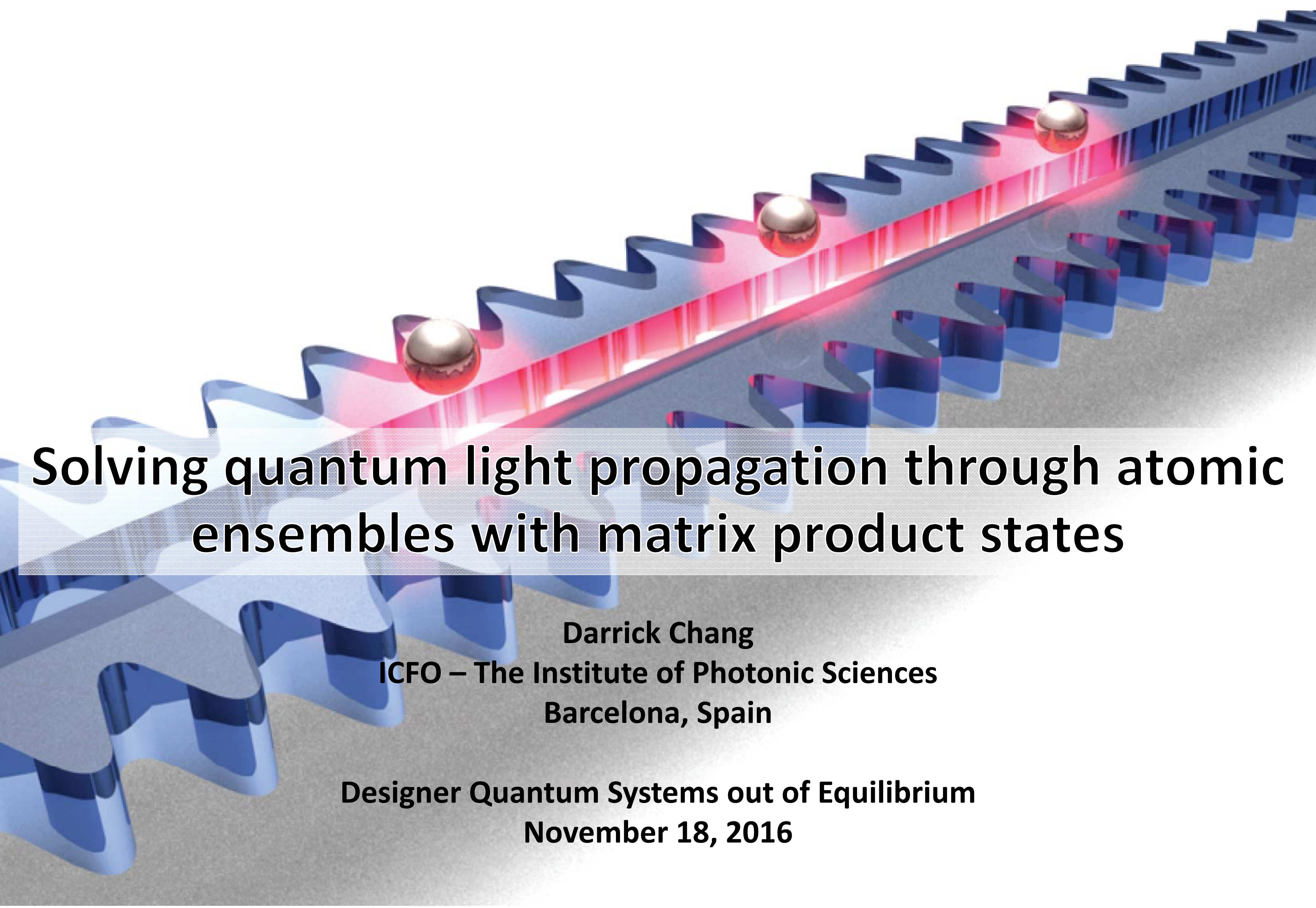
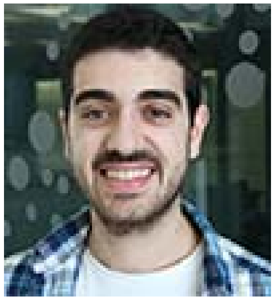


Solving quantum light propagation through atomic ensembles with matrix product states

Darrick Chang
ICFO – The Institute of Photonic Sciences
Barcelona, Spain

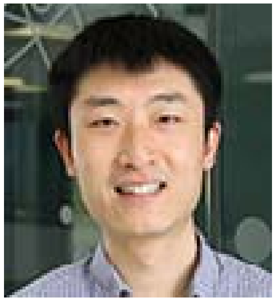
Designer Quantum Systems out of Equilibrium
November 18, 2016

Acknowledgments



Marcos
Lopez

Mariona
Moreno

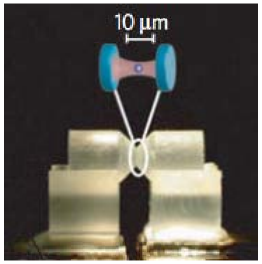


David
Li

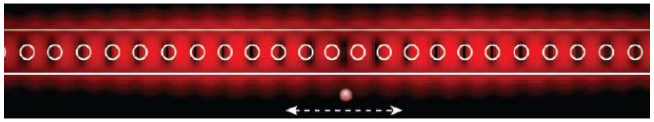
Motivation

- Many emerging experimental systems that can achieve strong interactions between photons
  - Quantum information processing or strongly correlated states of light

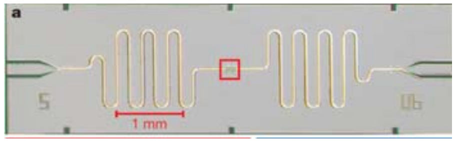
Cavity QED



Atoms / Fabry-Perot cavities

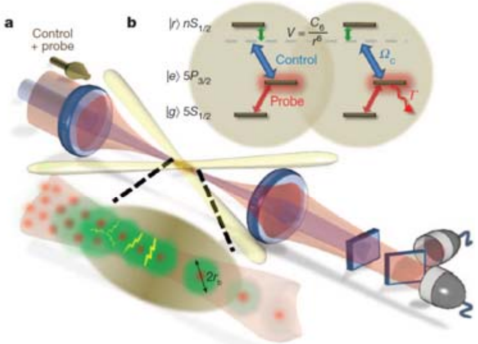


Atoms and solid-state emitters / nanophotonic cavities

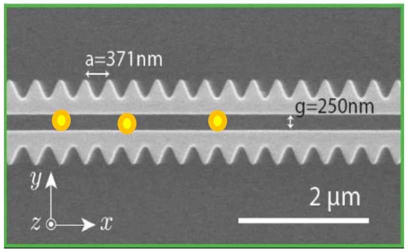


Circuit QED

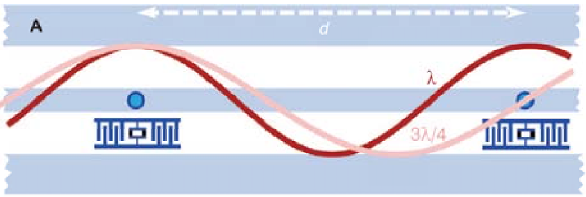
Beyond cavity QED



Atomic Rydberg gases



Atoms coupled to nanofibers and PhC waveguides

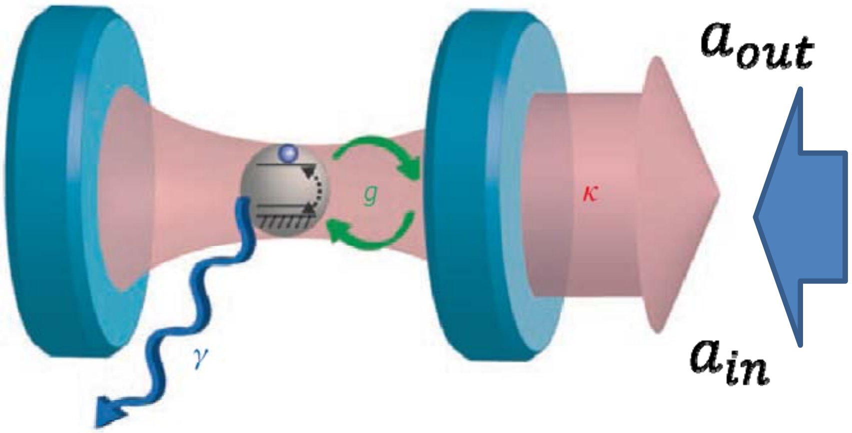


Waveguide QED

Theoretical approaches

- Theoretical complexity of cavity QED vs. non-cavity QED systems differs dramatically

- Cavity QED: “easy” to solve



Jaynes-Cummings model

$$H = g\left(\sigma_{eg} a + \sigma_{ge} a^{\dagger}\right)$$

- Limited Hilbert space $|e\rangle, |g\rangle \quad \otimes \quad |0\rangle, |1\rangle, |2\rangle, \ldots$

