Chain of Majorana fermions along a magnetic domain wall on a superconducting topological insulator

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T. Neupert, SO, & A. Furusaki, PRL 105, 206404 (2010)
"Chain of Majorana states from SC Dirac fermions at a magnetic DW"

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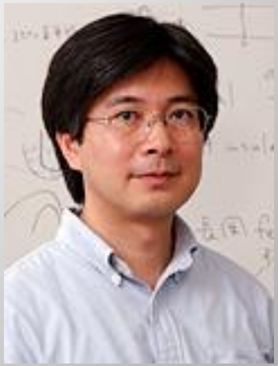


A.Furusaki
RIKEN

@RIKEN

Classification of topological insulators and superconductors

Synder-Ryu-Furusaki-Ludwig

→ *Ludwig's talk*

This talk

IQH, spinful chiral p-wave SC

Spinless chiral p-wave SC

SC quantum wire

Symmetry				d							
AZ	Θ	Ξ	Π	1	2	3	4	5	6	7	8
A	0	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$
AIII	0	0	1	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0
AI	1	0	0	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$
BDI	1	1	1	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$
D	0	1	0	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$
DIII	−1	1	1	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$	0
AII	−1	0	0	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$
CII	−1	−1	1	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0
C	0	−1	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0
CI	1	−1	1	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0

Singlet SC under field
d+id SC

SO-coupled SC

QSH

Z_2TI

SF ^{3}He B phase

from Hasan-Kane RMP

Majorana fermions

Ettore Majorana (1906-?)

Real spin-1/2 particles, that are their own anti-particles, devised for neutrinos (1937)

$$\gamma^{\dagger} = \gamma$$

Bogoliubov quasi-particles in SC: $\hat{\gamma} = a\hat{f}^{\dagger} + b\hat{f}$

$$\gamma^{\dagger} = \gamma \quad \text{if} \quad b = a^{*}$$

Accommodate Majorana zero modes at vortex cores (Kopnin et al. '91) of topological SC (Read, Green '00)

- sSC 2D Dirac (Jachiw, Rossi '81)
- ν=5/2 FQHE (Das Sarma et al. '05)
- chiral-pSC (Read, Green '00; Ivanov '01)
- SC quantum wires

Majorana fermions in atomic fermion quantum wires as platforms for quantum information processing

Non-Abelian statistics

Useful for fault-tolerant quantum computation   Nayak et al. RMP '08

Kitaev's model '03

$$H_{Kitaev} = -\sum_{j=1}^{N-1}\left(tf_j^\dagger f_{j+1} + |\Delta| e^{i\phi} f_j f_{j+1} + h.c.\right) - \mu \sum_{j=1}^{N} f_j^\dagger f_j$$

$$\rightarrow -it\sum_{j=1}^{N-1}\gamma_{B,j}\gamma_{A,j+1} \text{ when } \mu = 0,\ t = |\Delta| \qquad f_j = e^{-i\phi/2}\left(\gamma_{B,j} + \gamma_{A,j}^\dagger\right),\ \gamma_{\alpha,j} = \gamma_{\alpha,j}^\dagger,\ \left\{\gamma_{\alpha,j}, \gamma_{\alpha',j'}^\dagger\right\}$$

Tewari, Das Sarma, et al, '07
chiral p-wave SF atomic quantum wire
(p-wave Feshbach resonance in ^{6}Li & ^{40}K)

