When is a single many-body wavefcn “thermal”??

MPA Fisher

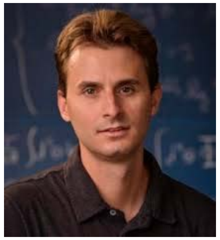
KITP Designer Quantum Systems conf; November 14, 2016

Eigenstate Thermalization Hypothesis (ETH): (Quantum stat mech T>0)
Eigenstates of generic **Hamiltonians** are thermal (“ergodic”)

Goal: Classify many-body *wavefunctions,* with no “recourse” to a Hamiltonian

Tarun Grover

Jim Garrison



Ryan Mishmash

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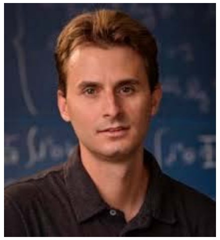
How to even define a "thermal wavefunction"?



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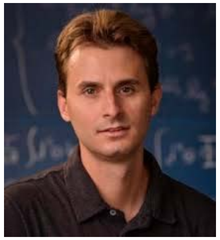


Ryan Mishmash

Perhaps by classifying “non-thermal” wave functions...
Towards this end, this talk...

Eigenstate Thermalization Hypothesis (ETH)

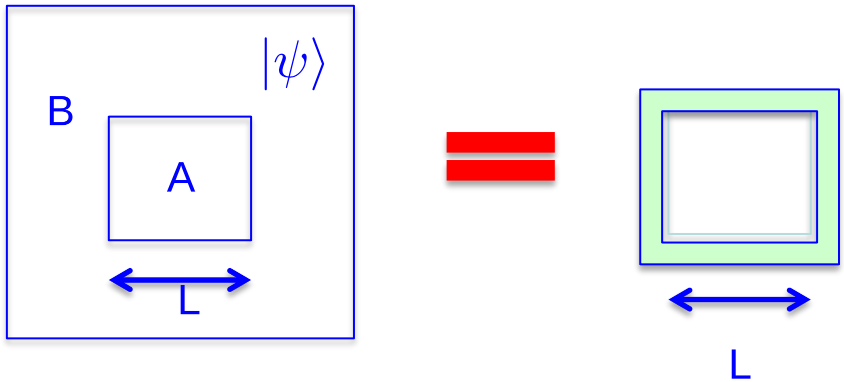
Josh Deutsch, Mark Srednicki

ETH: sub region of a single E/L^d eigenstate equivalent to thermal ensemble w/ heat bath

$$\hat{\mathcal{H}}|\psi\rangle = E|\psi\rangle$$

Regions A (“system”)
and B (“environment”)

$$\hat{\rho} = |\psi\rangle\langle\psi|$$



$$k_B T$$

Reduced density matrix in A

$$\hat{\rho}_A = Tr_B(\hat{\rho}) \approx \hat{\rho}_{th} = \frac{1}{Z} e^{-\beta \hat{H}}$$

eg equivalence of (local) observables: $\langle \hat{\mathcal{O}} \rangle_E = \langle \hat{\mathcal{O}} \rangle_{th}$

Eigenstates in narrow energy (density) window have identical correlations $\langle \hat{\mathcal{O}} \rangle_E = \langle \hat{\mathcal{O}} \rangle_{E+\Delta E}$

Equivalence of operator averages in narrow energy window

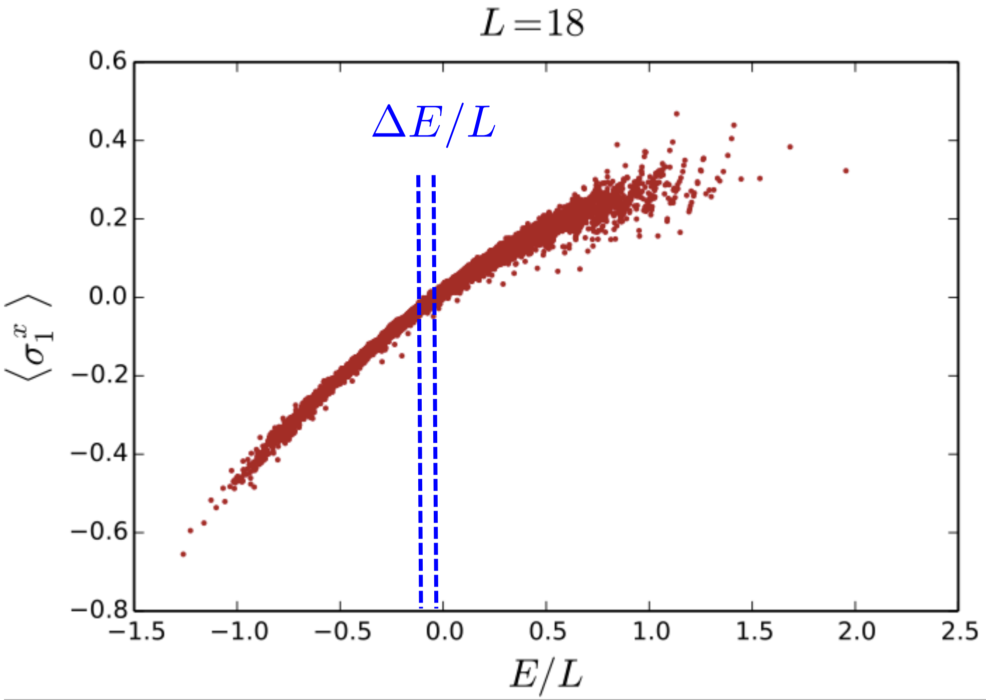
Kim, Ikeda and Huse, 2014
J. Garrison (unpublished)

Exact diagonalization
1d Quantum Ising chain

$$H = \sum_{i=1}^{L} \left(g\sigma_i^x + h\sigma_i^z + J\sigma_i^z\sigma_{i+1}^z \right)$$

(g, h, J) = (0.9, 0.8, 1)

$$\langle \psi_E | \sigma_1^x | \psi_E \rangle$$



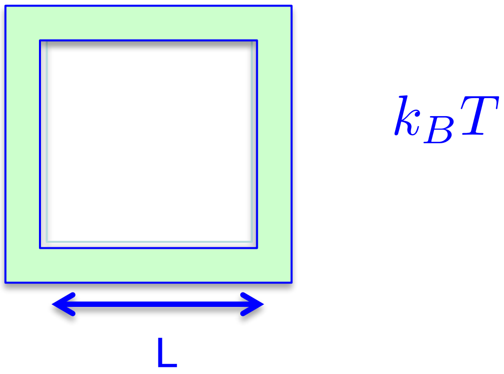
Operaror average the ”same” for all eigenstates in narrow energy window (ETH)

Entropy in ETH: Thermal "versus" entanglement

Thermal entropy:

Number of states,
extensive for T>0

$$S_{th} = -Tr[\hat{\rho}_{th} \ln \hat{\rho}_{th}] \sim L^d$$

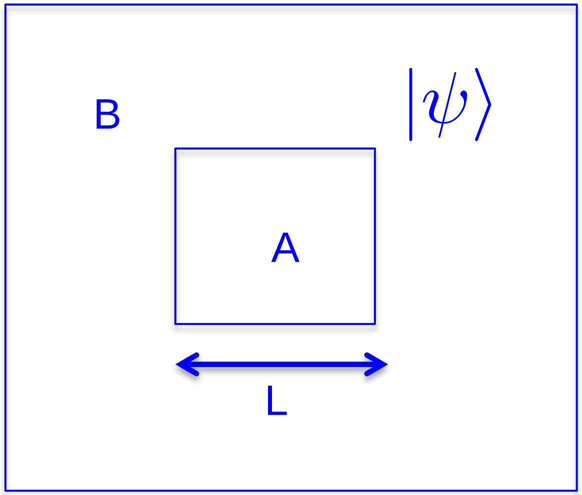


Entanglement Entropy: Single eigenstate

$$\hat{\mathcal{H}}|\psi\rangle = E|\psi\rangle$$
$$\hat{\rho} = |\psi\rangle\langle\psi|$$
$$\hat{\rho}_A = Tr_B(\hat{\rho})$$

Entanglement entropy:

$$S_A(L) = -Tr_A(\hat{\rho}_A \ln \hat{\rho}_A)$$



ETH: Equivalence of Thermal and entanglement entropies

$$S_A/L^d = S_{th}/L^d; \quad L \to \infty$$

Thermal entropy is state counting, entanglement entropy
depends on the properties of the states!

Ex: Entanglement entropy in Quantum Ising chain

1d Quantum Ising chain

$$H = \sum_{i=1}^{L} \left(g\sigma_i^x + h\sigma_i^z + J\sigma_i^z\sigma_{i+1}^z \right)$$

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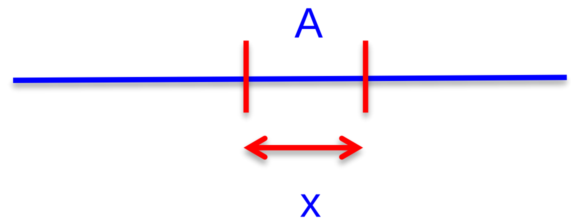
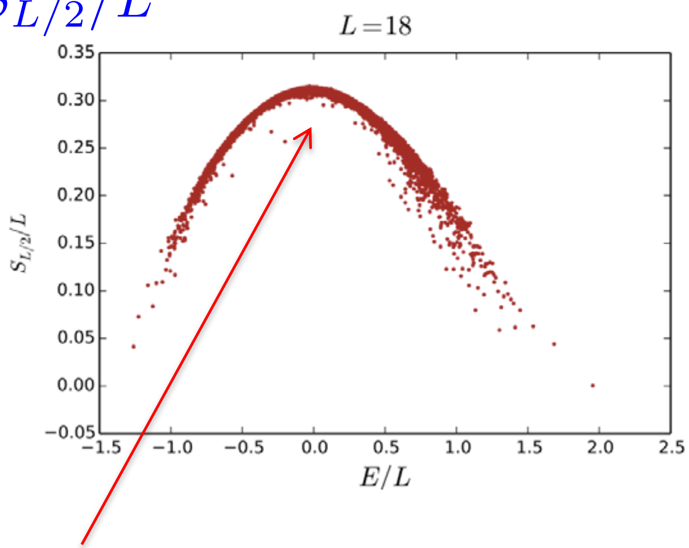


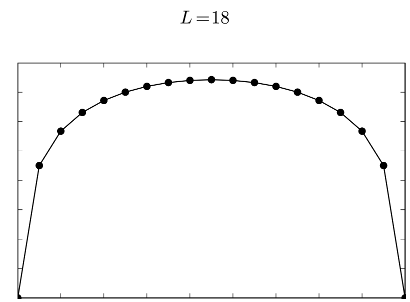
$S_{L/2}/L$



Maximum entropy state,
minimal (zero) local information $S_A/L_A \approx \ln 2$

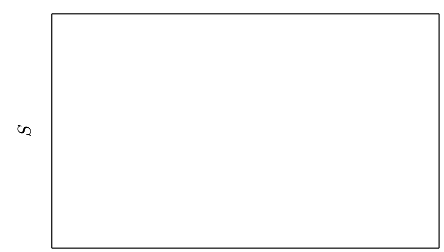
Ground state,
Area-law:

$$S_A \sim O(1)$$



"High-energy" state,
Volume law:

$$S_A \sim \frac{L}{2} - \left|x - \frac{L}{2}\right|$$



Can we deduce “thermalization” from one wavefcn? (w/out a Hamiltonian)

How to tell if a single wavefunction is "thermal"?

Can we classify “thermal” wavefunctions by their entanglement structure?
(as we do for ground states)

How should we define a “fully thermal” (ergodic) wavefunction?

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Perhaps, by what it isn’t (ie non-thermal...)?

When (and how) does ETH Break Down?

1) Fine-tuned Integrable models: Generalized Gibbs Ensemble (GGE).
Can one deduce integrability from one eigenstate?

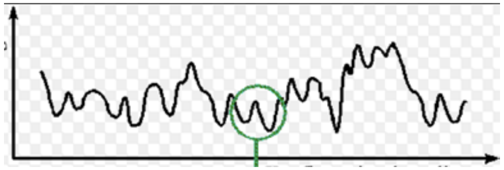
2) Many-body-localization (MBL) w/ disorder

I.V. Gornyi, A.D. Mirlin, D.G. Polyakov (2005)
D. Basko, I. Aleiner, B. Altshuler (2006)
V. Oganesyan, D. Huse (2007)
Pall and Huse (2010)
Bauer, Nayak (2013)

Isolated, interacting, quantum particles in **random potential**

- **Lack of thermalization** Particles don't serve as own thermal bath
- **Entanglement does not "propagate"**
- All eigenstates w/ **area law entanglement entropy** (thermal entropy is extensive)

$$S_A \sim L^{d-1} \qquad S_{th} \sim L^d$$



MBL is protected by "emergent" (i.e. hidden) integrals of motion (no fine tuning needed!)

Abanin et al,...

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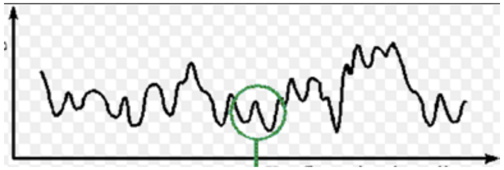

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Abanin et al,...

But... The random potential consitutes **classical (frozen) set of degrees of freedom**...
(as does the Floquet drive...)

