



Hauser-Raspe
Foundation



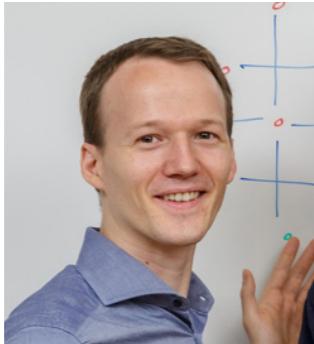
Adiabatic Quantum Computing with Parity Constraints

Wolfgang Lechner

Peter Zoller



Philipp Hauke



Alex Glätzle



Rick van Bijnen



Martin Leib



Markus Heyl



Lars Bonnes



Sci. Adv. 1, 1500838 (2015).

arXiv:1611.02594 (2016)

arXiv:1604.02359(2016) arXiv:1411.7933 (2014)

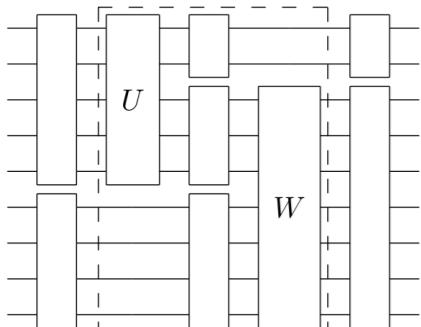
Quantum Computing

- Entanglement
- Superposition
- Measurement

... as a resource.

Universal Quantum Computer

Gate Models



One-Way Quantum Computing

Raussendorf, Briegel PRL (2001).

P. Walther et. al Nature (2005).

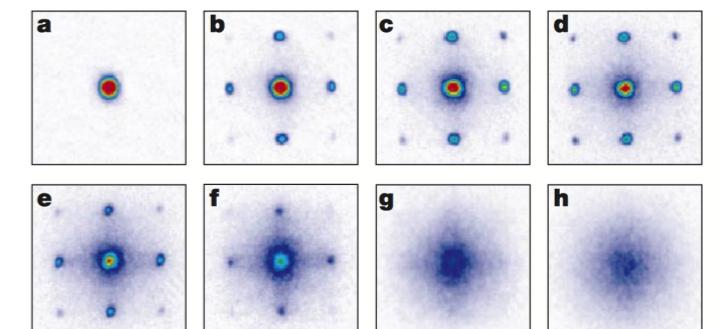
Adiabatic Quantum Computing

Farhi et. al., Science (2001).

Quantum Simulation

e.g. Hubbard Model

$$H = -t \sum_{\langle i,j \rangle} a_i^\dagger a_j + \frac{U}{2} \sum_i \hat{n}_i (\hat{n}_i - 1) - \mu \sum_i \hat{n}_i$$



D Jaksch, et. al. PRL 81, 3108 (1998).

M. Greiner, et. al. Nature, 415 (2002).

Adiabatic Quantum Computing

H. Nishimori et. al., PRE **58**, 5355 (1998).

E. Farhi et. al., Science **292**, 472 (2001).

$$H(t) = \left(1 - \frac{t}{T}\right) H(0) + \frac{t}{T} H_p$$

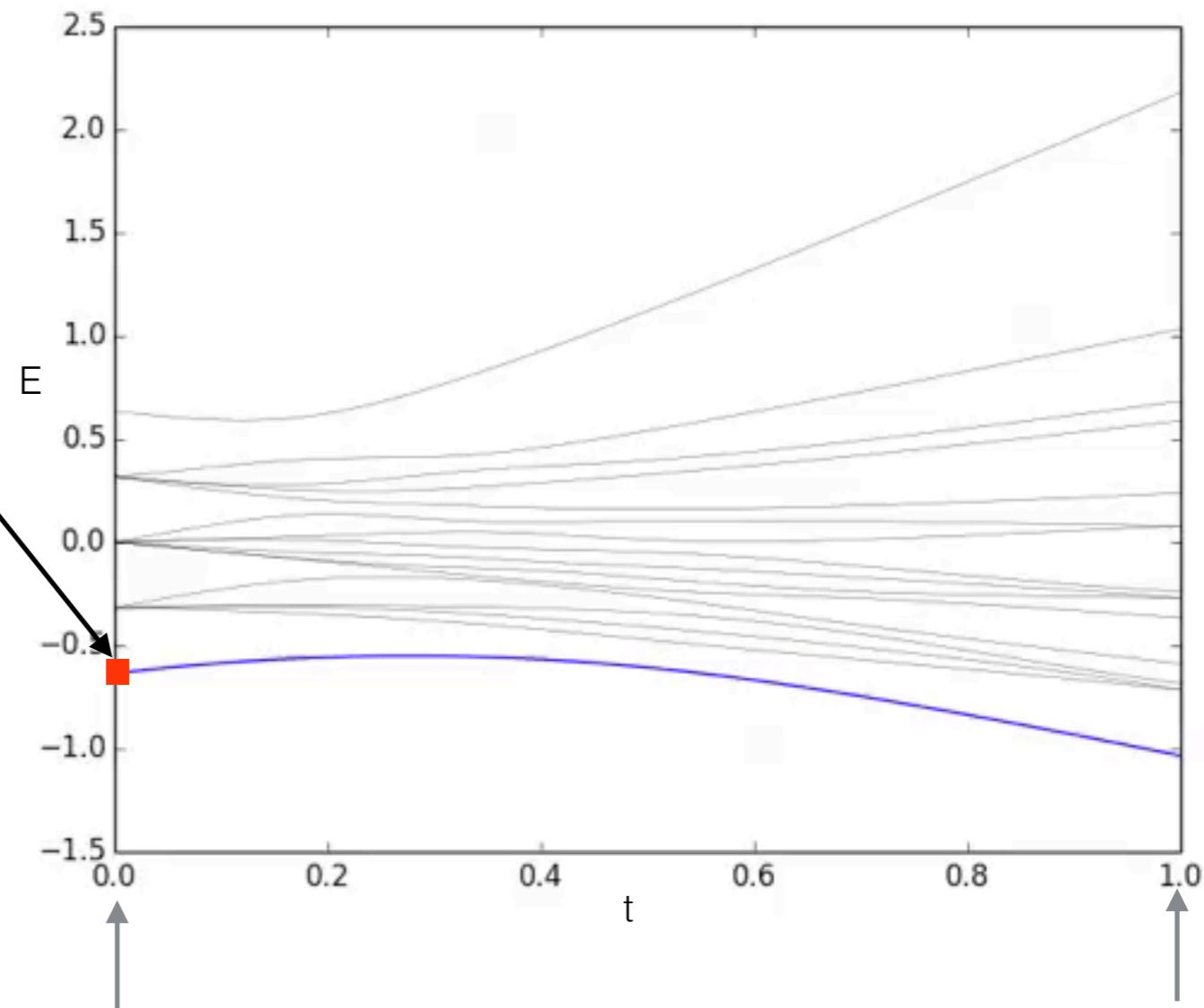
Trivial initial Hamiltonian Problem Hamiltonian

1. Prepare System in $|\psi_0\rangle$
which is the ground state of $H(0)$

2. Slowly evolve the system with

$$|\psi(t)\rangle = e^{-i \int_0^t H(t) dt} |\psi_0\rangle$$

3. Due to the adiabatic theorem, the final
state is the ground state of H_p



$$H_0 = \sum_i^N \sigma_x^{(i)}$$

$$H_p = \sum_{i < j} J_{ij} \sigma_z^{(i)} \sigma_z^{(j)}$$

Adiabatic Quantum Computing

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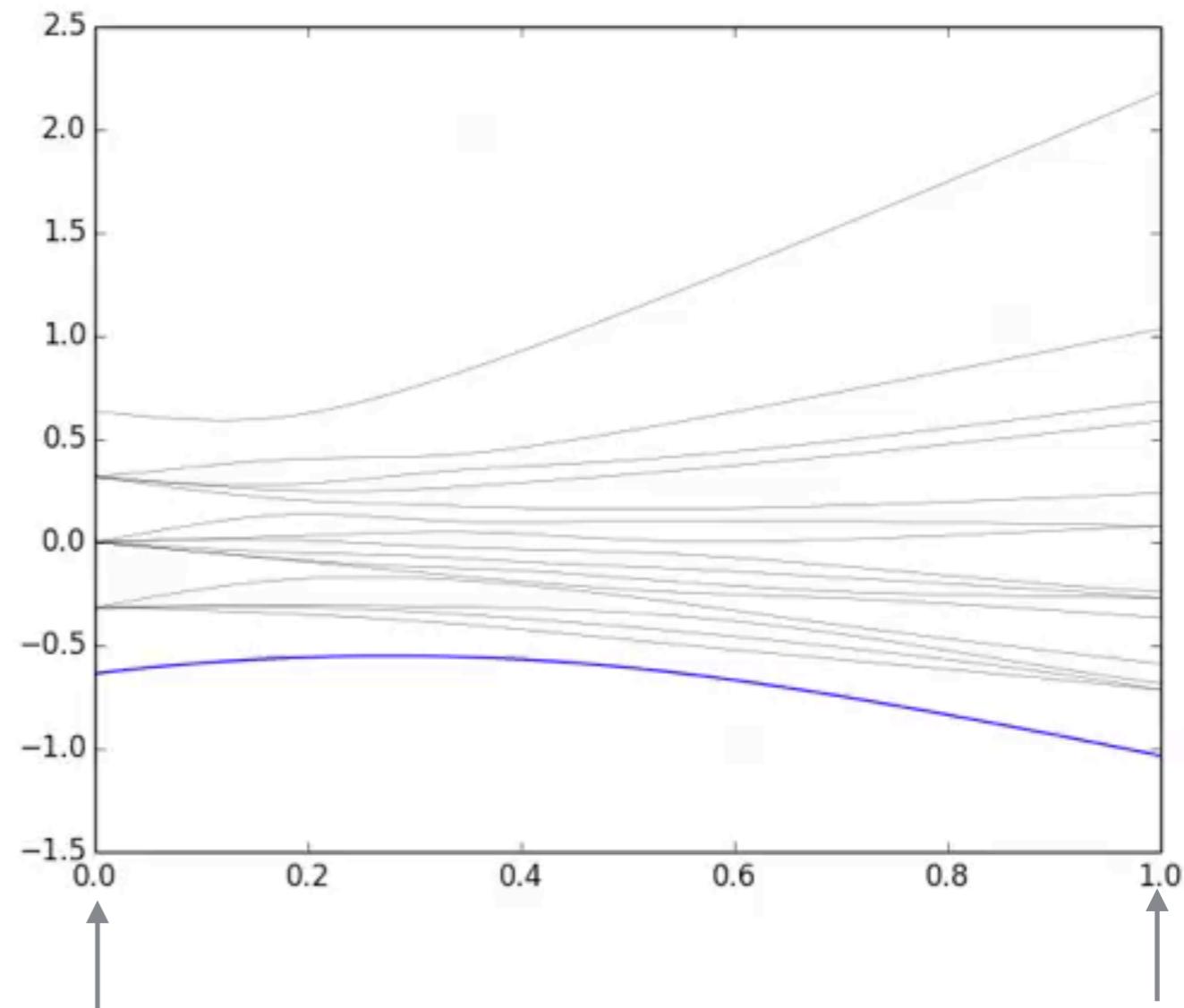
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$$H_0 = \sum_i^N \sigma_x^{(i)}$$

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Adiabatic Quantum Computing

- Adiabaticity:

mapping function $s(t)$ $i\frac{ds}{dt}\frac{d}{ds}|\psi(s)\rangle = H(s)|\psi(s)\rangle$

$$\frac{1}{\tau(s)} = \frac{ds}{dt} \ll \frac{\Delta^2}{|\frac{d}{ds}H(s)|}$$

- Coherence:

D. Wild, S. Gopalakrishnan, M. Knap, N. Y. Yao, M. D. Lukin
Phys. Rev. Lett. , 117, 150501 (2016).

- only scalable at zero-temperature
- at finite temperature, error correction is needed

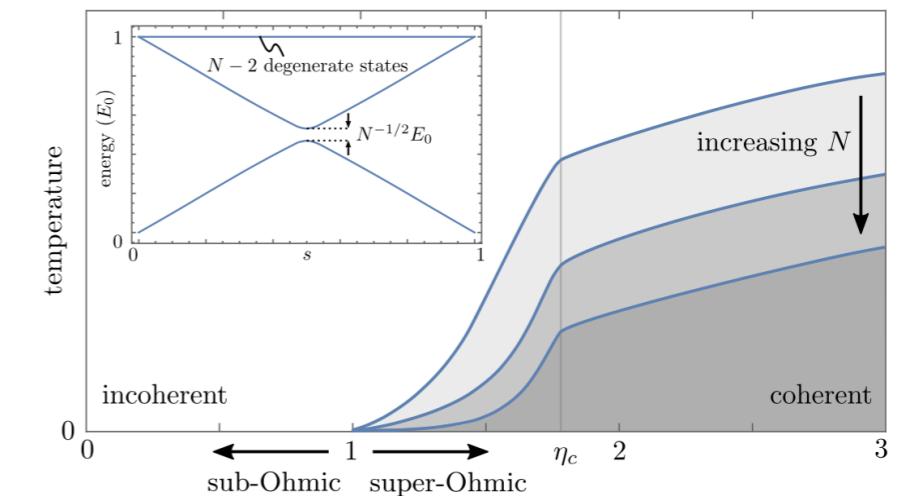
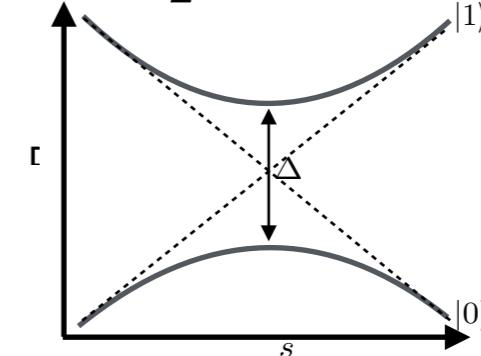
- Universality:

- Any gate model can be simulated with an adiabatic quantum computer.
Aharonov et. al. SIAM Review 50, 755 (2008).

- Grover Algorithm can be implemented optimally $\mathcal{O}(\sqrt{N})$.
Fahri and Gutmann, Phys. Rev. A 57, 2403 (1998).

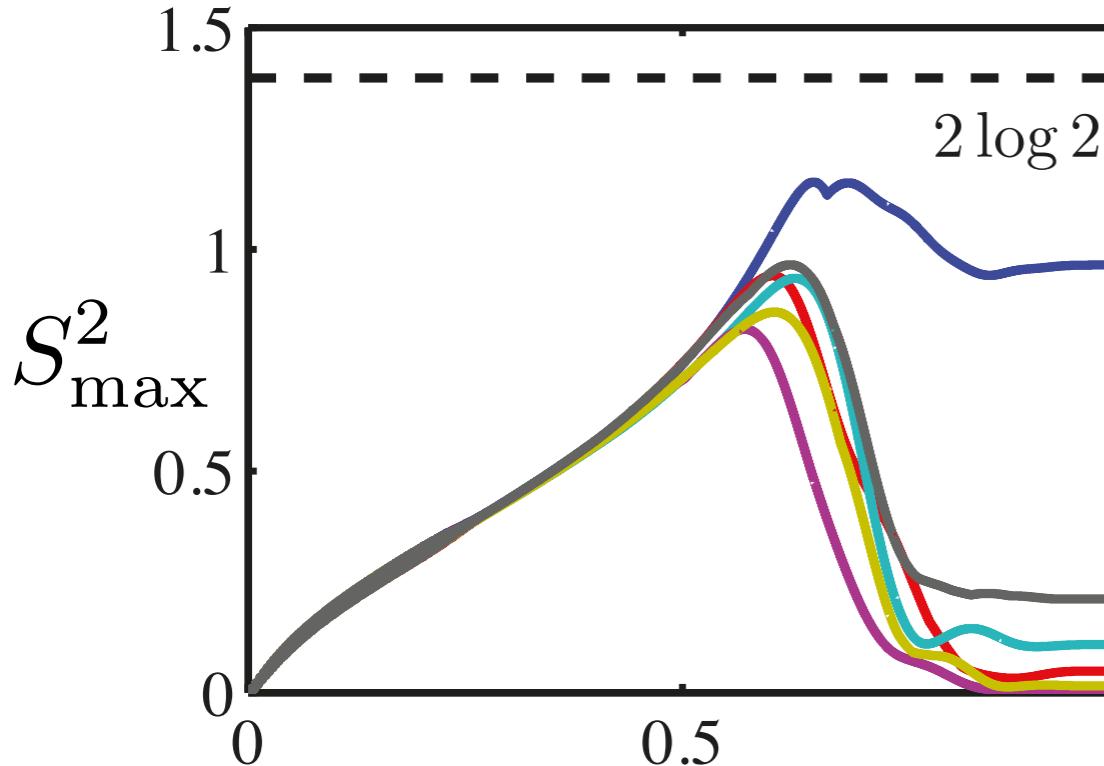
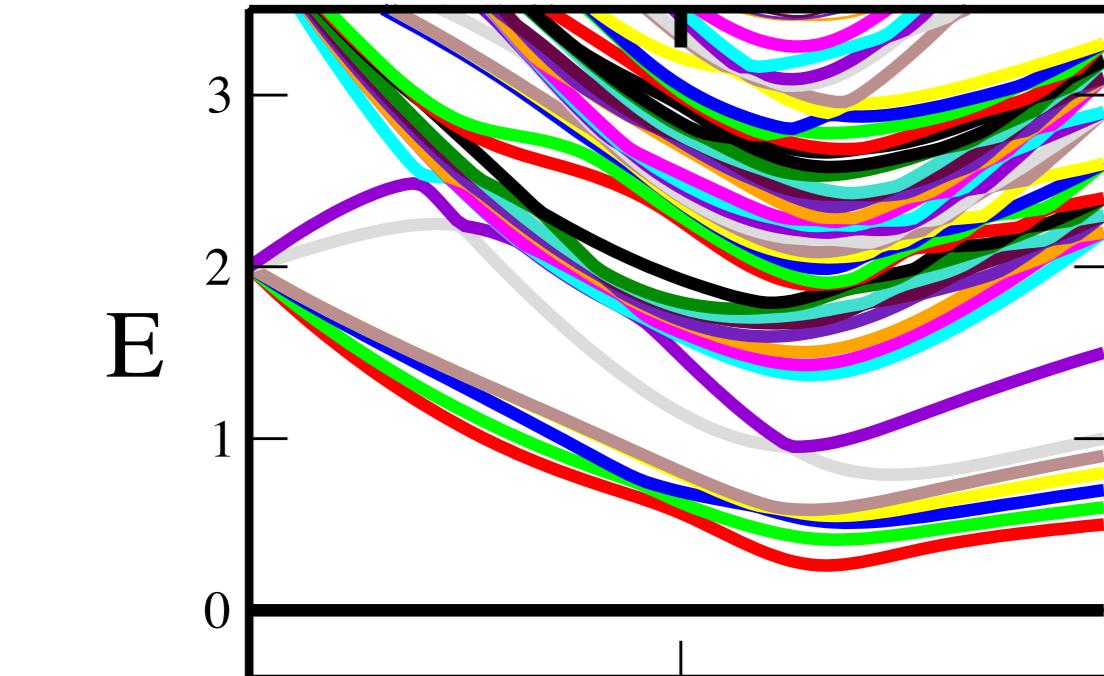
Landau-Zener problem

$$H(t) = \frac{1}{2}(1-s)\Delta\sigma_x + \frac{1}{2}s\sigma_z$$



Adiabatic Quantum Computing

- Entanglement as a resource:



$$\mathcal{H}(t) = A(t) \sum_i^N b_i \sigma_x^{(i)} + B(t) \left[\sum_{i=1}^N h_i \sigma_z^{(i)} + \sum_{i=1}^N \sum_{j=1}^i J_{ij} \sigma_z^{(i)} \sigma_z^{(j)} \right]$$

e.g : Entanglement Entropy:

$$S = -\text{Tr} [\rho_A \log \rho_A] \quad \rho_A = \text{Tr}_B [\rho]$$

Maximum of S for each combination of bipartition A and B at each time:

$$S_{\max}^2(t) = \max S(A, B)$$

non-adiabatic (irreversible)

adiabatic (reversible)

Adiabatic Quantum Computing

- Infinite Range Spin glass

$$\mathcal{H}(t) = A(t) \sum_i^N b_i \sigma_x^{(i)} + B(t) \left[\sum_{i=1}^N h_i \sigma_z^{(i)} + \sum_{i=1}^N \sum_{j=1}^i J_{ij} \sigma_z^{(i)} \sigma_z^{(j)} \right]$$

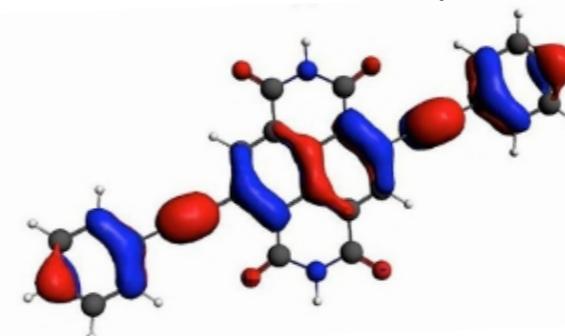
NP-hard problems

A. Lucas, *Frontiers in Physics* **2**, 00005 (2014).

- Traveling Salesman
- Knapsack
- Minimal Spanning Tree
- Number Partitioning
- Graph Coloring
- 3SAT
- Graph Partition
- ...

Quantum Chemistry

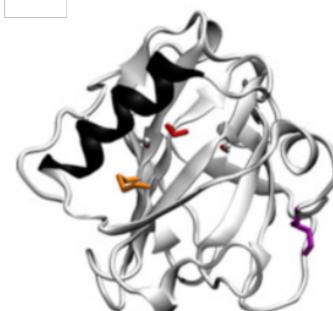
R. Babbush et. al., *Sci. Rep.* 4, 6603 (2014).



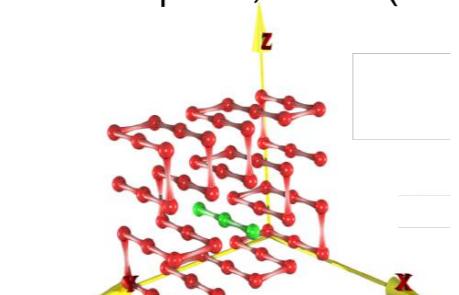
Picture: E. Meijer, University of Amsterdam.

Protein Folding

A. Perdomo-Ortiz et. al., *Sci. Rep.* 2, 571 (2012).



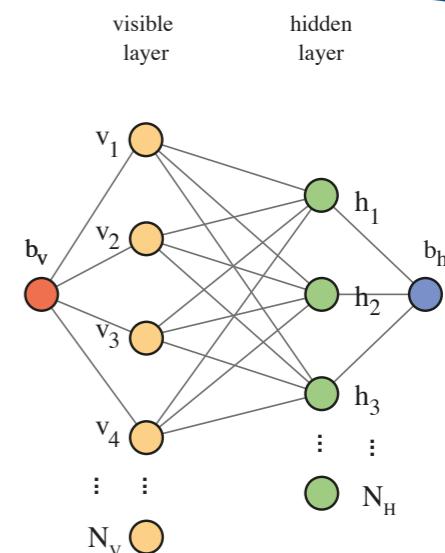
e.g. beta-lactoglobulin (milk protein)
Picture: Peter Bolhuis



I. Coluzza, et.al. *Biophys. J.* (2007).

Machine Learning

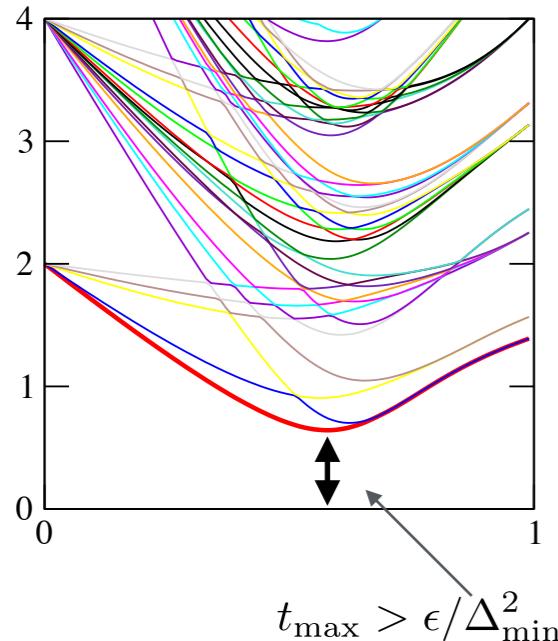
G. Hinton et. al.,
Restricted Boltzmann
Machines



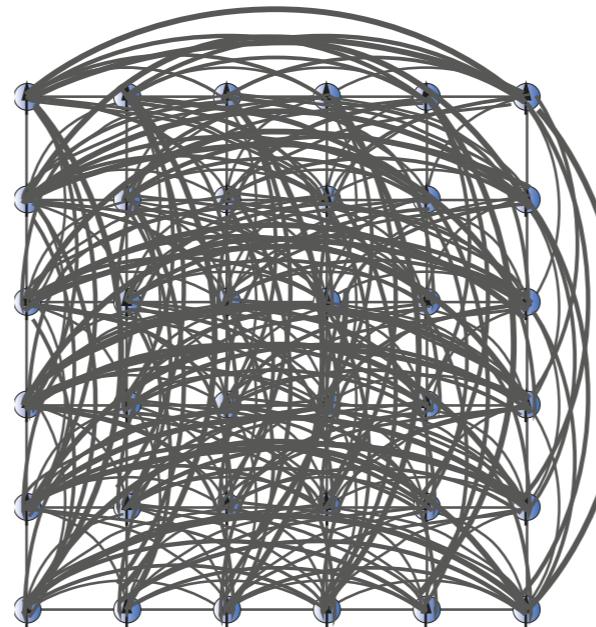
Adiabatic Quantum Computing

Challenges:

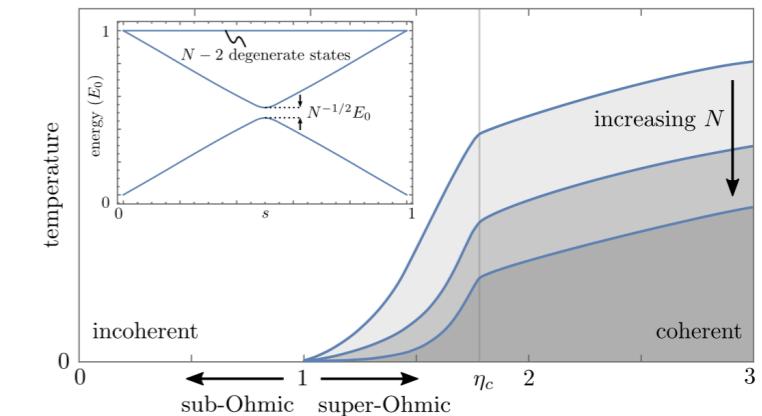
Minimal gap



Connectivity

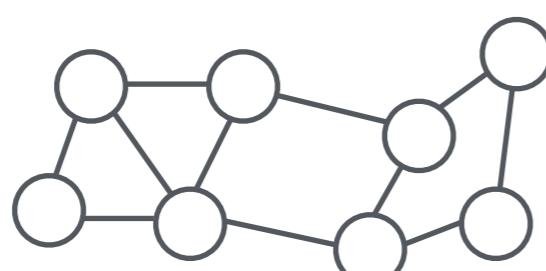


Finite temperature

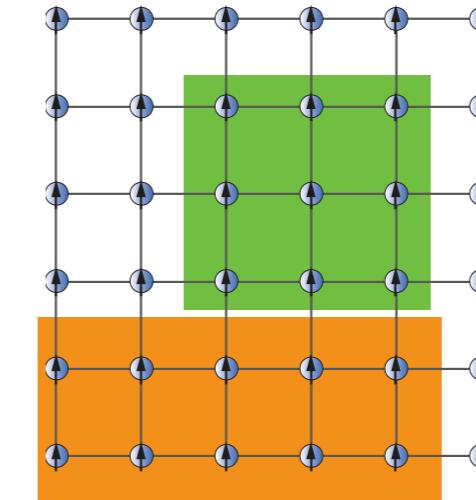


D. Wild et. al.,
Phys. Rev. Lett. , 117, 150501 (2016).

n-body terms

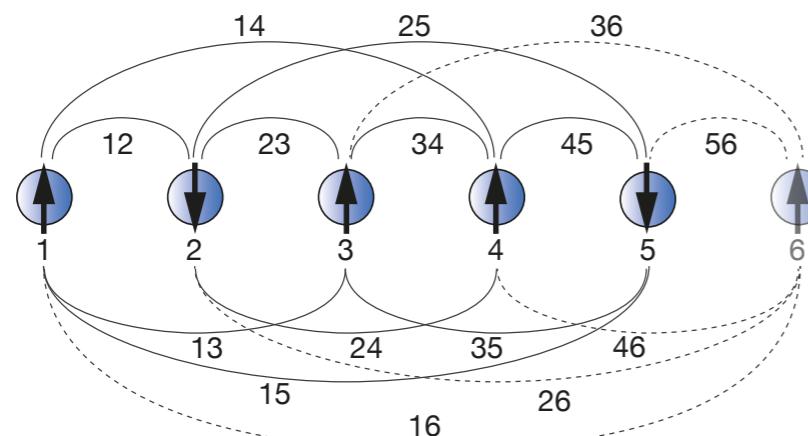


Fault tolerance



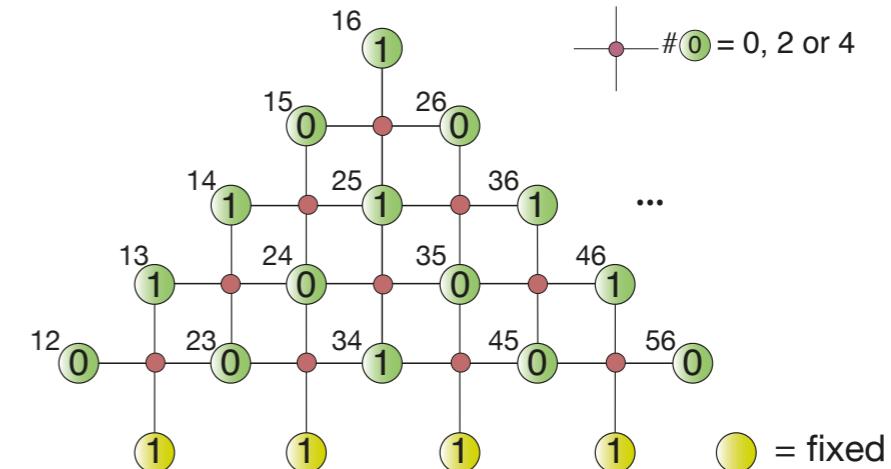
Parity Adiabatic Quantum Computing

Spin glass paradigm



$$\mathcal{H}(t) = A(t) \sum_i^N b_i \sigma_x^{(i)} + B(t) \left[\sum_{i=1}^N h_i \sigma_z^{(i)} + \sum_{i=1}^N \sum_{j=1}^i J_{ij} \sigma_z^{(i)} \sigma_z^{(j)} \right]$$