Braiding operations of fermions
→ Quanutm gates

Dissipative Mojorana braiding
Diehl et al. '11

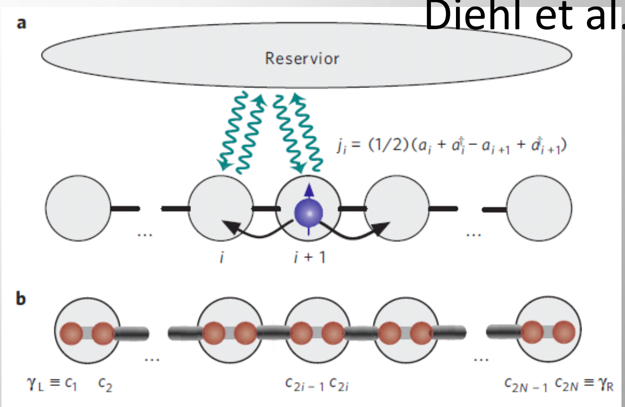
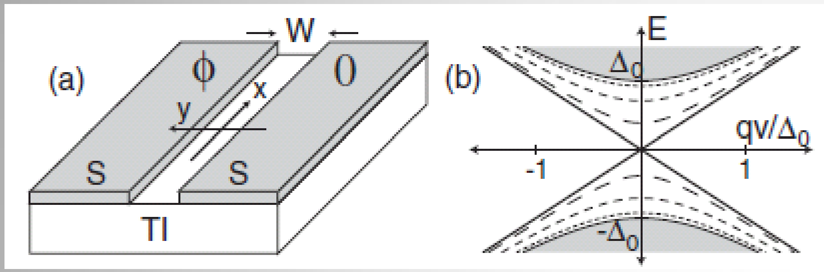


Figure 1 | Schematic set-up for the dissipative Majorana quantum wire.

Majorana fermions in solids

Fu-Kane '08, '09

Proximity effect of SC on the TI surface



$$\mathcal{H} = -iv\tau^z\sigma\cdot\nabla - \mu\tau^z + \Delta_0(\tau^x\cos\phi + \tau^y\sin\phi).$$

Majorana-fermion chain can be created without breaking T

Andreev bound states

Tri-junctions

Alicea et al. ’11: Semiconducting quantum wire

Majorana braiding with T-junctions

Sau et al. '10

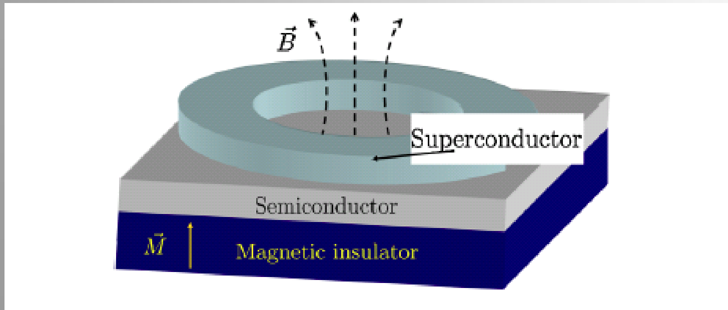


FIG. 1 (color online). Schematic picture of the proposed heterostructure exhibiting Majorana zero-energy bound state inside an ordinary vortex.

