From Luttinger Liquid to Non-Abelian Quantum Hall States

Jeffrey Teo and C.L. Kane

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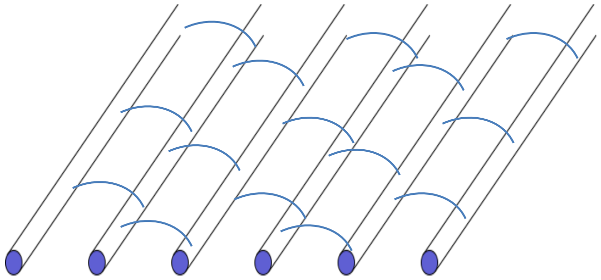
Outline

- Introduction to FQHE
  - Bulk-edge correspondence
- Abelian Quantum Hall States
  - Coupled wires
  - Laughlin and hierarchy states



- Non-Abelian Quantum Hall States
  - Coupled bundles of wires
  - Moore Read and Read Rezayi states

Abelian FQH States

Gapped (2+1)D-bulk

- Topological field theory

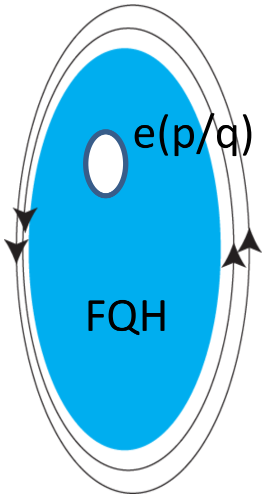
$$\mathcal{L}_{CS} = \frac{K_{IJ}}{4\pi} a_I \wedge da_J + \frac{e}{2\pi} t_I A \wedge da_I$$

- Bulk quasihole excitation
- Fractional charge
- Abelian statistics

Gapless (1+1)D-Edge

- Chiral Luttinger liquid

$$\mathcal{L}_{LL} = \frac{K_{IJ}}{4\pi} \partial_x \phi_I \partial_t \phi_J + \frac{e}{2\pi} t_I A_t \partial_x \phi_I$$



- Chiral multi-component Luttinger liquid

$$[\partial_x \phi_I^{qp}(x), \phi_J^{qp}(x')] = 2\pi i (K^{-1})_{IJ} \delta(x - x')$$

Wen, Zee, 92

Non-Abelian FQH States

Gapped (2+1)D-bulk

- Ground state wave function $z = x + iy$

$$\tilde{\Psi}_{GS} = \langle \psi(z_1) \dots \psi(z_N) \rangle$$

- Moore Read state

  Ising non-Abelian statistics

- Read Rezayi state

  $\mathbb{Z}_k$ non-Abelian statistics

  $\cup|$

  Fibonacci anyons

Gapless (1+1)D-Edge

- Chiral Conformal field theory $z = \tau + ix$

  Vertex operator

$$\psi(z) =: e^{i\phi(z)} :$$

- Charge + neutral mode

  c = 1 + 1/2

  Kac-Moody algebra

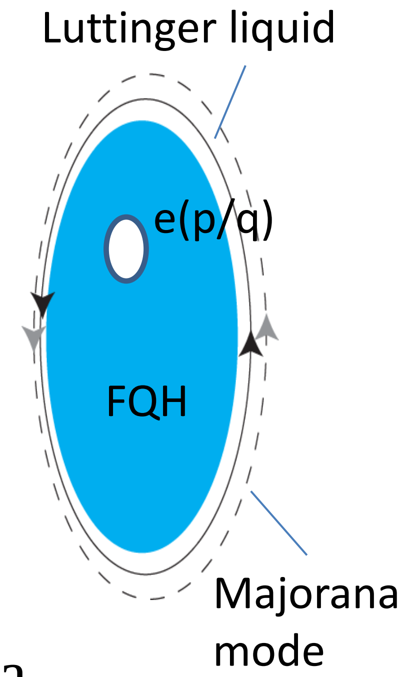
$$U(1)_q \times \mathbb{Z}_2 = SU(2)_2$$

- Charge + neutral mode

  c = 1 + 2(k-1)/(k+2)

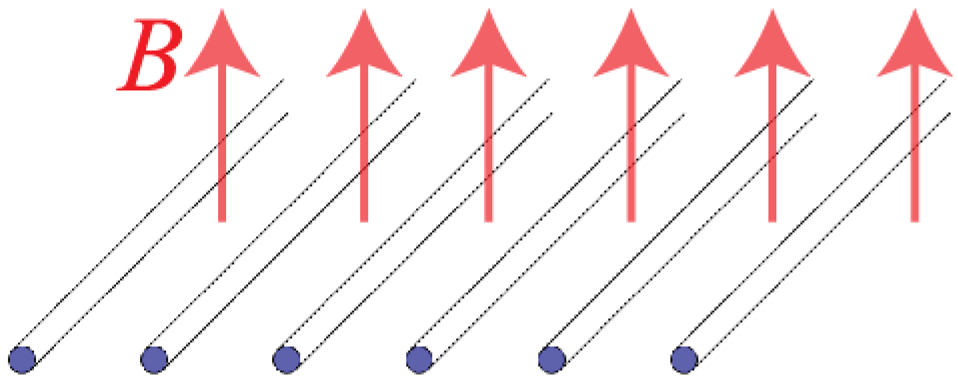
  Kac-Moody algebra

$$U(1)_q \times \mathbb{Z}_k = SU(2)_k$$



Moore, Read, 91; Read, Rezayi, 99

Coupled Wires Construction



- 1D Luttinger liquid

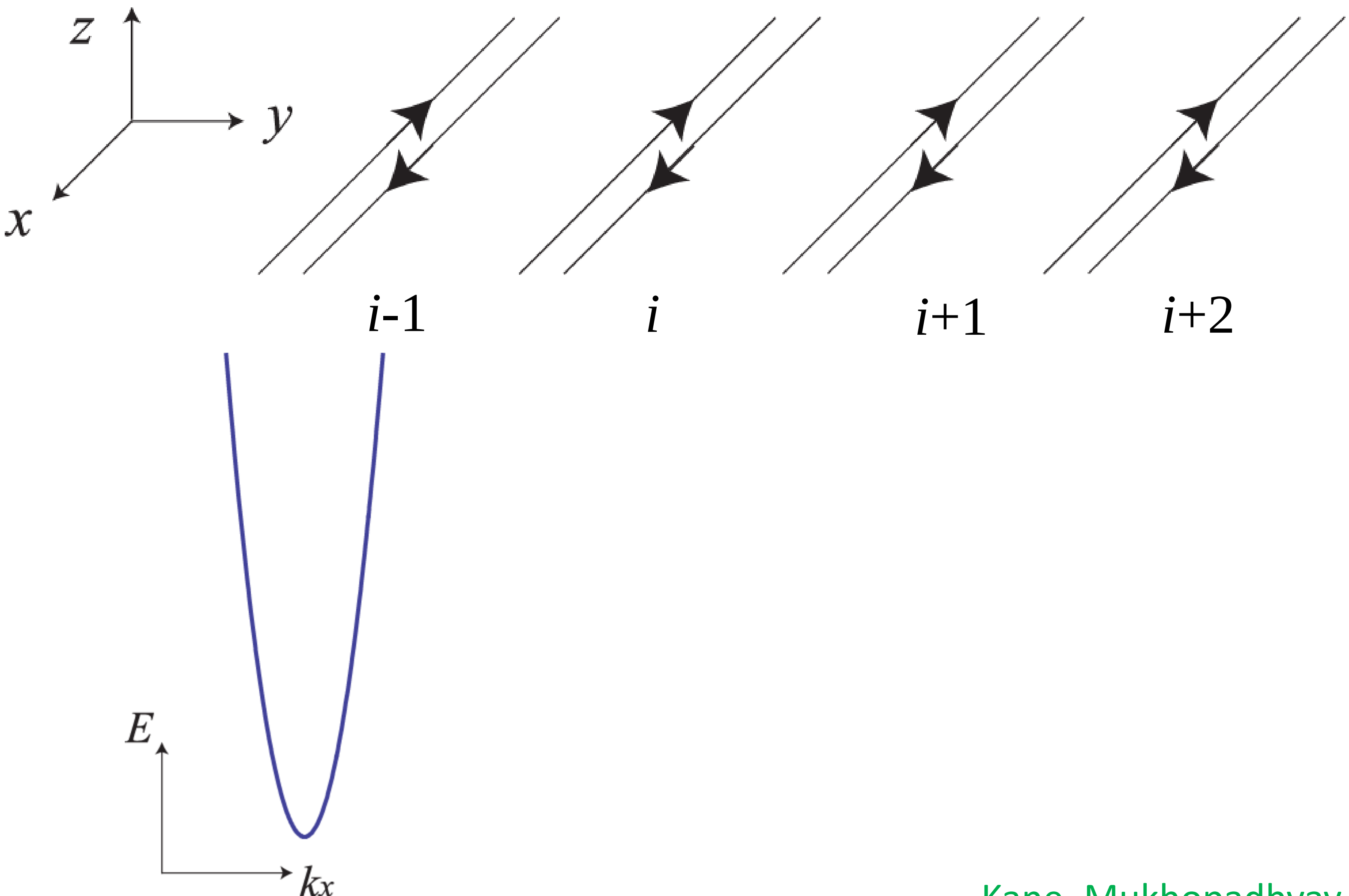
  simple description of interaction via abelian bosonization

