Quantized magnetization density in Floquet systems

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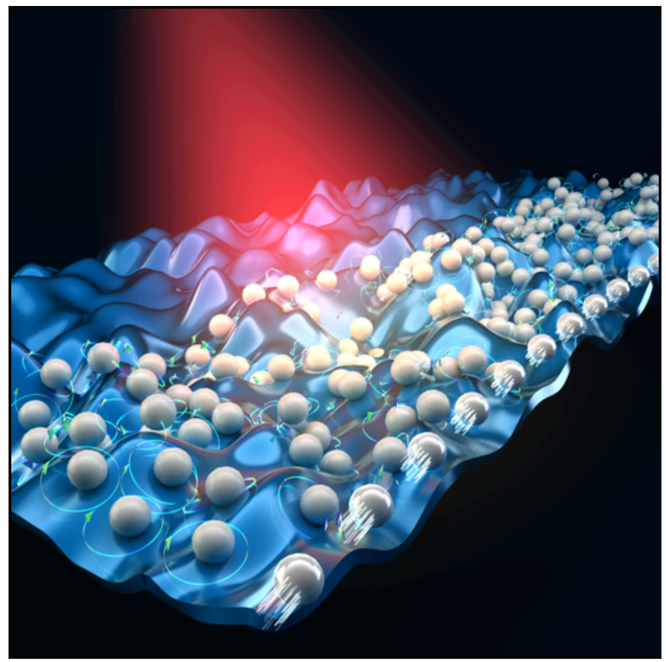
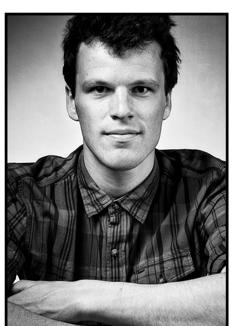




In collaboration with:
Frederik Nathan, Netanel Lindner, Erez Berg, and Gil Refael

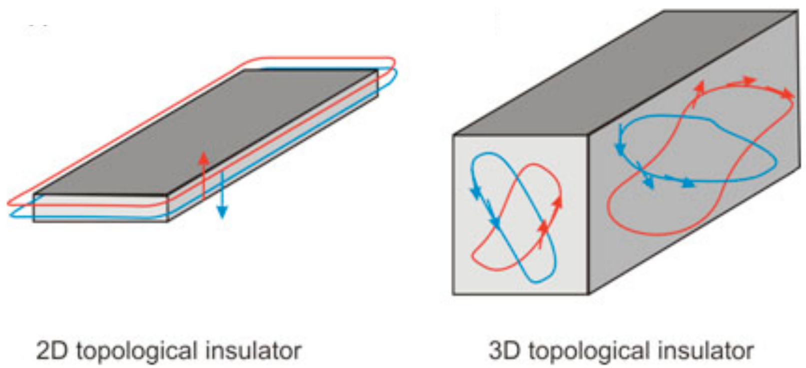
F. Nathan and MR, New Journal of Physics **17**, 125014 (2015).

F. Nathan, MR, N. H. Lindner, E. Berg, and G. Refael, arXiv:1610.03590 (2016).

Advances of the past decade bring new challenges, new tools

Theory

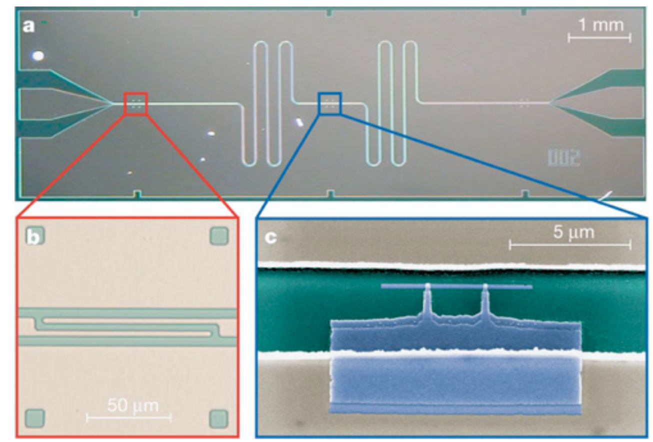
New phases, topological phenomena



M. Z. Hasan, SSRL Science Highlight, March 2009

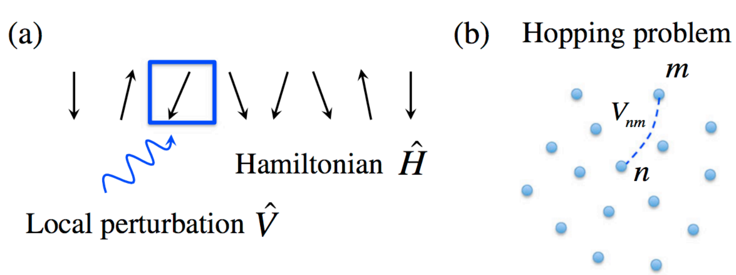
Experiment

Quantum control: MWs, lasers



A. Wallraff. *et al.*, Nature **431**, 162 (2004)

Quantum dynamics, thermalization



M. Serbyn, Z. Papic, and D. Abanin, Phys. Rev. X **5**, 041047 (2015).

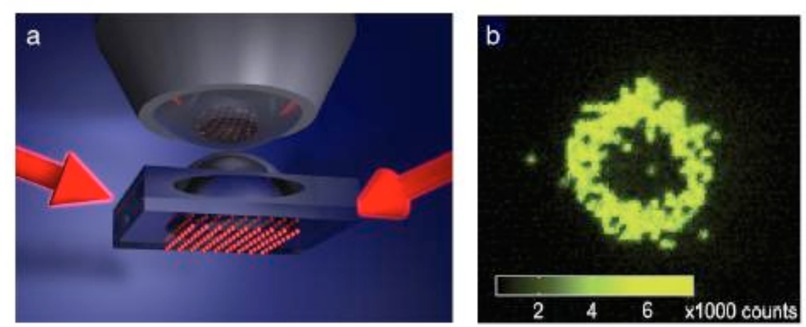


Image from `http://greiner.physics.harvard.edu`

What new types of robust many-body phenomena are possible in periodically-driven systems?

Past

$$\text{Pb} \rightarrow \text{Au}$$

Present

$$\text{GaAs} \rightarrow \text{HgTe?}$$

$$\rightarrow \quad \ldots \quad ?$$

The Plan

I. Micromotion and “Floquet-only” topology

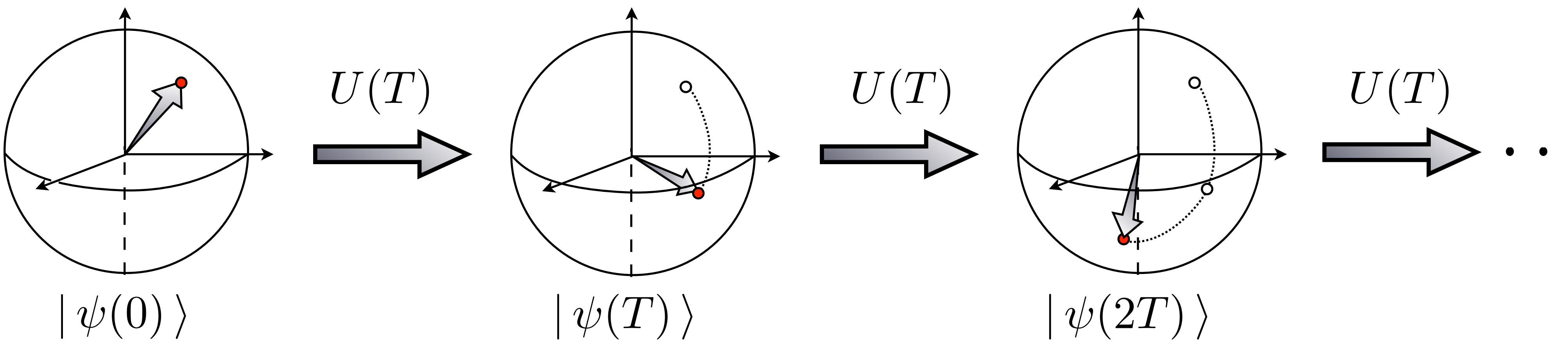
II. Quantized magnetization density in fully-localized Floquet systems

