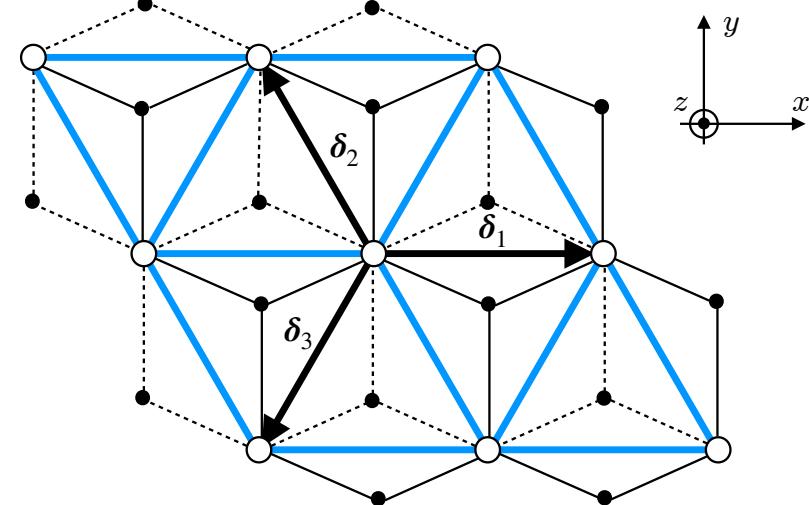
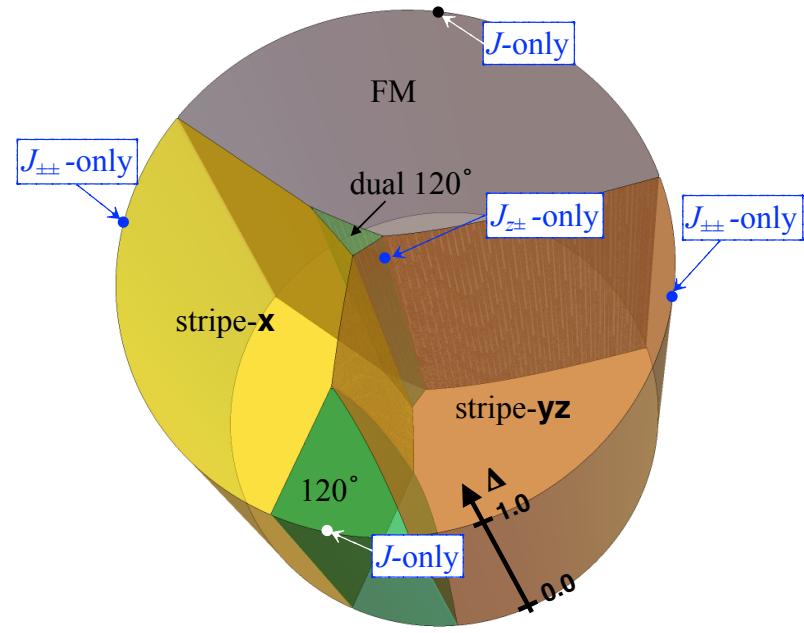


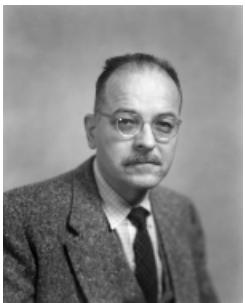
# Triangular-lattice antiferromagnets. Again.

Sasha Chernyshev

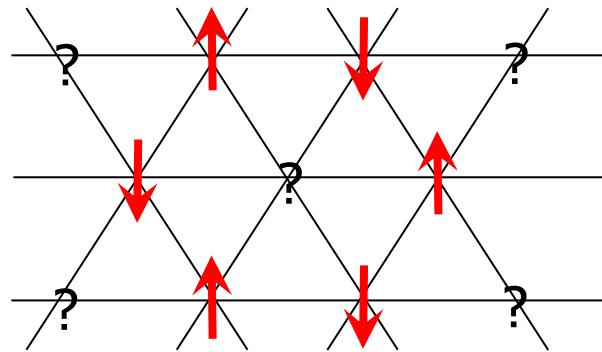
UCIrvine  
University of California, Irvine



# frustrated magnets: triangular foundation

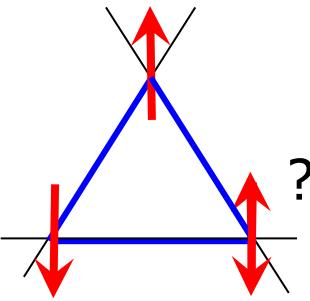


Ising

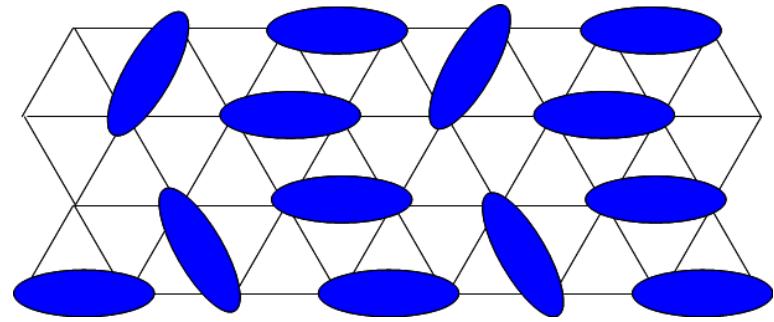


no unique ground state exists

G. H. Wannier, *Phys. Rev.* **79**, 357 (1950).



Heisenberg



exotic state **may** exist

P. W. Anderson, *Mat. Res. Bul.* **8**, 153 (1973).  
P. Fazekas and P. W. Anderson, *Phil Mag* **30**, 23 (1974).



# triangular AFs: deep(er) implications



VOLUME 79, NUMBER 2

JULY 15, 1950

## Antiferromagnetism. The Triangular Ising Net

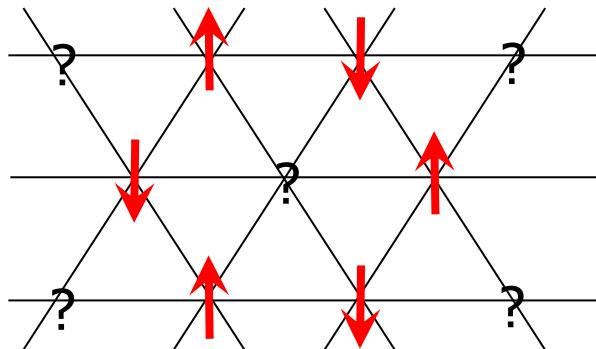
G. H. WANNIER

what it is in the ferromagnetic case. The entropy at absolute zero is finite, it equals

$$S(0) = R \frac{2}{\pi} \int_0^{\pi/3} \ln(2 \cos \omega) d\omega = 0.3383R.$$

The system is disordered at all temperatures and possesses no Curie point.

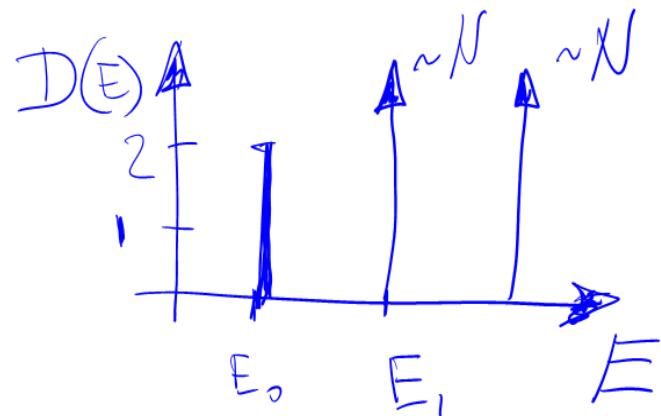
Ising



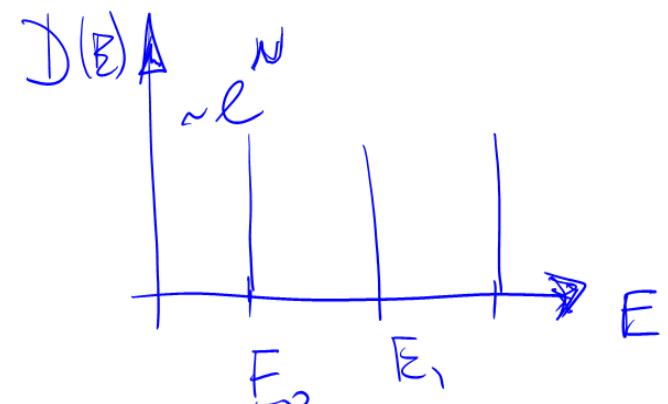
no unique ground state exists

G. H. Wannier, *Phys. Rev.* 79, 357 (1950).

unfrustrated  $\Rightarrow$  nondegenerate



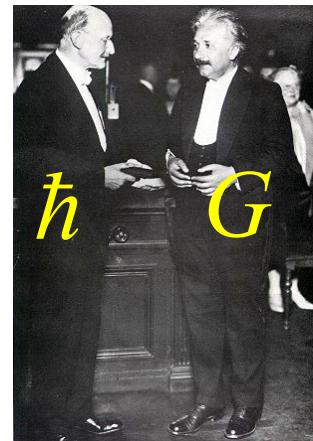
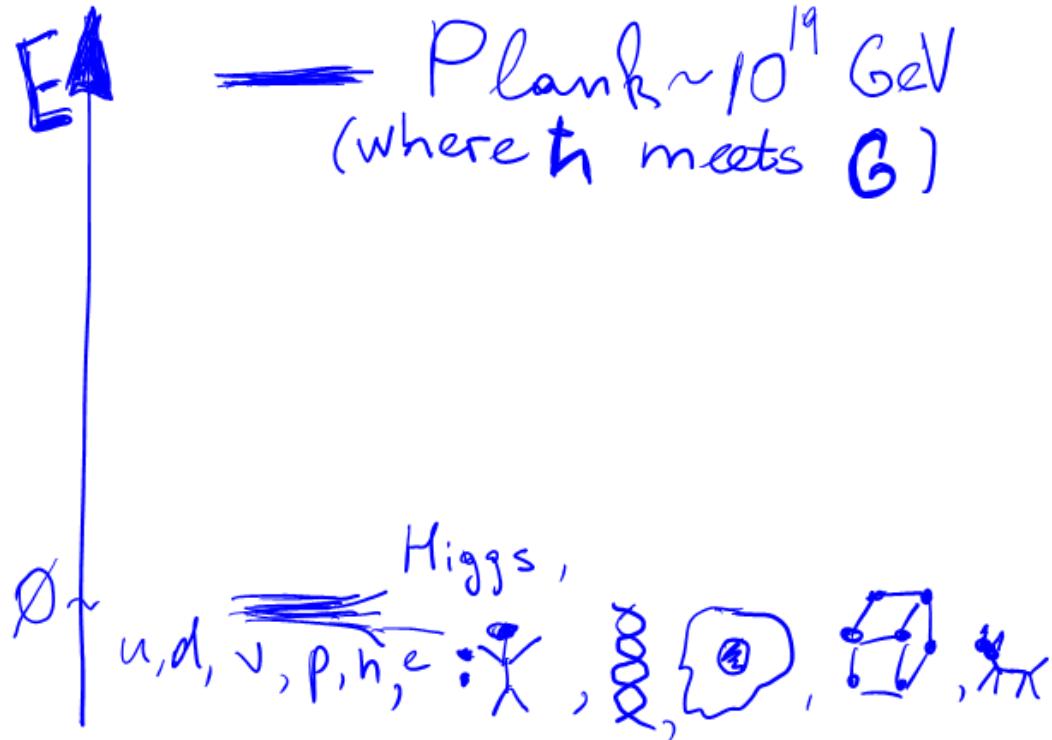
frustrated  $\Rightarrow$  degenerate



frustration yields degeneracy!

# degeneracy: the Bright side

- a parallel  $m_P = \sqrt{\frac{\hbar c}{G}}$



- massive degeneracy at the bottom → lots of possibilities
- fluctuations are important → **order-by-disorder** (Georgii Jackeli's talk, Wednesday)

# curiosities, I

VOLUME 79, NUMBER 2

JULY 15, 1950

## Antiferromagnetism. The Triangular Ising Net

G. H. WANNIER

what it is in the ferromagnetic case. The entropy at absolute zero is finite; it equals

$$S(0) = R \frac{2}{\pi} \int_0^{\pi/3} \ln(2 \cos \omega) d\omega = 0.3383R$$

The system is disordered at all temperatures and possesses no Curie point.

PHYSICAL REVIEW B

VOLUME 7, NUMBER 11

1 JUNE 1973

## ERRATA

---

Antiferromagnetism. The Triangular Ising Net,  
G. H. Wannier [Phys. Rev. 79, 357 (1950)]. It  
was kindly pointed out to me recently by Meijer<sup>1</sup>  
that the energy-versus-temperature plots differ by  
a small amount from similar plots constructed

where Houtappel needs three. The same page con-  
tains an incorrect number for the zero-point en-  
tropy of the antiferromagnetic net. The number in  
Eq. (37c) is 0.323066; the series given there is  
correct. Both corrections do not change the major



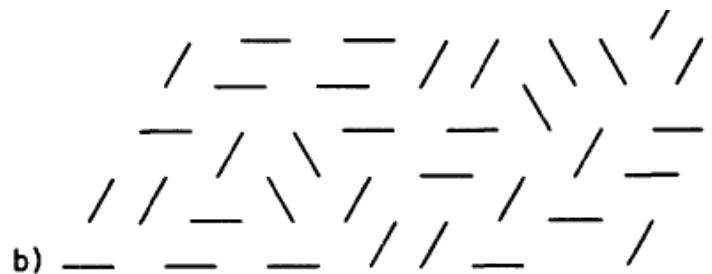
# curiosities, II

Mat. Res. Bull. Vol. 8, pp. 153-160, 1973.

## RESONATING VALENCE BONDS: A NEW KIND OF INSULATOR?\*

P. W. Anderson

The possibility of a new kind of electronic state is pointed out, corresponding roughly to Pauling's idea of "resonating valence bonds" in metals. As observed by Pauling, a pure state of this



\*\*This paper was originally intended for the Pauling Festschrift, Volume 7, Number 11 (November 1972).

Rumer (7) and Pauling (8) have given rules for determining which and how many of these configurations are independent: as we see from eqn. (9), in

7. G. Rumer, Nachr. d. Ges. d. Wiss. zu Gottingen, M. P. Klasse, 337 (1932).
8. L. Pauling, J. Chem. Phys. 1, 280 (1933).

**who is Mr. Rumer??**

P. W. Anderson, *Mat. Res. Bul.* **8**, 153 (1973).

P. Fazekas and P. W. Anderson, *Phil Mag* **30**, 23 (1974).

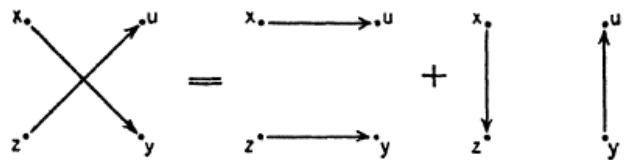
# curiosities, II

Zur Theorie der Spinvalenz.

Von

**Georg Rumer** in Moskau.

Vorgelegt von H. WEYL in der Sitzung am 22. Juli 1932.



APRIL, 1933

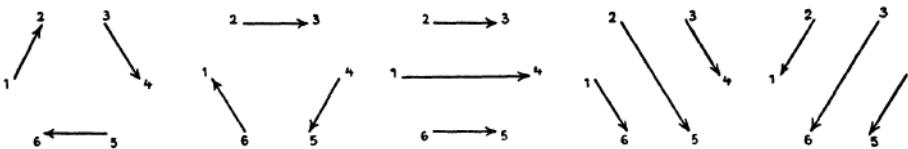
JOURNAL OF CHEMICAL PHYSICS

Eine für die Valenztheorie geeignete Basis  
der binären Vektorinvarianten.

Von

**G. Rumer** (Moskau), **E. Teller** und **H. Weyl** (Göttingen).

Vorgelegt von H. WEYL in der Sitzung am 28. Oktober 1932.



**Yuriii (George) Rumer**

## The Calculation of Matrix Elements for Lewis Electronic Structures of Molecules

LINUS PAULING, *California Institute of Technology*

(Received February 14, 1933)

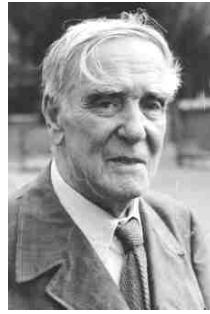
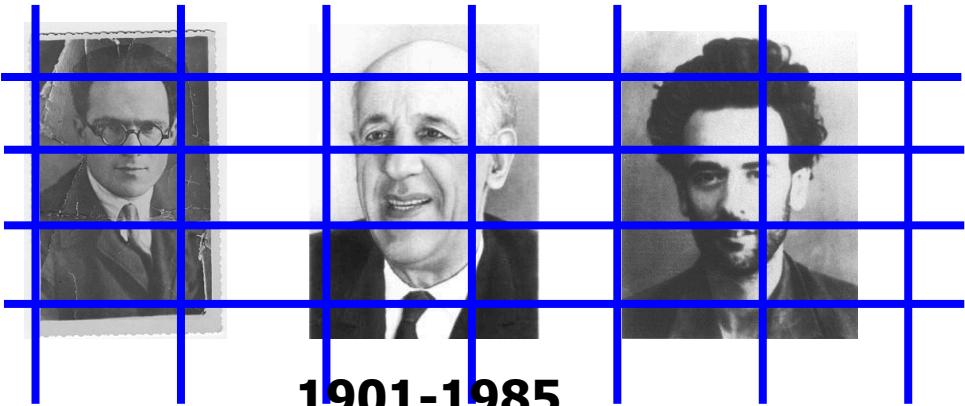
Starting from the discovery by Rumer that the eigenfunctions corresponding to different distributions of valence bonds in a molecule can be represented by plane diagrams which provide information regarding their mutual linear

independence, a very simple graphical method is developed for calculating the coefficients of the integrals occurring in the matrix elements involved in Slater's treatment of the electronic structure of molecules.

- 7. G. Rumer, *Nachr. d. Ges. d. Wiss. zu Gottingen, M. P. Klasse*, 337 (1932).
- 8. L. Pauling, *J. Chem. Phys.* 1, 280 (1933).



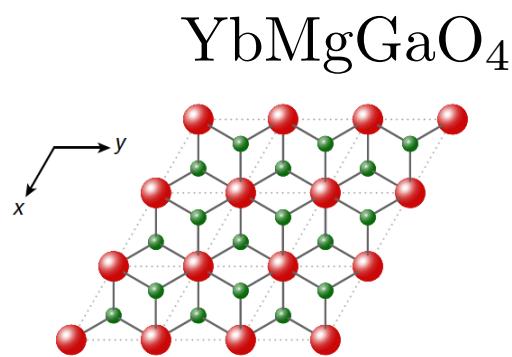
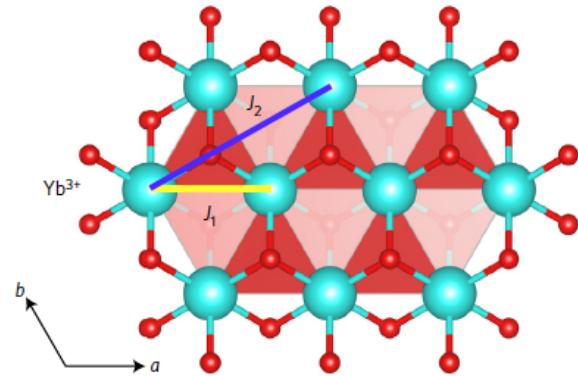
# curiosities, II



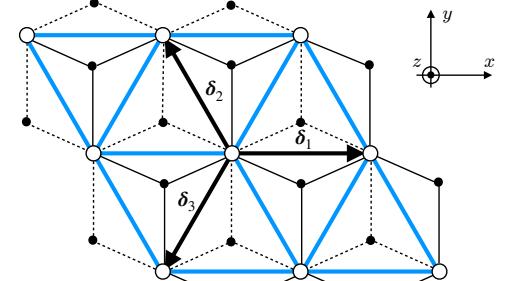
Novosibirsk science center



1953 – founder of (one of the) theory groups



NaYbCh<sub>2</sub> (Ch = O, S, Se)  
CeCd<sub>3</sub>As<sub>3</sub>



# most general\* n-n anisotropic model on the triangular lattice

- no exact (Kitaev-like) solution
  - combination of
    - bond-dependent (anisotropic) interactions, and
    - frustrating [triangular-lattice] geometry
- ⇒ **something interesting**

# the team

UCIrvine  
University of California, Irvine

Zhenyue Zhu



*DMRG guru (AI now)*

Steven White



Pavel Maksimov



*(quasi-)classic guy [now Dubna]*

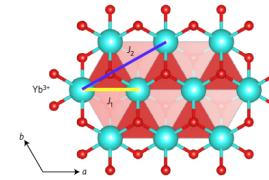
# papers

## I. disorder-induced **mimicry** in YMGO

PRL **119**, 157201 (2017)

PHYSICAL REVIEW LETTERS

week ending  
13 OCTOBER 2017



YbMgGaO<sub>4</sub>



### Disorder-Induced **Mimicry** of a Spin Liquid in YbMgGaO<sub>4</sub>

Zhenyue Zhu, P. A. Maksimov, Steven R. White, and A. L. Chernyshev

## II. phase diagram and **topography** of spin-liquid phase

PHYSICAL REVIEW LETTERS **120**, 207203 (2018)

### Topography of Spin Liquids on a Triangular Lattice

Zhenyue Zhu, P. A. Maksimov, Steven R. White, and A. L. Chernyshev

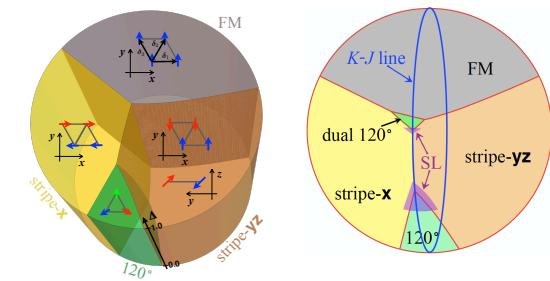


## III. accidental degeneracies, **duality** of spin liquids

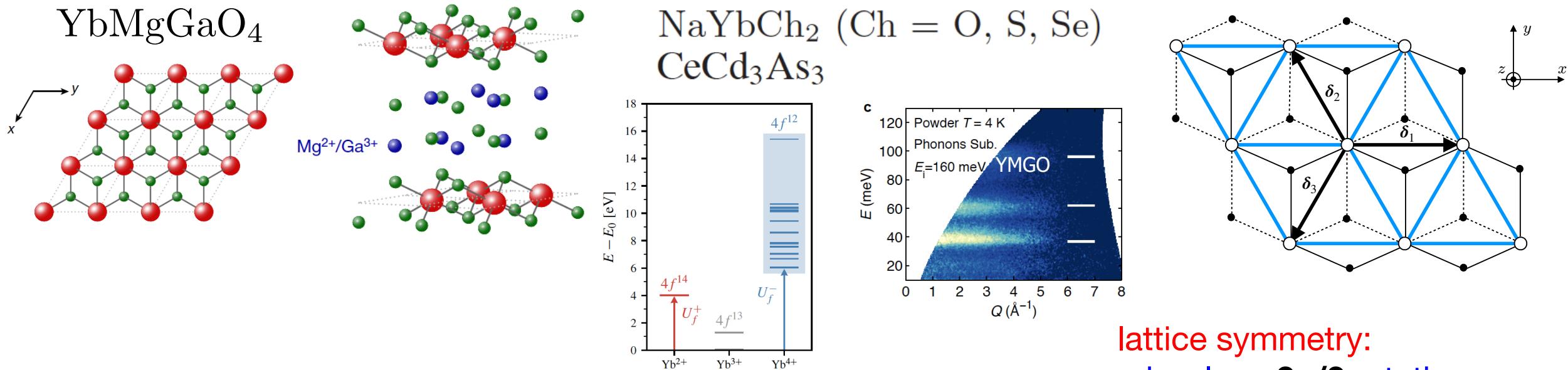
PHYSICAL REVIEW X **9**, 021017 (2019)

### Anisotropic-Exchange Magnets on a Triangular Lattice: Spin Waves, Accidental Degeneracies, and **Dual Spin Liquids**

P. A. Maksimov, Zhenyue Zhu, Steven R. White, and A. L. Chernyshev



# materials, rare-earth: common model



- rare-earth, Yb<sup>3+</sup>,  $J=7/2$ , + crystal-field splitting
- lowest doublet, effective  $S=1/2$
- anisotropic exchanges ... (compare to Heisenberg)
- octahedral environment ...
- lattice symmetries  $\Rightarrow$  four terms in the exchange matrix
- mostly nearest-neighbor exchanges ( $f$ -electrons)

lattice symmetry:

- in-plane  $2\pi/3$  rotation
- $\pi$  rotation around the bond
- site inversion

$$\hat{\mathcal{H}}_{12} = \mathbf{S}_1^0 \begin{pmatrix} J_{xx} & J_{xy} & J_{xz} \\ J_{yx} & J_{yy} & J_{yz} \\ J_{zx} & J_{zy} & J_{zz} \end{pmatrix} \mathbf{S}_2^0$$