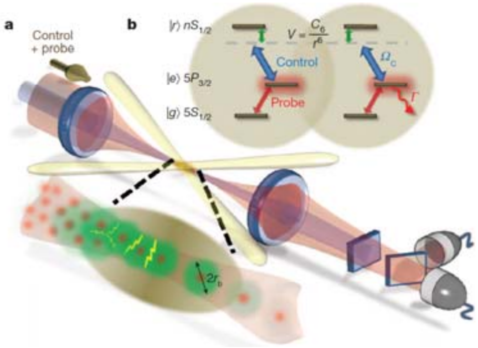
atoms **photons**

- Input-output equation: relates observable fields outside the cavity to dynamics inside

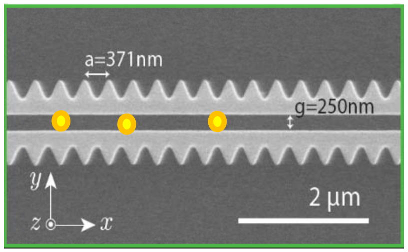
$$a_{out}(t) = a_{in}(t) + \sqrt{\kappa}\, a(t)$$

Theoretical approaches

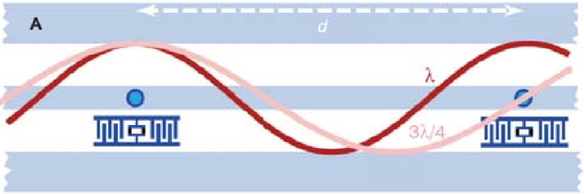
- Non-cavity QED systems



Atomic Rydberg gases



Atoms coupled to nanofibers and PhC waveguides



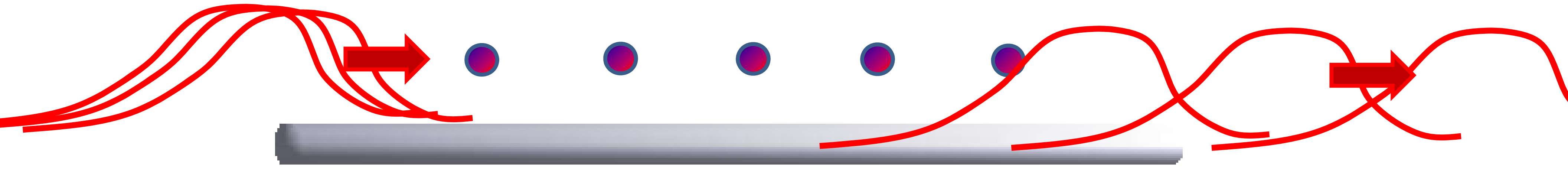
Waveguide QED

- Common feature: coupling to *propagating* fields
- One photon already represents an infinite Hilbert space!

$$\int d\omega \ f(\omega) a_\omega^\dagger |\mathrm{vac}\rangle$$

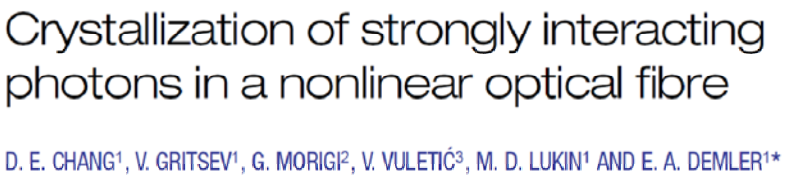
Many-body physics with light?

- Possible to prepare strongly correlated states of light?



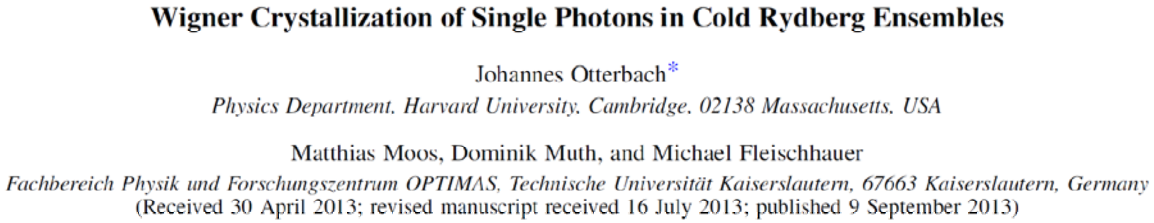
Crystal of photons?

- Many interesting proposals:



Crystallization of strongly interacting photons in a nonlinear optical fibre

D. E. CHANG[1], V. GRITSEV[1], G. MORIGI[2], V. VULETIĆ[3], M. D. LUKIN[1] AND E. A. DEMLER[1]*



Wigner Crystallization of Single Photons in Cold Rydberg Ensembles

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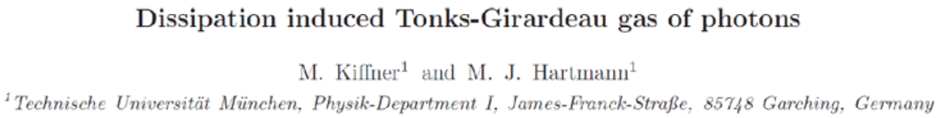
(Received 30 April 2013; revised manuscript received 16 July 2013; published 9 September 2013)



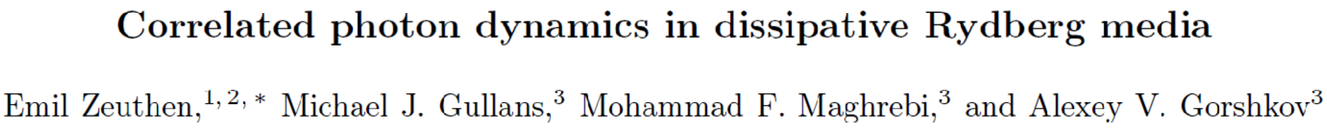
Dissipation induced Tonks-Girardeau gas of photons

M. Kiffner[1] and M. J. Hartmann[1]

[1] *Technische Universität München, Physik-Department I, James-Franck-Straße, 85748 Garching, Germany*

Highly nonlocal optical nonlinearities in atoms trapped near a waveguide

EPHRAIM SHAHMOON,[1,2,3,*] PJOTRS GRIŠINS,[4] HANS PETER STIMMING,[5,6] IGOR MAZETS,[4,6] AND GERSHON KURIZKI[1]



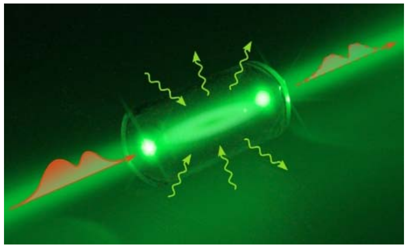
Correlated photon dynamics in dissipative Rydberg media

Emil Zeuthen,[1, 2, *] Michael J. Gullans,[3] Mohammad F. Maghrebi,[3] and Alexey V. Gorshkov[3]

- Challenge to analytically solve or numerically verify!
  - Continuum, driven, open, out-of-equilibrium, …

Theoretical approaches

- Work in regimes of classical optics or Gaussian physics



Photon storage (EIT), EPR entanglement of atomic ensembles and light, spin squeezing, …

- Computational black box for quantum case: discretize space and solve quantum discretized wave equations

$$\begin{matrix} |0_{z1}\rangle \\ |1_{z1}\rangle \\ |2_{z1}\rangle \\ \dots \end{matrix} \otimes \begin{matrix} |0_{z2}\rangle \\ |1_{z2}\rangle \\ |2_{z2}\rangle \\ \dots \end{matrix}$$

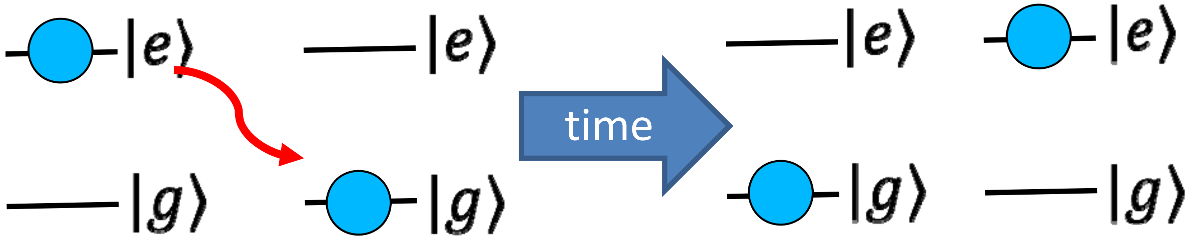
+ atoms

Assign local Hilbert space to each site

  - Hilbert space explodes, even for ***empty space!***
  - In practice, limited to 2-3 total excitations in system, excluding quantum jumps
- **Goal:** approach where limit of empty space becomes trivial