Part I

Micromotion and “Floquet-only” topology

Quasi-energy is conserved for system with discrete time translation symmetry

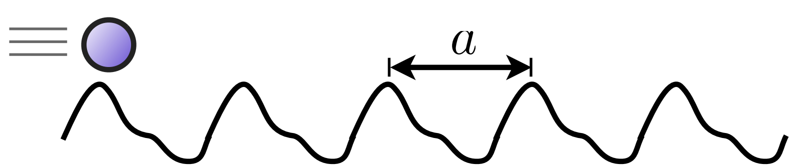
$$U(T) = \mathcal{T} e^{-i \int_0^T H(t) dt} \qquad H(t+T) = H(t)$$



$$U(T) | \psi_n \rangle = e^{-i \varepsilon_n T} | \psi_n \rangle$$

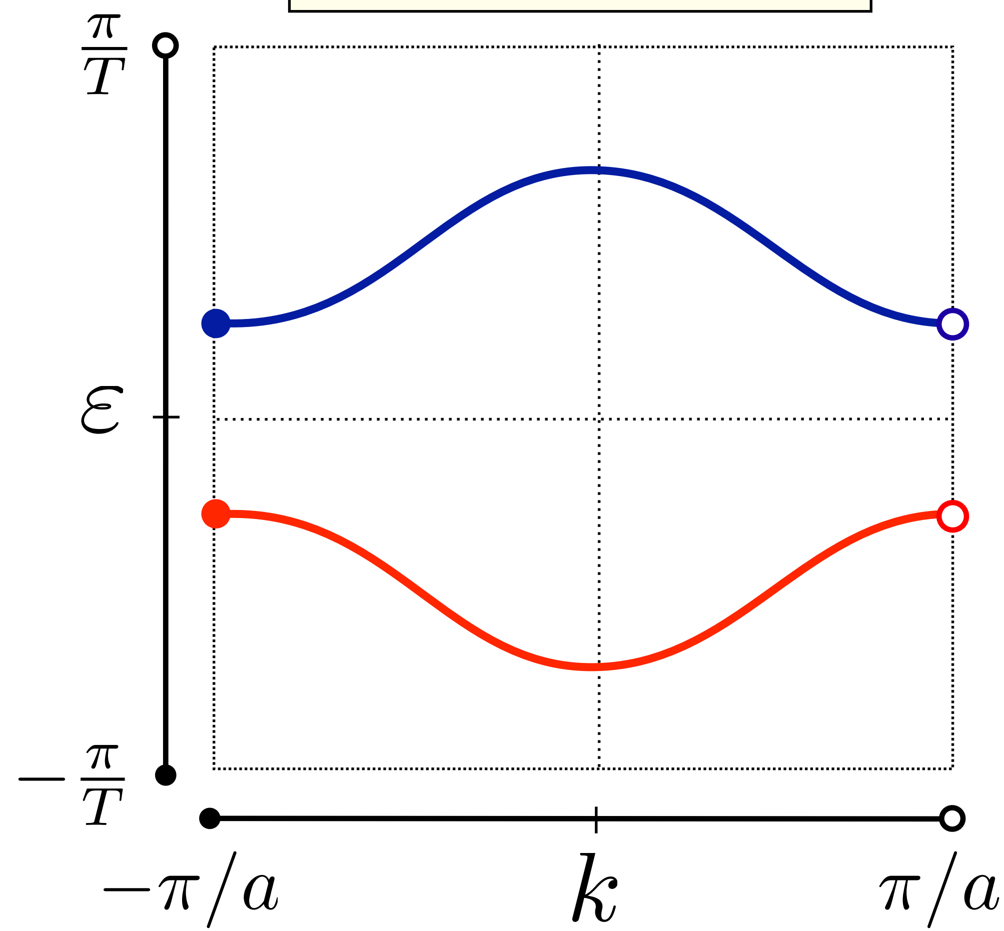
Eigenvalue invariant under $\varepsilon_n \to \varepsilon_n + 2\pi N/T$: quasi-energy lives on a circle

On a lattice find Floquet bands, similar to static system



$$V(x+a) = V(x)$$

$$U(T) \equiv e^{-iH_{\rm eff}T}$$

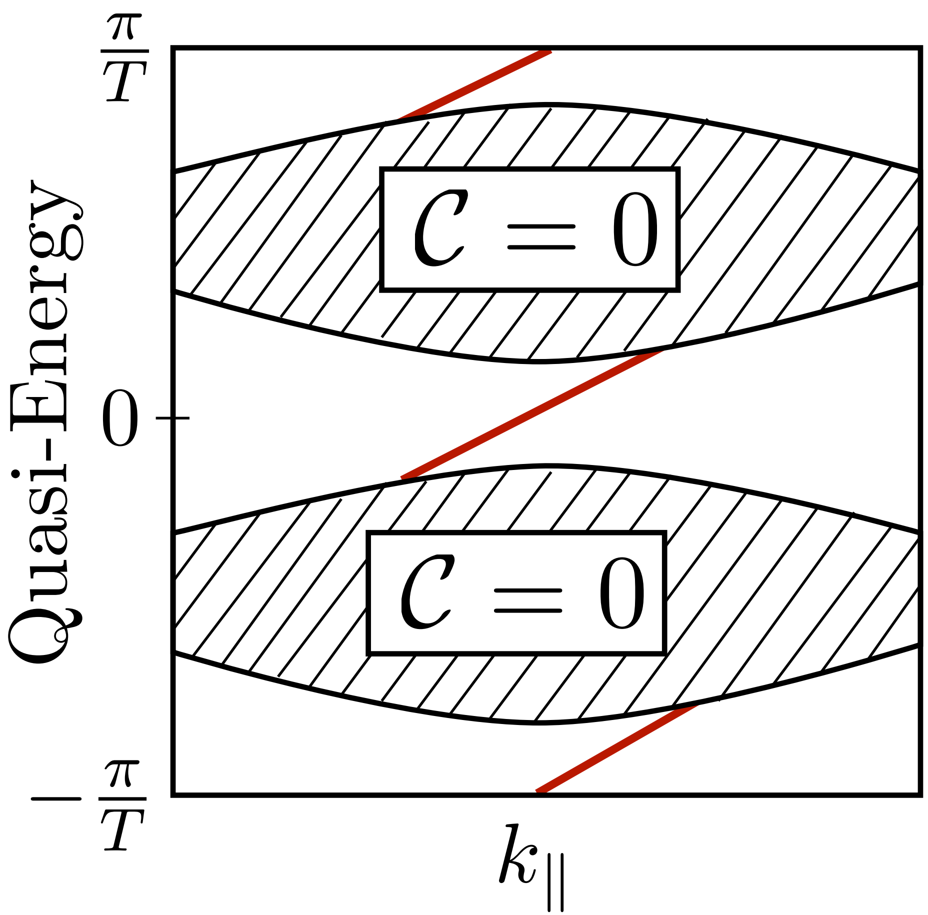


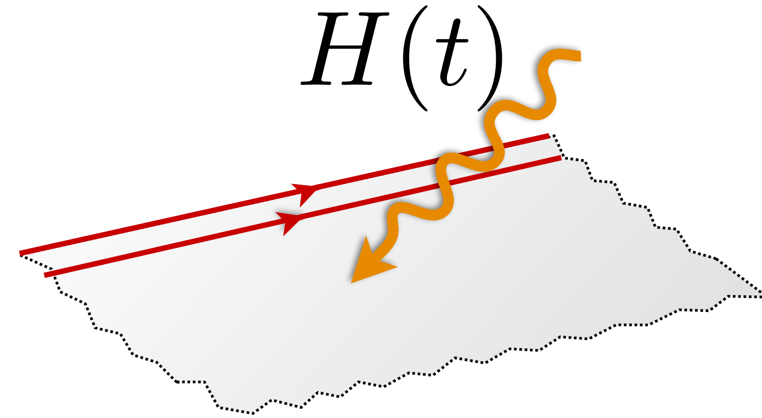
Suggests analogues of topological phenomena from static systems in driven systems

T. Kitagawa, E. Berg, MR, and E. A. Demler, Phys. Rev. B 82, 235114 (2010).

T. Oka and H. Aoki, Phys. Rev. B **79**, 081406 (2009). N. Lindner, G. Refael, and V. Galitski, Nature Physics **7**, 490 (2011).