# where do these model(s) come from?

$$\mathcal{H} = \sum_{\langle ij \rangle} J_{zz} S_i^z S_j^z + J_{\pm} (S_i^+ S_j^- + S_i^- S_j^+) + J_{\pm\pm} [\gamma_{ij} S_i^+ S_j^+ + \gamma_{ij}^* S_i^- S_j^-] - \frac{i J_{z\pm}}{2} [\gamma_{ij}^* S_i^+ S_j^z - \gamma_{ij} S_i^- S_j^z + (i \leftrightarrow j)]$$

$\gamma_{ij} = 1, e^{2\pi i/3}, e^{-2\pi i/3}$

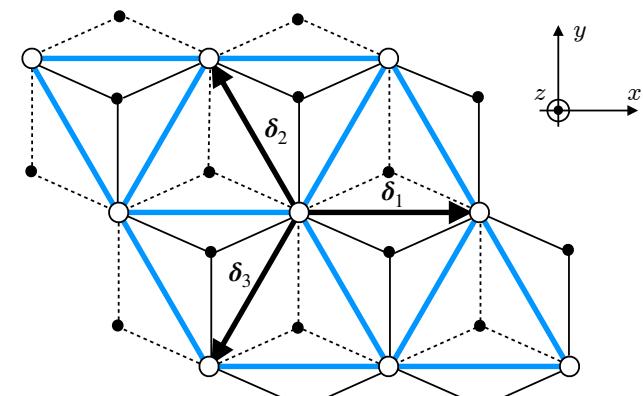
-- ["ice-like"]

$$\mathcal{H} = \sum_{\langle ij \rangle} J_{xy} (S_i^x S_j^x + S_i^y S_j^y) + J_z S_i^z S_j^z + J_c (\mathbf{e}_{ij} \cdot \mathbf{S}_i) (\mathbf{e}_{ij} \cdot \mathbf{S}_j) + J_{z\pm} [\mathbf{z} \cdot (\mathbf{e}_{ij} \times \mathbf{S}_i) S_j^z + \mathbf{z} \cdot (\mathbf{e}_{ij} \times \mathbf{S}_j) S_i^z]$$

-- ["clock/compass-like"]

$$\mathcal{H} = \sum_{\langle ij \rangle \gamma} J_0 \mathbf{S}_i \cdot \mathbf{S}_j + K S_i^\gamma S_j^\gamma + \Gamma (S_i^\alpha S_j^\beta + S_i^\beta S_j^\alpha) + \Gamma' (S_i^\gamma S_j^\alpha + S_i^\gamma S_j^\beta + S_i^\alpha S_j^\gamma + S_i^\beta S_j^\gamma)$$

-- ["generalized Kitaev",  
natural in cubic axes]



- triangular lattice symmetries  $\Rightarrow$  **four** terms in the exchange matrix
- different parametrizations

# model(s)?

$$\mathcal{H} = \sum_{\langle ij \rangle} J_{zz} S_i^z S_j^z + J_{\pm} (S_i^+ S_j^- + S_i^- S_j^+) + J_{\pm\pm} [\gamma_{ij} S_i^+ S_j^+ + \gamma_{ij}^* S_i^- S_j^-] - \frac{i J_{z\pm}}{2} [\gamma_{ij}^* S_i^+ S_j^z - \gamma_{ij} S_i^- S_j^z + (i \leftrightarrow j)]$$

$\gamma_{ij} = 1, e^{2\pi i/3}, e^{-2\pi i/3}$

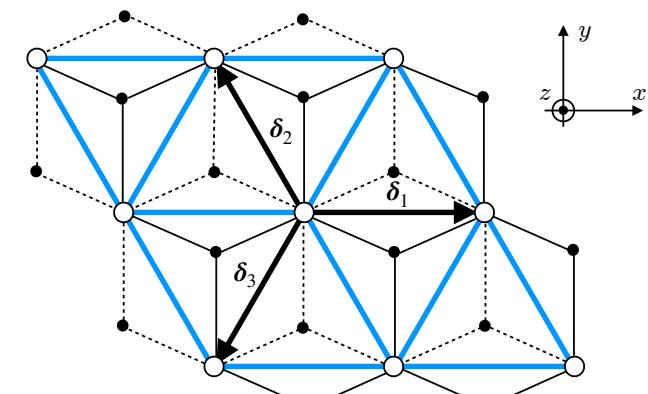
-- ["ice-like"]

$$\mathcal{H} = \sum_{\langle ij \rangle} J_{xy} (S_i^x S_j^x + S_i^y S_j^y) + J_z S_i^z S_j^z + J_c (\mathbf{e}_{ij} \cdot \mathbf{S}_i) (\mathbf{e}_{ij} \cdot \mathbf{S}_j) + J_{z\pm} [\mathbf{z} \cdot (\mathbf{e}_{ij} \times \mathbf{S}_i) S_j^z + \mathbf{z} \cdot (\mathbf{e}_{ij} \times \mathbf{S}_j) S_i^z]$$

-- ["clock/compass-like"]

$$\mathcal{H} = \sum_{\langle ij \rangle \gamma} J_0 \mathbf{S}_i \cdot \mathbf{S}_j + K S_i^\gamma S_j^\gamma + \Gamma (S_i^\alpha S_j^\beta + S_i^\beta S_j^\alpha) + \Gamma' (S_i^\gamma S_j^\alpha + S_i^\gamma S_j^\beta + S_i^\alpha S_j^\gamma + S_i^\beta S_j^\gamma)$$

-- ["generalized Kitaev",  
natural in cubic axes]



- triangular lattice symmetries  $\Rightarrow$  **four** terms in the exchange matrix
- different parametrizations

# what is your $J$ ?

$$\mathcal{H} = \sum_{\langle ij \rangle} [J_{zz} S_i^z S_j^z + J_{\pm} (S_i^+ S_j^- + S_i^- S_j^+) + J_{\pm\pm} (\gamma_{ij} S_i^+ S_j^+ + \gamma_{ij}^* S_i^- S_j^-)] - \frac{i J_{z\pm}}{2} [\gamma_{ij}^* S_i^+ S_j^z - \gamma_{ij} S_i^- S_j^z + (i \leftrightarrow j)]$$

$\gamma_{ij} = 1, e^{2\pi i/3}, e^{-2\pi i/3}$



$$\mathcal{H} = \sum_{\langle ij \rangle} [J_{xy} (S_i^x S_j^x + S_i^y S_j^y) + J_z S_i^z S_j^z] + J_c (\mathbf{e}_{ij} \cdot \mathbf{S}_i) (\mathbf{e}_{ij} \cdot \mathbf{S}_j) + J_{z\pm} [\mathbf{z} \cdot (\mathbf{e}_{ij} \times \mathbf{S}_i) S_j^z + \mathbf{z} \cdot (\mathbf{e}_{ij} \times \mathbf{S}_j) S_i^z]$$

$$\mathcal{H} = \sum_{\langle ij \rangle_\gamma} [J_0 \mathbf{S}_i \cdot \mathbf{S}_j + K S_i^\gamma S_j^\gamma + \Gamma (S_i^\alpha S_j^\beta + S_i^\beta S_j^\alpha) + \Gamma' (S_i^\gamma S_j^\alpha + S_i^\gamma S_j^\beta + S_i^\alpha S_j^\gamma + S_i^\beta S_j^\gamma)]$$

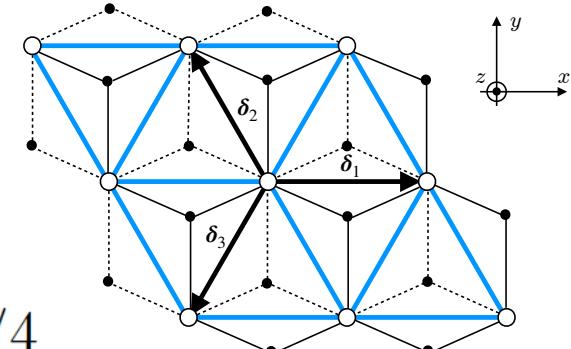
need standardization!

- in all three, “Heisenberg” or XXZ terms **are not** the same
- hence, the answer **“in what model?”? (\*)**
- no standards kept for “-”, “1/2”, etc. for  $J_{\pm\pm}$ - $J_{z\pm}$

# $\text{XXZ-J}_{++}\text{-J}_{z+}$ model

$$\hat{\mathcal{H}} = \sum_{\langle ij \rangle} \mathbf{S}_i^T \hat{\mathbf{J}}_{ij} \mathbf{S}_j$$

bond- $\boldsymbol{\delta}_1$ :  $\hat{\mathcal{H}}_{ij} = \mathbf{S}_i^T \hat{\mathbf{J}}_1 \mathbf{S}_j = \mathbf{S}_i^T \begin{pmatrix} J_{xx} & 0 & 0 \\ 0 & J_{yy} & J_{yz} \\ 0 & J_{zy} & J_{zz} \end{pmatrix} \mathbf{S}_j$



- parametrization:  $J_{zz} = \Delta \cdot J$ ,  $J = (J_{xx} + J_{yy})/2$ ,  $J_{\pm\pm} = (J_{xx} - J_{yy})/4$   
 $J_{zy} = J_{zy} = J_{z\pm}$
- other bonds:  $\hat{\mathbf{J}}_\alpha = \hat{\mathbf{R}}_\alpha^{-1} \hat{\mathbf{J}}_1 \hat{\mathbf{R}}_\alpha$        $\hat{\mathbf{R}}_\alpha = \begin{pmatrix} \cos \tilde{\varphi}_\alpha & \sin \tilde{\varphi}_\alpha & 0 \\ -\sin \tilde{\varphi}_\alpha & \cos \tilde{\varphi}_\alpha & 0 \\ 0 & 0 & 1 \end{pmatrix}$

$$\mathcal{H} = \sum_{\langle ij \rangle} J [S_i^x S_j^x + S_i^y S_j^y + \Delta S_i^z S_j^z] \quad \text{XXZ} \quad \tilde{\varphi}_\alpha = \{0, -2\pi/3, 2\pi/3\}$$

bond-dependent

$$+ 2J_{\pm\pm} [\cos \tilde{\varphi}_\alpha (S_i^x S_j^x - S_i^y S_j^y) - \sin \tilde{\varphi}_\alpha (S_i^x S_j^y + S_i^y S_j^x)]$$

$$+ J_{z\pm} [\cos \tilde{\varphi}_\alpha (S_i^y S_j^z + S_i^z S_j^y) - \sin \tilde{\varphi}_\alpha (S_i^x S_j^z + S_i^z S_j^x)]$$

- XXZ part  $\Rightarrow (*)$ :  $0 < \Delta < 1$
- $\Delta > 1$ , "Ising"-like
- $\Delta < 0$ , less physically motivated?



# $\text{XXZ-J}_{++}\text{-J}_{z+}$ model

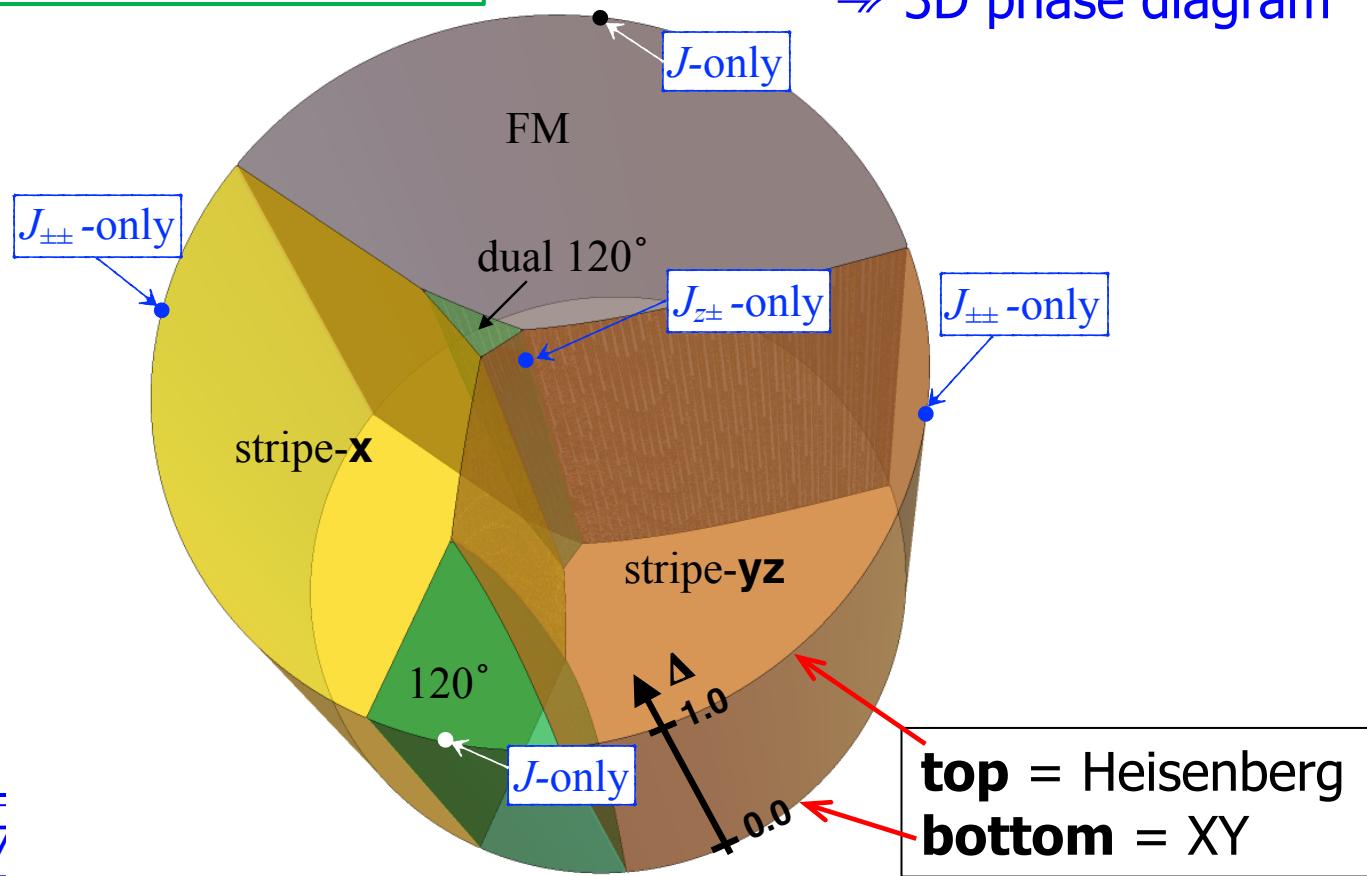
$$\mathcal{H} = \sum_{\langle ij \rangle} J [S_i^x S_j^x + S_i^y S_j^y + \Delta S_i^z S_j^z] \quad \text{xxz} \quad |\tilde{\varphi}_\alpha| = \{0, -2\pi/3, 2\pi/3\}$$

bond-dependent

$$+ 2J_{\pm\pm} [\cos \tilde{\varphi}_\alpha (S_i^x S_j^x - S_i^y S_j^y) - \sin \tilde{\varphi}_\alpha (S_i^x S_j^y + S_i^y S_j^x)]$$

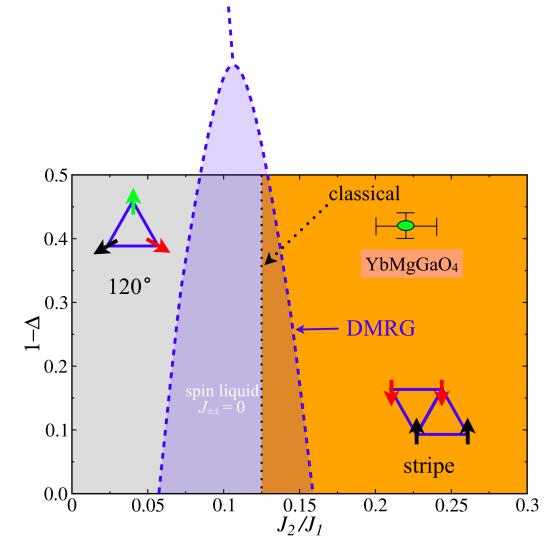
$$+ J_{z\pm} [\cos \tilde{\varphi}_\alpha (S_i^y S_j^z + S_i^z S_j^y) - \sin \tilde{\varphi}_\alpha (S_i^x S_j^z + S_i^z S_j^x)]$$

- XXZ part  $\Rightarrow (*)$ :  $\Delta < 1$
- four parameters  $\Rightarrow$  3D phase diagram



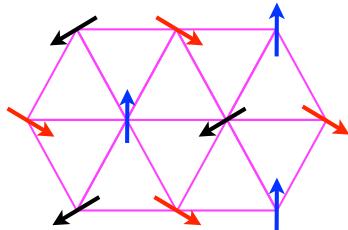
# quasiclassics

hints of an intermediate (non-magnetic?) states

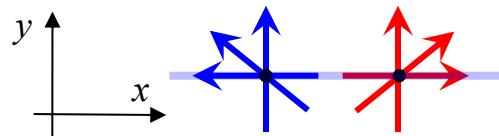


# what do they want to do?

- **what does each term want?**
- XXZ part, for  $0 \leq \Delta \leq 1$ ,  $J_1 > 0$ , wants a **120° state**



- **what do bond-dependent terms want?**
- one bond direction ( $x$ -axis) and only  $J_{\pm\pm}$ -term ( $>0$ )



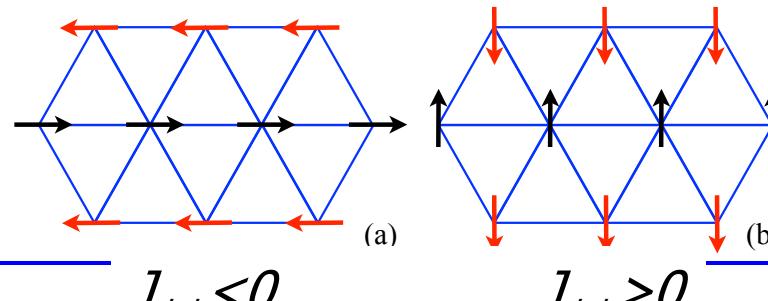
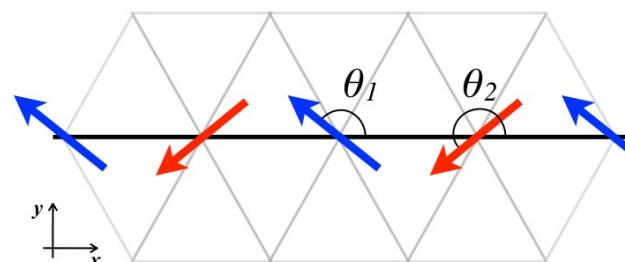
$$+2J_{\pm\pm}(S_1^{x_0}S_2^{x_0} - S_1^{y_0}S_2^{y_0})$$

- “hidden” U(1)  $\Rightarrow$  “counter-rotating” spins in  $x$ - $y$ -plane with  $\theta_1 + \theta_2 = \pi$
- incompatible for different bond directions ( $x$ - $y$ -planes are different)
- “locking”  $\Rightarrow$  **“bond-oriented” stripes**
- one of the bonds is from the “spiral” manifold
- $1+1/2+1/2=2$  out of 3 bonds are “happy”

$$\mathcal{H}_{XXZ} = J \sum_{\langle ij \rangle} (S_i^x S_j^x + S_i^y S_j^y + \Delta S_i^z S_j^z),$$

$$\begin{aligned} \mathcal{H}_{bd} = & \sum_{\langle ij \rangle} 2J_{\pm\pm} (\cos \tilde{\varphi}_\alpha [x, y]_{ij} - \sin \tilde{\varphi}_\alpha \{x, y\}_{ij}) \\ & + J_{z\pm} (\cos \tilde{\varphi}_\alpha \{y, z\}_{ij} - \sin \tilde{\varphi}_\alpha \{x, z\}_{ij}) \end{aligned}$$

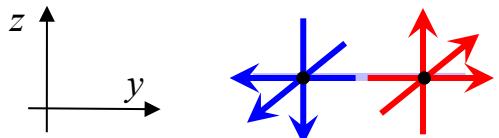
$$\begin{aligned} [a, b]_{ij} &= S_i^a S_j^a - S_i^b S_j^b \\ \{a, b\}_{ij} &= S_i^a S_j^b + S_i^b S_j^a \end{aligned}$$



stripe-x

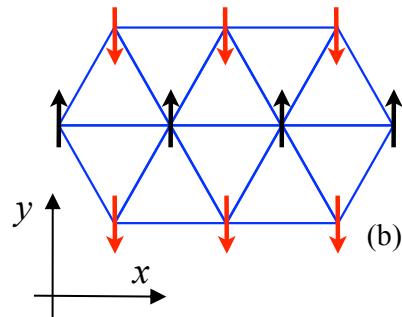
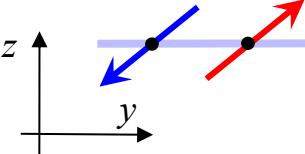
# what do they want to do?

- what do bond-dependent terms want?
- one bond ( $x$ -axis) and only  $J_{z\pm}$ -term ( $\langle \rangle \neq 0$ )



$$J_{z\pm} \left( S_1^{z_0} S_2^{y_0} + S_1^{y_0} S_2^{z_0} \right)$$

- ⇒ “counter-rotating” spins in  $y$ - $z$ -plane with  $\theta_1 + \theta_2 = 3\pi/2$
- ⇒ **stripes tilted out of x-y-plane**
- can be compatible with the ones induced by  $J_{\pm\pm}$ -term



stripe-yz

$$\mathcal{H}_{XXZ} = J \sum_{\langle ij \rangle} (S_i^x S_j^x + S_i^y S_j^y + \Delta S_i^z S_j^z),$$

$$\mathcal{H}_{bd} = \sum_{\langle ij \rangle} 2J_{\pm\pm} \left( \cos \tilde{\varphi}_\alpha [x, y]_{ij} - \sin \tilde{\varphi}_\alpha \{x, y\}_{ij} \right)$$