Outline of new approach

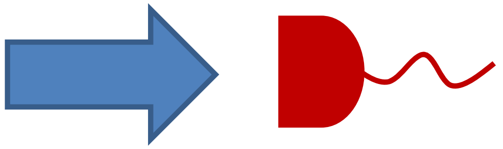
- Real degrees of freedom are just the atoms, encoded in a discrete "spin" Hamiltonian



$$H_{\mathrm{eff}} \sim G(r_i, r_j)\sigma_{eg}^{(i)}\sigma_{ge}^{(j)}$$

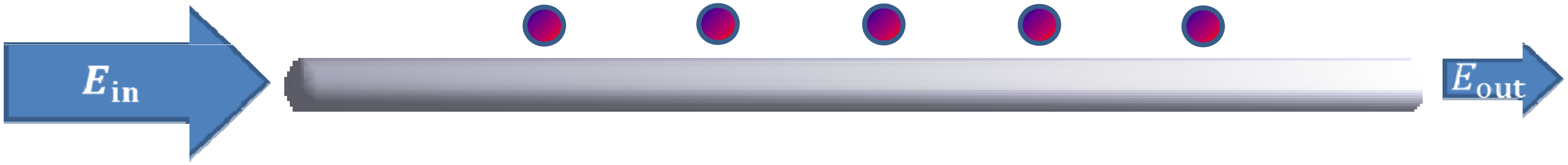
- Re-construct quantum field from generalized input-output equation

$\rho_{\mathrm{atoms}}(t)$

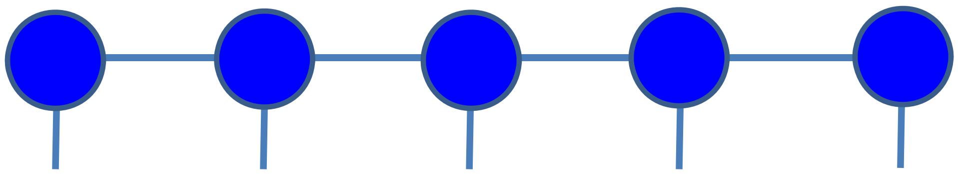


Photodetectors $\langle E^{\dagger}(t)E^{\dagger}(t')E(t')E(t)\rangle$

- Most systems mappable to effective 1D model with $N < 100$ atoms



- Numerically solve for $\rho_{\mathrm{atoms}}(t)$ using matrix product states

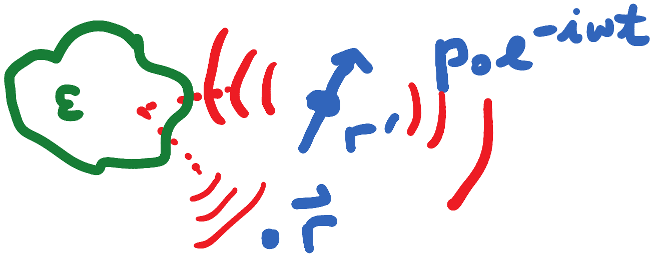


Electromagnetic Green's function

- Formalism: quantum atom-light interactions encoded in (classical) E&M Green's function

$$\left[(\nabla\times\nabla\times) - \omega^2\epsilon(r,\omega)/c^2\right]G_{\alpha\beta}(r,r',\omega) = \delta(r-r')\otimes I$$

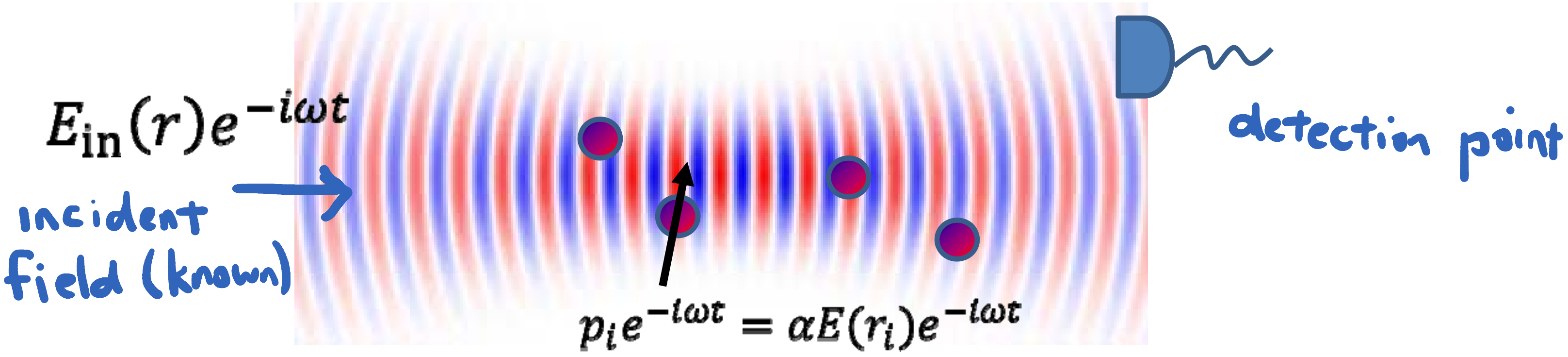
- G describes electric field at point r, of a normalized oscillating dipole at r'



α, β encode vector nature of dipole source and field

The fields: classical analogy

- Classical scattering from polarizable particles



- Know the radiation pattern for a dipole
- Can calculate the total field

$$\hat{E}(r,\omega) = \hat{E}_{\text{in}}(r,\omega) + \alpha \underbrace{\sum G(r, r_i, \omega)\hat{p}_i(\omega)}_{\text{Becomes convolution in time domain}}$$

  - Classical and quantum fields propagate the same way

- Generalized "input-output" equation *in time* for atoms

Field encoded in atoms!

$$\hat{E}(r,t) = \hat{E}_{in}(r,t) + \mu_0 \omega_{eg}^2 d_0^2 \sum_i G(r, r_i, \omega_{eg}) \sigma_{ge}^i(t)$$

Atomic dynamics

- Substitute field into equations of motion for atoms
  - Master equation for atoms alone
- Coherent evolution (emission and re-absorption)

$$H_{\text{eff}} = -\mu_0 d_0^2 \omega_{eg}^2 \sum_{i,j} \underline{(\operatorname{Re} G(r_j, r_i, \omega_{eg}))\sigma_{eg}^i \sigma_{ge}^j}$$

- Dissipation (spontaneous emission)

$$\dot{\rho} = L[\rho] = -\mu_0 d_0^2 \omega_{eg}^2 \sum_{i,j} \underline{(\operatorname{Im} G(r_j, r_i, \omega_{eg}))\,(\sigma_{eg}^i \sigma_{ge}^j \rho} + \rho \sigma_{eg}^i \sigma_{ge}^j - \underline{2\sigma_{ge}^j \rho \sigma_{eg}^i)}$$

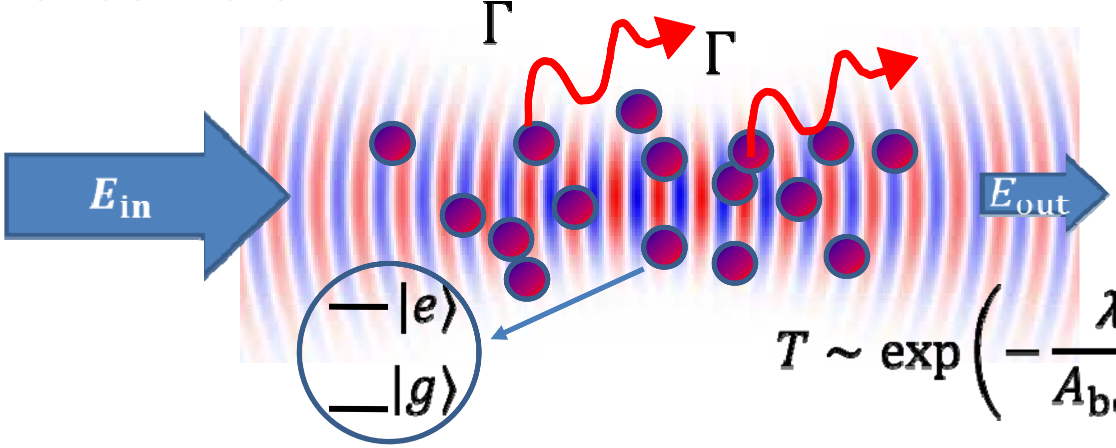
- In short (non-Hermitian Hamiltonian):

$$H_{\text{eff}} = -\mu_0 d_0^2 \omega_{eg}^2 \sum_{i,j} G(r_j, r_i, \omega_{eg})\sigma_{eg}^i \sigma_{ge}^j$$

+ quantum jumps

Mapping to 1D

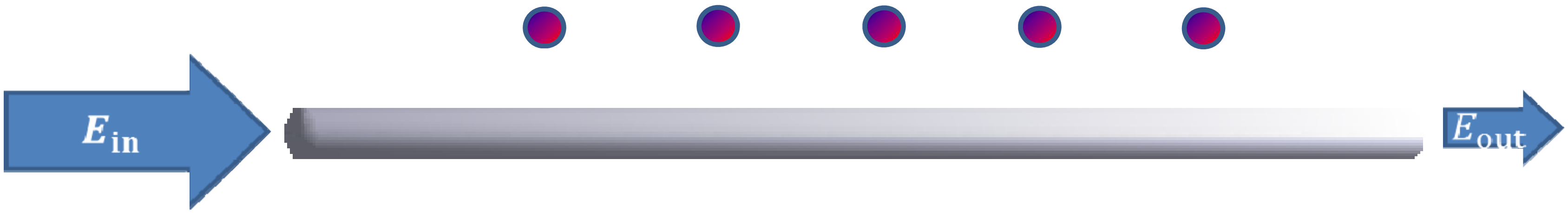
- Atomic ensemble



$$T \sim \exp\left(-\frac{\lambda^2}{A_{\text{beam}}} N_{\text{atoms}}\right) \equiv \exp(-\text{OD})$$