- Interwire many-body tunneling => FQH states

  solvable intermediate between microscopic electronic model and effective field theory

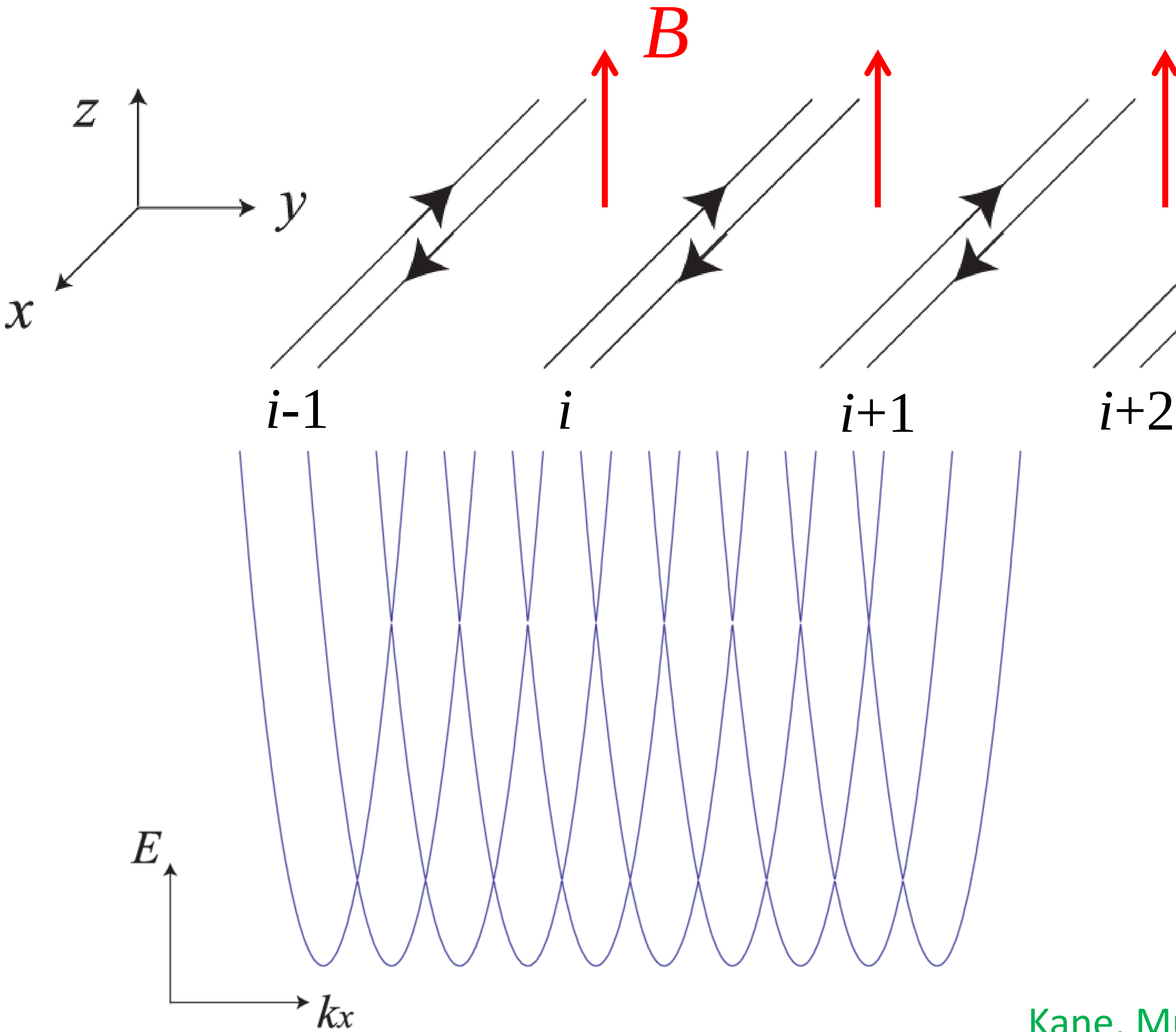
- Representation of chiral edge CFT and quasiparticle excitations