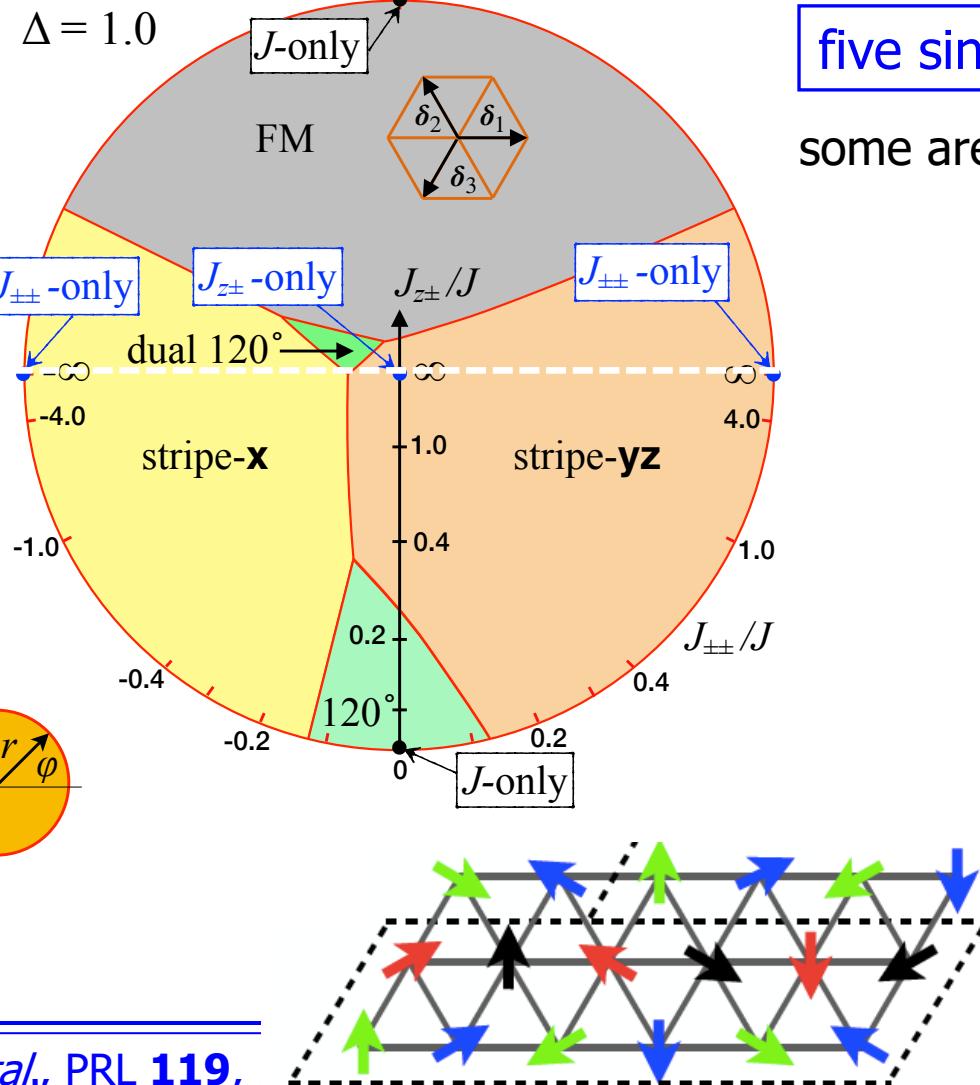
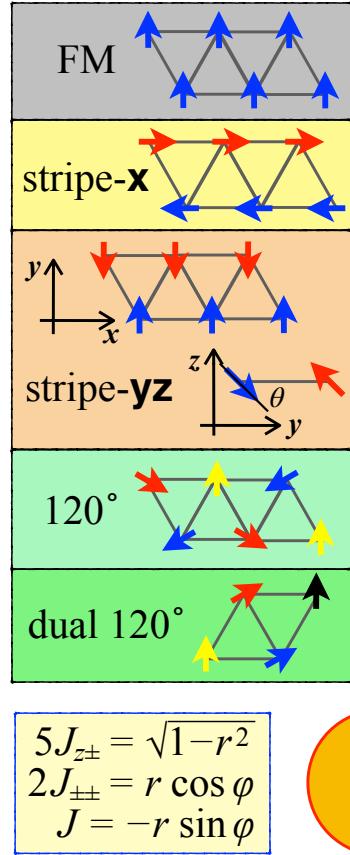
$$+ J_{z\pm} \left( \cos \tilde{\varphi}_\alpha \{y, z\}_{ij} - \sin \tilde{\varphi}_\alpha \{x, z\}_{ij} \right)$$

$$[a, b]_{ij} = S_i^a S_j^a - S_i^b S_j^b$$

$$\{a, b\}_{ij} = S_i^a S_j^b + S_i^b S_j^a$$

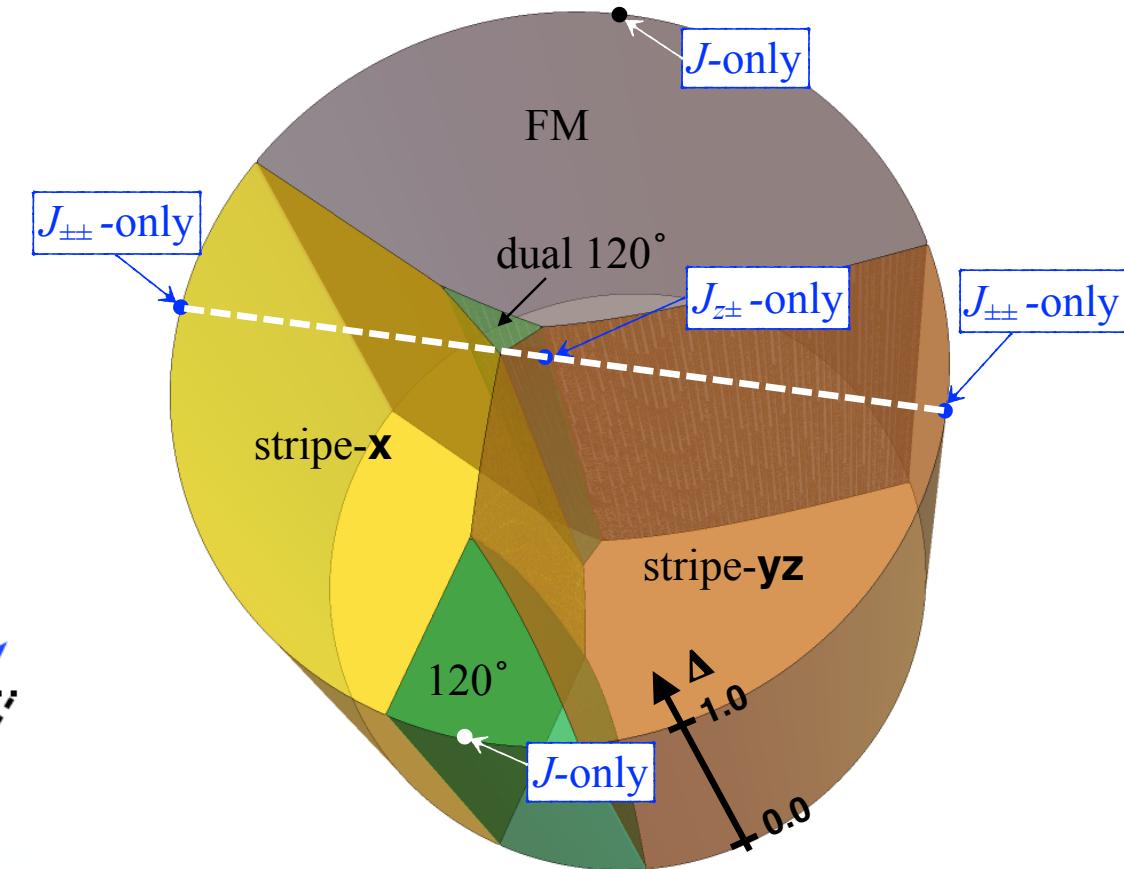
# 0<sup>th</sup>-order phase diagram: classical, single-Q

- parametrization of  $\Delta = \text{const}$  "slice":  $J_{z\pm}$ -radial,  $J_{\pm\pm}$ -polar



five single-Q states

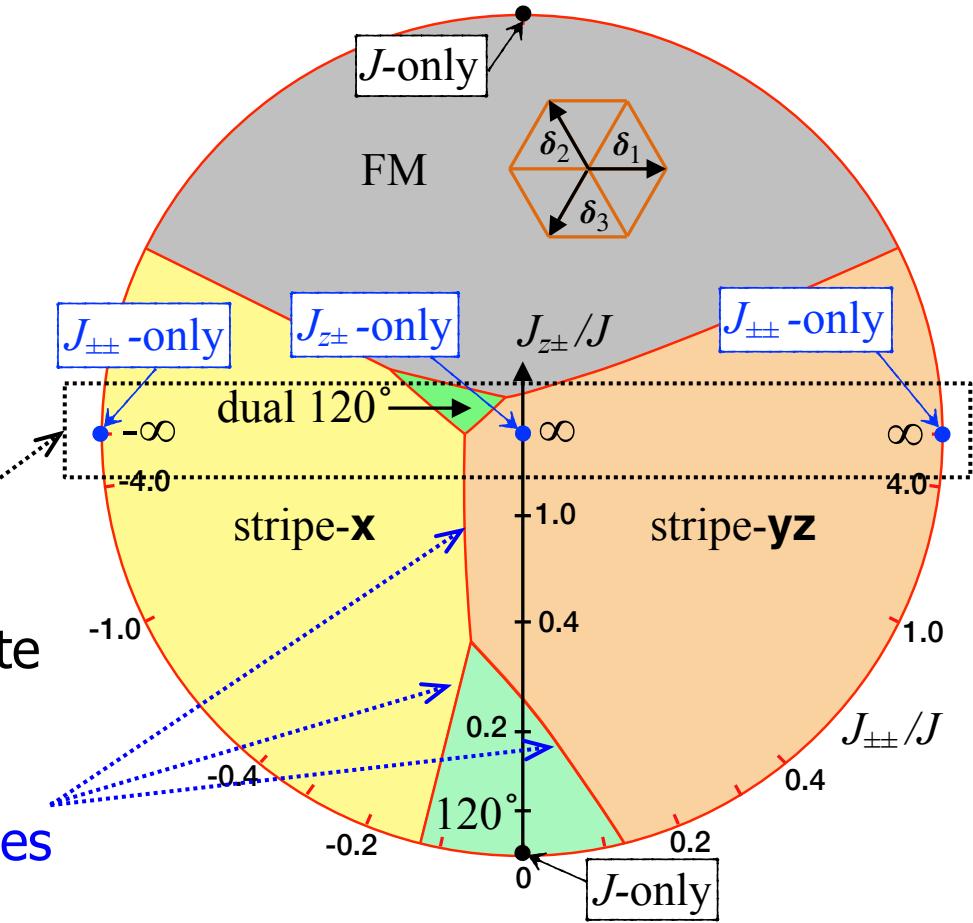
some are obvious, some are not ...



# look for a spin liquid?

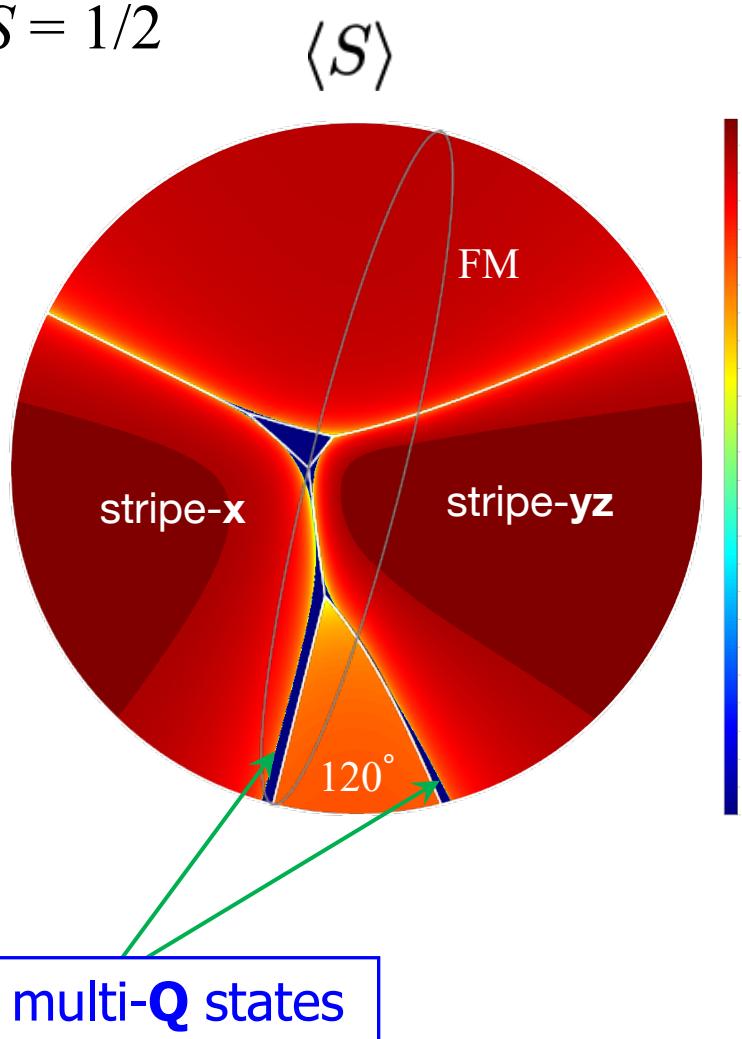
by analogy ...

- **Kitaev-like logic:** bond-dependent terms dominate
- **conventional** logic:  
intermediate regions between ordered phases

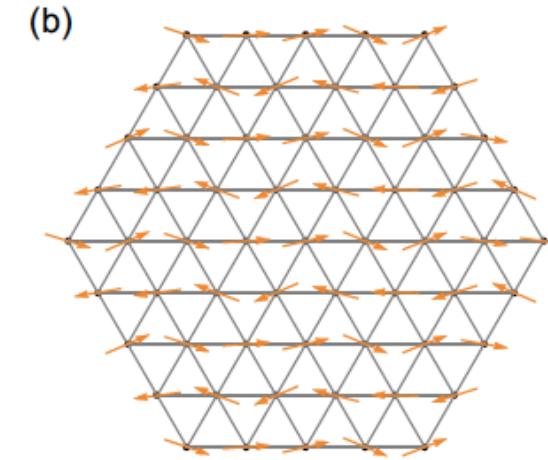
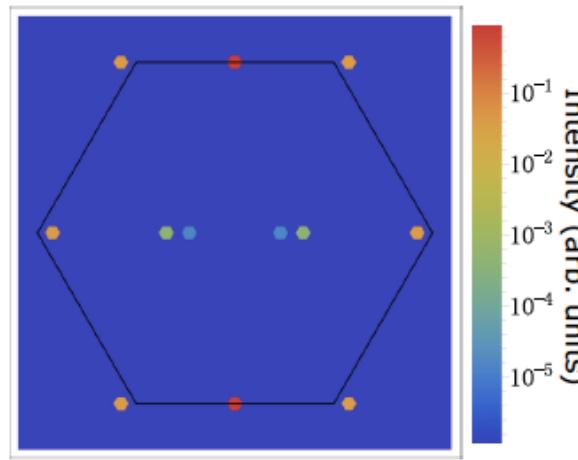


# quasiclassics: strongly-ordered phases ...

$S = 1/2$



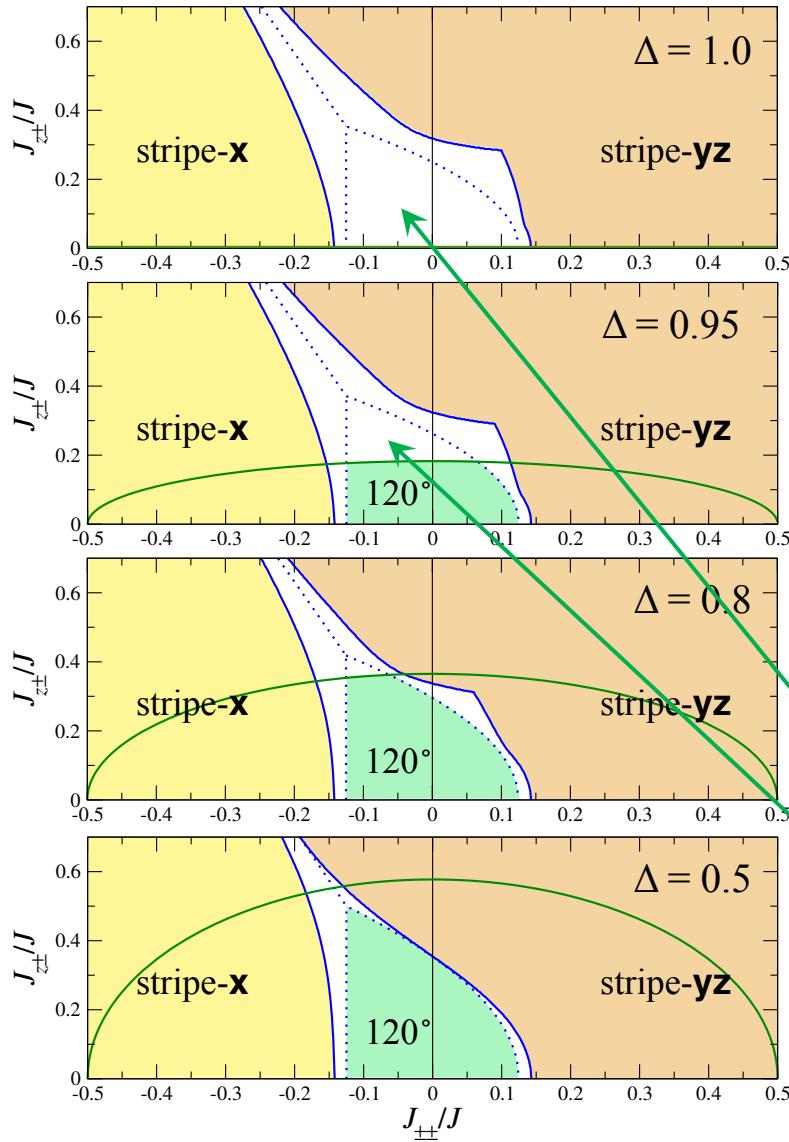
- cut at  $XXZ$ -anisotropy  $\Delta=0.5$  (middle of the cylinder)
- **large ordered moment:**  
leaves little (almost no) hope for a spin liquid ...
- some intermediate multi- $\mathbf{Q}$  states (modulated stripes)
- FM and  $120^\circ$  phases: preserve U(1), order-by-disorder



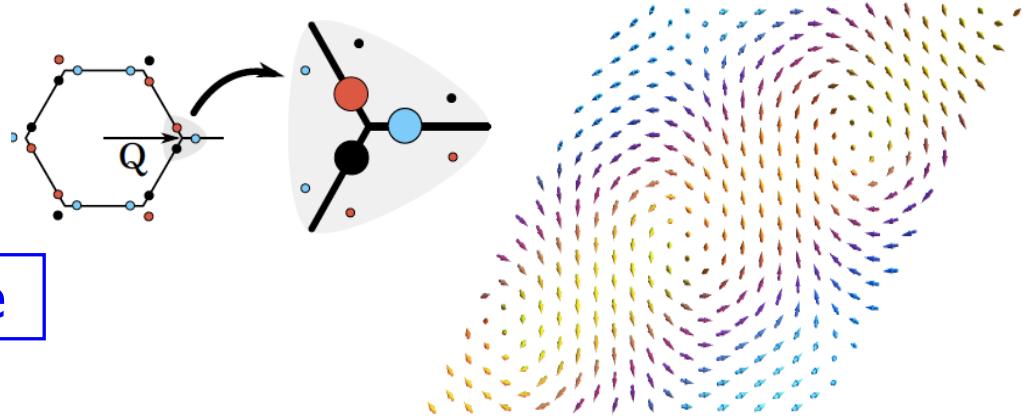
C. Liu, R. Yu, and X. Wang, Phys. Rev. B **95**, 174424 (2016).

- spectrum instability boundaries match numerical (MC), and indicate the same extra  $\mathbf{Q}$  vectors

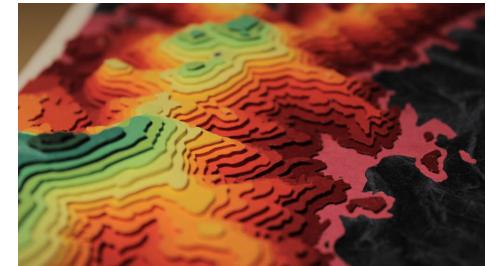
# quasiclassics: spectrum instabilities ...



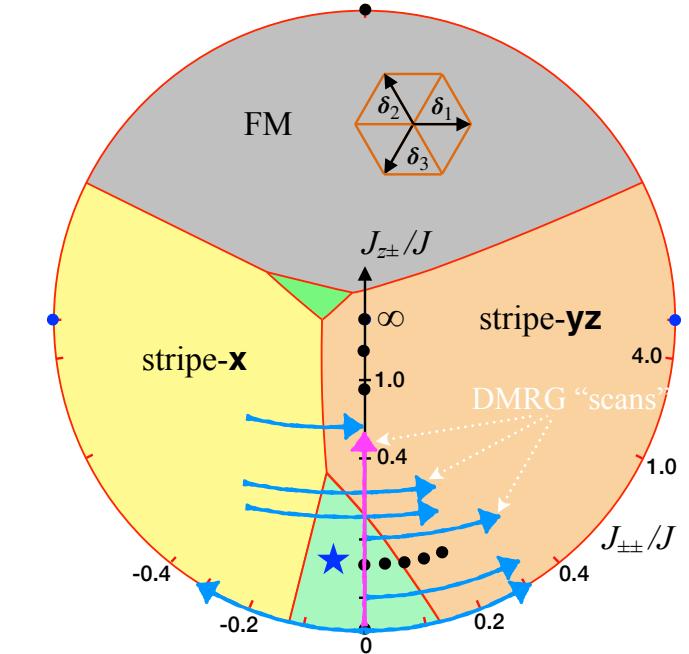
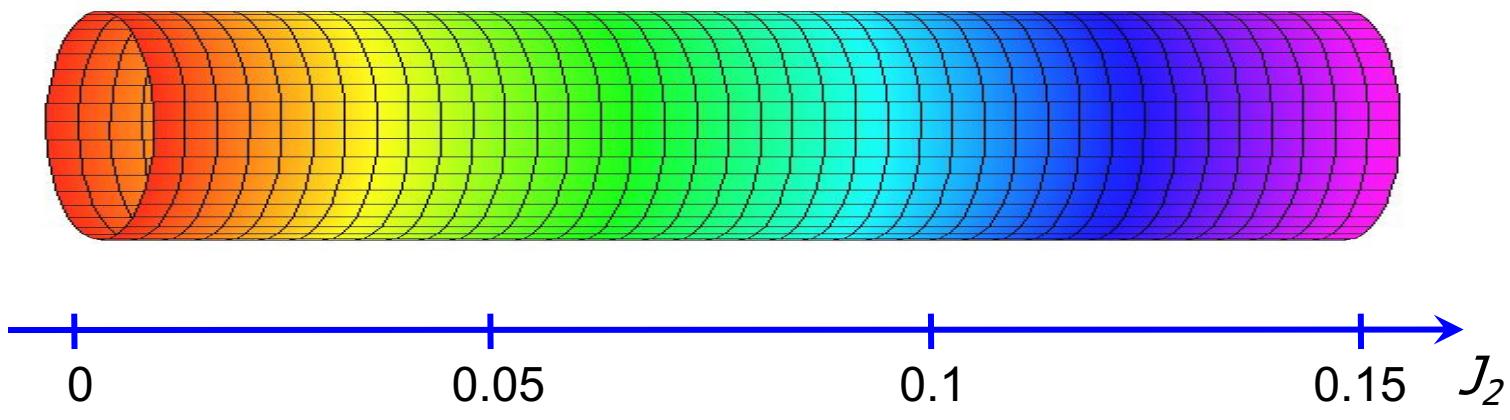
- same cut at  $XXZ$ -anisotropy  $\Delta=0.5$ , Cartesian coordinates
- spectrum instability:  $\varepsilon_k^2 < 0$  at some  $\mathbf{k}$  point(s)
- stripe spectra: unstable within their nominal boundaries (toward multi- $\mathbf{Q}$ , modulated stripes)
- $120^\circ$  spectrum: stable well **beyond** its boundaries
- such overlap of stable spectra of the competing phases: recipe for a **direct transition**
- larger  $\Delta$ , stability to  $J_{z\pm}$  shrinks:  
another multi- $\mathbf{Q}$ , modulated  $120^\circ$  state [ $Z_2$  vortex]
- **may be an opening an SL in  $S=1/2$  case?**



