Anomalous Hall Effect in a Multiband Chiral Superconductor (e.g. Sr$_2$RuO$_4$)

Catherine Kallin, McMaster
Ed Taylor and CK, arXiv:111.4471
KITP (TI&SC) Dec 14, 2011
Anomalous or spontaneous Hall Effect

- AHE also occurs in topological insulators and metals

- Superconducting case is different. dc effect (in $\text{Re}(\sigma_H)$) is not quantized and not described by a Berry’s curvature.

- Results apply in general to multiband chiral superconductors, but will focus on Sr$_2$RuO$_4$ and chiral p-wave, where results are relevant to Kerr effect at optical frequencies.
Experiments on \( \text{Sr}_2\text{RuO}_4 \)

Evidence for odd-parity, triplet pairing:

- half-quantum vortices [R. Budakian et al. (2011)]

Evidence for broken time-reversal symmetry:

- polar Kerr effect [J. Xia et al., *PRL* 97, 167002 (2006.]
- Josephson [F. Kidwingira et al., *Science* 314, 1271 (2006)]

\[ \text{Triplet with BTRS + SRO xtal symmetry + energetics } \rightarrow \text{chiral p-wave order} \]
Chiral p-wave superconductivity

\[ d(k) = \sin k_x \pm i \sin k_y \frac{z}{k_F} \]

Breaks time-reversal symmetry; chirality = ±1

\( k_x + ik_y \) degenerate with \( k_x - ik_y \) → can have domains

\( d \parallel z \) (or c) ↔ \( S_z = 0 \) or equal spin pairing in xy (ab) plane

Chiral edge states exist in any open geometry

Chiral p-wave state has topological order, analogous to 5/2 Moore-Read QH state, characterized by Chern number = ±1. (Read & Green 2000)
Spontaneous supercurrents for chiral p-wave

Equilibrium supercurrent within $\xi$ of surface

(for single domain)

Screening current within $\lambda+\xi$ of surface

$\rightarrow$ Magnetic field $B \sim 10G$ within $\lambda$ of surface and $B \sim 20G$ at domain walls.

Stone and Roy (2004)
Matsumato and Sigrist (1999)
He3 scanning SQUID signal across ab face of Sr$_2$RuO$_4$ single crystal at $T=0.27$K


Smaller SQUIDs, Hall bar probes, micron samples $\rightarrow$ still no spontaneous fields observed

Experiments put upper bounds on edge currents which are $\sim$ 3 orders of magnitude smaller than predicted.
Muon spin resonance sees internal fields below $T_c$

Interpreted as due to fields at domain walls $\rightarrow$ domains $\sim$15 microns in size.

Other possibilities within chiral p-wave:
- impurities
- fields induced by muon

But difficult to reconcile with null measurements of edge and surface fields.

[Also W. Higemoto et al. unpublished]
Linearly polarized light is reflected as elliptically polarized light, with rotation of polarization axis by Kerr angle.

Cooled in (a) 93 G  (b) -43 G
[\omega=0.8\text{ev}; \Theta=65 \text{ nanorads}]

Kerr angle determined by $\sigma_{xy}(\omega)$. Expt (Sagnac interferometer) measures contribution from $\sigma_H = (\sigma_{xy} - \sigma_{yx})/2$.

In system with Galilean invariance: $j_s = \frac{ie^2}{s/m} E$ \hspace{1cm} \sigma_{xy} = 0$

$\rightarrow$ No Kerr effect without breaking translation symmetry but broken translation symmetry and BTRS insufficient

Lowest order Born scattering ($n_i U^2$) gives zero:

Higher order scattering can also contribute:

Requires p-h asymmetry.

Goryo identified diagrams of order $n_i U^3$ (skew scattering) which contribute.

J. Goryo, PRB 78, 060501 (2008).

Thought to be dominant contribution in Sr$_2$RuO$_4$. Estimate: $\theta_K \sim 40 \mathrm{nrads}$ for $l_{\text{imp}} \sim 1000 \AA$.

Lutchyn, Nagornykh, Yakovenko, PRB 80, 104508 (2009).
Intrinsic contributions to $\sigma_H$ and $\theta_K$

- $\sigma_H(q,\omega)$ at finite $q$ – difficult to probe experimentally: Goryo and Ishikawa, Phys. Lett. A 246, 549 (1998).

- Effect due to edges (related to edge current): Furusaki, Matsumoto, Sigrist, PRB 64, 054514 (2001).


These are much too small to explain polar Kerr experiments on Sr$_2$RuO$_4$.

All the above, plus disorder calculations, used a single band model. **Can multibands give an intrinsic effect?**
Sr$_2$RuO$_4$ band structure

Ru d-orbitals

$d_{xy}$

$d_{yz}$

$d_{xz}$
Sr$_2$RuO$_4$ band structure

Ru d-orbitals

\[ d_{xy} \]
\[ d_{yz} \]
\[ d_{xz} \]

\[ \gamma \]

\[ t'' \approx 0.1t \]

\[ \alpha \]
\[ \beta \]
Can a multiband model resolve some of the puzzles?


Find intraorbital p-wave pairing for \( d_{xz} \) and \( d_{yx} \).

(a) Bands and pairing phases with no \( t'' \).

Relative phase of intraorbital pairing is \( \pi/2 \).

(b) Bands and pairing phases with \( d_{xz}-d_{yx} \) hopping, \( t'' \).
Chiral p-wave SC on 2d xy band gives one chiral mode at each edge of cylinder. Chern number is ±1.

Chiral p-wave SC on 1d xz and yz bands gives one non-chiral mode at each edge of cylinder.

Hole and electron bands \(\rightarrow\) Chern number is 0 \(\rightarrow\) not topologically protected.