Integer Quantum Hall



Kane, Mukhopadhyay, Lubensky, 02

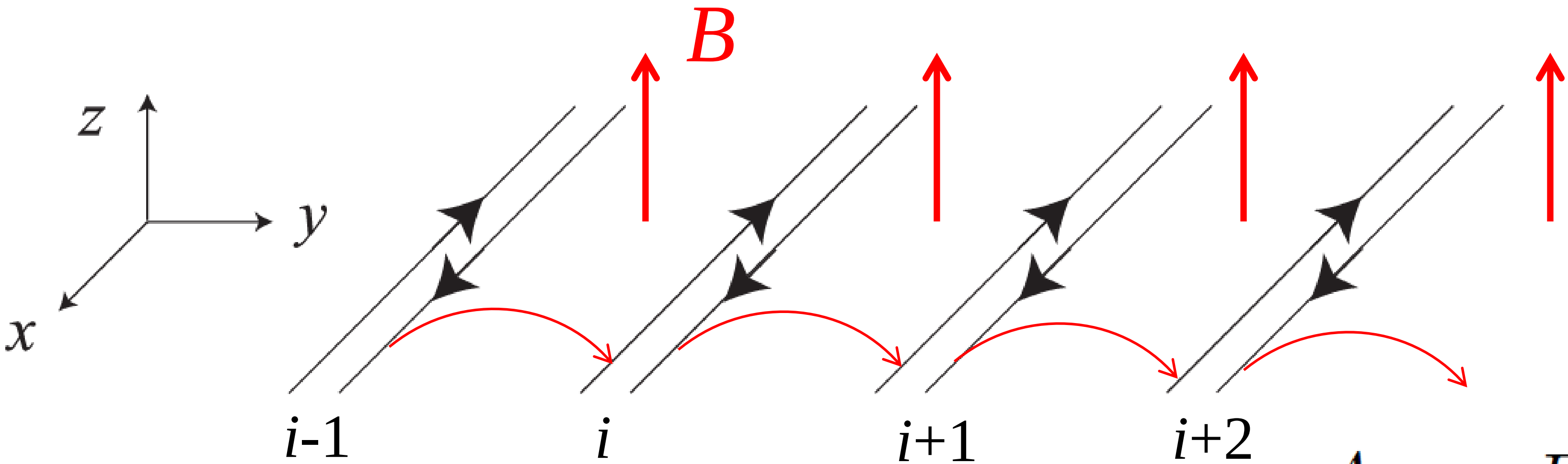
Integer Quantum Hall



$$A_x = By$$

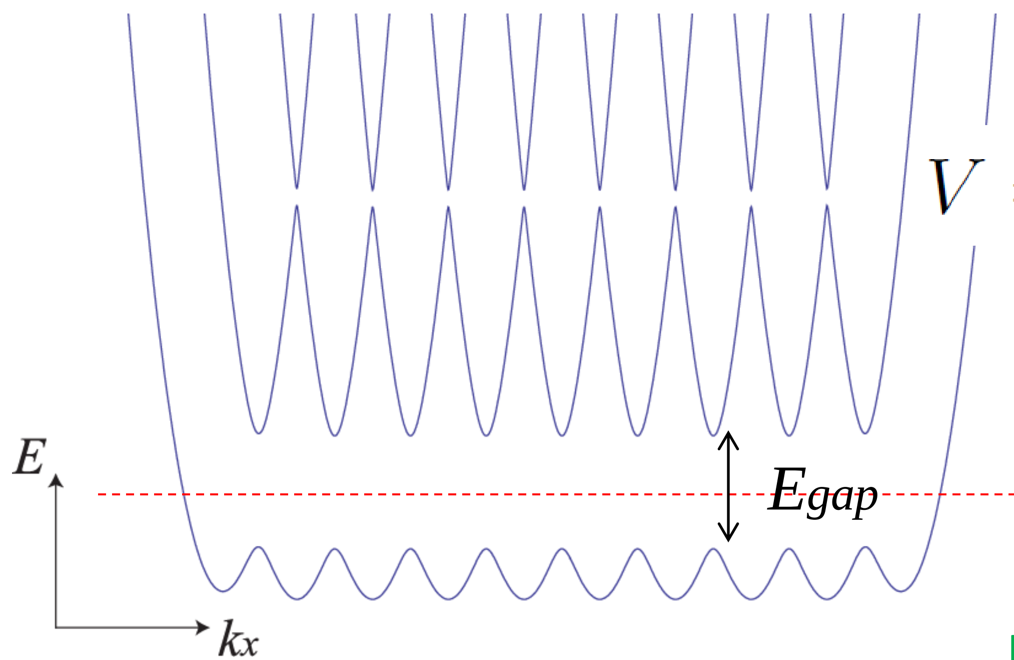
Kane, Mukhopadhyay, Lubensky, 02

Integer Quantum Hall



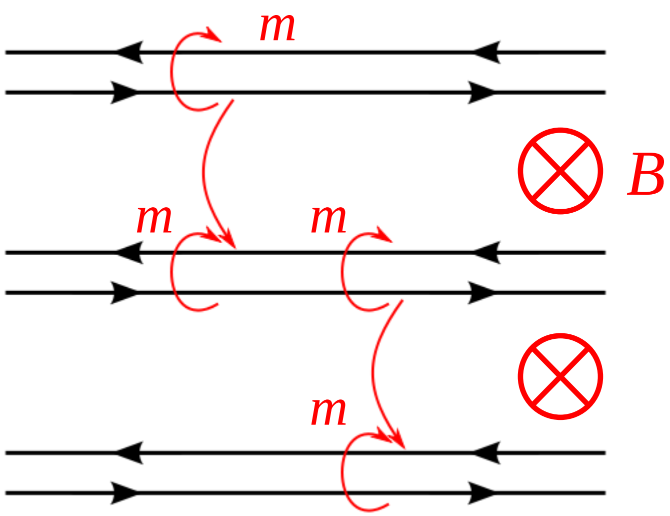
$$A_x = By$$

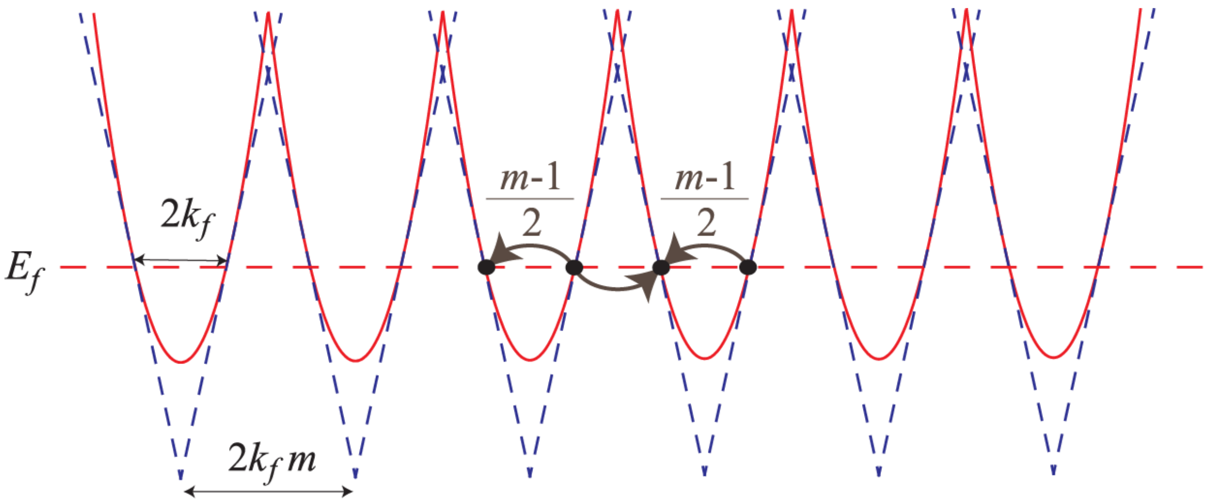
$$V = -t(\psi_i^R {\psi_{i+1}^L}^\dagger + h.c.)$$



Kane, Mukhopadhyay, Lubensky, 02

Laughlin States $\nu = 1/m$





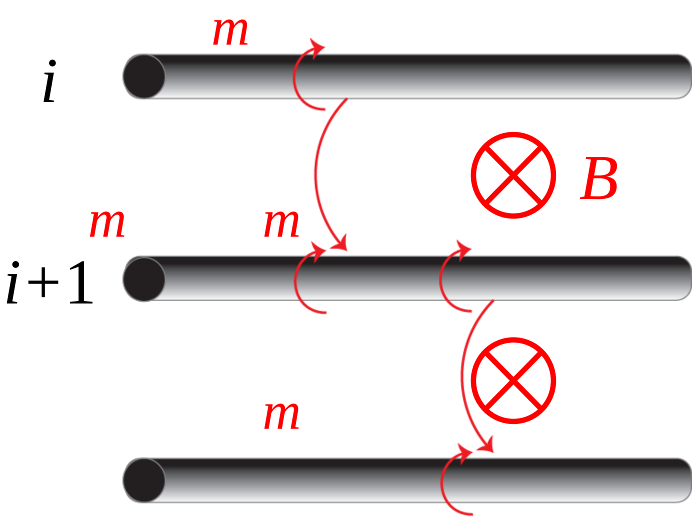
$$\psi^{R/L} \sim e^{i(\varphi \pm \theta)}$$

m odd for fermions

$$\begin{aligned} V &= -t[(\psi_i^R {\psi_{i+1}^L}^\dagger)(\psi_i^R {\psi_i^L}^\dagger)^{\frac{m-1}{2}}(\psi_{i+1}^R {\psi_{i+1}^L}^\dagger)^{\frac{m-1}{2}} + h.c.] \\ &= -2t\cos(\varphi_i - \varphi_{i+1} + m(\theta_i + \theta_{i+1})) \end{aligned}$$

Kane, Mukhopadhyay, Lubensky, 02

Laughlin States $\nu = 1/m$



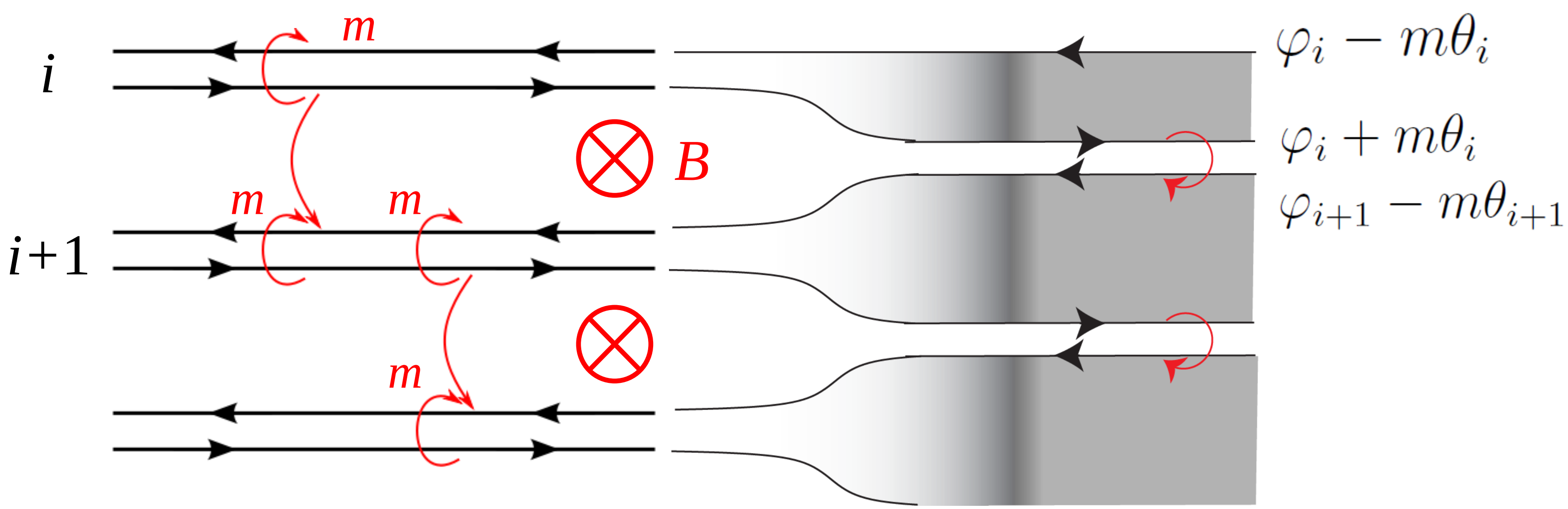
$$\Phi \sim e^{i\varphi} \qquad \rho(x) = \bar{\rho} + \sum_n e^{i2n\theta(x)}$$

m even for bosons

$$\begin{aligned} V &= -t[(\Phi_i \Phi_{i+1}{}^{\dagger}) e^{im\theta_i} e^{im\theta_j} + h.c.] \\ &= -2t\cos(\varphi_i - \varphi_{i+1} + m(\theta_i + \theta_{i+1})) \end{aligned}$$

Kane, Mukhopadhyay, Lubensky, 02

Laughlin States $\nu = 1/m$



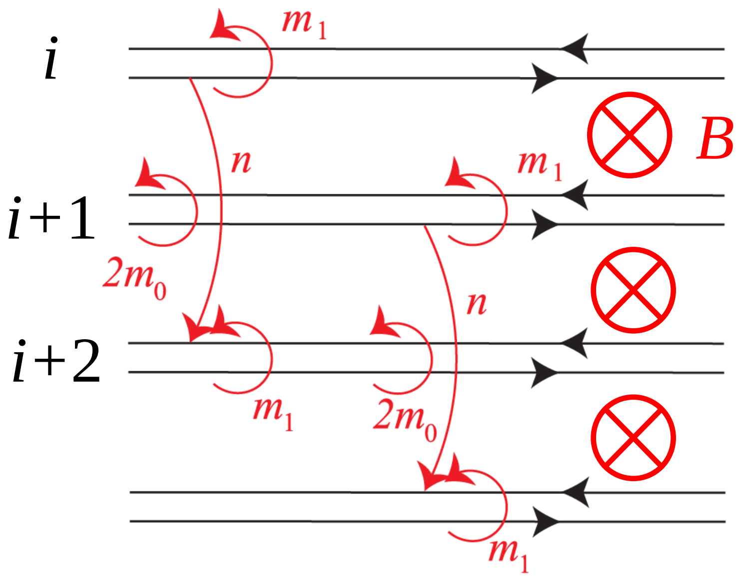
New electron operators $\phi^{R/L} = \varphi \pm m\theta$