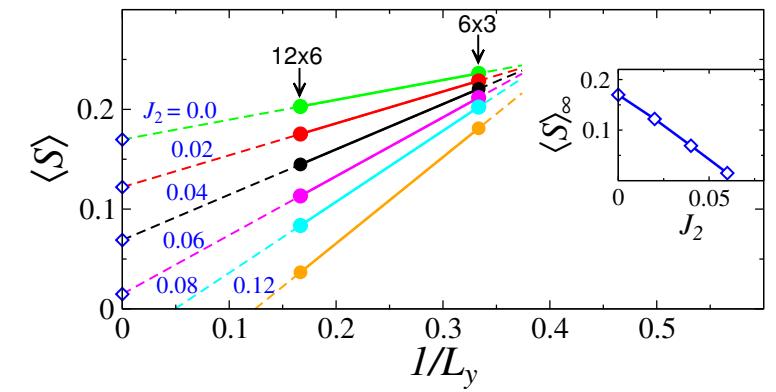
# DMRG



# $S=1/2$ , DMRG: 3D phase space, how?

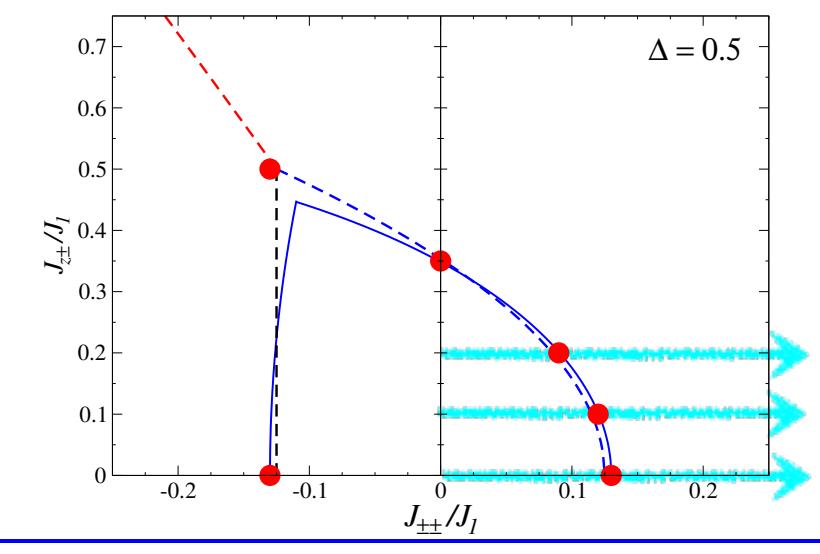
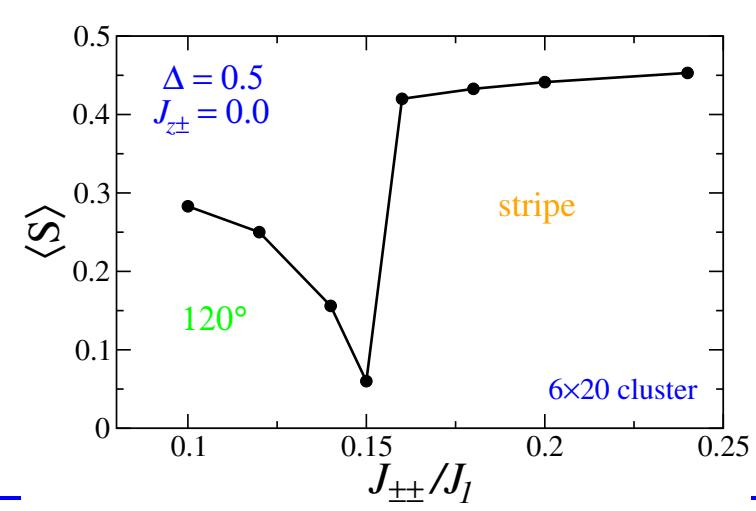
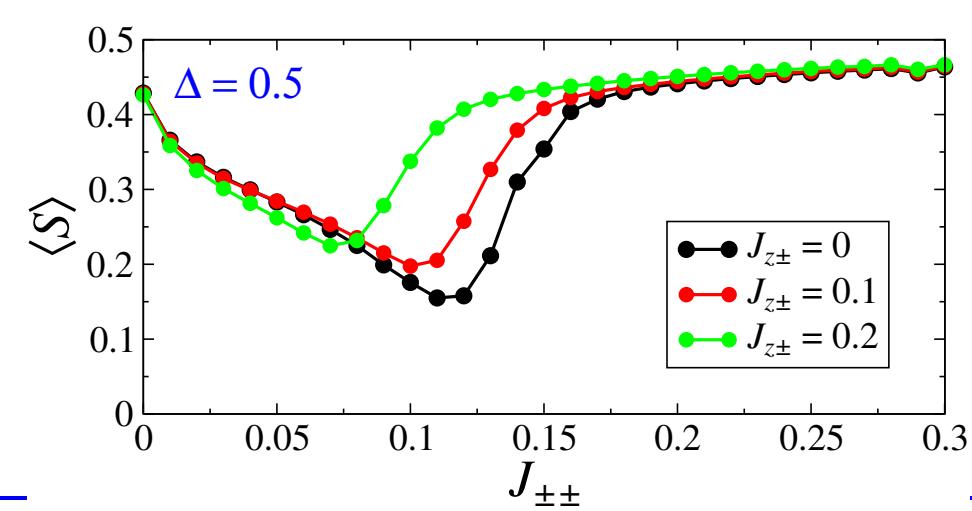
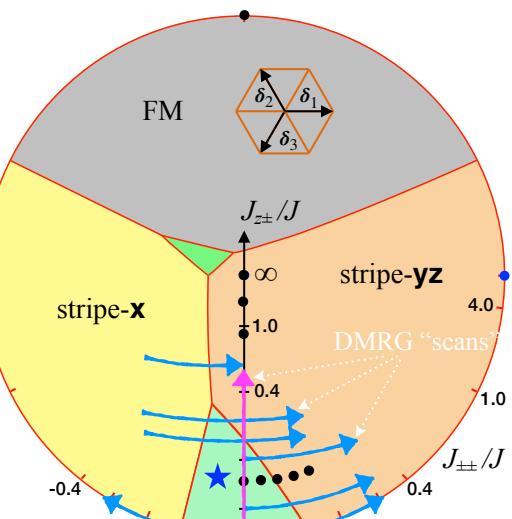
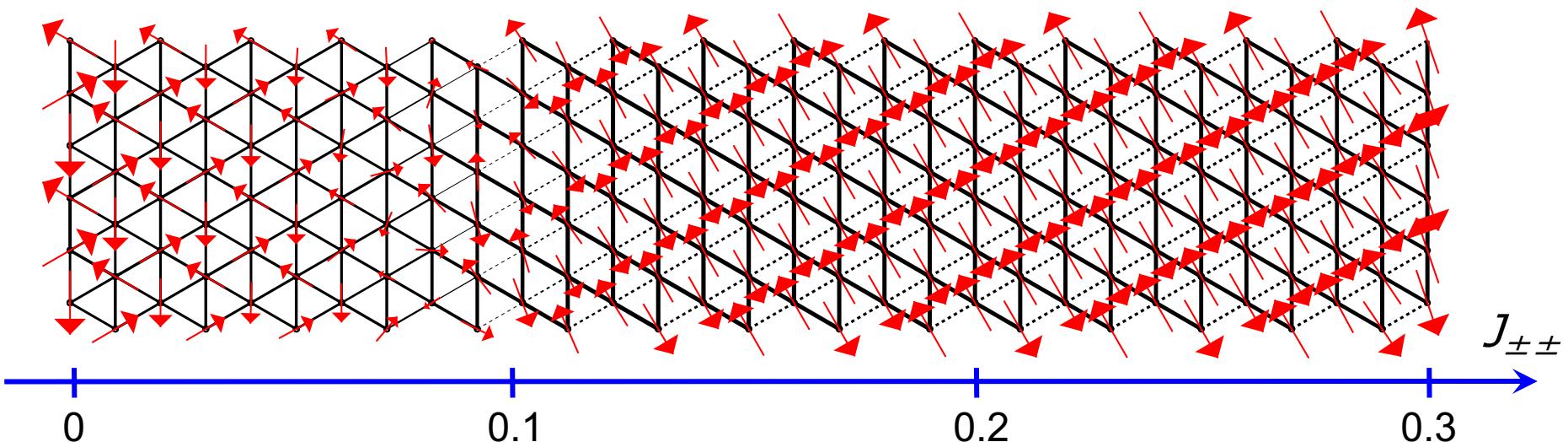


- **1D long-cylinder scans [“scans”]:** vary one parameter, play with BCs/range, read orders
- “all-fixed” parameters [“**non-scans**”], 12x6, 20x6 clusters
- $1/L_y$ -scaling, fixed aspect-ratio clusters
- decay of correlations off boundary, exp vs power-law
- $S(\mathbf{q})$  structure factor

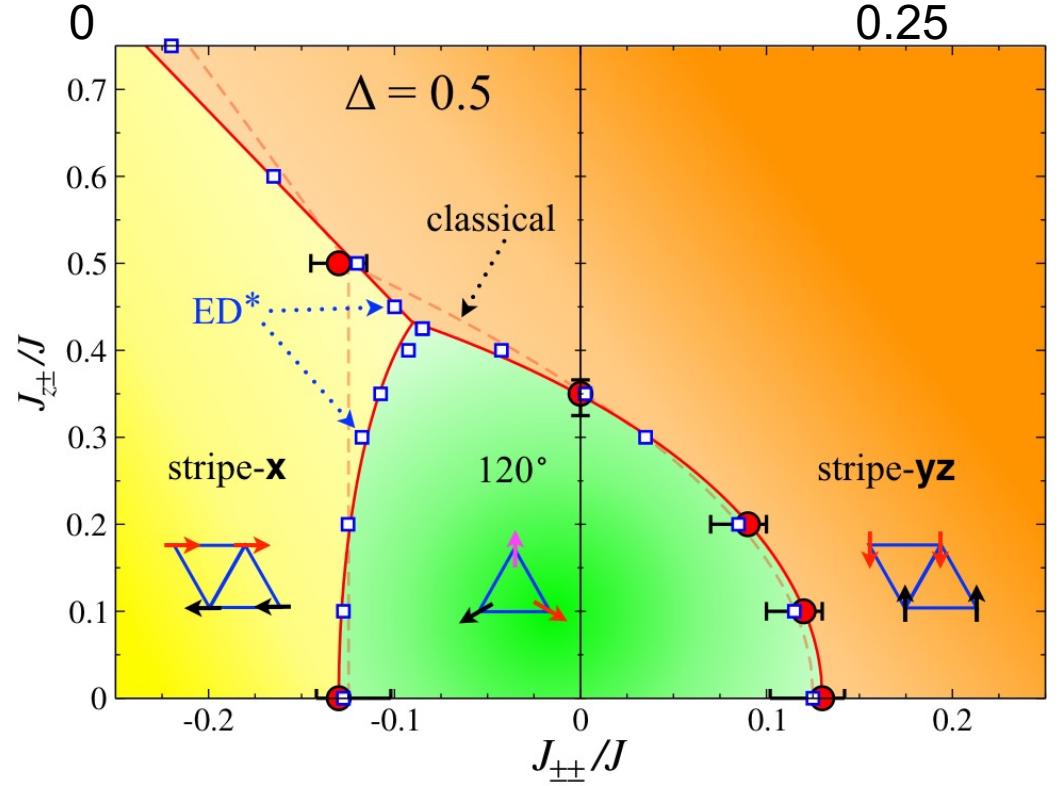
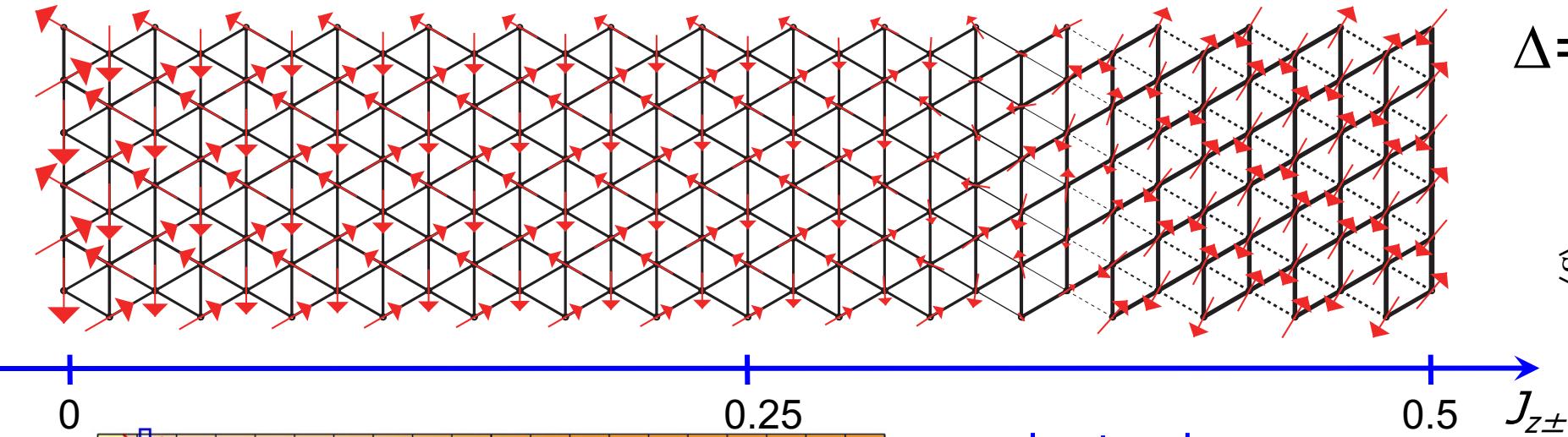


# DMRG “scans”, non-scans, I

- visual assessment; non-scan verification

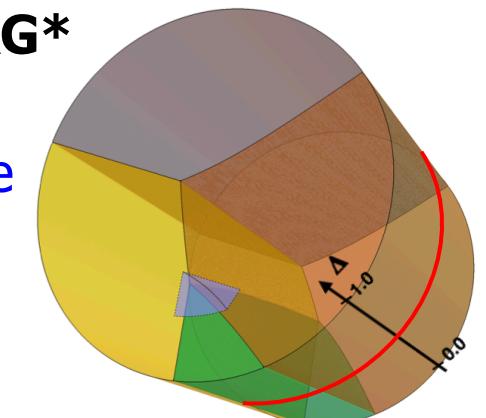
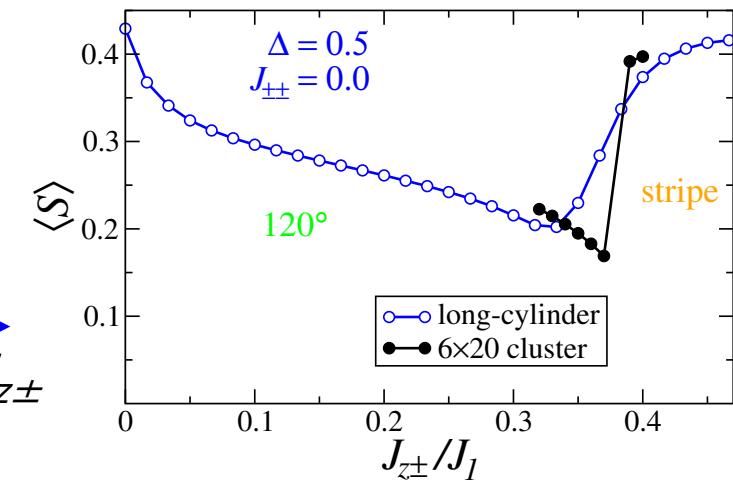


# DMRG “scans”, non-scans, II



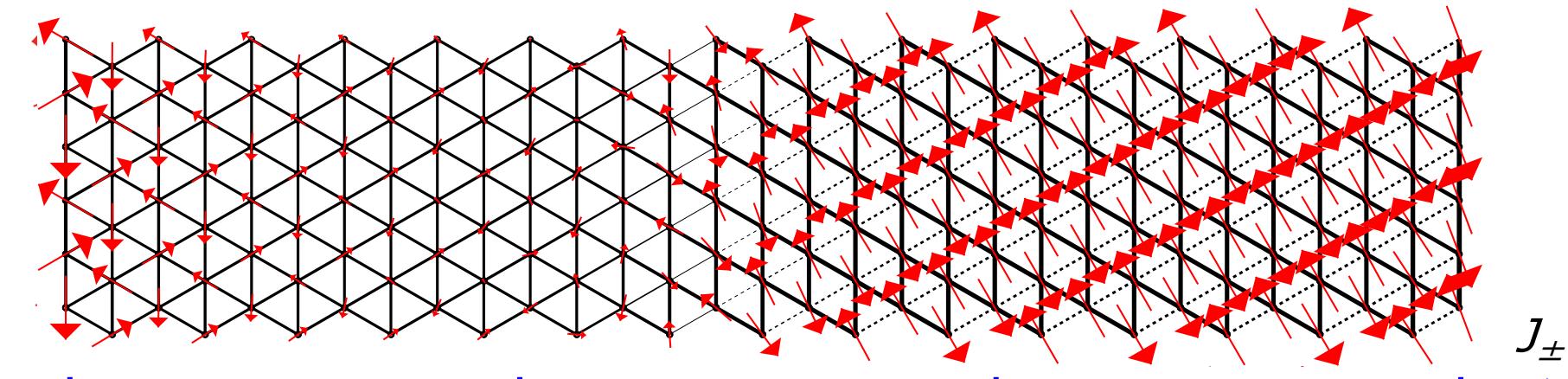
- robust order
- only direct transitions
- close agreement with classical/LSWT
- **in agreement with prior ED/DMRG\***
- continuity: expect  $\Delta < 0.5$  be the same

$\Delta = 0.5$

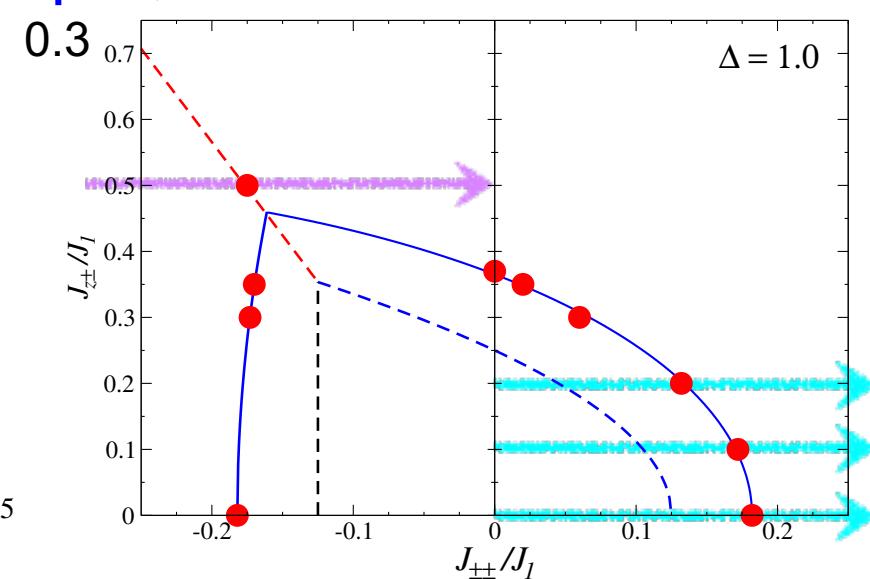
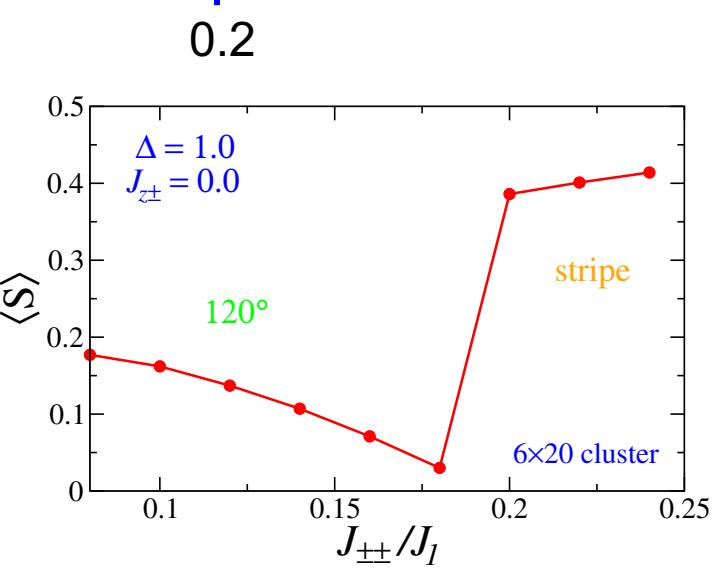
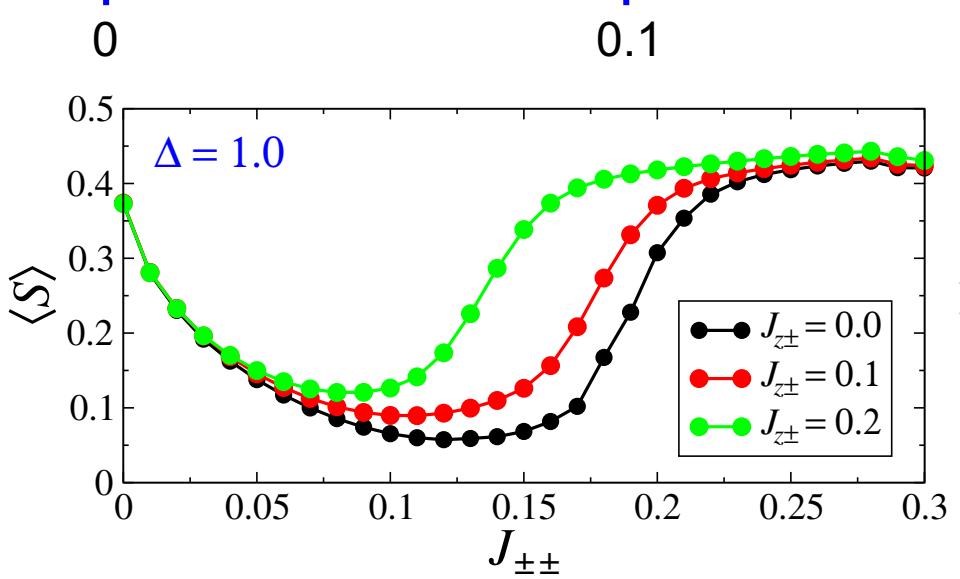
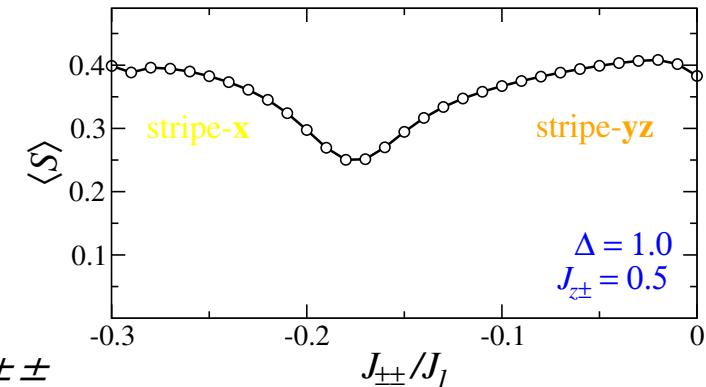


# DMRG, more scans, non-scans

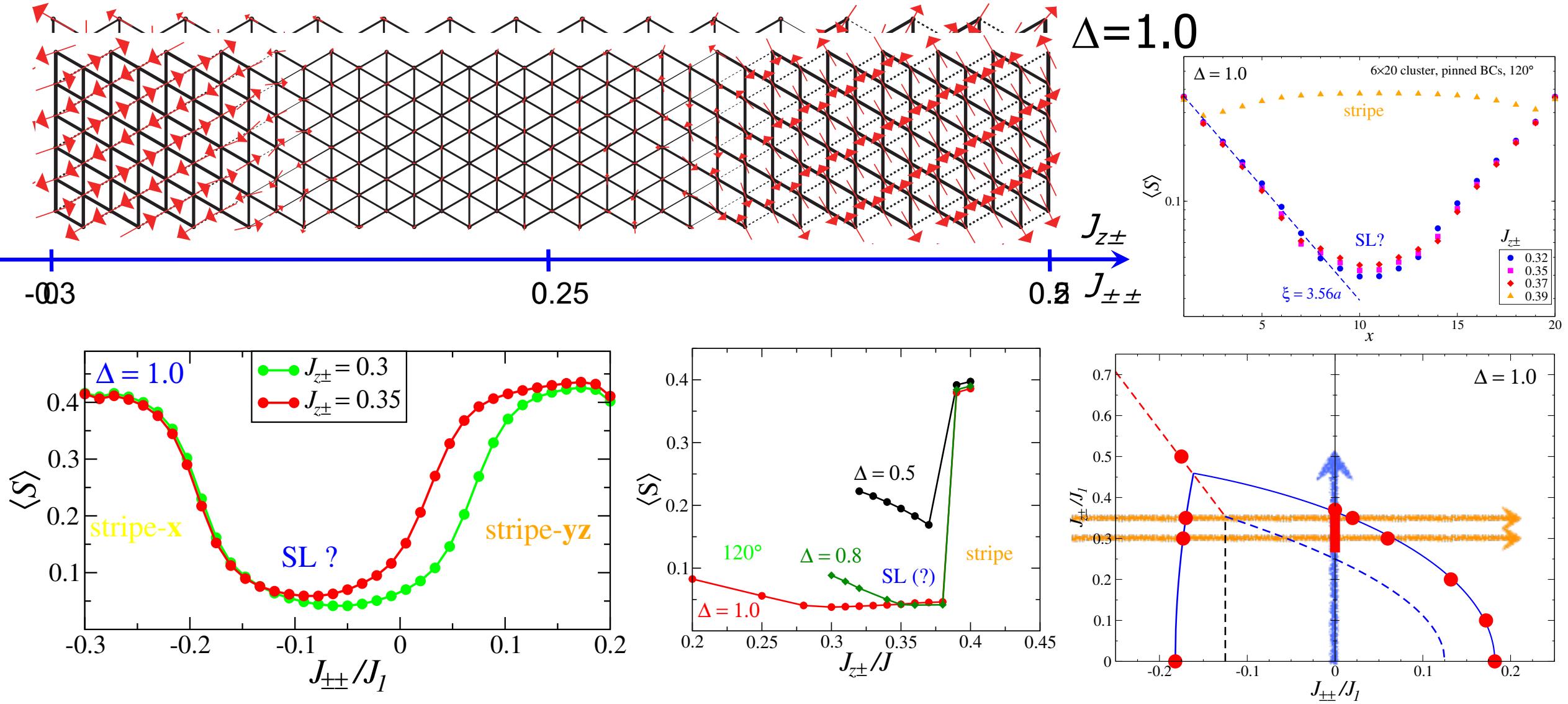
- 120° region is expanded; weaker 120° order, but looks ordered