New features arise for topology, bulk/boundary correspondence in driven systems





Chiral edge modes for $\mathcal{C} = 0$ bands

T. Kitagawa, E. Berg, MR, and E. A. Demler, Phys. Rev. B 82, 235114 (2010).

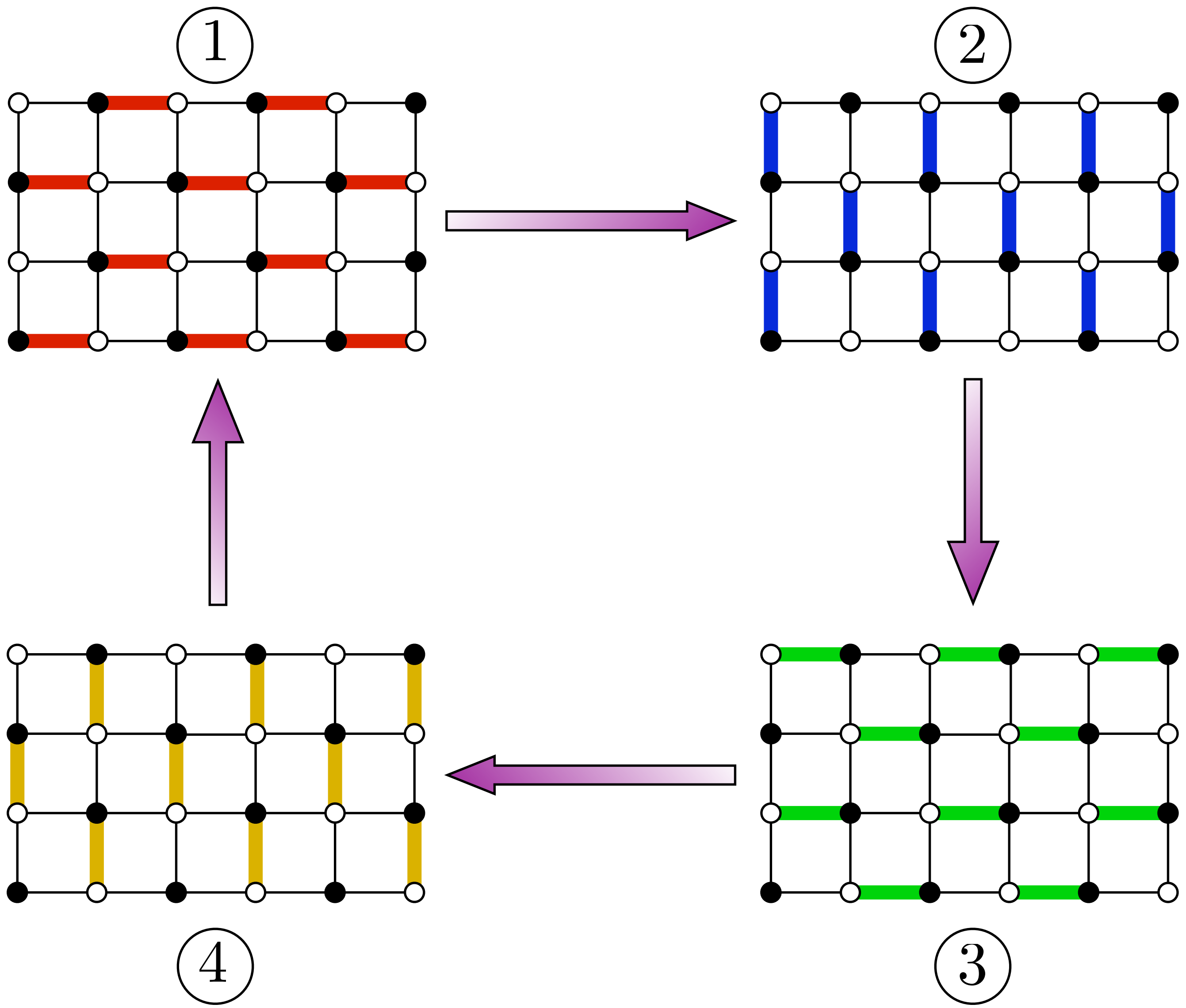
MR, N. Lindner, E. Berg, and M. Levin, Phys. Rev. X **3**, 031005 (2013).

Other examples (Floquet-Majorana, π spin glass, SPTs, …):

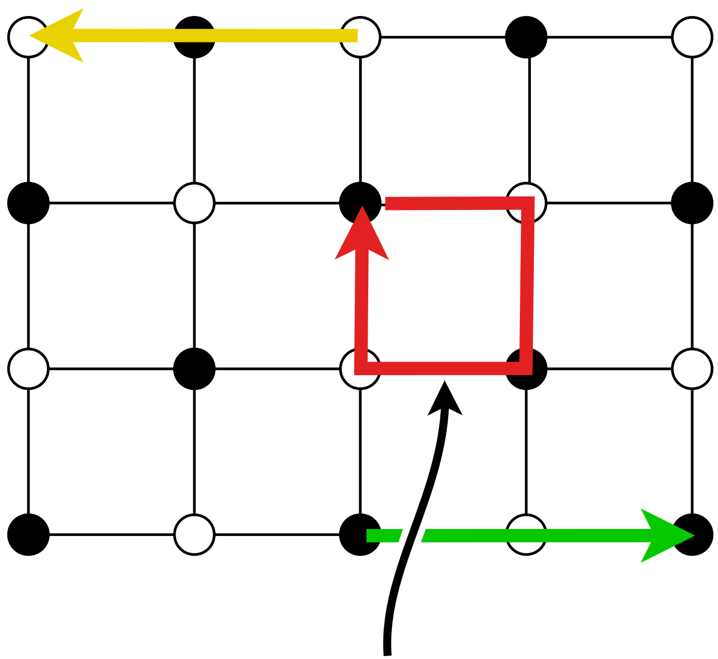
L. Jiang *et al.*, Phys. Rev. Lett. **106**, 220402 (2011). V. Khemani *et al.*, Phys. Rev. Lett. **116**, 250401 (2016).

von Keyserlingk and Sondhi; Potter, Morimoto and Vishwanath; Else and Nayak; Roy and Harper; …

New phase illustrated by model with modulated hoppings

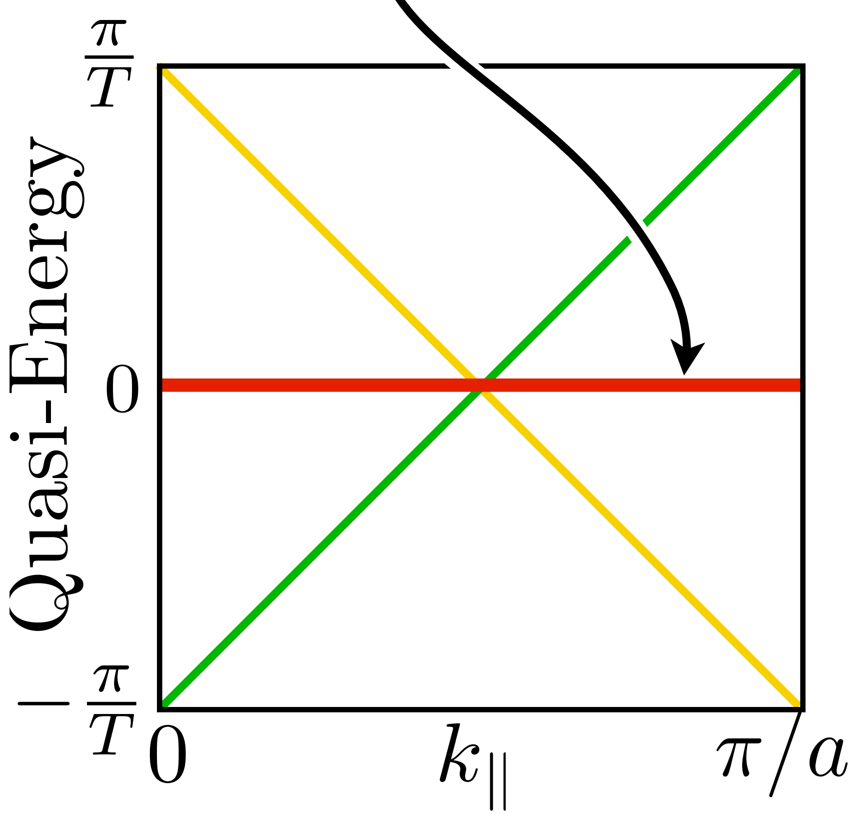


Bulk evolution trivial, chiral modes propagate along edges



Particle returns to initial position after cycle

Bulk Floquet operator is trivial: $U(T) = \mathbb{1}$



MR, N. Lindner, E. Berg, and M. Levin, Phys. Rev. X **3**, 031005 (2013).

