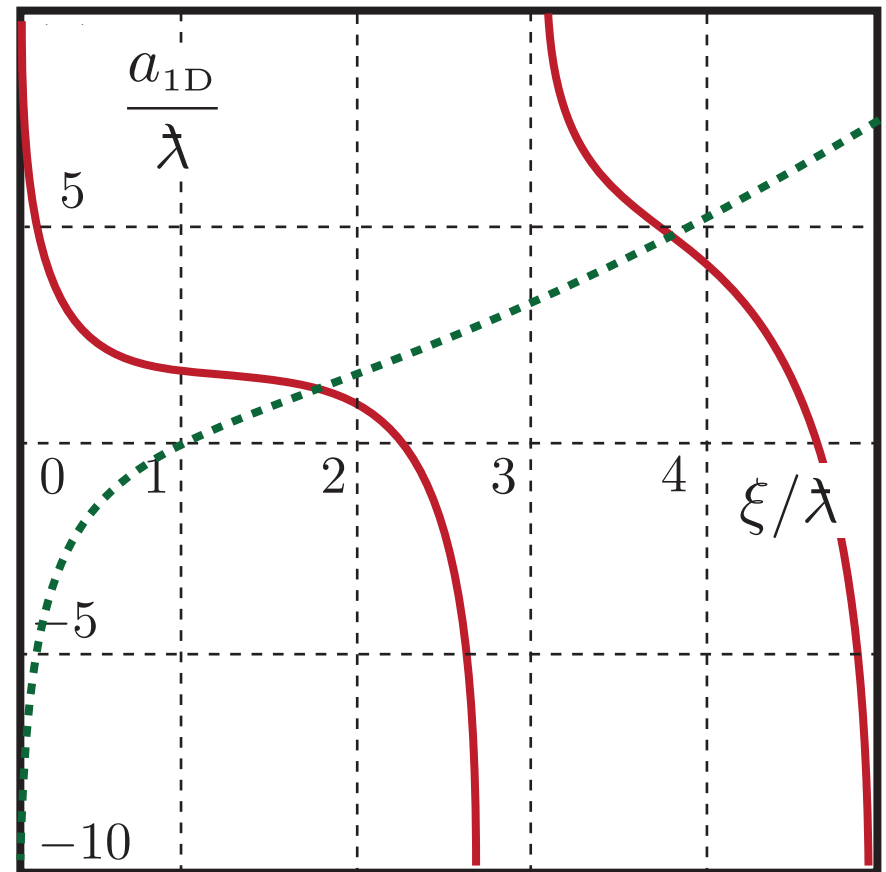
$$a_{1D} = \frac{3}{\pi} \left( -\frac{\bar{\chi}^5}{C_6} \right)^{1/6} \frac{\hbar^2}{\alpha m}$$

- repulsive interaction

zero crossing for the  
1D scattering length

- attractive interaction

scattering resonances  
for each additional bound  
state appearing

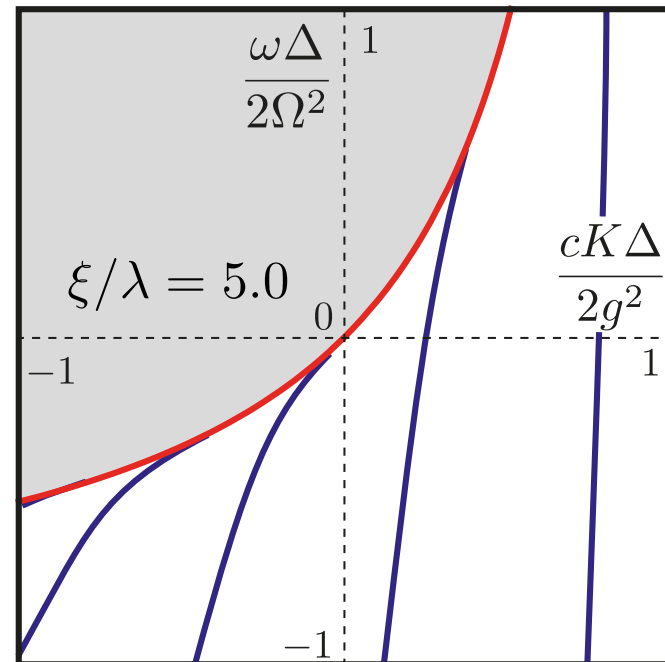
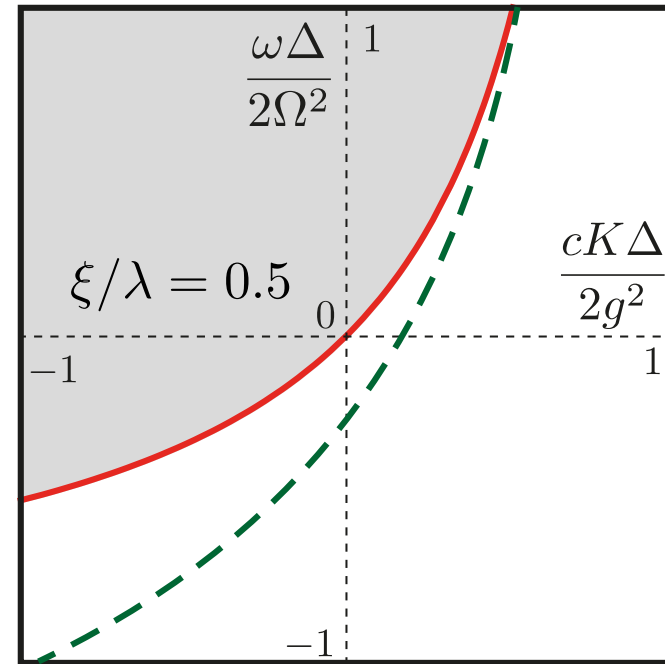