Parity constraints

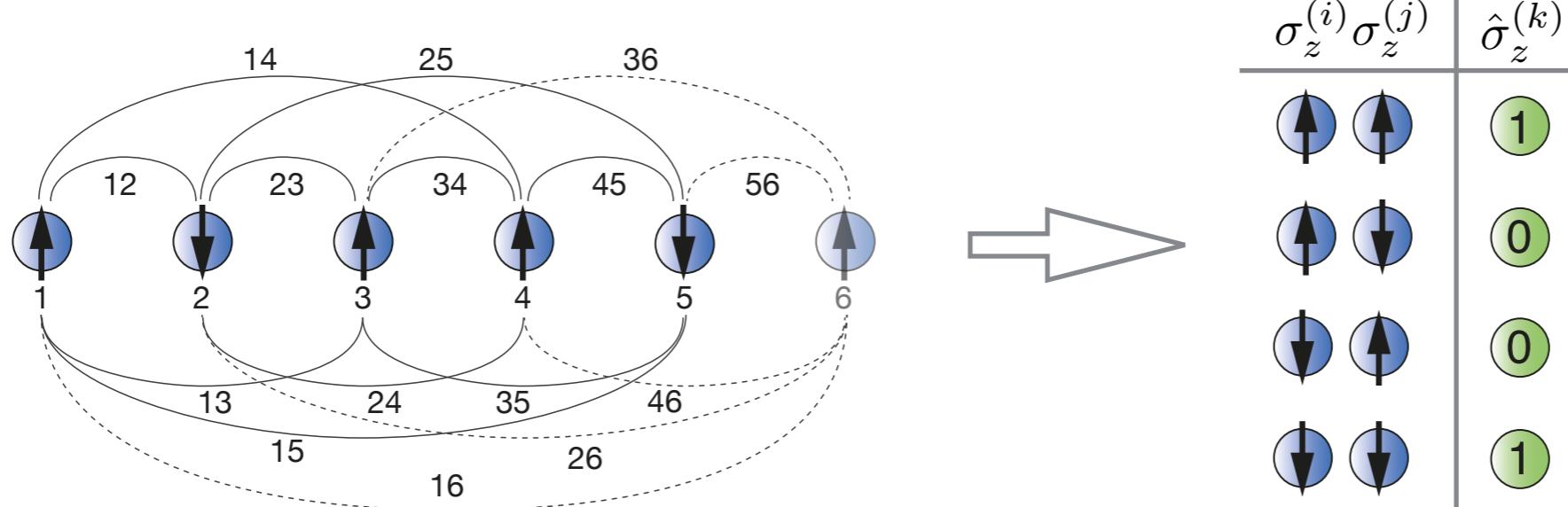


$$\mathcal{H}(t) = A(t) \sum_{i=1}^K b_i \sigma_x^{(i)} + B(t) \sum_{i=1}^K J_i \sigma_z^{(i)} + C(t) \sum_{l=1}^{K-N} C_l$$

WL, P. Hauke, P. Zoller, Science Advances 1, 1500838 (2015).

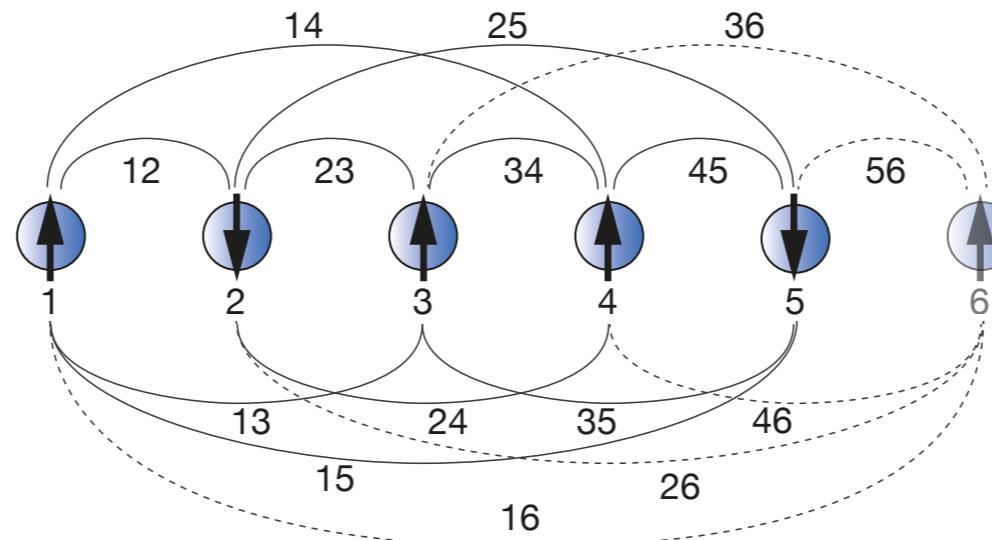
- abstraction of logical from physical qubits
- encode **all-to-all interaction matrix** in local fields
- architecture comprises **problem-independent, local interaction**
- can be realized in current **qubit platform**
(e.g. flux qubits, transmon qubits, ultracold atoms, hybrids systems,...)
- **classical error tolerance** from redundant encoding
- **k-body terms**

All-to-all programmable architecture



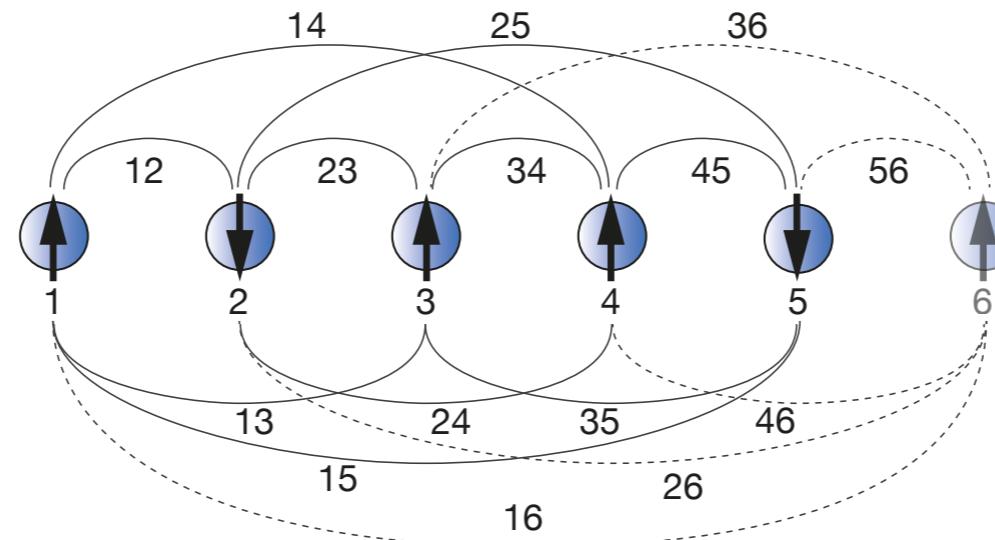
Physical qubits $\hat{\sigma}_z^{(k)}$ represent parity of **logical qubits**.

All-to-all programmable architecture



$$\mathcal{H} = \sum_{i=1}^N \sum_{j=1}^i J_{ij} \sigma_z^{(i)} \sigma_z^{(j)} \Rightarrow \sum_{i=1}^K J_i \hat{\sigma}_z^{(i)}$$

All-to-all programmable architecture



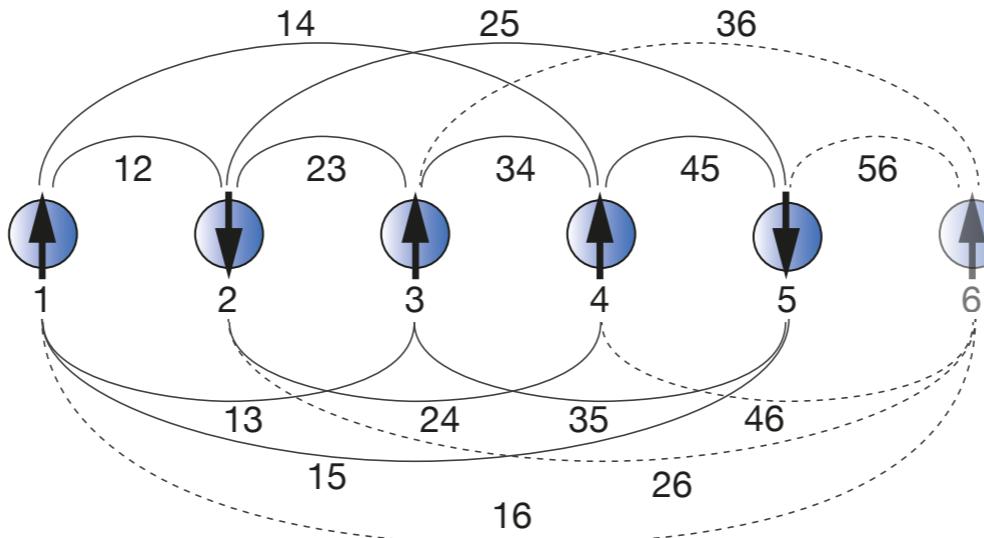
$$\mathcal{H} = \sum_{i=1}^N \sum_{j=1}^i J_{ij} \sigma_z^{(i)} \sigma_z^{(j)} \Rightarrow \sum_{i=1}^K J_i \hat{\sigma}_z^{(i)}$$

$\dots \rightarrow$
 N degrees of freedom

$$K = \frac{N(N - 1)}{2}$$

$\dots \rightarrow$
degrees of freedom

All-to-all programmable architecture

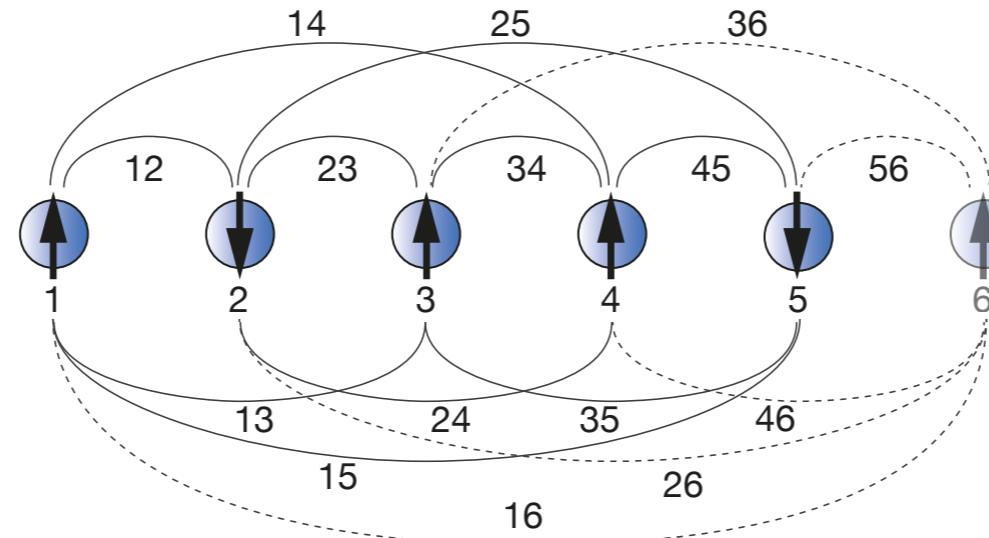


$$\mathcal{H} = \sum_{i=1}^N \sum_{j=1}^i J_{ij} \sigma_z^{(i)} \sigma_z^{(j)} \Rightarrow \sum_{i=1}^K J_i \hat{\sigma}_z^{(i)} + \sum_{l=1}^{K-N} C_l$$



K-N constraints

All-to-all programmable architecture



$$\mathcal{H} = \sum_{i=1}^N \sum_{j=1}^i J_{ij} \sigma_z^{(i)} \sigma_z^{(j)} \Rightarrow \sum_{i=1}^K J_i \hat{\sigma}_z^{(i)} + \sum_{l=1}^{K-N} C_l$$

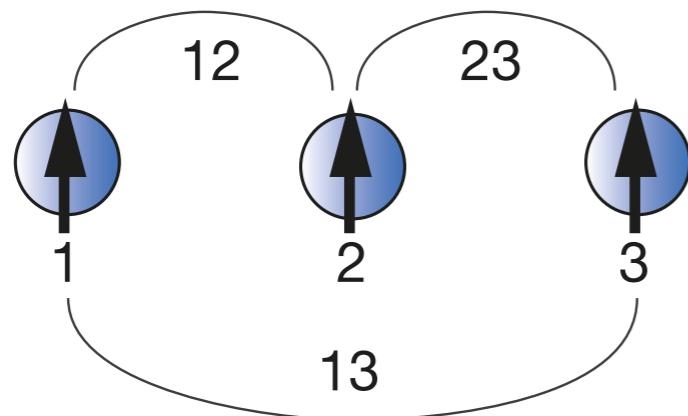


K-N constraints

What are these constraints?

Constraints

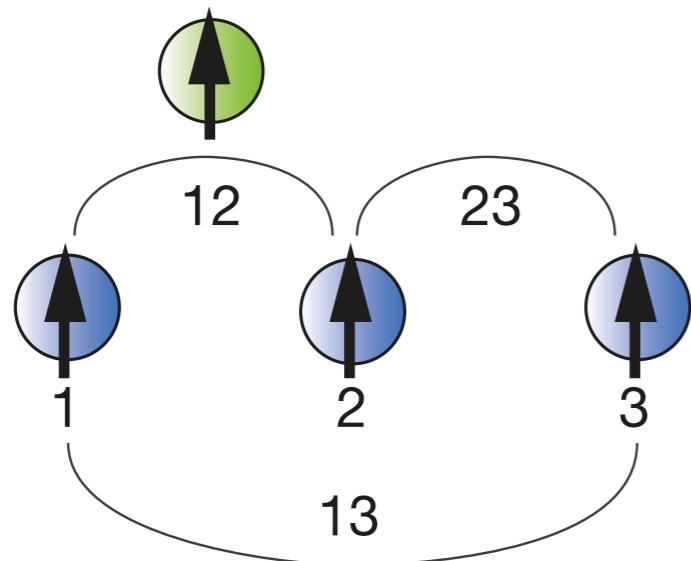
Conditions on closed loops



$\sigma_z^{(1)}$	$\sigma_z^{(2)}$	$\sigma_z^{(3)}$	$\hat{\sigma}_z^{(12)}$	$\hat{\sigma}_z^{(13)}$	$\hat{\sigma}_z^{(23)}$
1	1	1			

Constraints

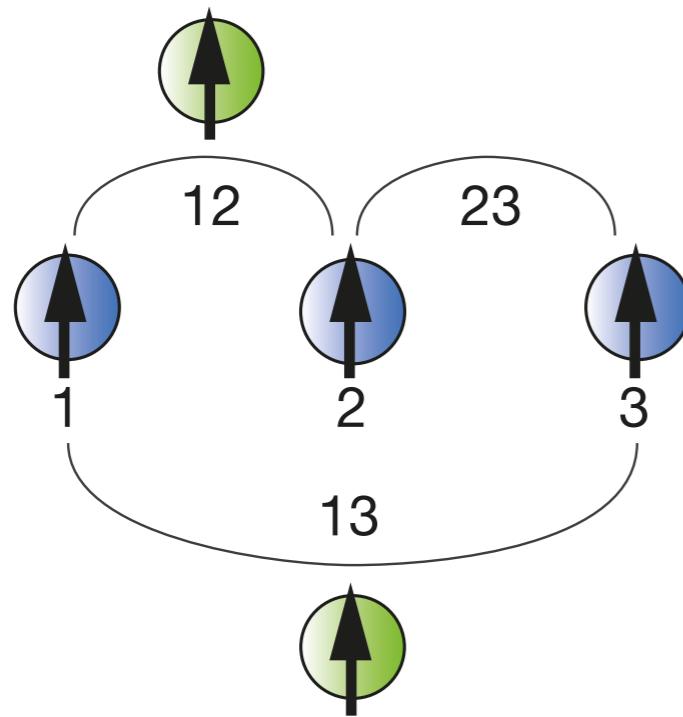
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1	1	1	1	1	1

Constraints

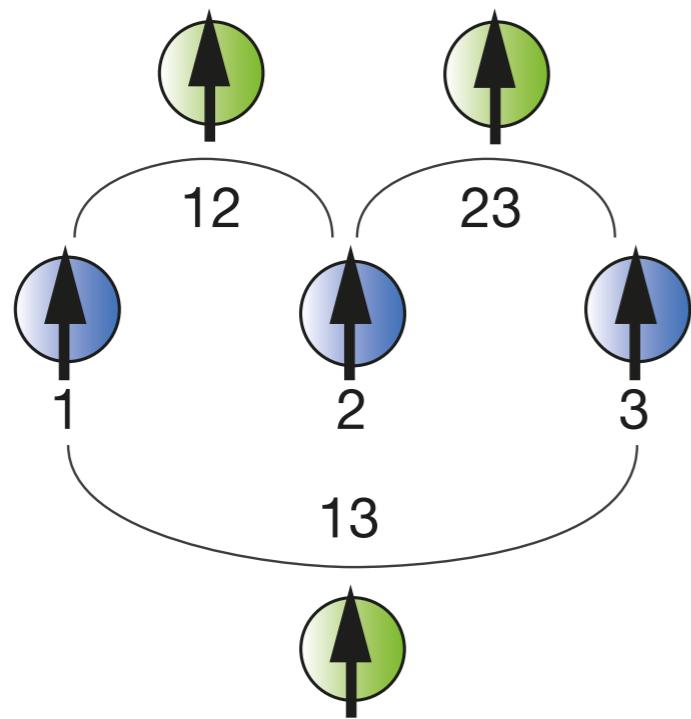
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1	1	1	1	1	1

Constraints

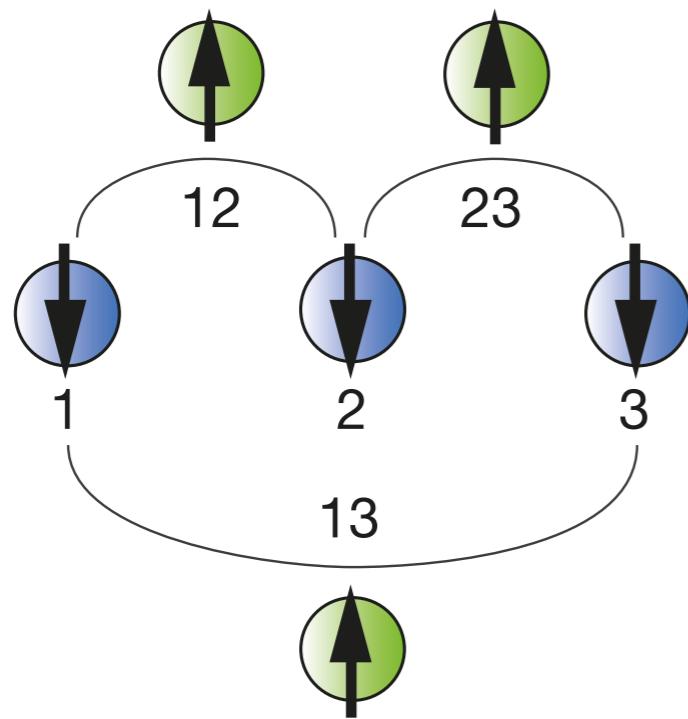
Conditions on closed loops



$\sigma_z^{(1)}$	$\sigma_z^{(2)}$	$\sigma_z^{(3)}$	$\hat{\sigma}_z^{(12)}$	$\hat{\sigma}_z^{(13)}$	$\hat{\sigma}_z^{(23)}$
1	1	1	1	1	1

Constraints

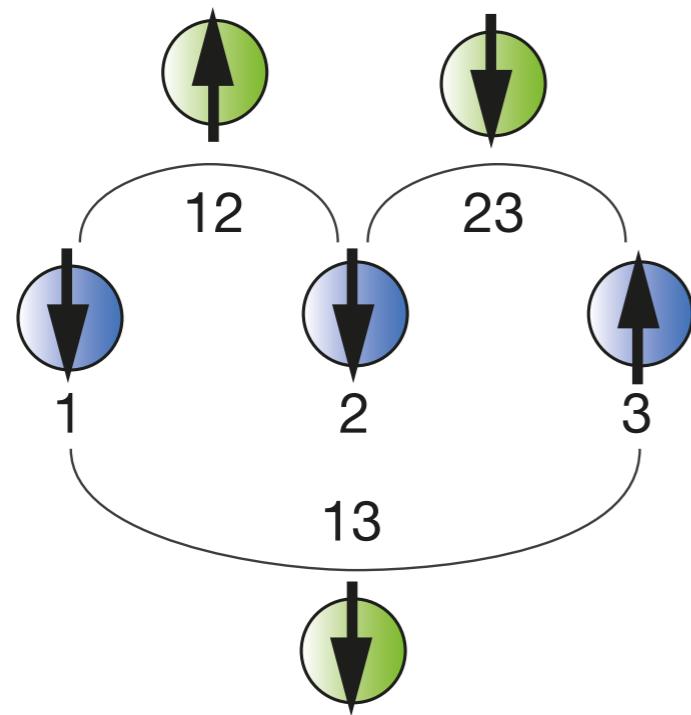
Conditions on closed loops



$\sigma_z^{(1)}$	$\sigma_z^{(2)}$	$\sigma_z^{(3)}$	$\hat{\sigma}_z^{(12)}$	$\hat{\sigma}_z^{(13)}$	$\hat{\sigma}_z^{(23)}$
1	1	1	1	1	1
0	0	0	1	1	1

Constraints

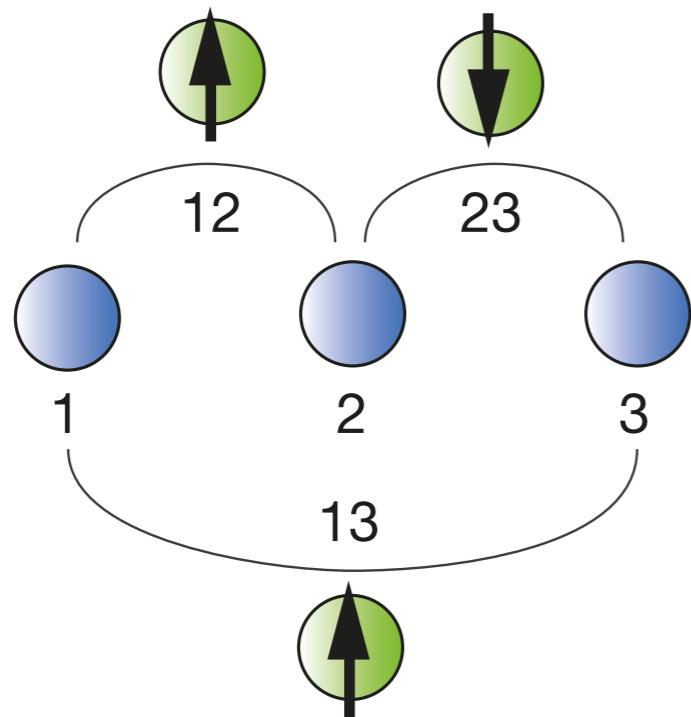
Conditions on closed loops



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1	1	1	1	1	1
0	0	0	1	1	1
0	0	1	1	0	0

Constraints

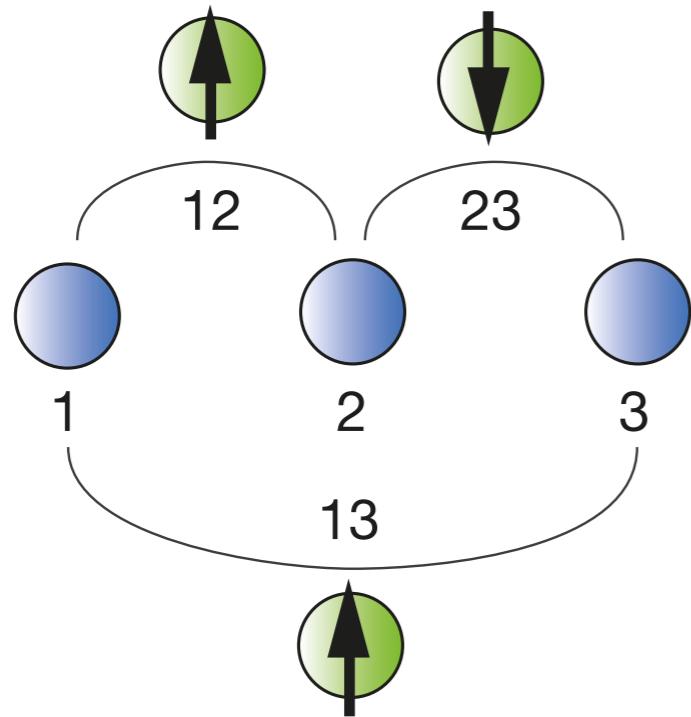
Conditions on closed loops



$\sigma_z^{(1)}$	$\sigma_z^{(2)}$	$\sigma_z^{(3)}$	$\hat{\sigma}_z^{(12)}$	$\hat{\sigma}_z^{(13)}$	$\hat{\sigma}_z^{(23)}$
1	1	1	1	1	1
0	0	0	1	1	1
0	0	1	1	0	0
...			...		
1	1	0	1	1	0

Constraints

Conditions on closed loops

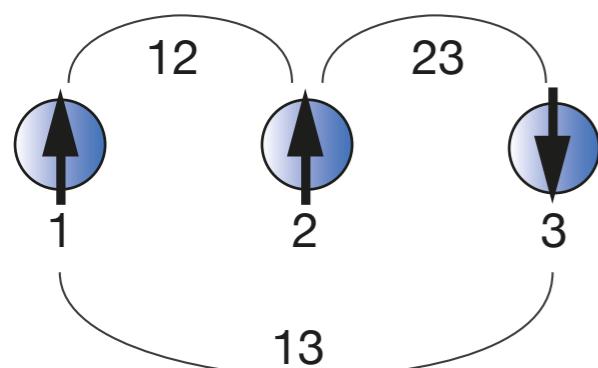


$\sigma_z^{(1)}$	$\sigma_z^{(2)}$	$\sigma_z^{(3)}$	$\hat{\sigma}_z^{(12)}$	$\hat{\sigma}_z^{(13)}$	$\hat{\sigma}_z^{(23)}$
1	1	1	1	1	1
0	0	0	1	1	1
0	0	1	1	0	0
...			...		
			X		
				1	1
					0

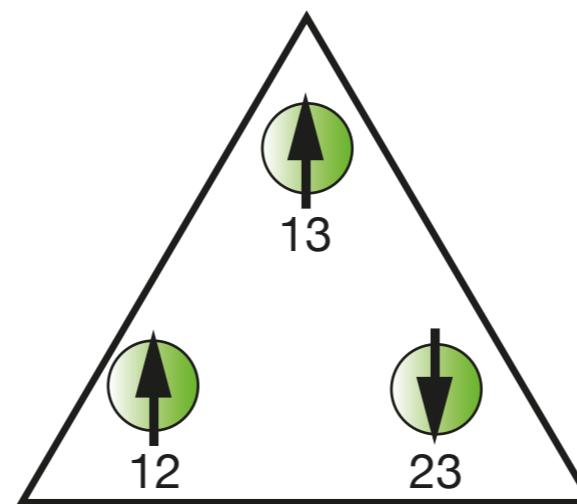
In **each closed loop**, the number of **spin-down** has to be an **even** number or 0.