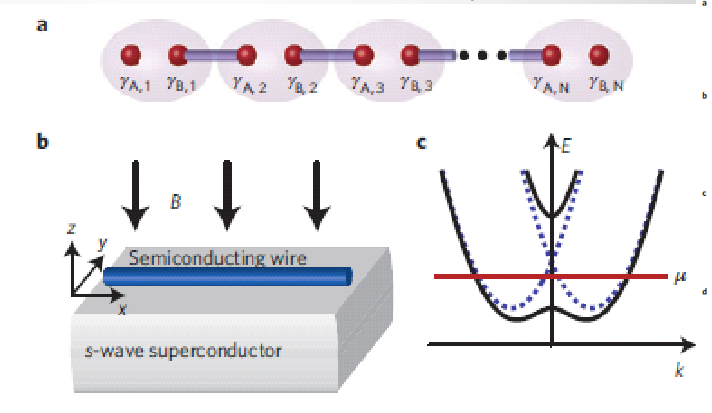
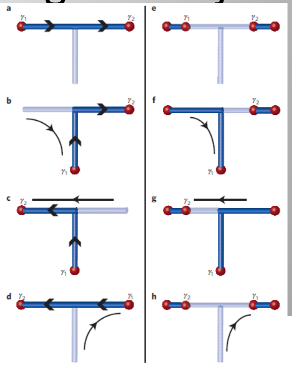
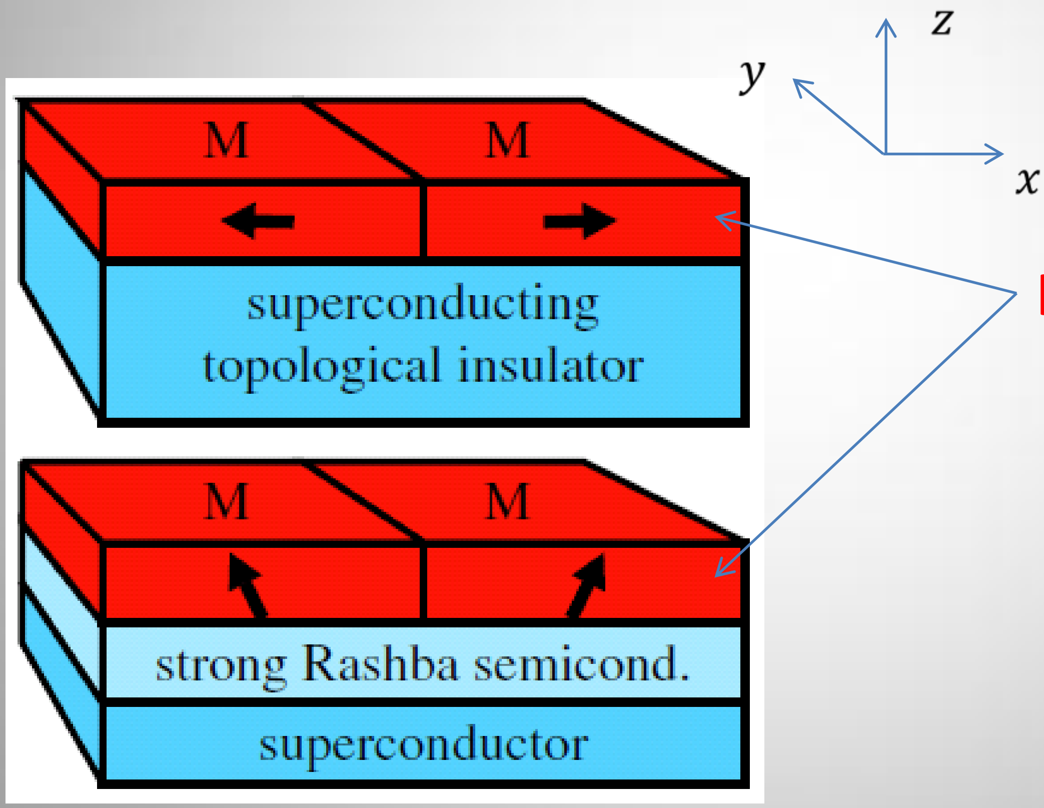


Figure 1 | Majorana fermions appear at the ends of a 1D ‘spinless’ p-wave superconductor, which can be experimentally realized in semiconducting



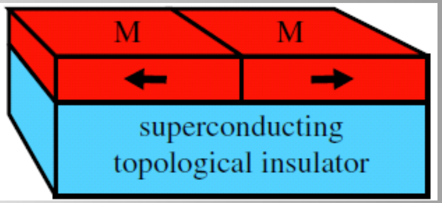
Interface of a SC TI with ferromagnet

Effects of in-plane Zeeman fields on the surface Dirac fermions



Interface of SC 2D Dirac fermions with ferromagnet

(2D Dirac fermions) + (exchange field)



$$\hat{h}(\mathbf{r}) = \hat{\psi}^{\dagger}(\mathbf{r})\{[iv\mathbf{D}\times\mathbf{e}_z - \boxed{\mathbf{M}(\mathbf{r})}]\cdot\boldsymbol{\sigma} - \mu\}\hat{\psi}(\mathbf{r})$$

$$+\Delta(\mathbf{r})\hat{\psi}^{\dagger}(\mathbf{r})i\sigma_2[\hat{\psi}^{\dagger}(\mathbf{r})]^T + H.c.$$ + (s-wave SC)

$$\mathbf{r} = (x, y), \quad \mathbf{D} = \nabla_{\mathbf{r}} - ie\mathbf{A}(\mathbf{r})$$

Consider the type-II limit (κ → infinity) !