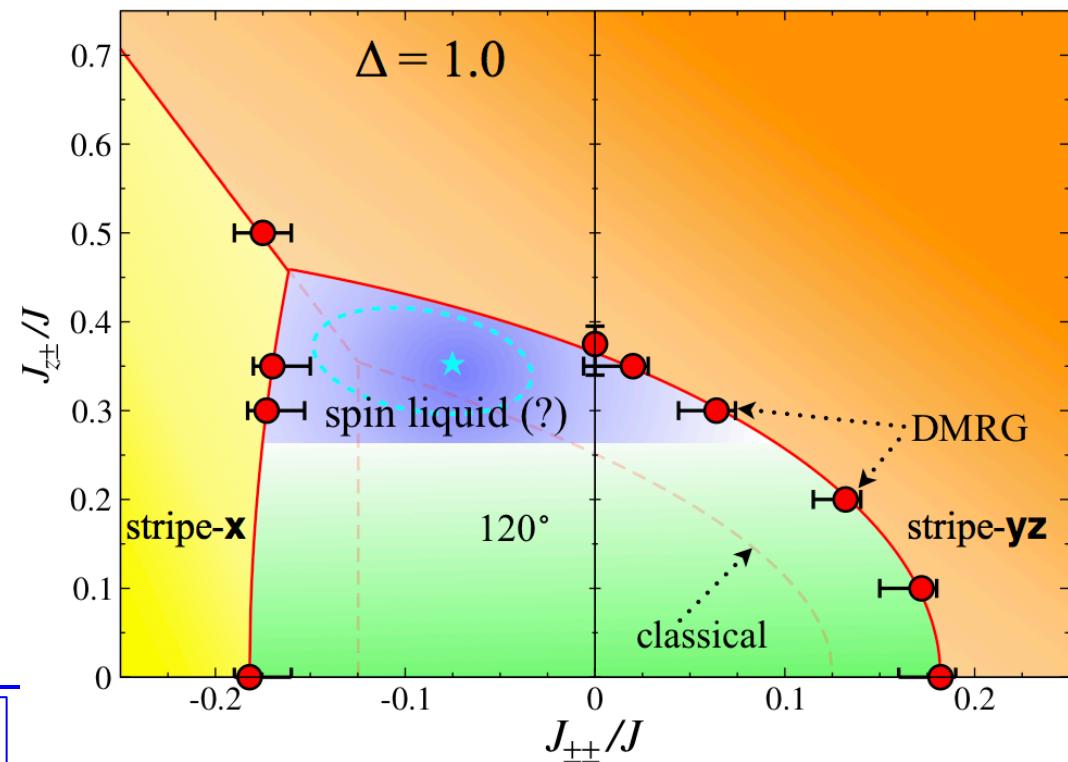
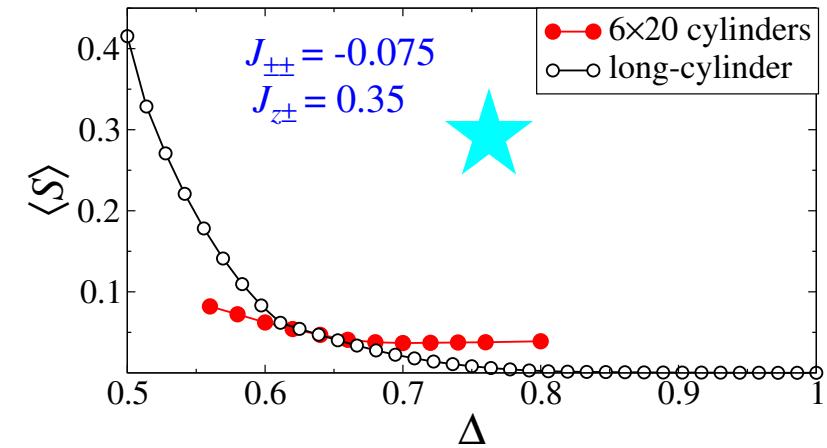
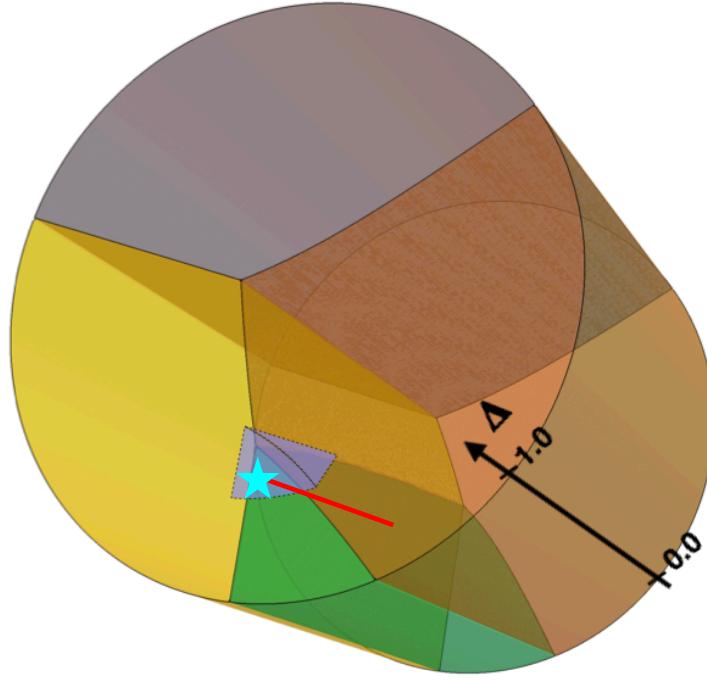
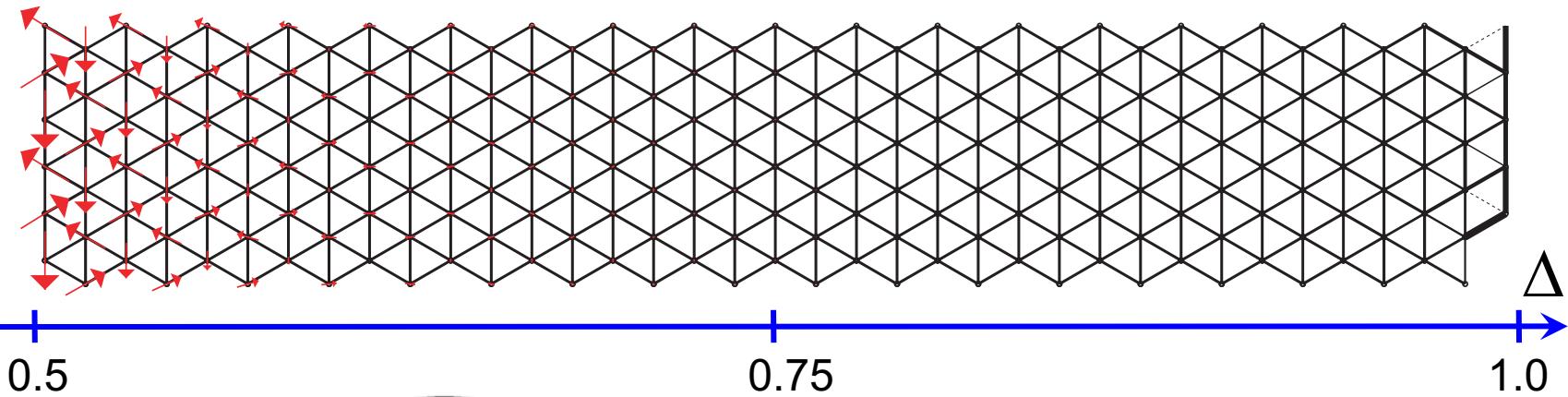
$\Delta=1.0$



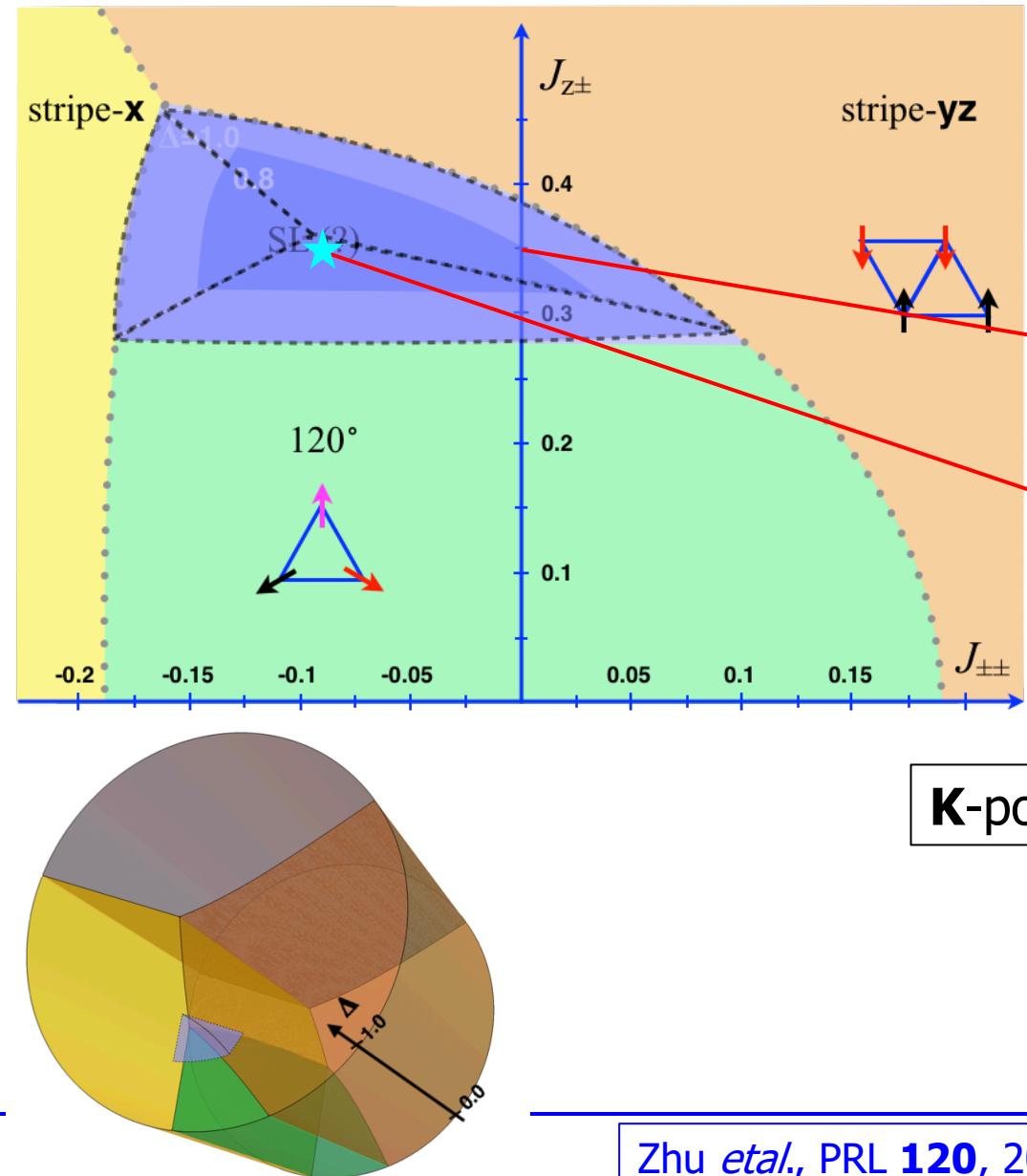
# DMRG, spin liquid?



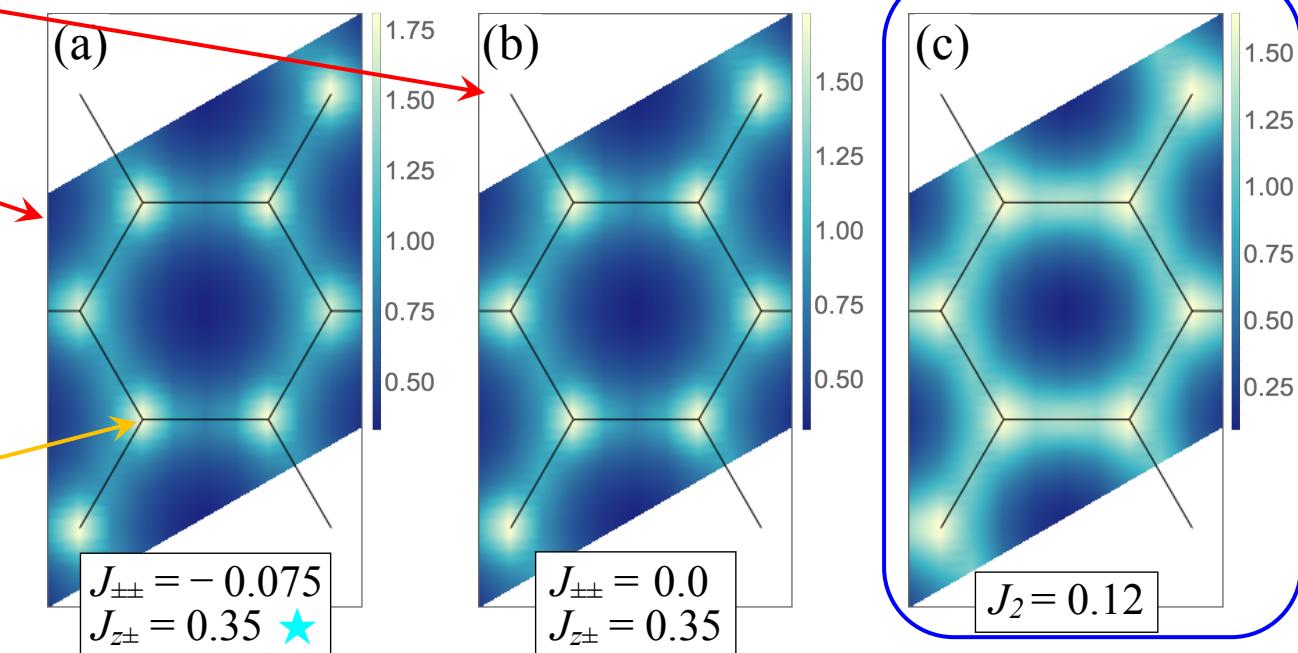
# spin liquid #1



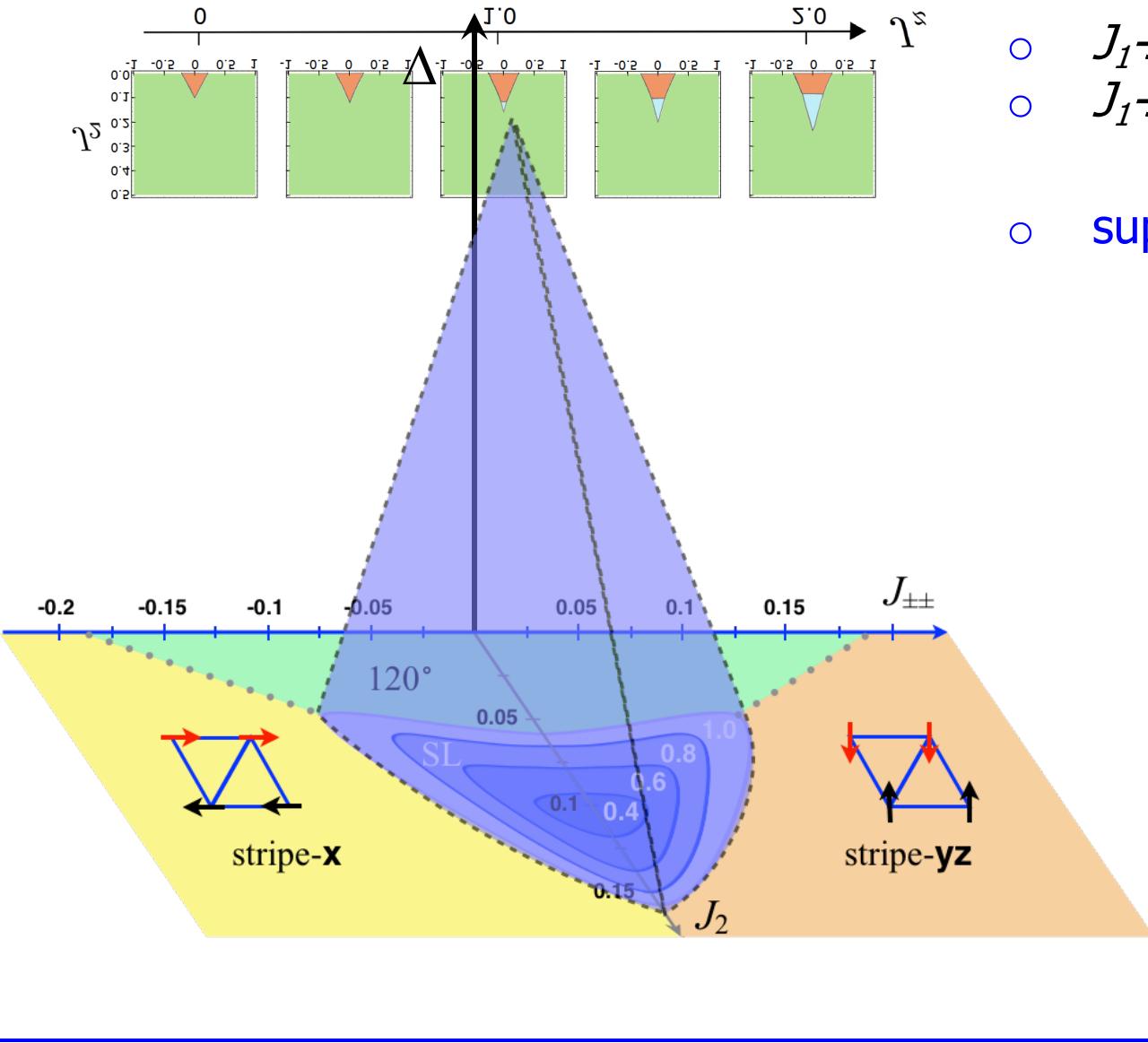
# spin liquid #1: topographic map



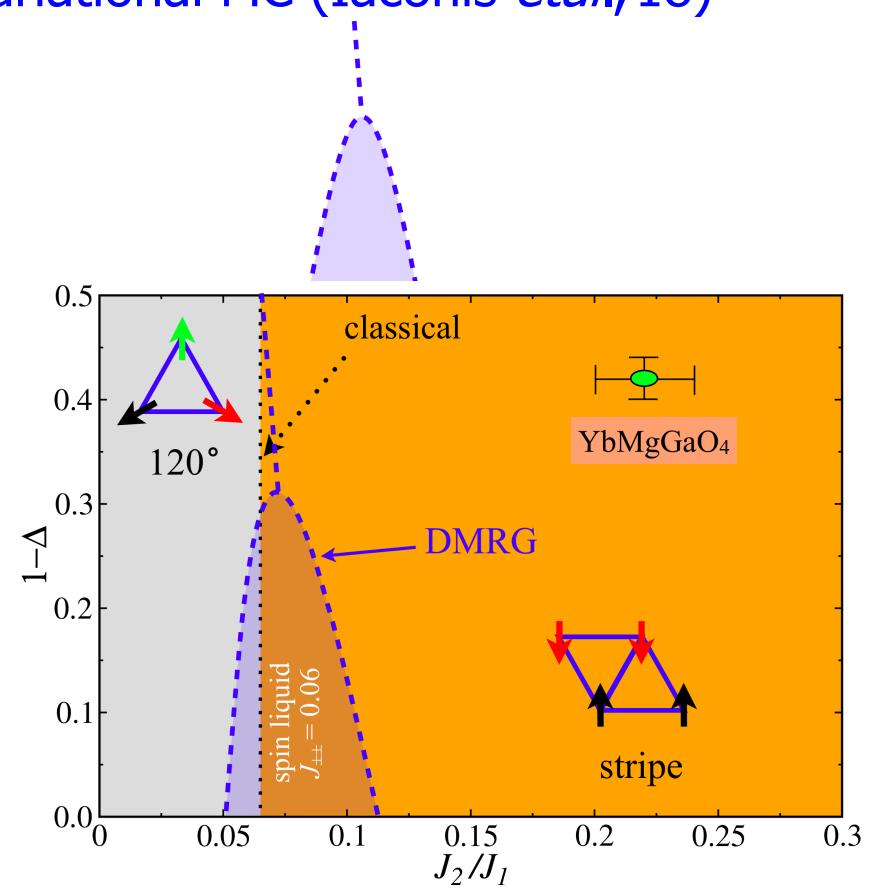
- $S(\mathbf{q})$ : "molten 120", connection to  $J_1$ - $J_2$



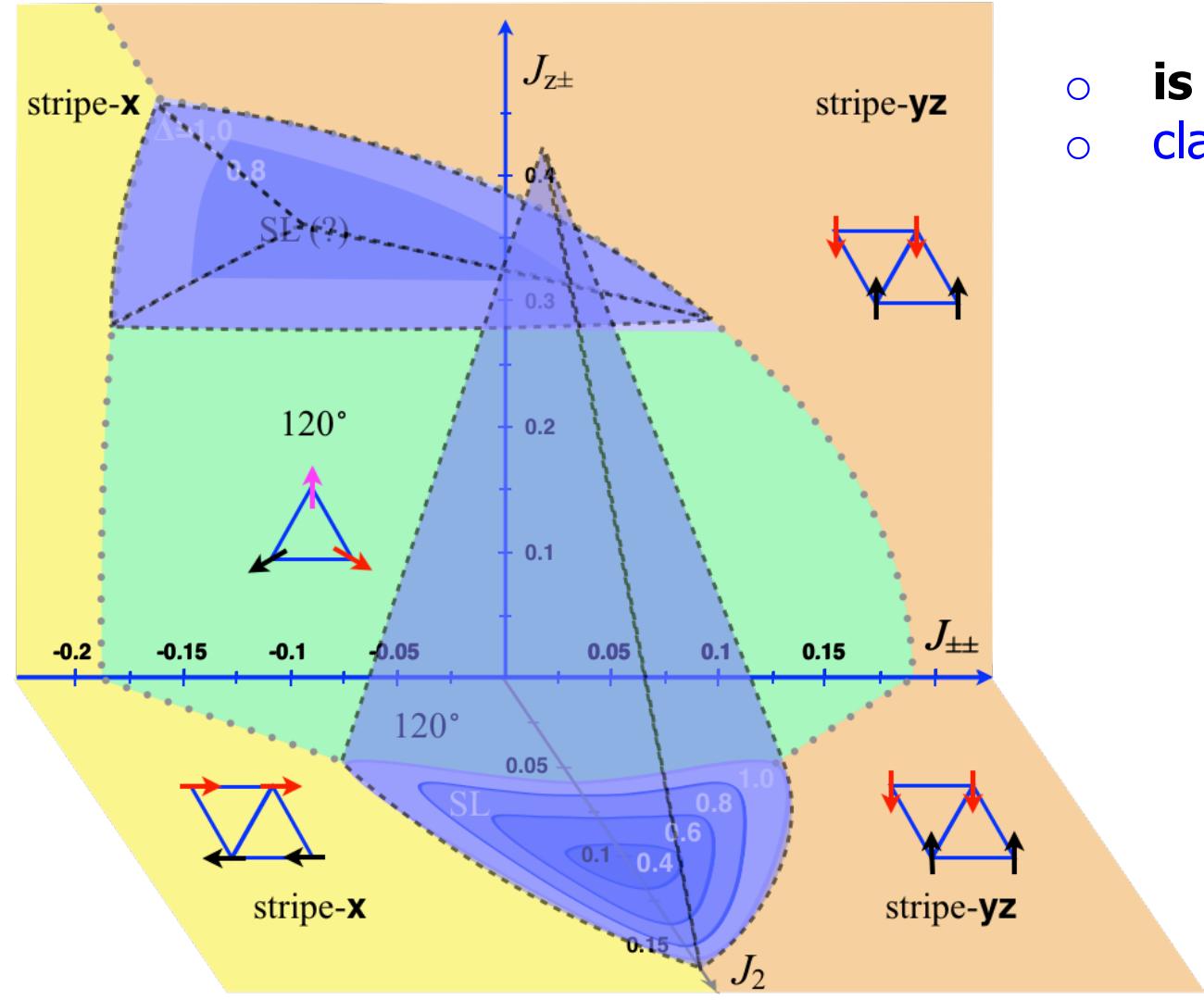
# $J_1$ - $J_2$ -XXZ- $J_{\pm\pm}$ , DMRG?