Constraints (Implementation)

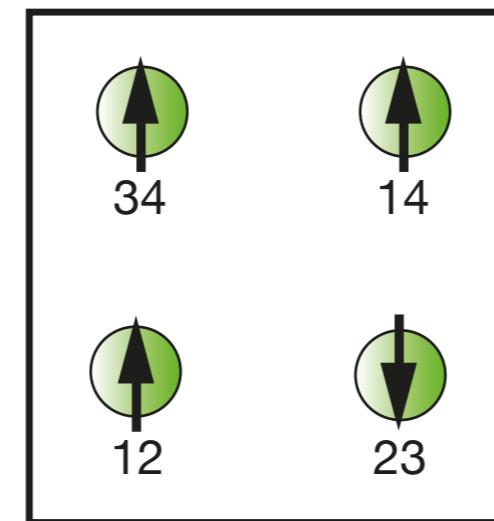
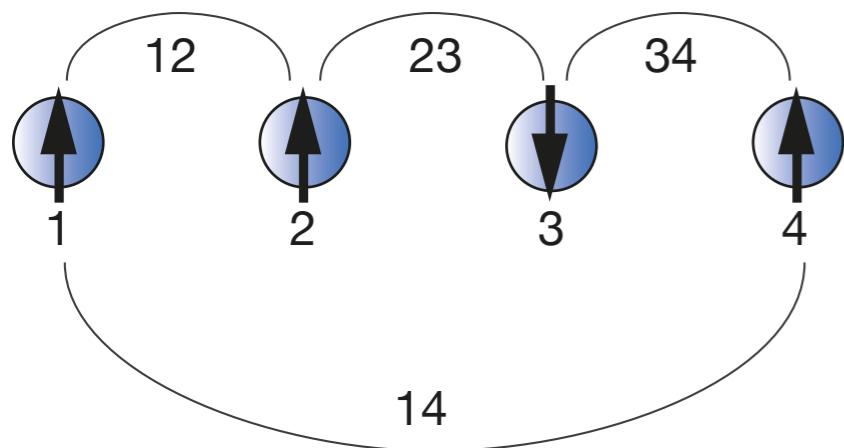
Logical Qubits



Physical Qubits

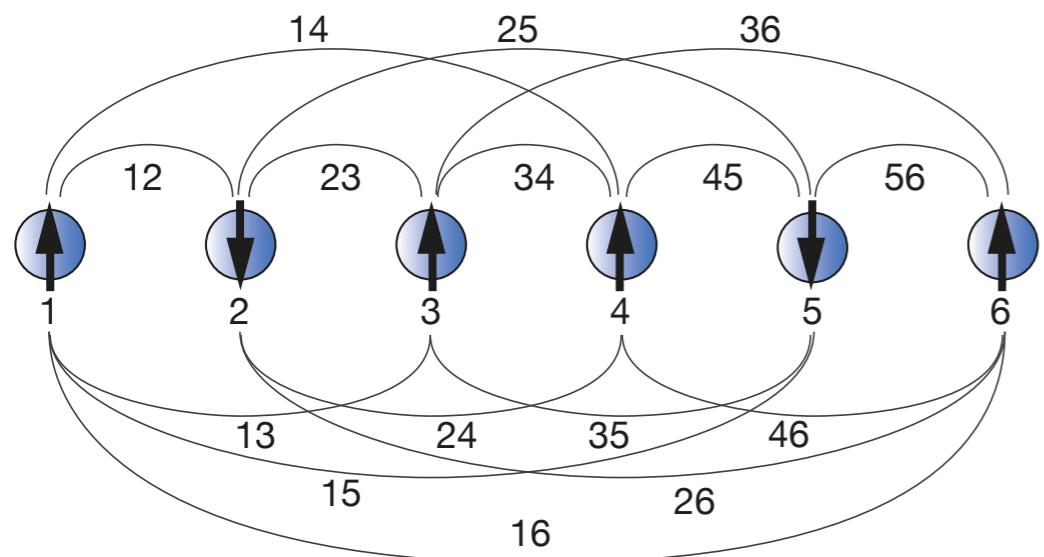


$$C_l = -C \hat{\sigma}_z^{(12)} \hat{\sigma}_z^{(23)} \hat{\sigma}_z^{(13)}$$

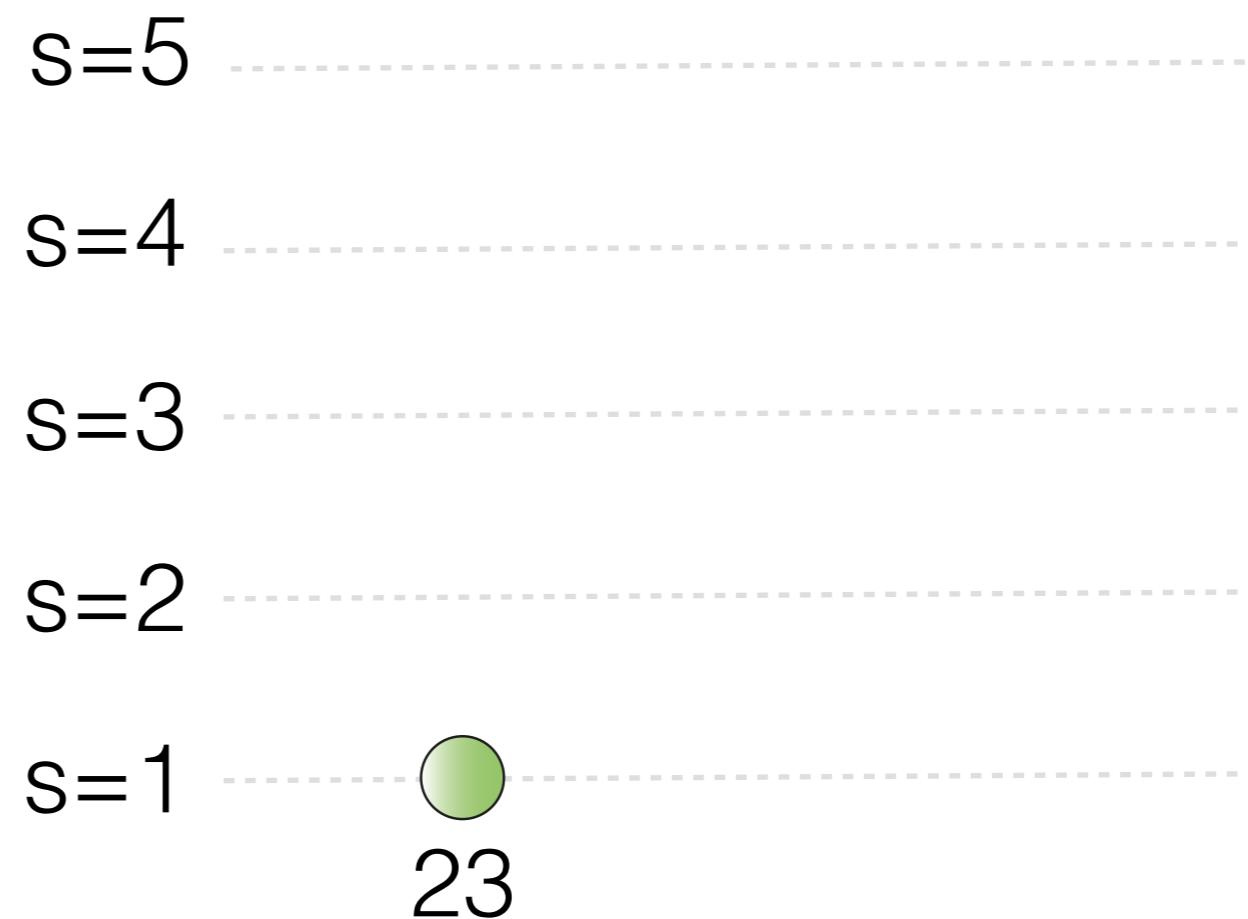
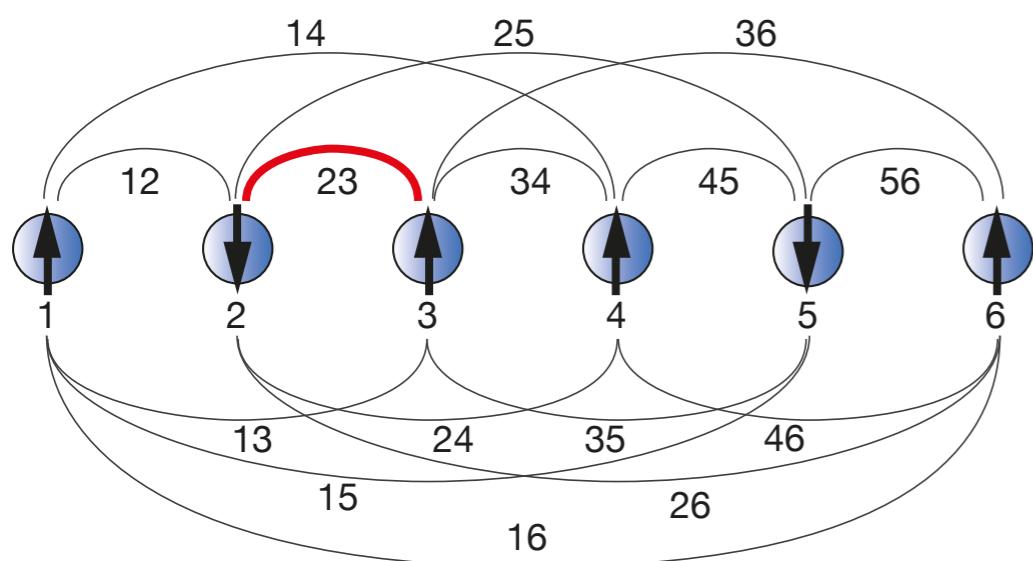


$$C_l = -C \hat{\sigma}_z^{(12)} \hat{\sigma}_z^{(23)} \hat{\sigma}_z^{(34)} \hat{\sigma}_z^{(14)}$$

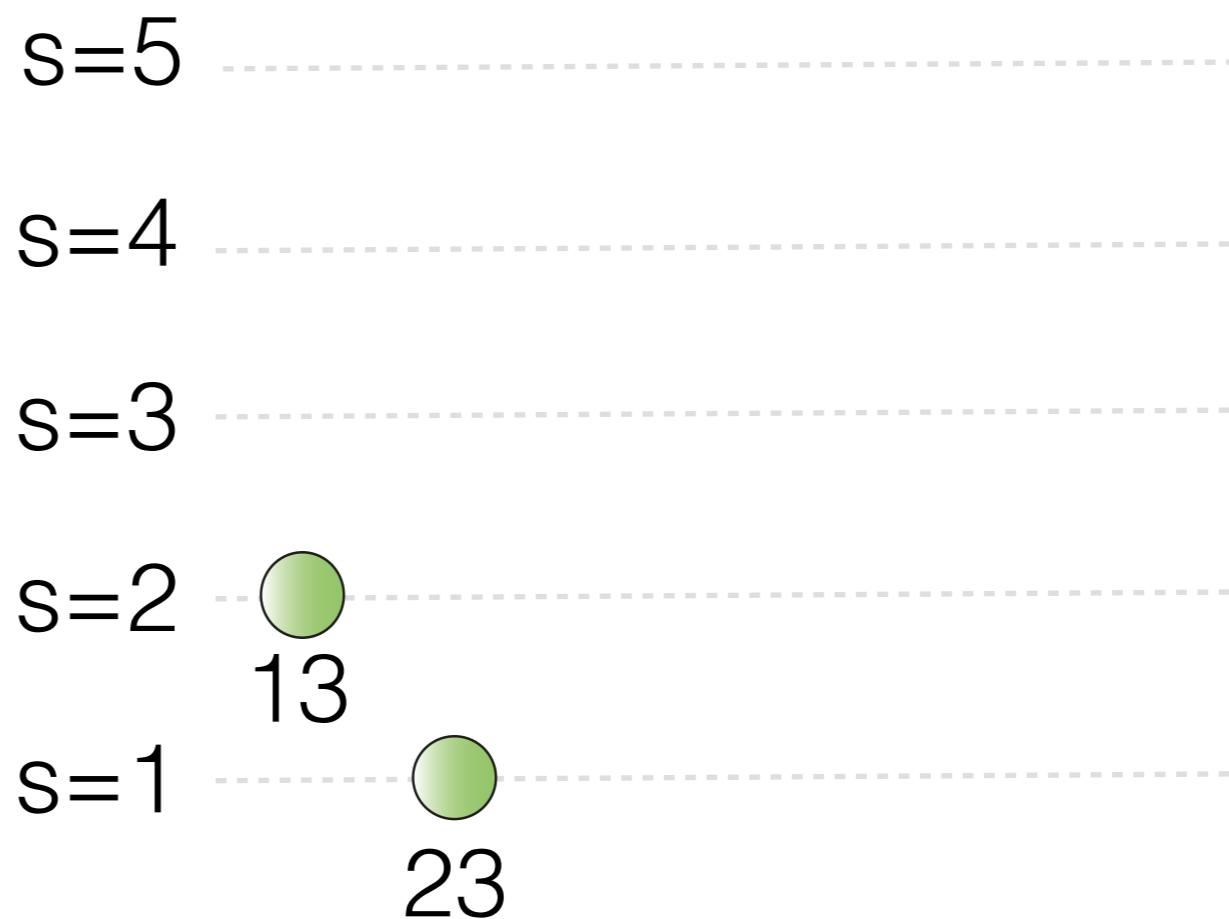
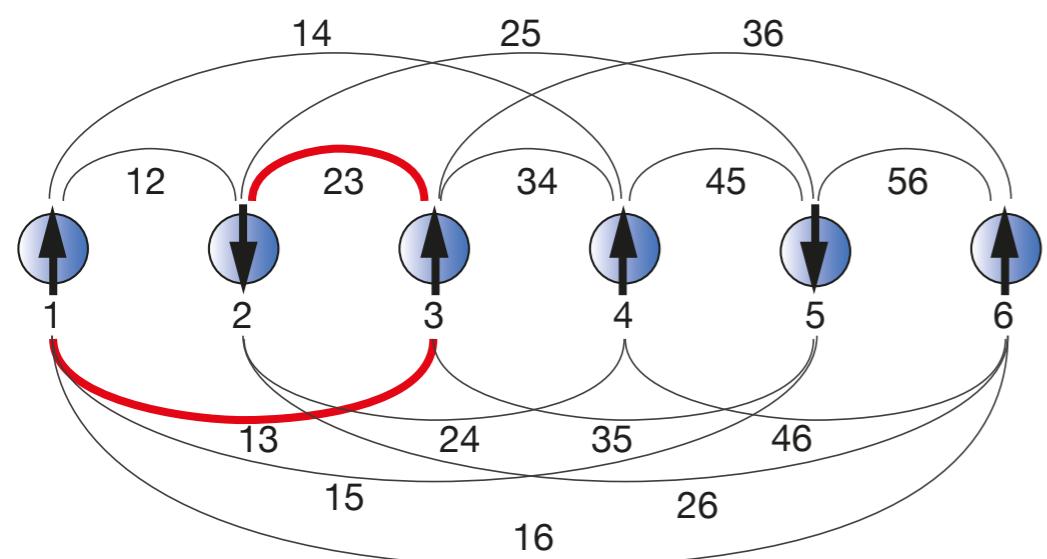
Constraints



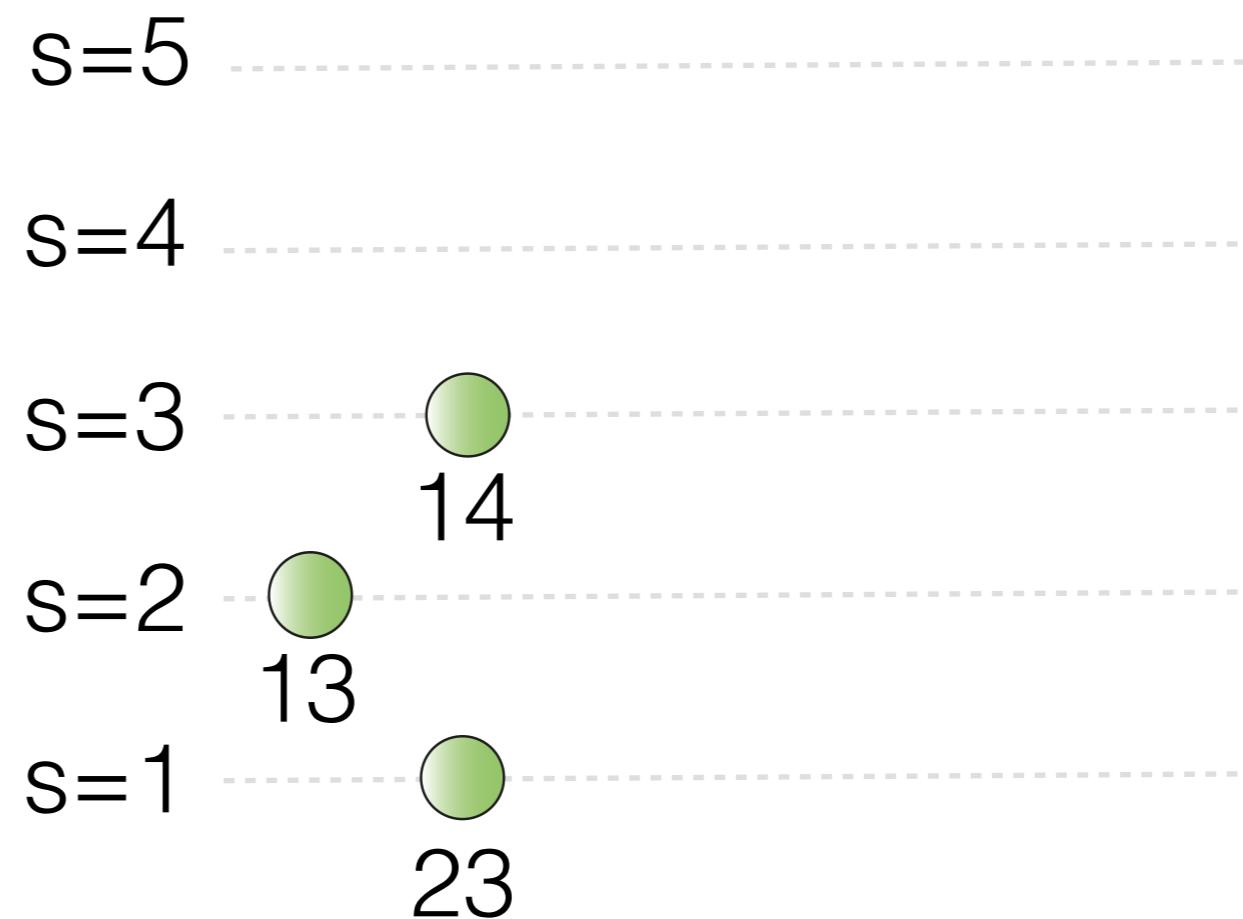
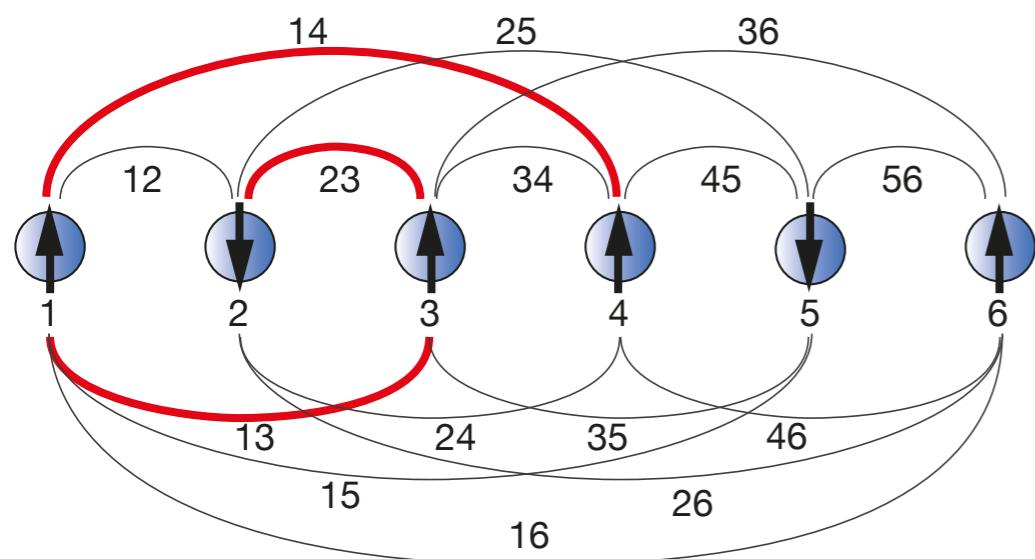
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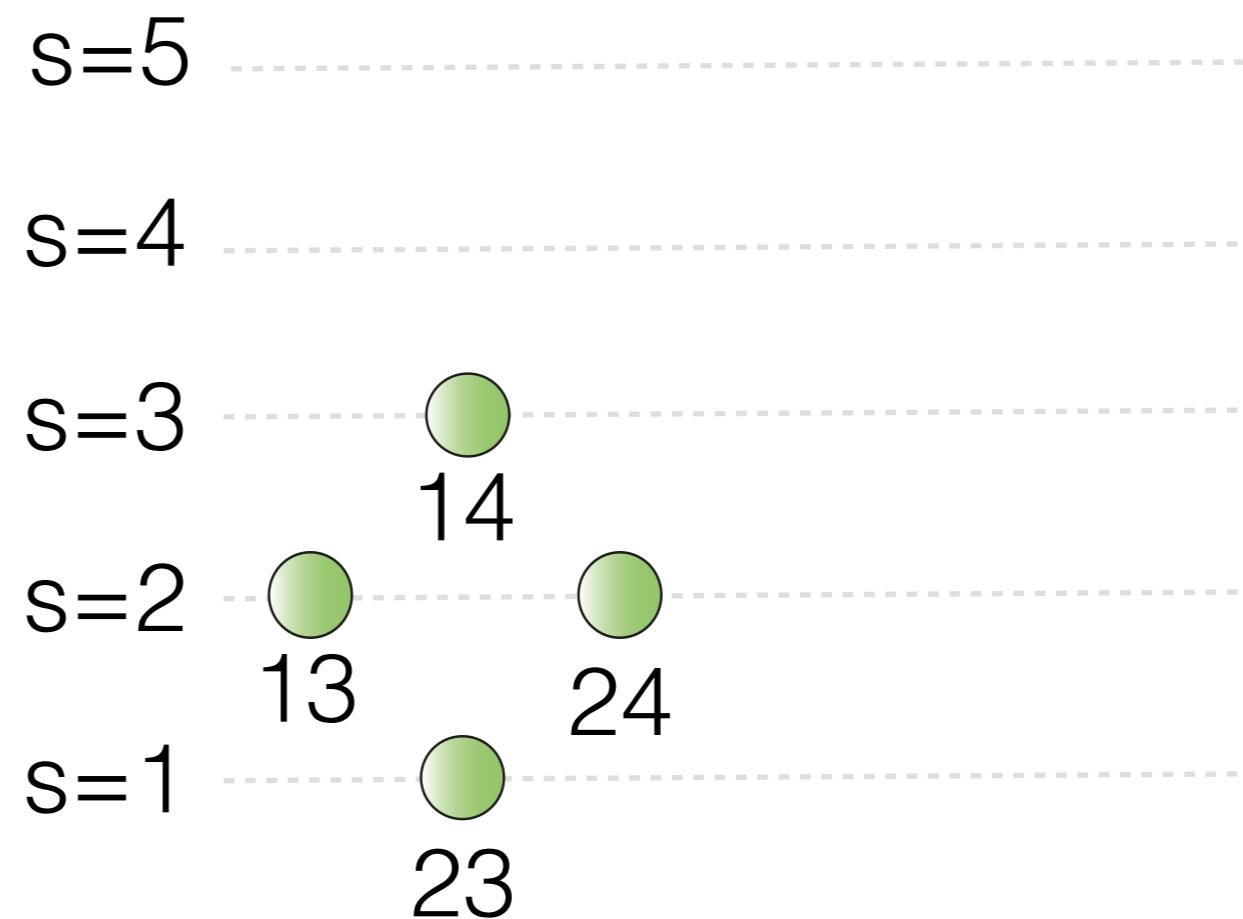
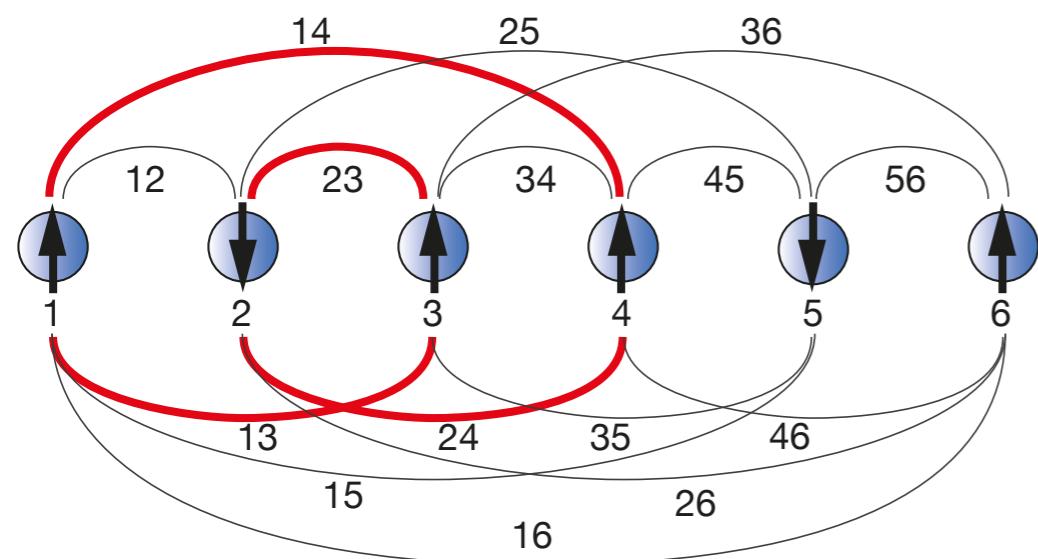
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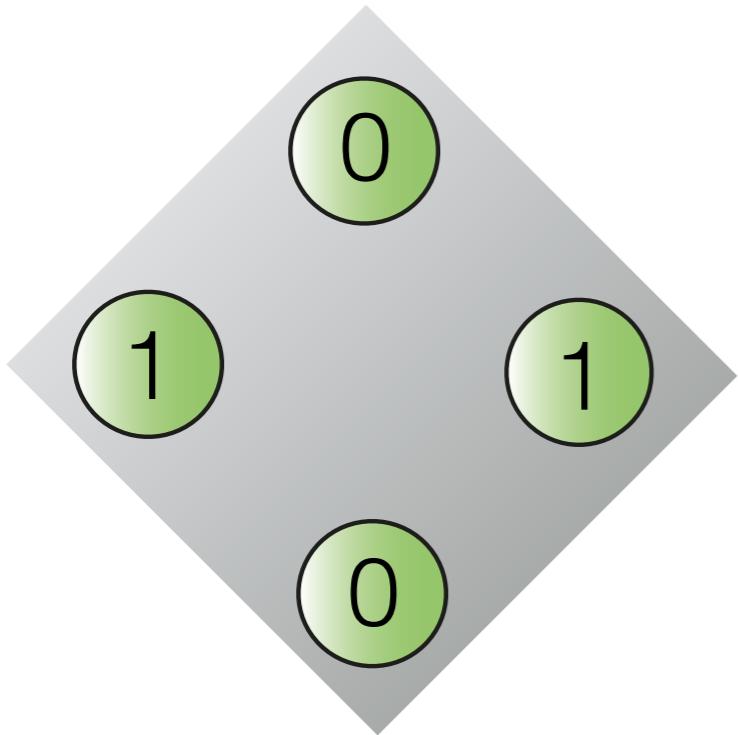
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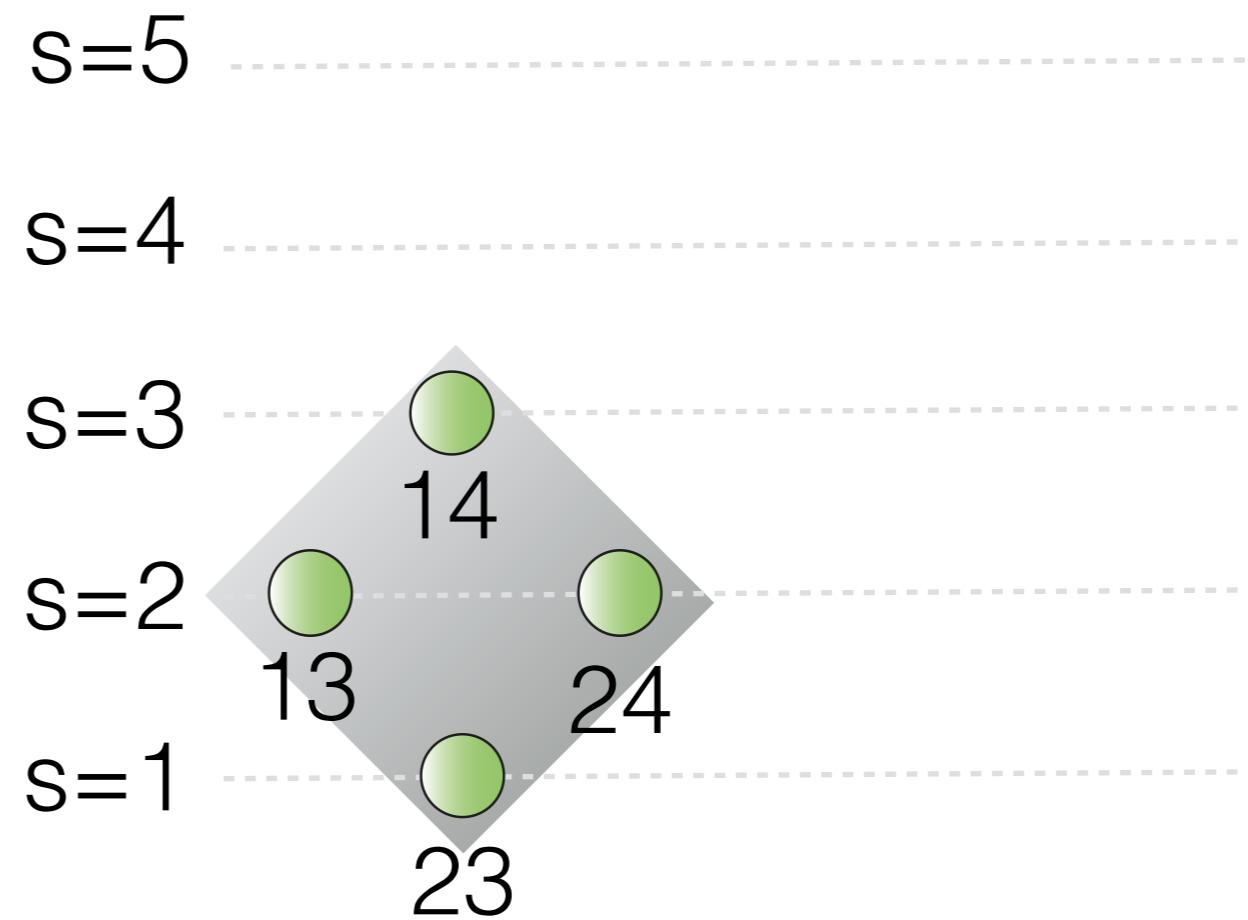
Constraints