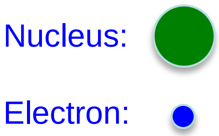
Can ETH Breakdown in “Quantum-only” (all) systems?

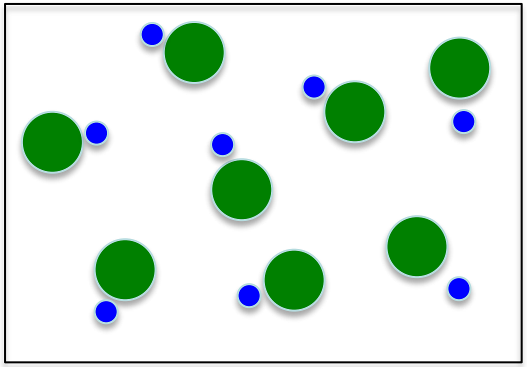
When might this happen?

Perhaps when “almost classical” and very quantum systems interact...

“Quantum-only” system of ions (almost classical) and electrons (very quantum)

Liquid of Neutral atoms, isolated in box, prepared in many-body eigenstate, energy E

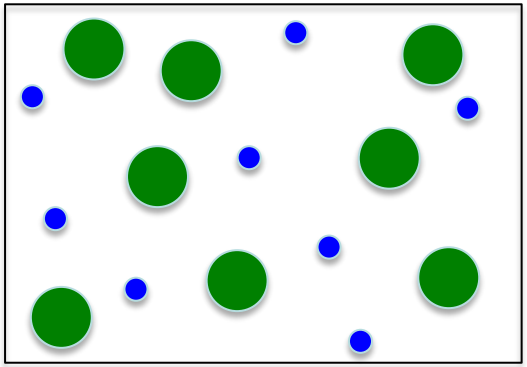




Atomic-Liquid

Vary energy E (the “temperature”)

- T ~ 10-1,000 K **atomic-liquid**
- T > 10 eV, **ionized-plasma**



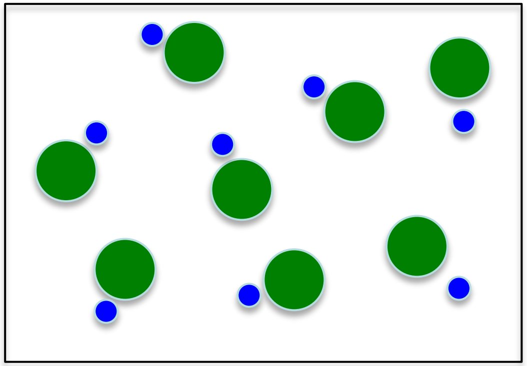
Ionized-Plasma

Question: Are eigenstates of **atomic-liquid** qualitatively the same as the **ionized-plasma** eigenstates?

Atomic-Liquid vs ionized-plasma?

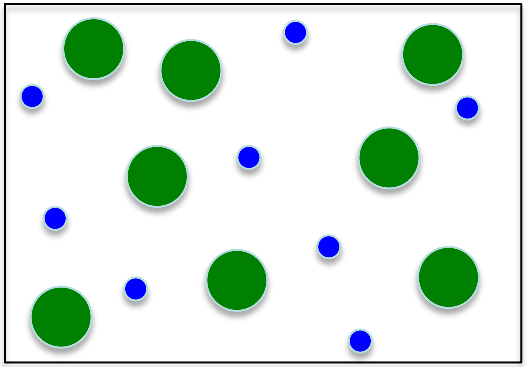
Standard answer: Atomic-liquid and ionized plasma are "equivalent"

- Atoms weakly ionized (even at low "T")
- Low density of ionized electrons "should" thermalize



Atomic-Liquid

But...
How does one (or can one) define "ionized"?
How to characterize thermalization of "ionized" electrons?



Ionized-Plasma

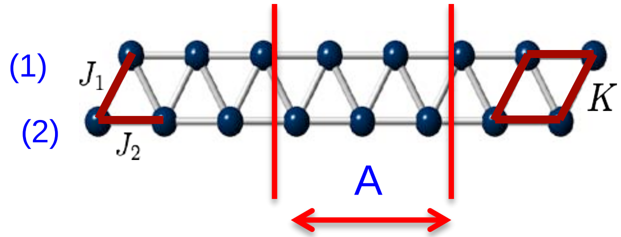
Plan:

- New wavefcn diagnostic ("a" characterization of ionization)
- New class of "non-thermal" quantum states

"Quantum Disentangled States"

Throwing away information (partial tracing) versus extracting information (projective measurement)

(Global) Full measurement



E>0 eigenstate w/ volume law entanglement: $|\Psi\rangle$

$$S_A = -Tr_A[\hat{\rho}_A \ln \hat{\rho}_A] = sL^d \qquad \hat{\rho}_A = Tr_B|\Psi\rangle\langle\Psi|$$

$$\hat{\mathcal{H}} = \sum_{ij} J_{ij}\vec{S}_i \cdot \vec{S}_j + ...$$

Measure spin on every site (global full measurement)
get direct product state, fully disentangled

$$|\Psi\rangle \rightarrow |\Psi'\rangle = |\tilde{S}\rangle \qquad S_A = sL^d \rightarrow S'_A = 0$$

(Global) Partial Measurement:

Ex: Measure spin on leg (2) but not on leg (1)

$|\Psi\rangle \rightarrow |\Psi'_1\rangle_{\tilde{S}_2}$ Wf for spins in leg (1)

Partial Tracing: Throwing away information

What is entanglement entropy of post (partial) measured wf ??

New wf diagnostic: "Post-measurement entanglement"

Illustrate w/ Atomic-liquid

E >0 volume-law wavefcn, $\Psi(R, r)$

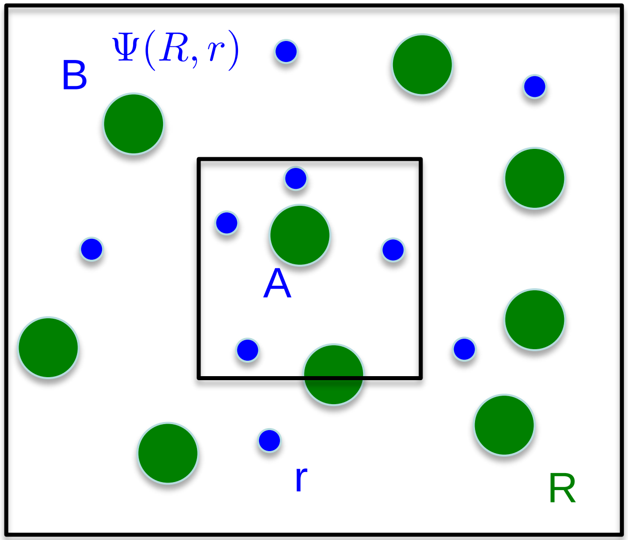
(i) **Measure** nuclei (but not electron) positions, (**partial measurement**) find $\underline{R}$

(ii) Wavefcn of (unmeasured) electrons,

$$\psi_e(r) \sim \Psi(\underline{R}, r)$$

(iii) Compute bi-partition entanglement entropy of electron wf (w/ fixed nuclei, $\underline{R}$)

(iv) Repeat (i)-(iii) and average: post-measurement entanglement entropy $S_A^{r/R}$



(R, r denote nuclei/electron coordinates)

Interest: Size scaling of $S_A^{r/R}$

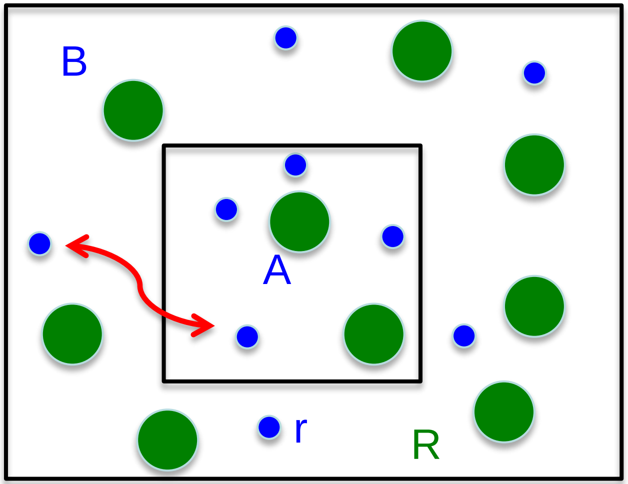
Scaling of post-measurment entanglement entropy?

***If* Volume Law:** $$S_A^{r/R} \sim L^d$$

Electrons thermalized (entangled) even after measuring nuclei positions,

Then: Thermalized wavefunction

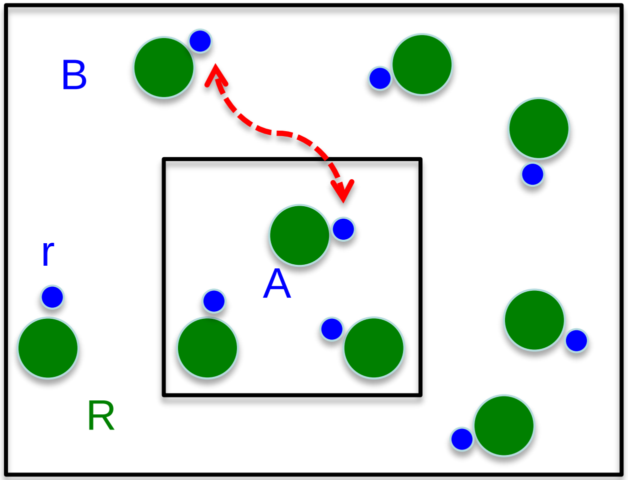
ionized-pasma, fully
thermal w/ volume laws



***IF* Area Law:** $$S_A^{r/R} \sim L^{d-1}$$

Measuring nuclei positions has disentangled ("localized") electrons

Then: "Quantum Disentangled liquid" wavefcn



Quantum Disentangled States: Def'n

Volume law entangled wf $S_A^{\Psi} \sim L^d$

Perform (some) **global partial-measurement**, wf of un-measured d.o.f. $|\Psi\rangle \longrightarrow |\psi\rangle$

Entanglement-entropy of post-measurement wf $|\psi\rangle \Rightarrow S_A^{\psi}$

Quantum Disentangled State: post-measurement wf has area law $S_A^{\psi} \sim L^{d-1}$

Partial Measurment induces full locality

In QDL a partial measurement effectively measures all degrees of freedom.

Thermal State: wf post-measurment still entangled (non-local, volume law) $S_A^{\psi} \sim L^d$

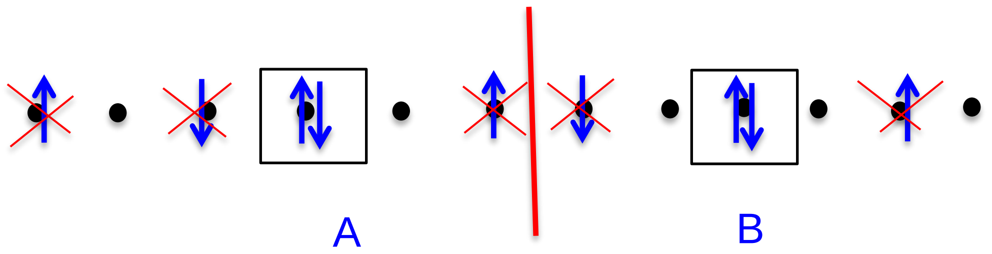
Do Quantum Disentangled States exist as eigenstates of generic Hamiltonians?

Fermion Hubbard models (U>0, 1/2 filling):

$$\hat{\mathcal{H}} = -t\sum_{ij}(\hat{c}_i^\dagger \hat{c}_j + h.c.) + u\sum_i n_{i\uparrow}n_{i\downarrow} + ... \qquad |\Psi\rangle$$

Measure Spin on each site, but not charge $\quad |\Psi\rangle \rightarrow \mathcal{P}|\Psi\rangle = |\psi\rangle$

After measurement, "doublon" (charge) wf $\quad |\psi\rangle$



Entanglement entropy of "charge-wf" (post spin-measurement) $\quad S_{c/s}$

$S_{c/s} \sim L^d$ Thermal Eigenstate

$S_{c/s} \sim L^{d-1}$ Quantum Disentangled Eigenstate

Numerics on 1d “Hubbard” chains

J. Garrison, R. Mishmash,, MPAF (2016)

$$\hat{\mathcal{H}} = -t \sum_{i,\sigma} (\hat{c}_{i\sigma}^{\dagger} \hat{c}_{i+1\sigma} + h.c.) + U \sum_{i} \hat{n}_{i\uparrow} \hat{n}_{i\downarrow} + V \sum_{i} \hat{n}_{i} \hat{n}_{i+1}$$

- U>0, half-filling
- n.n. repulsion, V, destroys integrability
- Compute *all* eigenstates from ED