Edge K-M algebra $[\partial_x \phi(x), \phi(x')] = 2\pi i m \delta(x - x')$

=> fractional charge e/m, fraction statistics

Kane, Mukhopadhyay, Lubensky, 02

Hierarchy States

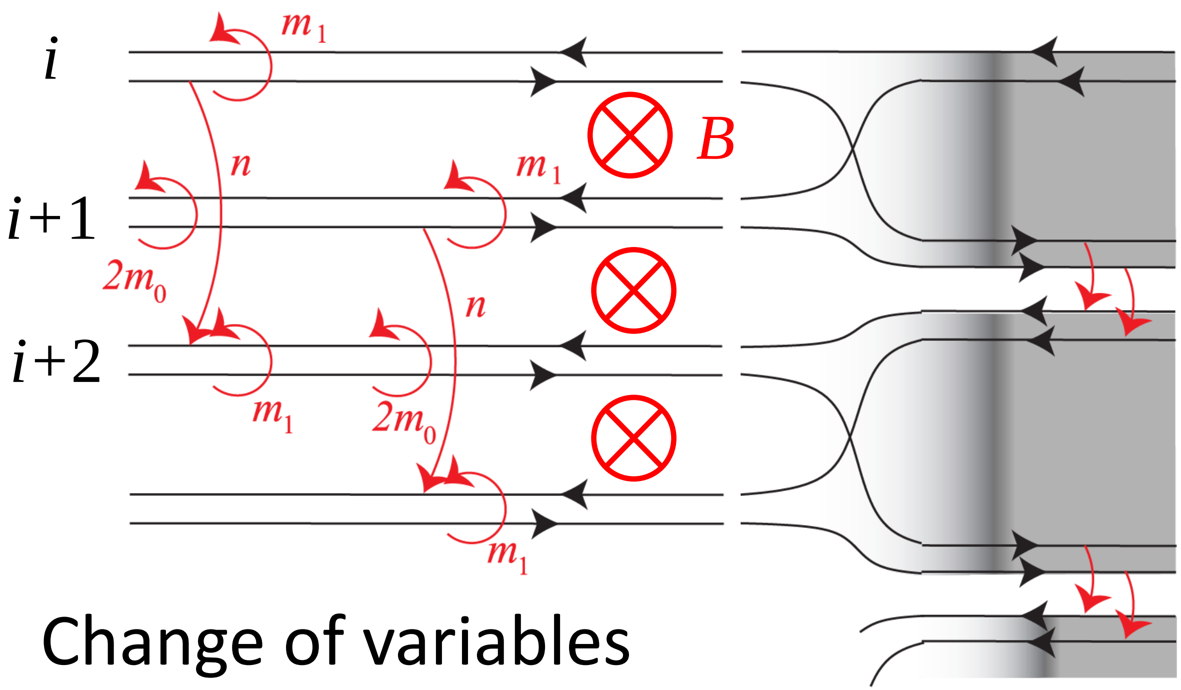


$$\nu = \frac{2n}{m_0 + m_1}$$

$$V = -t\cos\left[n(\varphi_{i+2} - \varphi_i) + 2m_0\theta_{i+1} + m_1(\theta_i + \theta_{i+2})\right]$$

Kane, Mukhopadhyay, Lubensky, 02

Hierarchy States



$$n\varphi_i - m_1\theta_i$$
$$n\varphi_{i+1} - 2m_0\theta_i - m_1\theta_{i+1}$$

$$n\varphi_i + m_1\theta_i + 2m_0\theta_{i+1}$$
$$n\varphi_{i+1} + m_1\theta_{i+1}$$
$$n\varphi_{i+2} - m_1\theta_{i+2}$$
$$n\varphi_{i+3} - 2m_0\theta_{i+2} - m_1\theta_{i+3}$$

Change of variables

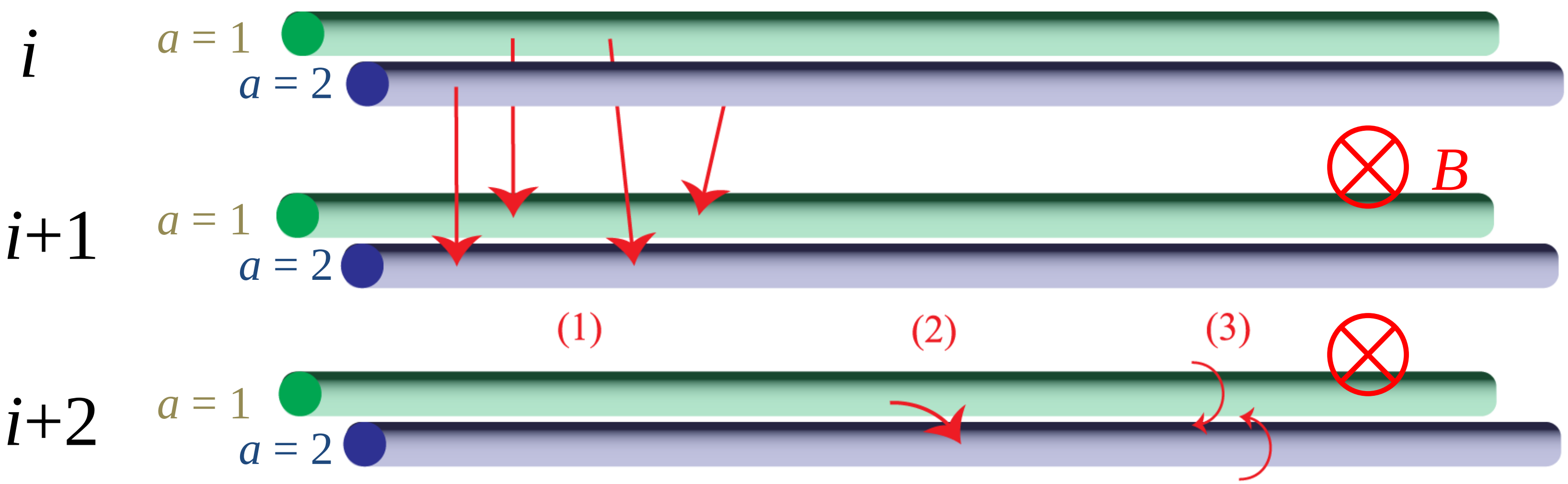
$$\begin{cases} \phi_1^L = n\varphi_i - m_1\theta_i \\ \phi_2^L = n\varphi_{i+1} - 2m_0\theta_i - m_1\theta_{i+1} \end{cases}$$

$$K = n\begin{pmatrix} m_1 & m_0 \\ m_0 & m_1 \end{pmatrix}$$

K-M algebra $[\partial_x\phi_I(x), \phi_J(x')] = -2\pi i K_{IJ}\delta(x - x')$

Kane, Mukhopadhyay, Lubensky, 02

Moore Read State ($\nu = 1$ boson)



(1) boson hopping in field (2) phase locking (3) crystal locking

$$V^{(1)} = t\sum_{ab}\cos[\varphi^a_{i+1} - \varphi^b_i + 2(\theta^a_i + \theta^b_{i+1})]$$

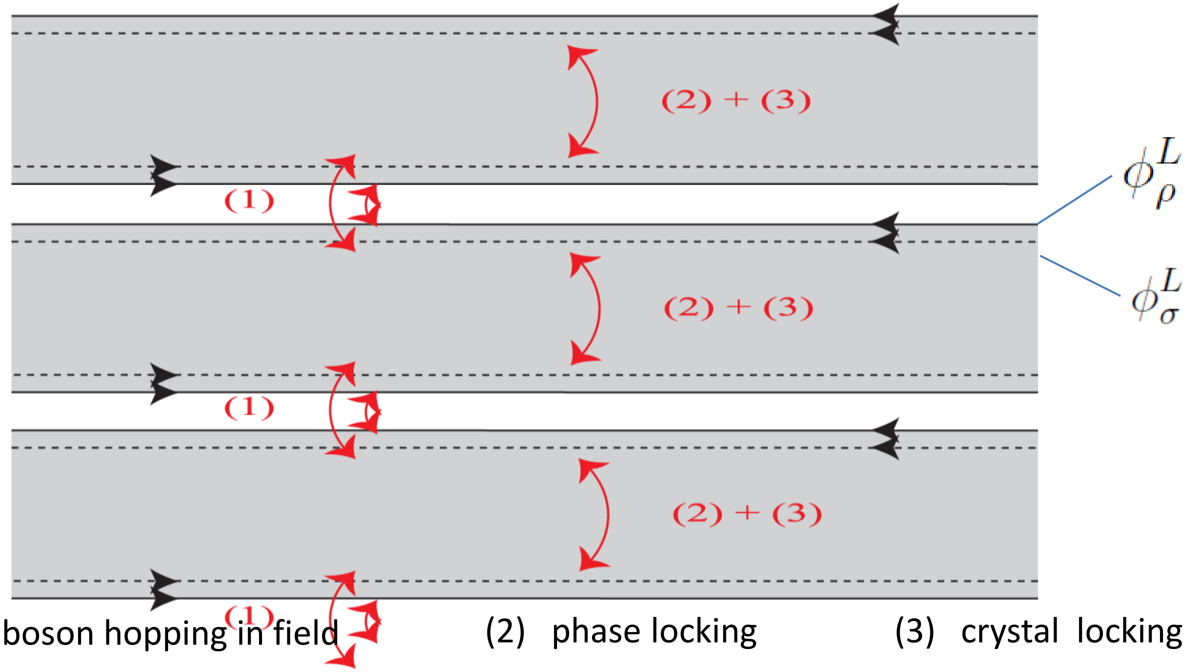
$$V^{(2)} = u\cos(\varphi^1_i - \varphi^2_i)$$

$$V^{(3)} = v\cos(2\theta^1_i - 2\theta^2_i)$$

JYT, Kane, 11

(inspired by Fradkin, Nayak, Schoutens, 99)