Constraints

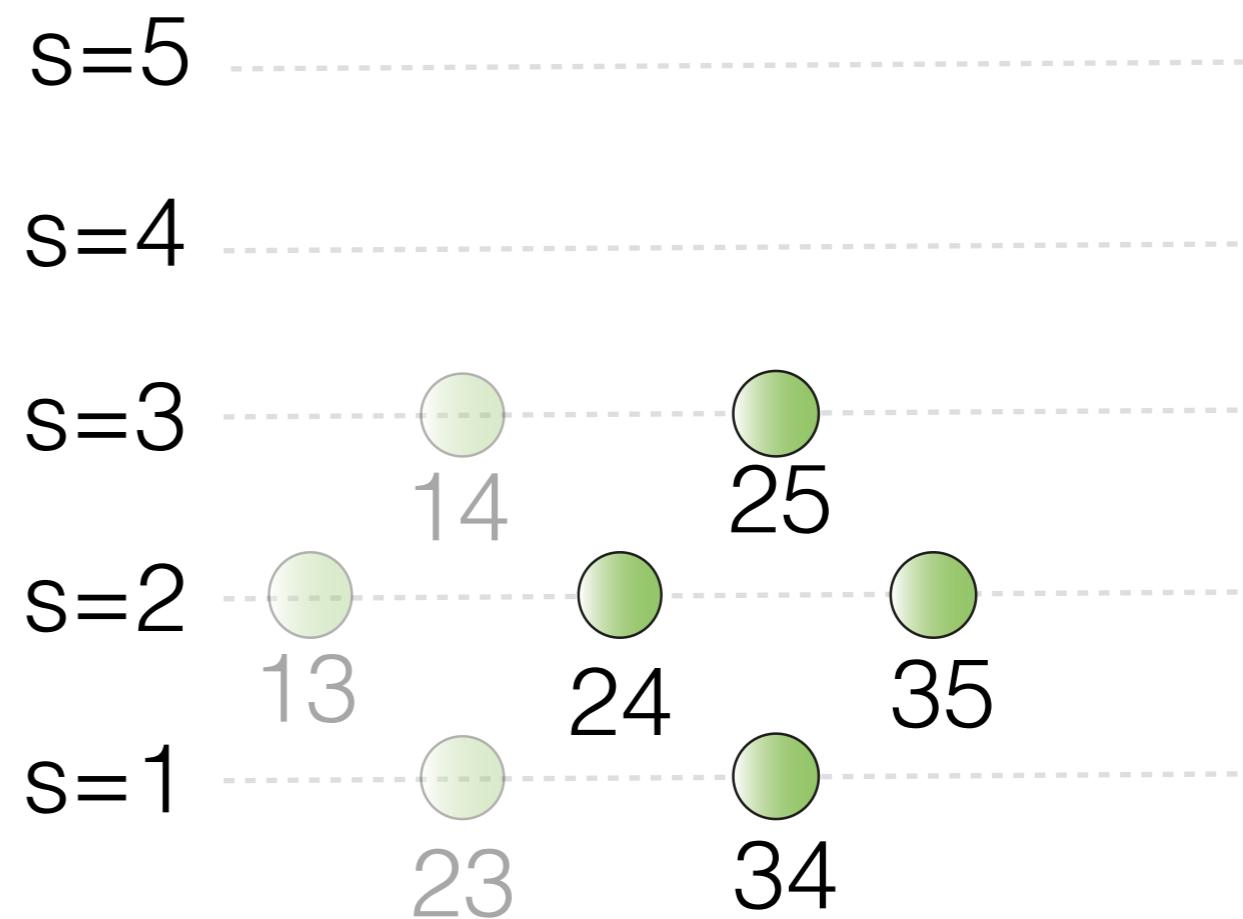
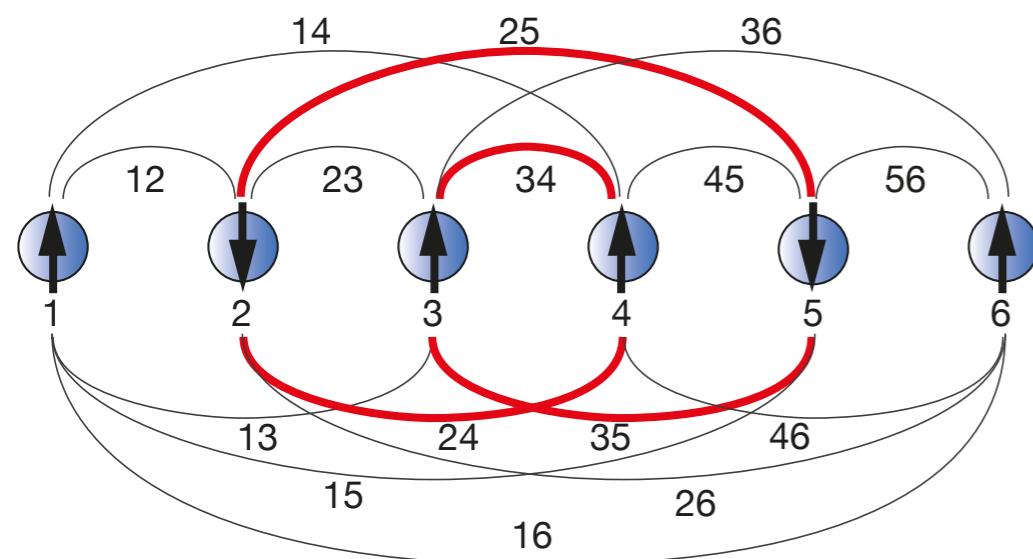


Number of spin-down
is even

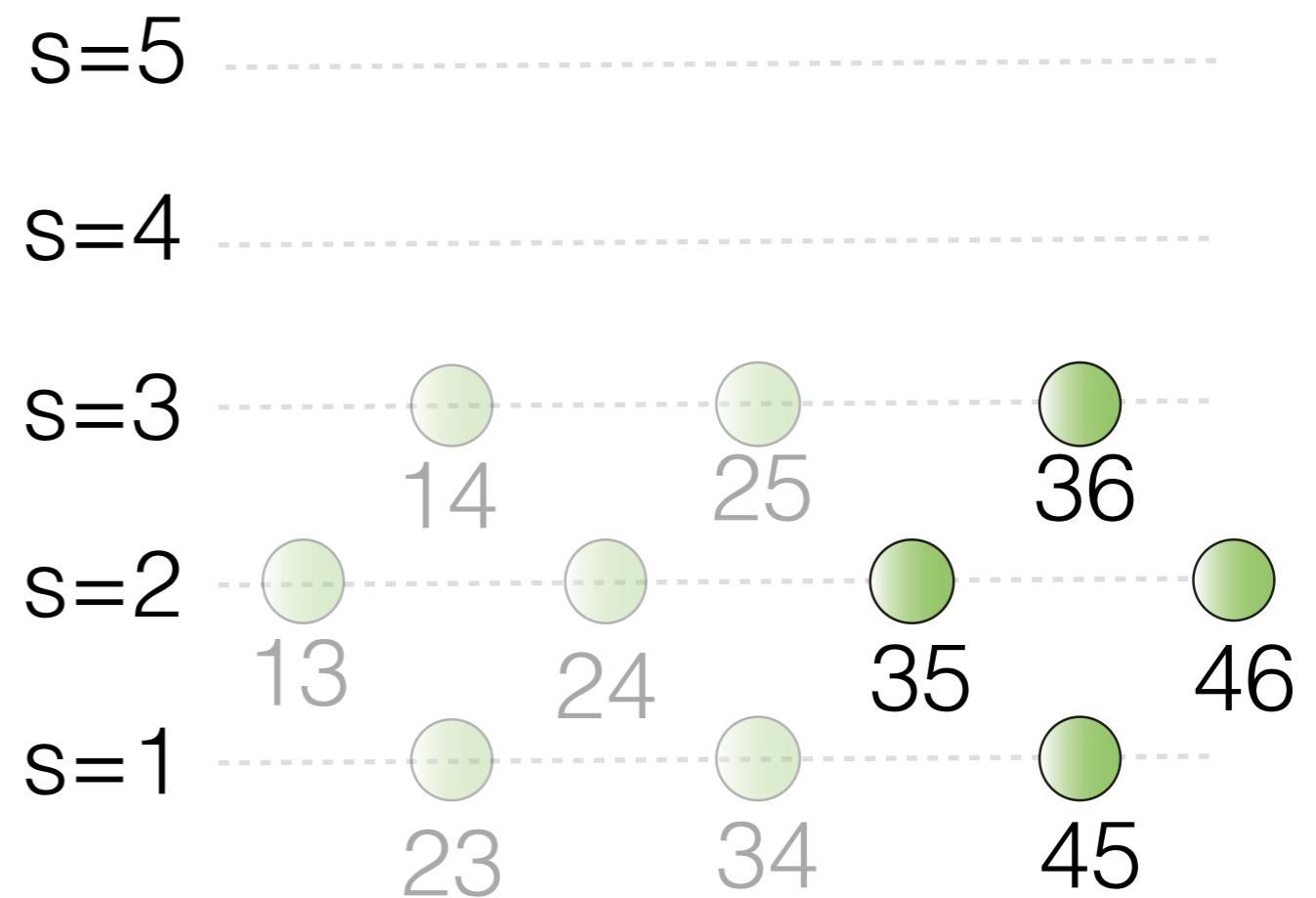
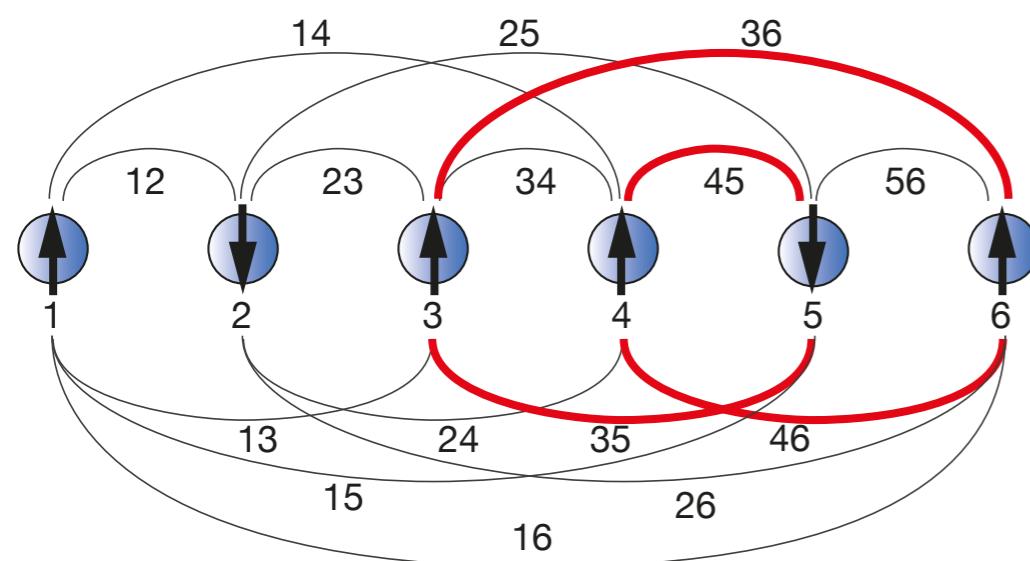


$$C_l = -C \left(\sum_{m=n,e,s,w} \tilde{\sigma}_z^{(l,m)} + S_z^l \right)^2 = -C \tilde{\sigma}_z^{(l,n)} \tilde{\sigma}_z^{(l,e)} \tilde{\sigma}_z^{(l,s)} \tilde{\sigma}_z^{(l,w)}$$

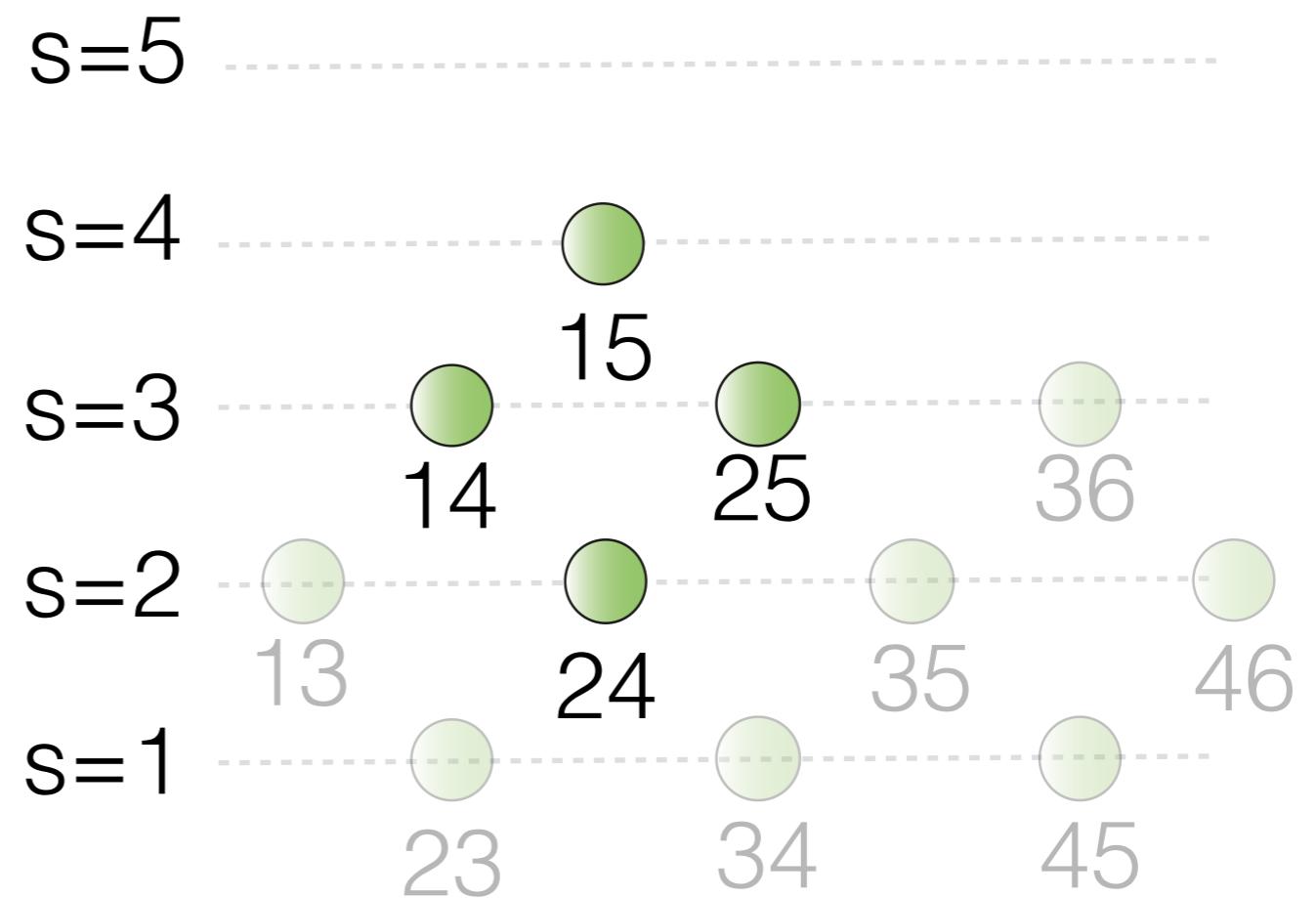
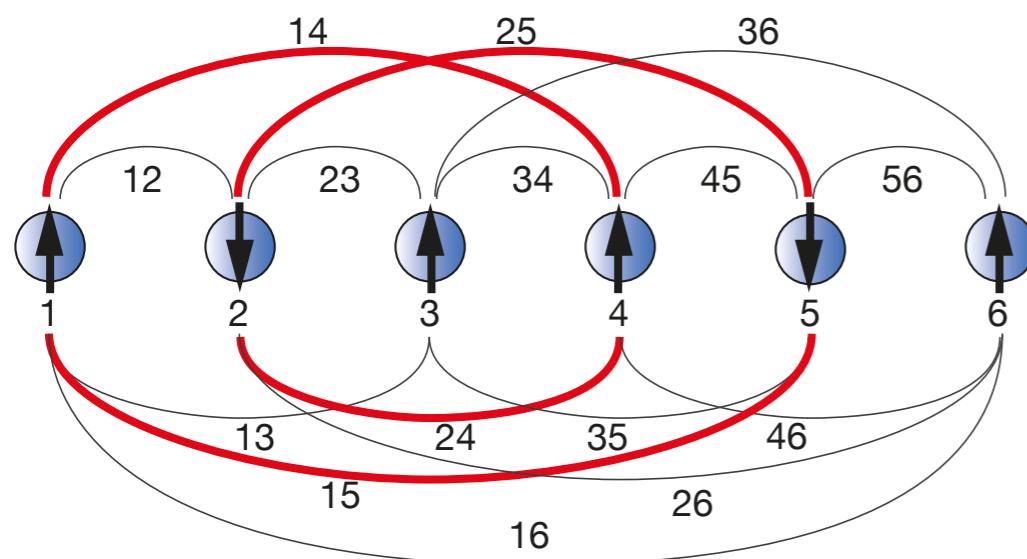
Constraints



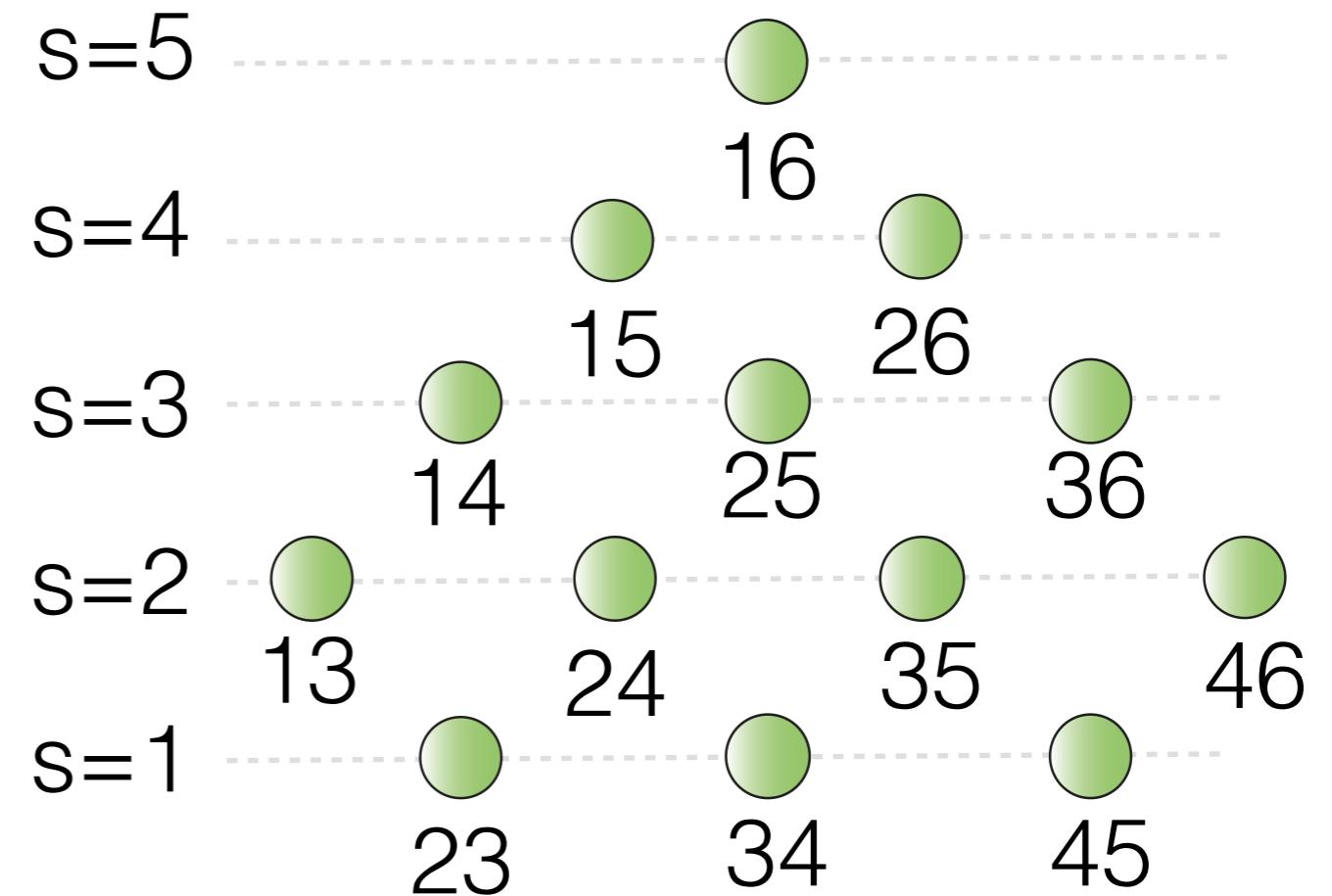
Constraints



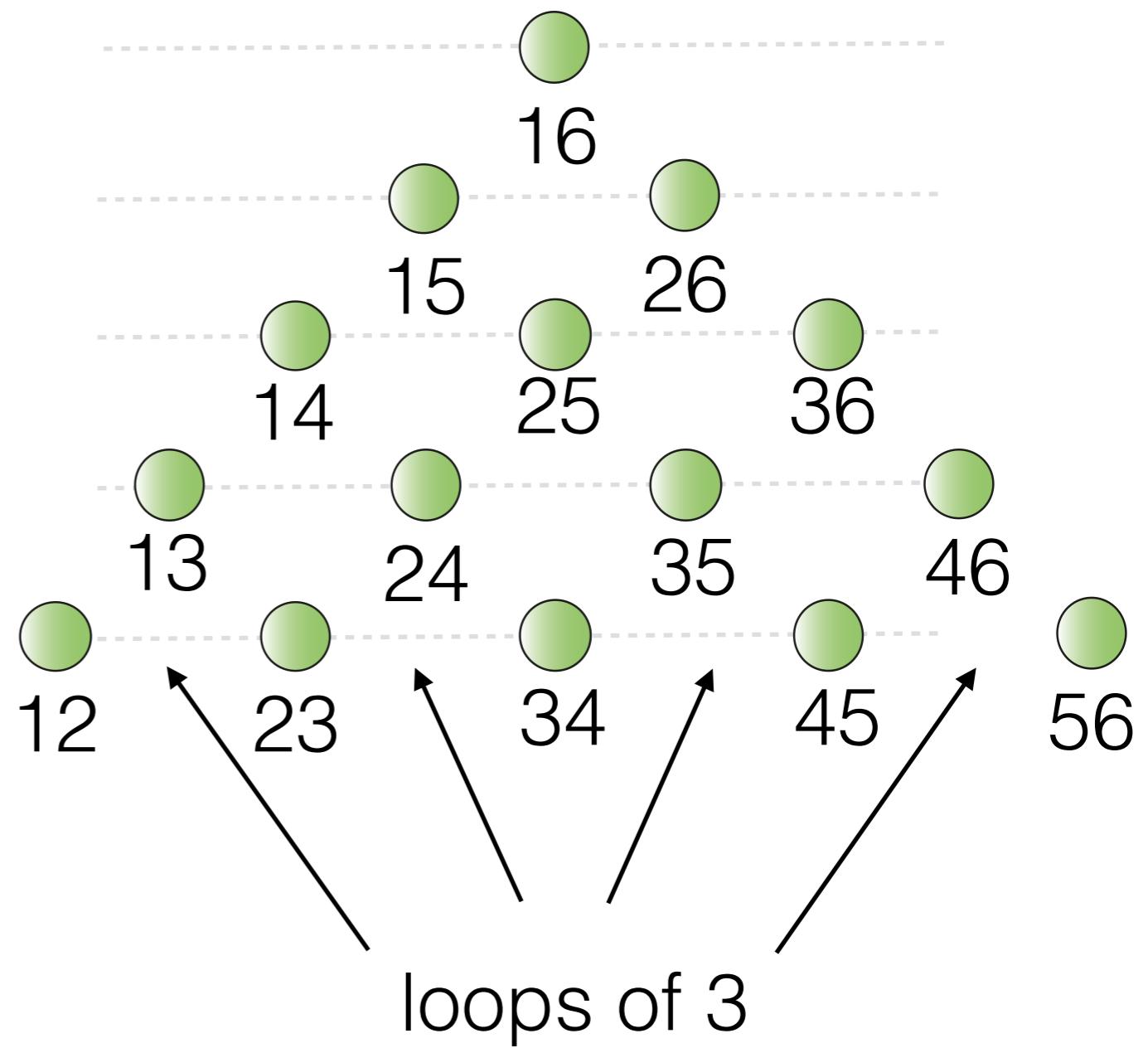
Constraints



Constraints



Constraints

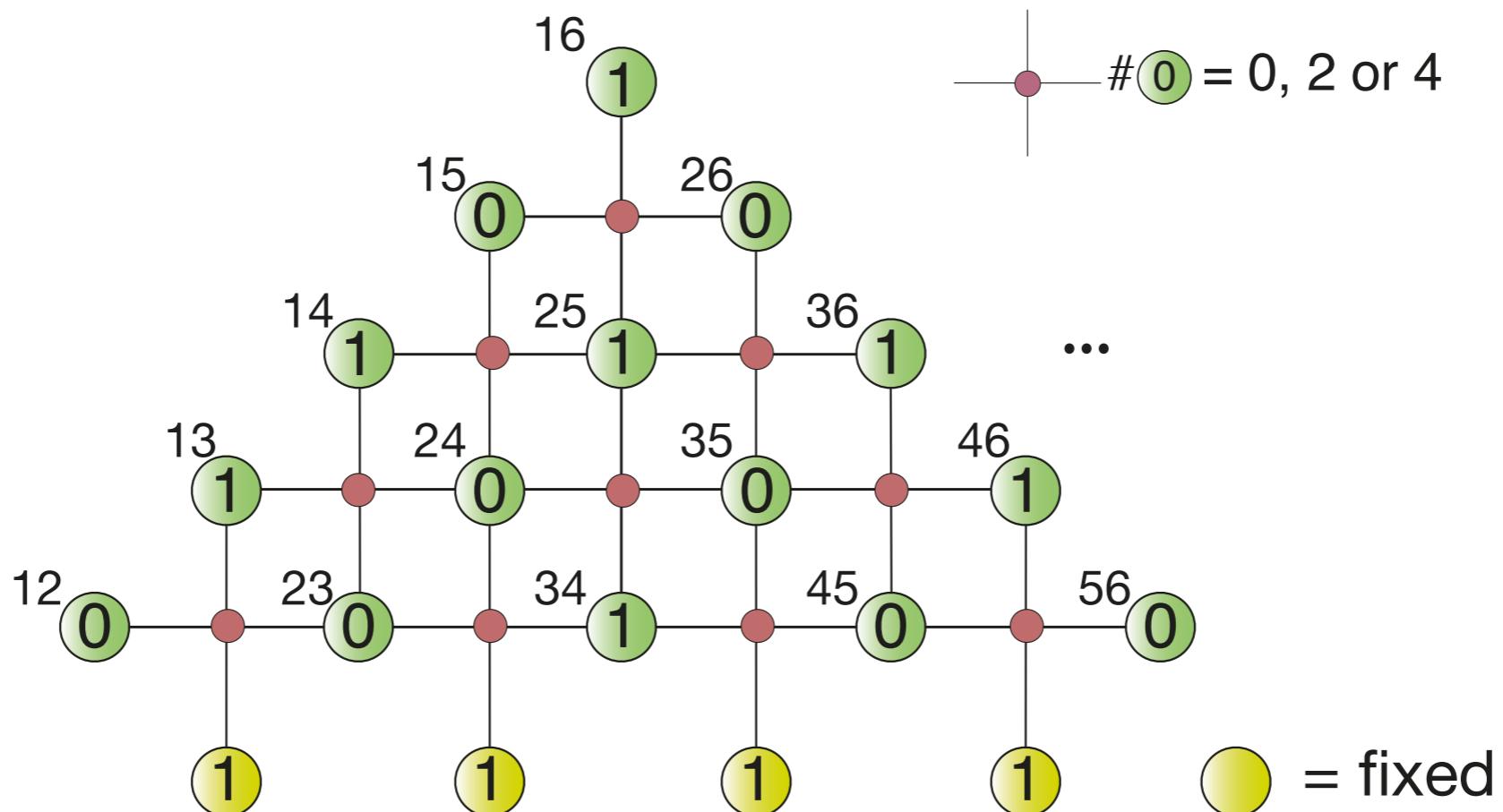


Parity AQC

$$\mathcal{H}(t) = A(t) \sum_{i=1}^K b_i \sigma_x^{(i)} + B(t) \sum_{i=1}^K J_i \sigma_z^{(i)} + C(t) \sum_{l=1}^{K-N} C_l$$