$(\text{N}_{\text{atoms}} \sim 10^6, \text{OD} < 10^2)$

- Write down effective 1D equation: $(\partial_t + \partial_z)E(z,t) \sim iP_{ge}(z,t)$
- Approximate: independent emission into other modes
- Idea: invent an equivalent 1D “waveguide” with huge interaction probability and $\text{N} \sim 10^2$ atoms



1D spin model

- Spin model in 1D

$$H_{\text{eff}} = -\mu_0 d_0^2 \omega_{eg}^2 \sum_{i,j} G(r_j, r_i, \omega_{eg}) \sigma_{eg}^i \sigma_{ge}^j \Longrightarrow -\frac{i\Gamma_{1D}}{2} \sum_{i,j} e^{ik|z_i - z_j|} \sigma_{eg}^i \sigma_{ge}^j$$

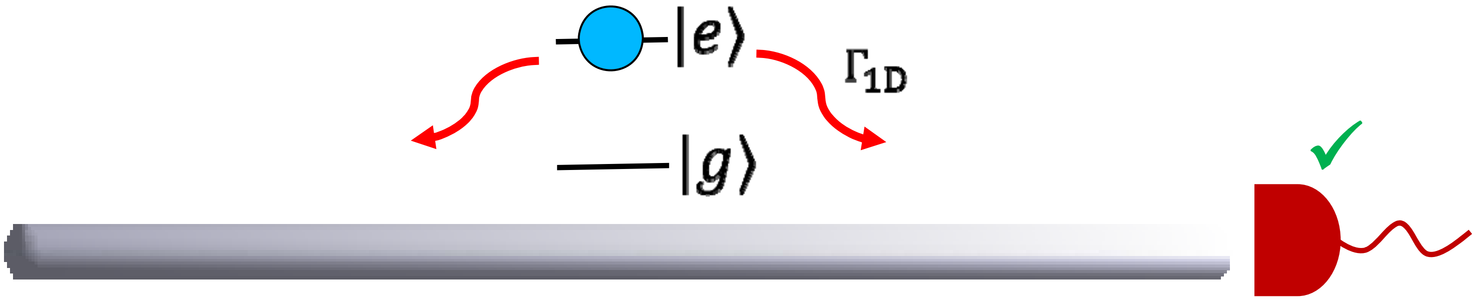
Plane wave propagation in 1D

- Single atom:

$$H_{\text{eff}} = -\frac{i\Gamma_{1D}}{2} \sigma_{ee}$$

- Γ_{1D} is single-atom spontaneous emission rate

- Not very physical – photon always emitted into 1D channel



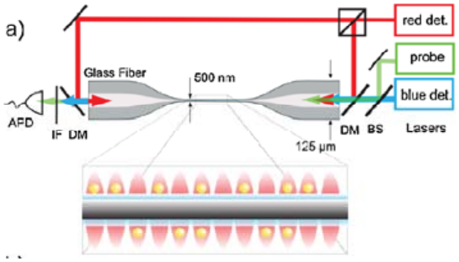
1D spin model

- Add phenomenological decay rate into other channels

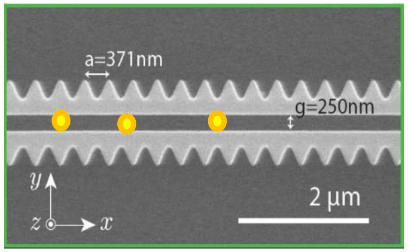
$$H_{\mathrm{eff}} = -\frac{i\Gamma_{1\mathrm{D}}}{2}\sum_{i,j}\exp(ik|z_i - z_j|)\,\sigma_{eg}^{i}\sigma_{ge}^{j} \quad -\frac{i\Gamma'}{2}\sum_{i}\sigma_{ee}^{i}$$

+ quantum jumps

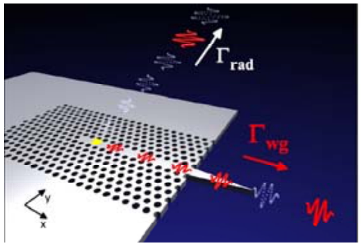
- Model connects quantitatively to experimental 1D systems



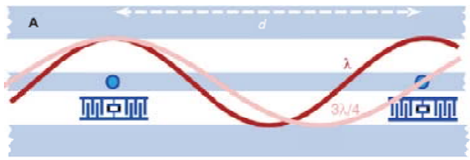
Atoms - nanofibers



Atoms - PhC waveguides



QD - PhC waveguides



Waveguide QED

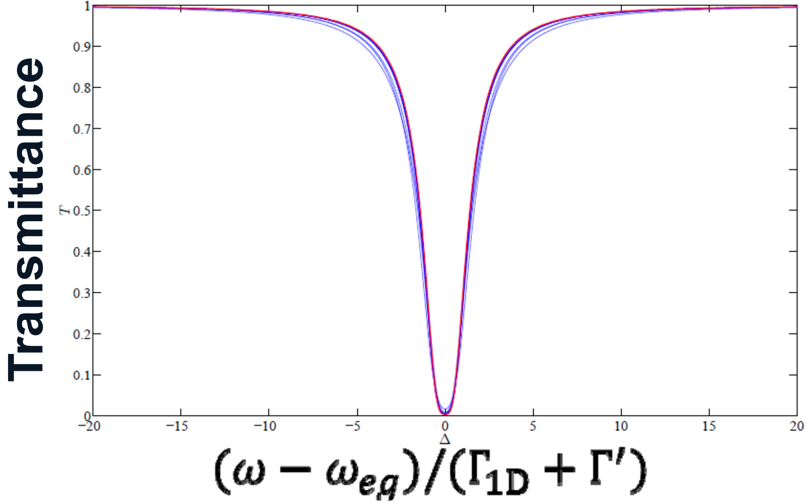
$\Gamma_{1D}/\Gamma' \sim 0.05$

$\Gamma_{1D}/\Gamma' \sim 1$

$\Gamma_{1D}/\Gamma' > 10$

Calculating output fields

- Apply input-output formalism to calculate fields
  - Randomly positioned two-level atoms, many runs



Linear transmission spectrum

N=10 atoms, $\frac{\Gamma_{1D}}{\Gamma'} = 0.3$

Randomly distributed on 100 sites (10 runs)

- On resonance:

$$T \approx \exp(-\mathrm{OD})$$

$$\mathrm{OD} = 2N_{\mathrm{atoms}}\Gamma_{1D}/\Gamma' \quad (\text{for } \Gamma_{1D} < \Gamma')$$

- Establishes connection with free-space ensembles with much higher N_{atoms}!

Dynamics with MPS

- Exact dynamics of spin Hamiltonian on full Hilbert space: $N_{\text{atoms}} < 20$

- Larger systems: use matrix product states

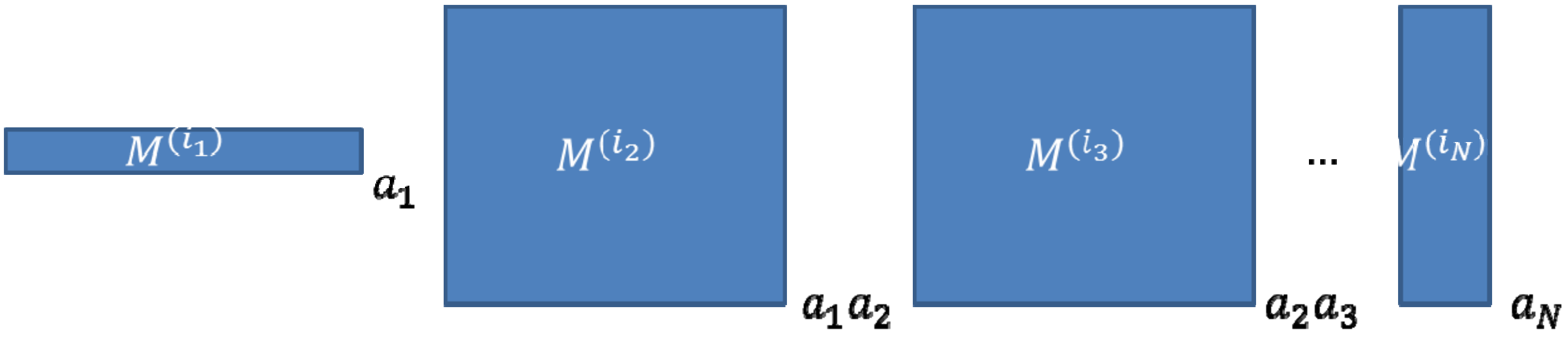
- MPS in a nutshell: $|\psi\rangle = \sum_{\{i=\uparrow,\downarrow\}} c_{i_1 i_2 i_3 \dots i_N} |i_1 i_2 i_3 \dots i_N\rangle$