“universal” low energy scattering length

# Far detuned regime ( $\Omega \ll |\Delta|$ )

## Bound state structure

- bound state energy depends on interaction strength and center of mass momentum
- requires self-consistent evaluation
- appearance of several bound states
- bound states have a higher group velocity higher than



# Many-body theory

Effective theory for Rydberg  
slow light polaritons

kinetic energy

$$H = \int dx \psi^\dagger \left( -i\hbar v_g \partial_z - \frac{\hbar^2}{2m} \partial_z^2 \right) \psi$$
$$+ \frac{1}{2} \int dx dy V^{\text{eff}}(x-y) \psi^\dagger(x) \psi^\dagger(y) \psi(y) \psi(x) + \dots$$

two-body interaction

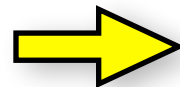
higher body  
interactions

Validity:

- low energy and momentum regime
- three-body interaction
- suppressed for weak interactions:
- suppressed in dilute regime:

$$\xi/\lambda \ll 1$$

$$n_d \xi \ll 1$$



- Lieb-Liniger model  $a_{1D} < 0$
- Super Tonks-Girardeau  $a_{1D} > 0$

# Many-body theory

Experimental probe  
of many-body interactions?

parameter regime, where mass  
can be neglected

$$H = \int dx \psi^\dagger \left( -i\hbar v_g \partial_z - \frac{\hbar^2}{2m} \partial_z^2 \right) \psi + \frac{1}{2} \int dx dy V^{\text{eff}}(x-y) \psi^\dagger(x) \psi^\dagger(y) \psi(y) \psi(x) + \dots$$

two-body interaction

higher body  
interactions

Exact solvable theory for  
arbitrary input

(Bienias, HPB, arXiv 2016)

- two photon solution

$$\phi^{\text{out}}(x, y, t) = e^{-i\varphi(x-y)} \phi^{\text{in}}(x - ct', y - ct')$$

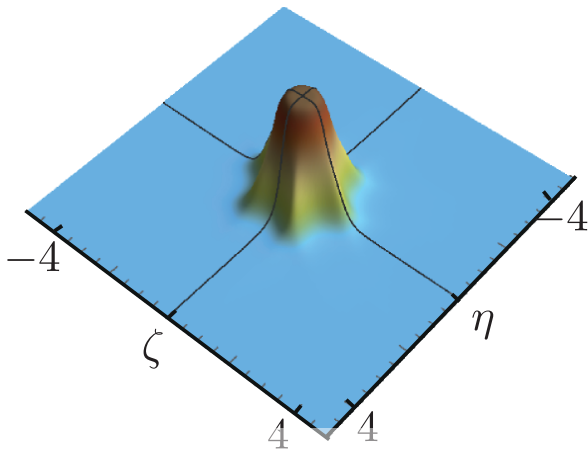
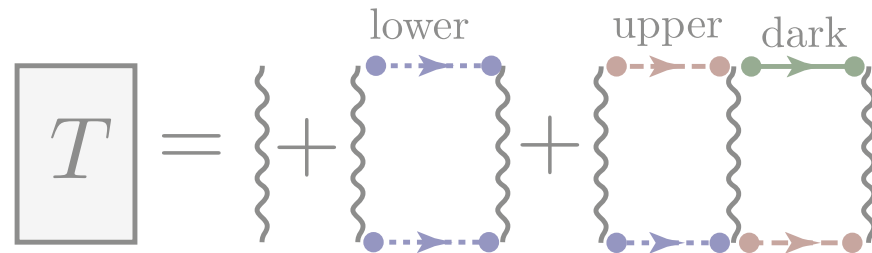
- effective interaction  
accessible in homodyne  
detection

$$\varphi(u) = \frac{1}{\hbar c} \int_{-\infty}^{\infty} dw \tilde{n}(w+u) \tilde{n}(w) \tilde{V}(w+u, w)$$

# Outline

## Many-body theory

- microscopic derivation
- two-photon: bound states and scattering states

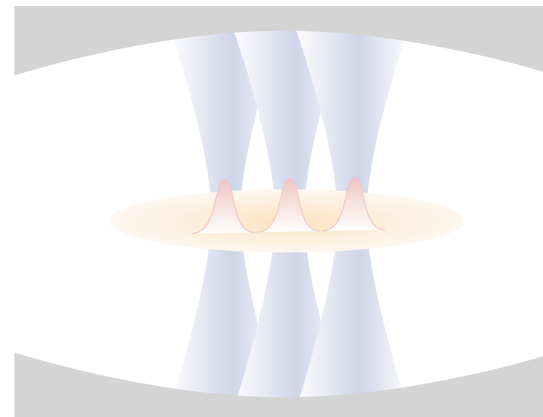


## Three-body interactions

- demonstration of strong influence

## Applications

- extension into two-dimensions
- multi-mode cavity
- phase gates





# Three-body interactions

(Jachymski, Binas, HPB, PRL 2016; see also Gullans et al, PRL 2016)