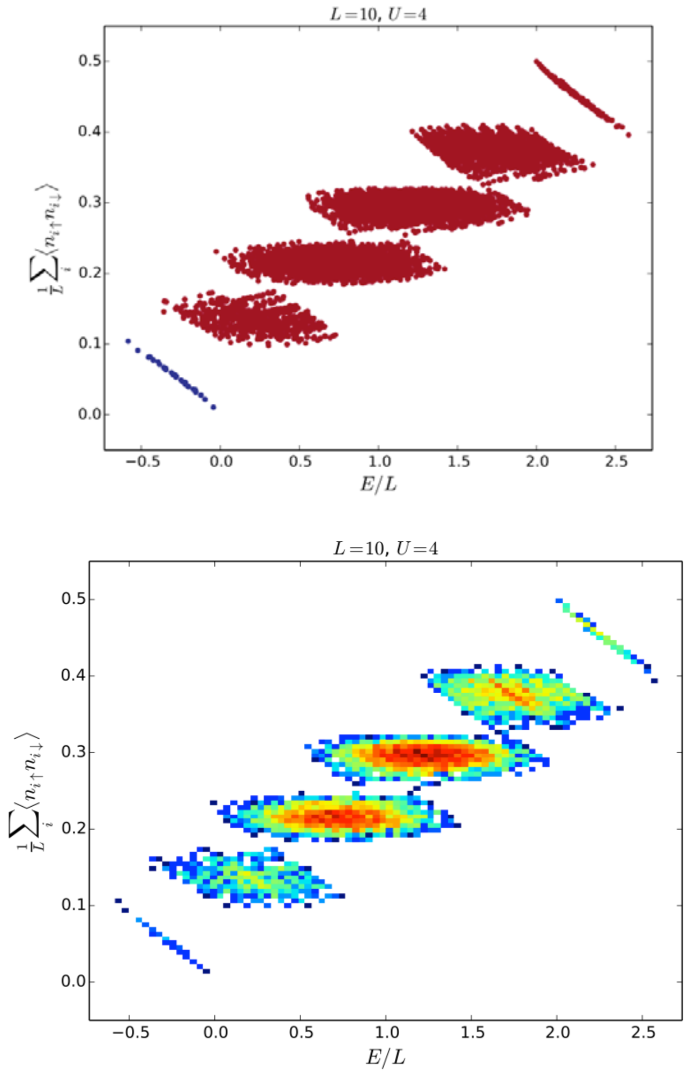
First: Extract mean-double occupancy for each eigenstate
(at many different values of U, V=0,3)

$$\mathcal{D} = \langle \hat{n}_{j\uparrow} \hat{n}_{j\downarrow} \rangle$$

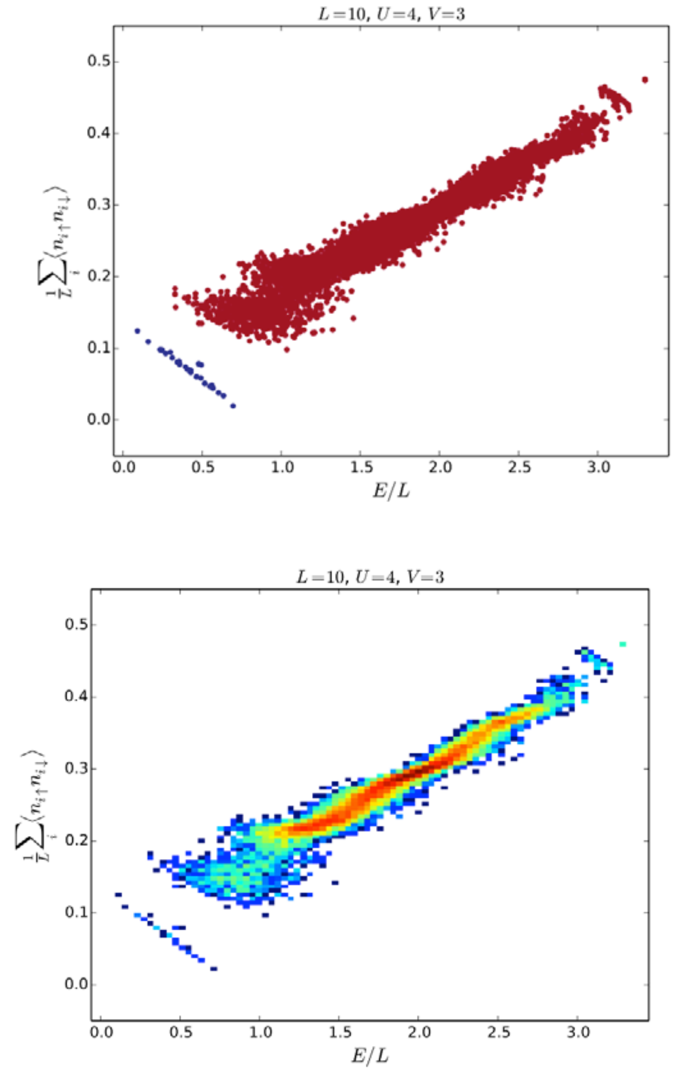
Movie time!

Integrable versus non-integrable

Hubbard (integrable)

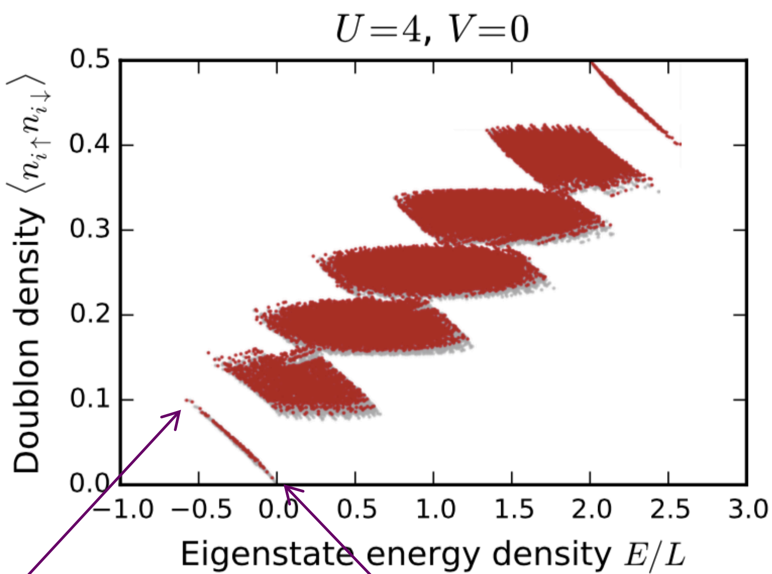


Non-zero V (non-integrable)



Larger system size L=12

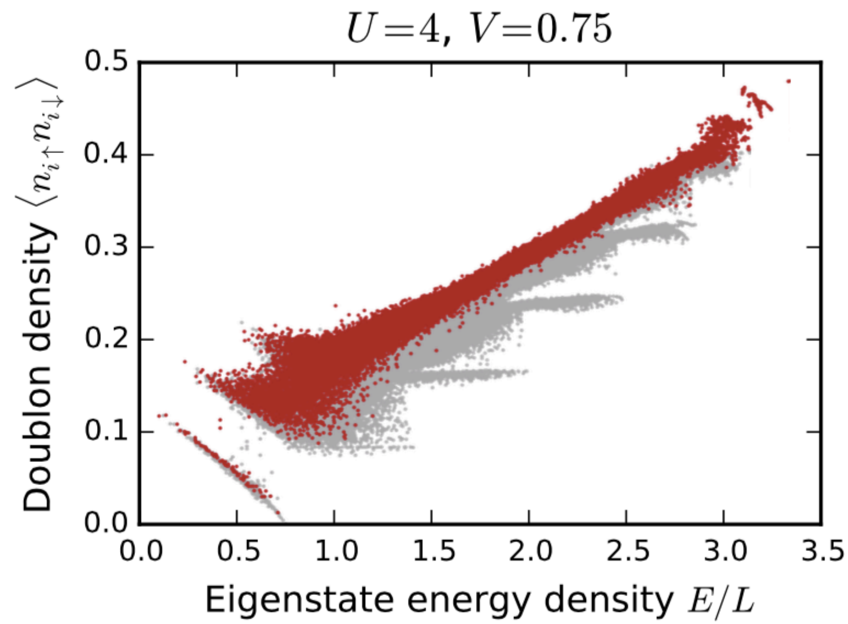
Hubbard (integrable)



Heisenburg AFM

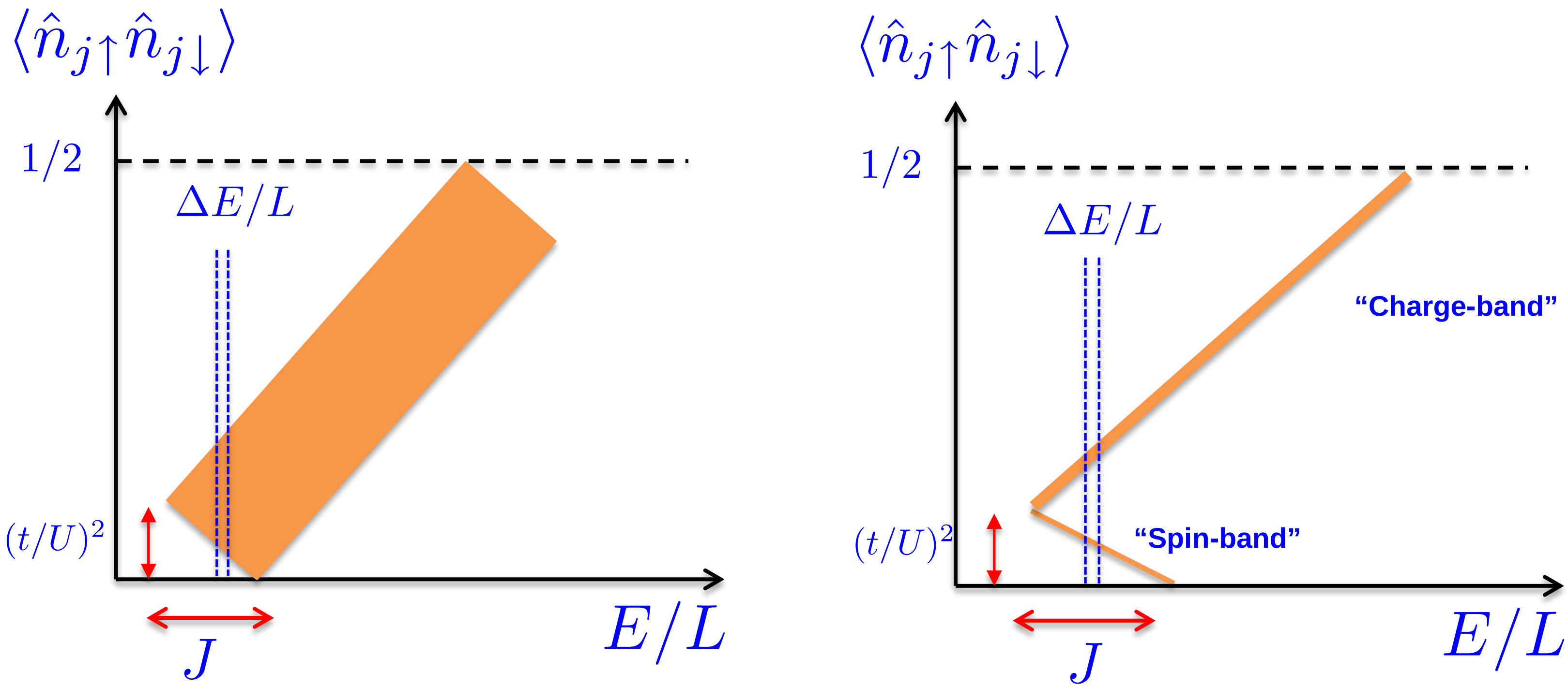
Heisenburg FM

Non-zero V (non-integrable)



Total spin singlets in red

Two sectors at same energy "spin" and "charge"??



Conjecture for 1d Hubbard (large U=4) in thermo limit??

Conjecture for 1d Hubbard + V (large U=4, V=3/4) in thermo limit??