Moore Read State ($\nu = 1$ boson)



(1) boson hopping in field (2) phase locking (3) crystal locking

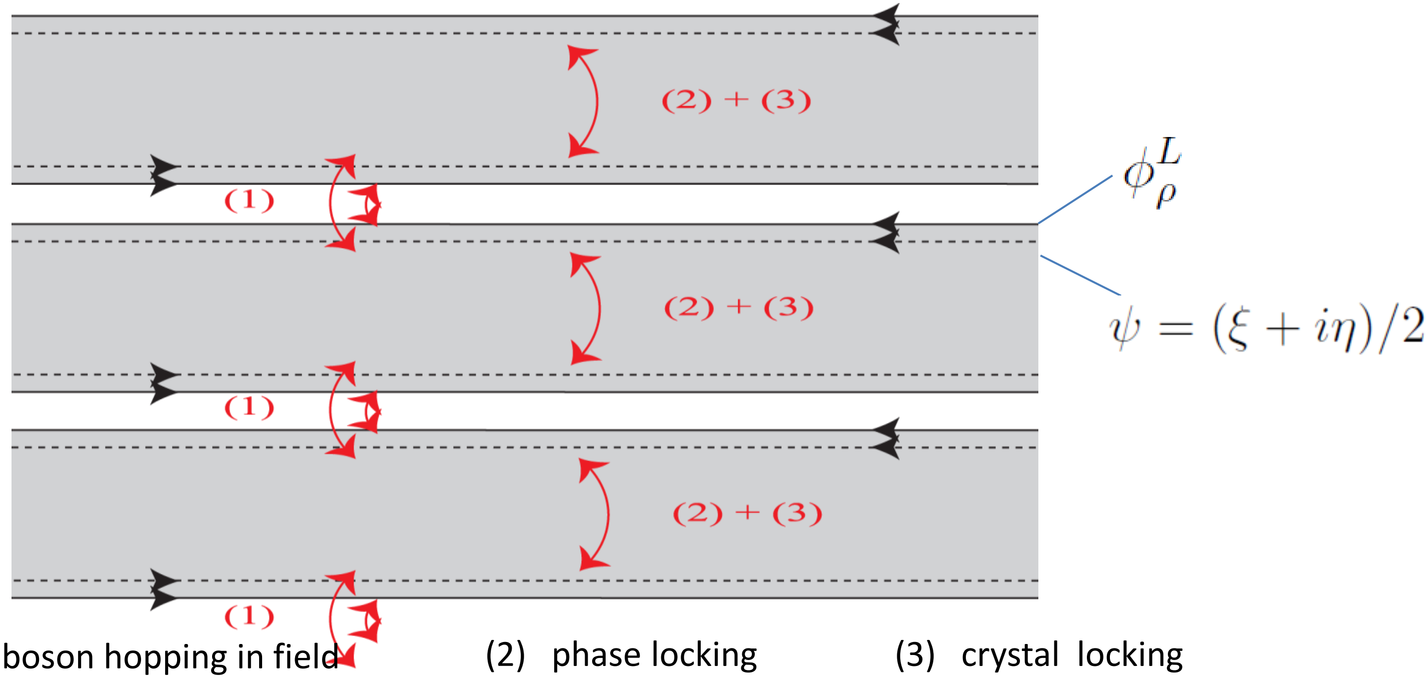
Change of variables $\phi_{i,a}^{R/L} = \varphi_i^a \pm 2\theta_i^a$

$$\begin{cases} \phi_{i,\rho}^{R/L} = \phi_{i,1}^{R/L} + \phi_{i,2}^{R/L} \\ \phi_{i,\sigma}^{R/L} = \phi_{i,1}^{R/L} - \phi_{i,2}^{R/L} \end{cases}$$

$$\begin{aligned} V^{(1)} &= t\cos(\phi_{i+1,\rho}^R - \phi_{i,\rho}^L)\cos\phi_{i,\sigma}^R\cos\phi_{i+1,\sigma}^L \\ V^{(2)} &= u\cos(\phi_{i,\sigma}^R + \phi_{i,\sigma}^L) \\ V^{(3)} &= v\cos(\phi_{i,\sigma}^R - \phi_{i,\sigma}^L) \end{aligned}$$

JYT, Kane, 11

Moore Read State ($\nu = 1$ boson)



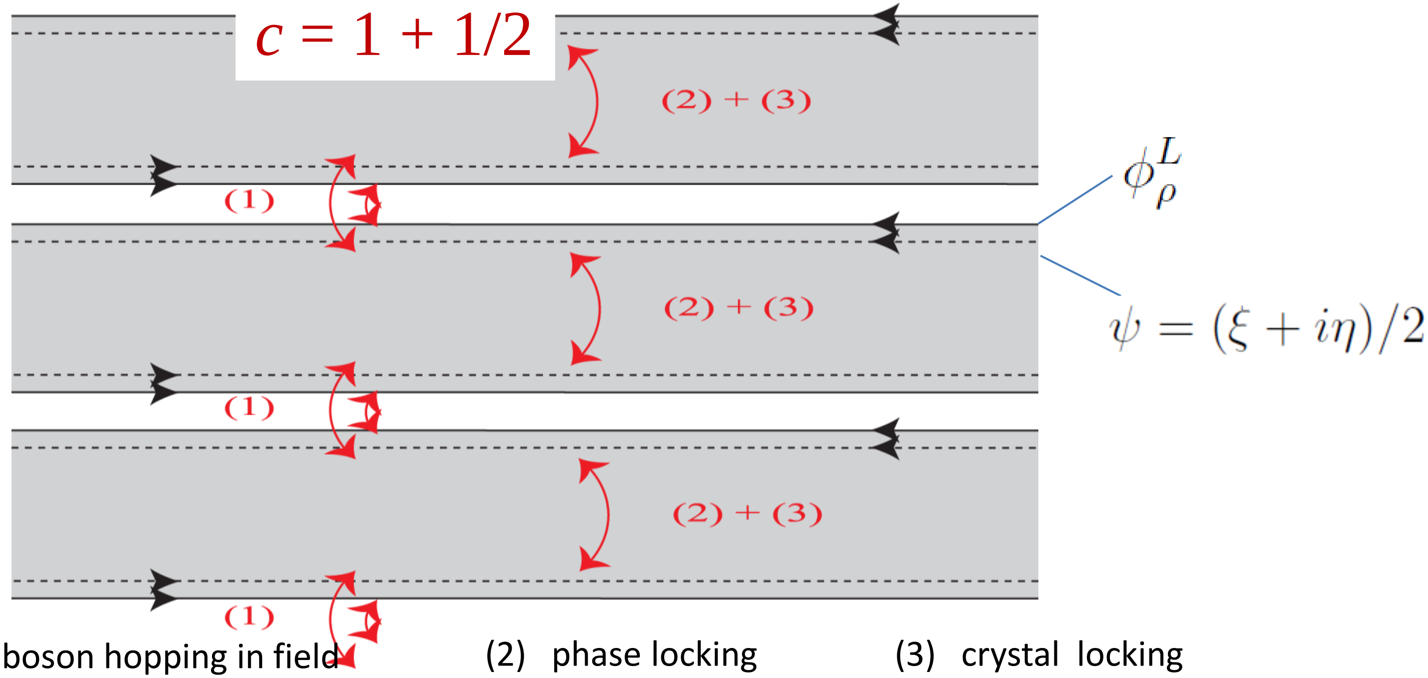
Fermionize $$e^{i\phi_\sigma} \sim \psi = (\xi + i\eta)/2$$

$$V^{(1)} = t\cos(2\tilde{\theta}_{i+1/2,\rho})\xi_i^R \xi_{i+1}^L$$

$$V^{(2)} + V^{(3)} = (u+v)\eta_i^R \eta_i^L + (u-v)\xi_i^R \xi_i^L$$

JYT, Kane, 11

Moore Read State ($\nu = 1$ boson)