Exact wavefunction

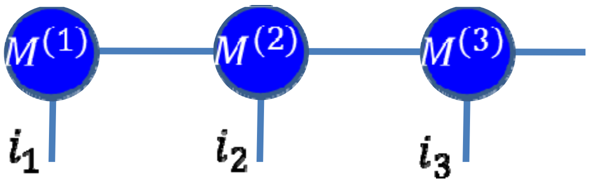
Rank-N tensor

Re-shape tensor

$$c_{i_1 i_2 \dots i_N} = M^{(i_1)} M^{(i_2)} \dots M^{(i_N)}$$

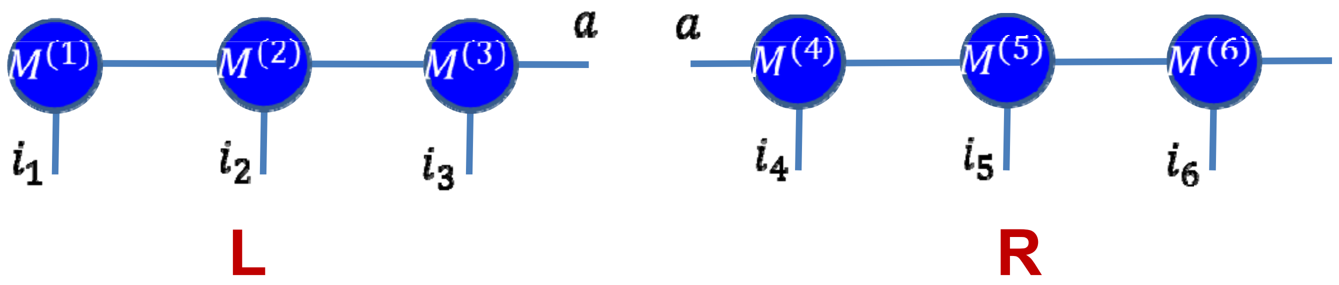


- Representation:



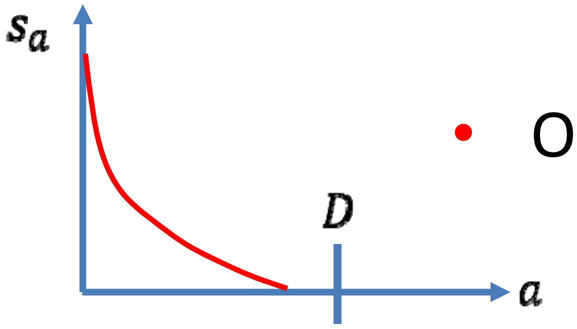
Dynamics with MPS

- Exact representation: matrix size D grows exponentially with N
- Approximation: truncate at max size D x D
- Interpretation:



$$|\psi\rangle = \sum_a s_a |a_L\rangle |a_R\rangle$$ **Schmidt decomposition**

- Good representation if s_a decays rapidly (low entanglement entropy)



- Our case: probably aided by dissipation

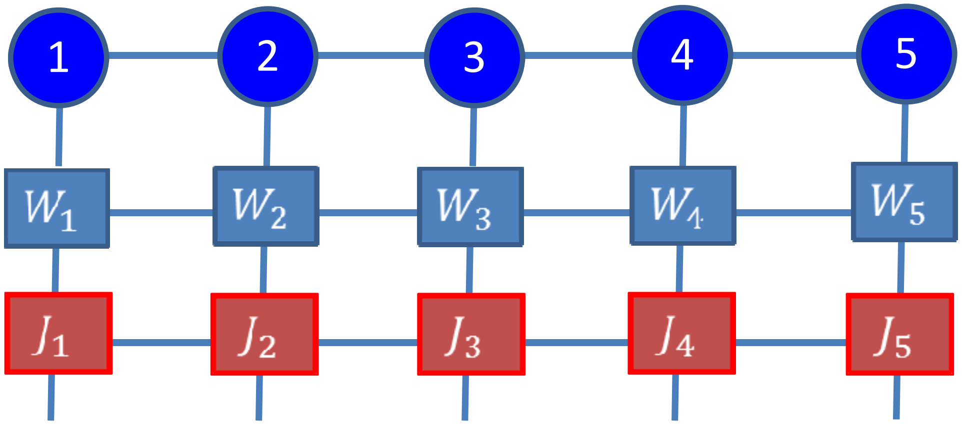
Dynamics with MPS

- Procedure for dynamics

Atom #

Apply MPO

Jump?

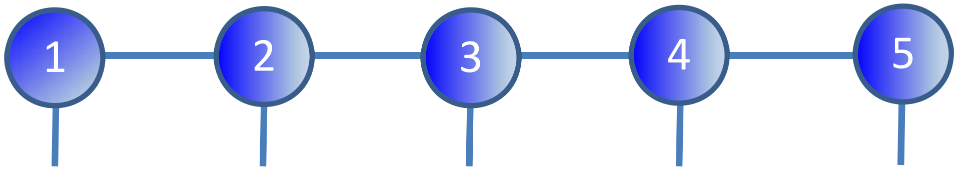


Initial state $|\psi(t_0)\rangle$

$$|\psi(t_0+\Delta t)\rangle \approx (1 - iH_{\text{eff}}\Delta t)|\psi(t_0)\rangle$$

- Size of MPS grows after evolution step

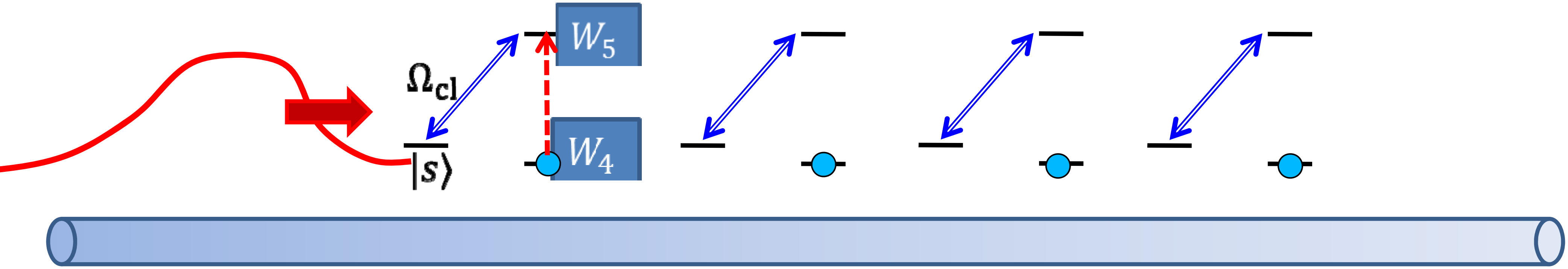
Compress and repeat



Test case: vacuum induced transparency

- Does the formalism work??
- Few cases of known many-body solutions
  - Exception: vacuum induced transparency

- Electromagnetically induced transparency:



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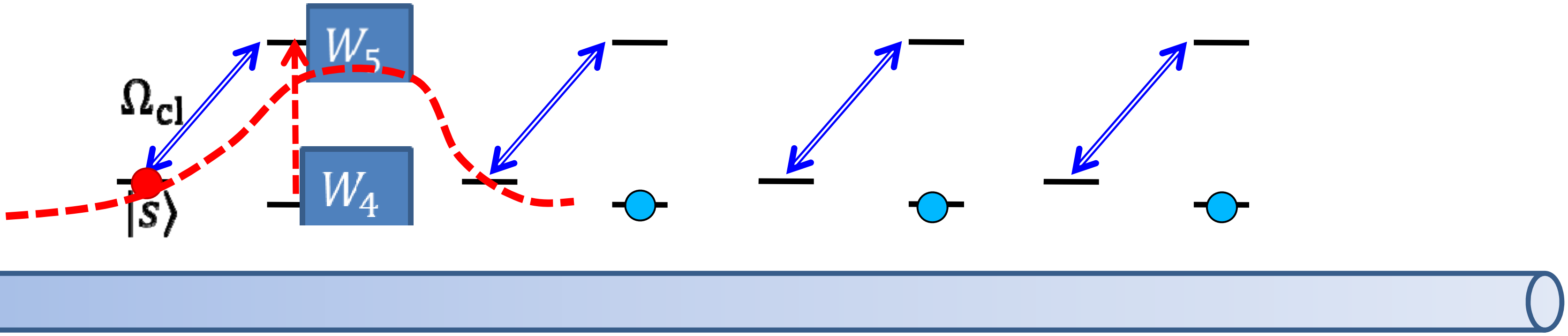
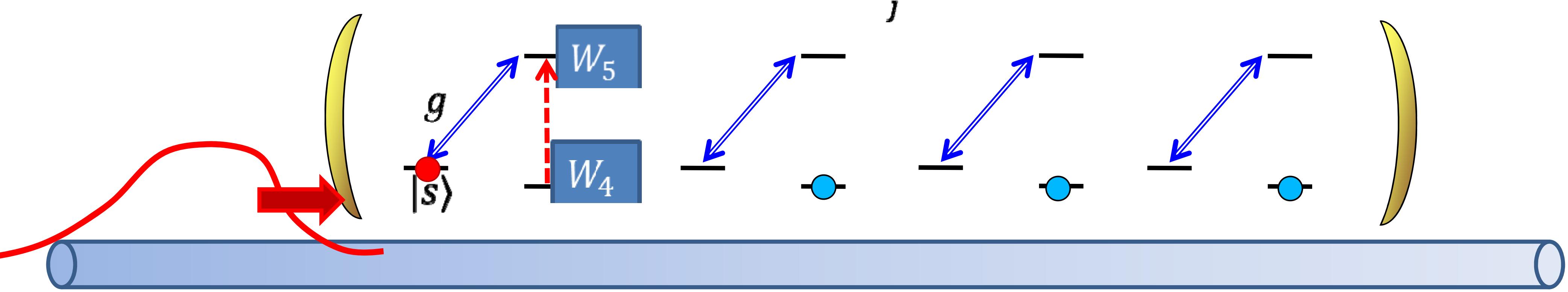