→ Orbital mag. field irrelevant to the field-induced change of SC order parameters

Under the uniform inplane field $\boldsymbol{M}(\boldsymbol{r}) = \boldsymbol{M}$

→ Dirac cone and the total momentum of Cooper pairs shift by $\boldsymbol{q} = \boldsymbol{M}\times\boldsymbol{e}_z/v$.

Santos et al. '10

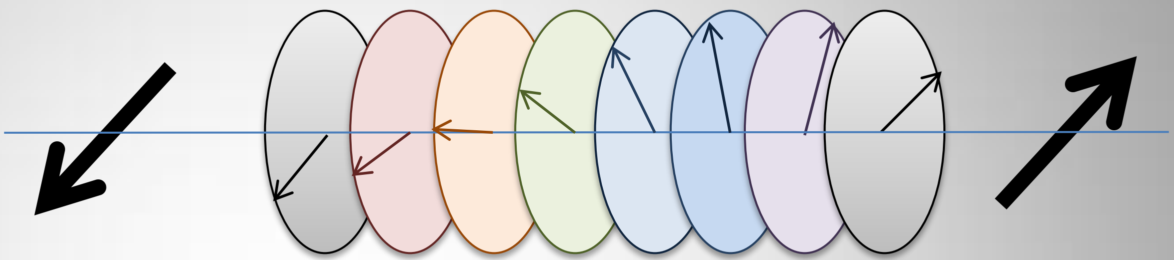
More complicated cases of domain walls

→ GL theory

Usual ferromagnetic domain walls

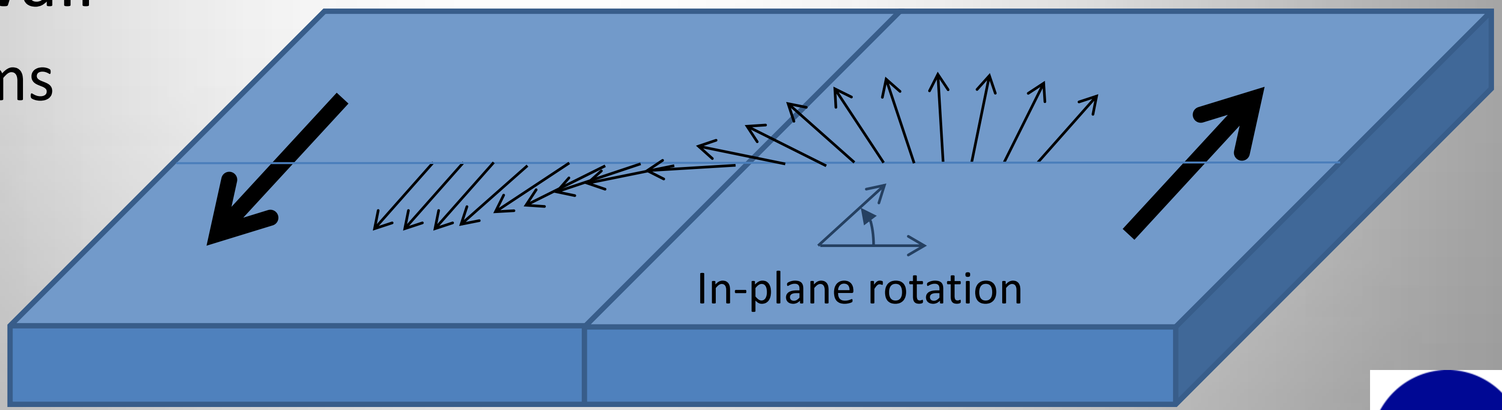
- Bloch wall

in the bulk



- Neel wall

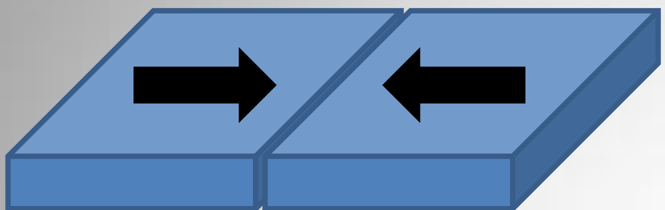
in thin films

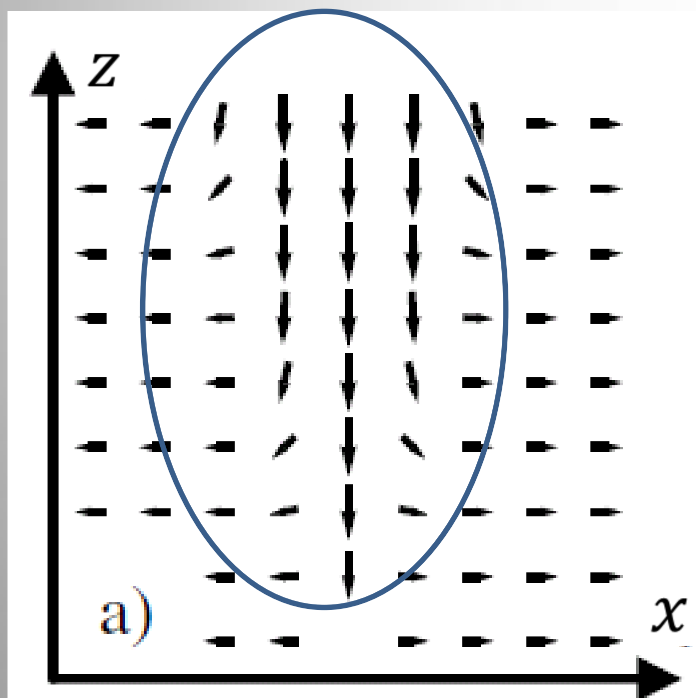


Avoid monopoles