Simple estimation of interaction strength inside the blockade radius: (far detuned regime  $|\Delta| \gg |\Omega|$ )

Probability to find 1 Rydberg state and (n-1) Photons:

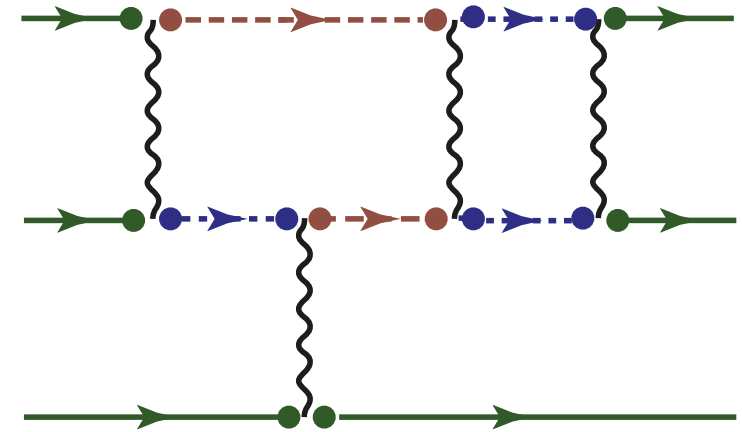
$$ng^2 \frac{\Omega^{2(n-1)}}{(g^2 + \Omega^2)^n}$$

Dispersive energy shift of a photon inside the blockade regime:

$$-\frac{\hbar g^2}{\Delta}$$

Total dispersive energy shift:

$$-\frac{\hbar g^2}{\Delta} (n-1) ng^2 \frac{\Omega^{2(n-1)}}{(g^2 + \Omega^2)^n}$$



Two-body:

$$-\frac{2\hbar\Omega^2}{\Delta} \frac{g^4}{(\Omega^2 + g^2)^2}$$

Three-body interactions:

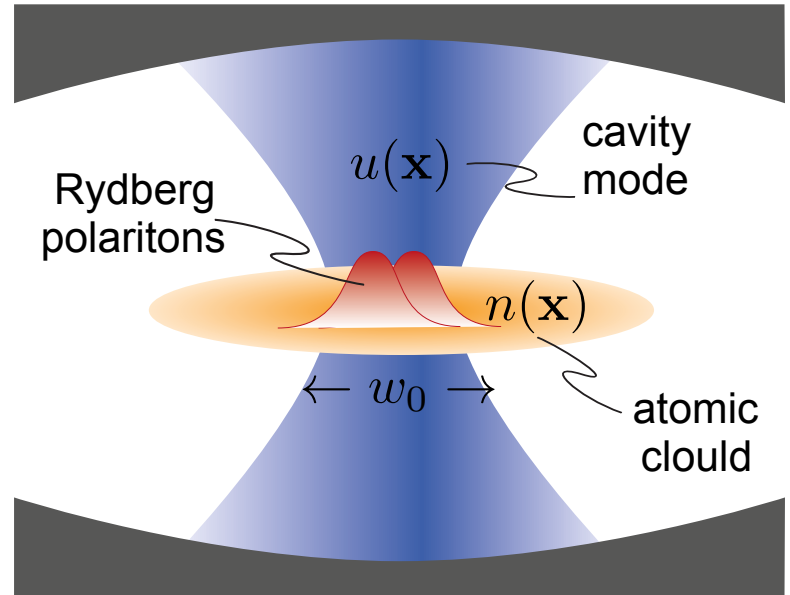
$$-\frac{6\hbar\Omega^2}{\Delta} \frac{\Omega^2 g^4}{(\Omega^2 + g^2)^3} + 3 \frac{2\hbar\Omega^2}{\Delta} \frac{g^4}{(\Omega^2 + g^2)^2} = \frac{6\hbar\Omega^2}{\Delta} \frac{g^6}{(\Omega^2 + g^2)^3}$$

repulsive and very strong for slow light polaritons

# Single mode cavity

Energy shift for two polaritons in a cavity

- large cavity mode:  $w_0 \gg \xi$



$$\Delta E = \int d\mathbf{x}d\mathbf{y} |h(\mathbf{x})|^2 |h(\mathbf{y})|^2 V_{\text{eff}}^{(2)}(\mathbf{x} - \mathbf{y})$$

polariton wave function:

$$h(\mathbf{x}) = \sqrt{n(\mathbf{x})}u(\mathbf{x})$$

Effective interaction

$$\alpha = \frac{g^2}{\Omega^2 + g^2} : \text{Probability for the polariton in the Rydberg state}$$

$$V_{\text{eff}}^{(2)}(\mathbf{x}) = \alpha^2 \frac{V(\mathbf{x})}{1 - \bar{\chi}V(\mathbf{x})}$$