- Photon hybridizes with a spin excitation, $|g\rangle \rightarrow |s\rangle$
- Reduced group velocity $v_g \propto \Omega_{\rm cl}^2 \ll c$ and transparency window

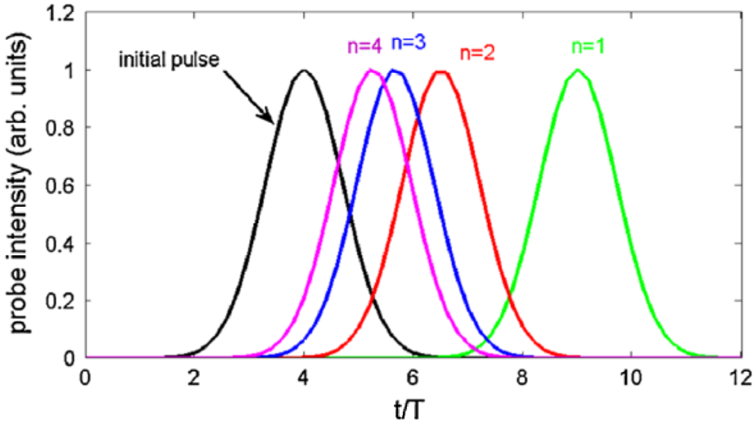
Vacuum induced transparency

- Replace classical control field with cavity vacuum

$$H_{\mathrm{cav}} = g a_{\mathrm{cav}} \sum_j \sigma_{es}^{(j)}$$



- Pulse propagation acquires nonlinearity of atom-cavity interaction
  - *Photon number-dependent* group velocity, $v_g \propto n_{\mathrm{ph}}$



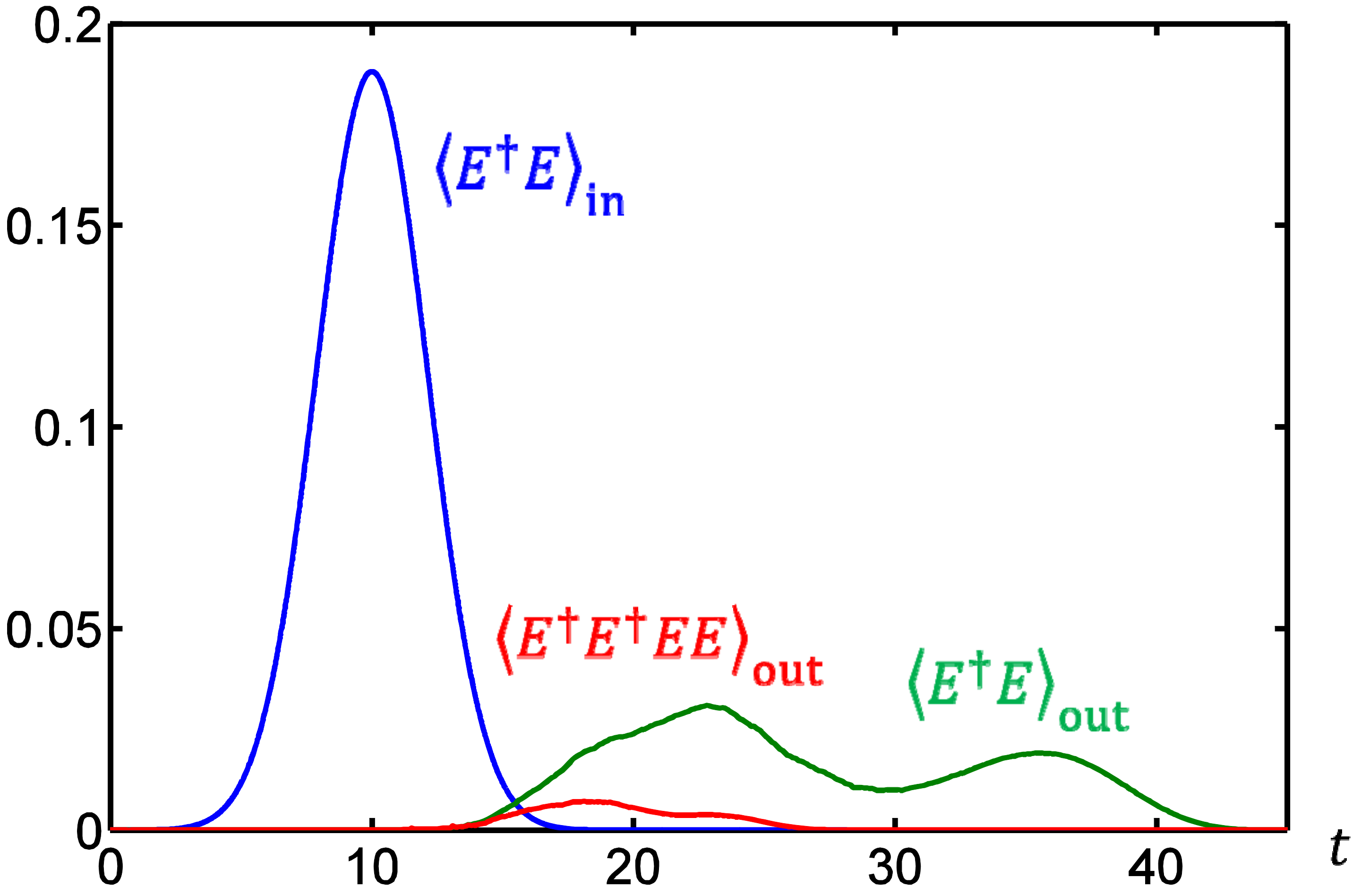
- Spatial separation of photon number

Theory: **Fleischhauer (PRL, 2010)**

Experiment: **Vuletic (Science, 2011)**

MPS simulations

- $N_{\text{atoms}} = 100, \Gamma_{1D} = 2, \Gamma' = 1, g = 2, \kappa = 0.03$ (OD=400)
- Coherent state input:



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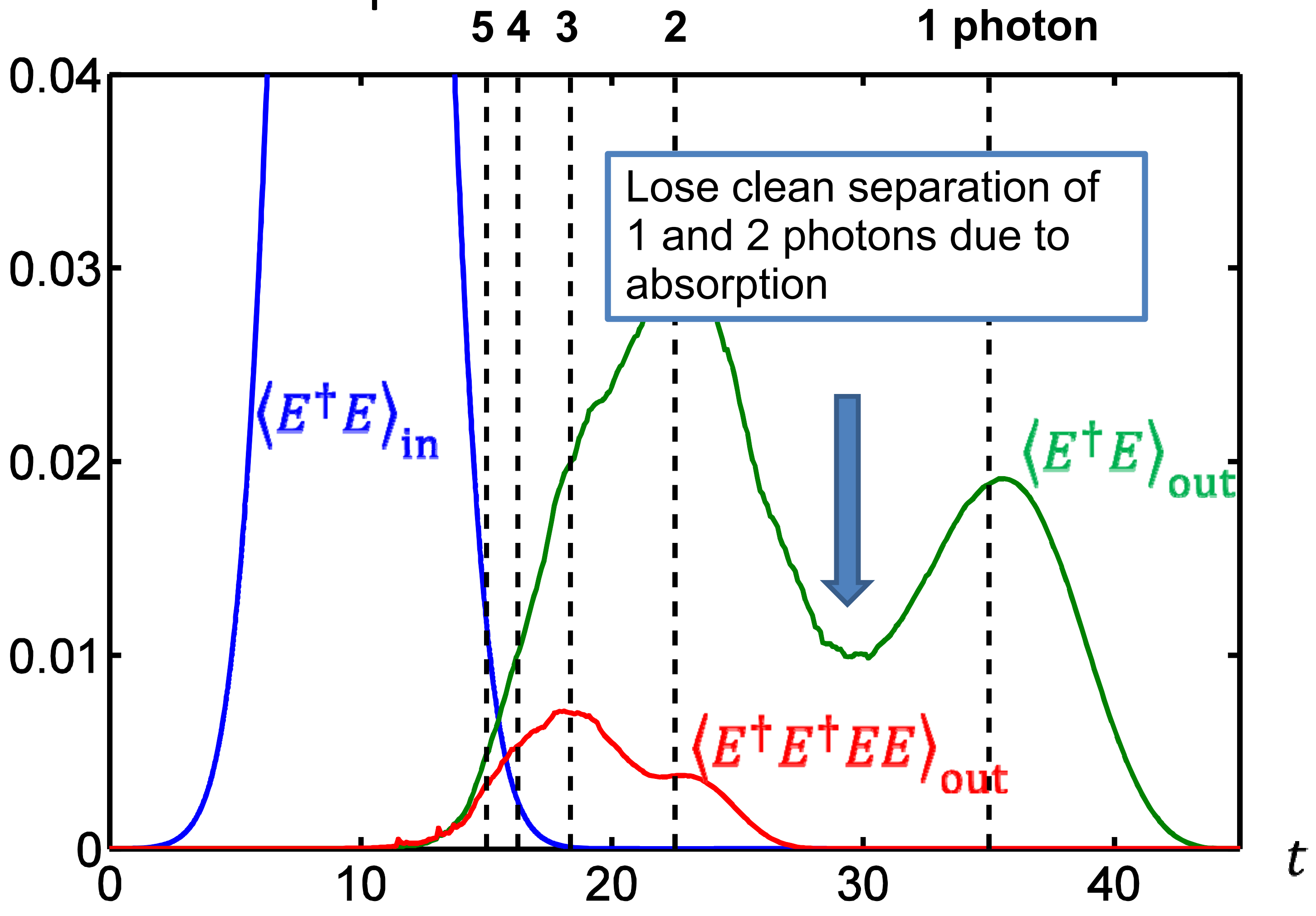
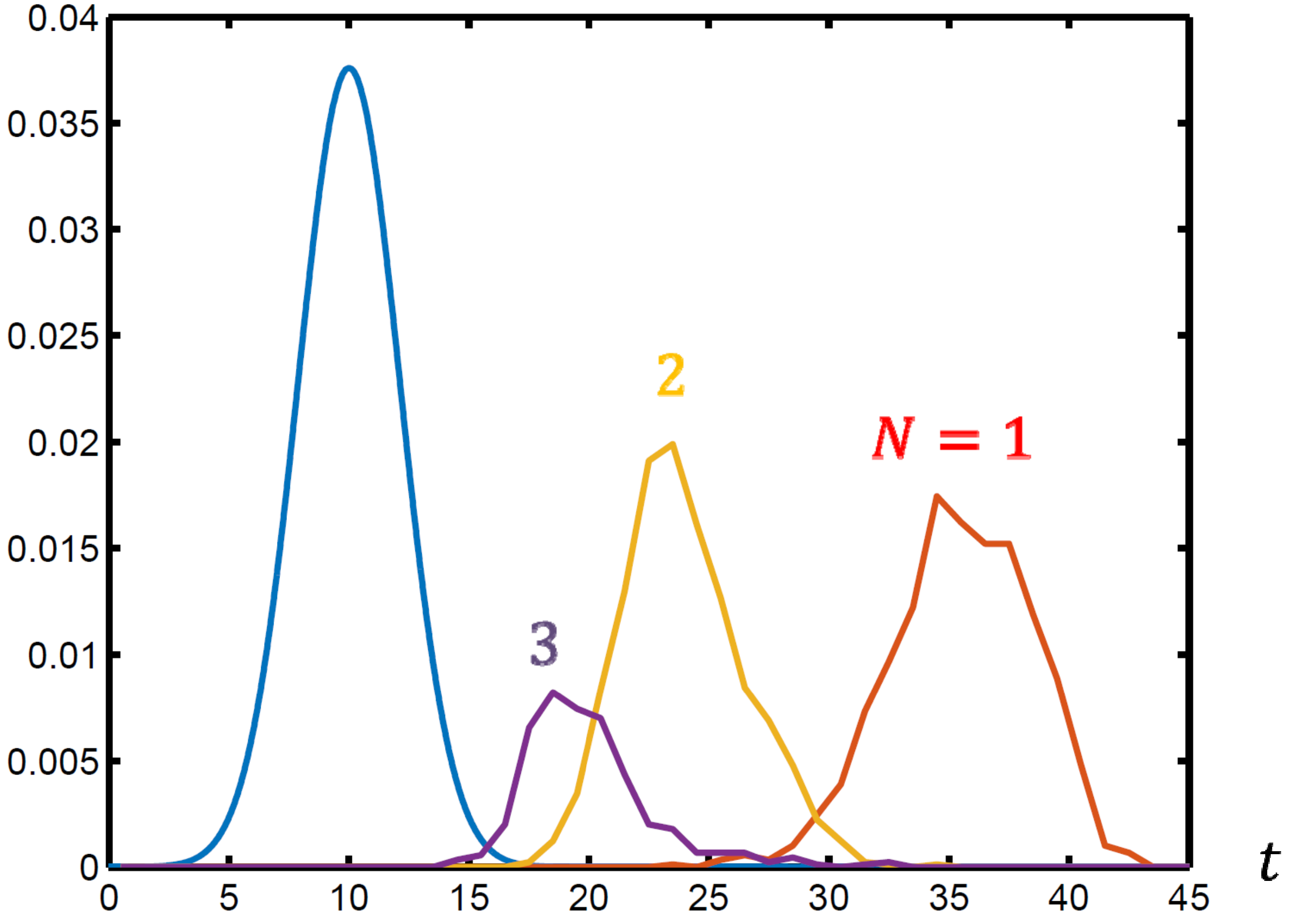


Lose clean separation of 1 and 2 photons due to absorption

MPS simulations

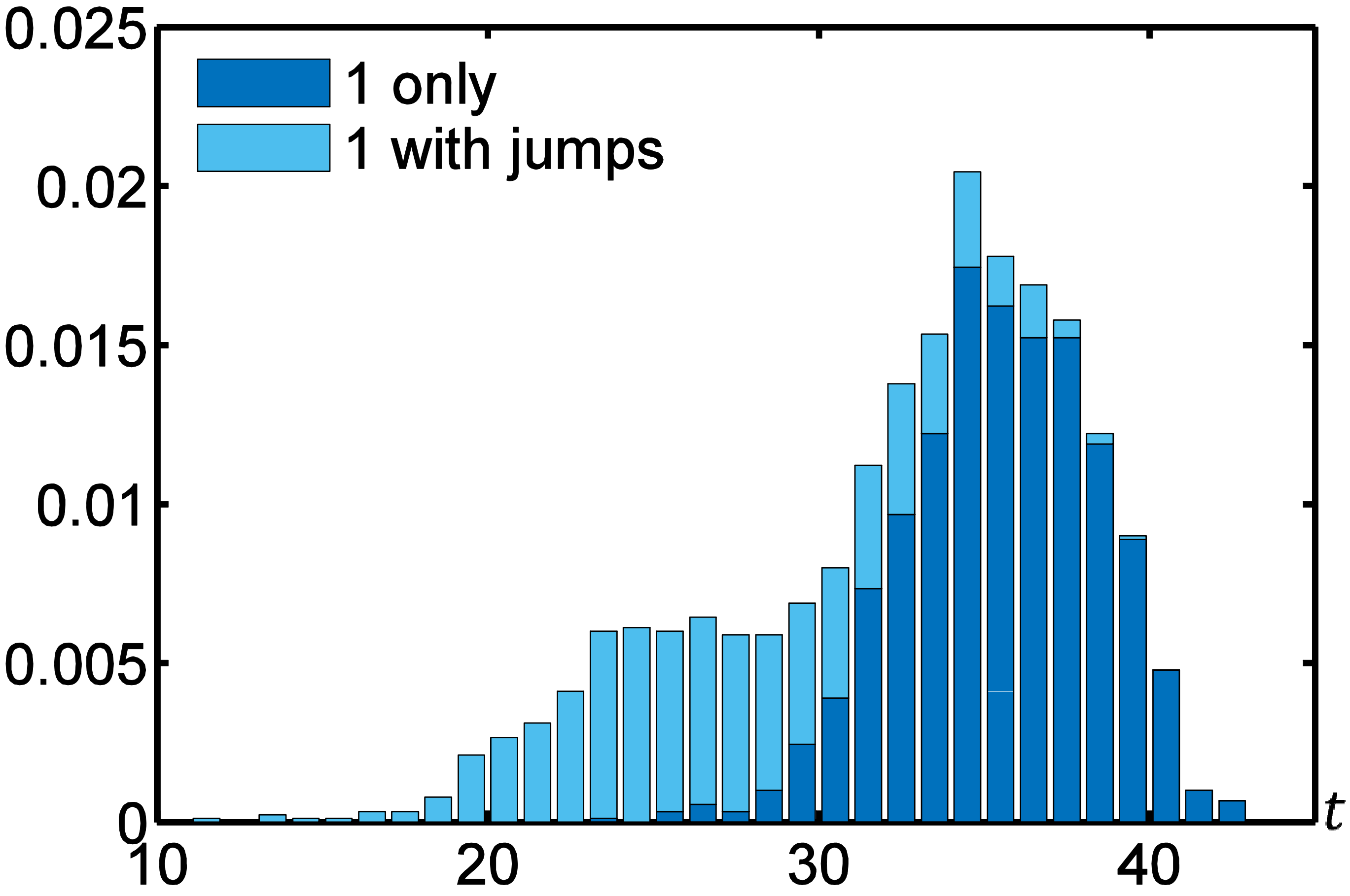
- $N_{\text{atoms}} = 100, \Gamma_{1D} = 2, \Gamma' = 1, g = 2, \kappa = 0.03$
- Effect of partial absorption