Head-to-head/tail-to-tail domain wall

Possible in ferromagnetic thin films with large magnetic anisotropy





Divergence-free

$$\mathbf{M}(\mathbf{r}, z) = B \sum_{\tau=\pm} \frac{\mathbf{e}_x - s\tau\mathbf{e}_z}{2} \tanh \frac{x + s\tau z}{l_{DW}}$$

l_{DW} Domain-wall thickness

s Spatial anisotropy characterizing the magnetization change

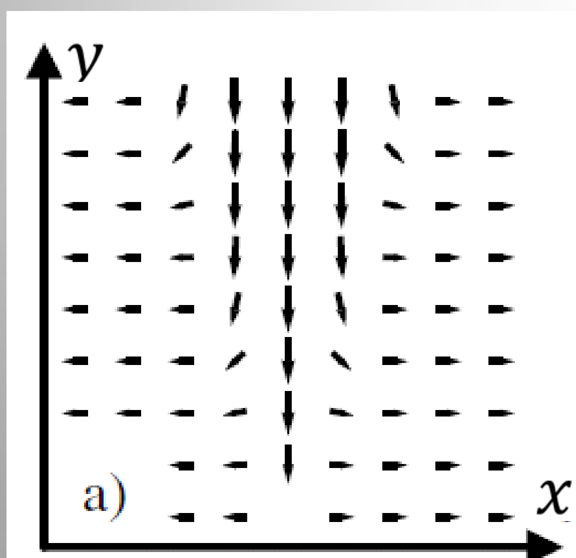
Modulation of SC order parameters: 1 a case of head-to-head domain wall

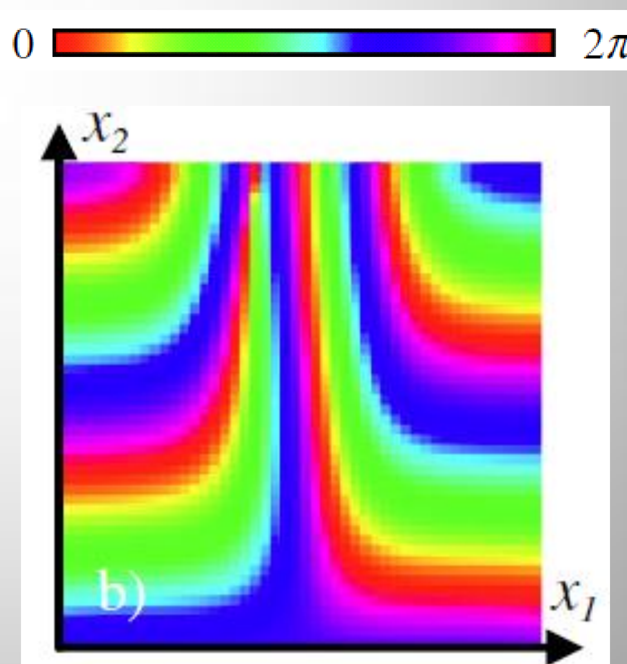
$$F = \int d^2\mathbf{r}\left[\alpha\,|\Delta(\mathbf{r})|^2 + \beta\,|\mathbf{D}_\Delta \Delta(\mathbf{r})|^2 + u\,|\Delta(\mathbf{r})|^4\right],$$