(1) boson hopping in field (2) phase locking (3) crystal locking

Fermionize $e^{i\phi_\sigma} \sim \psi = (\xi + i\eta)/2$

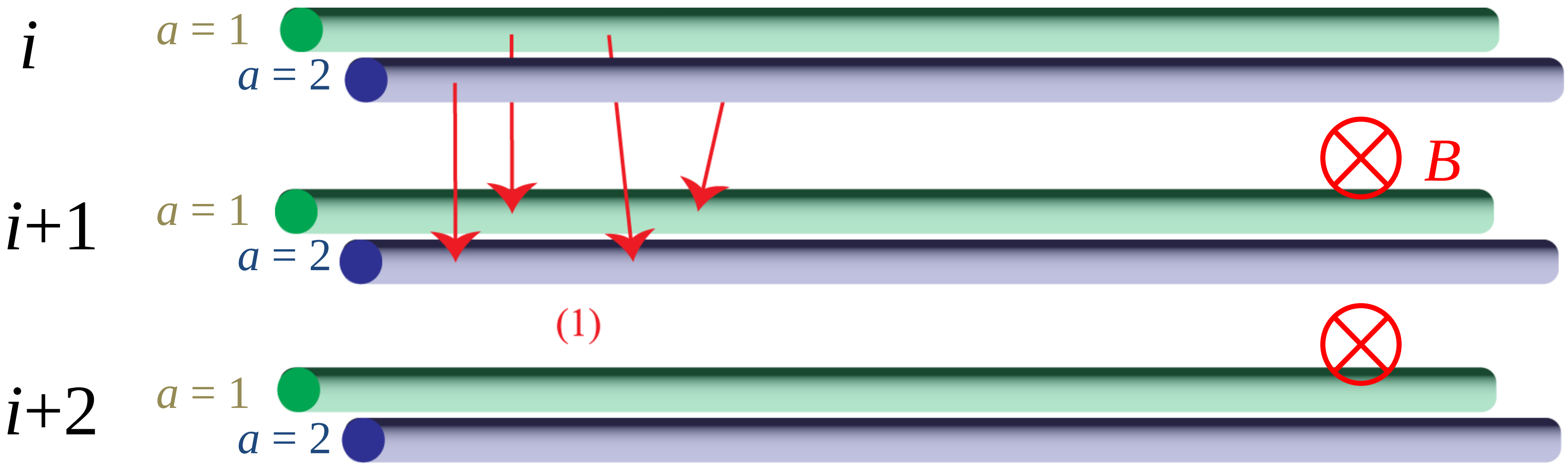
$$V^{(1)} = t\cos(2\tilde{\theta}_{i+1/2,\rho})\xi_i^R \xi_{i+1}^L$$

$$V^{(2)} + V^{(3)} = (u+v)\eta_i^R\eta_i^L + (u-v)\xi_i^R\xi_i^L$$

JYT, Kane, 11

$t > |u - v|$ => Moore Read State

Coset Construction



(1) boson hopping in field

$SU(2)_2$ current algebra

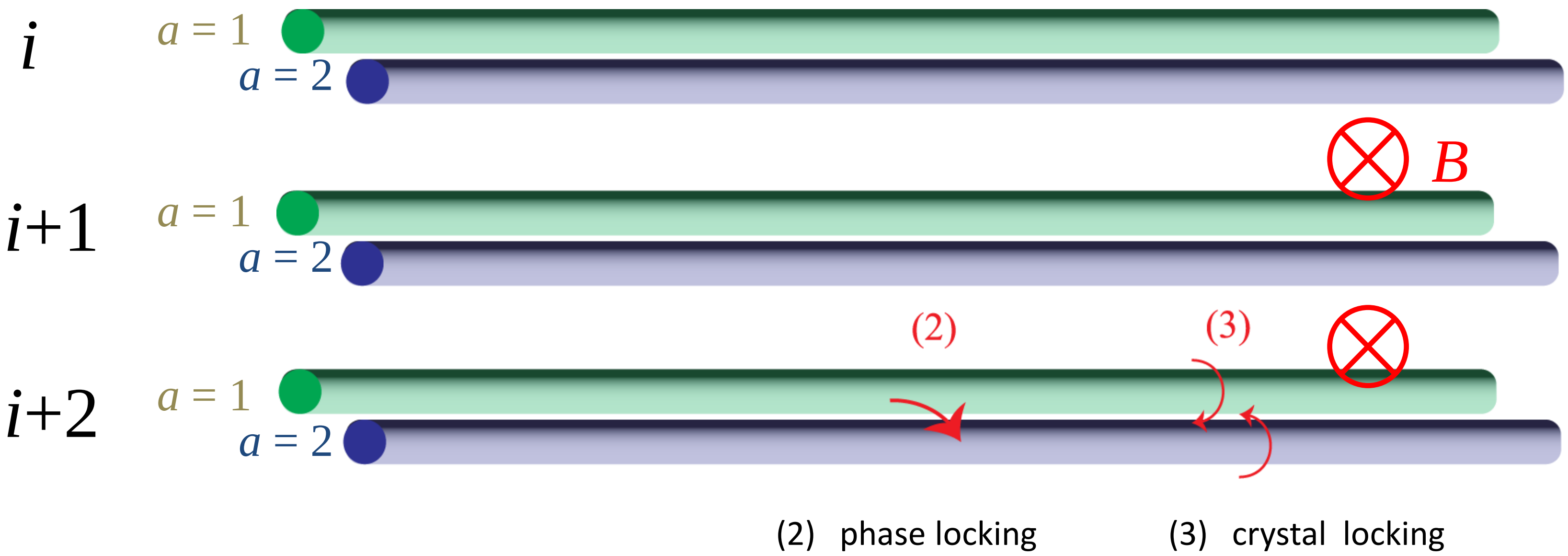
$$J_a^z \sim i\partial\phi_a, \quad J_a^{\pm} \sim e^{\pm i\phi_a}$$
$$\mathbf{J} = \mathbf{J}_{a=1} + \mathbf{J}_{a=2}$$

gapped by

$$V^{(1)} \sim t\mathbf{J}_i^L \cdot \mathbf{J}_{i+1}^R$$

JYT, Kane, 11

Coset Construction



Remaining

$$\frac{SU(2)_1 \times SU(2)_1}{SU(2)_2}$$

gapped by

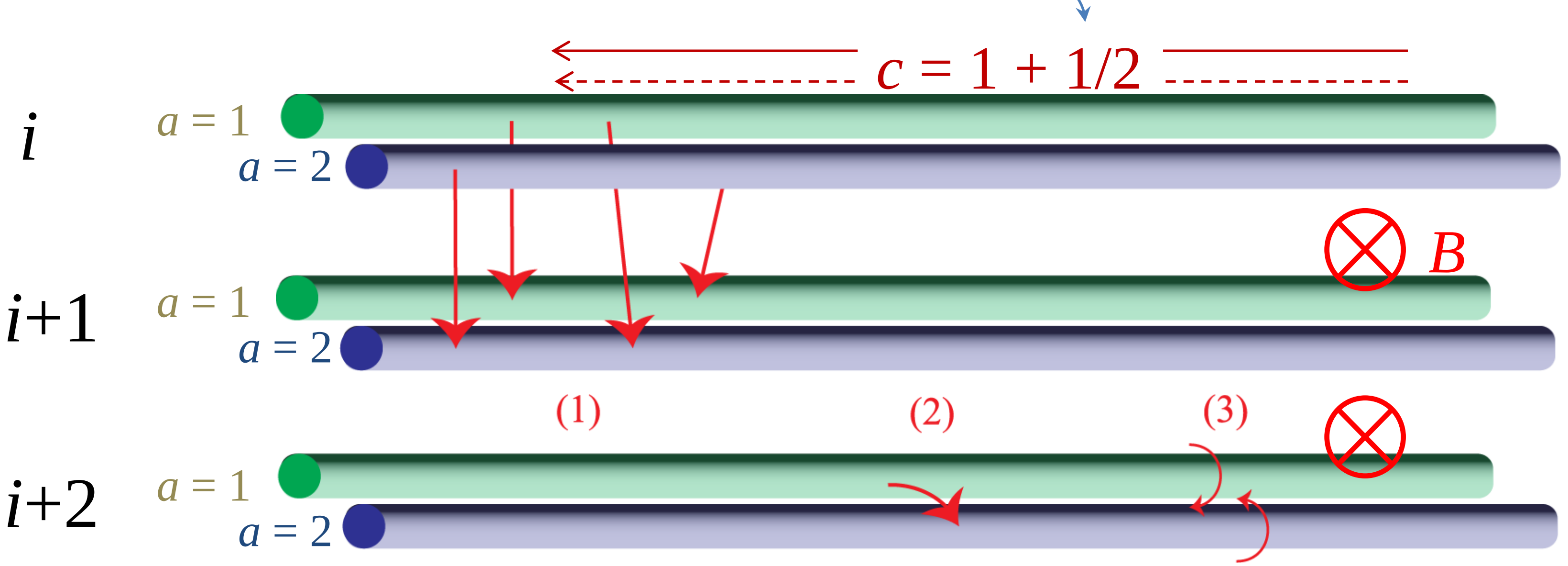
$$V^{(2)} + V^{(3)} = (u+v)\eta_i^R \eta_i^L$$