Breakdown of ETH?? "Hot spin-states" w/ same energy as "Cold charge states"

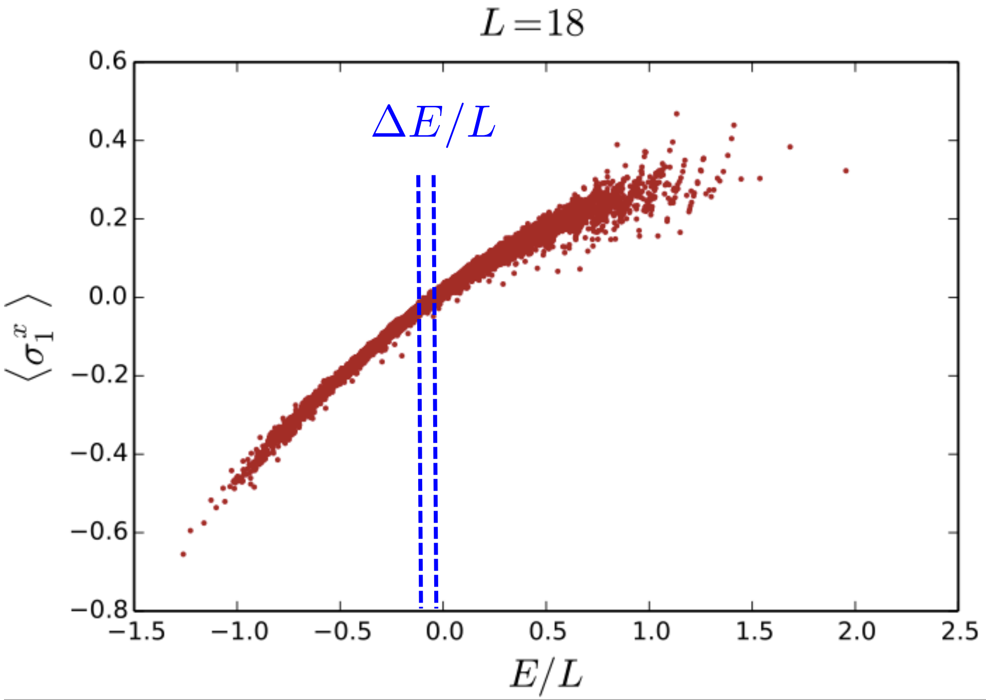
Qualitatively different than Quantum Ising chain

$$H = \sum_{i=1}^{L} \left(g\sigma_i^x + h\sigma_i^z + J\sigma_i^z\sigma_{i+1}^z \right)$$

Kim, Ikeda and Huse, 2014
J. Garrison (unpublished)

Exact diagonalization

$$\langle \psi_E | \sigma_1^x | \psi_E \rangle$$

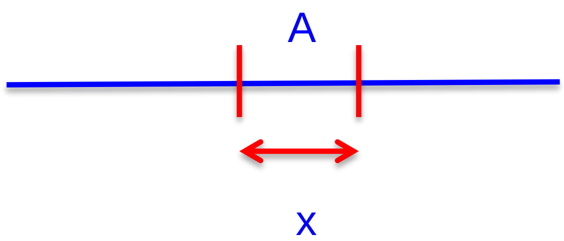


Operaror average “same” for all
eigenstates in narrow energy window (ETH)

Entanglement entropies for Hubbard+V

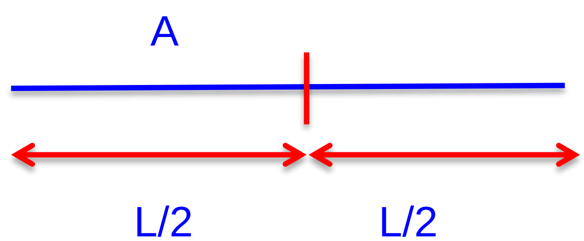
J. Garrison, R. Mishmash, MPAF arXiv;1606.05650

- Compute entanglement entropy for eigenstates $S(x)$
- Measure eigenstate Spin**,** but not charge, compute post-measurement entanglement entropy of **“charge-wf”** $S_{c/s}(x)$



Focus first on L/2 + L/2 partition, for each eigenstate

$S(L/2)$ $S_{c/s}(L/2)$



Full/post-measurement entanglement entropy: L=10

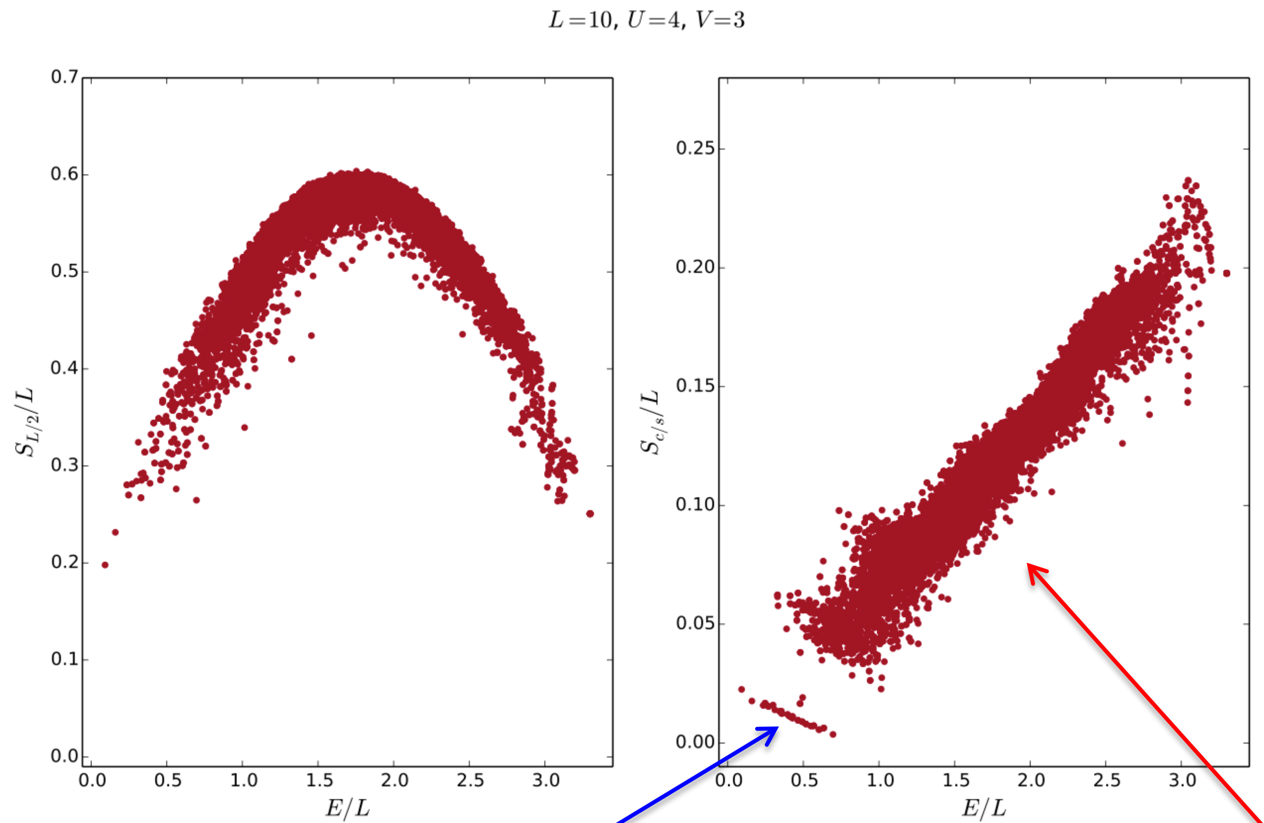
L=10 site Hubbard, V=3

Full Entanglement entropy

$$S(L/2)$$

Post-spin-measurement entropy of "charge-wf"

$$S_{c/s}(L/2)$$



"Charge-band"

"Spin-band" -
Post-spin-measurment, "charge-wf" low entanglement

Full/post-measurement entanglement entropy: L=12

L=12 site Hubbard, U=4, V=3/4

Full Entanglement entropy

$S(L/2)$

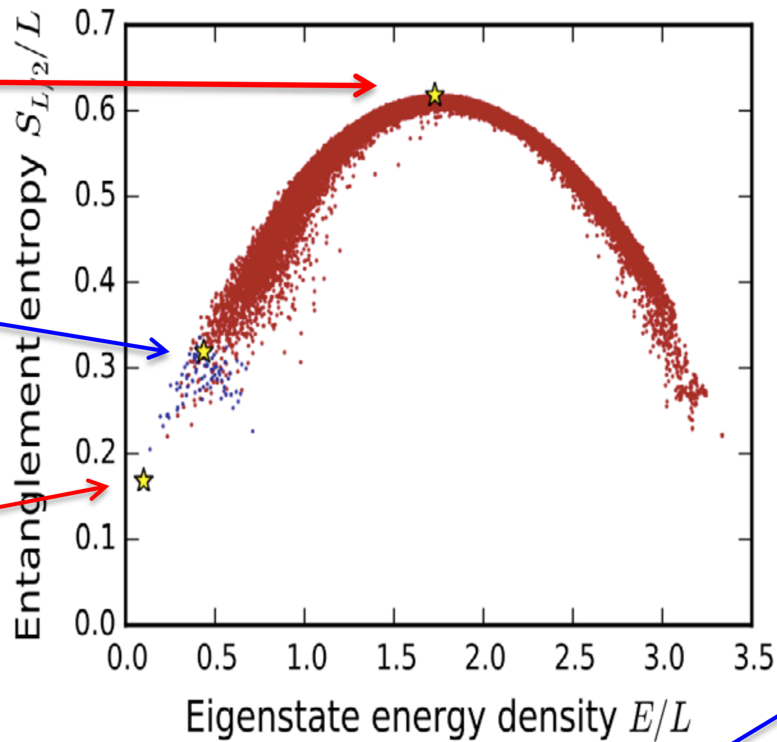
Post-spin-measurement entropy of “charge-wf”

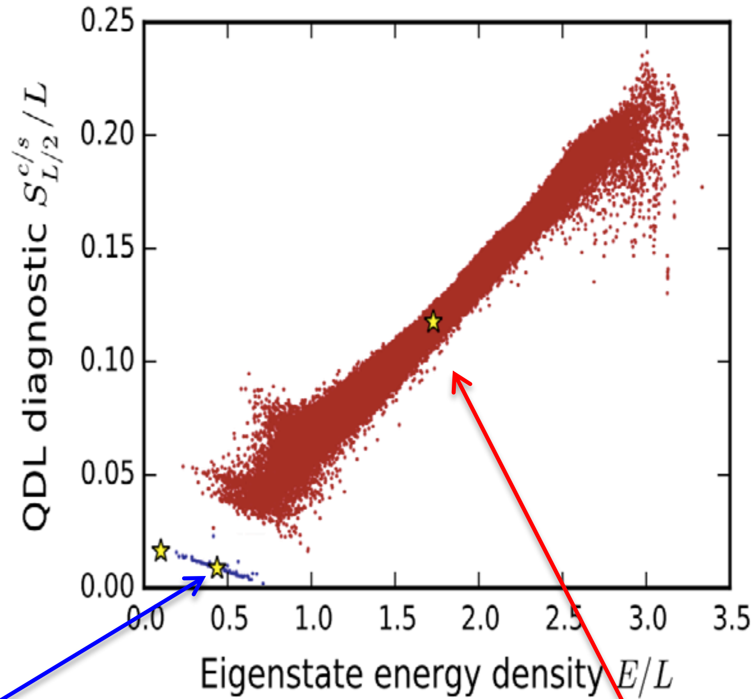
$S_{c/s}(L/2)$

High entropy state, “charge-band”

High entropy state “spin-band”

Ground state





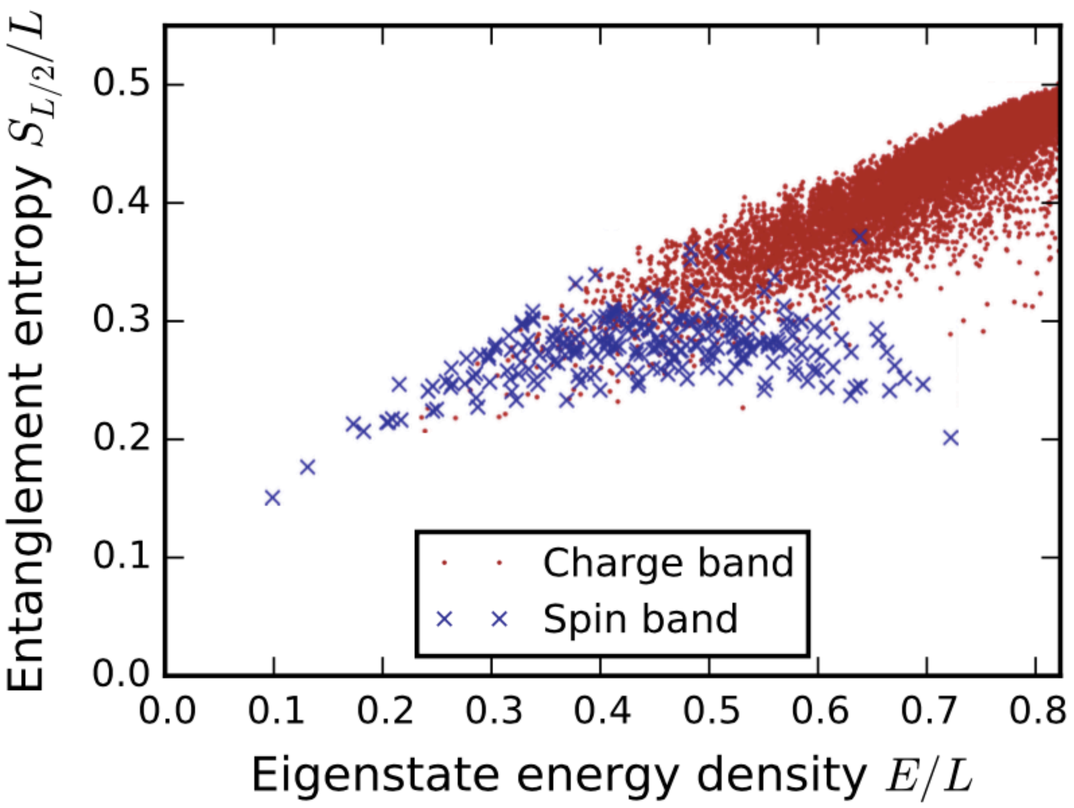
“Spin-band” -
Post-spin-measurment, “charge-wf” low entanglement

“Charge-band”

"Spin" band "Charge" band for L=14

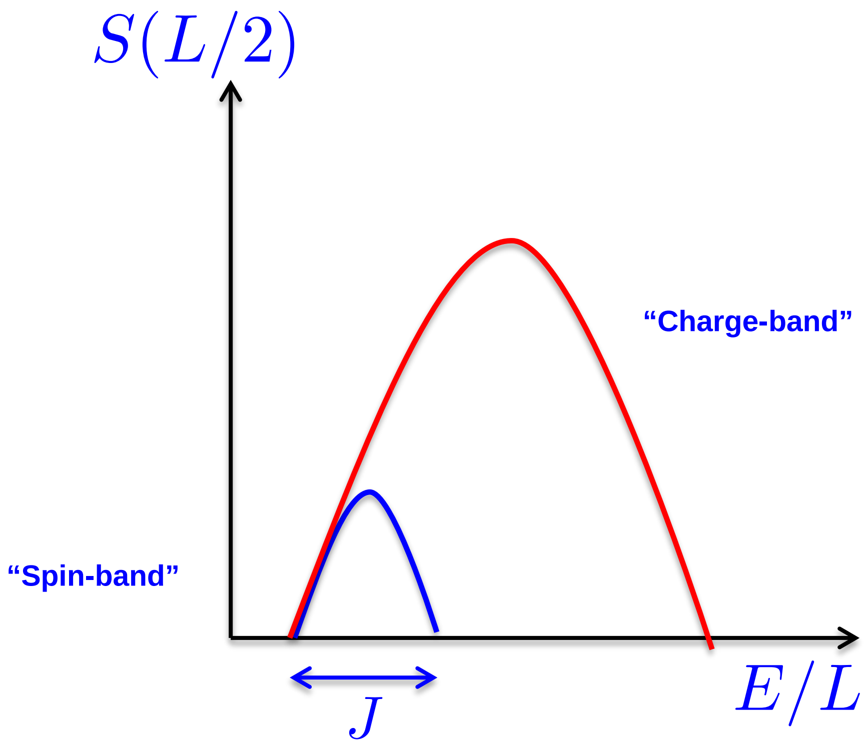
Full Entanglement entropy

$S(L/2)$



L=14 U=4, V=3/4

Co-existing “entropy-bands”?