## Derivation

- two photon wave function
- set of coupled equations
- expansion in small energy shift

$$\omega\phi_0 = -2\nu\phi_0 + \int dx \phi_1(x)h^*(x),$$

$$\omega\phi_1(x) = -(\nu + \frac{1}{\nu})\phi_1(x) + 2h(x)\phi_0 + 2 \int dy \phi_2(x, y)h^*(y),$$

$$\omega\phi_2(x, y) = -\frac{2}{\nu}\phi_2(x, y) + \frac{h(x)\phi_1(y) + h(y)\phi_1(x)}{2} + V(x - y)\phi_2(x, y).$$

# Three-body interaction

- Energy shift for three polaritons in the cavity

$$\Delta E = \int d\mathbf{x}_1 d\mathbf{x}_2 d\mathbf{x}_3 |h(\mathbf{x}_1)|^2 |h(\mathbf{x}_2)|^2 |h(\mathbf{x}_3)|^2 U(\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3)$$

- includes two- and three-body interactions

$$U(\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3) = \sum_{i<j} V_{\text{eff}}^{(2)}(\mathbf{x}_i - \mathbf{x}_j) + V_{\text{eff}}^{(3)}(\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3)$$

- analytical expression for three-body interaction

$$V_{\text{eff}}^{(3)}(\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3) = \alpha^3 \sum_{i<j} \frac{V_3(\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3) - V(\mathbf{x}_i - \mathbf{x}_j)}{1 - \bar{\chi} V(\mathbf{x}_i - \mathbf{x}_j)}$$

$$V_3(\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3) = \frac{\sum_{i<j} V(\mathbf{x}_i - \mathbf{x}_j)}{3 - 2\chi \sum_{i<j} V(\mathbf{x}_i - \mathbf{x}_j)}$$

- analog derivation

$$\omega\phi_0 = -3\nu\phi_0 + \int dx \phi_1(x)h^*(x),$$

$$\omega\phi_1(x) = -(2\nu + \frac{1}{\nu})\phi_1(x) + 3h(x)\phi_0 + 2 \int dy \phi_2(x, y)h^*(y),$$

$$\omega\phi_2(x, y) = -(\nu + \frac{2}{\nu})\phi_2(x, y) + h(x)\phi_1(y) + h(y)\phi_1(x) + 3 \int dz \phi_3(x, y, z)h^*(z) + V(x - y)\phi_2(x, y),$$

$$\omega\phi_3(x, y, z) = -\frac{3}{\nu}\phi_3(x, y, z) + h(z)\phi_2(x, y) + h(y)\phi_2(x, z) + h(x)\phi_2(y, z) + W(x, y, z)\phi_3(x, y, z).$$

# Three-body interactions

## Three body interactions

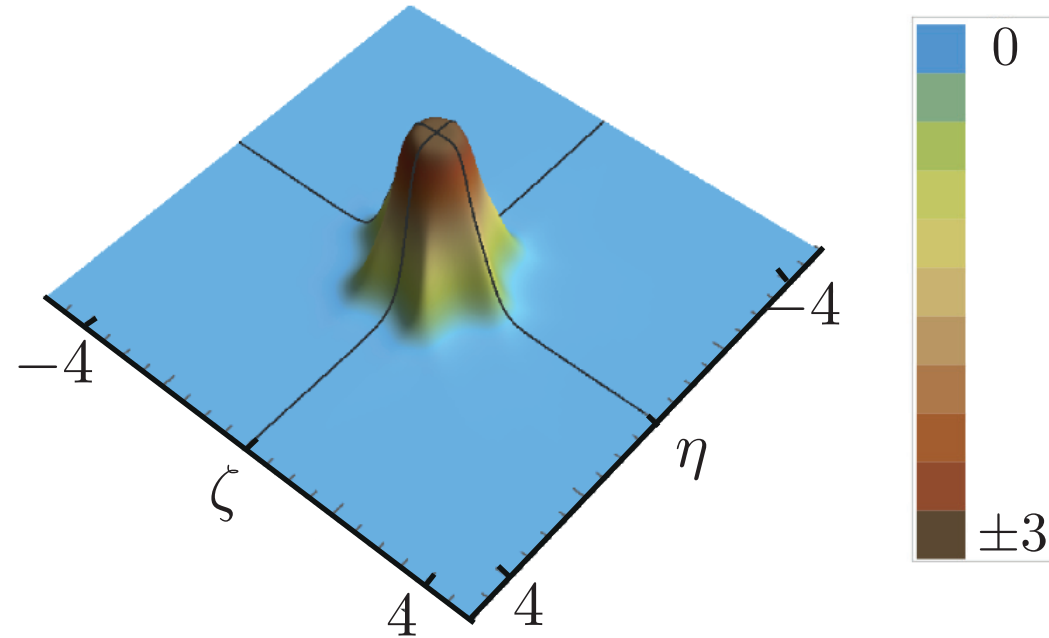
- Jacobi coordinates:

$$\eta = \frac{\mathbf{x}_1 - \mathbf{x}_2}{\sqrt{2} \xi}$$

$$\zeta = \frac{\sqrt{2}}{\sqrt{3} \xi} \left( \frac{\mathbf{x}_1 + \mathbf{x}_2}{2} - \mathbf{x}_3 \right)$$