JYT, Kane, 11

Coset Construction

Chiral CFT

$$SU(2)_2 = U(1)_1 \times \mathbb{Z}_2$$



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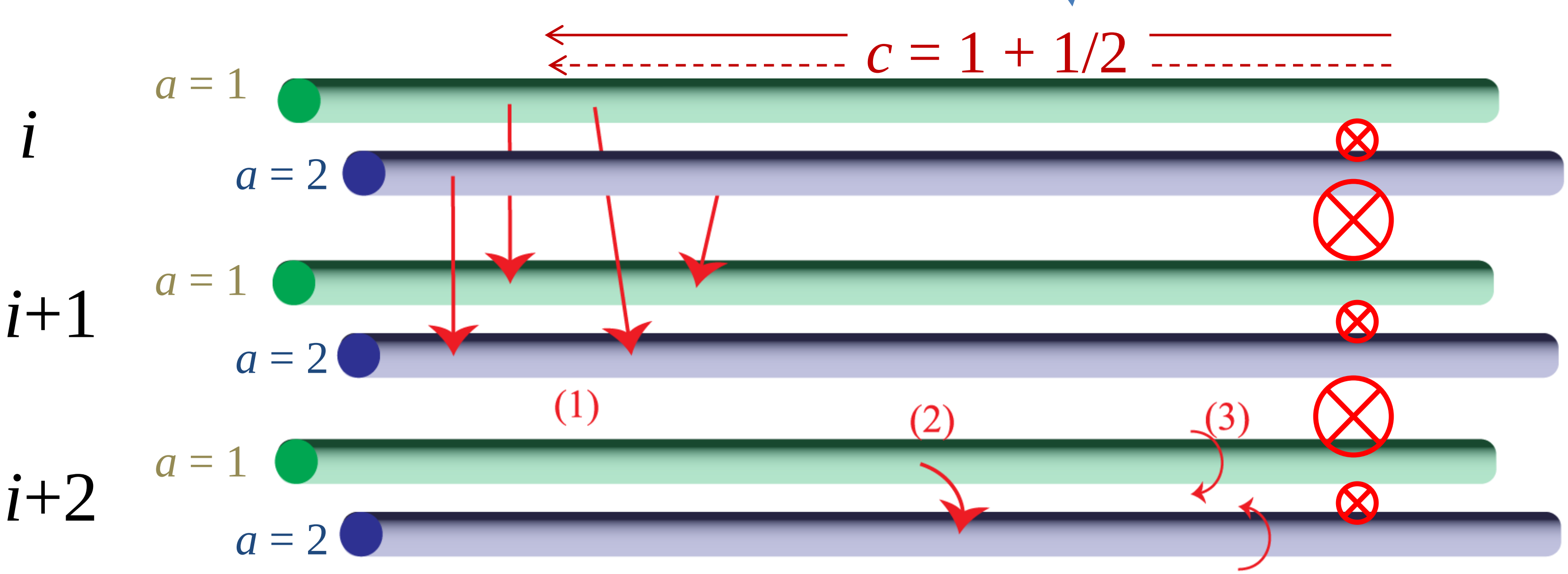
q-Pfaffian States

Chiral CFT

$$SU(2)_2 = U(1)_{1+q} \times \mathbb{Z}_2$$

$$\nu = \frac{1}{1+q}$$

q even for boson
q odd for fermion



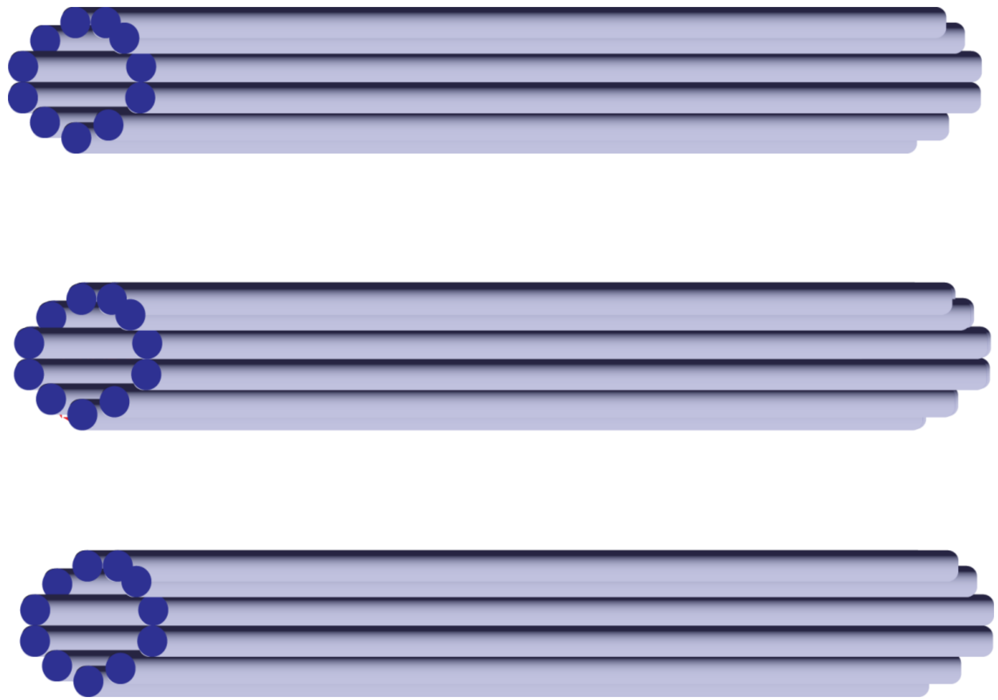
JYT, Kane, 11

Read Rezayi States

i $a = 1,\ldots,k$

$i+1$ $a = 1,\ldots,k$

$i+2$ $a = 1,\ldots,k$



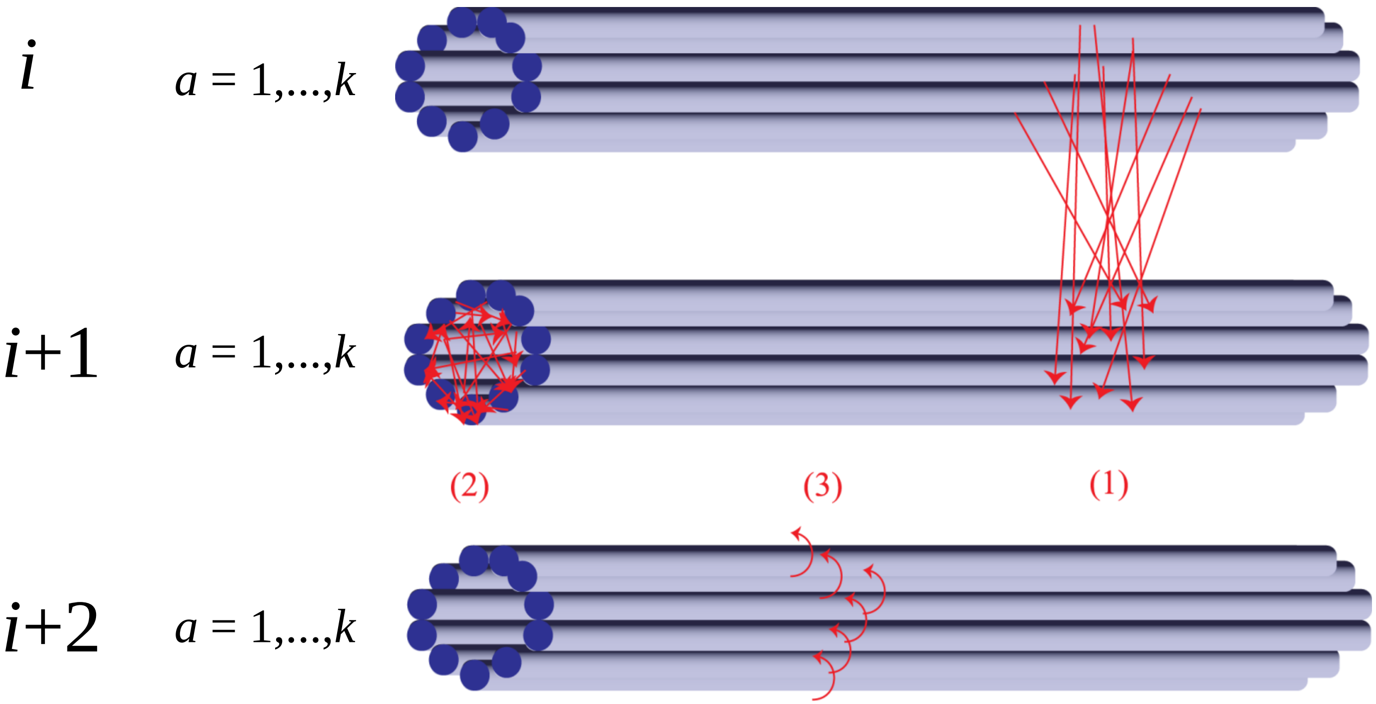
$$\nu = \frac{k}{2 + kq}$$

$$T = T_{U(1)_q} + T_{SU(2)_k/U(1)} + T_{[SU(2)_1^k]/SU(2)_k}$$

Central charge $$k = 1 + \frac{2(k-1)}{k+2} + \frac{k(k-1)}{k+2}$$

JYT, Kane, 11

Read Rezayi States



$$\nu = \frac{k}{2+kq}$$

$$\phi_{i,a}^{R/L} = \frac{1}{\sqrt{k}}\phi_{i,\rho}^{R/L} + \vec{d}_a \cdot \vec{\phi}_{i,\sigma}^{R/L} \qquad \vec{\phi}_{\sigma}^{R/L} = \vec{\varphi}_\sigma \pm \vec{\theta}_\sigma$$