Conjecture: Breakdown of ETH, co-existing
“hot spin-states” w/ “cold charge states”

Focus on 3 eigenstates

L=12 site Hubbard, U=4, V=3/4

Full Entanglement entropy

$$S(L/2)$$

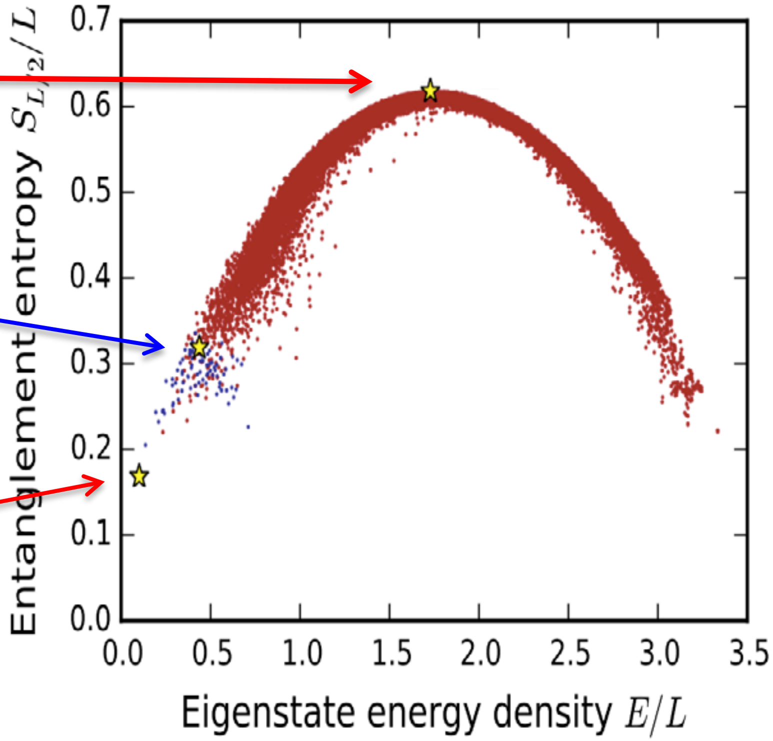
Post-spin-measurement entropy of "charge-wf"

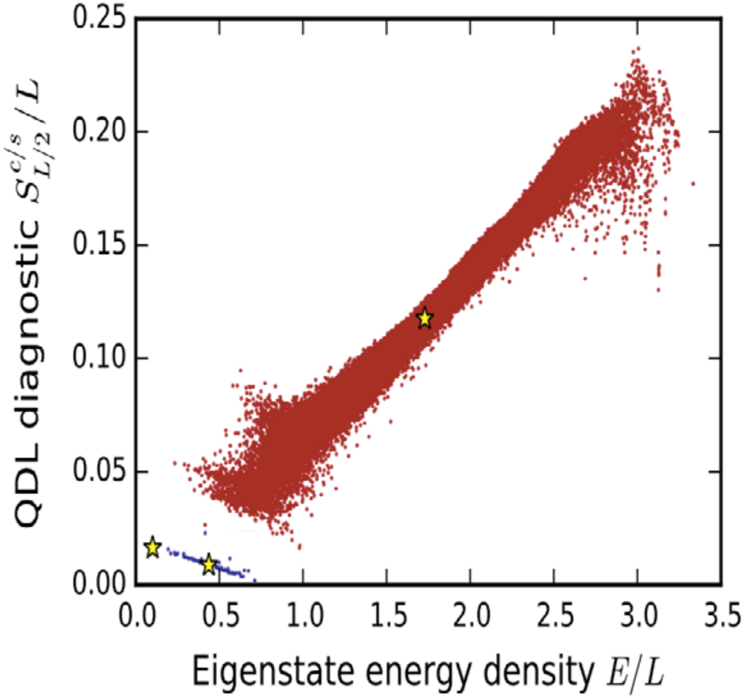
$$S_{c/s}(L/2)$$

High entropy state, "charge-band"

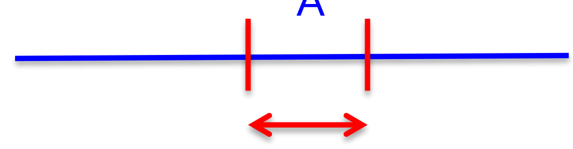
High entropy state "spin-band"

Ground state





Entropy scaling for 3 eigenstates

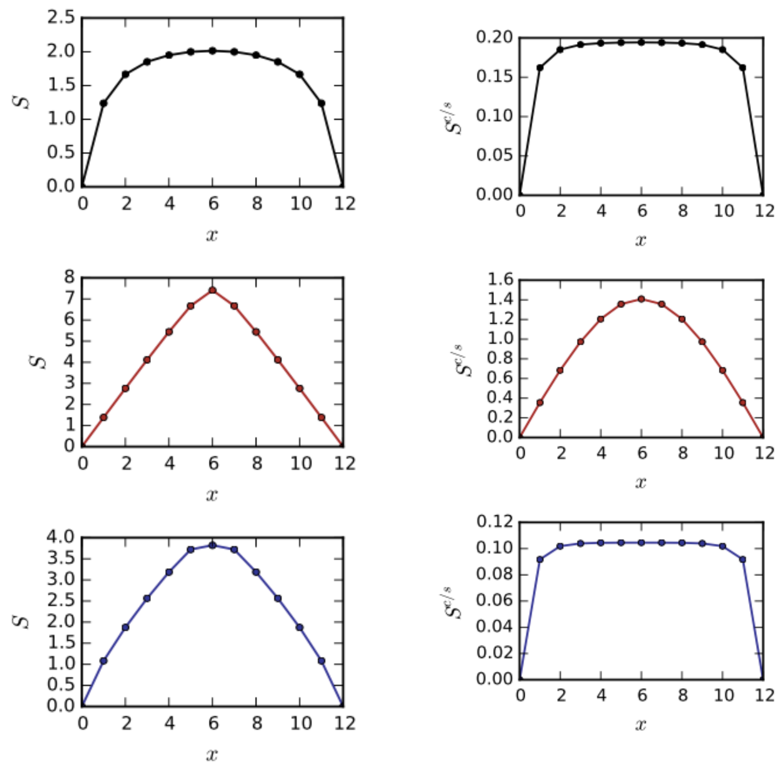


Full Entropy

$$S(x)$$

Post-measurement entropy

$$S_{c/s}(x)$$



Ground state

Area-law (ln(L))

Area-law

High entropy state from "charge-band"

Volume-law

Volume-law

High entropy state from "spin-band"

Volume-law

Area-law!

Possible "spin-band" of Quantum-disentangled states, area law charge entropy post-spin-measurement

Summary

Diagnostic: entanglement entropy of post partial-measured wf

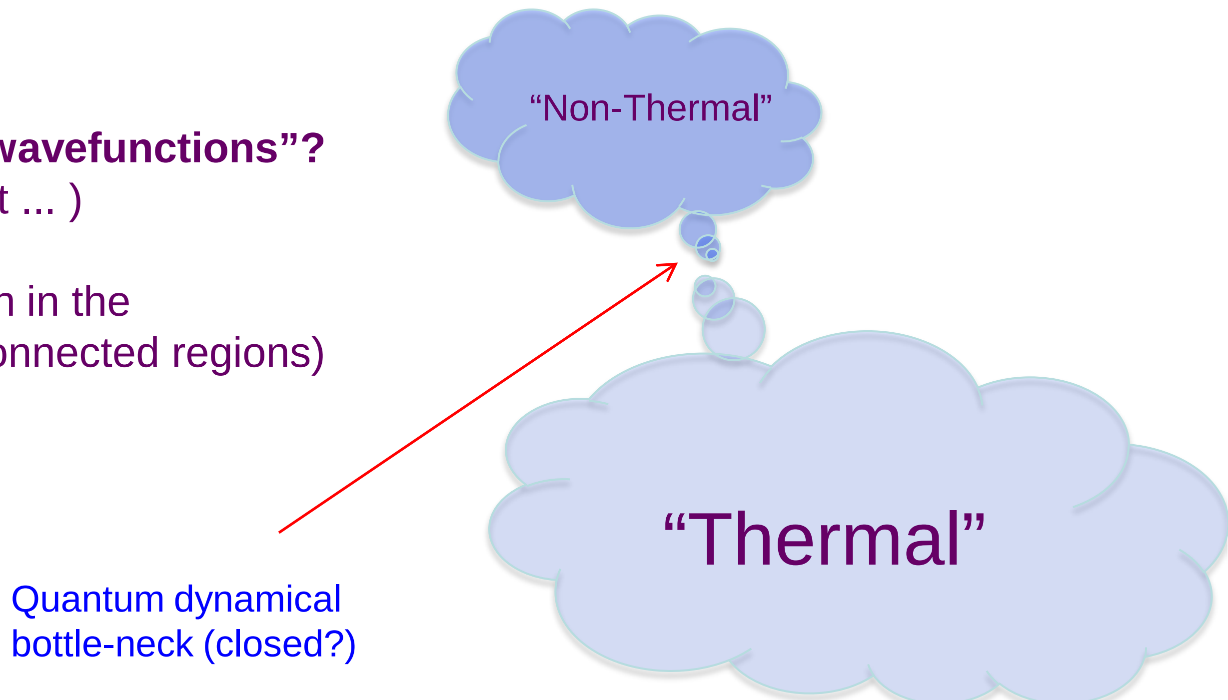
Quantum Disentangled States: Volume-law wf w/ area law entanglement following partial-measurement – **“exposing hidden locality”**

In QDL a partial measurement effectively measures all degrees of freedom

Open/future:

How to define **“fully thermal wavefunctions”?**
(besides not MBL, not QDL, not ...)

Physics: Dynamical localization in the many-body Hilbert space (disconnected regions)



Experimental systems?

(Not so) Cold Atoms

- Heavy/light QDL?   Li-6, Cs-133 mixtures
- Spin/Charge QDL? 1d Fermion Hubbard in optical lattice
  (making partial measurements w/ optical microscope)

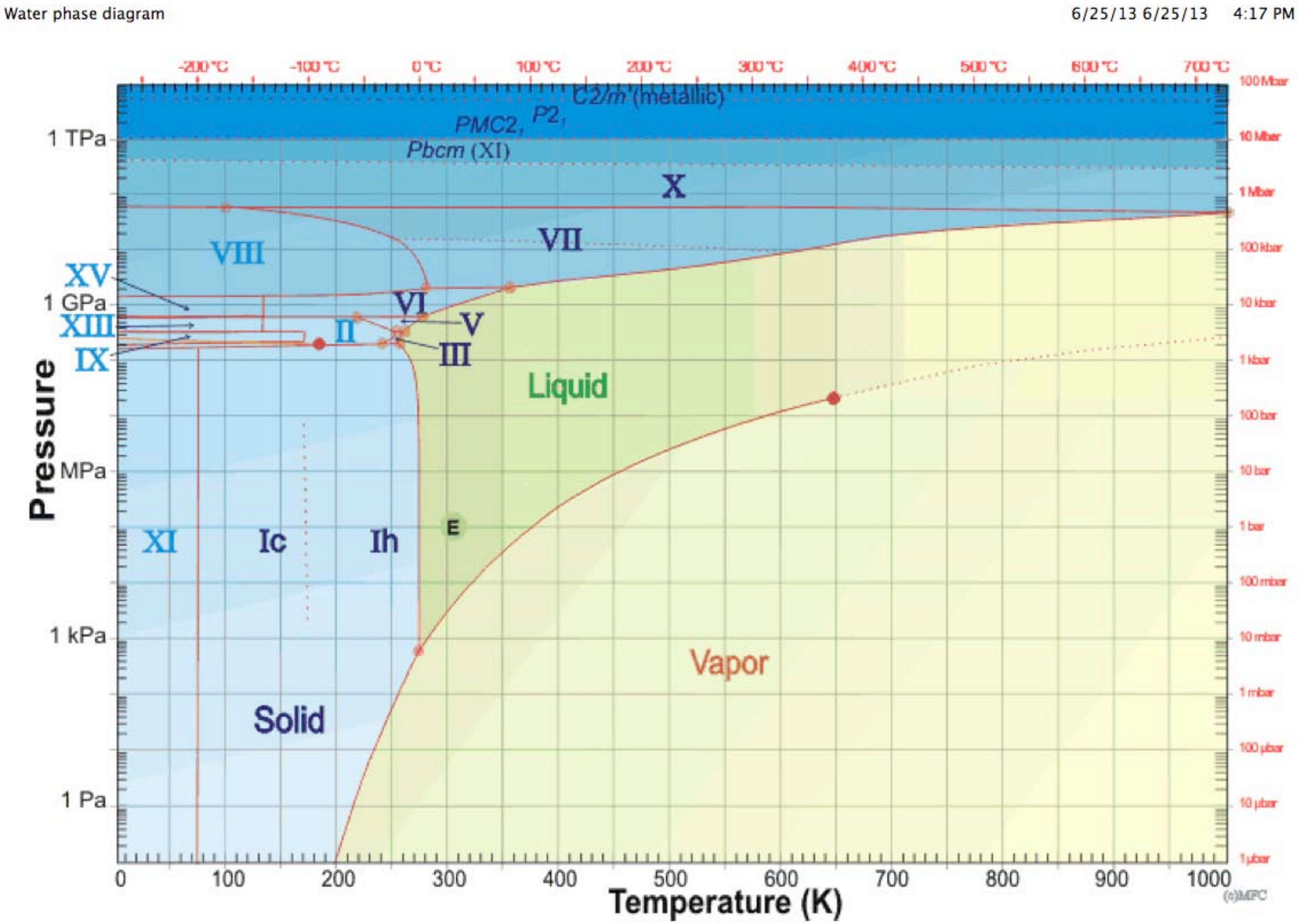
Electron Mott Insulators: U>0 Hubbard

- Undoped Mott insulators, excite spin-excitations staying in “spin-band”? Electrical conductivity vanish?