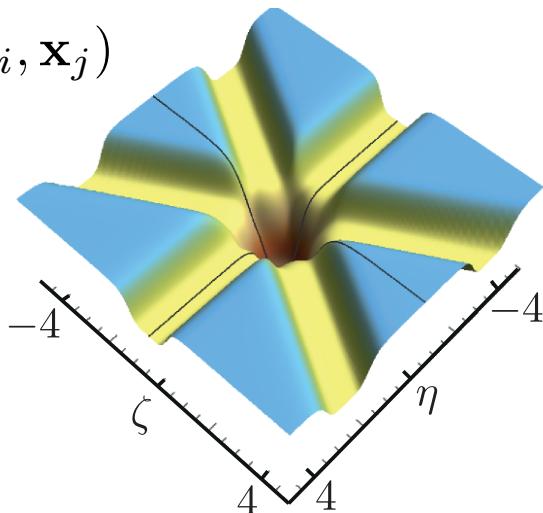
- strong repulsion inside the blockade radius

- compensates the two-body attraction for  $\Omega^2 \ll g^2$



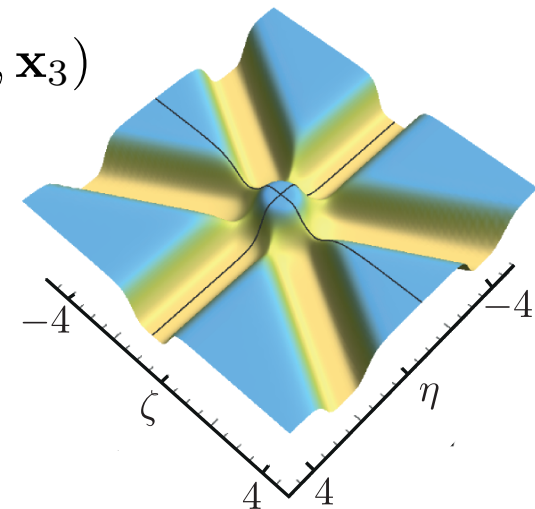
## Two-body contribution

$$\sum_{i < j} V_{\text{eff}}^{(2)}(\mathbf{x}_i, \mathbf{x}_j)$$



## Combined interaction

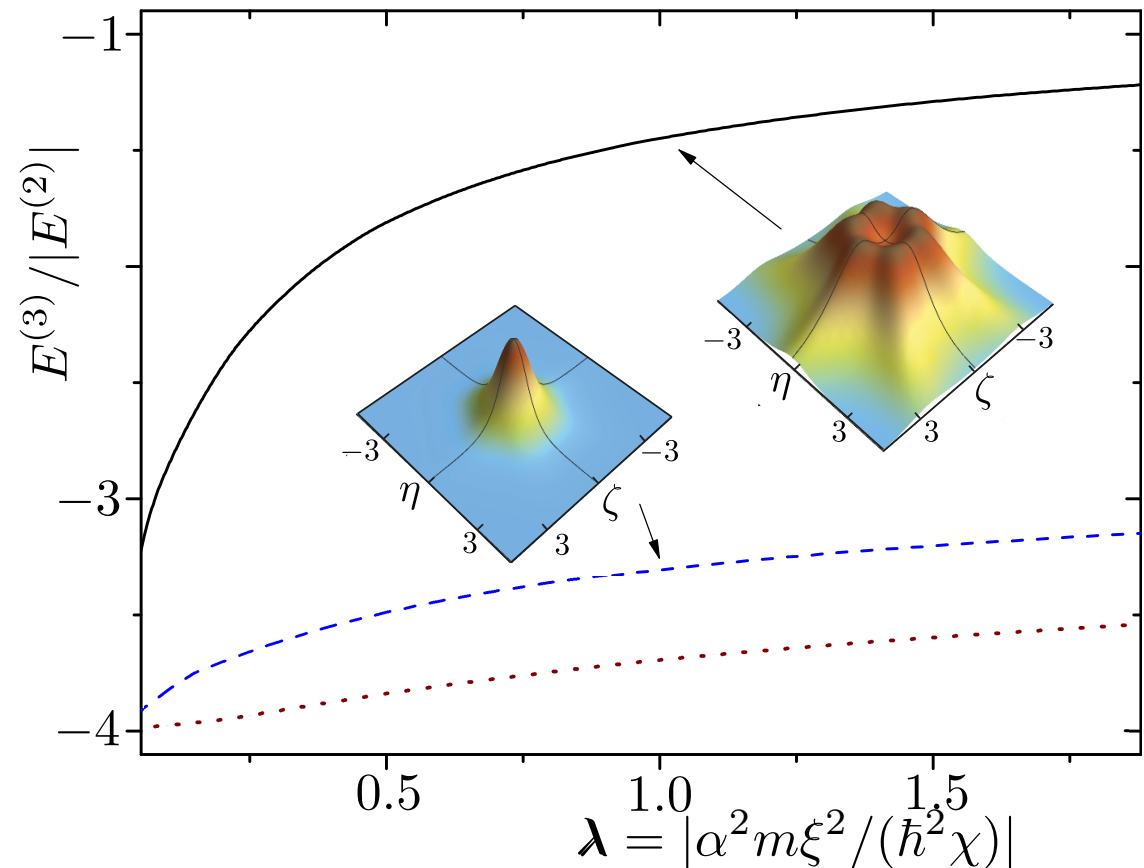
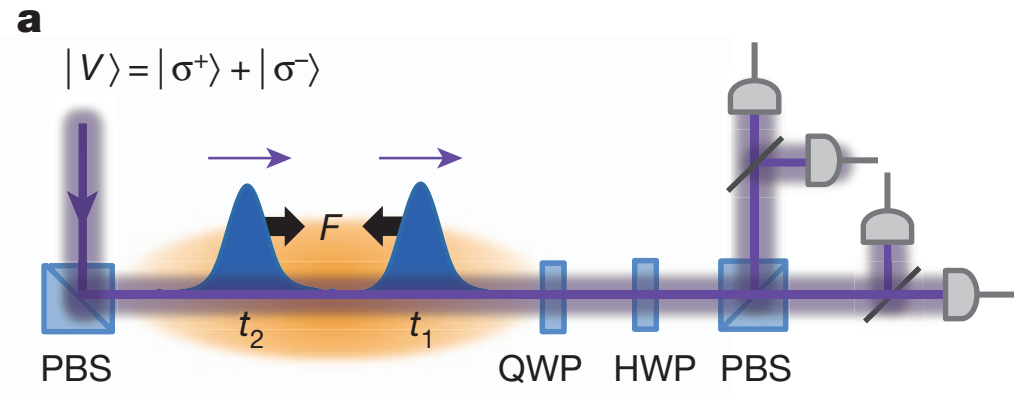
$$U(\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3)$$