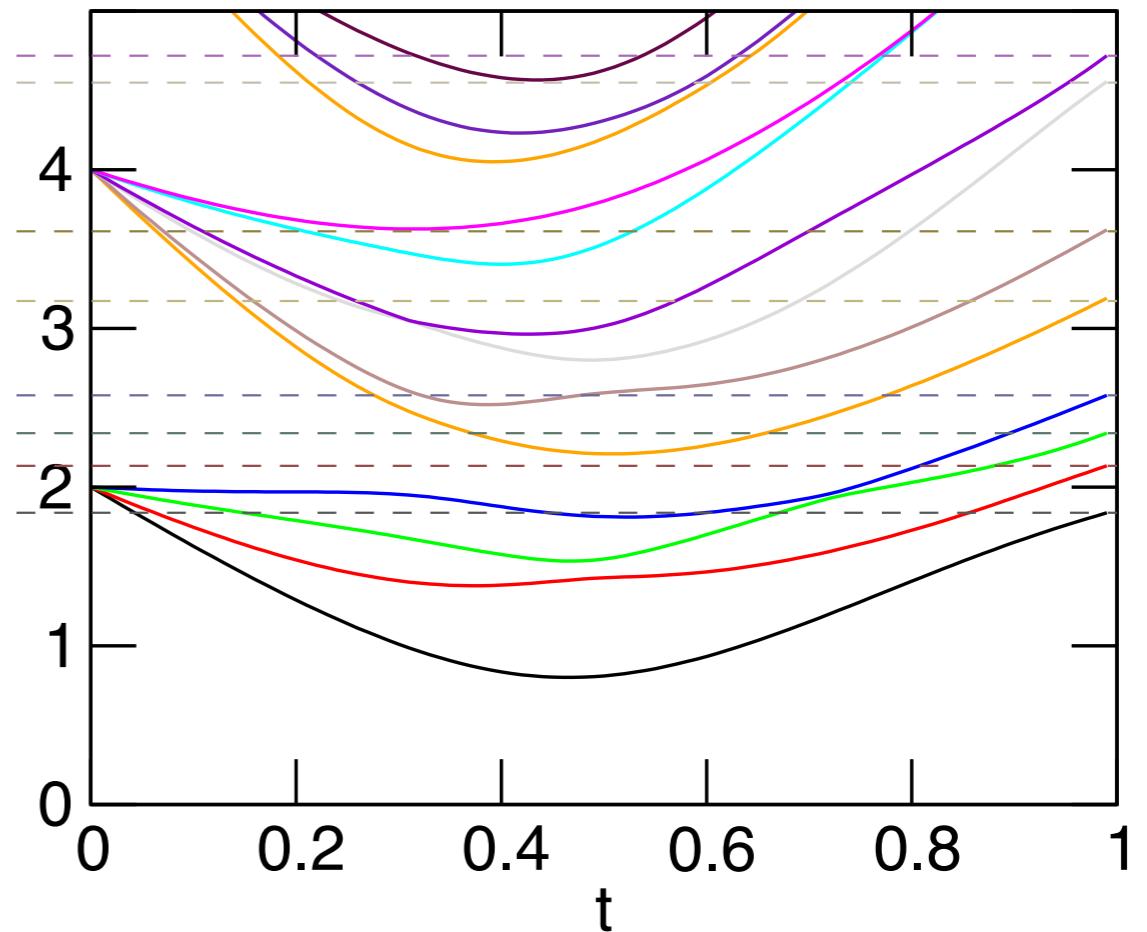
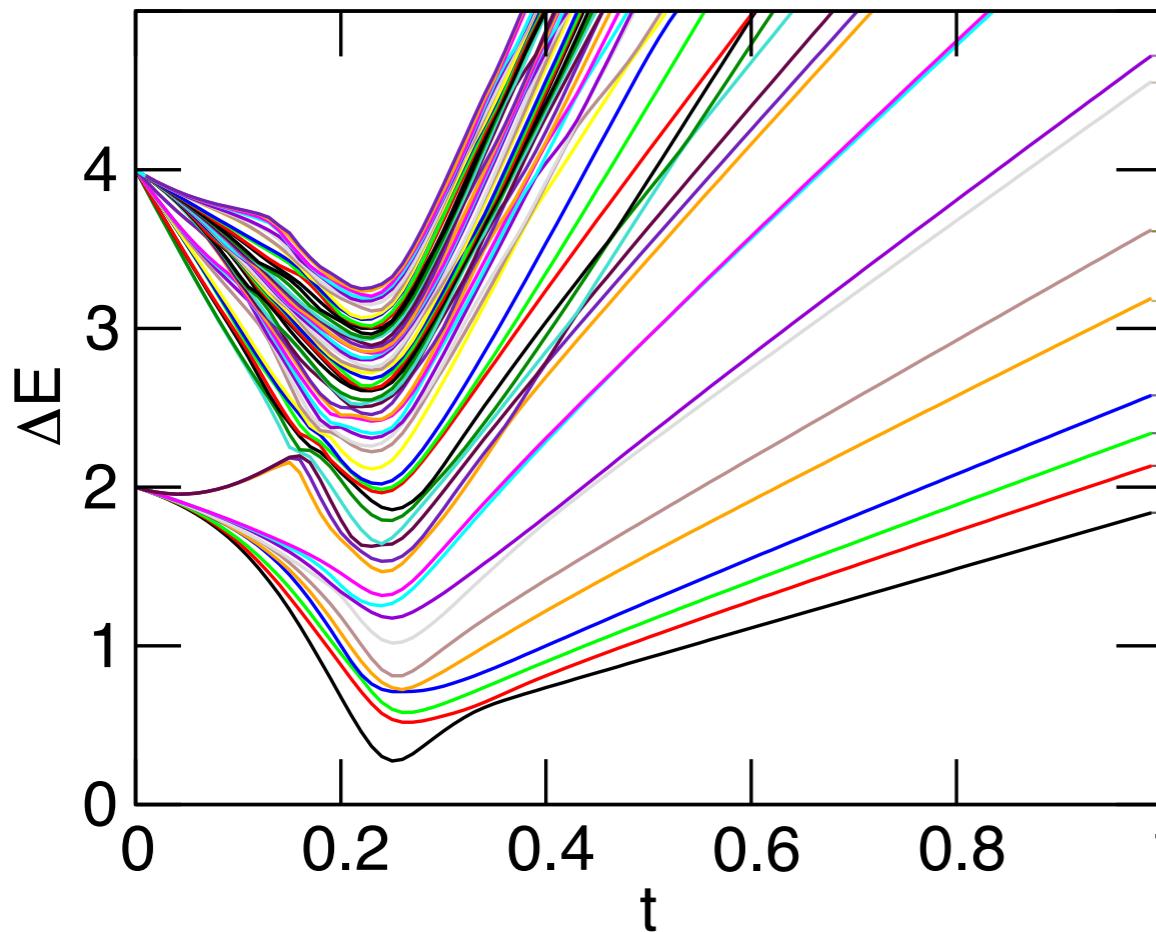
$$K = \frac{N(N-1)}{2}$$

$$C_l = -C \tilde{\sigma}_z^{(l,n)} \tilde{\sigma}_z^{(l,e)} \tilde{\sigma}_z^{(l,s)} \tilde{\sigma}_z^{(l,w)}$$



Spectrum

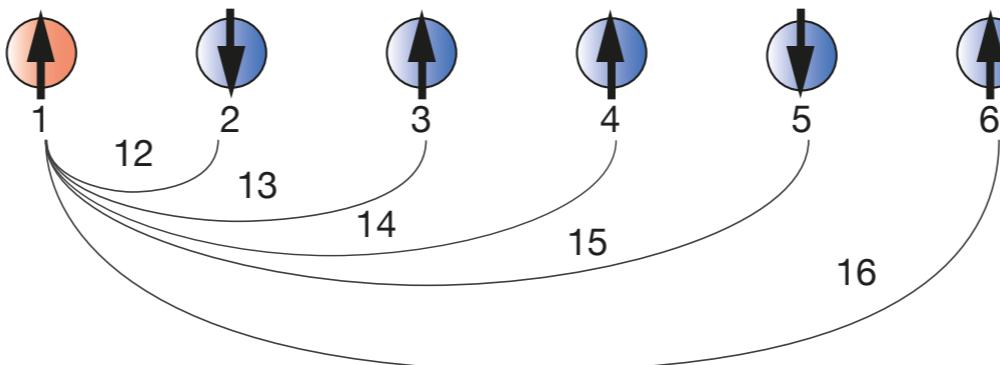
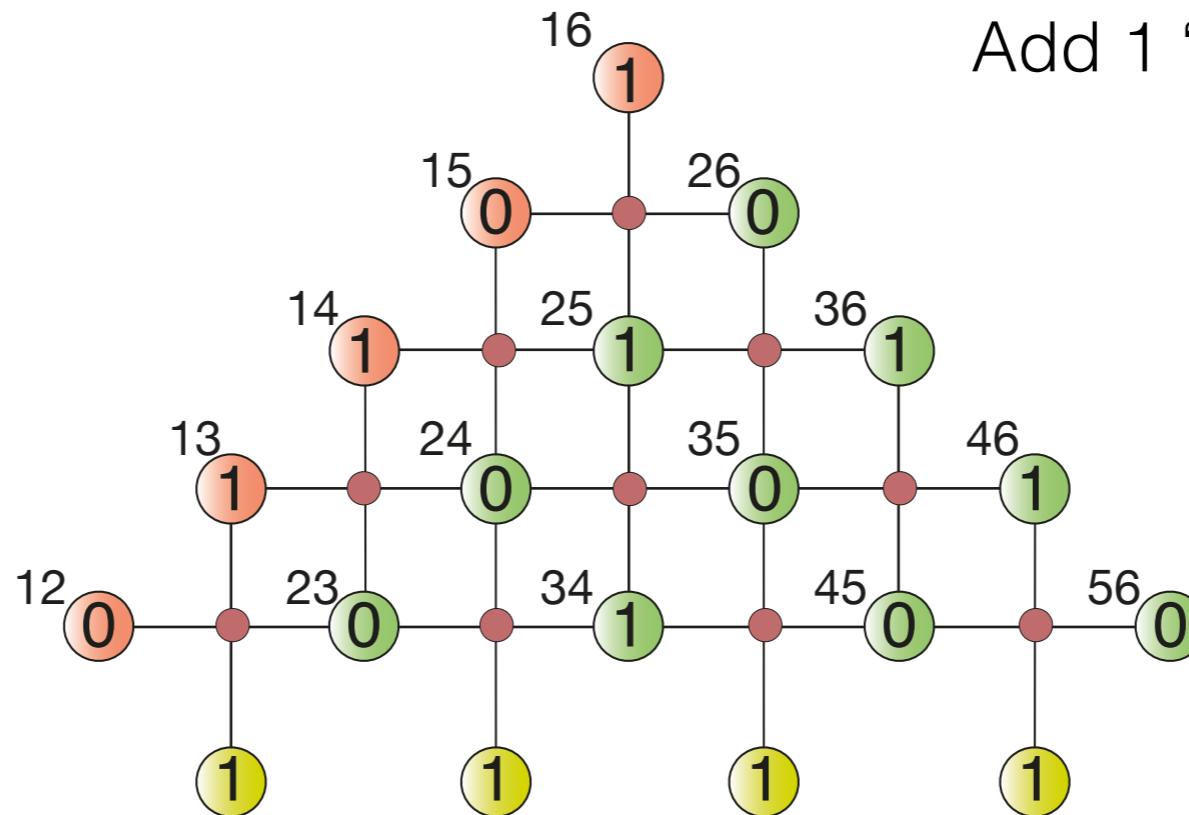
$$H(t) = A(t) \sum_{k=1}^K h_k \tilde{\sigma}_x^{(k)} + B(t) \left[\sum_{k=1}^K J_k \tilde{\sigma}_z^{(k)} + \sum_{l=1}^{K-N} C_l \right]$$



Magnetic Field term

$$H = \sum_{i=1}^N h_i \sigma_z^{(i)} + \sum_{i=1}^N \sum_{j=1}^i J_{ij} \sigma_z^{(i)} \sigma_z^{(j)}$$

Add 1 “row” of qubits.

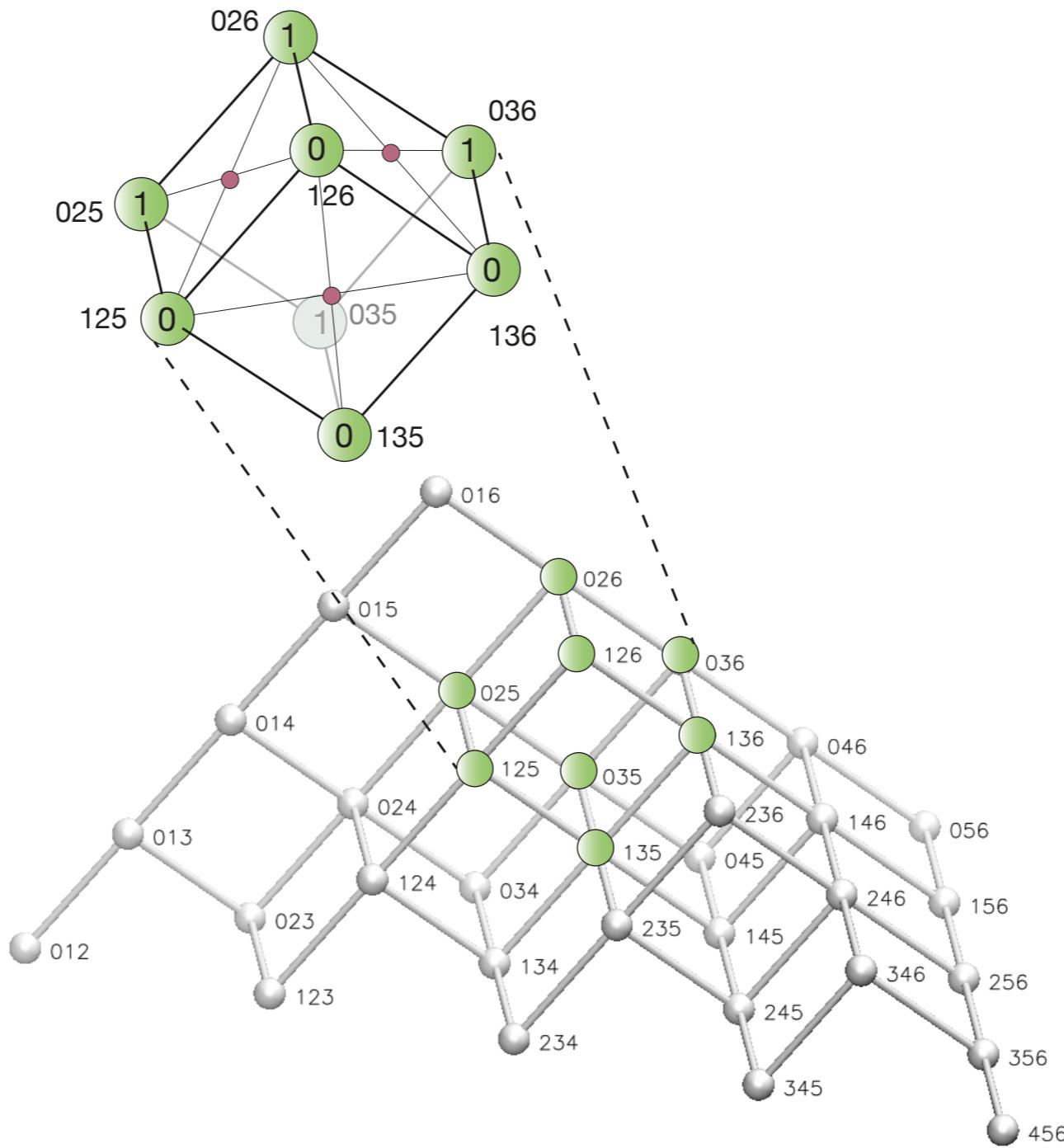


Three-Body interactions

$$H = \sum_{i=1}^N h_i \sigma_z^{(i)} + \sum_{i=1}^N \sum_{j=1}^i J_{ij} \sigma_z^{(i)} \sigma_z^{(j)} + \sum_i \sum_j \sum_k J_{ijk} \sigma_z^{(i)} \sigma_z^{(j)} \sigma_z^{(k)}$$

i	j	k	$\sigma_z^{(i)} \sigma_z^{(j)} \sigma_z^{(k)}$
↑	↑	↑	= 1
↑	↑	↓	= 0
↑	↓	↑	= 0
↑	↓	↓	= 1
↓	↑	↑	= 0
↓	↑	↓	= 1
↓	↓	↑	= 1
↓	↓	↓	= 0

0 = 0, 2 or 4



Error correction

Fernando Pastawski, John Preskill, Physical Review A 93, 052325 (2015).

Belief propagation algorithm

Parity is preserved for any closed loop: $0 = (12) \oplus (23) \oplus (13) = (12) \oplus (24) \oplus (14) = \dots = (12) \oplus (2N) \oplus (1N)$

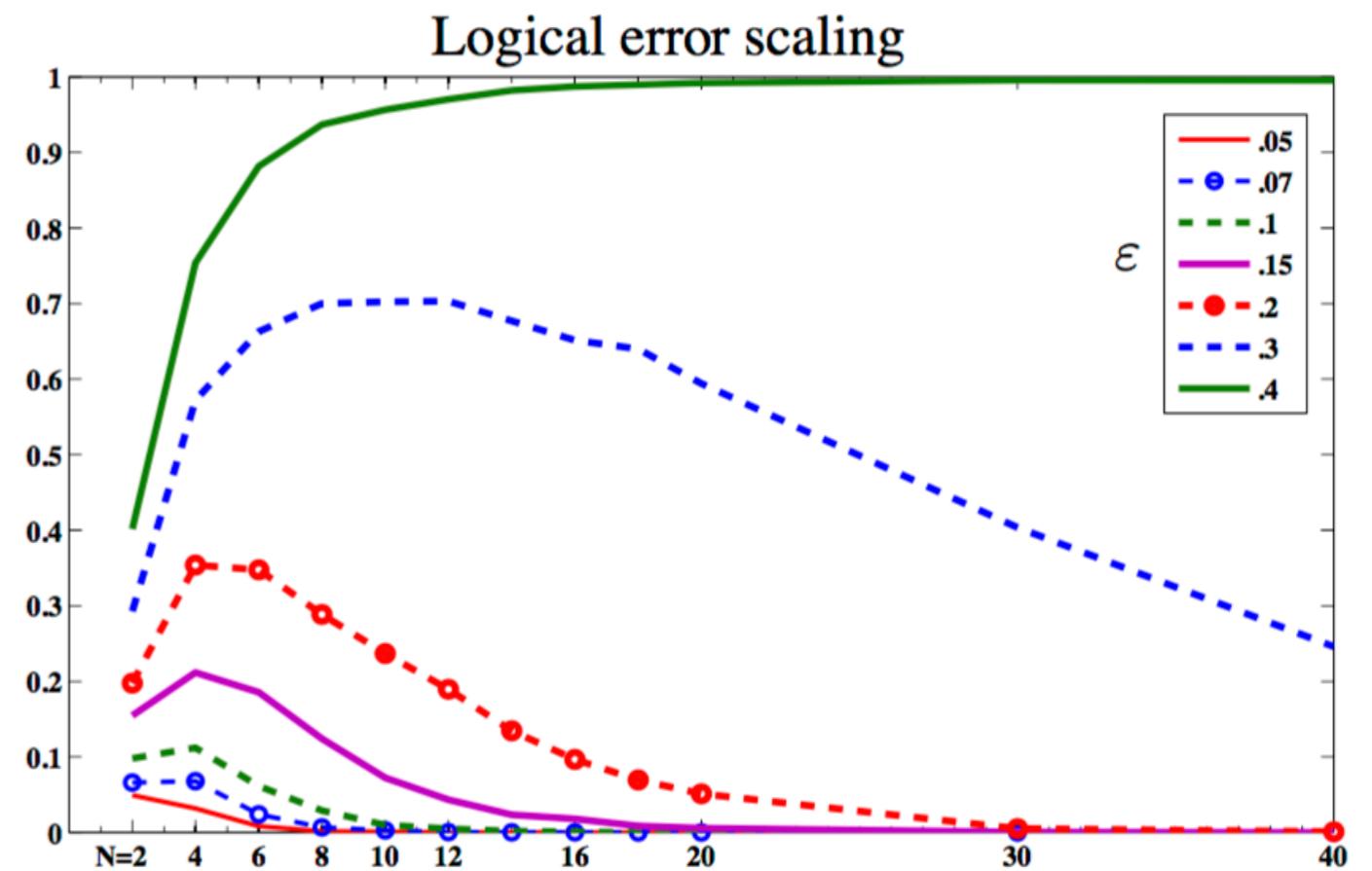
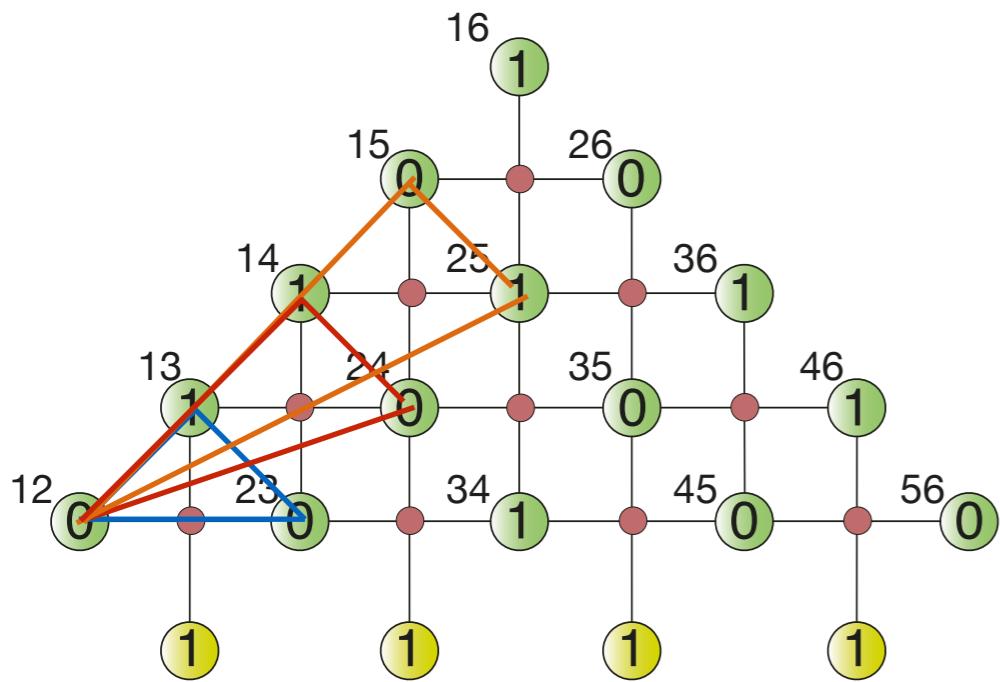
Estimate value of (12) from

$$g_{12} = (23) \oplus (13) = (24) \oplus (14) = \dots \quad \text{loops of 3}$$

$$g_{12} = (23) \oplus (34) \oplus (14) = (23) \oplus (35) \oplus (15) = \dots = \quad \text{loops of 4}$$

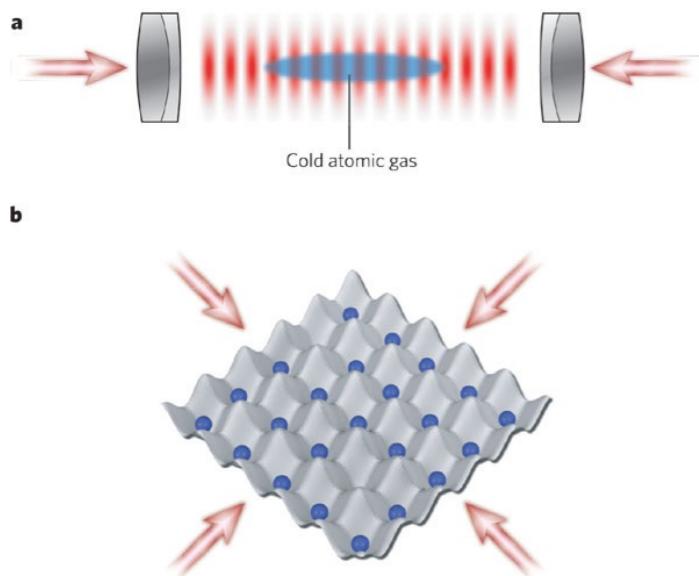
$$g_{12} = (23) \oplus (34) \oplus (45) \oplus (15) = \dots \quad \text{loops of 5}$$

...