$$\mathbf{D}_\Delta = \nabla - 2i(e\mathbf{A} + \mathbf{a}/v),$$

$\mathbf{a} = \mathbf{e}_z \times \mathbf{M}$ fictitious vector pot. due to the exchange field M

$$\mathbf{b} = \nabla \times \mathbf{a} = \mathbf{0} \ @ \ z = 0$$





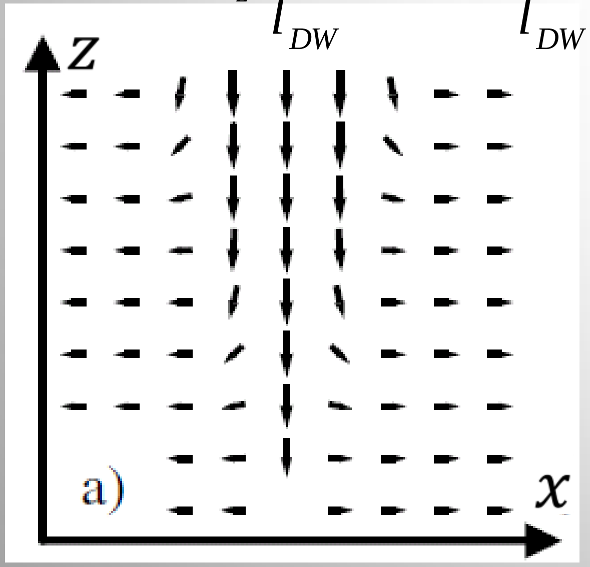
Modulation of SC order parameters: 2 a case of head-to-head domain wall

$$F = \int d^2\mathbf{r}\left[\alpha\,|\Delta(\mathbf{r})|^2 + \beta\,|\mathbf{D}_\Delta \Delta(\mathbf{r})|^2 + u\,|\Delta(\mathbf{r})|^4\right],$$

$$\mathbf{D}_\Delta = \nabla - 2i(e\mathbf{A} + \mathbf{a}/v),$$

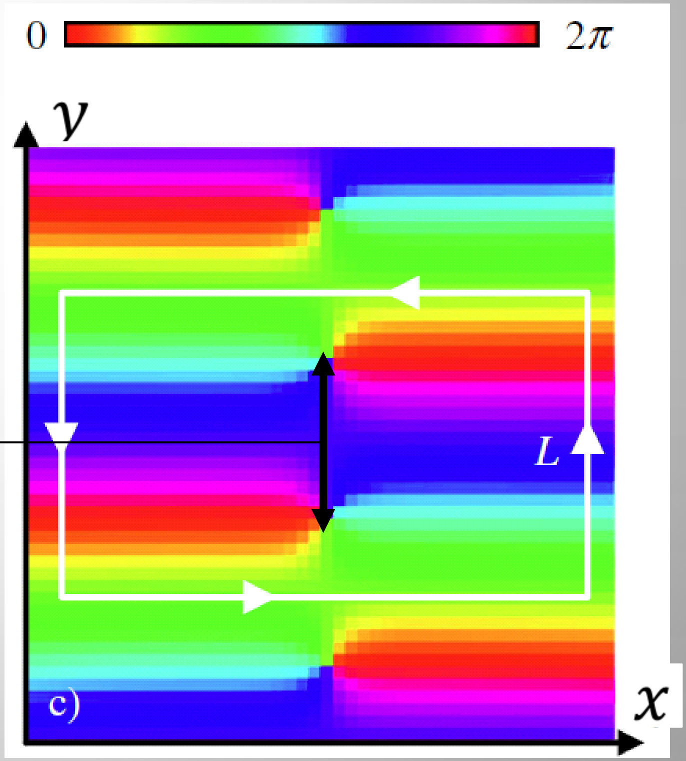
$$\mathbf{a} = \mathbf{e}_z \times \mathbf{M}$$