Atomic/Molecular Liquids: eg water

- Heavy/light, Oxygen/protons, conductivity of water vanish?
- Charge/spin: Nuclear-charge/nuclear-spin

Water: Phase Diagram



Electrical Conductivity of Water

Water STP

$$\sigma_{salt} = 10^{-1} S/cm \qquad \rho_{salt} = 10\ \Omega - cm$$

$$\sigma_{tap} = 10^{-4} S/cm$$

$$\sigma_{pure} \sim 10^{-7} S/cm$$

"Supercritical Water", pressure up to 500 kbar
(pure water under "shock")

$$\sigma_{STP} \sim 10^{-7} S/cm$$

$$\sigma_{50\,kbar} \sim 10^{-4} S/cm$$

$$\sigma_{200\,kbar} \sim 20 S/cm \quad (T \sim 5,000K)$$

Water at STP in QDL???
Water at 200 kbar in conventional phase?

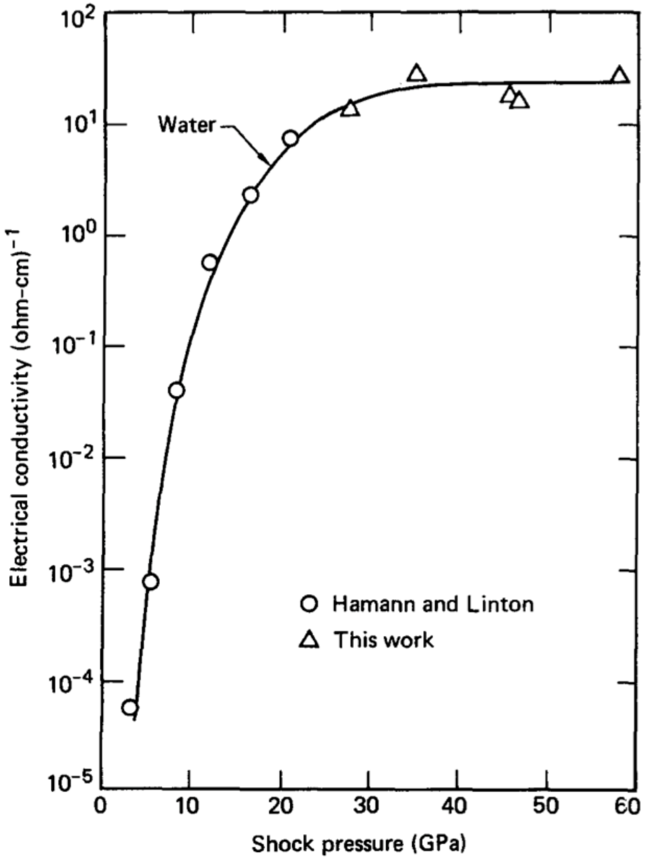
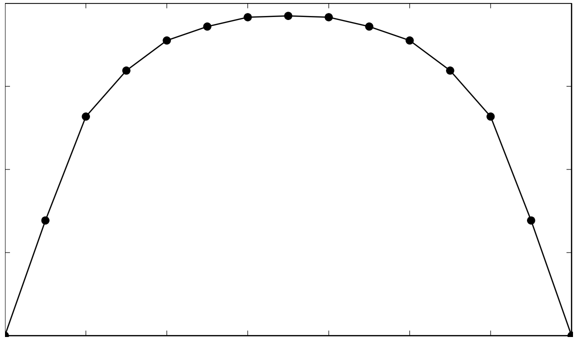


FIG. 10. Electrical conductivity vs shock pressure for liquid H_2O (10 GPA = 100 kbar).

S(x)-vs-x for J1-J2 s=1/2 model

Ground state (VBS state)



Maximally entangled state

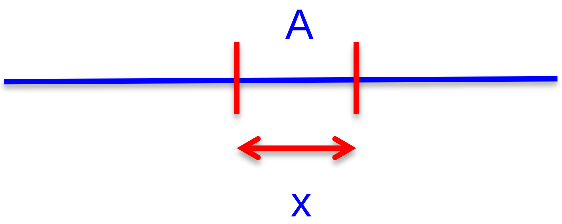
Entropy scaling for 3 eigenstates

Full Entropy

$$S(x)$$

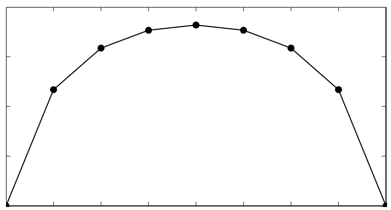
Post-measurement Spin entropy

$$S_{s/c}(x)$$



Ground state

Area-law



Area-law

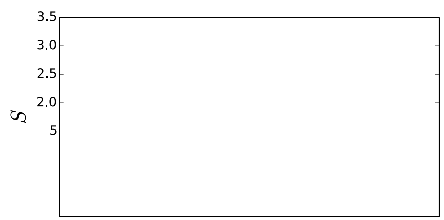
E>0 eigenstate,
not in “band”

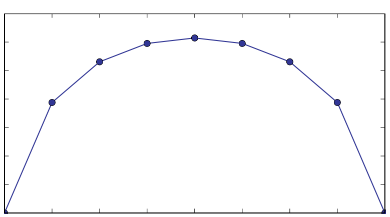
Volume-law

Volume-law

E>0 Eigenstate
from “band”

Volume-law





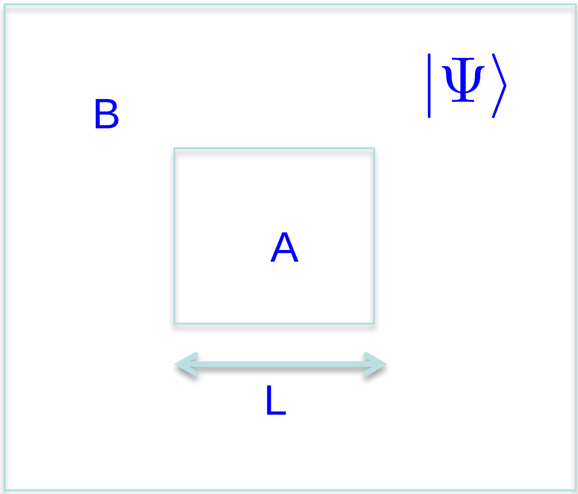
Area-law?

Possible band of Quantum-disentangled states, “liquid of pairs”
w/ area-law post-measurement spin-entropy?

Quantum Disentangled Eigenstates

Consider finite-energy density eigenstate of a many-body Hamiltonian, which has a volume law entanglement entropy $S_A^{\Psi} \sim L^d$

$$\hat{\mathcal{H}}|\Psi\rangle = E|\Psi\rangle$$



Perform (some) local **partial** measurement, and obtain projected wavefunction for unmeasured degrees of freedom

$$|\Psi\rangle \longrightarrow |\psi\rangle$$

Compute spatial entanglement entropy of post-measurement wavefunction S_A^{ψ}

“Conventional Thermal state”: Post-measurment entanglement is still **volume** law $S_A^{\psi} \sim L^d$

“Quantum Disentangled state”: **Area** law post-measurement entanglement $S_A^{\psi} \sim L^{d-1}$

In Quantum Disentangled Liquid a *partial* local measurement induces *full* locality

Full measurement in spatially extended system: Disentangles

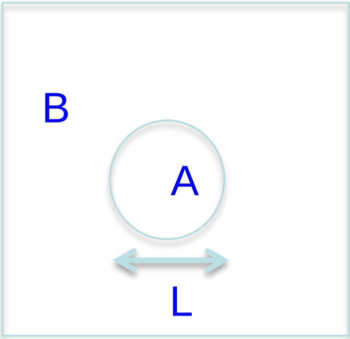
Heisenberg model with s=1/2 $\hat{\mathcal{H}} = \sum_{ij} J_{ij} \vec{S}_i \cdot \vec{S}_j + \ldots$

Hilbert space: Direct product of up/down spin on each site $|\{s_j\}\rangle = \prod_{j=1}^{N} |s_j\rangle_j \qquad s_j = \pm 1$

Finite energy-density eigenstate:
Volume law entanglement,

$$|\Psi\rangle = \sum_{\{s_j\}} \phi(\{s_j\}) |\{s_j\}\rangle$$

$$\hat{\rho}_A = Tr_B |\Psi\rangle\langle\Psi| \qquad S_A = -Tr_A[\hat{\rho}_A \ln \hat{\rho}_A] = sL^d$$



Make Full (Local) Measurement: Measure spin on each and every site, find $\{\tilde{s}_j\}$

$$|\Psi\rangle \longrightarrow |\Psi'\rangle = |\{\tilde{s}_j\}\rangle$$ (a direct product state)

After measurement: Wavefcn fully disentangled

$$S_A = sL^d \rightarrow S'_A = 0$$

Local Measurement collapses “non-local” volume law state, inducing “locality”

Local **Partial**-Measurements

Hubbard model for electrons $\hat{\mathcal{H}} = -t \sum_{ij} (\hat{c}_i^\dagger \hat{c}_j + h.c.) + u \sum_i n_{i\uparrow} n_{i\downarrow} + ...$

On-site Hilbert space: Up/down spin, empty, doubly-occupied $|\uparrow\rangle, |\downarrow\rangle, |0\rangle, |\uparrow\downarrow\rangle$

***Partial*-Measurement: Measure spin** on each site

Single-site wf: $|\psi\rangle = \phi_\uparrow |\uparrow\rangle + \phi_\downarrow |\downarrow\rangle + \phi_0 |0\rangle + \phi_{\uparrow\downarrow} |\uparrow\downarrow\rangle$

$$|\psi\rangle \longrightarrow \begin{cases} \mathcal{P}_\uparrow |\psi\rangle = \phi_\uparrow |\uparrow\rangle \\ \mathcal{P}_\downarrow |\psi\rangle = \phi_\downarrow |\downarrow\rangle \\ \mathcal{P}_0 |\psi\rangle = \phi_0 |0\rangle + \phi_{\uparrow\downarrow} |\uparrow\downarrow\rangle \end{cases}$$

Spin projection operators

$$\mathcal{P}_\uparrow = |\uparrow\rangle\langle\uparrow|$$
$$\mathcal{P}_\downarrow = |\downarrow\rangle\langle\downarrow|$$
$$\mathcal{P}_0 = |0\rangle\langle 0| + |\uparrow\downarrow\rangle\langle\uparrow\downarrow|$$

After measuring spin, get a charge wavefunction (doublon/holon)

***Partial*-Measurement: Measure charge** on each site

$$|\psi\rangle \longrightarrow \begin{cases} \mathcal{P}_0 |\psi\rangle = \phi_0 |0\rangle \\ \mathcal{P}_1 |\psi\rangle = \phi_\uparrow |\uparrow\rangle + \phi_\downarrow |\downarrow\rangle \\ \mathcal{P}_2 |\psi\rangle = \phi_{\uparrow\downarrow} |\uparrow\downarrow\rangle \end{cases}$$

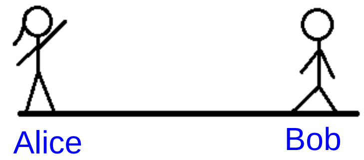
Charge projection operators:

$$\mathcal{P}_0 = |0\rangle\langle 0|$$
$$\mathcal{P}_1 = |\uparrow\rangle\langle\uparrow| + |\downarrow\rangle\langle\downarrow|$$
$$\mathcal{P}_2 = |\uparrow\downarrow\rangle\langle\uparrow\downarrow|$$

After measuring charge, get a spin wavefunction (up/down)

Entanglement, locality, information and measurment

Alice and Bob, each with a s=1/2 particle



Direct product: $|\psi\rangle = |\uparrow\rangle_A \otimes |\downarrow\rangle_B$

Bob's measurement, no effect on Alice.
Alice has full **local quantum information**.