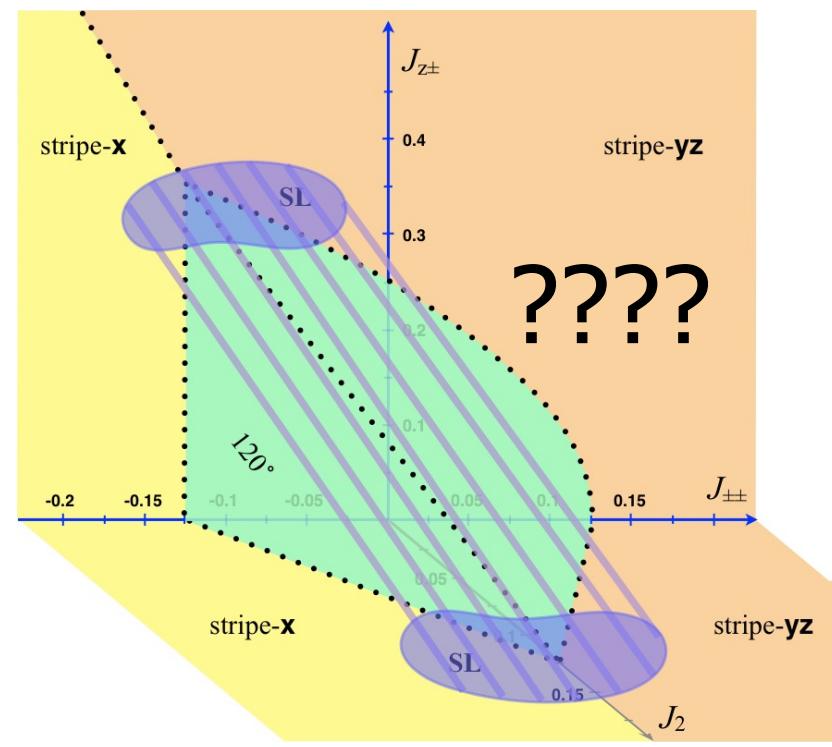
- $J_1$ - $J_2$  model has a spin liquid phase [ $J_2 = 0.06 \dots 0.15$ ]
- $J_1$ - $J_2$ -XXZ- $J_{\pm\pm} \Rightarrow$  **extended spin liquid phase**  
(extent in  $\Delta = [1.0 \dots 0.3]$ ,  $J_{\pm\pm} = [-0.1 \dots 0.1]$  )
- supported by variational MC (Iaconis *etal.*, '18)



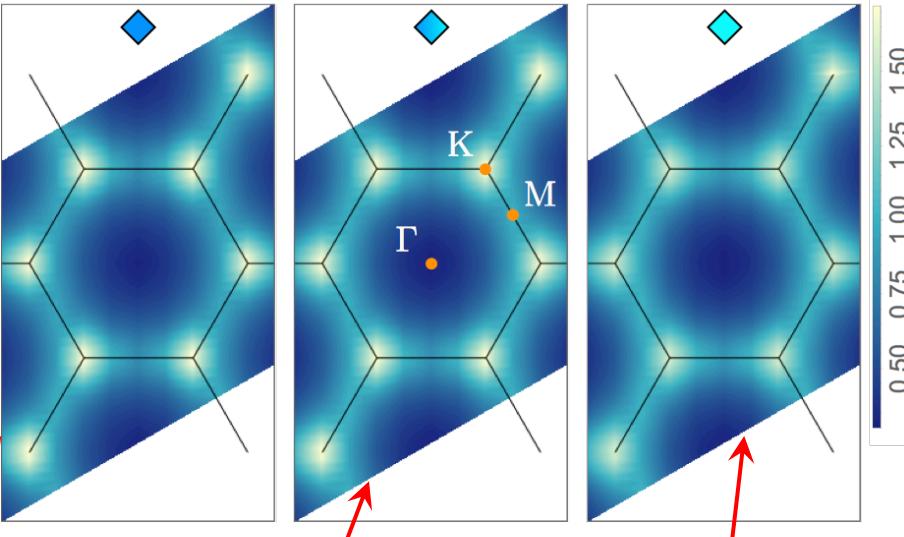
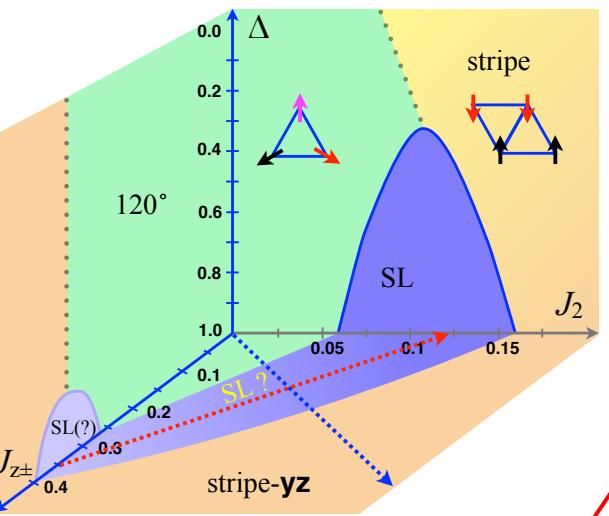
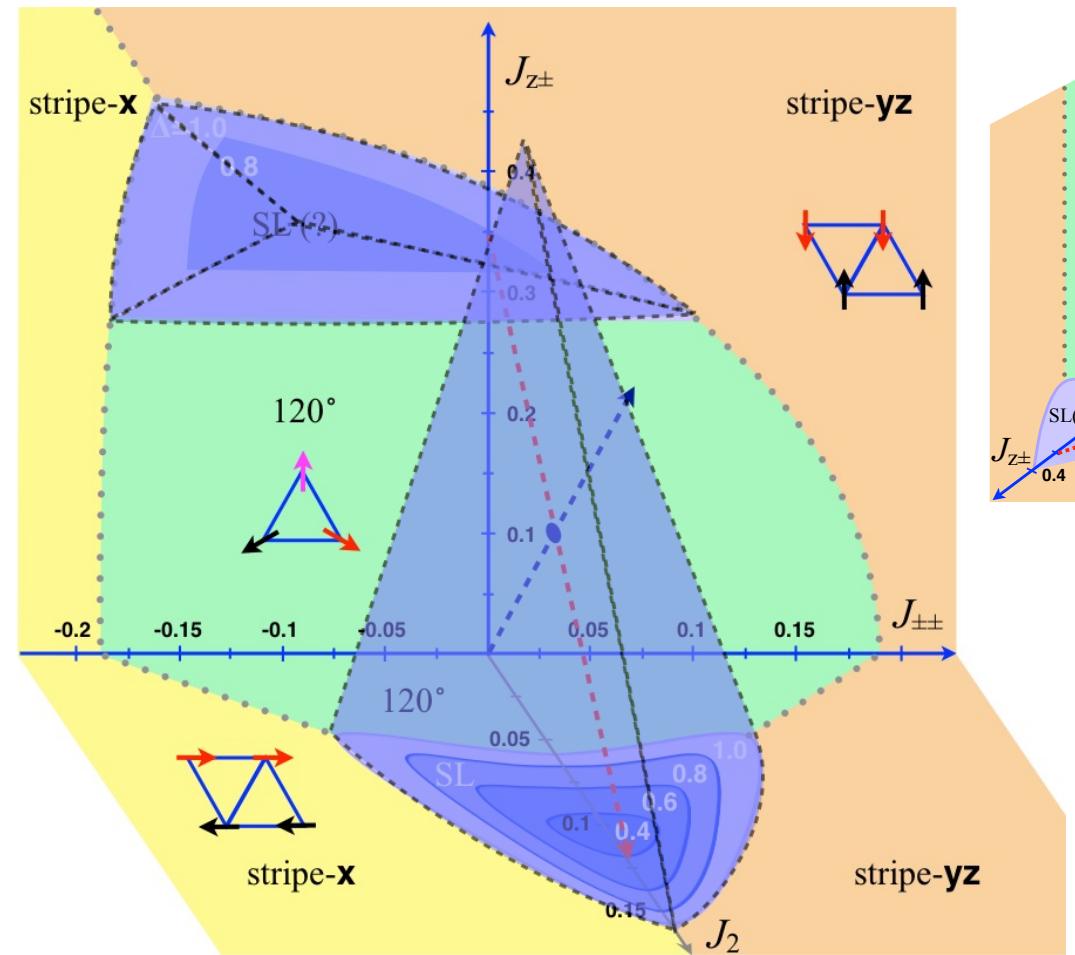
# connection with the SL in $J_1$ - $J_2$ -XXZ- $J_{\pm\pm}$ ?



- **is there a connection between SL phases?**
- classically, connection along the rim of the 120° phase

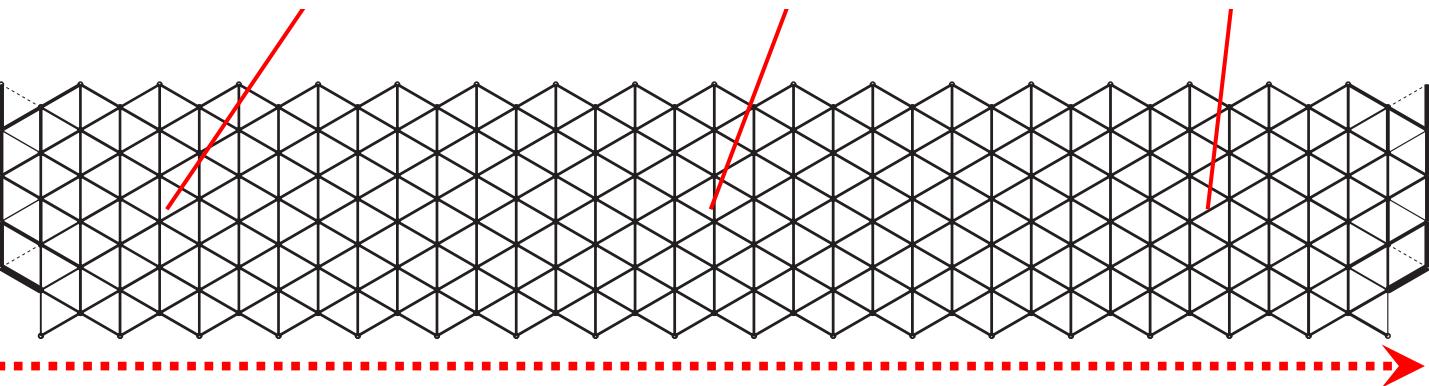


# 4D connection with the SL in $J_1$ - $J_2$ -XXZ

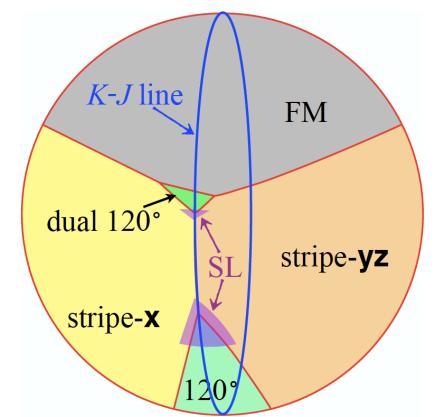


o **⇒ anisotropic and  $J_1$ - $J_2$  spin liquids are isomorphic (= same phase)**

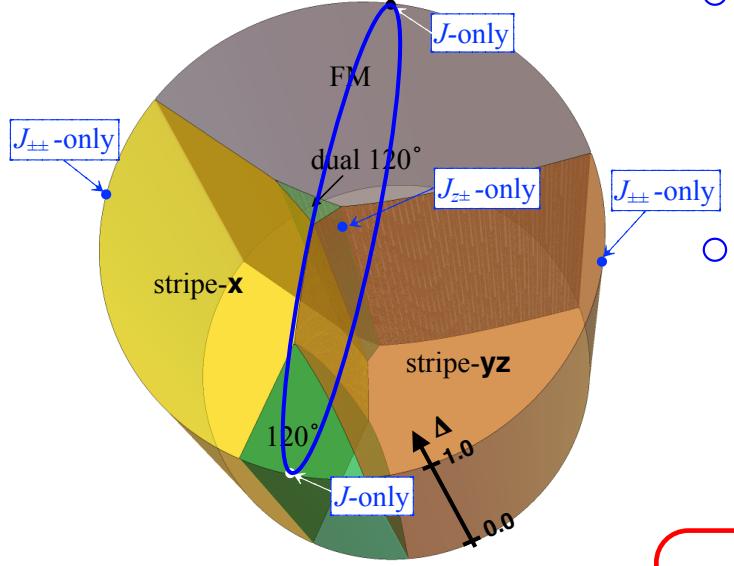
o **no valence bond order or chirality**



# duality

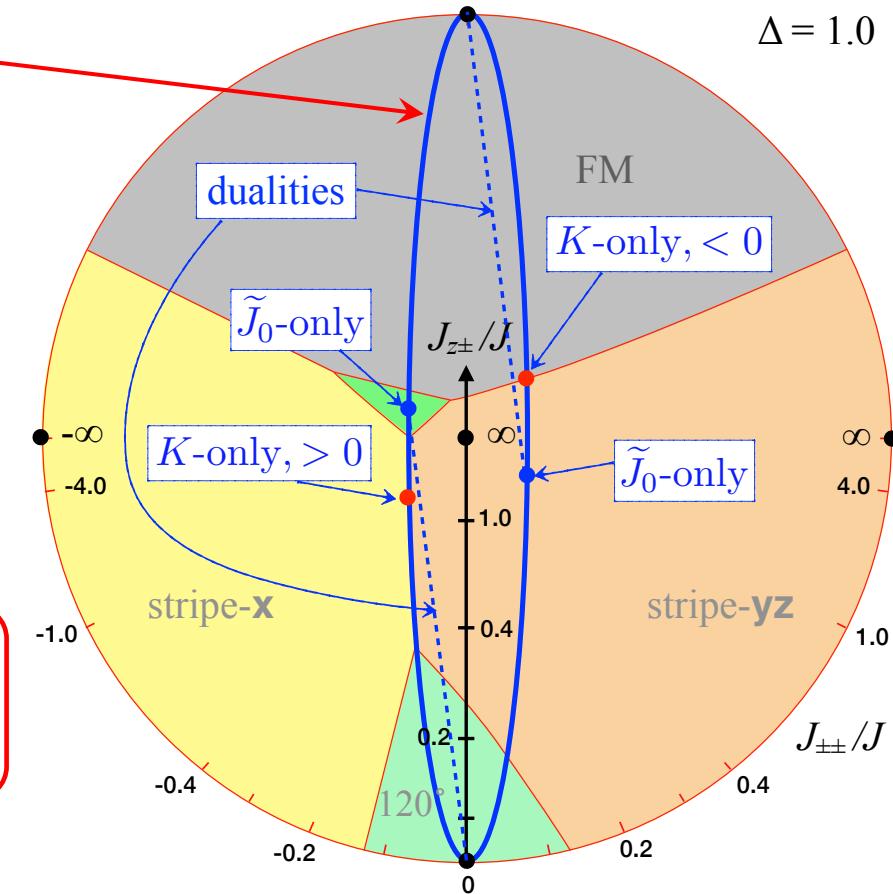


# $J$ - $K$ - $\Gamma$ - $\Gamma'$ -model correspondence



- for  $\Delta=1.0$  and  $J_{z\pm}=2\sqrt{2}J_{\pm\pm}$  line  
the model becomes  $J$ - $K$  model ( $\Gamma=\Gamma'=0$ )
- \*elsewhere (not on that line):  
 $\Rightarrow$  equivalent to  $J$ - $K$ - $\Gamma$ - $\Gamma'$  model

$$\mathcal{H} = -J_0 \sum_{\langle ij \rangle} \mathbf{S}_i \cdot \mathbf{S}_j - K \sum_{\gamma \parallel \langle ij \rangle} S_i^\gamma S_j^\gamma$$

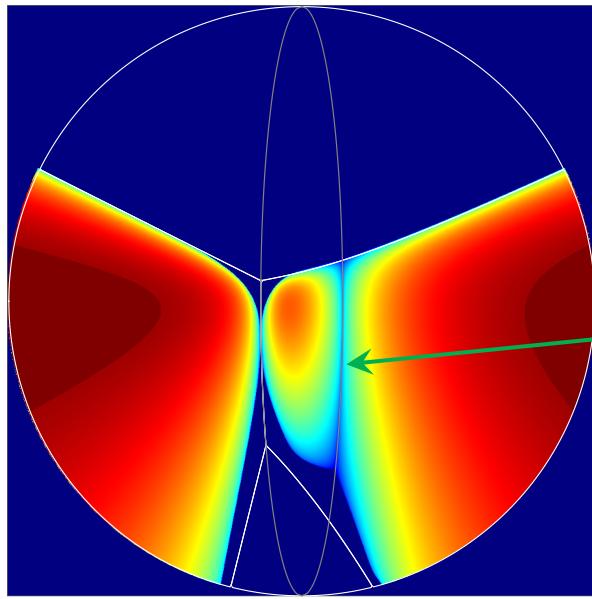


$$\begin{aligned} \mathcal{H} = \sum_{\langle ij \rangle} & J [S_i^x S_j^x + S_i^y S_j^y + \Delta S_i^z S_j^z] \\ & + 2J_{\pm\pm} [\cos \tilde{\varphi}_\alpha (S_i^x S_j^x - S_i^y S_j^y) - \sin \tilde{\varphi}_\alpha (S_i^x S_j^y + S_i^y S_j^x)] \\ & + J_{z\pm} [\cos \tilde{\varphi}_\alpha (S_i^y S_j^z + S_i^z S_j^y) - \sin \tilde{\varphi}_\alpha (S_i^x S_j^z + S_i^z S_j^x)] \end{aligned}$$

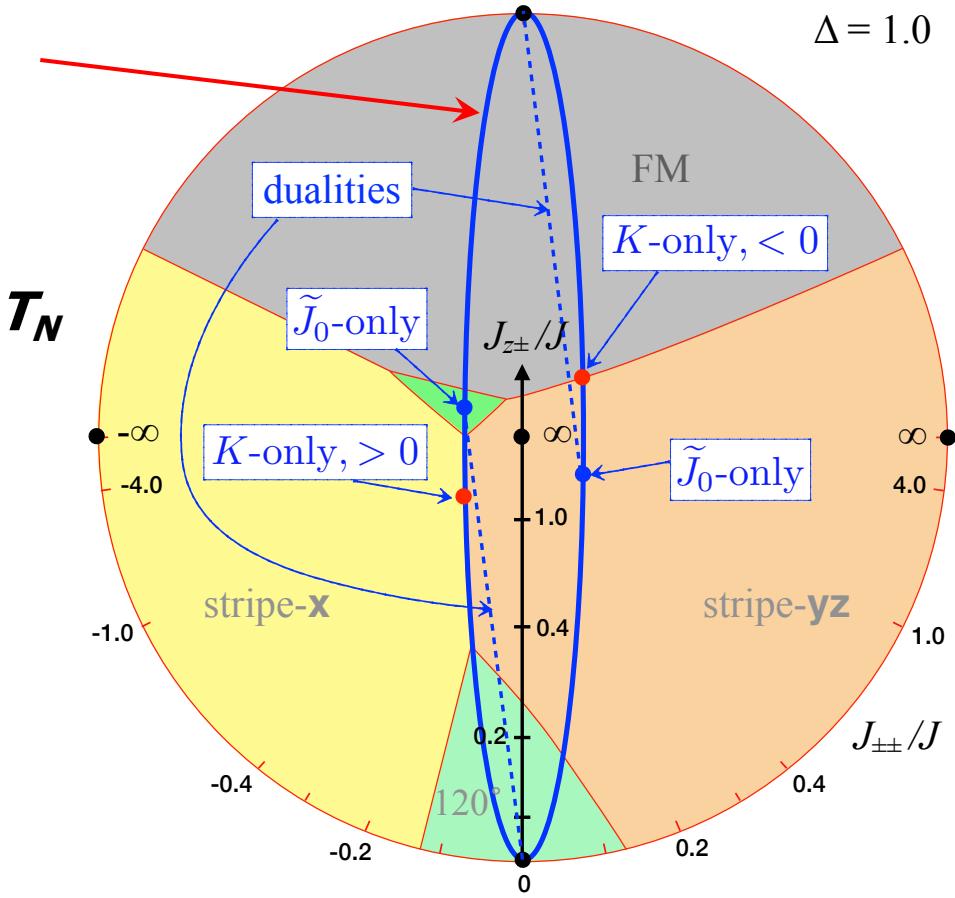
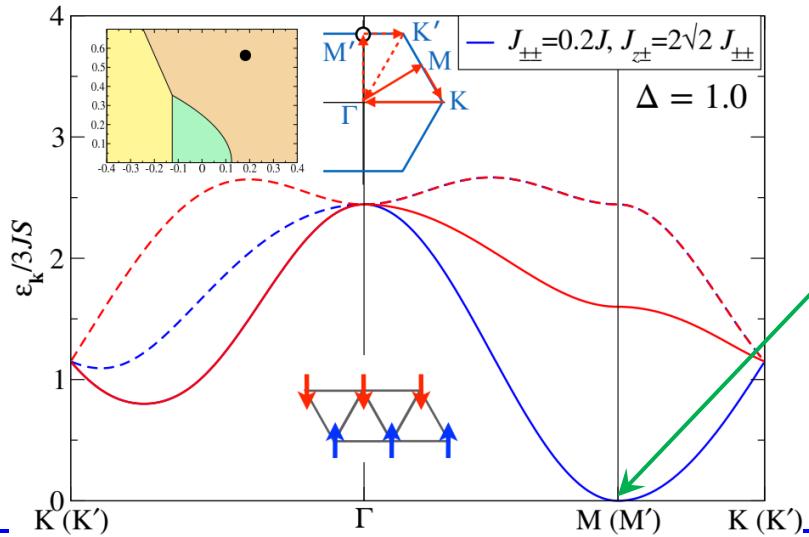
$$\begin{aligned} \Rightarrow \mathcal{H} = \sum_{\langle ij \rangle_\gamma} & J_0 \mathbf{S}_i \cdot \mathbf{S}_j + K S_i^\gamma S_j^\gamma + \Gamma (S_i^\alpha S_j^\beta + S_i^\beta S_j^\alpha) \\ & + \Gamma' (S_i^\gamma S_j^\alpha + S_i^\gamma S_j^\beta + S_i^\alpha S_j^\gamma + S_i^\beta S_j^\gamma) \end{aligned}$$



# $J$ - $K$ - $\Gamma$ - $\Gamma'$ -model correspondence



- **$J$ - $K$**  model: emergent continuous symmetries (in the classical limit)
- **⇒ pseudo-Goldstone modes**
- **⇒ Mermin-Wagner, "scars" in  $T_N$**

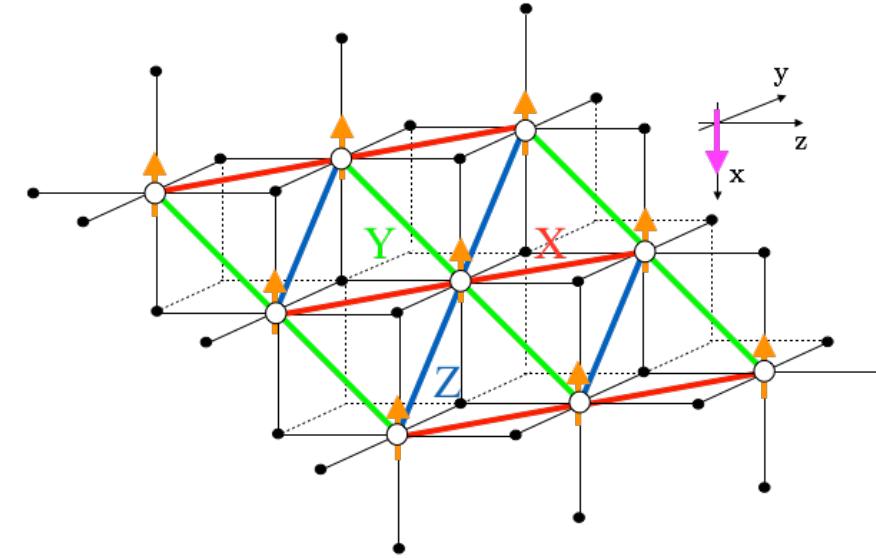
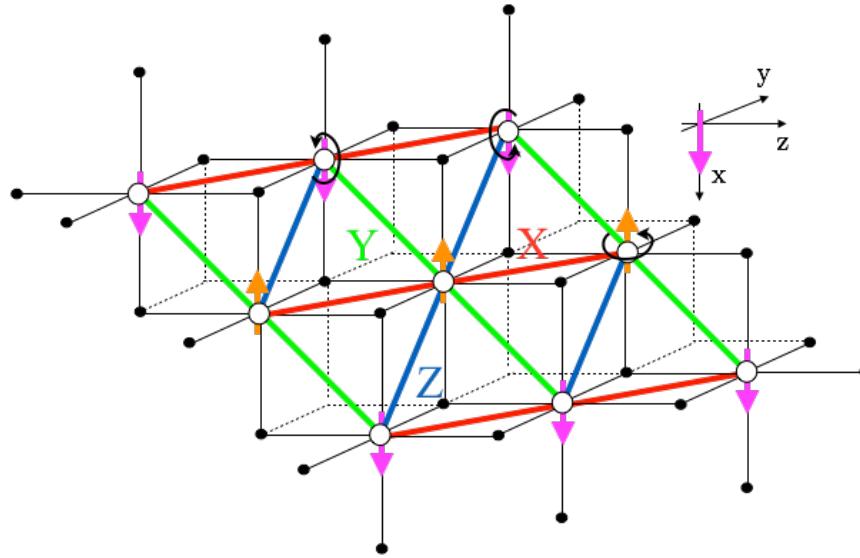


$$\mathcal{H} = -\tilde{J}_0 \sum_{\langle ij \rangle} \mathbf{S}_i \cdot \mathbf{S}_j - K \sum_{\gamma \parallel \langle ij \rangle} S_i^\gamma S_j^\gamma$$



# Klein dualities

- four-sublattice transformation:  $\mathbf{S}(\mathbf{r})$ ,  $\mathbf{R}_X(\pi)\mathbf{S}(\mathbf{r}+\delta_1)$ ,  $\mathbf{R}_Y(\pi)\mathbf{S}(\mathbf{r}+\delta_2)$ ,  $\mathbf{R}_Z(\pi)\mathbf{S}(\mathbf{r}+\delta_3)$
- that leaves the model invariant (e.g., stripe-yz  $\Rightarrow\Rightarrow$  FM)



$$\mathcal{H} = -J_0 \sum_{\langle ij \rangle} \mathbf{S}_i \cdot \mathbf{S}_j - K \sum_{\gamma \parallel \langle ij \rangle} S_i^\gamma S_j^\gamma$$

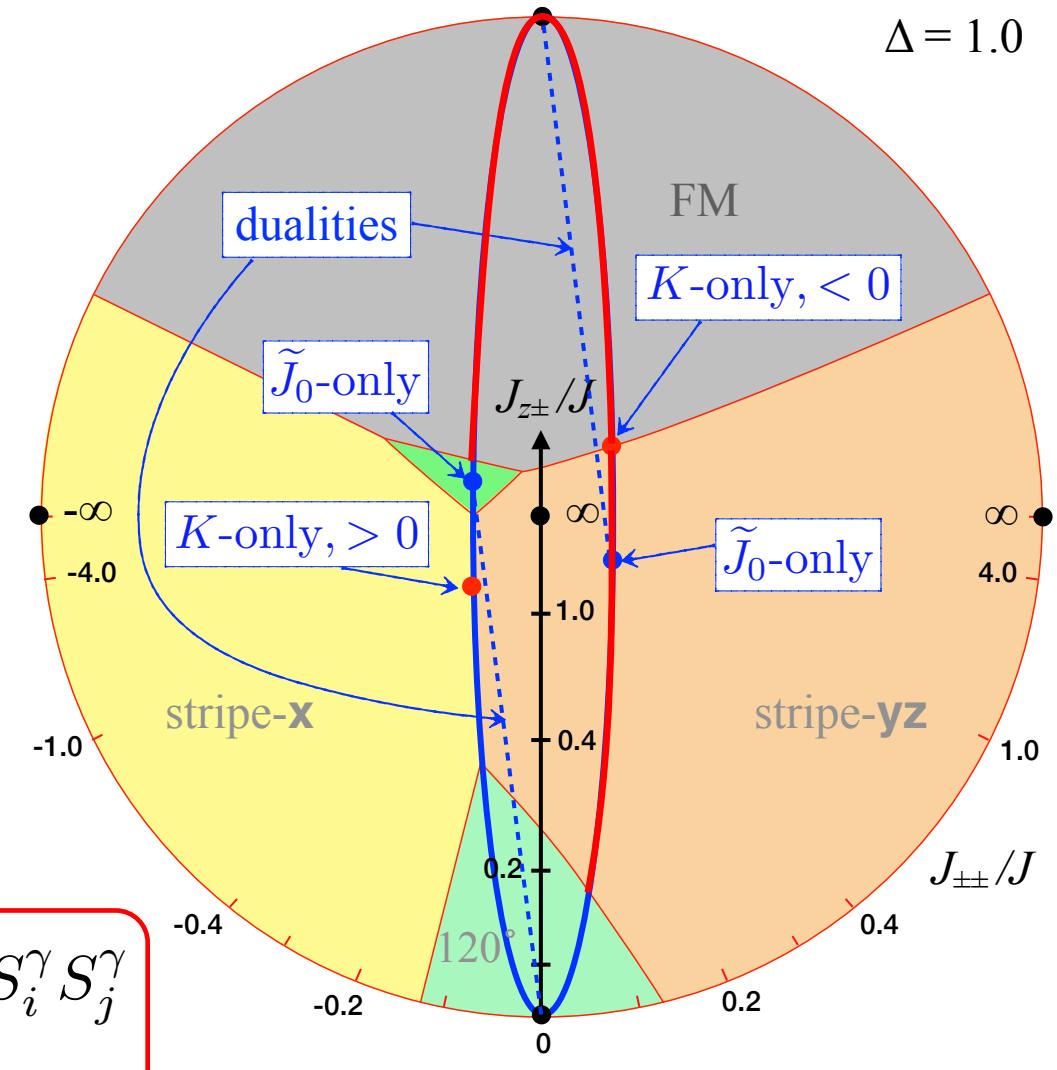


$$\mathcal{H} = -\tilde{J}_0 \sum_{\langle ij \rangle} \mathbf{S}_i \cdot \mathbf{S}_j - \tilde{K} \sum_{\gamma \parallel \langle ij \rangle} S_i^\gamma S_j^\gamma$$

$$\tilde{J}_0 = -J_0, \quad \tilde{K} = 2J_0 + K,$$

# Klein dualities: stripe-yz to FM

- dualities of different sectors
- yz-stripe sector  $\Rightarrow$  **duality to FM**