“Phase band” picture reveals what distinguishes topology in driven and non-driven systems

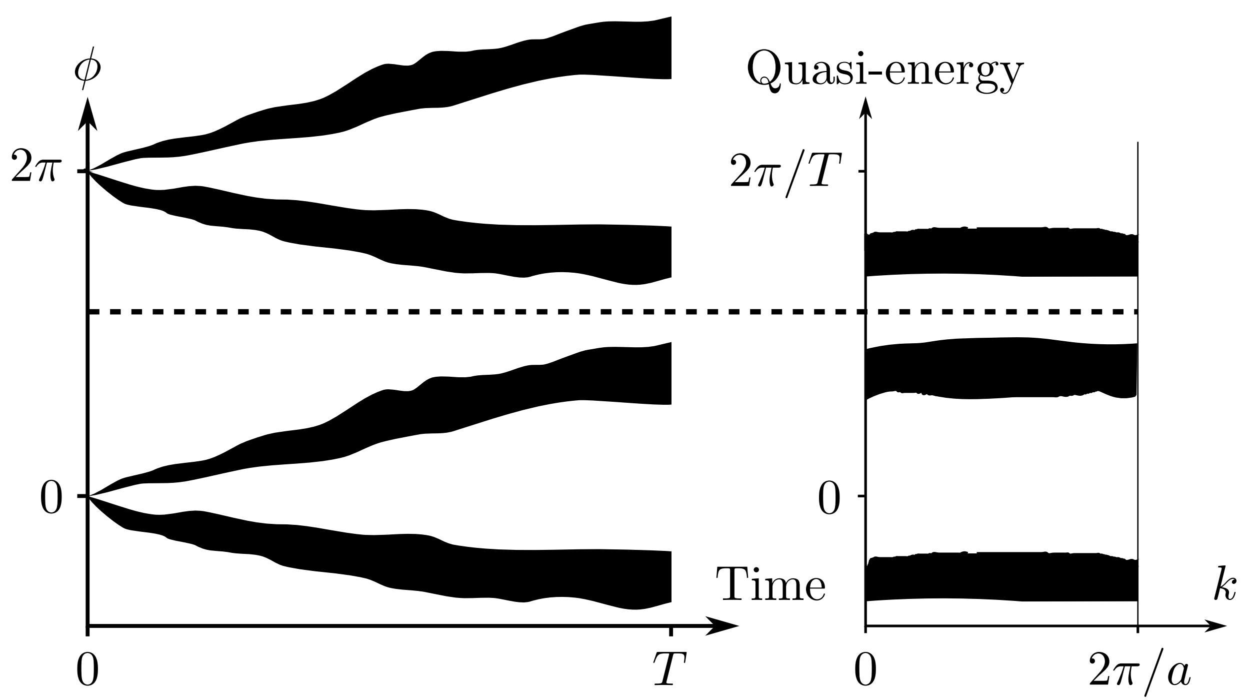
number of bands

$$U(\mathbf{k},t) = \sum_{n=1}^{N} P_n(\mathbf{k},t)e^{-i\phi_n(\mathbf{k},t)}$$

band projector

phase eigenvalue

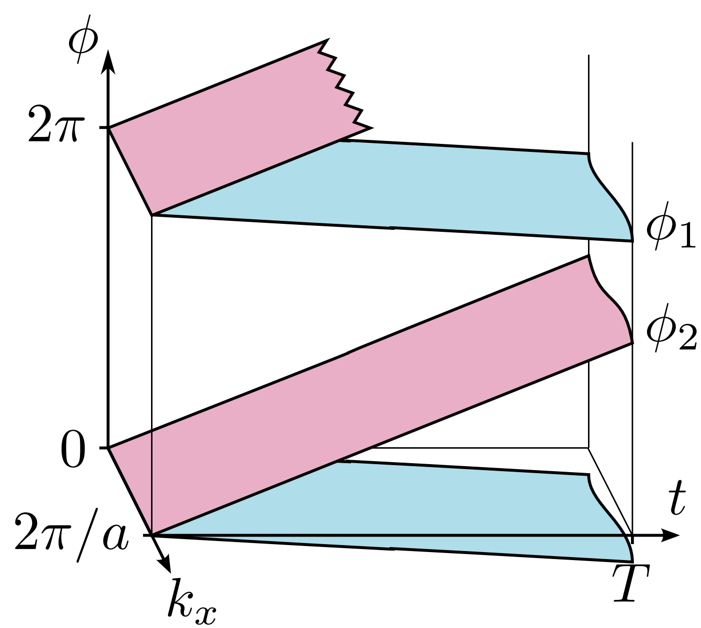
<u>Example: two bands</u>



In non-driven system, phases wind *linearly* in time

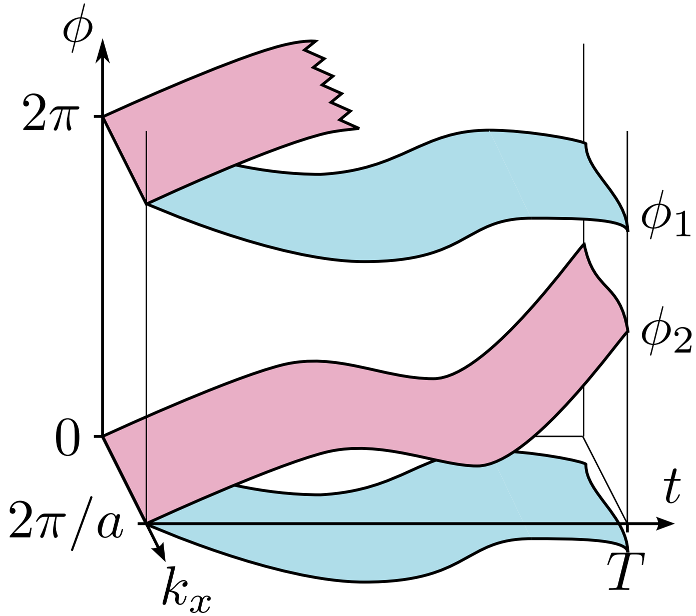
$$H|\,\psi_n\,\rangle = E_n|\,\psi_n\,\rangle, \quad U(t) = \sum_n e^{-iE_n t}|\,\psi_n\,\rangle\langle\,\psi_n\,|$$

<u>Example: two bands, one dimension</u>

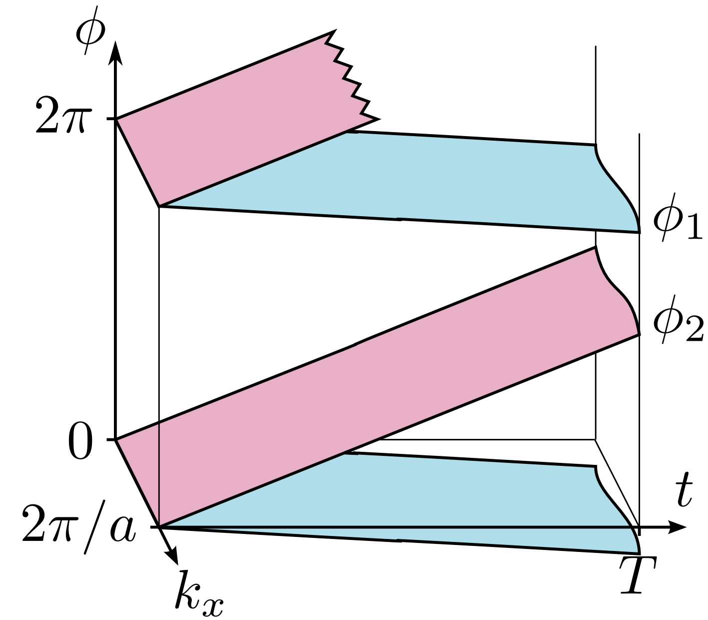


Topological equivalence: continuously adjust H(t) to "straighten" phase bands of driven system