(k - 1) - vector

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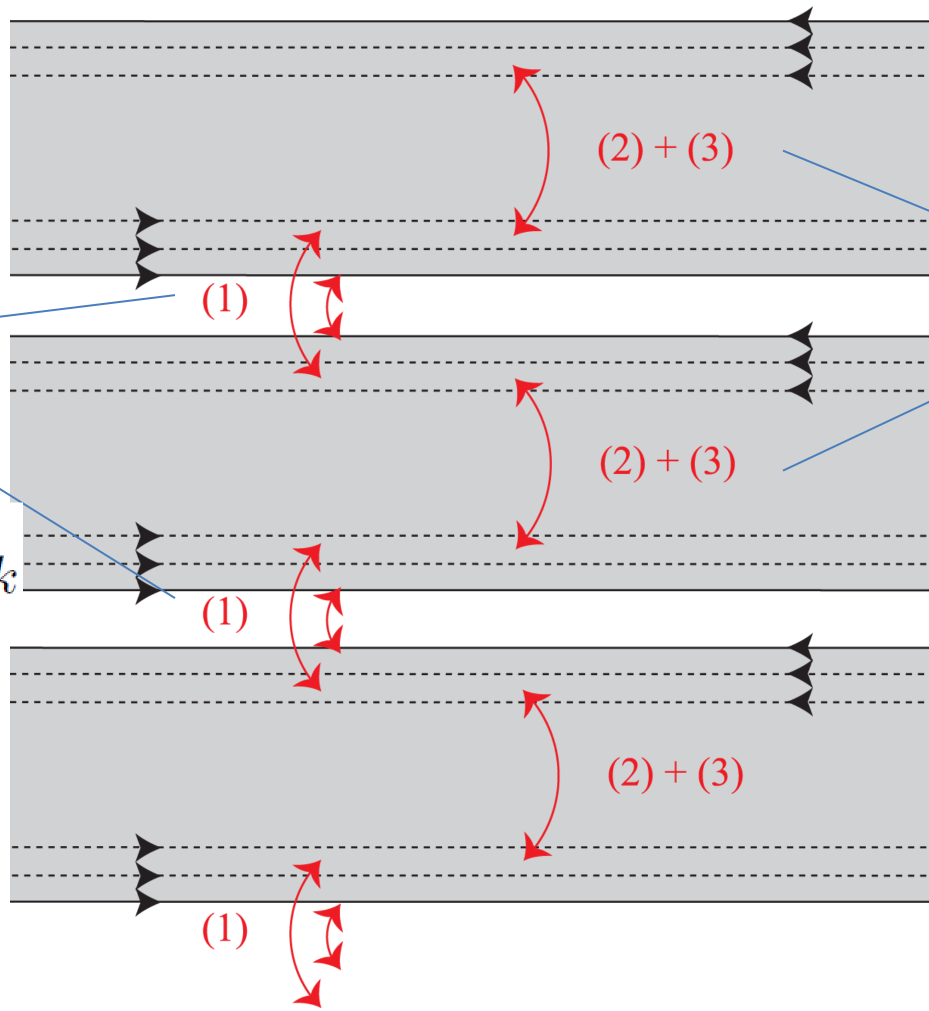
Read Rezayi States

Chiral CFT on edge: charge + $\mathbb{Z}_k$ neutral mode



Gap out

$$SU(2)_k = U(1)_q \times \mathbb{Z}_k$$

Gap out

$$\frac{{SU(2)_1}^k}{SU(2)_k}$$

JYT, Kane, 11

Conclusion

Abelian bosonization of coupled electron wires leads to:

- Abelian FQH:

  nearest wire interaction => Laughlin states

  next nearest interaction => Hierarchy states

- Non-Abelian FQH:

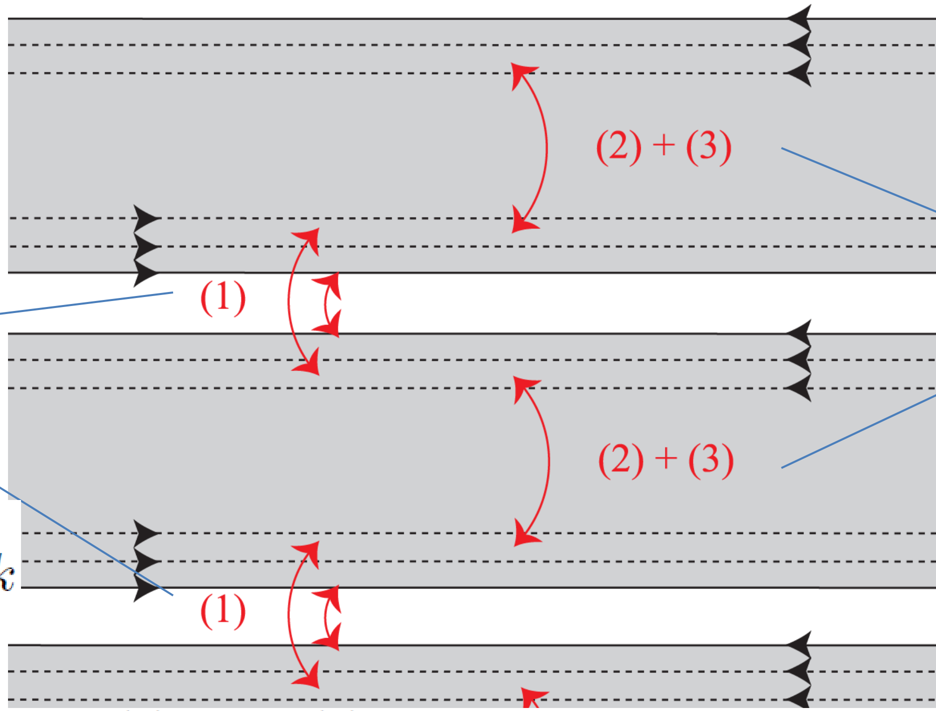
  conformal sectors gapped out separately by inter-bundle and intra-bundle coupling

  => Moore Read, Read Rezayi states

Outlook:

- Other FQH states
- Fractional Chern insulator, Fractional topological insulator

Read Rezayi States



Gap out

$SU(2)_k$

$= U(1)_q \times \mathbb{Z}_k$

Gap out

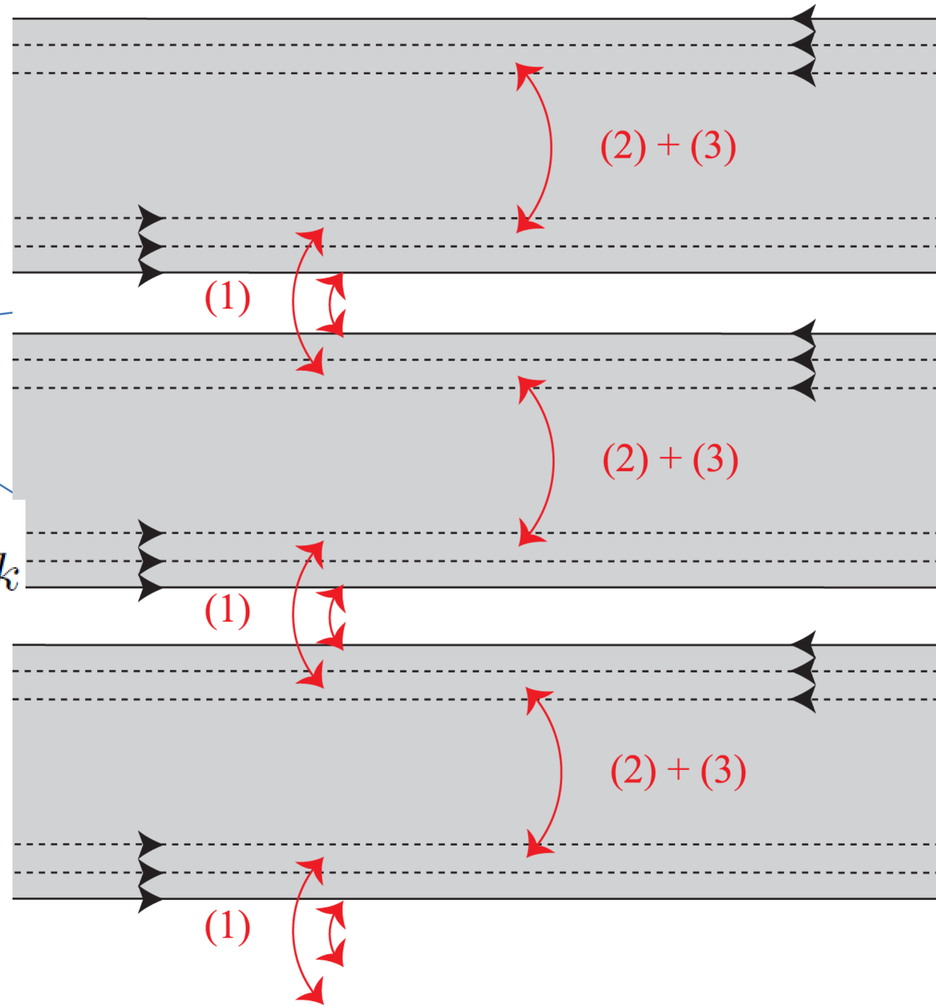
$$\frac{{SU(2)_1}^k}{SU(2)_k}$$

$\mathbb{Z}_k$ - model

$$\begin{aligned} \mathcal{H}_\sigma &= \mathcal{H}_\sigma^0 + V^{(2)} + V^{(3)} \\ &= \frac{v_f}{2\pi}\left[(\partial_x \vec{\theta}_\sigma)^2 + (\partial_x \vec{\varphi}_\sigma)^2\right] \\ &\quad + u\sum_{ab} \cos\sqrt{2}(\vec{d}_a - \vec{d}_b)\cdot\vec{\theta}_\sigma + v\sum_{ab}\cos\sqrt{2}(\vec{d}_a - \vec{d}_b)\cdot\vec{\varphi}_\sigma \end{aligned}$$

JYT, Kane, 11

Read Rezayi States



Gap out

$$SU(2)_k = U(1)_q \times \mathbb{Z}_k$$

Gap out

$$\frac{SU(2)_1{}^k}{SU(2)_k}$$

$$V^{(1)} = t\mathbf{J}_i^L \cdot \mathbf{J}_{i+1}^R$$

JYT, Kane, 11