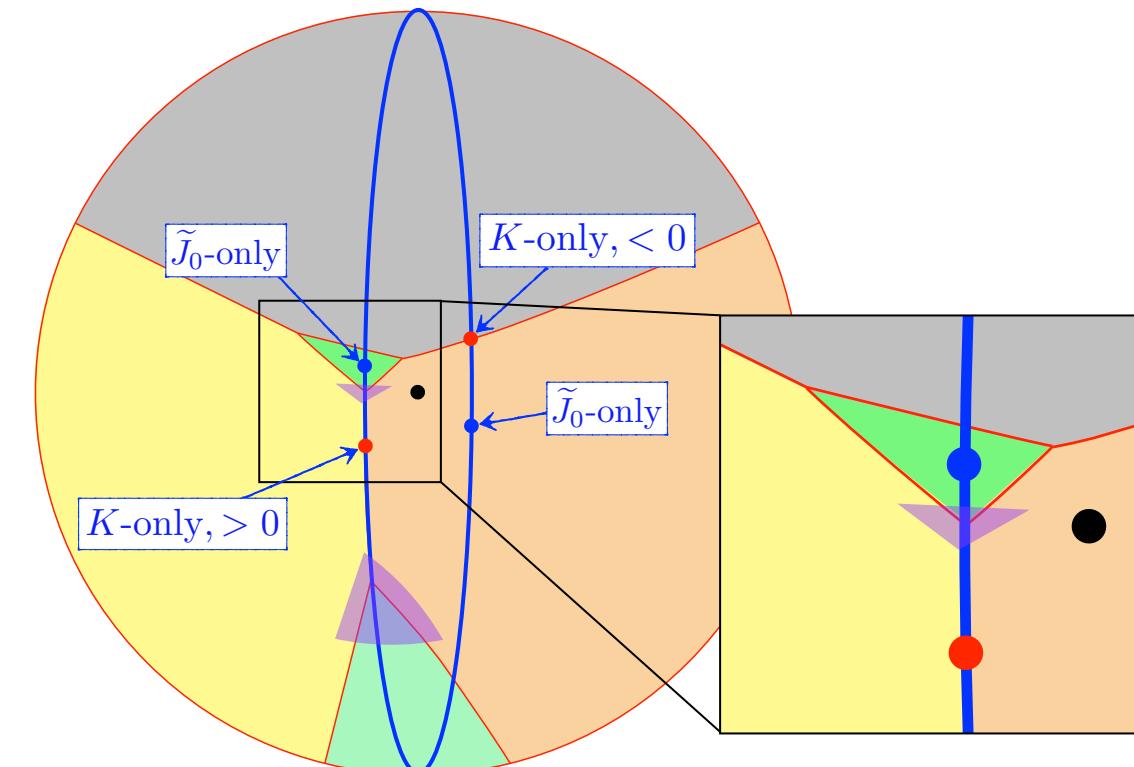


$$\mathcal{H} = -J_0 \sum_{\langle ij \rangle} \mathbf{S}_i \cdot \mathbf{S}_j - K \sum_{\gamma \parallel \langle ij \rangle} S_i^\gamma S_j^\gamma$$

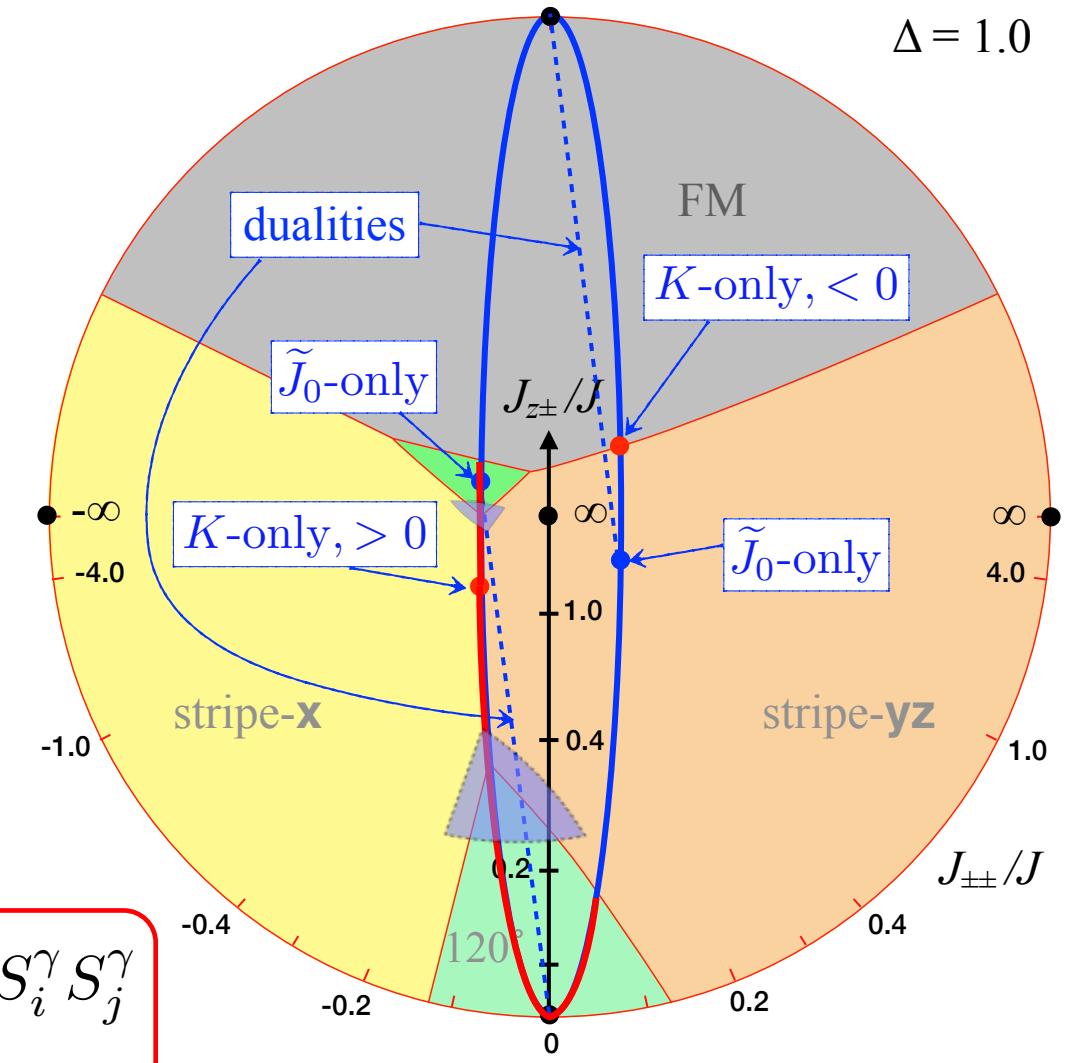


# Klein dualities: spin liquid #2

- dualities of different sectors
- there should exist a **dual SL phase**

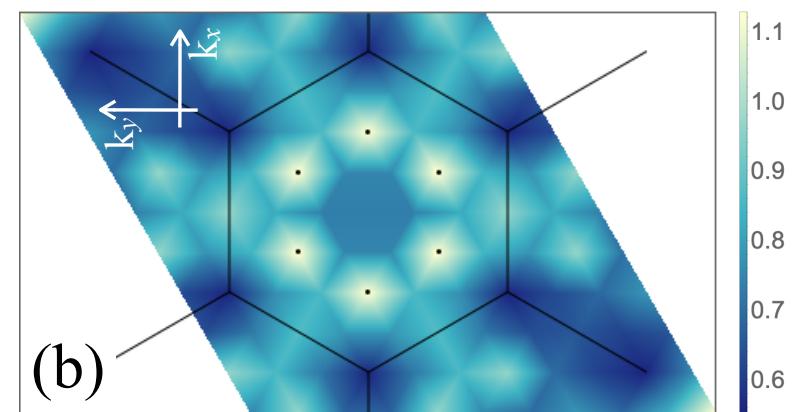
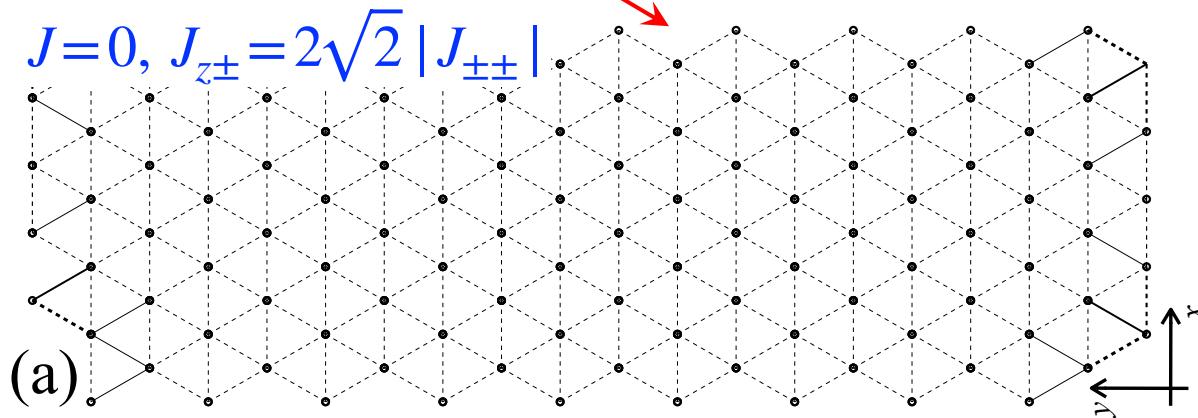
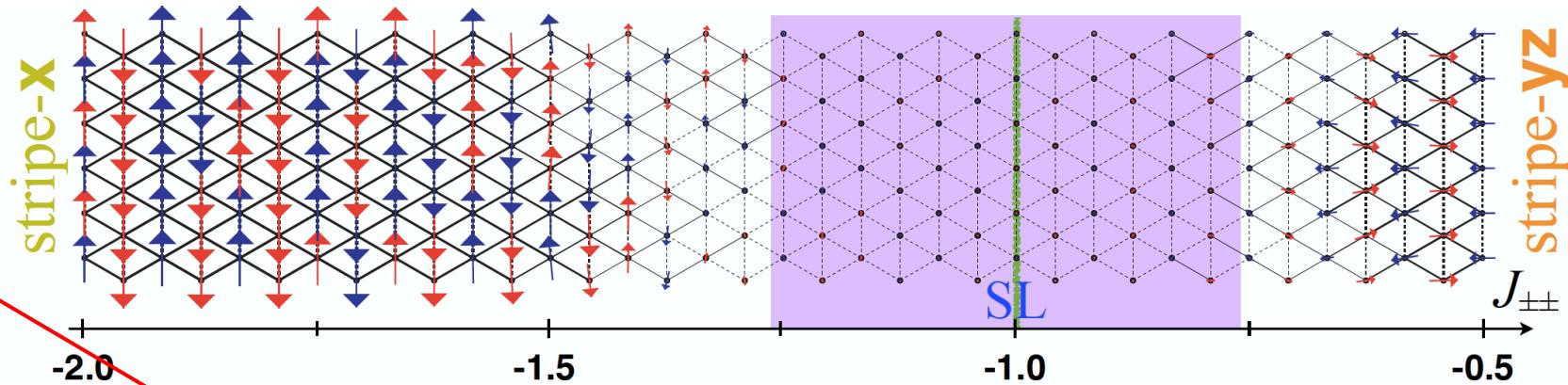
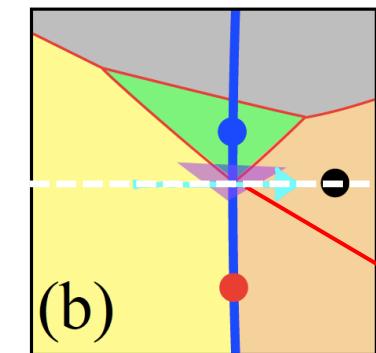
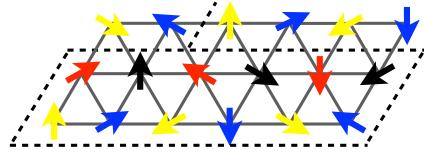


$$\mathcal{H} = -J_0 \sum_{\langle ij \rangle} \mathbf{S}_i \cdot \mathbf{S}_j - K \sum_{\gamma \parallel \langle ij \rangle} S_i^\gamma S_j^\gamma$$



# spin liquid #2

- DMRG “scans”, “non-scans”,  $S(\mathbf{q})$
- SL = molten dual 120-phase
- mesmerizing, **all-anisotropic** model (only  $J_{\pm\pm}$ - $J_{z\pm}$ ) has an SL, exact solution?



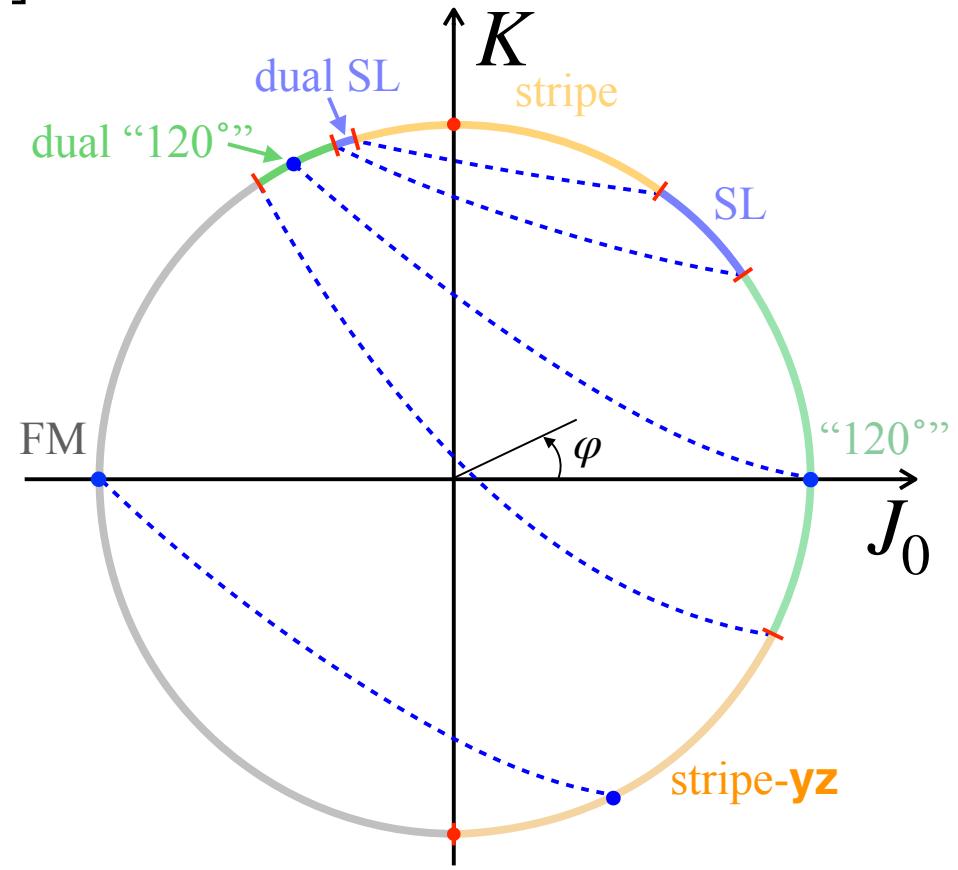
# by-product: better phase diagram for K-J model

- phase diagram of the triangular-lattice  $J$ - $K$  model [updated]:  
**no Kitaev-like solution, two (dual) SL phases (!)**

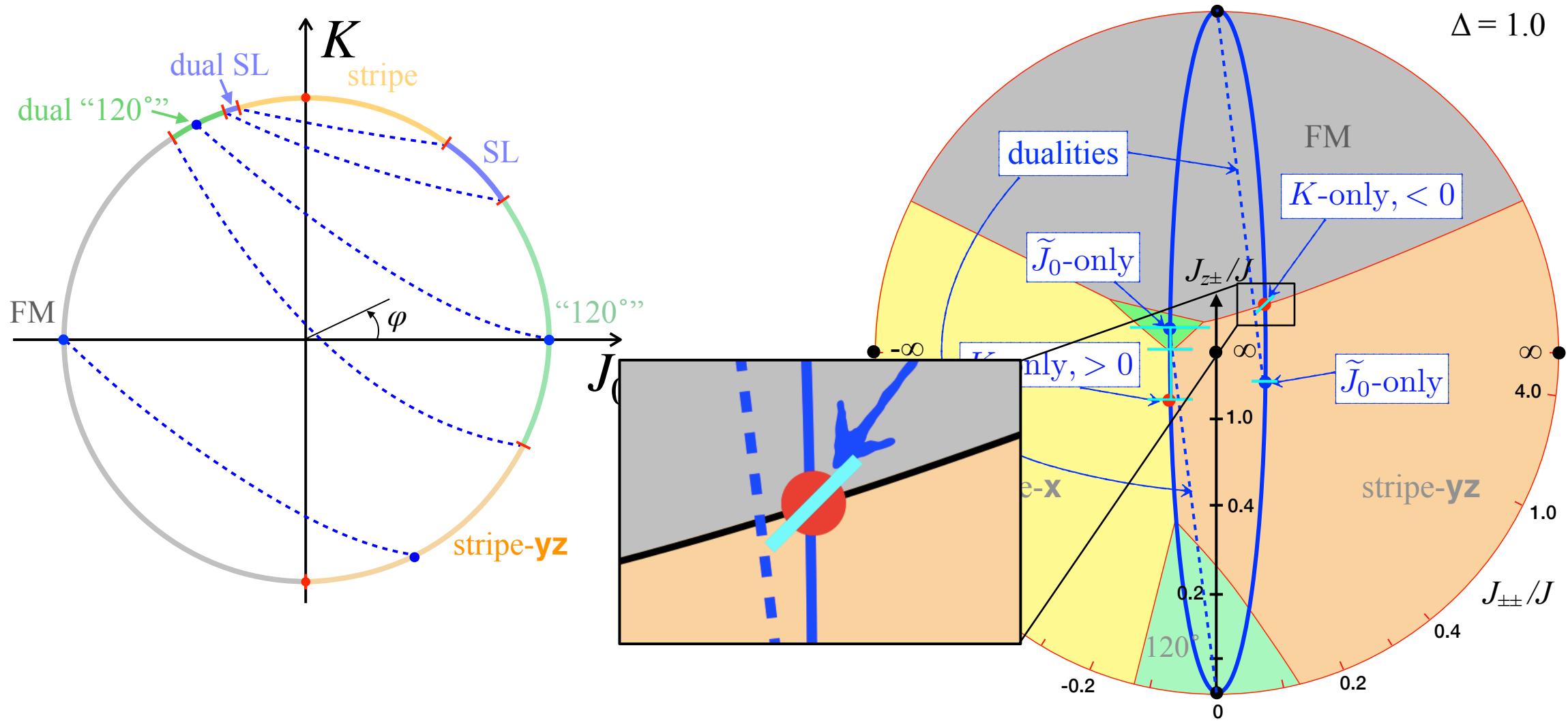
$$\mathcal{H} = -J_0 \sum_{\langle ij \rangle} \mathbf{S}_i \cdot \mathbf{S}_j - K \sum_{\gamma \parallel \langle ij \rangle} S_i^\gamma S_j^\gamma$$



- “mesmerizing point”, **not integrable (ED)**

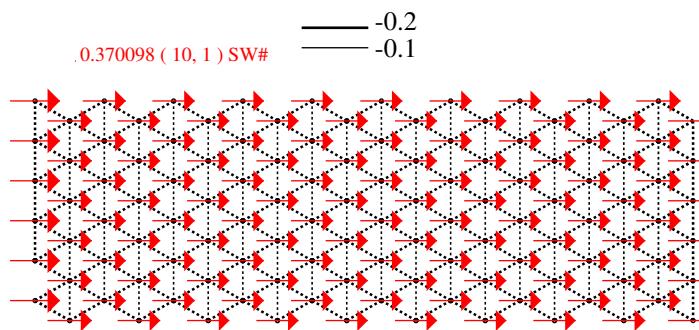
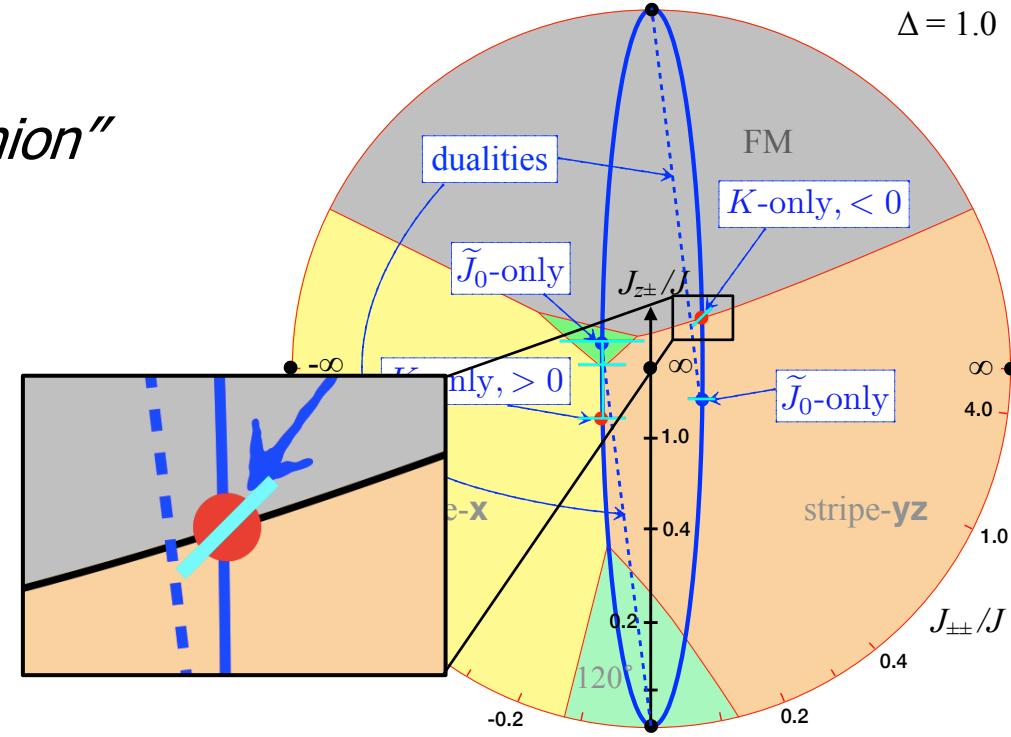
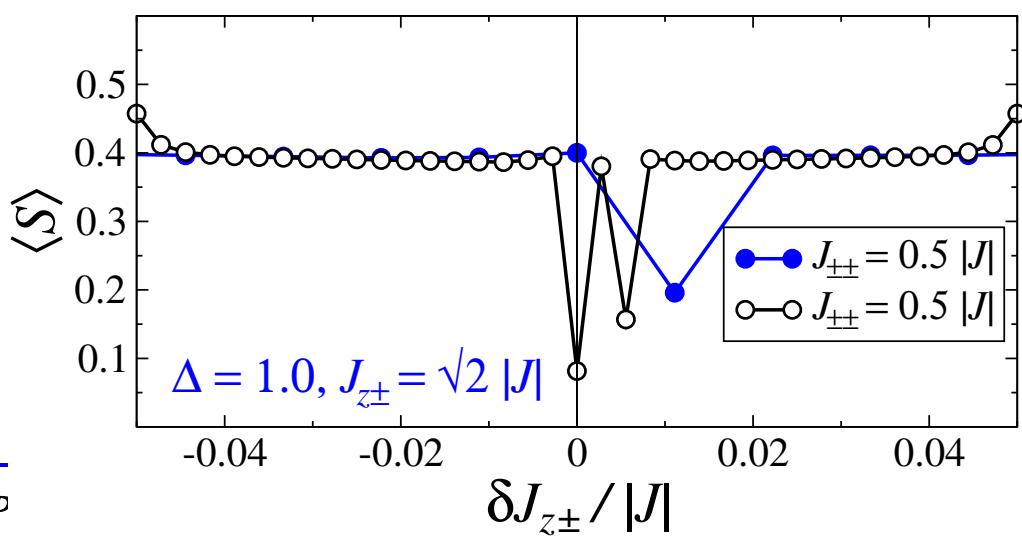
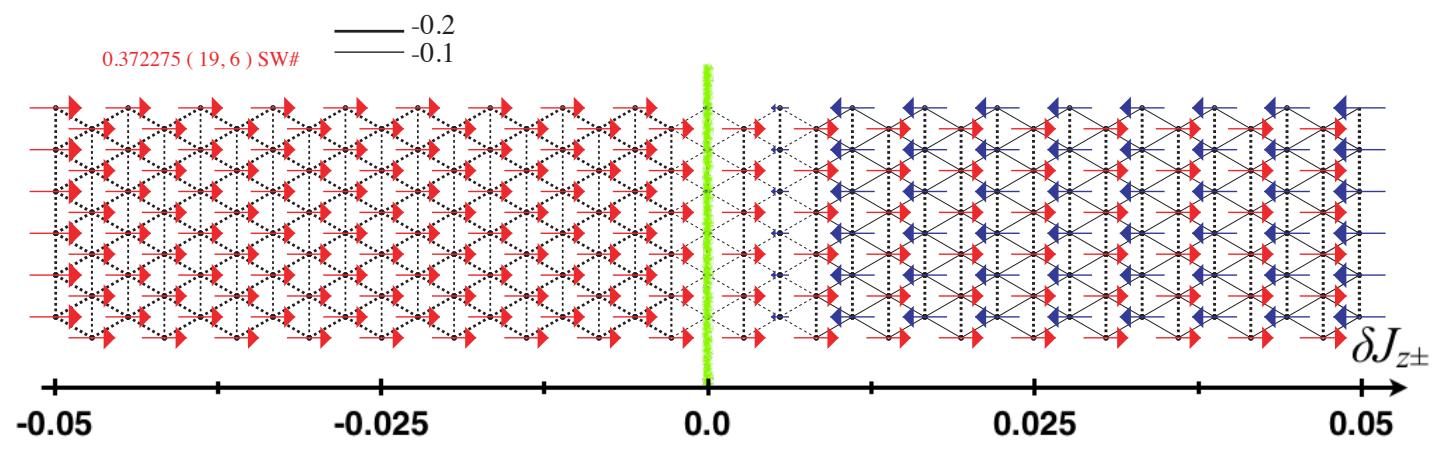


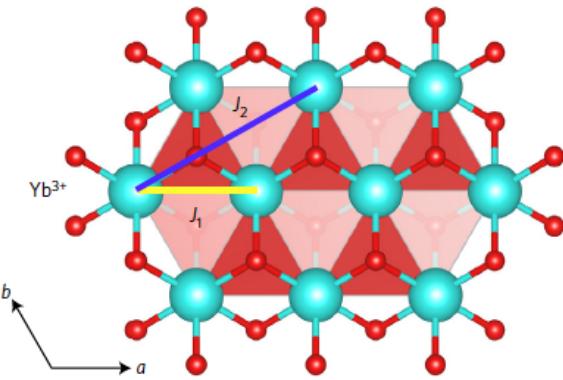
# some unpublished scans ...