Driven system evolution



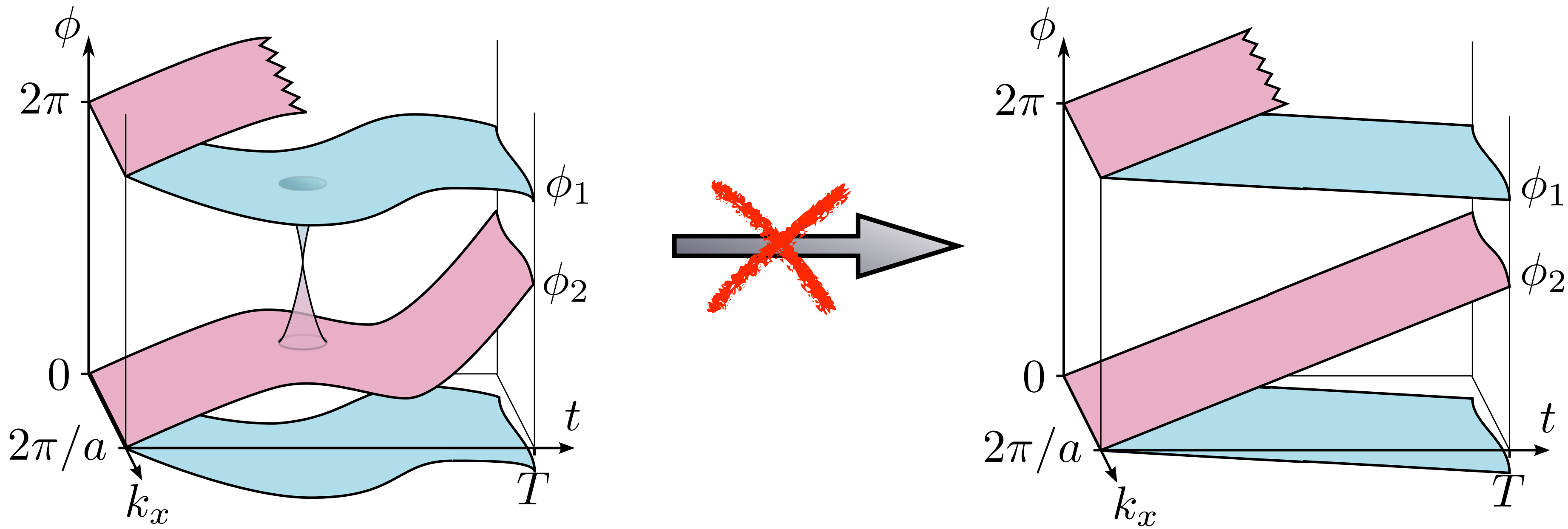
Equivalent non-driven evolution



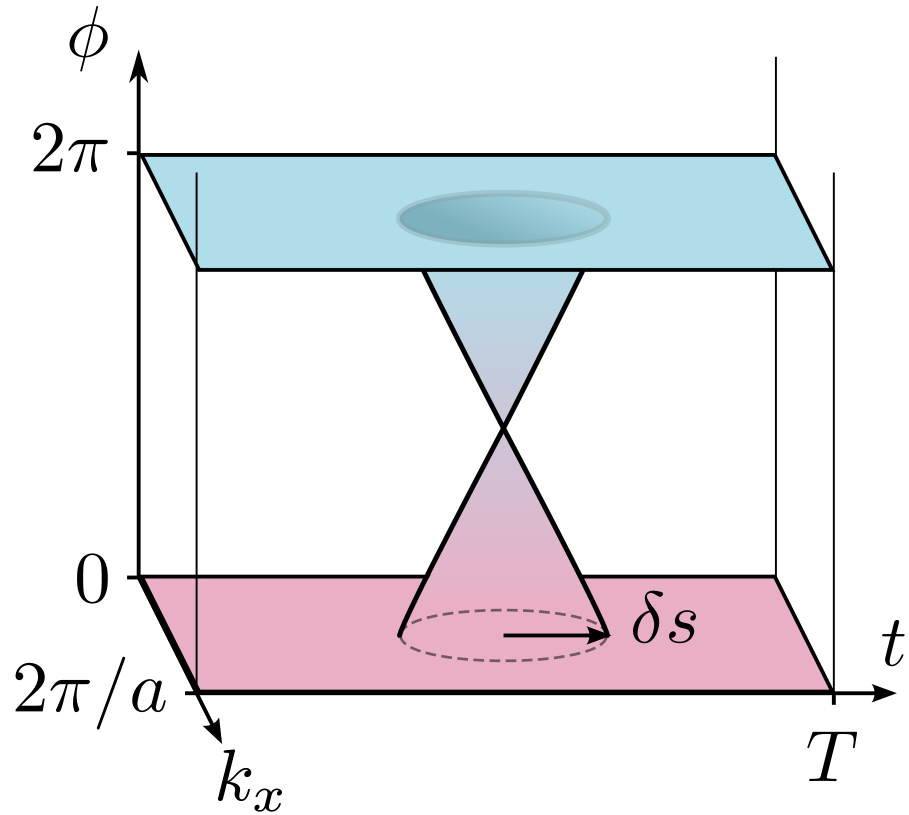
“Topological singularities” may break relation between driven and non-driven evolution

Driven system evolution

Equivalent non-driven evolution



Similar to Weyl nodes, topological singularities can be shifted but not removed by local perturbations



$$U(\mathbf{s}) \sim e^{-i(\mathbf{s}-\mathbf{s}_0)\cdot\boldsymbol{\sigma}}, \quad \mathbf{s} = (k_x, k_y, t)$$

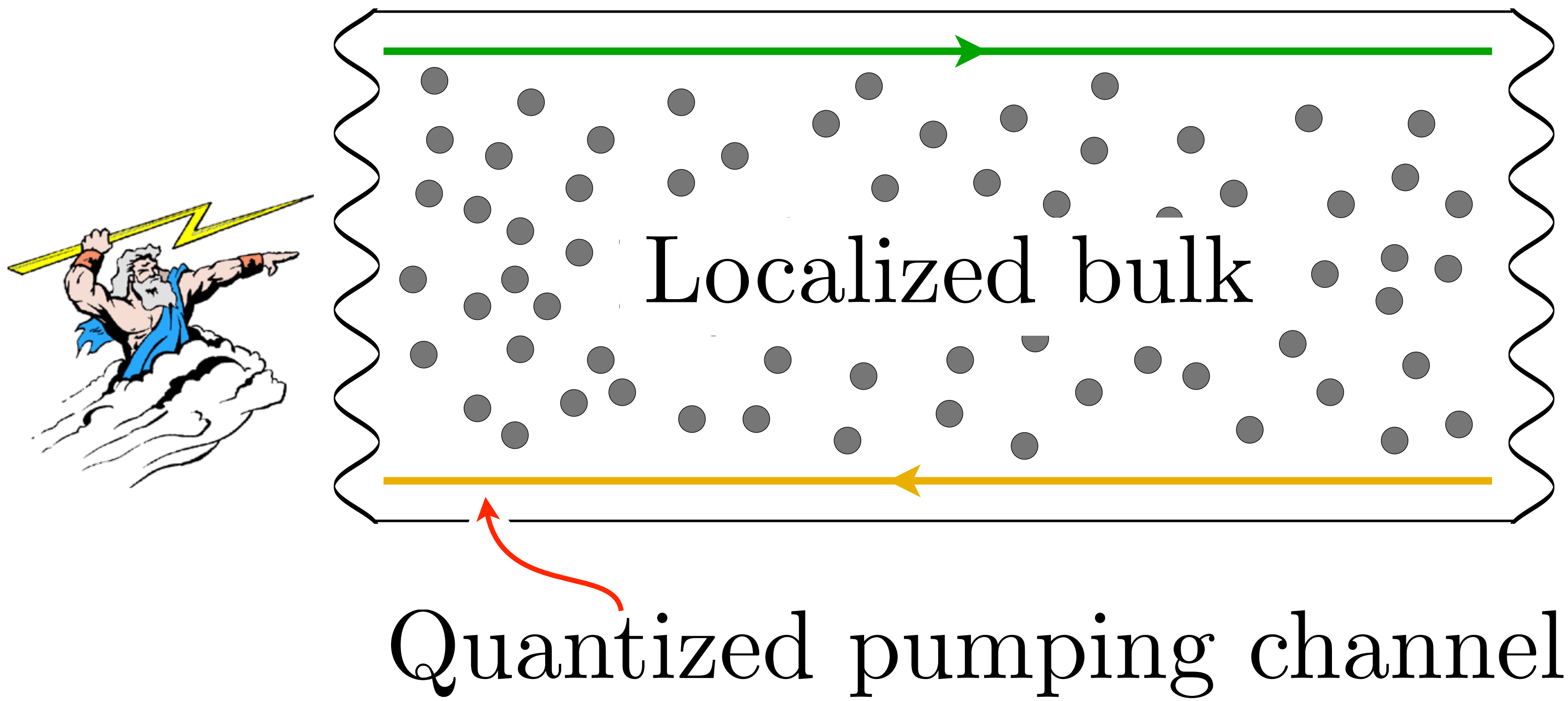
* Singularity-based classification captures topology of all FTIs and "anomalous" variants