$$\mathbf{b} = \nabla \times \mathbf{a} = \mathbf{e}_z \frac{B}{l_{DW}} \operatorname{sech}^2 \frac{x}{l_{DW}} \;@\; z = 0$$



Inter-vortex distance

$$\frac{\pi}{q} = \frac{\pi v}{B}$$



Bogoliubov de-Gennes equation: a case of an isolated vortex

$$H_{BdG}\Psi(\mathbf{r}) = \varepsilon\Psi(\mathbf{r})$$

$$H_{BdG} = \begin{pmatrix} -\mu & v\partial^{+} & \Delta & 0 \\ v\partial^{-} & -\mu & 0 & \Delta \\ \Delta^{*} & 0 & \mu & -v\partial^{+} \\ 0 & \Delta^{*} & -v\partial^{-} & \mu \end{pmatrix}$$

in the Nambu representation

$$\hat{\Psi}^{\dagger}(\mathbf{r}) = \left(\hat{\psi}^{\dagger}(\mathbf{r}), -i\hat{\psi}^{\mathrm{T}}(\mathbf{r})\sigma_y\right)$$

$$\partial^{\pm} = \pm\partial_x + i\partial_y$$

Solution:

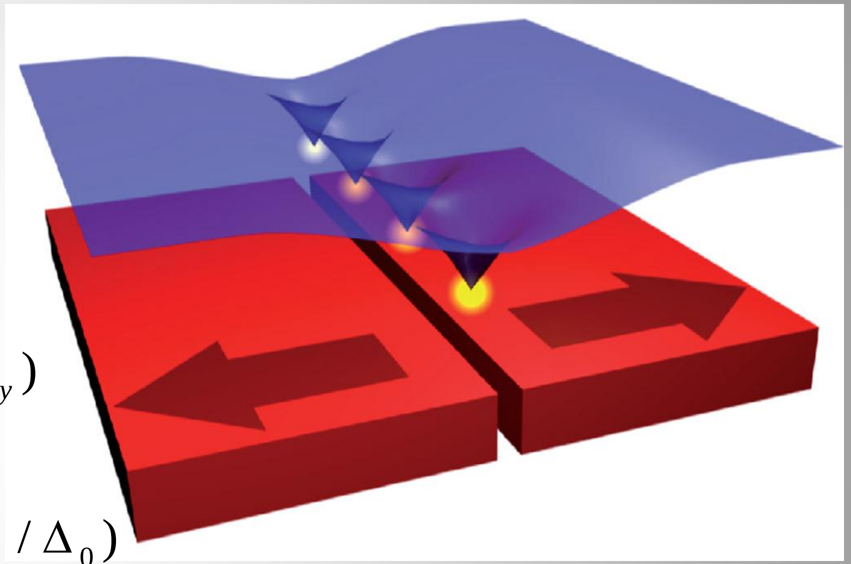
$$\Psi_{\pm}(\mathbf{r}) = \frac{1}{\sqrt{N}}\begin{pmatrix} 1 \\ -\frac{\mu}{2v}(x+iy) \\ \pm\frac{\mu}{2v}(x-iy) \\ \pm 1 \end{pmatrix}\exp\left[-\frac{1}{v}\int_0^{|x|}dx'\Delta(x')\right]$$

Bergman, Hur '09

A single vortex to a chain of vortices

Tight-binding Majorana fermions

$$\hat{H}_{MF} = i\sum_j (t+\delta t_j)\phi_j\phi_{j+1}$$



N.N. transfer integral

$$t = \int d^2\mathbf{r}\Psi_+^\dagger(\mathbf{r}+\frac{q}{2}\mathbf{e}_y)H_{BdG}\Psi_-(\mathbf{r}-\frac{q}{2}\mathbf{e}_y)$$

$$\approx C\mu\left(\frac{\Delta_0}{|B|}\right)^{3/2}\exp\left(-\frac{\pi}{2}\frac{\Delta_0}{|B|}\right)+O(\mu^2/\Delta_0)$$

depends linearly on the chemical potential of underlying Dirac fermions.