Singlet state: $|\psi\rangle = \frac{1}{\sqrt{2}}[|\uparrow\rangle_A \otimes |\downarrow\rangle_B - |\downarrow\rangle_A \otimes |\uparrow\rangle_B]$

Bob's measurement affects Alice. Alice has **no local quantum information** about "her" spin - two spins are "entangled"

Entanglement entropy quantifies local quantum information

Direct product state – complete local info: $S_A = S_B = 0$
Entangled Singlet state – no local info: $S_A = S_B = \ln(2)$

Local Measurement: Causes spatial disentanglement

For singlet state Bob **measures** his spin:

$$|\psi\rangle = \frac{1}{\sqrt{2}}[|\uparrow\rangle_A \otimes |\downarrow\rangle_B - |\downarrow\rangle_A \otimes |\uparrow\rangle_B] \Rightarrow \begin{cases} |\psi'\rangle = |\uparrow\rangle_A \otimes |\downarrow\rangle_B \\ |\psi'\rangle = |\downarrow\rangle_A \otimes |\uparrow\rangle_B \end{cases}$$

After measurment, direct product state

$$S_A = S_B = \ln(2) \Rightarrow S'_A = S'_B = 0$$

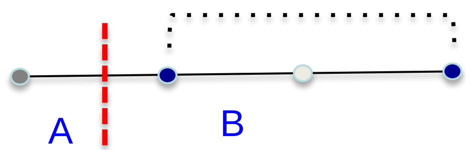
Measuring spin disentangles charge

4-site Hubbard, 2 up, 2 down spin electrons

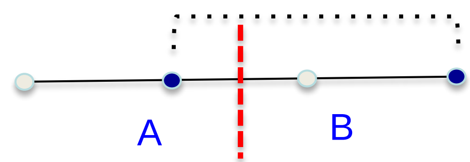
Measure spin on each site: $|\Psi\rangle \longrightarrow |\Psi'\rangle$

Suppose spin measurement gives (+1,0,-1,0),
wf after measurement $|\Psi'\rangle = a_1|\uparrow, 0, \downarrow, \uparrow\downarrow\rangle + a_2|\uparrow, \uparrow\downarrow, \downarrow, 0\rangle$

Bipartition and compute entanglement entropy



$S_A = 0$



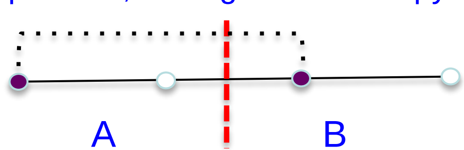
$S_A = \ln 2; \quad (a_1 = a_2 = 1/\sqrt{2})$

Measuring spin (partially) disentangles charge

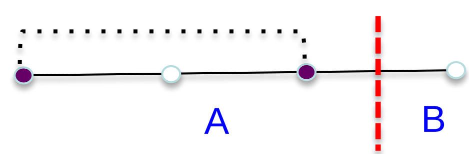
Measuring charge (partially) disentangles spin $|\Psi\rangle \longrightarrow |\Psi'\rangle$

Suppose charge measurement gives (1,0,1,2) leaving spin wf $|\Psi'\rangle = c_1|\uparrow, 0, \downarrow, \uparrow\downarrow\rangle + c_2|\downarrow, 0, \uparrow, \uparrow\downarrow\rangle$

Bipartition, entanglement entropy of spin wf



$S_A = \ln 2; \quad (c_1 = c_2)$



$S_A = 0$

Can partial measurments fully disentangle?

Do Quantum-disentangled eigenstates exist with area law “spin-wf”?

Can Charge measurement on volume-law wf induce complete locality, **area law of post-measurment “spin-wf”?**

$$S_{s/c} \sim L^{d-1}$$

Do Quantum-disentangled eigenstates exist with area law “charge-wf”?

Can Spin measurment on volume-law wf induces complete locality, **area law of post-measurment “charge-wf”?**

$$S_{c/s} \sim L^{d-1}$$

QDL in Hubbard Model?

Bi-partite Hubbard, half-filling

$$\hat{\mathcal{H}} = -t \sum_{ij} (\hat{c}^\dagger_{i\sigma} \hat{c}_{j\sigma} + h.c.) + U \sum_i n_{i\uparrow} n_{i\downarrow}$$

Charge/spin duality between positve and negative U Hubbard

$$\begin{aligned} \hat{c}_{j\uparrow} &\longrightarrow \hat{c}_{j\uparrow} \\ \hat{c}_{j\downarrow} &\longrightarrow (-1)^j \hat{c}^\dagger_{j\downarrow} \end{aligned} \qquad \hat{\mathcal{H}}(U) \longrightarrow \hat{\mathcal{H}}(-U)$$

Duality between charge/spin and spin/charge entanglement entropies

$$S_{c/s}(U) = S_{s/c}(-U)$$

Implication: Free Fermions are not in a QDL phase!

$$S_{c/s}(0) = S_{s/c}(0) \sim L^d$$

Large U>0: Measuring spin almost determines charge, since doublons are rare,
Possible charge-disentangled QDL?

Large U<0, measuring charge almost determines state since mostly Cooper pairs
Possible spin-disentangled QDL?

“Doublons” in the Hubbard Model

Near-neighbor Hubbard, H, in any dimension

$$\hat{\mathcal{H}} = -t\sum_{ij}(\hat{c}^\dagger_{i\sigma}\hat{c}_{j\sigma} + h.c.) + U\sum_i n_{i\uparrow}n_{i\downarrow}$$

MacDonald, Girvin, Yoshioka (1988)

Order-by-order in t/U, can perform a unitary transformation on H which eliminates terms coupling states w/ differing number of doubly occupied sites

$$H' = e^{iS}He^{-iS}$$

H' is “block-diagonalized” into decoupled “doublon-sectors”

Example: At leading order in t/U

$$iS = (T_1 - T_{-1})/U + \mathcal{O}(t/U)^2$$

$T_{\pm 1}$ Terms in kinetic energy which change number of doubly occupied sites by ± 1

In sector w/ zero doublons, at leading order in t/U, get t-J model $H' = H_{tJ}$

“Doublons” at half-filling

At half-filling get spin-model

$$H' = J_2 \sum_{ij} \vec{S}_i \cdot \vec{S}_j + J_4 \sum_{ijkl} (P_{ijkl} + h.c.) + ... \qquad J_n \sim t(t/U)^{n-1}$$

Naively, can transform spin-model eigenstates into Hubbard eigenstates with the unitary,

$$|\psi_{Hubbard}\rangle = e^{iS} |\psi_{spin}\rangle$$

Questions:

- Does the t/U expansion converge
  - for ground state of spin-model?
  - for thermal eigenstates of spin-model??
- Is the operator S local in space (for U>>t)? Is exp(iS) a “local unitary”?
- Role of integrability of 1d Hubbard model?
- Generalization to 1d non-integrable model, eg w/ n.n. interaction, V

If a volume law Hubbard eigenstate can be transformed via a local unitary into a (volume law) spin-wf, then a spin-measurement of wf will disentangle, giving an area law post-measurement charge-entanglement entropy (ie it is a QDL state)

$$S_{c/s} \sim L^{d-1}$$

Equivalence of local correlators in ETH

Kim, Ikeda and Huse, 2014

$$H = \sum_{i=1}^{L} \left(g\sigma_i^x + h\sigma_i^z + J\sigma_i^z\sigma_{i+1}^z \right)$$

$$H = \sum_{i=1}^{L} \left[-t(b_{i+1}^\dagger b_i + b_i^\dagger b_{i+1}) + V n_i n_{i+1} \right] + \sum_{i=1}^{L} \left[-t'(b_{i+2}^\dagger b_i + b_i^\dagger b_{i+2}) + V' n_i n_{i+2} \right]$$

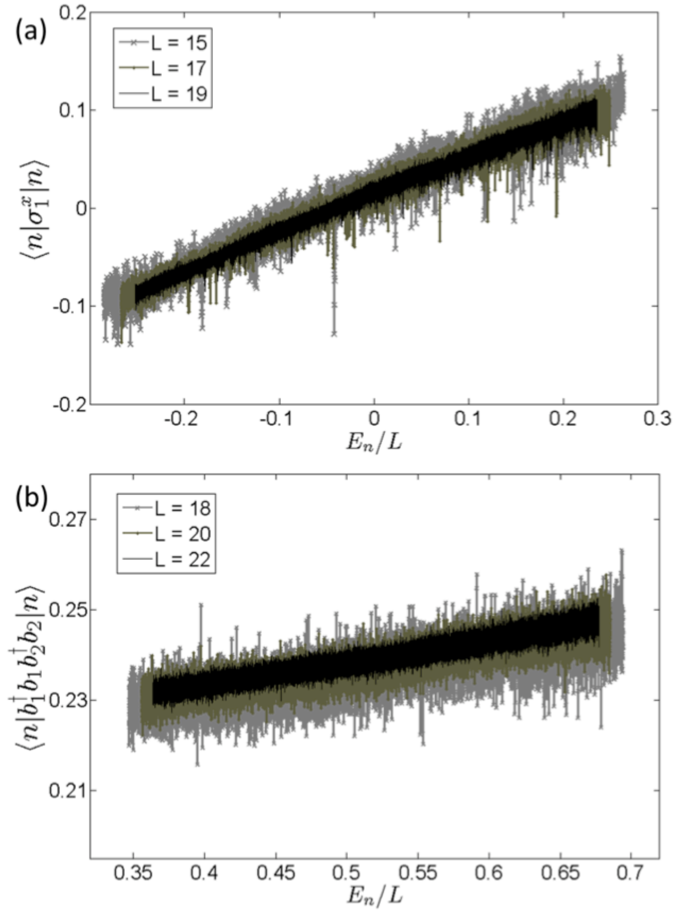
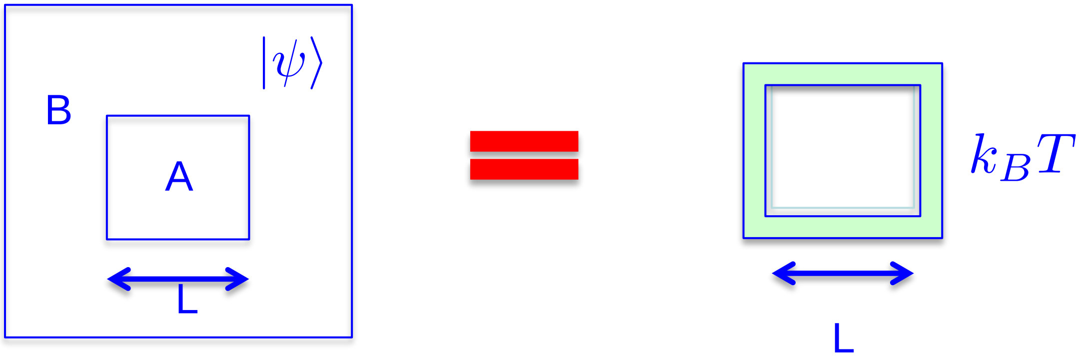


FIG. 1: Diagonal elements of a few-body operator in energy eigenbasis vs. energy density. The darker, the larger the system size. (a) Ising Hamiltonian (Eq. (7)). The operator is σ_1^x. (b) Hard-core boson Hamiltonian (Eq. (8)). The operator is $n_1 n_2 = b_1^\dagger b_1 b_2^\dagger b_2$. For both cases, the fluctuations become smaller as the system size increases.

Eigenstate Thermalization Hypothesis (ETH)

Josh Deutsch, Mark Srednicki

ETH = Microcanonical ensemble for $E/L^d > 0$ eigenstate
“equivalent”, in region A, to canonical ensemble w/ heat bath



Equivalence of (non-local) Thermal and entanglement entropies

$$S_A/L^d = S_{th}/L^d; \quad L \to \infty$$

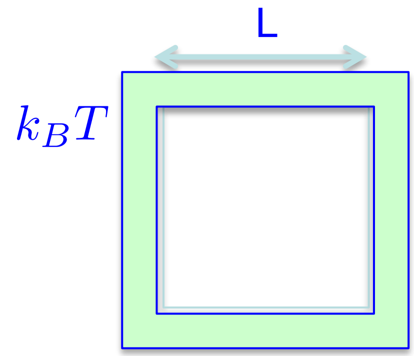
Thermal entropy is state counting, entanglement entropy
depends on the properties of the states!