F. Nathan and MR, New Journal of Physics **17**, 125014 (2015).

Part II

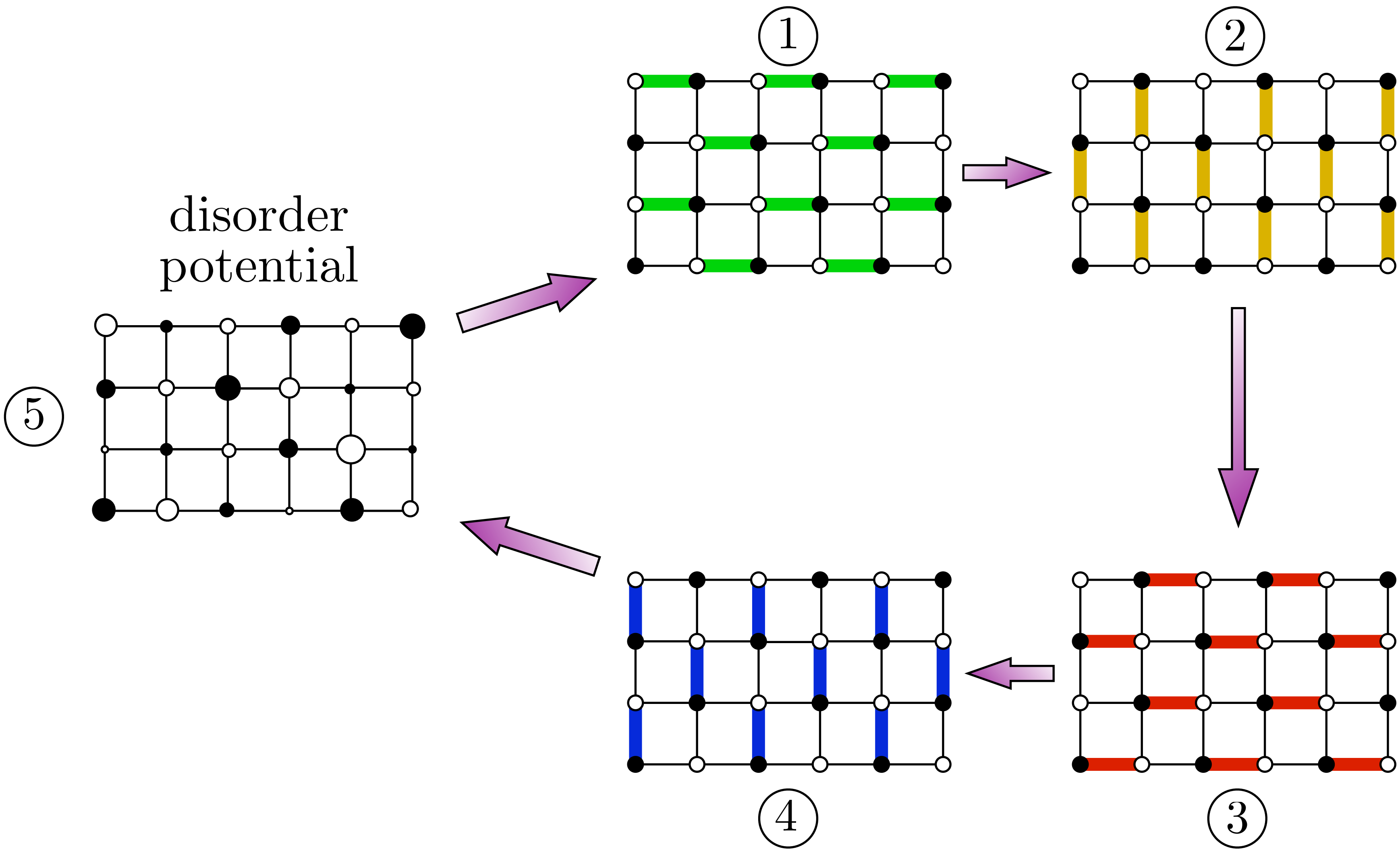
Quantized magnetization density in fully-localized Floquet systems

Anomalous Floquet-Anderson Insulator: fully localized bulk with propagating chiral edge states



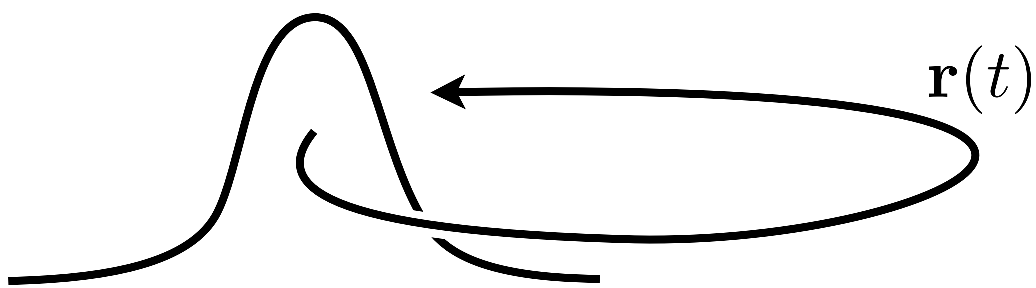
P. Titum, E. Berg, MR, G. Refael, and N. H. Lindner, Phys. Rev. X **6**, 021013 (2016)

Disorder localizes all bulk states



Magnetization characterizes micromotion within localized Floquet states

$$M(t) = \frac{1}{2}\mathbf{r}(t) \times \partial_t \mathbf{r}(t)$$

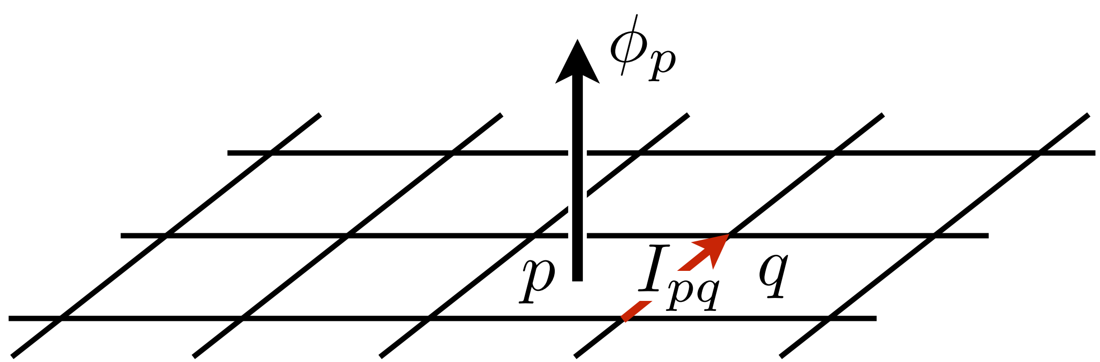


Alternative form:

$$M(t) = -dH(t)/dB$$

For stationary states, magnetization density satisfies lattice version of Ampere's law

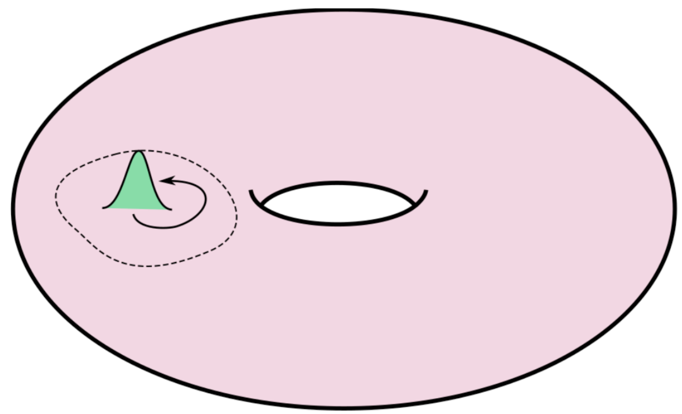
$$m_p(t) = -\frac{\partial H}{\partial \phi_p}, \quad \phi_p = \int_p d^2 r B(\mathbf{r})$$



$$\mathbf{j} = \nabla \times \mathbf{m} \quad \Longleftrightarrow \quad \langle I_{pq} \rangle_\tau = \langle m_p \rangle_\tau - \langle m_q \rangle_\tau$$

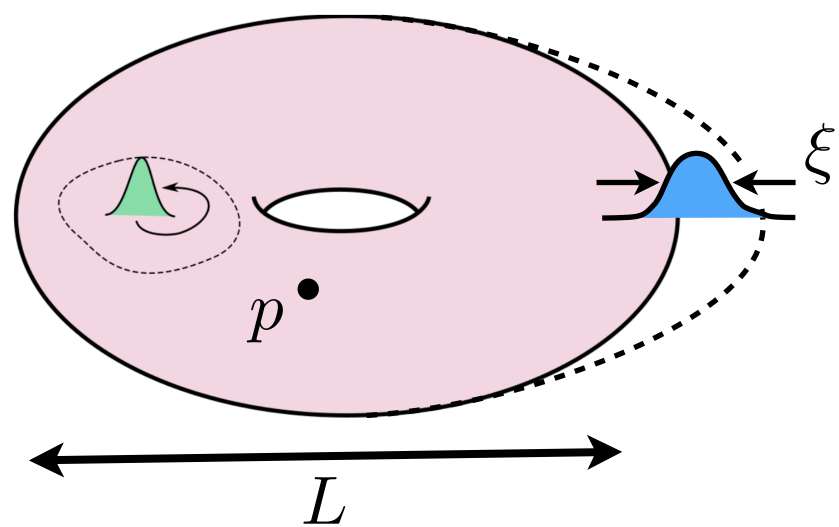
$$\langle A \rangle_\tau = \frac{1}{\tau} \int_0^\tau \langle \psi(t) | A(t) | \psi(t) \rangle$$

Magnetization density of fully localized, fully filled Floquet system is homogeneous



$$I_{pq} = 0 \;\Rightarrow\; m_p = \bar{m}$$

Because all states are localized, magnetization density exponentially insensitive to system size



$$\langle m_p \rangle = \bar{m}_\infty + \mathcal{O}(e^{-L/\xi})$$

Asymptotic value $\bar{m}_\infty$ is quantized, topological!