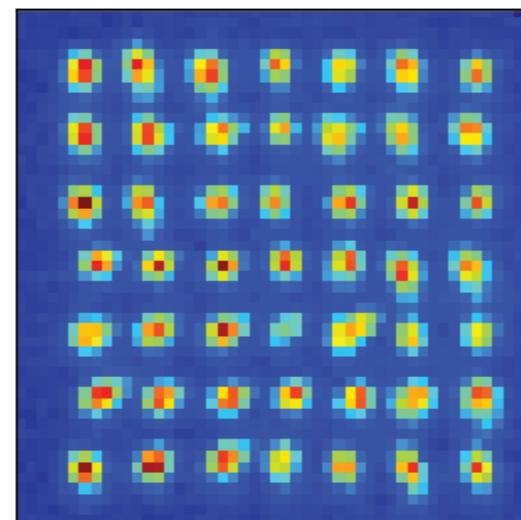
New platforms for Quantum Annealing

Ultracold atoms in optical lattices



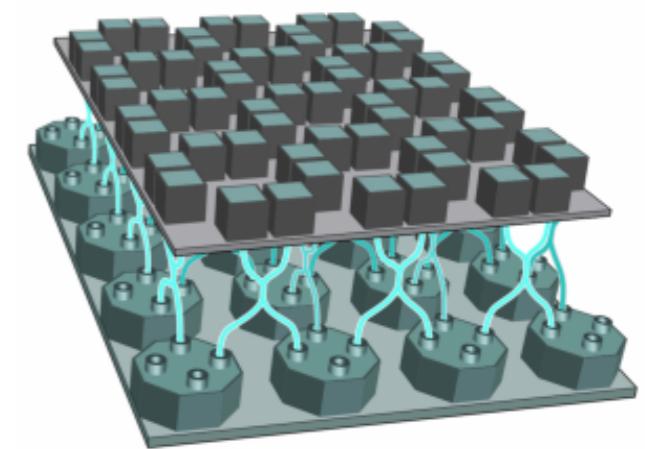
Bloch, Munich

Rydberg atoms



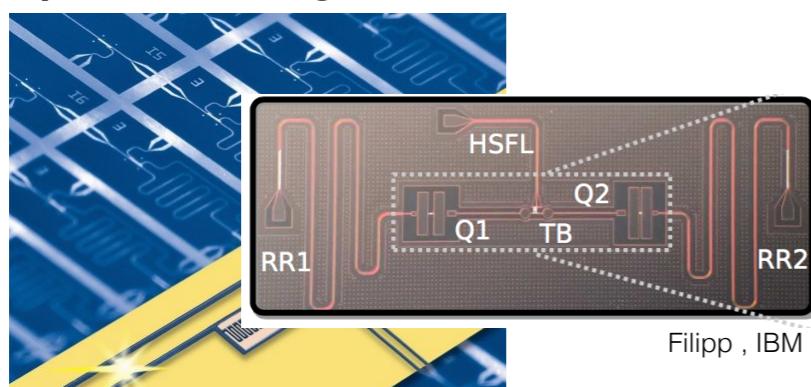
Saffman, Madison

Hybrid Ion-traps



S. Benjamin, Oxford

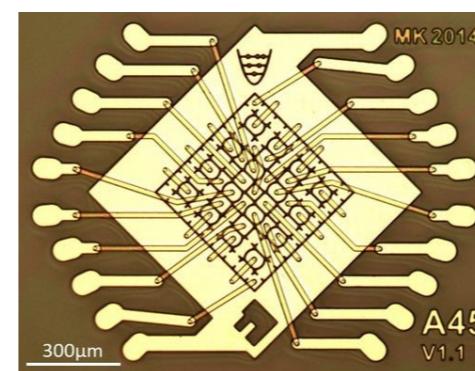
Superconducting Qubits



A. Wallraff

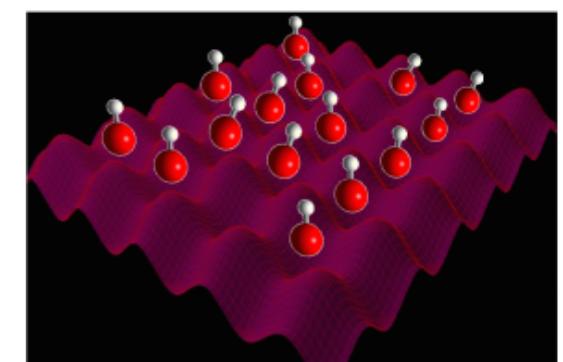
Filipp , IBM

Ions in surface traps



Blatt, Innsbruck

Polar Molecules

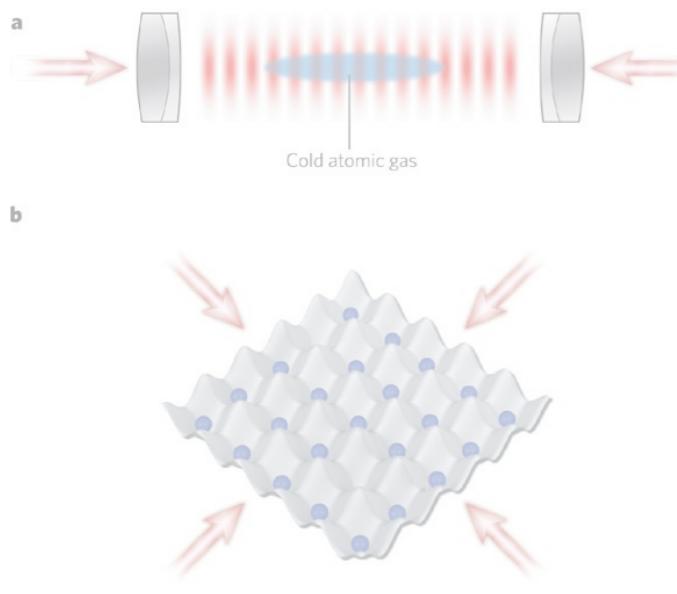


J. Ye, Boulder

New platforms for Quantum Annealing

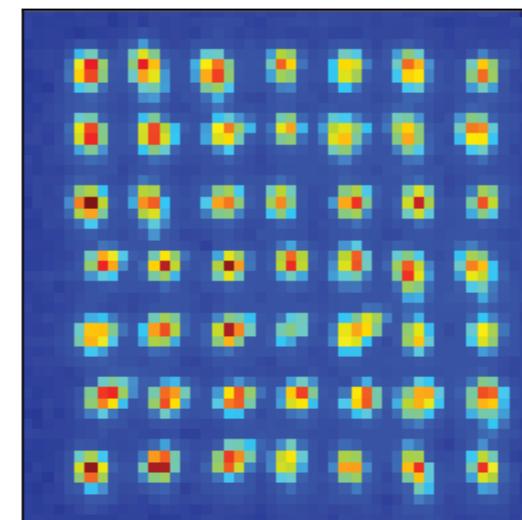
A. Glätzle, R. van Bijnen, P. Zoller and WL, arXiv:1611.02594 (2016).

Ultracold atoms in optical lattices



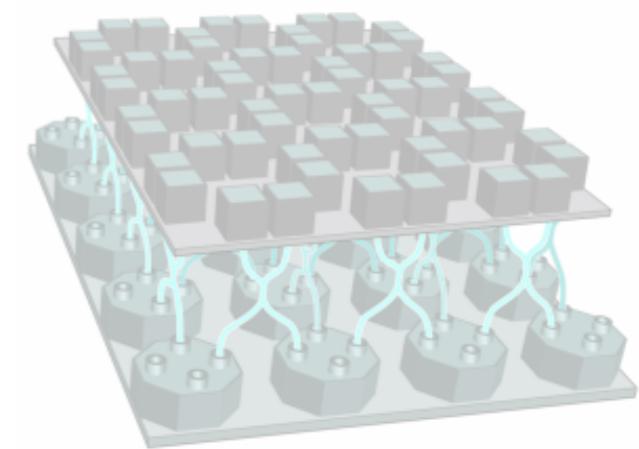
Bloch, Munich

Rydberg atoms



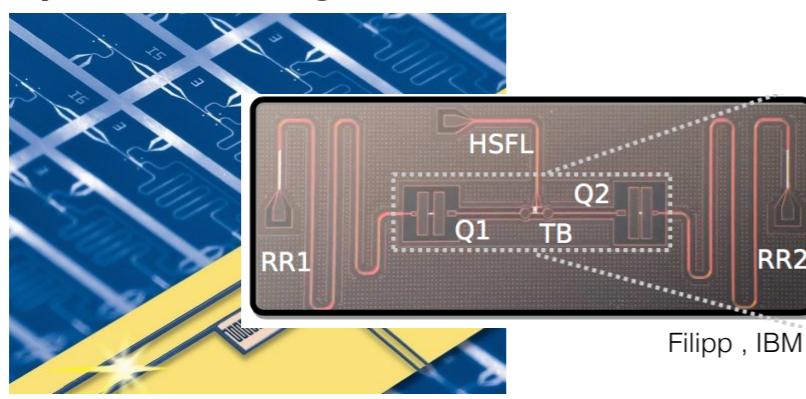
Saffman, Madison

Hybrid Ion-traps



S. Benjamin, Oxford

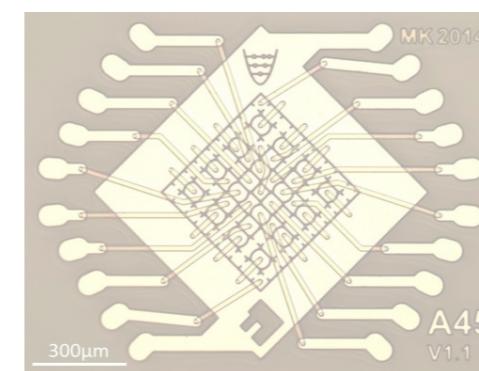
Superconducting Qubits



A. Wallraff

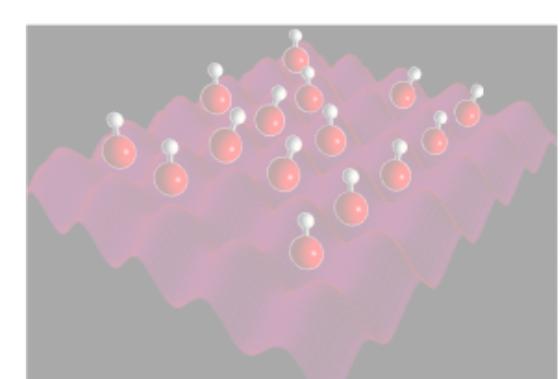
Filipp , IBM

Ions in surface traps



Blatt, Innsbruck

Polar Molecules

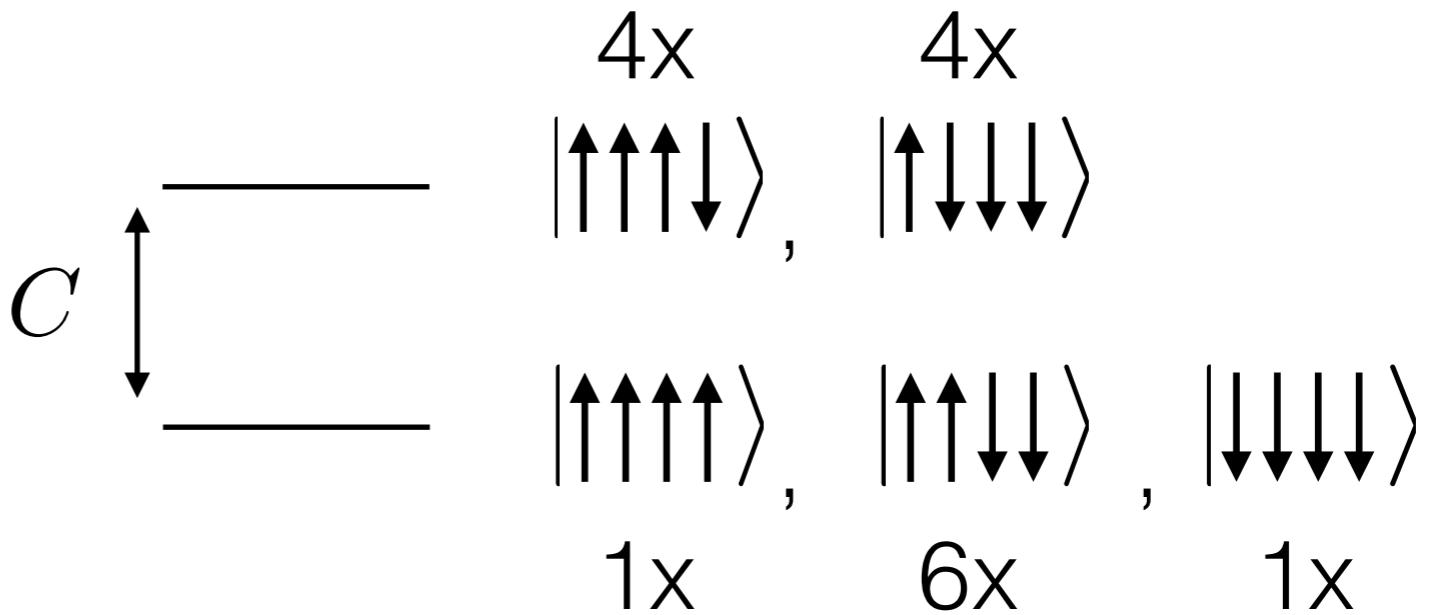
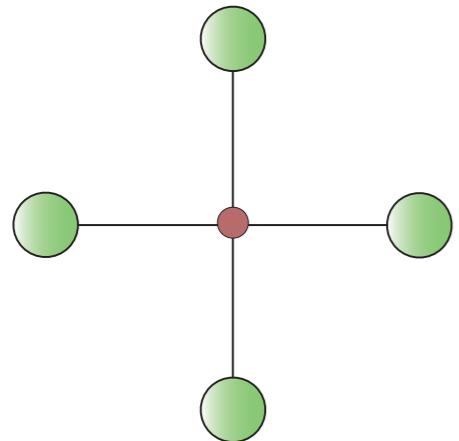


J. Ye, Boulder

Martin Leib, P. Zoller, and WL, arXiv:1604.02359(2016).

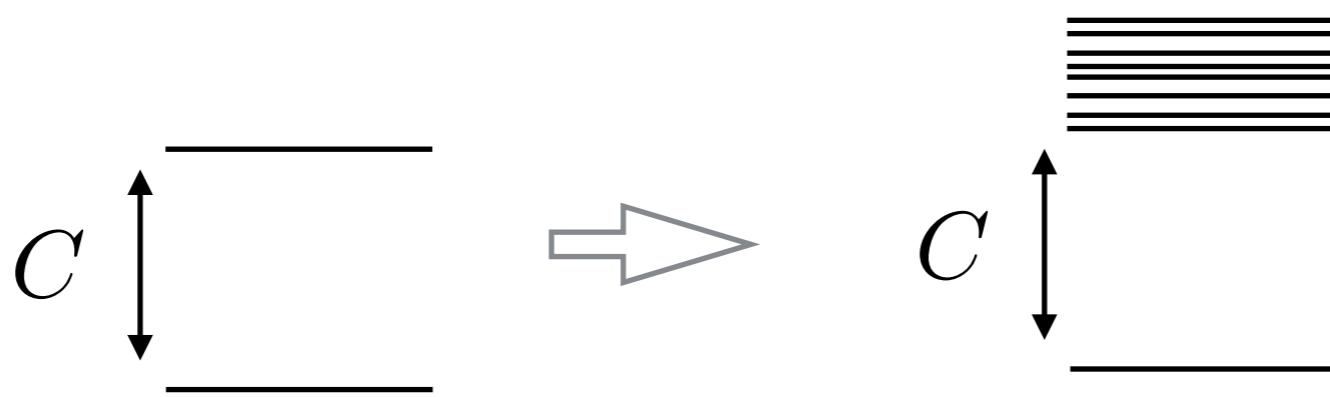
Strategies for constraint design with Rydberg atoms

4-body constraints



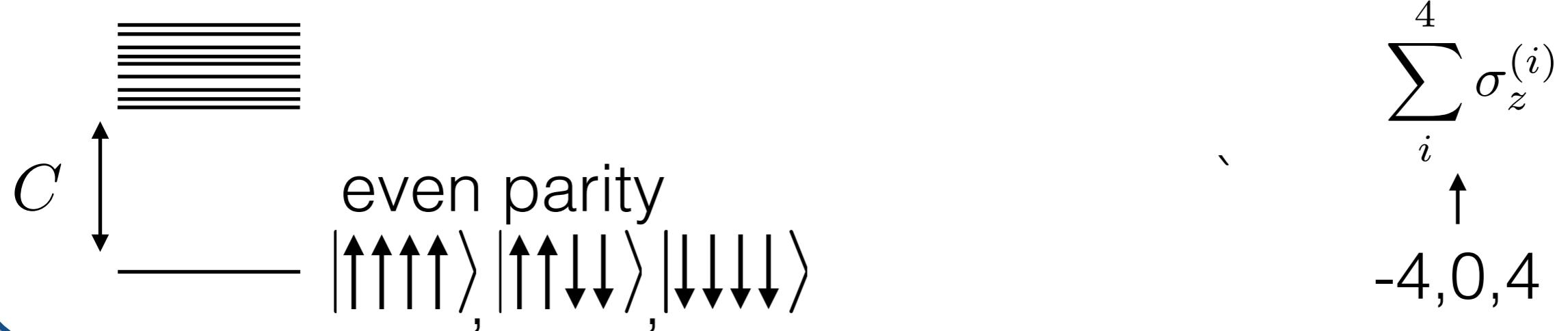
$$H_c = -C \sigma_z^1 \sigma_z^2 \sigma_z^3 \sigma_z^4$$

Unphysical states do not matter



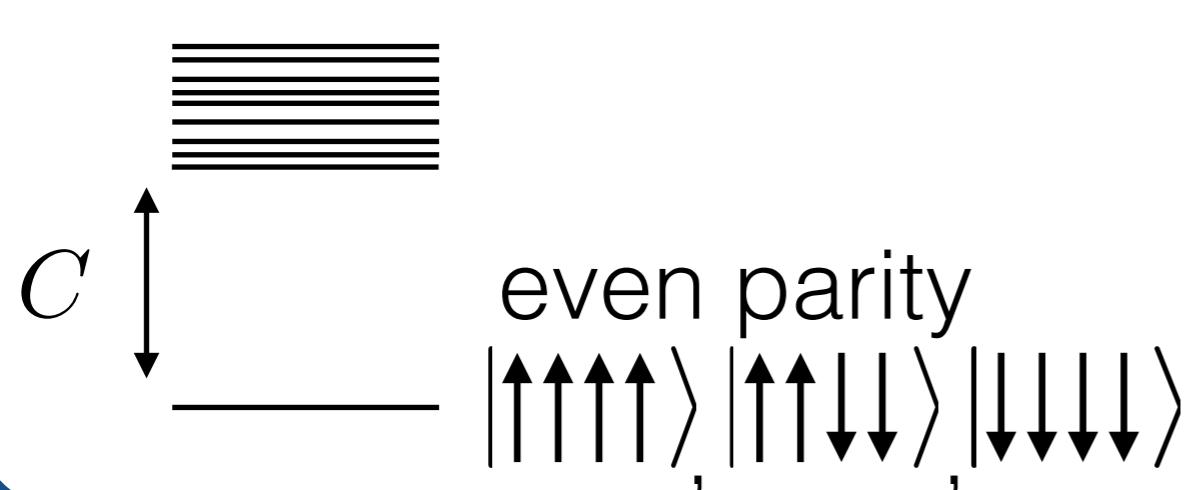
Strategies for constraint design with Rydberg atoms

Andrea Rocchetto, Simon C. Benjamin and Ying Li, Science Advances 2, e1601246 (2016).



Strategies for constraint design with Rydberg atoms

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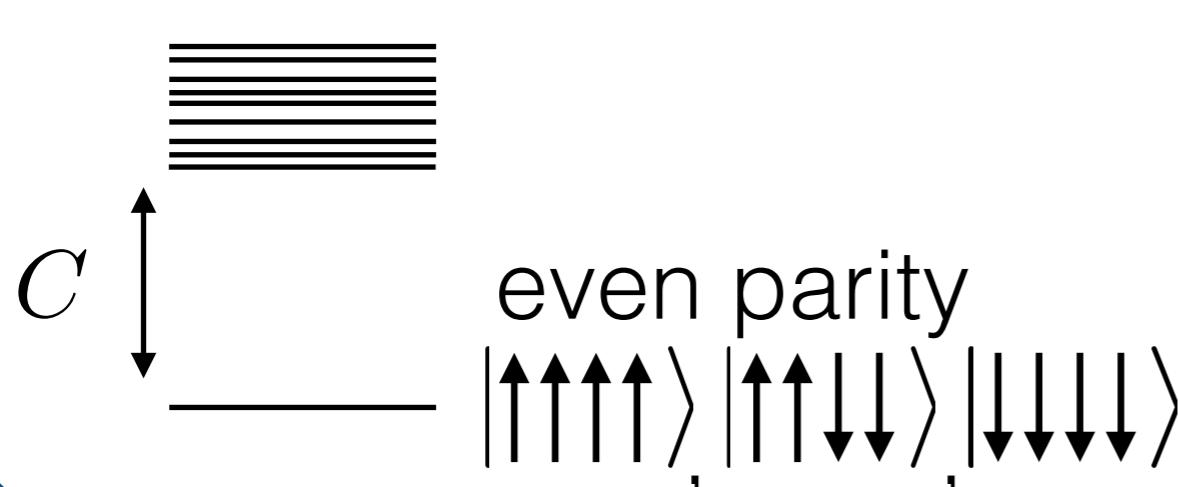


$$S = \left(4S_z + \sum_i^4 \sigma_z^{(i)} \right)^2$$

↑ ↑
qutrit -4,0,4

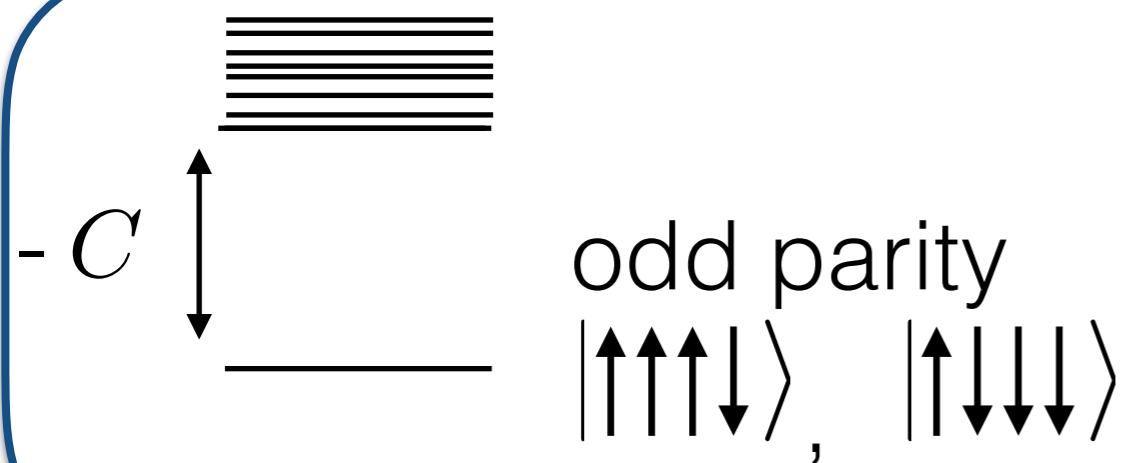
Strategies for constraint design with Rydberg atoms

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$$S = \left(4S_z + \sum_i^4 \sigma_z^{(i)} \right)^2$$

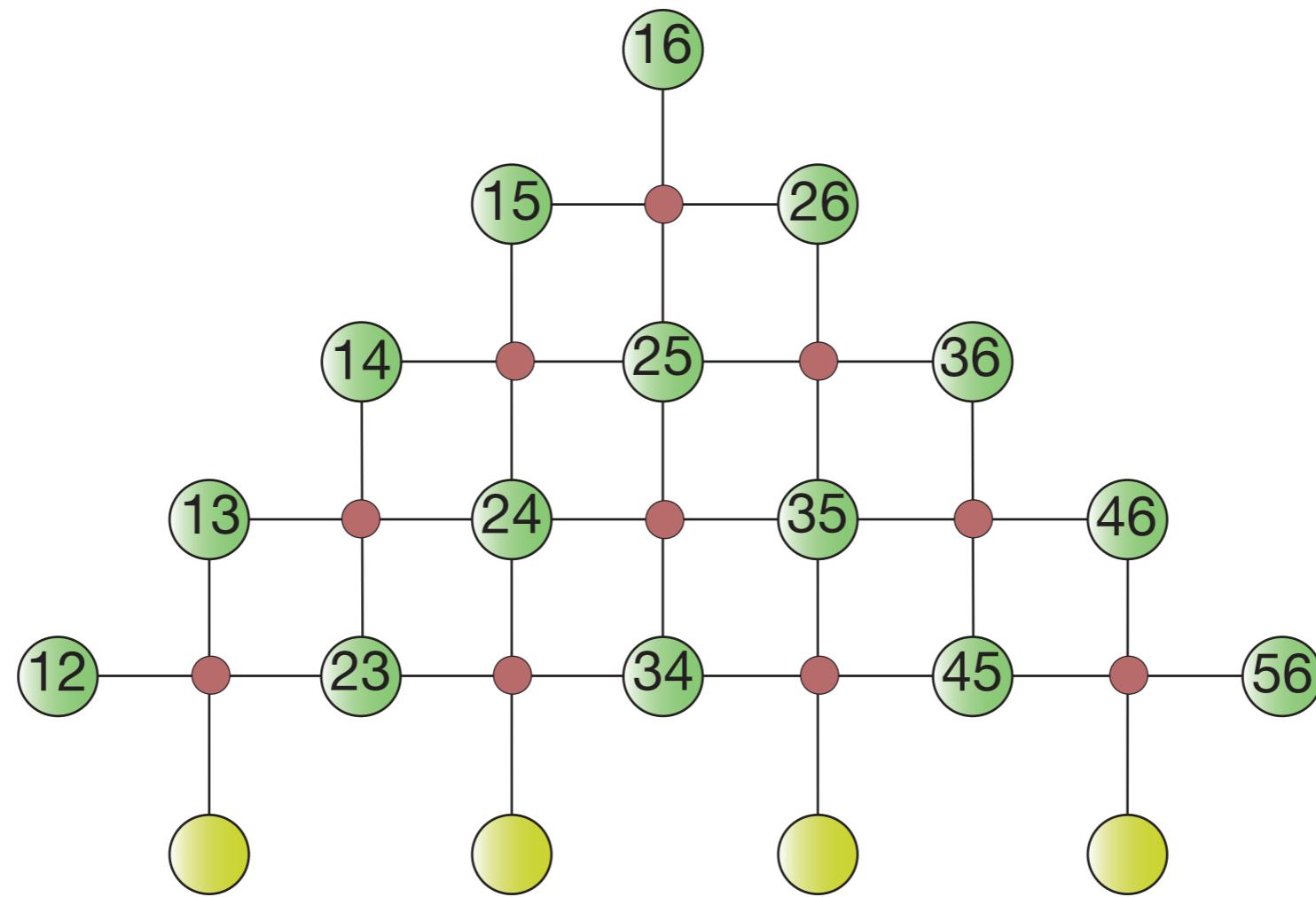
↑ ↑
qutrit -4,0,4



$$S = \left(2\sigma_z^{(a)} + \sum_i^4 \sigma_z^{(i)} \right)^2$$

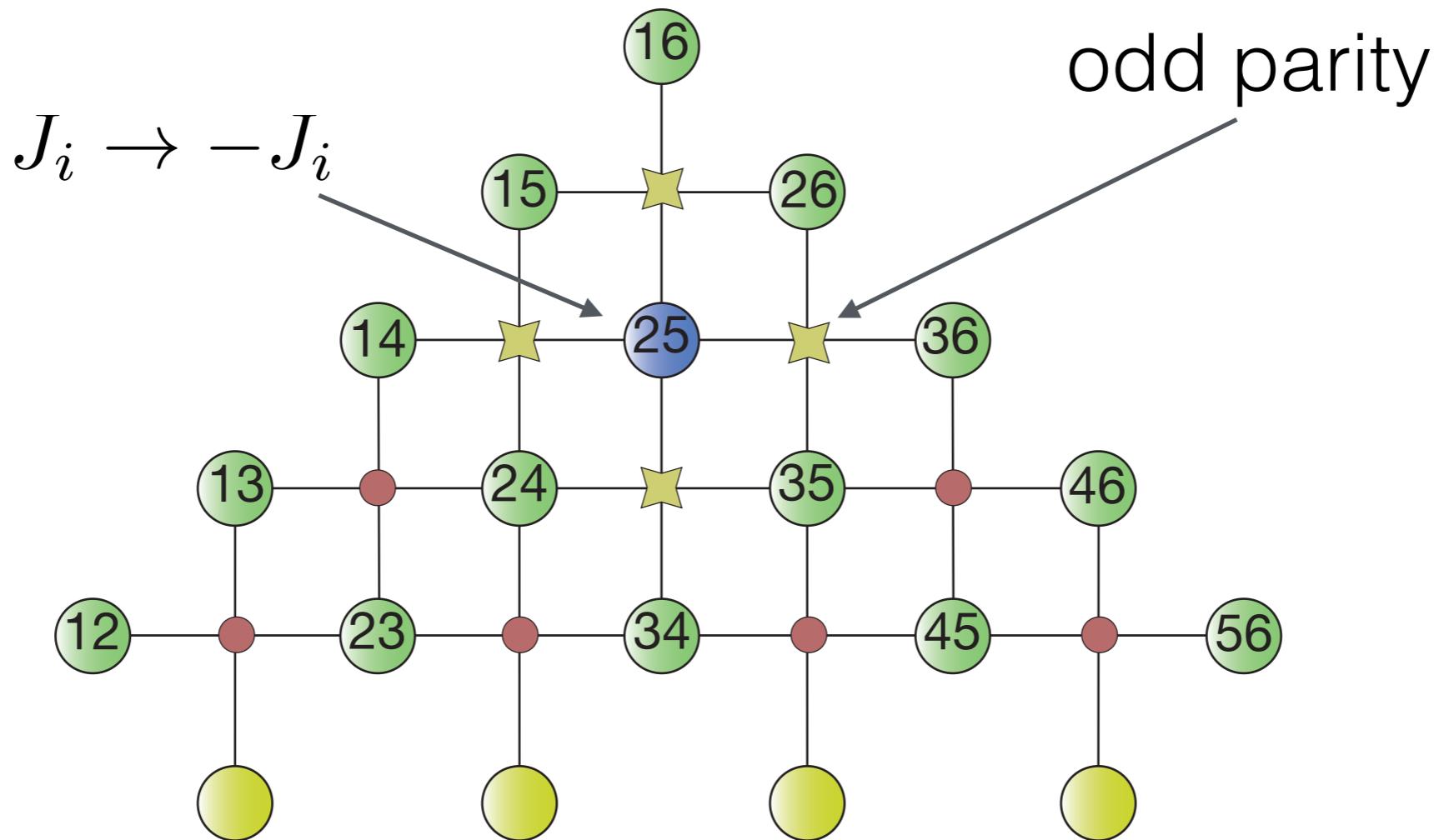
↑ ↑
qubit -2,2

Odd parity representation



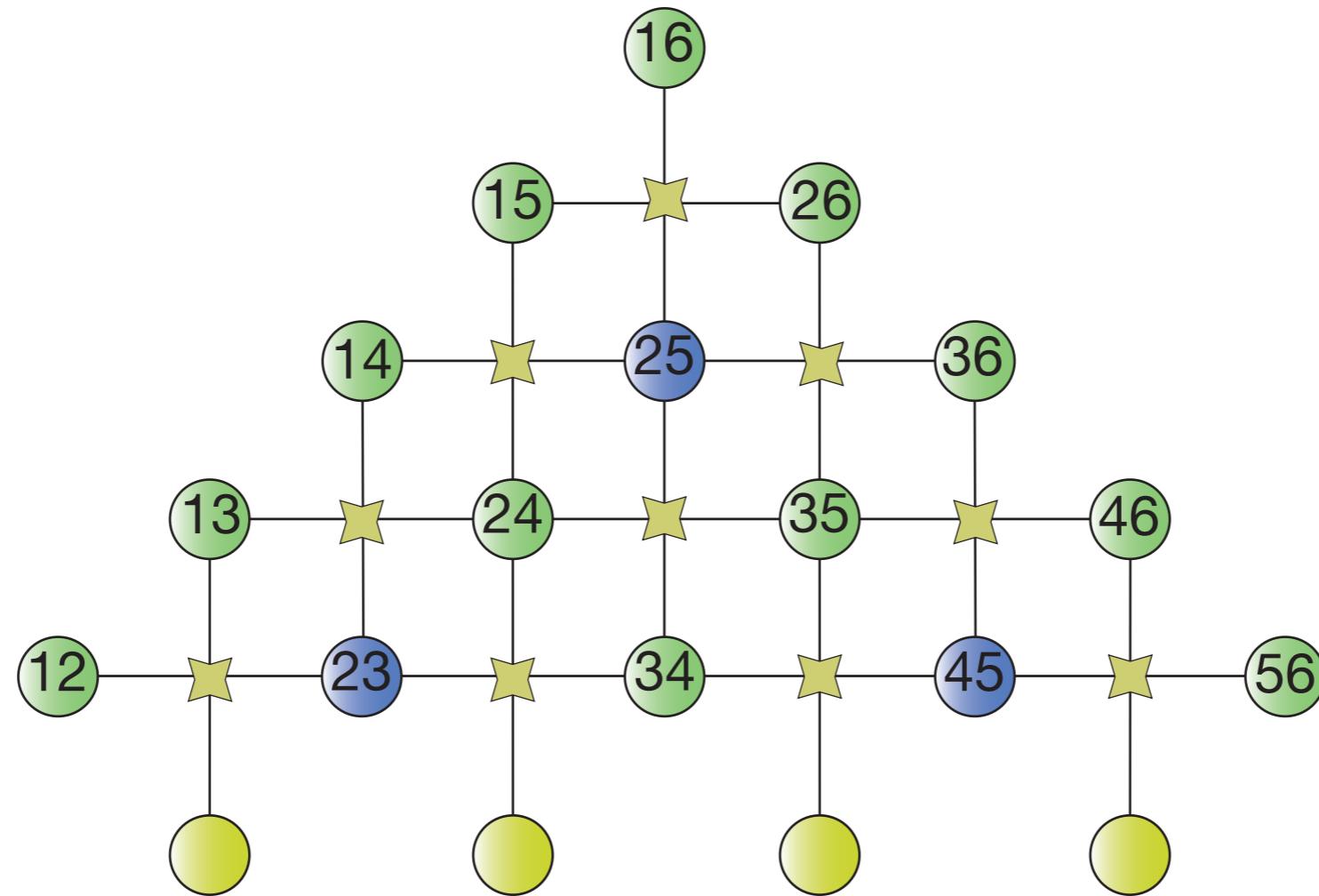
Odd parity representation

Andrea Rocchetto, Simon C. Benjamin and Ying Li, Science Advances 2, e1601246 (2016).