Post-selection – no jumps out of waveguide, N detected photons in transmission

MPS simulations

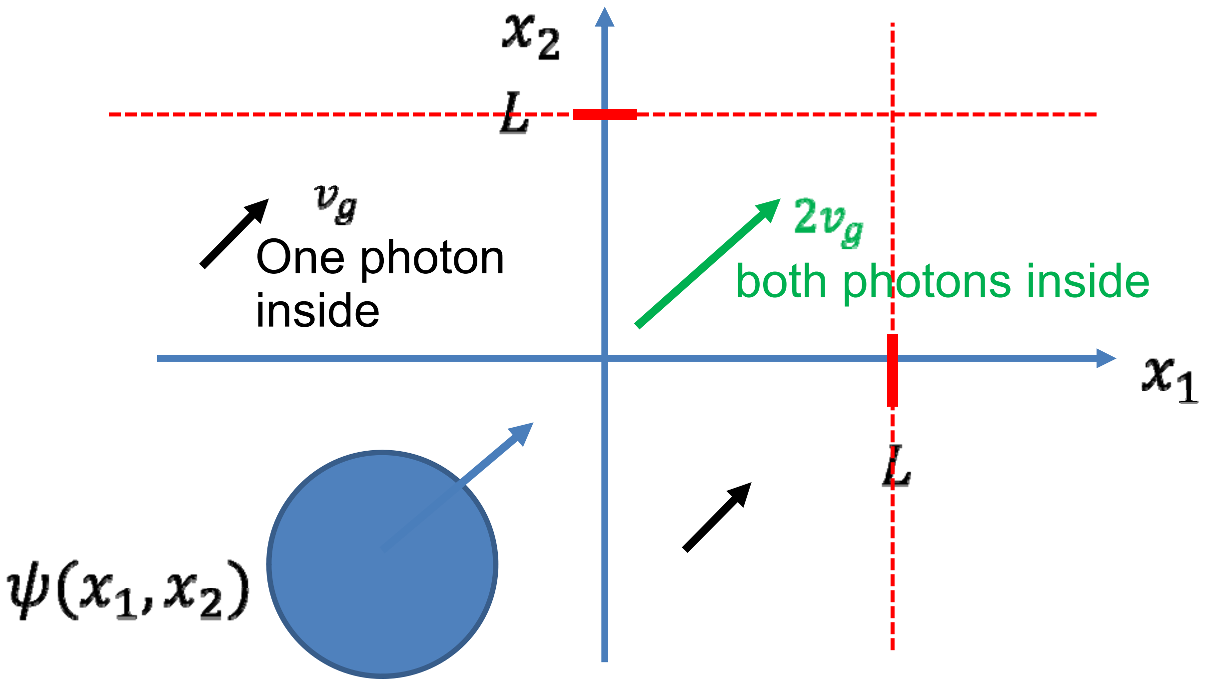
- $N_{\mathrm{atoms}} = 100, \Gamma_{1D} = 2, \Gamma' = 1, g = 2, \kappa = 0.03$
- Effect of partial absorption



Post-selection – jumps/no jumps out of waveguide, 1 detected photon in transmission

Two-time correlation functions

- Distortion of a two-photon wavepacket



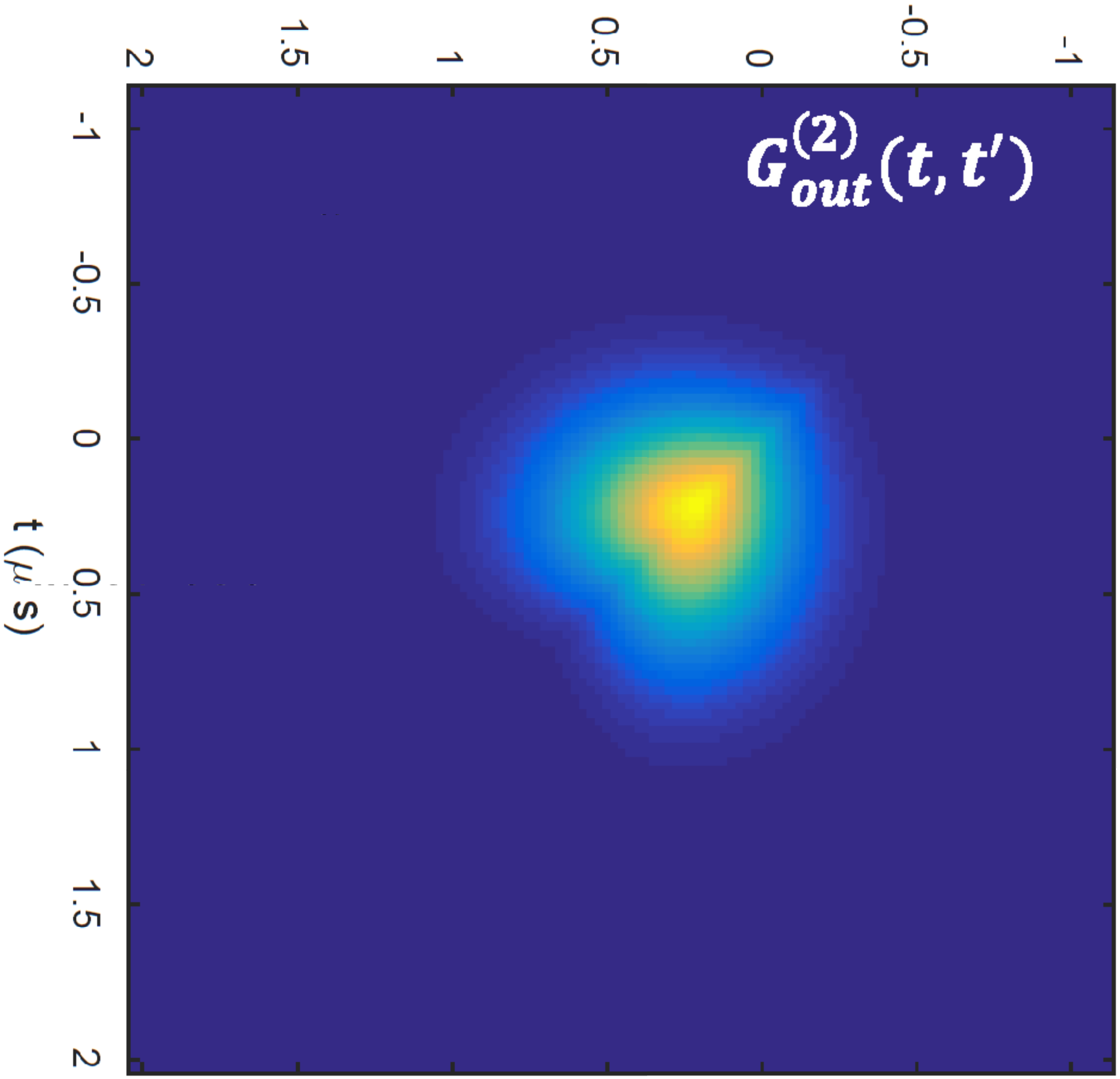
One photon inside

both photons inside

Incoming pulse
(starts outside system)

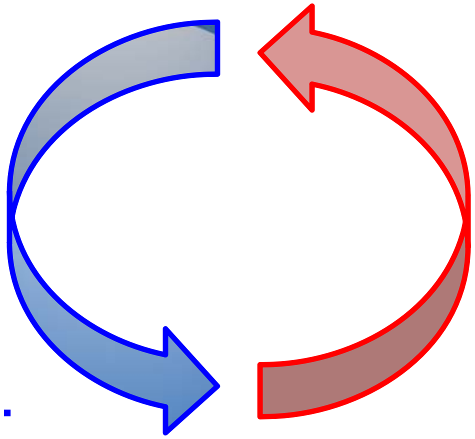
Two-time correlation functions

- Distortion of a two-photon wavepacket



Outlook

- Promising technique to solve for many-photon dynamics



Effective theories / analytical solutions

H.-P. Buchler, A. Gorshkov, T. Pohl, …

Numerical techniques

- Why does it work, and when does it fail?
  - Nature of entanglement growth in dissipative systems

- Atom-light interactions as a quantum spin model
  - Other interesting consequences?