Eigenstates in narrow energy (density) window have identical correlations of local operators

$$\langle \hat{\mathcal{O}} \rangle_E = \langle \hat{\mathcal{O}} \rangle_{E+\Delta E}$$

Quantum Statistical Mechanics

Canonical Ensemble, w/ heat bath



Thermal density matrix $\hat{\rho}_{th} = \frac{1}{Z} e^{-\beta \hat{\mathcal{H}}}$

(Local) Observables $\langle \hat{\mathcal{O}} \rangle_{th} = Tr[\hat{\rho}_{th} \hat{\mathcal{O}}]$

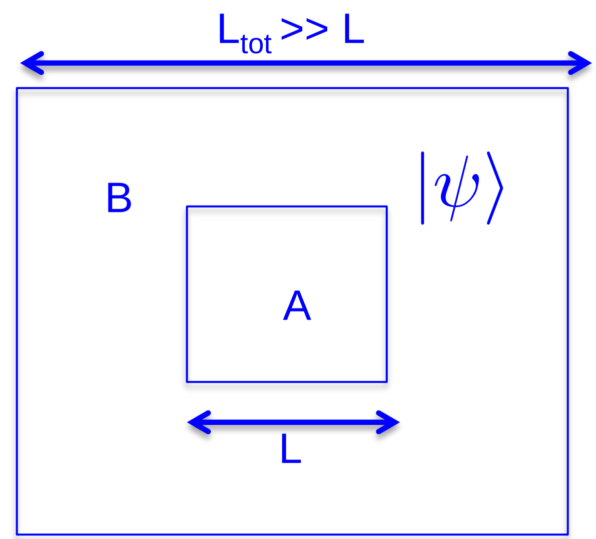
Internal Energy (extensive) $U = Tr[\hat{\rho}_{th} \hat{\mathcal{H}}]$ $U/L^d > 0$

Microcanonical Ensemble: Isolated, single eigenstate

$$\hat{\mathcal{H}} |\psi\rangle = E |\psi\rangle$$

$$\hat{\rho} = |\psi\rangle\langle\psi|$$

$$|\psi\rangle$$



Spatial partition: Regions A ("system") and B ("environment")

Observables inside "system" A $\langle \hat{\mathcal{O}} \rangle_E = Tr[\hat{\rho} \hat{\mathcal{O}}]$

Equivalence of Canonical and Microcanonical Ensemble:

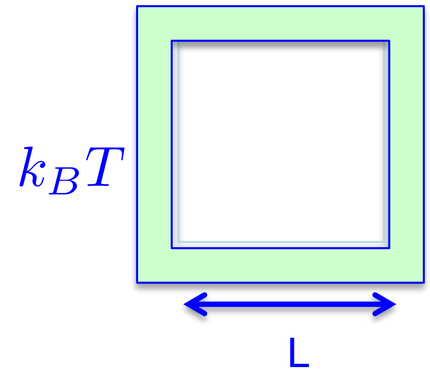
For "all" states w/ energy E,
observables are equivalent

$$E/L^d = U/L^d$$

$$\langle \hat{\mathcal{O}} \rangle_E = \langle \hat{\mathcal{O}} \rangle_{th}$$

Entropy: Thermal "versus" entanglement

Entropy is "non-local"



Thermal entropy, w/ heat bath:

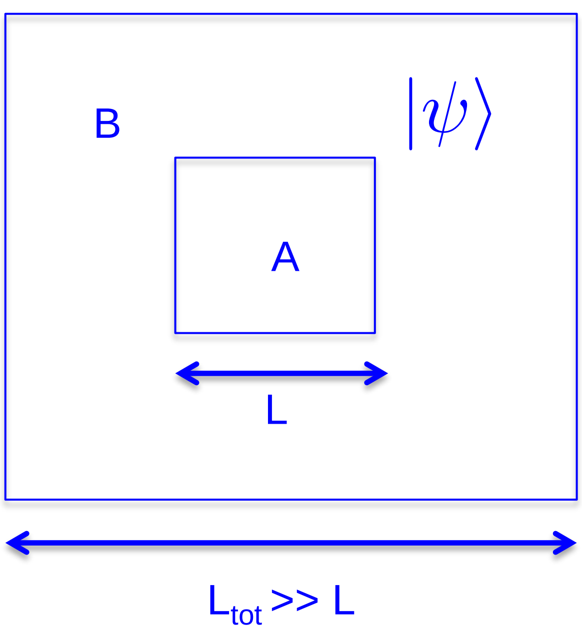
Number of states,
extensive for T>0

$$S_{th} = -Tr[\hat{\rho}_{th} \ln \hat{\rho}_{th}]$$

$$S_{th} \sim L^d$$

$$\hat{\rho}_{th} = \frac{1}{Z} e^{-\beta \hat{\mathcal{H}}}$$

Entanglement Entropy: Isolated, single eigenstate $|\psi\rangle$



$$\hat{\mathcal{H}}|\psi\rangle = E|\psi\rangle$$

$$\hat{\rho} = |\psi\rangle\langle\psi|$$

Reduced density matrix in A $\hat{\rho}_A = Tr_B(\hat{\rho})$

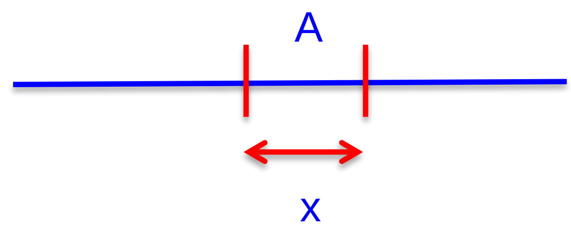
Entanglement entropy: $S_A(L) = -Tr_A(\hat{\rho}_A \ln \hat{\rho}_A)$

Ex: Entanglement entropy in Quantum Ising chain

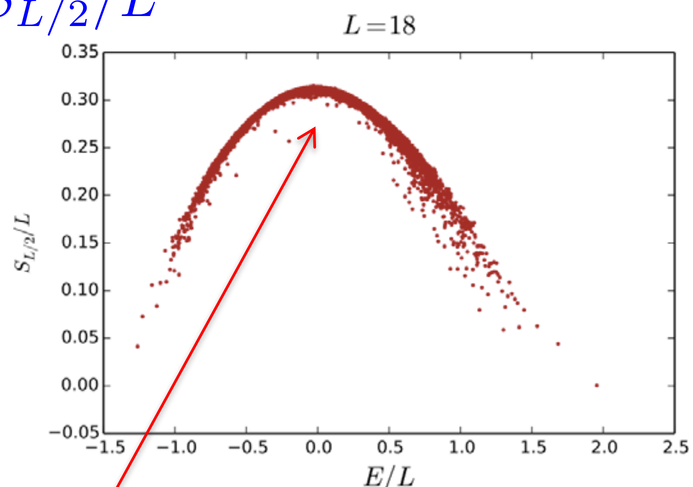
1d Quantum Ising chain

$$H = \sum_{i=1}^{L} \left(g\sigma_i^x + h\sigma_i^z + J\sigma_i^z\sigma_{i+1}^z \right)$$

(g, h, J) = (0.9, 0.8, 1)



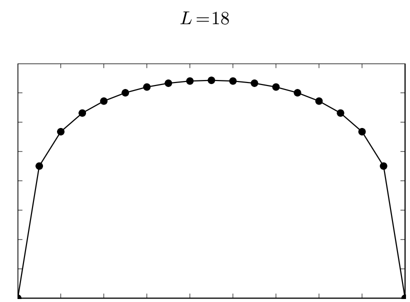
$S_{L/2}/L$



Maximum entropy state,
minimal (zero) local information $S_A/L_A \approx \ln 2$

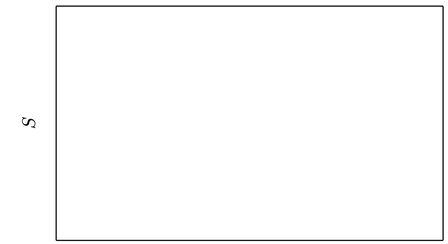
Ground state,
Area-law:

$S_A \sim O(1)$



"High-energy" state,
Volume law:

$$S_A \sim \frac{L}{2} - |x - \frac{L}{2}|$$



"Doublons" at half-filling

U>>t Hubbard: In t/U expansion unitary transformation on
H to eliminate doubly occupancy, spin Hamiltonian:

MacDonald, Girvin, Yoshioka (1988)

$$\hat{H}_{spin} = \hat{\mathcal{U}}\ \hat{H}_H\ \hat{\mathcal{U}}^\dagger$$

$$\hat{H}_{spin} = J_2 \sum_{ij} \vec{S}_i \cdot \vec{S}_j + J_4 \sum_{ijkl} (P_{ijkl} + h.c.) + ...$$

$$J_n \sim t(t/U)^{n-1}$$

Naively, can transform spin-model eigenstates into Hubbard eigenstates with the unitary,

$$|\psi_H\rangle = \hat{\mathcal{U}}\ |\psi_{spin}\rangle$$

Questions:

- Does t/U expansion converge
  - for ground state of spin-model?
  - for thermal eigenstates of spin-model??
- Is U a "local unitary"?

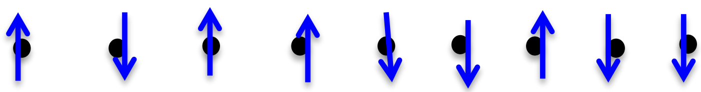
If volume law Hubbard eigenstate can be transformed via local unitary into (vol law) spin-wf, then measuring spin of Hubbard wf will disentangle the charge, giving (a post-measurement) area-law charge-entanglement entropy (ie it is a QDL state)

$$S_{c/s} \sim L^{d-1}$$

Subtlety: Mixing of large U "Bands"?

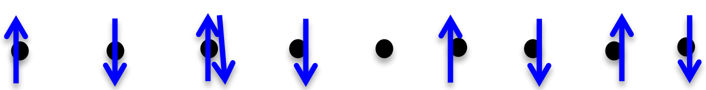
- Start w/ infinite U
- Spin-sector

$$E_0 = 0$$



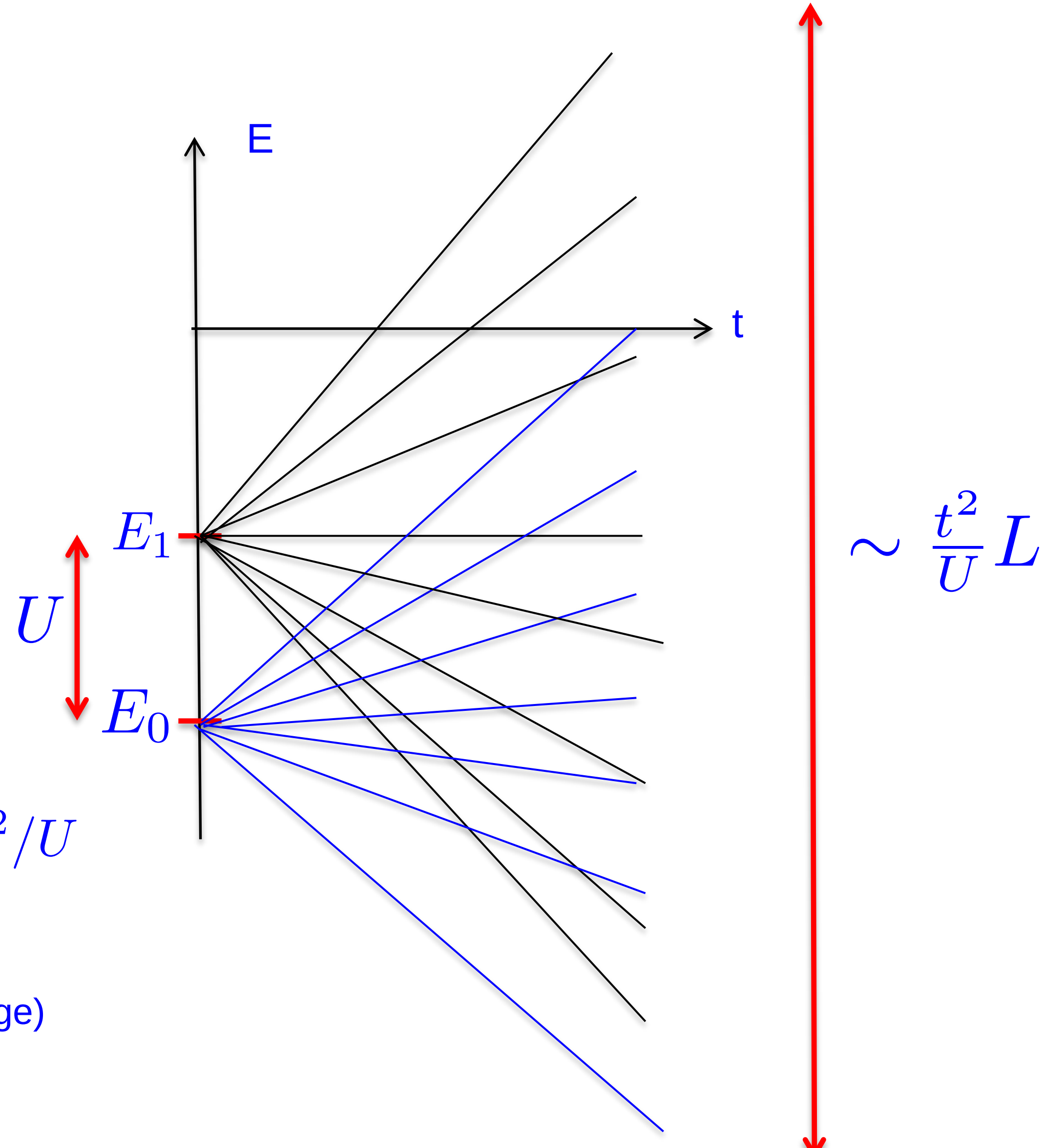
- Band w/ a single doublon (charge)

$$E_1 = E_0 + U$$



- Perturb in t/U, spin model

$$H_s = J_2 \sum_{ij} \vec{S}_i \cdot \vec{S}_j \qquad J_2 \sim t^2/U$$



Does the spin band "mix" with doublon (charge) bands in the same energy range?