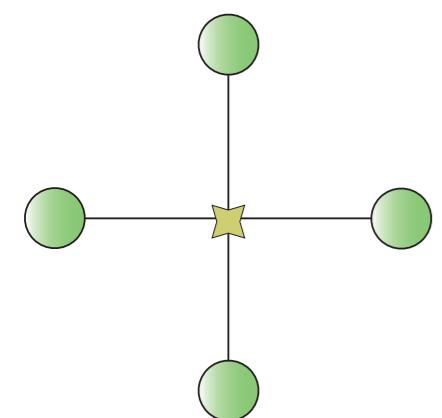
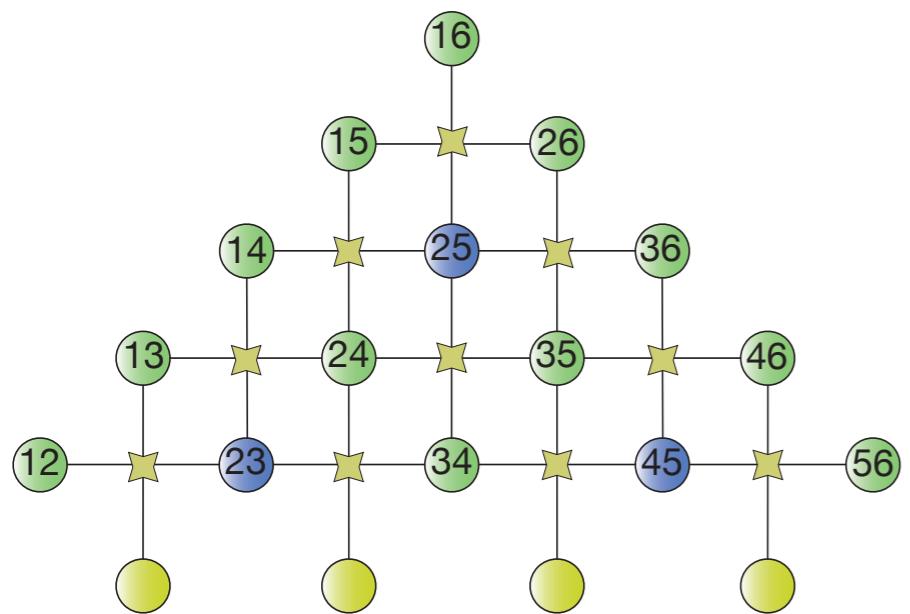
Odd parity representation

Andrea Rocchetto, Simon C. Benjamin and Ying Li, Science Advances 2, e1601246 (2016).

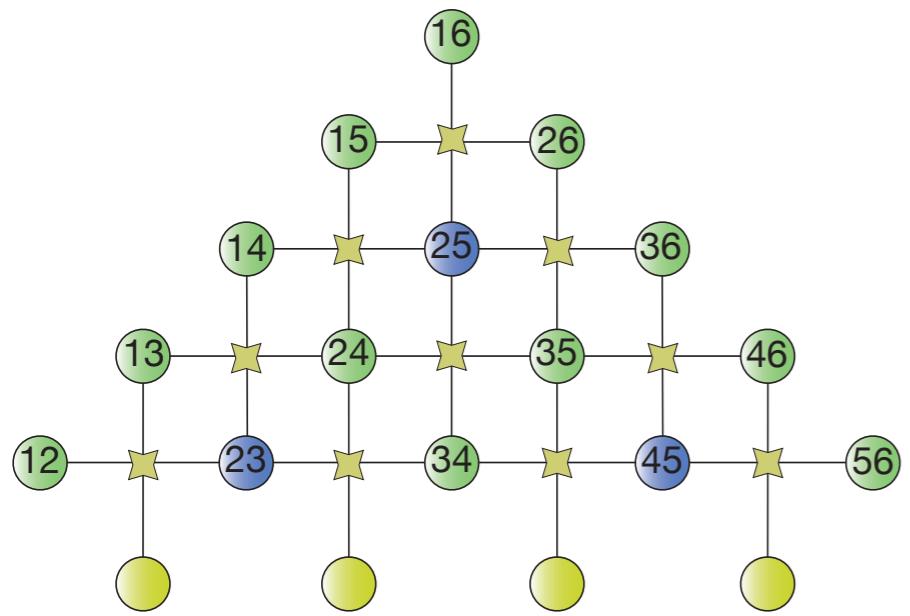


Odd parity representation

$$S = \left(2\sigma_z^{(a)} + \sum_i^4 \sigma_z^{(i)} \right)^2$$



Odd parity representation

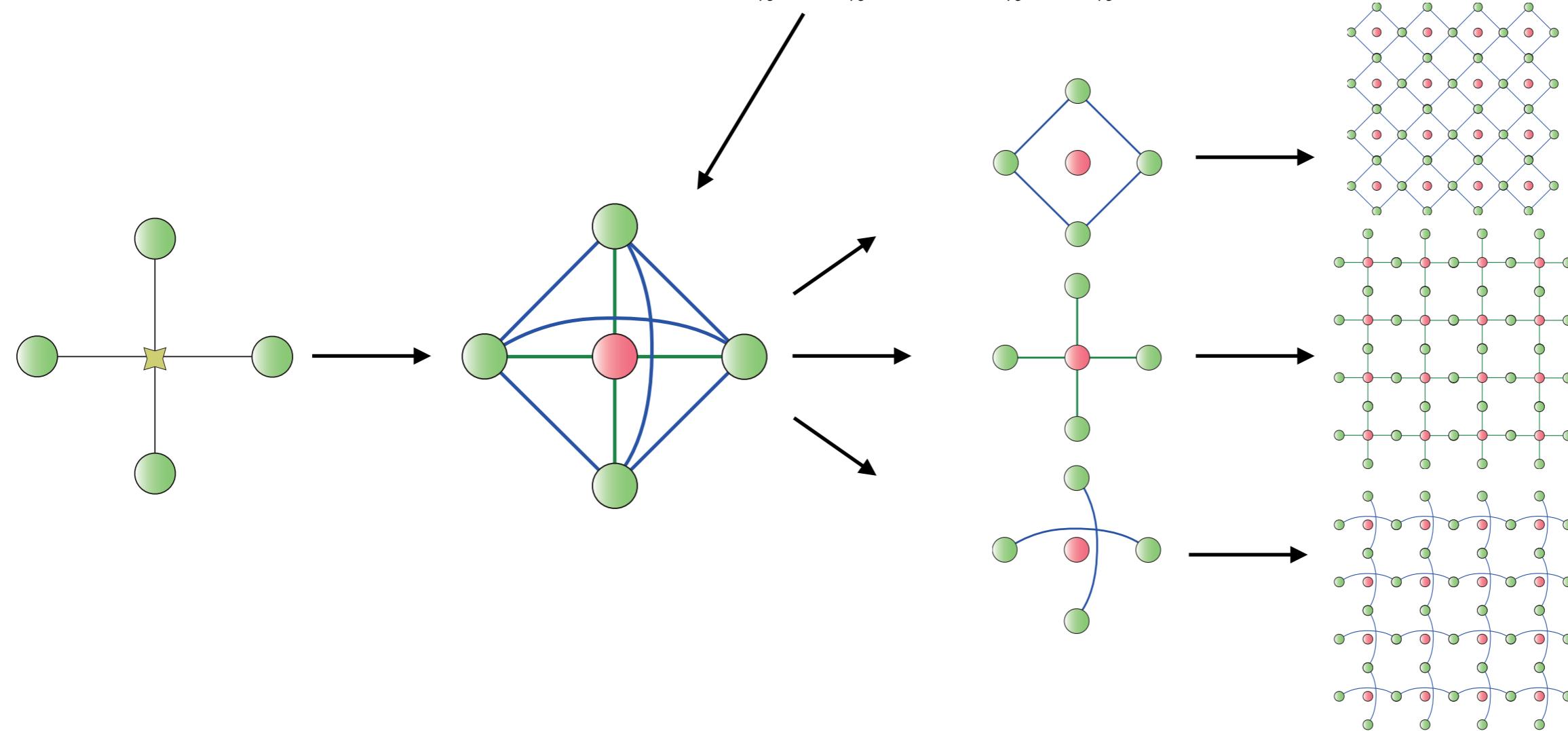


$$S = \left(2\sigma_z^{(a)} + \sum_i^4 \sigma_z^{(i)} \right)^2$$

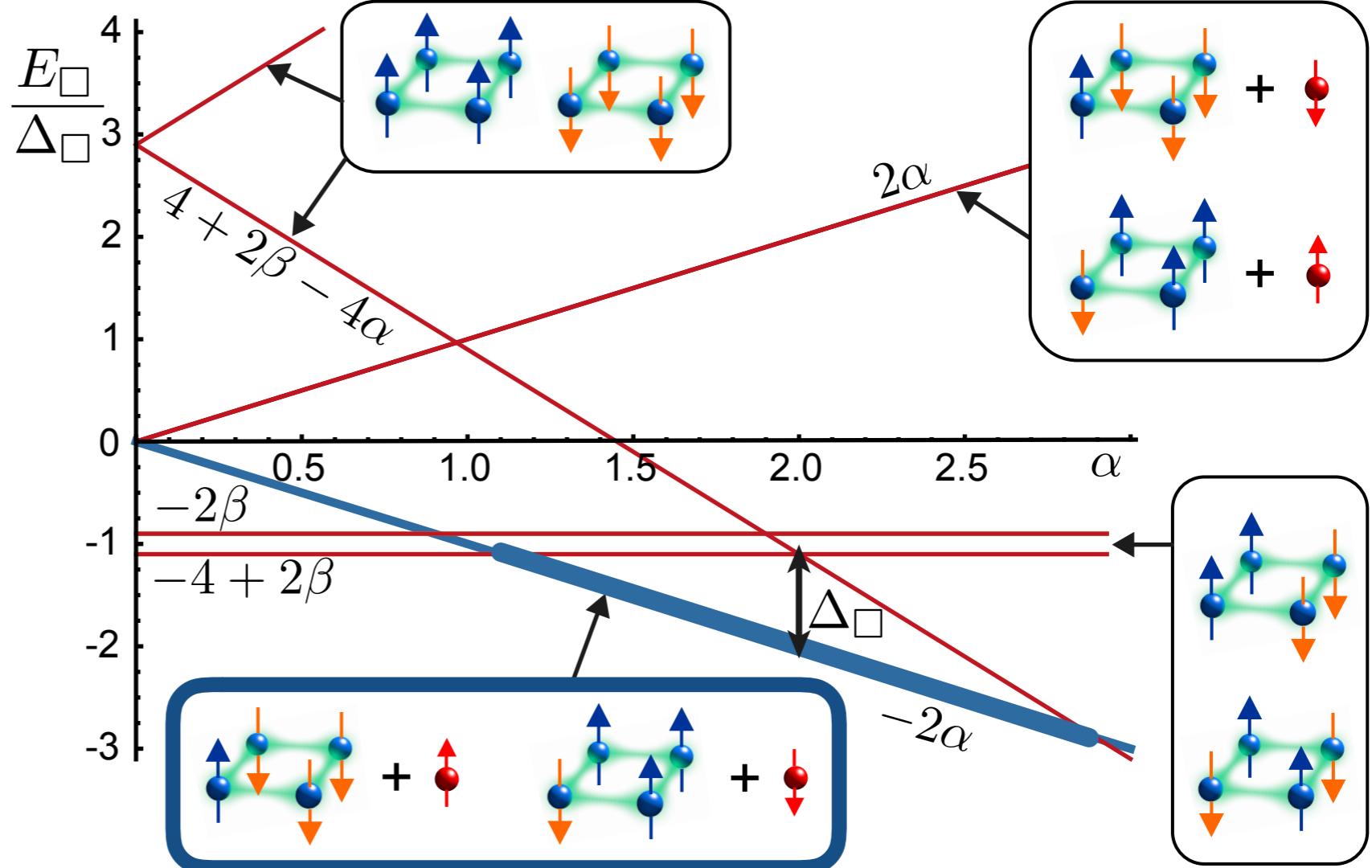
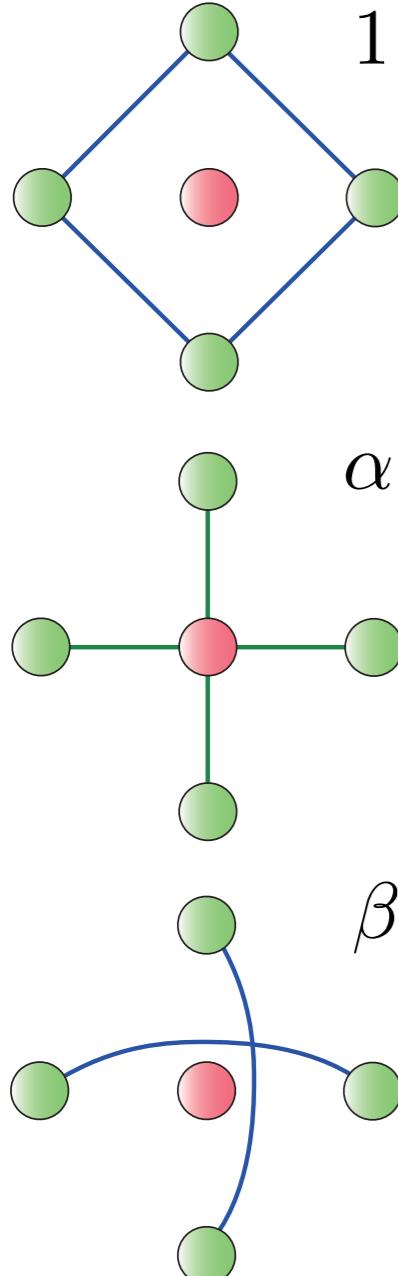
$$2\sigma_z^{(a)}\sigma_z^{(1)} + 2\sigma_z^{(a)}\sigma_z^{(2)} + 2\sigma_z^{(a)}\sigma_z^{(3)} + 2\sigma_z^{(a)}\sigma_z^{(4)} +$$

$$\sigma_z^{(1)}\sigma_z^{(2)} + \sigma_z^{(1)}\sigma_z^{(3)} + \sigma_z^{(1)}\sigma_z^{(4)} + \sigma_z^{(2)}\sigma_z^{(3)} +$$

$$\sigma_z^{(2)}\sigma_z^{(4)} + \sigma_z^{(3)}\sigma_z^{(4)}$$

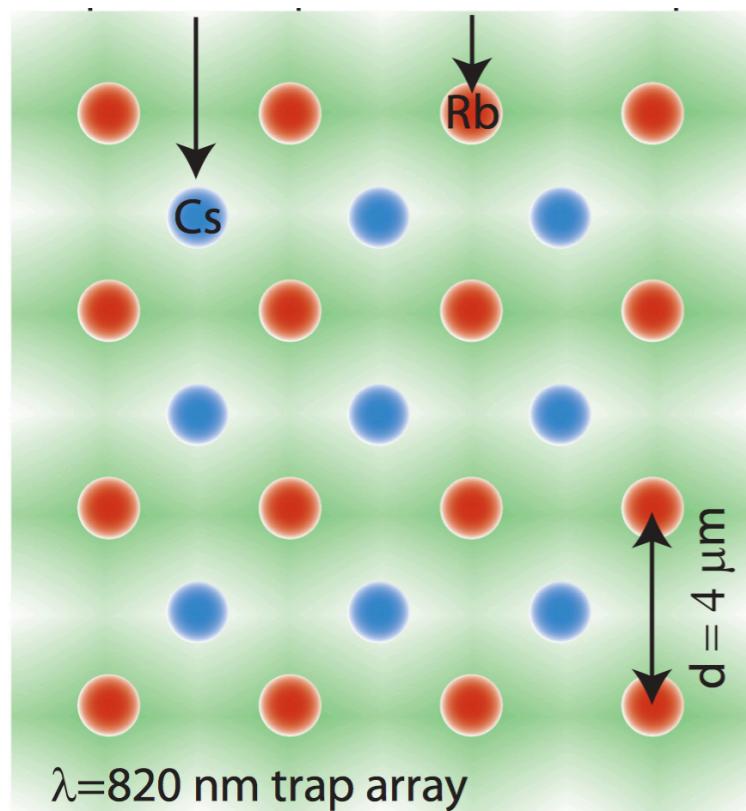


Error Robustness

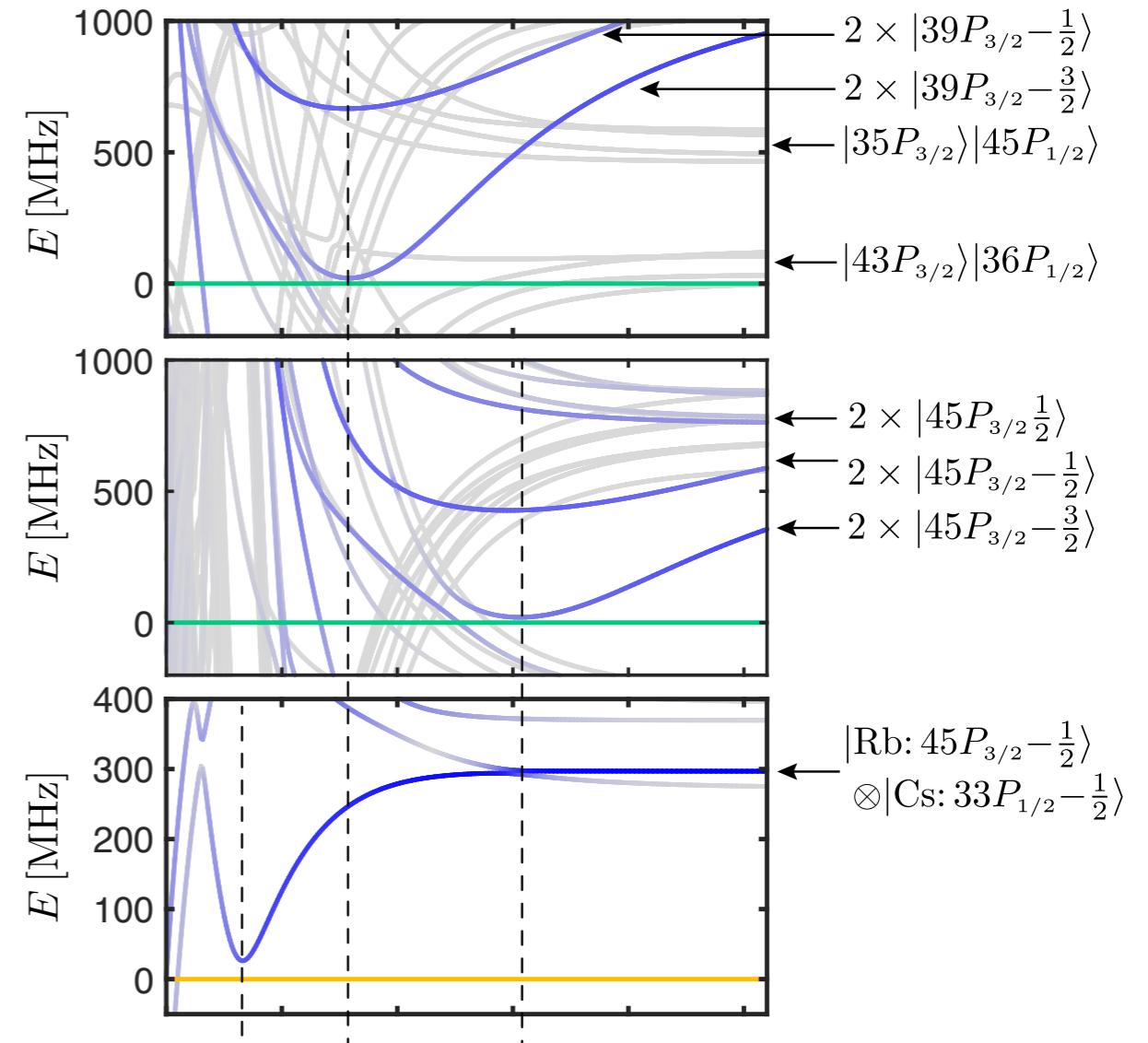
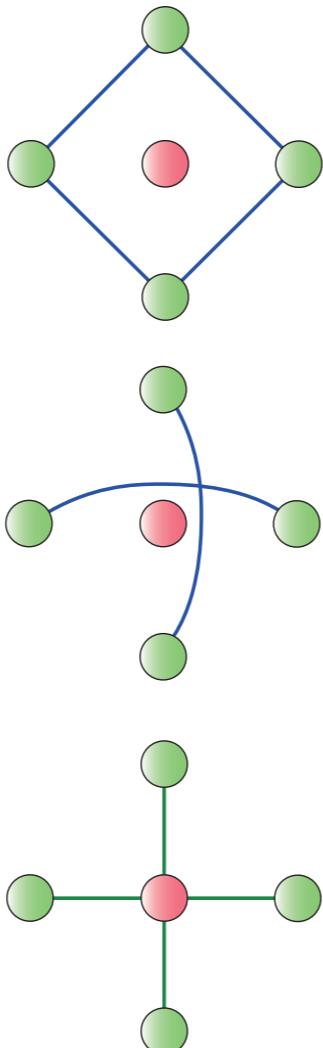


Neutral Atoms in Optical Lattices

A. Glätzle, R. van Bijnen, P. Zoller and WL, arXiv:1611.02594 (2016).



I. I. Beterov and M. Saffman,
Phys. Rev. A 92, 042710 (2015).

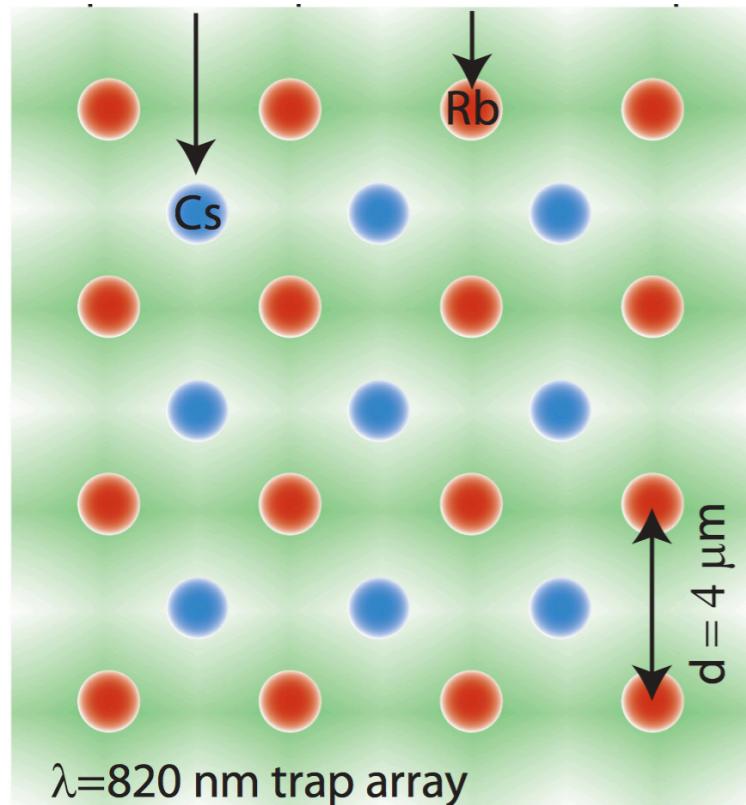


$$\hat{H} = \hat{H}_A^{(1)} \otimes \hat{I} + \hat{I} \otimes \hat{H}_A^{(2)} + \hat{V}_{dd}$$

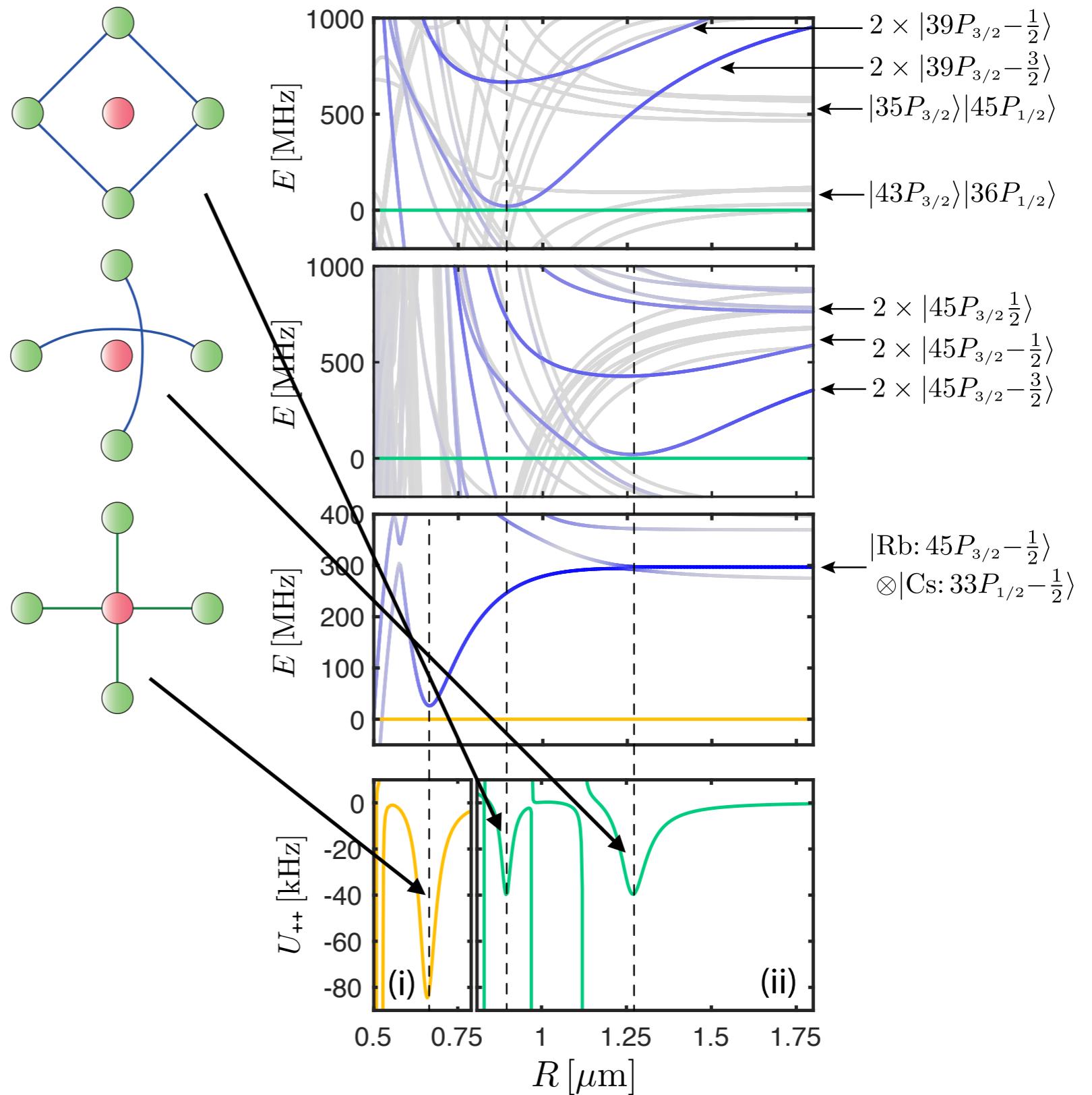
R. M. W. van Bijnen and T. Pohl
Phys. Rev. Lett. 114, 243002 (2015)