The sign change of t → topological phase transition: D → BDI → D

Z_2 → Z → Z_2

The amplitude is controlled by the magnitude of the exchange field **B**

E.J. Weinberg's index theorem

Dirac fermions coupled to Higgs fields and vector potential, decaying faster than 1/r

→

The lower bound of the number of zero modes:

(vortices – anti-vortices)

$$n = \frac{1}{2\pi}\int_C dl_i \frac{\epsilon_{ab}\phi_a(\partial_i\phi)_b}{|\phi|^2}$$

E.J. Weinberg, prd 24, 2669 ('81)

c.f. Jackiw-Rossi

In our case, only vortices or anti-vortices appear.

Candidates

- Cu-doped Bi_2Se_3, Pd-doped Bi_2Te_3, TlBiTe

Hasan, …

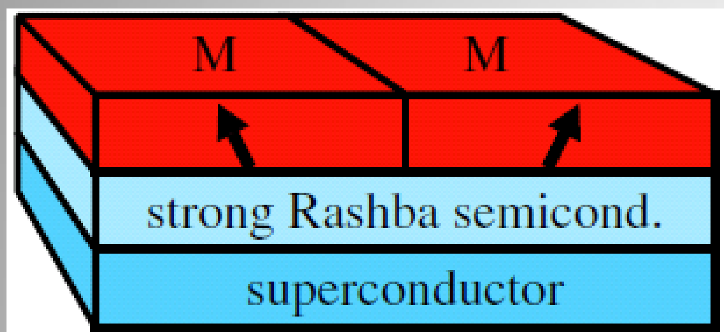
- LaPtBi (3D TI to SC)

Chadov et al.

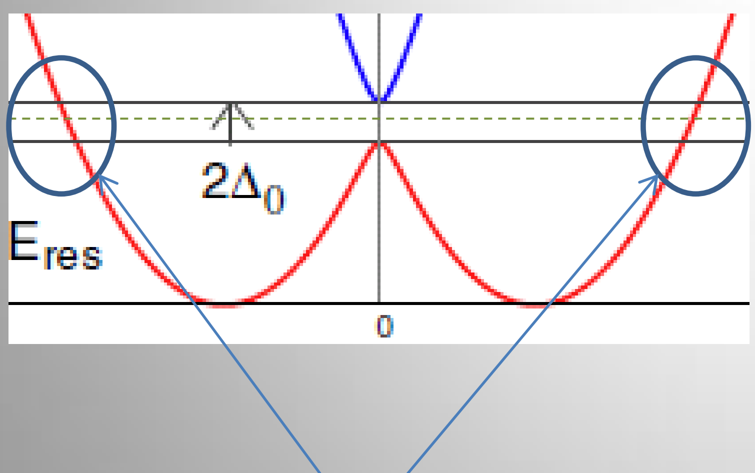
Topological SC? Or,

even if they are topologically less nontrivial without chiral Majorana edge modes, the interface with ferromagnetic DW induces a Majorana chain!!

A possible alternative realization



Out-of-plane component M_z is crucial to gap out the Γ point level.



μ must be inside the $2\Delta_0 = 2M_z$ energy gap window.

Modeled by the same Hamiltonian

Summary

- Magnetic texture (DW) accommodates a chain of vortices, each of which accomodates Majorana fermions.
- Topological transition with the chemical potential varied across the Dirac nodal point.
- STM/STS
- Thermal transport

(Yet other contributions from phonons, magnons, …)

c.f. Beenakker

More nontrivial effects could be devised on the interface with insulating textured magnets