Could explain absence of observable edge/surface currents (and low-lying excitations)
Is there an intrinsic AHE in a multiband chiral superconductor?

\[ H(\ ) = \frac{xy(\ ) - yx(\ )}{2} \]

\[ xy(\ n) = \frac{ie^2T}{k,\ n} \text{tr} \ \hat{v}_x G_0(k,\ n)\hat{v}_y G_0(k,\ n + \ n) \]

\[ \hat{v}_i = \frac{(k)}{k_i} \quad \text{(or} \ \frac{k_i}{m} \quad \text{for free electrons)} \]

\[ H \quad \text{vanishes for single band} \]

Consider two-band (two-orbital) case

\[ H_0 = \begin{pmatrix}
1(k) & 12(k) \\
12(k) & 2(k)
\end{pmatrix}; \quad i = i \quad \text{Intraorbital pairing:} \quad 11, \ 22 \quad \Rightarrow 4 \times 4 \ G_0 \]

\[ \text{Interorbital pairing:} \quad 12 \]

The velocity matrix also has off-diagonal terms (interorbital transitions)
Is there an intrinsic AHE in a multiband chiral superconductor?

Nonzero contributions involve transitions between orbitals ($\varepsilon_{12}$ or t") or between bands ($\Delta_{12}$) and require different relative OP phases.

I.e. $\Delta(k) = e^{i\theta} [\Delta'(k) + i\Delta''(k)] \rightarrow$ relative phase $= \phi(k) = \tan^{-1}[\Delta''(k)/\Delta'(k)]$

Changes sign with chirality.

$$ n_H = \frac{e^2}{2} \sum_k ((v_{11} v_{22}) \times v_{12}) \frac{E}{E_+} \left[ \text{Im} \begin{pmatrix} \ast & 22 \\ 11 & 12 \end{pmatrix} + \text{Im} \begin{pmatrix} \ast & 12 \\ 22 & 11 \end{pmatrix} + \text{Im} \begin{pmatrix} \ast & \ast \\ 11 & 12 \end{pmatrix} \right] \times \left[ \begin{pmatrix} E_1 & E_2 \\ +E_1 & +E_2 \end{pmatrix} \right] $$
\[ T=0 \left( \frac{H}{H} \right) = 2e^2 \frac{\text{Im}\langle c_{k_1}^+ c_{k_2} \rangle [\langle v_{11} v_{22} \rangle v_{12}]_z}{(w + i e)^2 (E + E_+)^2} \]

\[ t = 10t'' = 1\text{eV} \]

\[ \phi_0 = 0.23\text{meV} \]

Applied to Raghu et al. quasi-1d model for Sr2RuO4

Interobital coherence

Sign depends on chirality

Requires p-h asymmetry
Effect requires particle-hole asymmetry.

Note: photon with polarization in ab (xy) plane cannot cause transitions involving $d_{xy}$ orbitals $\rightarrow \gamma$ band only enters indirectly through coherence factors
$t = 10t'' = 1\text{eV}$

$0 = 0.23\text{meV}$

$T=0$

For $T>0$, also have qp scattering, i.e. $\delta(\omega-(E_1-E_2))$

Small numbers due to $(\Delta/t)^2$

Absorption only for $\omega \sim 2t''$ and larger (not $2\Delta$)

\[
\sigma_H[e^2/h] = 2e^2 \sum_k \frac{((v_{11} - v_{22}) \times v_{12})_z}{E E_+(E_+ + E_+)} \frac{\text{Im}(v_{11}^{\ast} v_{22})}{(E_+ + E_+)^2 (E_+ + E_+)^2}
\]

\[
= \frac{e^2}{h} 16 \frac{2t'' t}{t'^2} \int dx dy \frac{\sin^2 x \sin^2 y (\cos y \sin^2 x + \cos x \sin^2 y)}{E E_+(E_+ + E_+)} \left[ (E_+ + E_+)^2 \left( E_+ + E_+ \right)^2 \right]
\]

Applied to Raghu et al. quasi-1d model for Sr2RuO4
Polar Kerr Effect

\[ \kappa (\ ) = \frac{4}{d} \text{Im} \left( \frac{H(\ )}{n(n^2 - 1)} \right), \quad n(\ ) = \sqrt{4 i (\ ) / n} \]

\[ \frac{1}{\hbar \omega} = 0.4 \text{eV} \]

\[ \frac{1}{\hbar \omega} = 0.8 \text{eV} \]

\[ \sim 35 \text{ nrads at } \omega = 0.8 \text{eV} \]

[same parameters as used in skew-scattering estimate which gave 43 nrads.]
• Results very similar with further neighbor hoppings. Also whether there are low-lying excitations has little effect on T=0 result.

• Spin-orbit coupling does not give any new effect but can play the role of interorbital hopping.

• Adding γ band will not change much, as it plays a passive role. Need substantial superconductivity on quasi-1d bands.

• Quasi-1d model seems to maximize the effect. Much smaller effect if SC primarily on γ band.
In conclusion

• Identified an intrinsic contribution to the polar Kerr effect (and spontaneous Hall effect) which is generic to multiband chiral superconductors provided there is interband pairing with a different relative phase than intraband pairing $[\text{Im}(\Delta^{*}_{aa}\Delta_{ab})]$, and broken p-h symmetry.

• The quasi-1d model for $\text{Sr}_2\text{RuO}_4$ gives $\theta_K$ comparable to experiment. By contrast, if SC is primarily on the $\gamma$ band, the Kerr angle would likely be reduced by more than an order of magnitude.

• Experiments with controlled disorder may determine if the observed Kerr angle is of intrinsic or extrinsic origin. Intrinsic would imply the SC is of a multiband nature. Other discrepancies point toward a multiband model of chiral p-wave.