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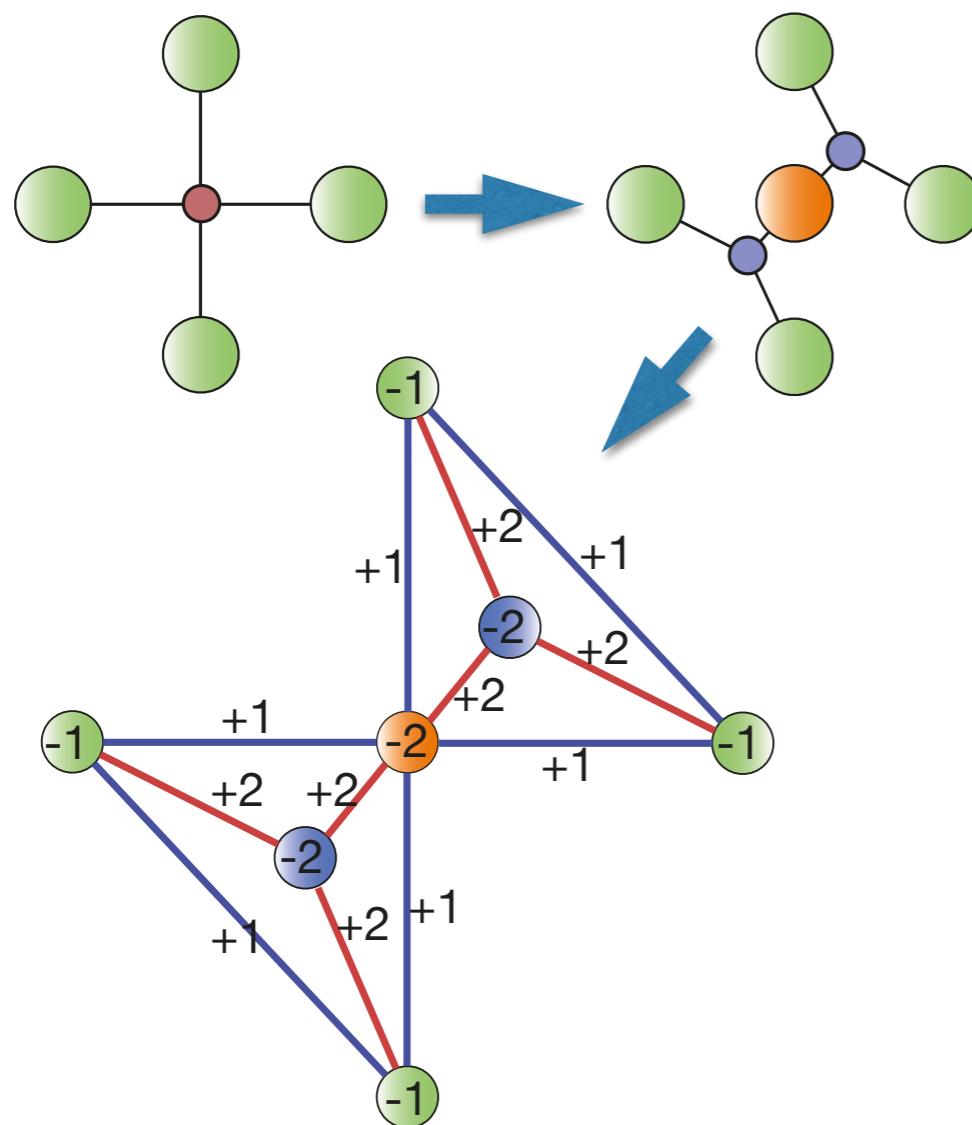


I. I. Beterov and M. Saffman,
Phys. Rev. A 92, 042710 (2015).

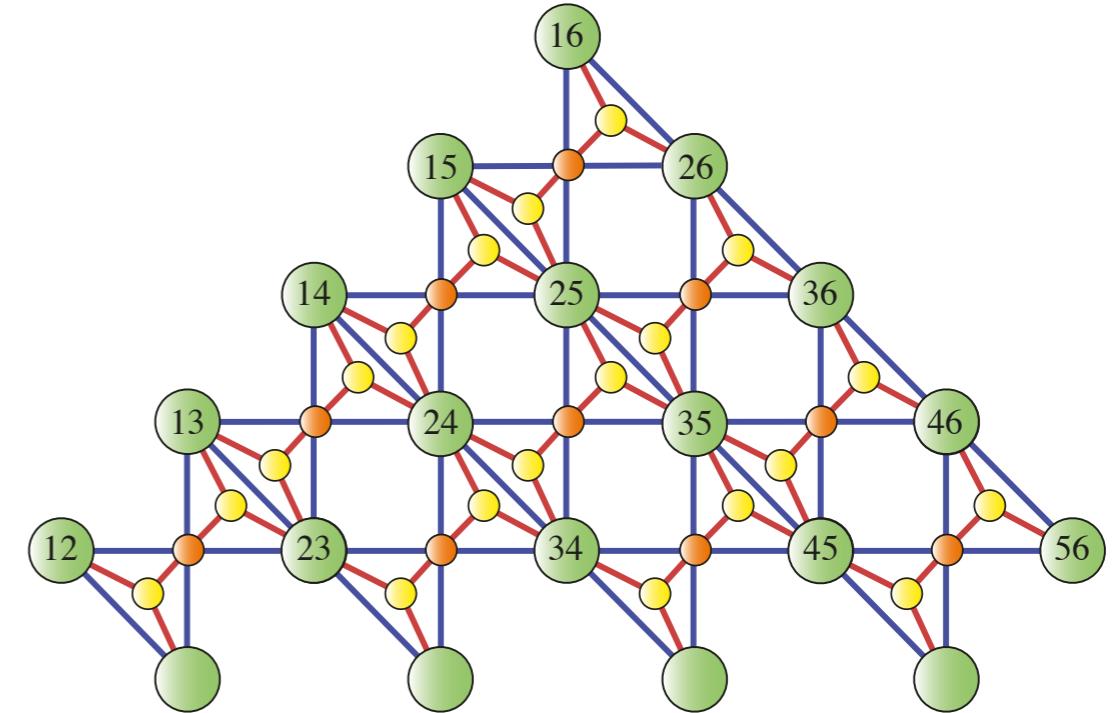


Transmon Implementations

Martin Leib, P. Zoller, WL, arXiv:1604.02359(2016).



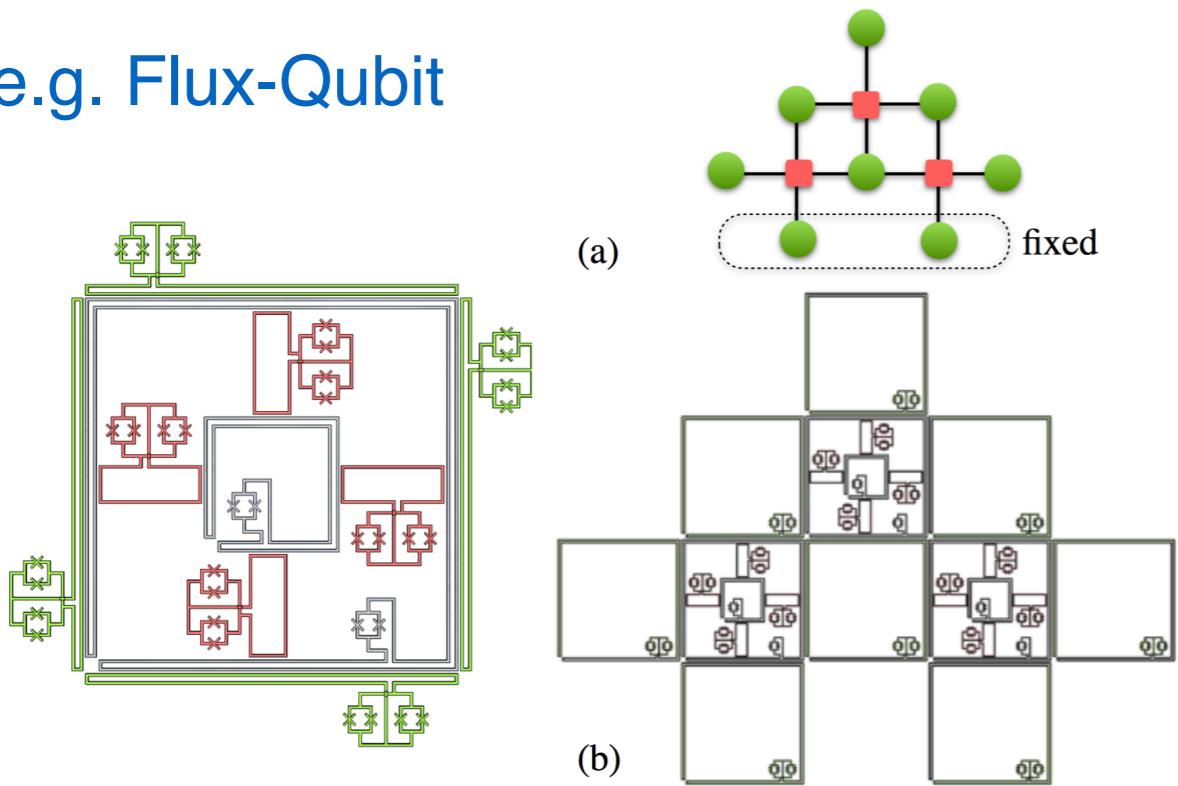
$$\sigma_z^{(1)} \sigma_z^{(2)} \sigma_z^{(3)} \rightarrow \sigma_z^{(1)} \sigma_z^{(2)} + \sigma_z^{(2)} \sigma_z^{(3)} + \sigma_z^{(3)} \sigma_z^{(1)} + \sum_{i=1}^3 [2\sigma_z^{(i)} \sigma_z^{(a)} - \sigma_z^{(i)}] - 2\sigma_z^{(a)}$$



- rotating frame
- only **pair interactions**
- **no crossings**
- all **local fields** have the **same sign**
- all **interactions** have the **same sign**
- programming [0,2]

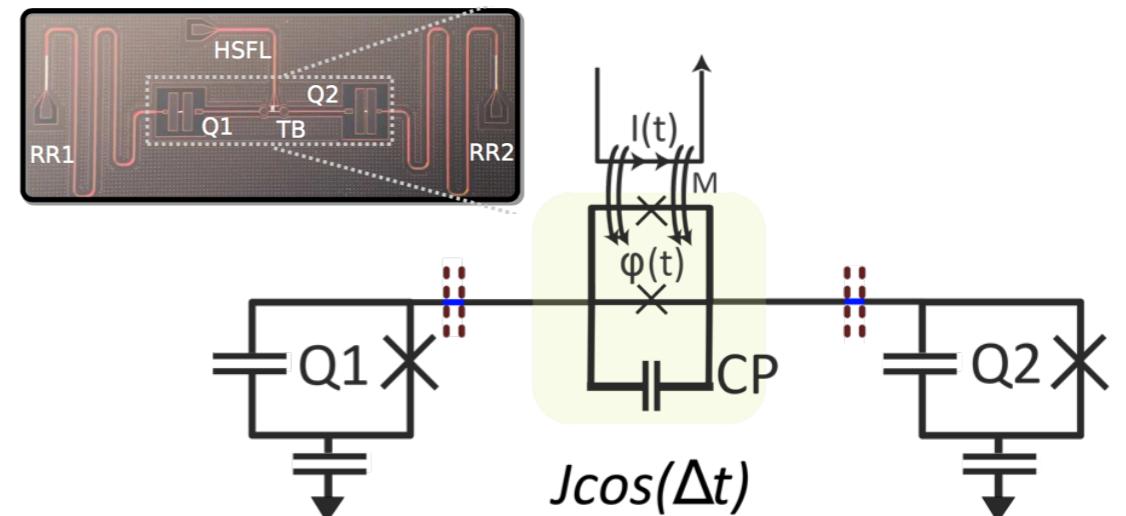
Quantum Circuit for LHZ

e.g. Flux-Qubit



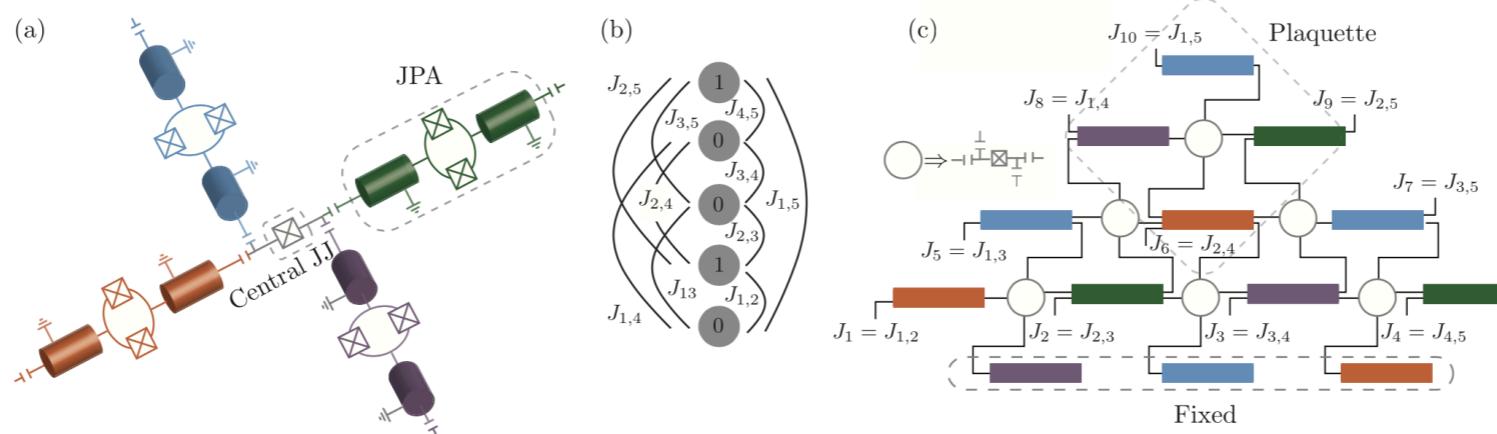
Nicholas Chancellor, Stefan Zohren, Paul A. Warburton
arXiv:1603:09521 (2016).

e.g. Driven fixed frequency Transmons



D. C. McKay, S. Filipp, A. Mezzacapo, F. Solgun,
J. Chow, and J. M. Gambetta. arXiv:1604.03076 (2016).

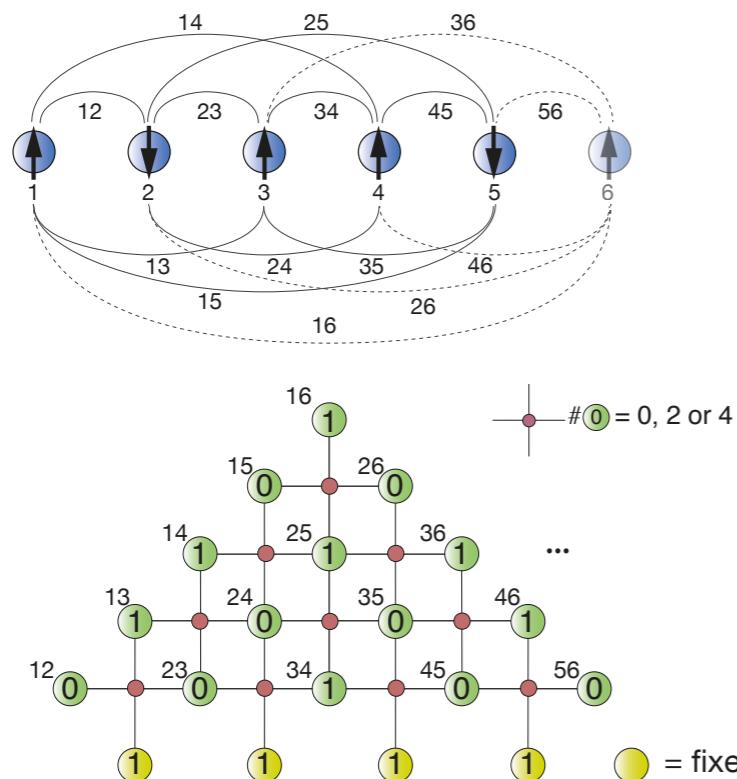
e.g. Driven Kerr-nonlinearities



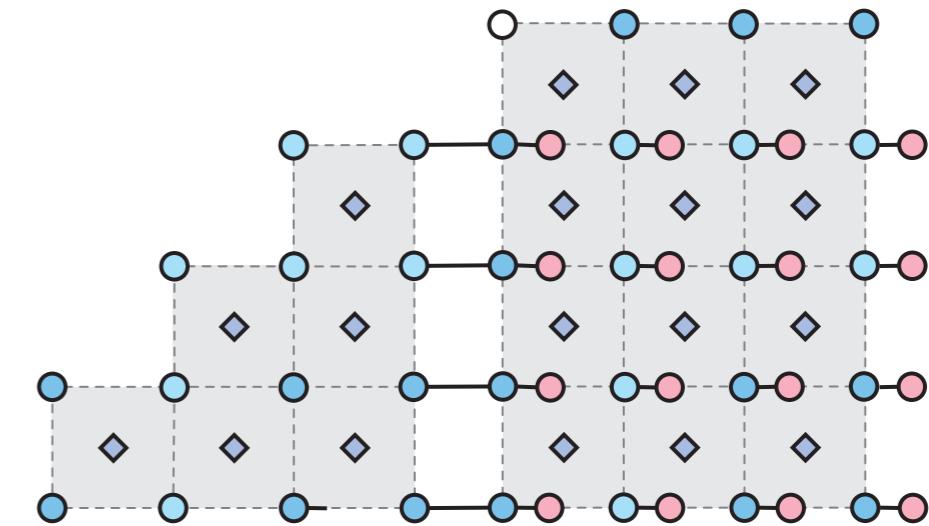
Shruti Puri, Christian Kraglund Andersen, Arne L. Grimsmo, Alexandre Blais, arXiv:1609.07117 (2016).

Outlook

Infinite range spin-glass vs.
finite range lattice gauge

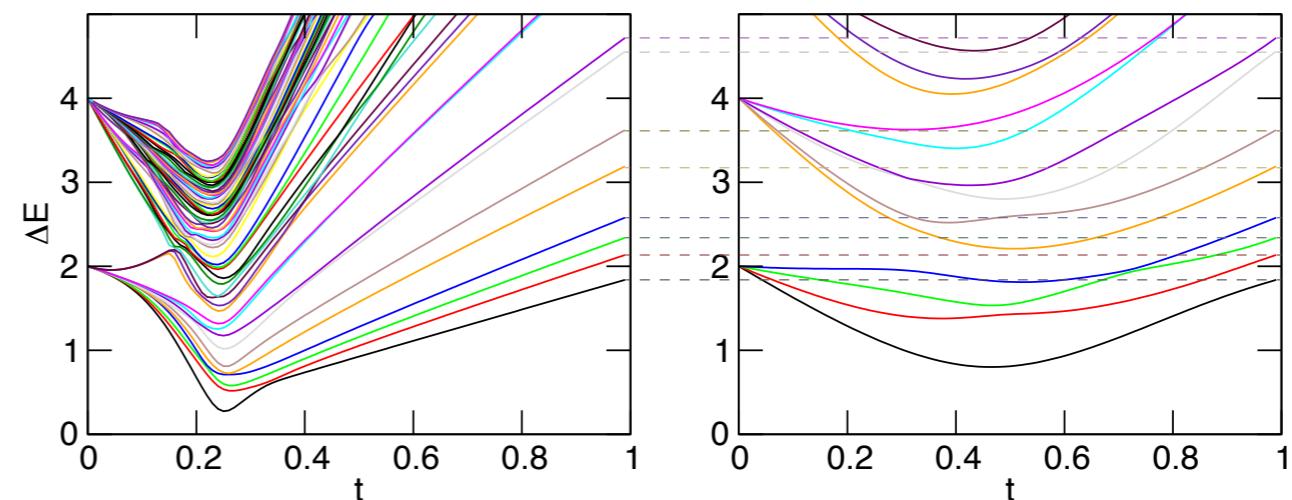


Quantum Machine Learning

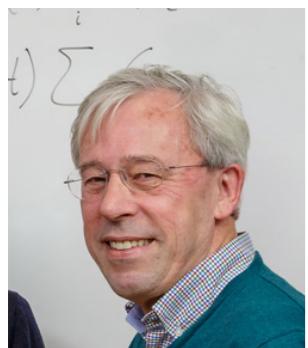


Dynamical Weights

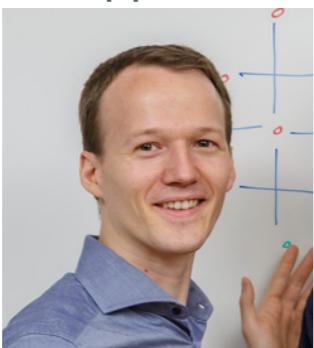
Coherent Annealing



Peter Zoller



Philipp Hauke



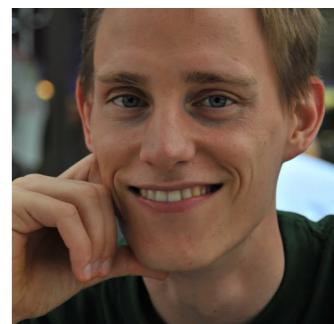
Alex Glätzle



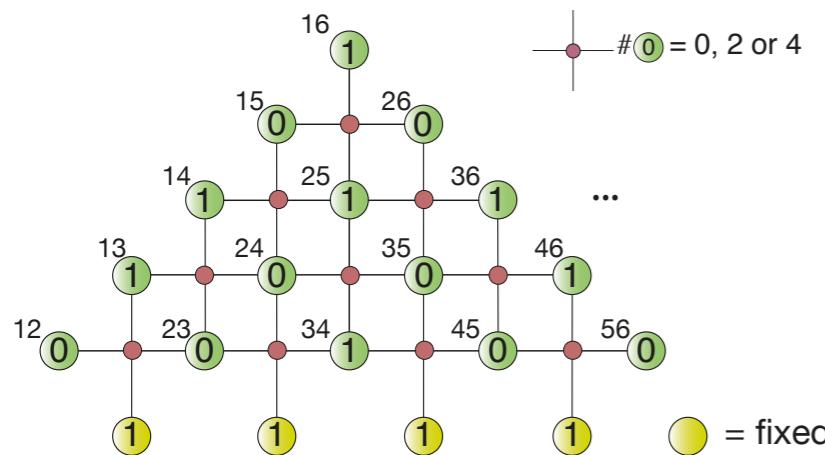
Rick van Bijnen



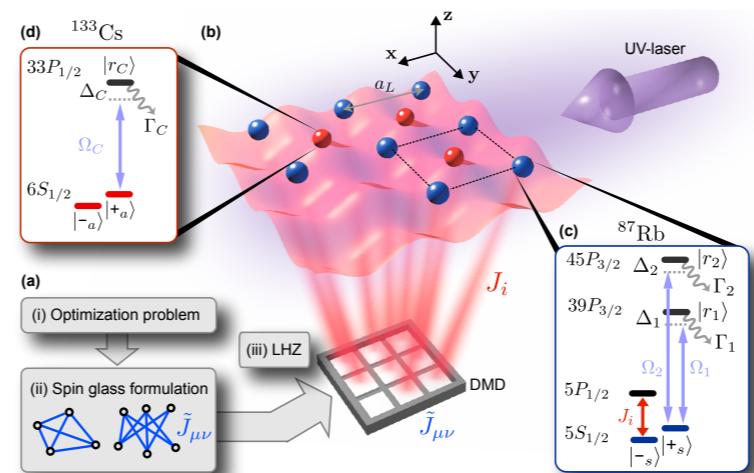
Martin Leib



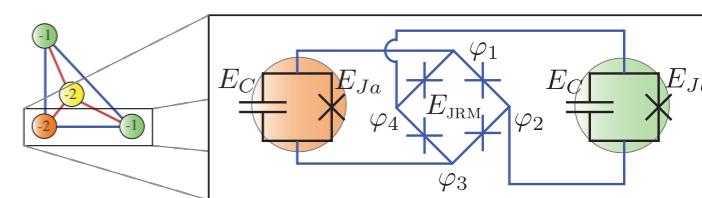
Science Advances 1, 1500838 (2015).



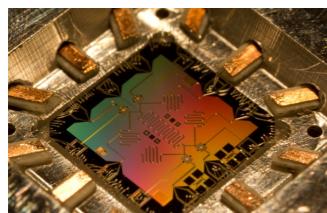
arXiv:1611.02594 (2016).



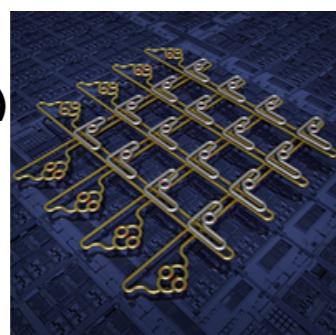
arXiv:1611.02594 (2016).



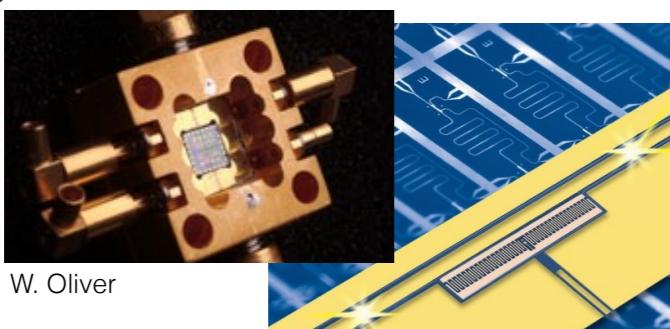
Superconducting Qubits (Transmons)



J. Martinis

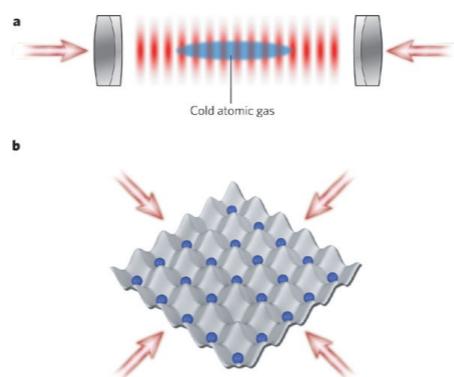


S. Boixo



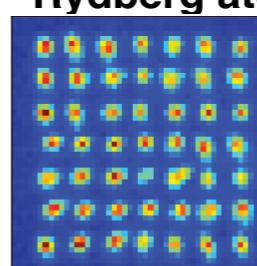
W. Oliver

Ultracold atoms in optical lattices



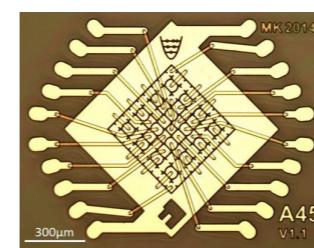
I. Bloch et al. *Nature* 453, 016

Rydberg atoms



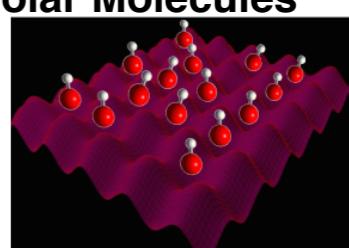
Saffman, Madison

Ions in surface traps



Blatt.

Polar Molecules



J. Ye.

A. Wallraff