# Three-body bound states

## Rydberg slow light polaritons in 1D

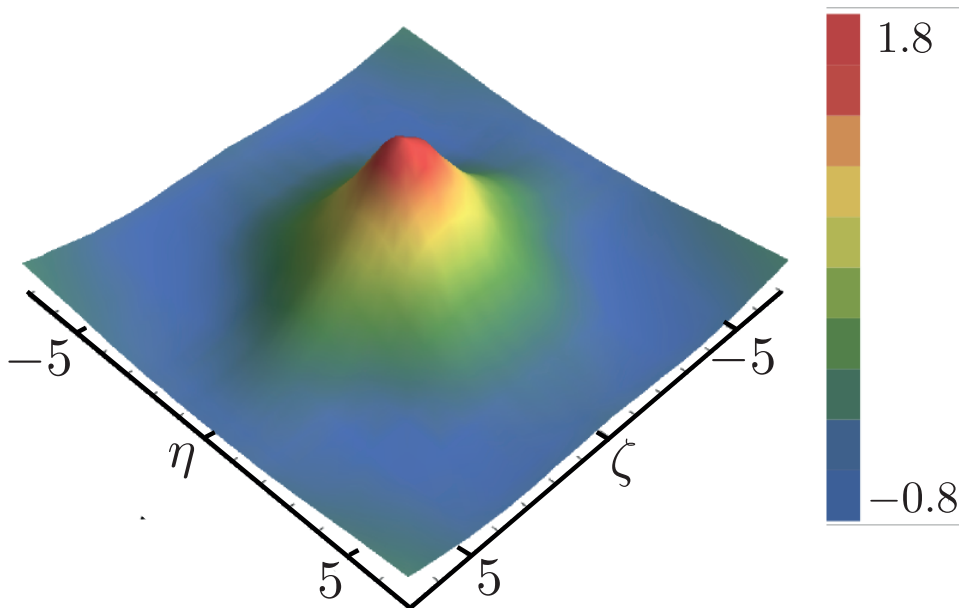
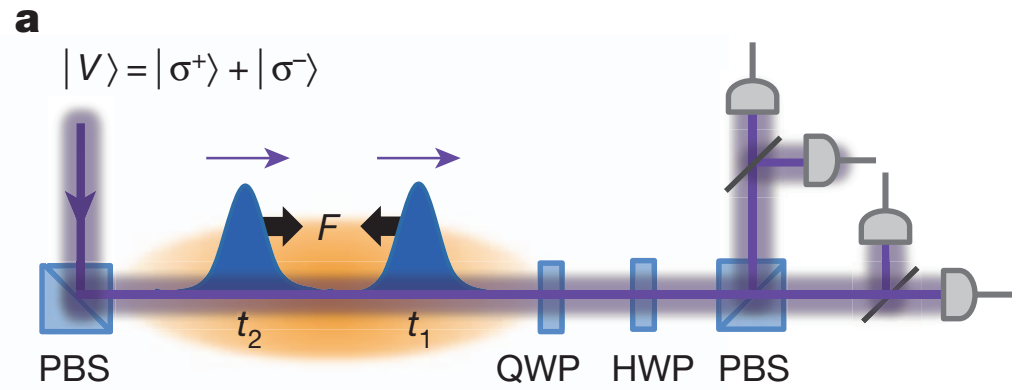
- experimental observation of two-body bound states
- two-body interactions provide also three-body bound state (Lieb-Liniger model)
- for  $\delta$  - function interaction
  - $B$  : two-body bound state energy
  - $4B$  : three-body bound state energy
- strong modifications by three-body interaction
  - three body bound state for arbitrary interactions
  - characteristic shape on short distances due to repulsion