Useful identity:

$$\log|U(T)| = -i\int_0^T dt\,\mathrm{Tr}[H(t)]$$

Peierls phases off-diagonal, do not affect trace:

$$\sum_n \varepsilon_n(B_0) = \sum_n \varepsilon(0) + \frac{2\pi\nu}{T}$$

field of 1 flux quantum through system, $B_0 A = 2\pi$

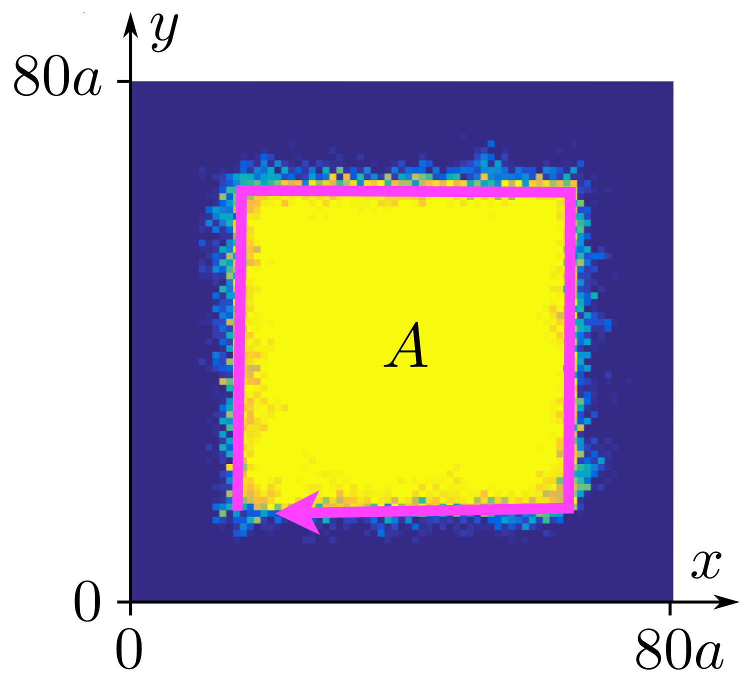
$[\varepsilon_n(B_0) - \varepsilon_n(0)]/B_0 + \mathcal{O}(1/A^2)$

Total magnetization $M = \bar{m}A = -\sum_n d\varepsilon_n/dB$, giving:

$$\bar{m}_\infty = \frac{\nu}{T}$$

$$|U(T)| = e^{-i\sum_n \varepsilon_n T}$$

Quantization of magnetization density implies quantized current flows around finite filled region



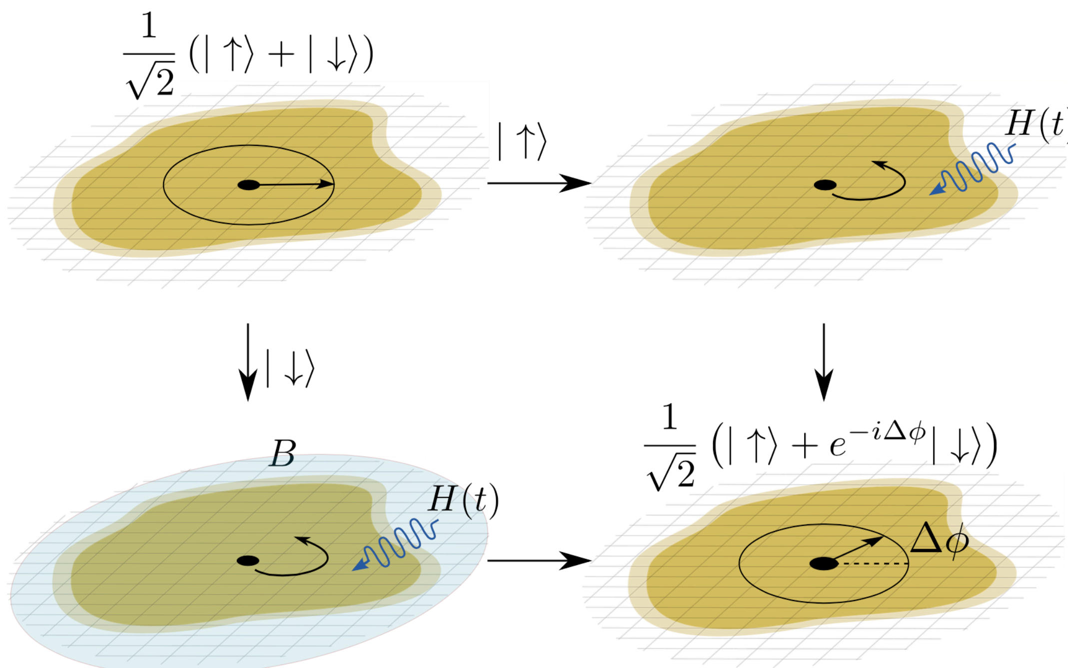
Ampere's law for long-time averaged current, magnetization:

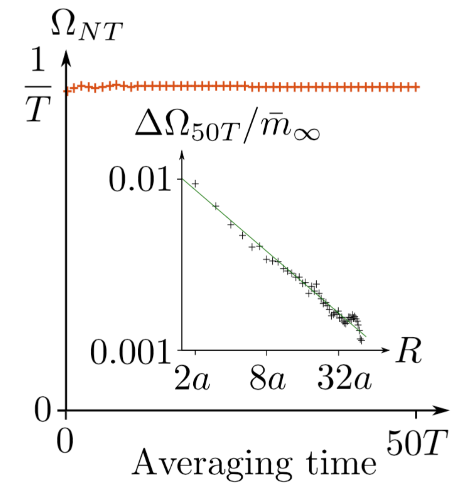
$$\langle\langle M \rangle\rangle = \langle\langle I \rangle\rangle A, \quad \langle\langle I \rangle\rangle = \frac{\bar{m}_\infty}{T}$$

$$\langle\langle O \rangle\rangle = \lim_{\tau \to \infty} \langle O \rangle_\tau$$

Quantized magnetization density can be observed for neutral atoms in interference experiment

Use atoms with two internal states, spin-dependent magnetic field





$$\Delta\phi = \langle M\rangle_T BT$$

Spin rotation angle proportional to magnetization density!

$$\langle \overline{\sigma}_y(NT)\rangle = \Omega_{NT} B a^2 NT$$

$$\Omega_{NT} = \langle\langle \overline{m} \rangle\rangle + \mathcal{O}\left(\frac{1}{NT}\right)$$

F. Nathan, MR, N. H. Lindner, E. Berg, and G. Refael, arXiv:1610.03590 (2016).

Summary and open questions

Periodic driving brings new features in topology, many-body dynamics

Micromotion is crucial in the "new" phenomena of Floquet systems

Outlook: interactions bring many new challenges, opportunities

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Support for this work provided by:

MARIE CURIE FELLOWSHIPS VILLUM FONDEN erc

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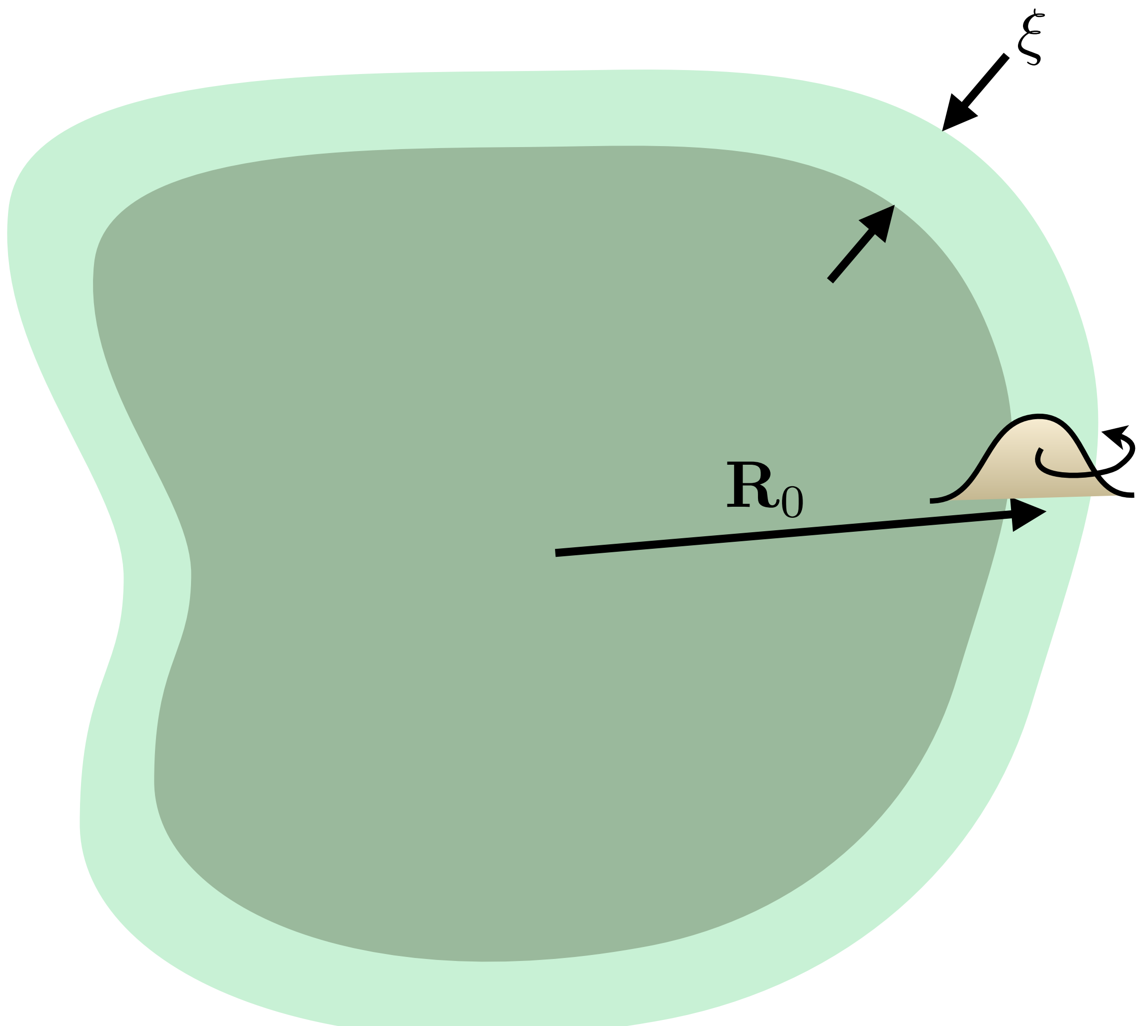
Transient correction to magnetization density measurement



$$M = \tfrac{1}{2}\mathbf{r} \times \dot{\mathbf{r}}$$

$$\mathbf{r}(t) = \mathbf{R}_0 + \delta\mathbf{r}$$

$$\langle M \rangle_{NT}^{(j)} \sim \tfrac{1}{2}\mathbf{R}_0 \times \langle \delta\dot{\mathbf{r}} \rangle_{NT}$$

$$\mathcal{O}\left(\frac{\xi}{NT}\right)$$

Number of particles in strip:

$$\mathcal{N}_{\mathrm{strip}} \sim R\xi/a^2$$

For random signs of net velocity:

$$\sum \langle M \rangle_{NT}^{(j)} \sim \sqrt{\frac{R\xi}{a^2}} \cdot \frac{R\xi}{NT}$$

Stability of AFAI demonstrated by perturbative expansion

$$e^{-iH_\lambda^{\text{eff}}T} = \mathcal{T}e^{-i\int_0^T dt\,[H_0(t)+\lambda D(t)]}$$

Static effective Hamiltonian (bulk)

$$H_\lambda^{\text{eff}} = H_{(0)}^{\text{eff}} + D_{\text{eff}}$$

Clean + staggered potential in 5th segment

$$H_0(t) = H_{\text{clean}}(t) + V(t)$$

Disorder (periodic or static)

$$D(t)$$

$$D_{\text{eff}} = \lambda D_{\text{eff}}^{(1)} + \lambda^2 D_{\text{eff}}^{(2)} + \cdots\,, \quad D_{\text{eff}}^{(n)} = \sum_{i,j}\Delta_{i,j}^{(n)} c_i^\dagger c_j$$

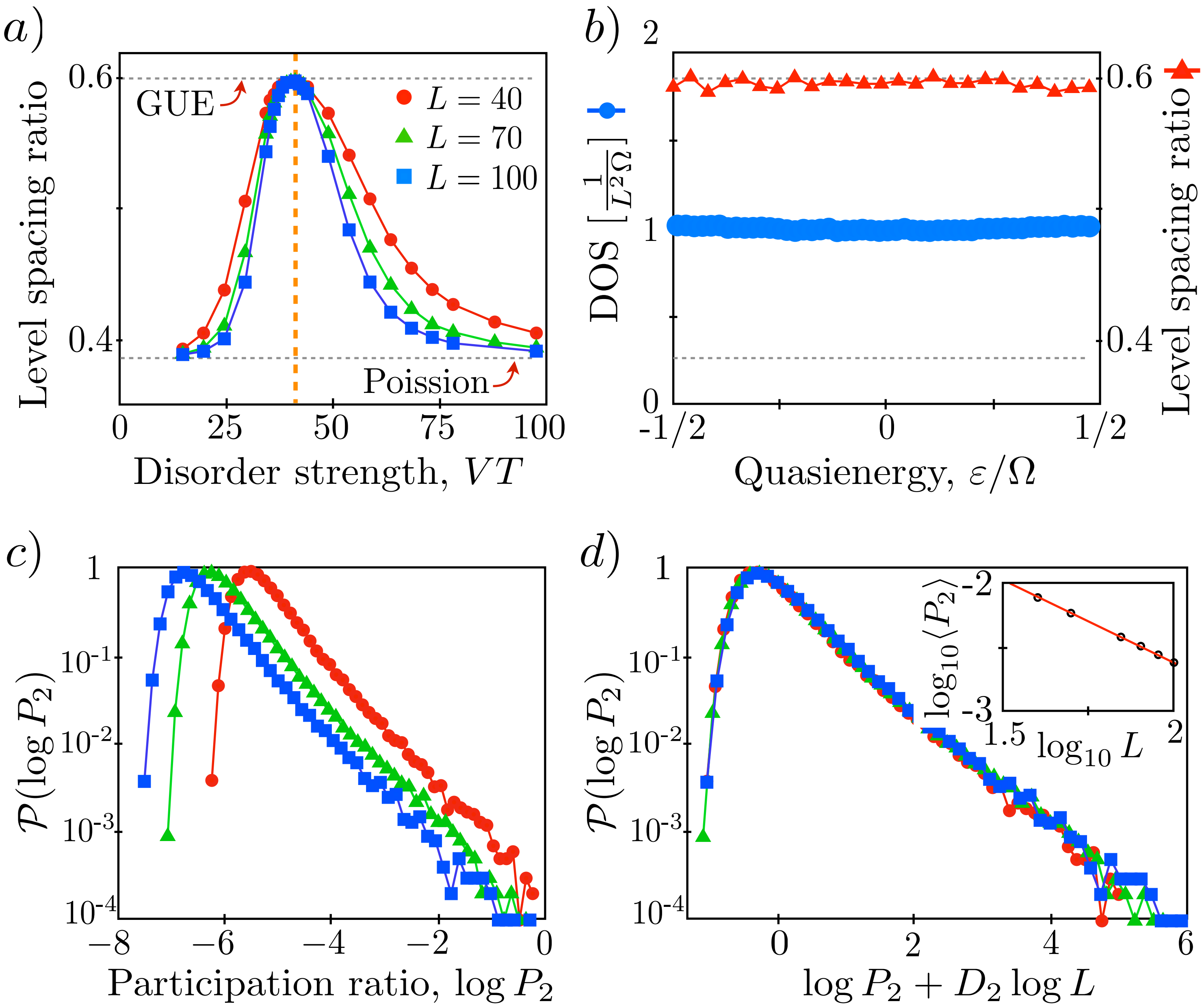
Expand both exponentials, using "interaction pictures" and match orders

$$\int_0^T dt\, D_{\text{eff}}^{(1)}(t) = \int_0^T dt\, \mathcal{D}(t)$$

$$\vdots$$

Find effective Hamiltonian with disorder, hopping decaying exponentially with r_{ij}

At strong disorder, find topological transition to trivial phase



Inverse participation ratio:

$$P_2 = \sum_{\mathbf{r}} |\psi(\mathbf{r})|^4$$

Finite size scaling:

$$\langle P_2 \rangle \sim L^{-D_2}$$

Fractal dimension from fit:

$$D_2 \approx 1.3$$

For integer quantum Hall case, see:

A. W. W. Ludwig, M. P. A. Fisher, R. Shankar, G. Grinstein, Phys. Rev. B **50**, 7526 (1994).