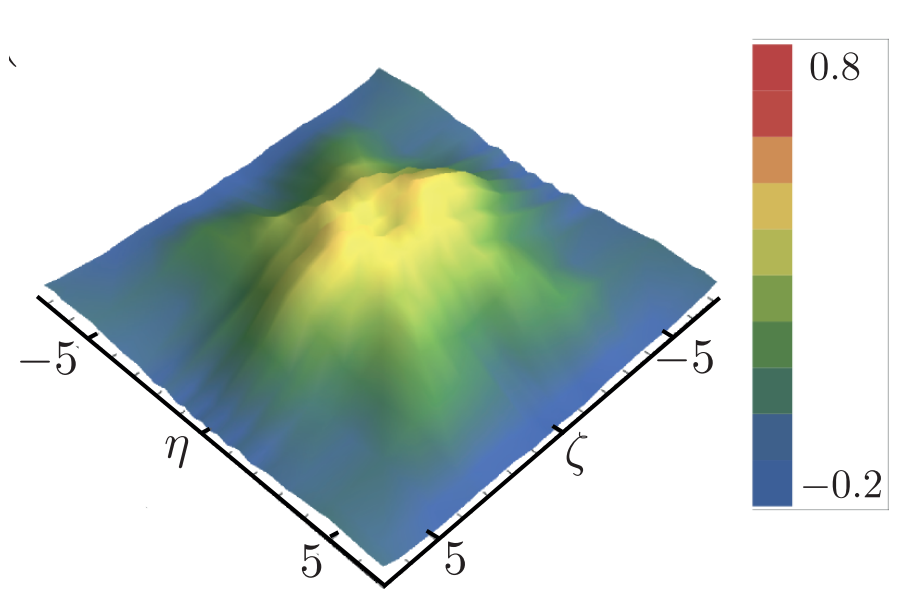
# Three-body correlations

Intensity correlations for the transmitted light

- $g_2$  characteristic peak for two-body bound state
- $g_3$  characteristic behavior of the bound state in the bunching of photons



without three-body interaction

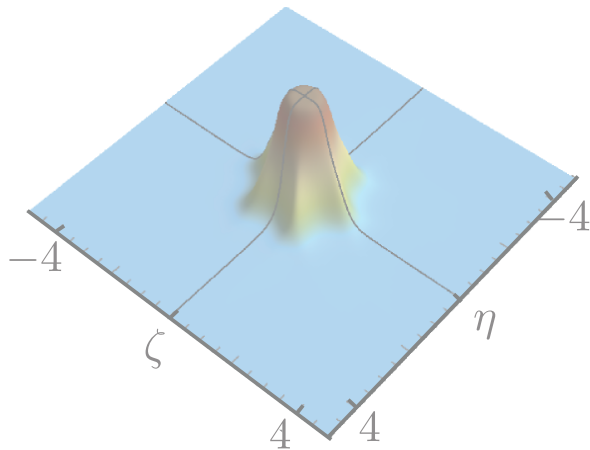
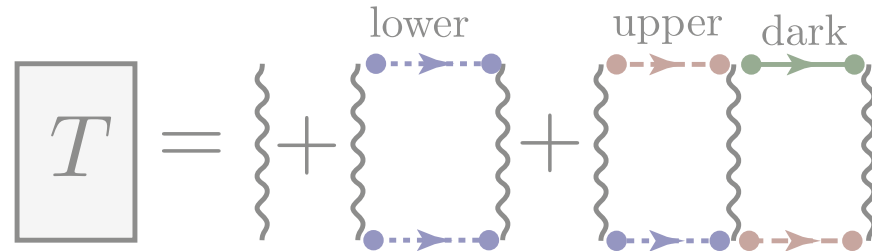


with three-body interaction

# Outline

## Many-body theory

- microscopic derivation
- two-photon: bound states and scattering states

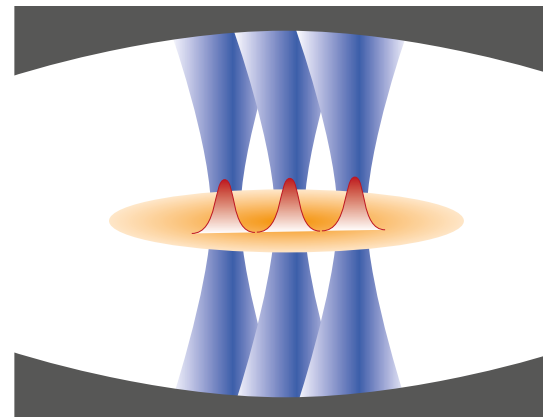


## Three-body interactions

- demonstration of strong influence

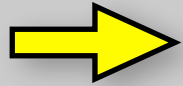
## Applications

- extension into two-dimensions
- multi-mode cavity
- phase gates



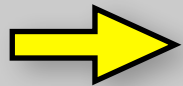
# Extension into two dimensions

Optical multi mode cavity: - several transverse near degenerate modes



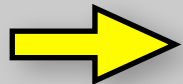
Exp: Confocal cavity  
- harmonic oscillators for photons  
- only even oscillator functions

- kinetic energy of the polaritons:

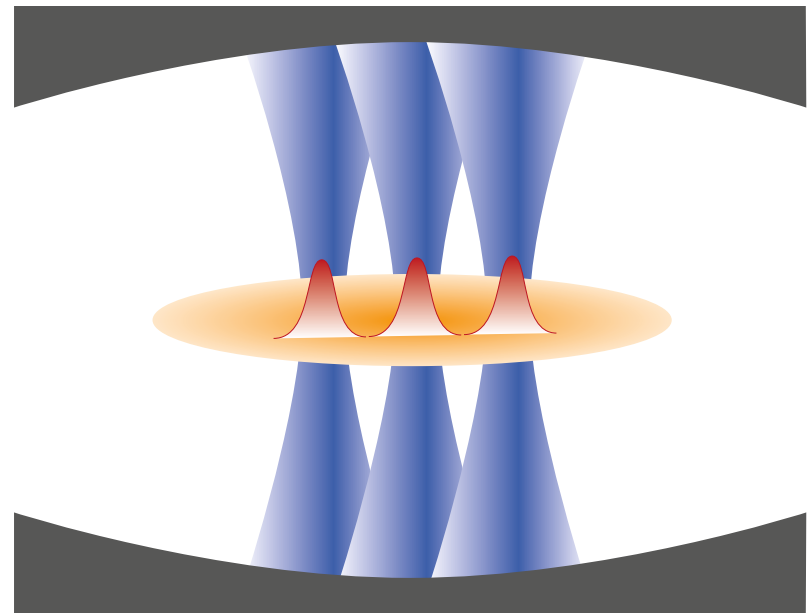


cavity mode spectrum reduced by slow light velocity

- interaction energy



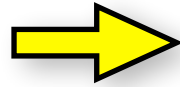
effective interaction



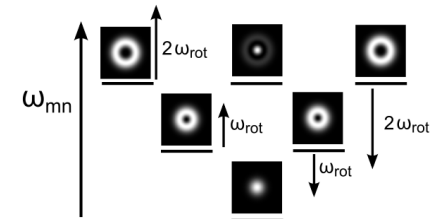
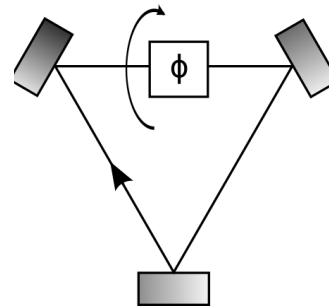


# Extension into two dimensions

The design of the cavity determines the kinetic energy of the photons:

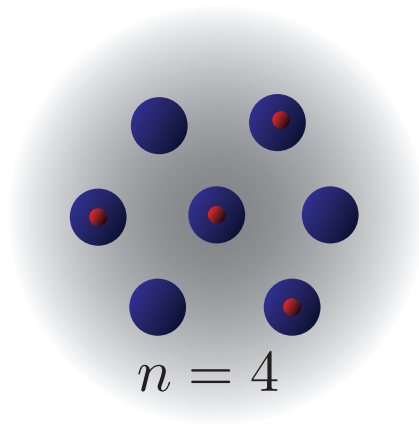


Ring cavity with a phase shift



Example: Confocal cavity

- dominating interaction energy
- harmonic oscillators for photons
- only even oscillator functions
- ordered structure for strong interactions



- low energy photonic modes are Laughlin states

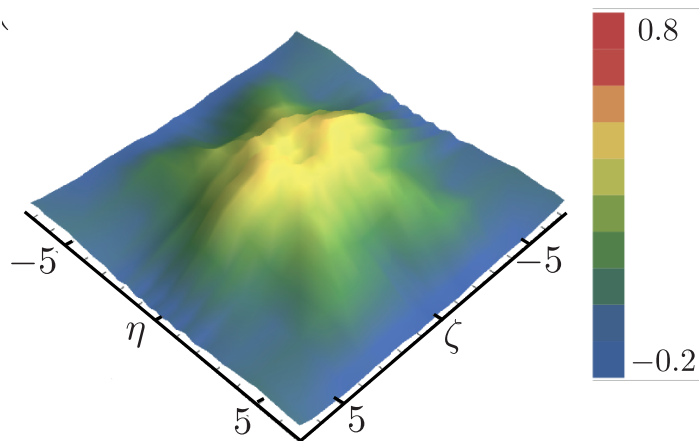


- topological states of matter?
- novel states of matter?

# Conclusions

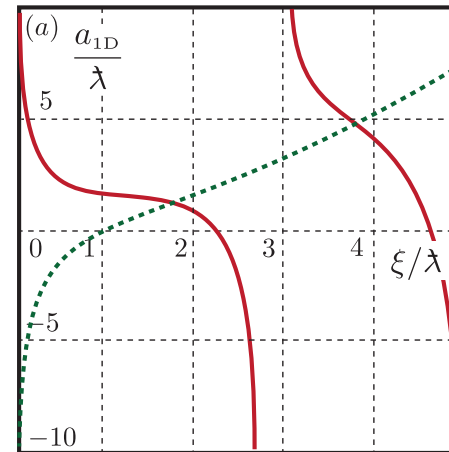
## Theoretical framework for analyzing Rydberg slow light polaritons

- effective theory for slow light polaritons alone
- two-particle properties
- low energy many-body Hamiltonian



## Applications

- tool engineer interesting states of quantum matter in 2D



## Three-body interaction

- correction to many-body theory
- applications on bound states and correlations