# some unpublished scans ...

- **self-dual point** (Georgii Jackeli's talk, Wednesday)
- message to the referee on  
"mass production of spin liquids in an assembly line fashion"

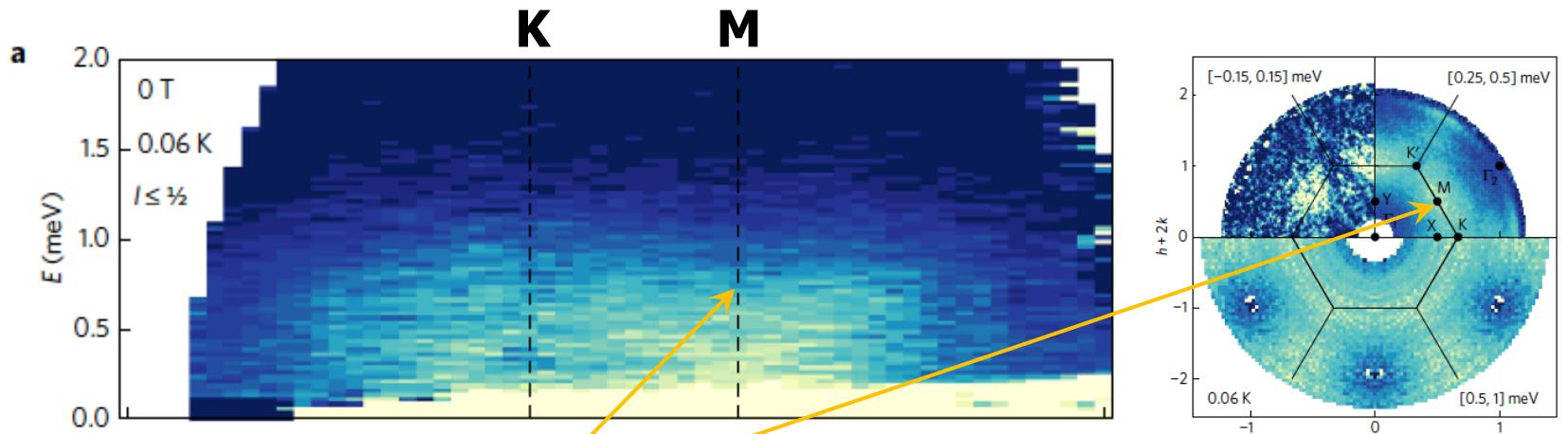




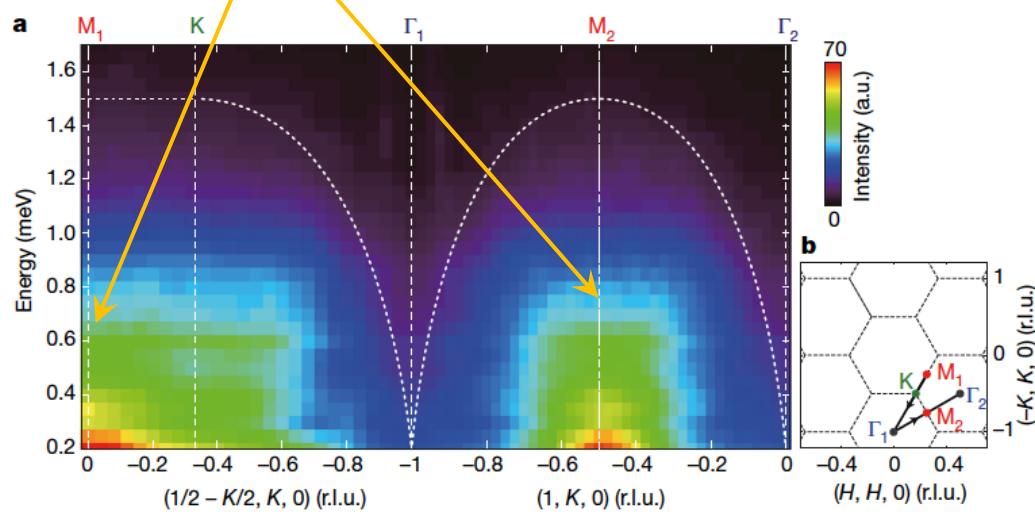
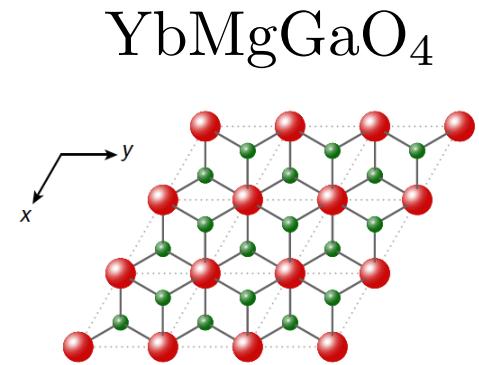
mimicry



# experiments, I



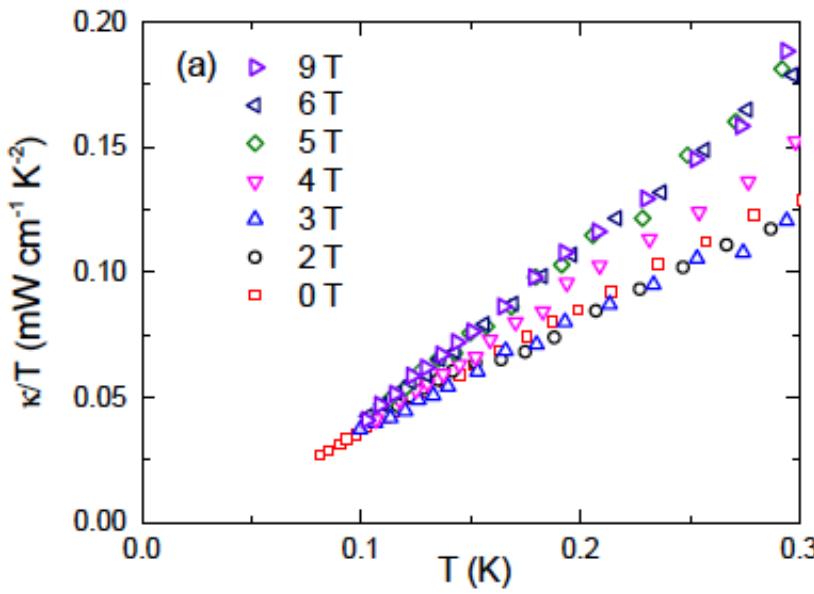
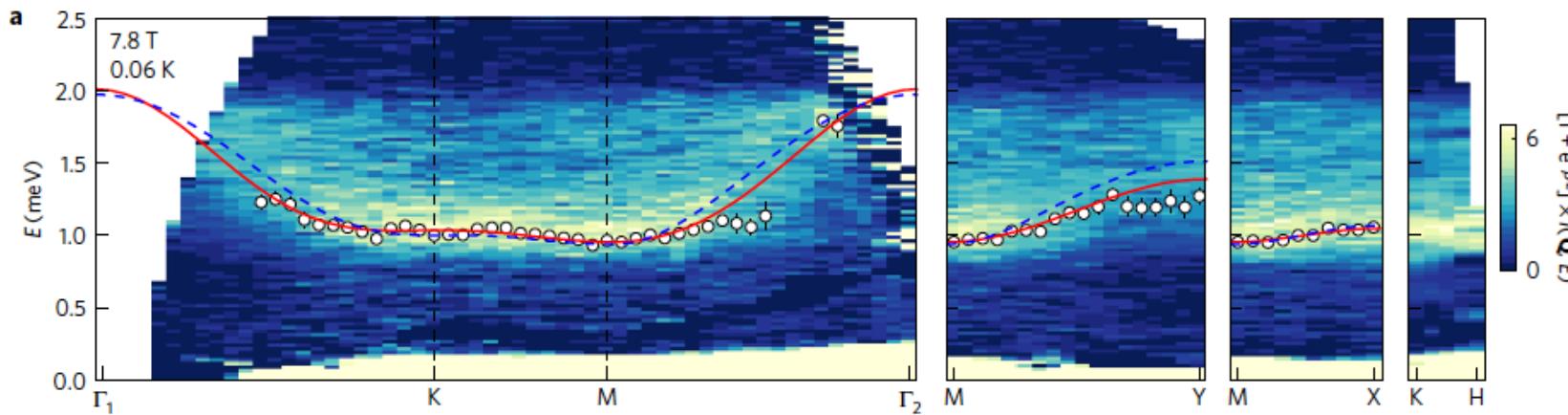
broad intensity at **M**-points



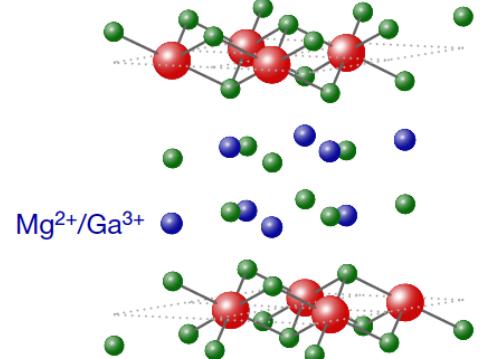
- no order
- broad features in  $S(\mathbf{q}, \omega)$
- spin-liquid?



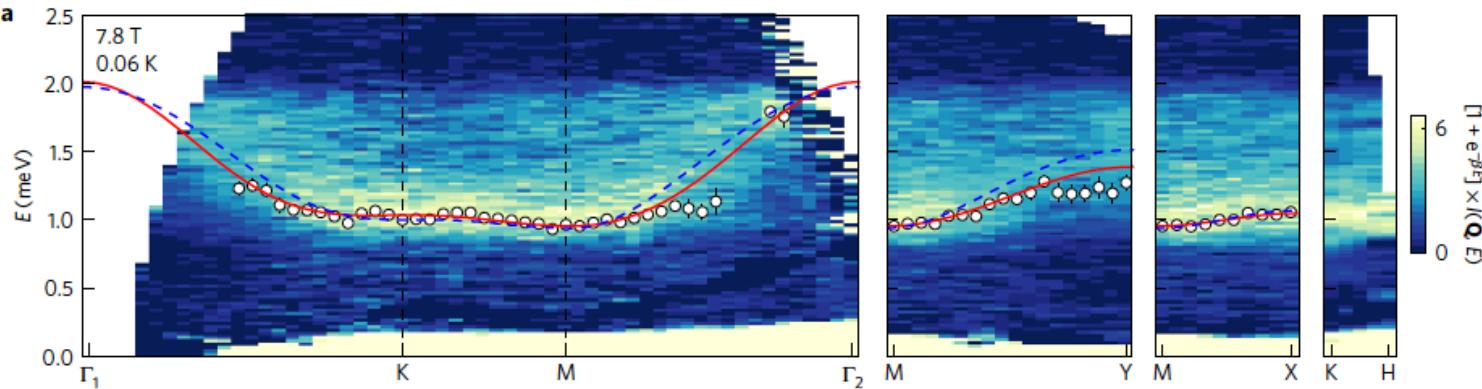
# experiments, II



- features remain broad above saturation
- no thermal conductivity by spin excitations
- low- $T$  freezing
- disorder? [mixing of interplane  $\text{Ga}^{3+}$  and  $\text{Mg}^{2+}$ ]



# (early) parameters: simpler model!



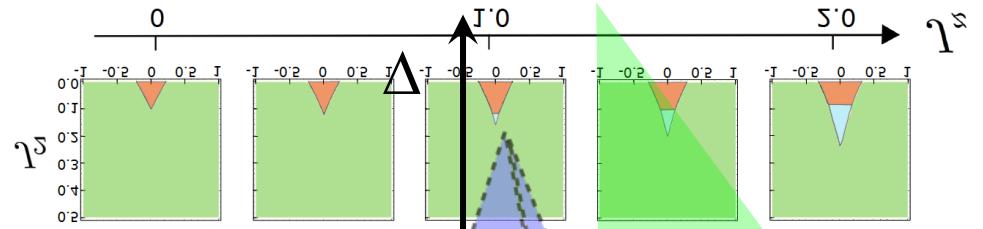
- **neutrons, ESR:**  $J_1 = 0.218(8)\text{meV}$ ,  
 $\Delta=0.58(2)$ ,  $\mathbf{J}_2 = \mathbf{0.22(2)J}_1$ ,  
 $J_{\pm\pm} = 0.06J_1$ ,  $J_{z\pm} \approx 0.0$
- **mostly  $J_1$ - $J_2$  XXZ**  
+ **subleading**  $J_{\pm\pm}$   
+ **negligible**  $J_{z\pm}$

$$\mathcal{H} = \mathcal{H}_{\text{XXZ}}^{J_1-J_2} + \mathcal{H}_{\text{pd}}$$

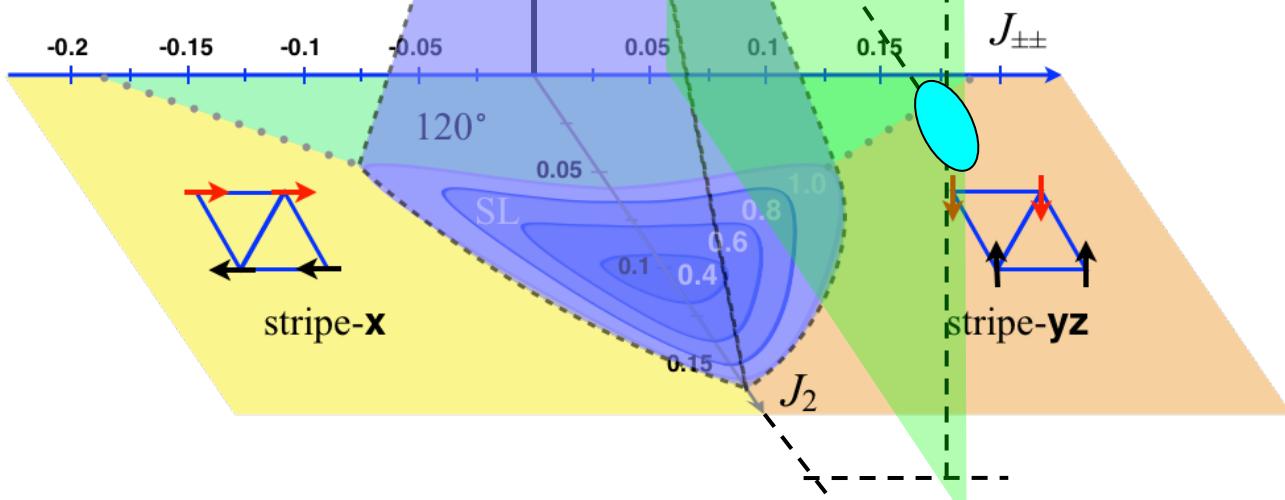
$$\mathcal{H}_{\text{XXZ}}^{J_1-J_2} = \sum_{\langle ij \rangle_n} J_n (S_i^x S_j^x + S_i^y S_j^y + \Delta S_i^z S_j^z)$$

$$\mathcal{H}_{\text{pd}} = J_{\pm\pm} \sum_{\langle ij \rangle} (e^{i\tilde{\varphi}_\alpha} S_i^+ S_j^+ + e^{-i\tilde{\varphi}_\alpha} S_i^- S_j^-)$$

# $J_1$ - $J_2$ -XXZ- $J_{\pm\pm}$ , DMRG?

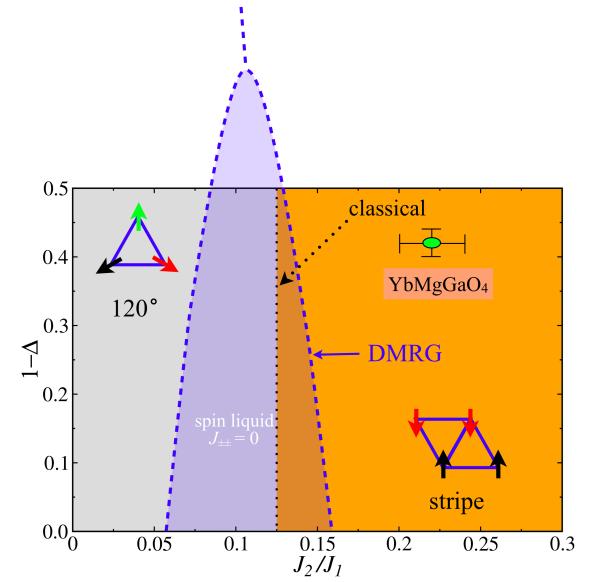
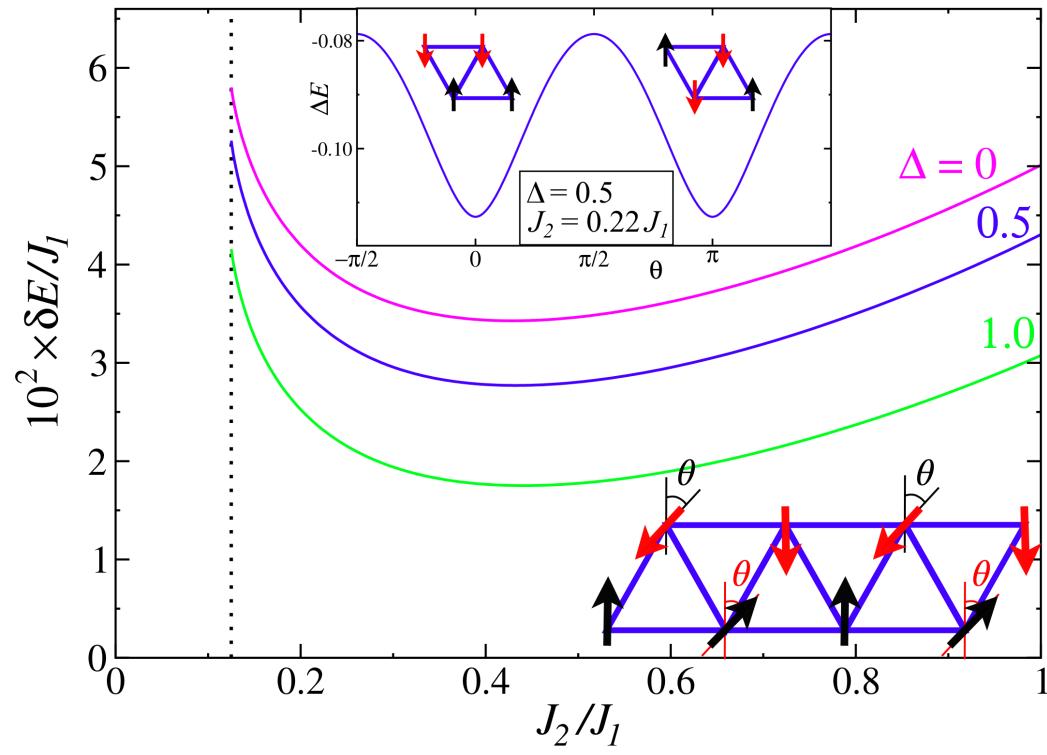


- $J_1$ - $J_2$  model has a spin liquid phase [ $J_2 = 0.06 \dots 0.15$ ]
- $J_1$ - $J_2$ -XXZ- $J_{\pm\pm} \Rightarrow$  **extended spin liquid phase**  
(extent in  $\Delta = [1.0 \dots 0.3]$ ,  $J_{\pm\pm} = [-0.1 \dots 0.1]$ )
- supported by variational MC (Iaconis *et al.*, '18)
- YMGO?: **well outside an SL, in a stripe phase**  
with a large ordered moment  $\langle S \rangle \approx 0.42$  [DMRG]



# how can stripe look like an “SL”? a scenario:

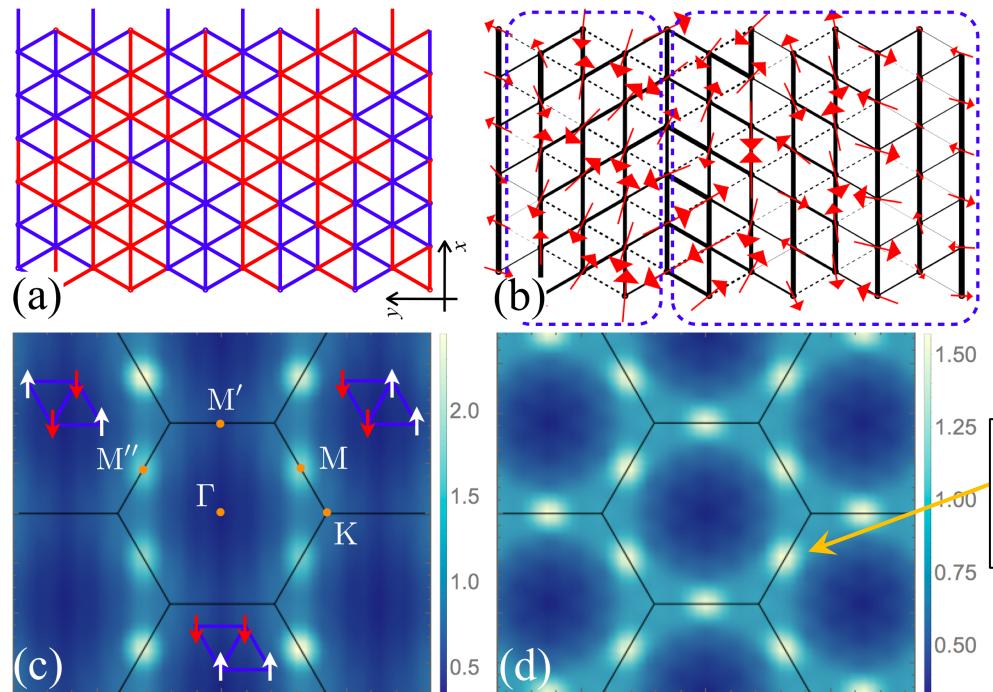
- pure  **$J_1$ - $J_2$ -XXZ** limit: stripes are selected by an order-by-disorder effect [from a classically degenerate manifold of (spiral) states]
- $\Rightarrow$  tunneling barrier between stripe different phases: about **0.03 $J_1$**  per spin



# + disorder ... = SL mimicry

$$\hat{\mathcal{H}}_{12} = \mathbf{S}_1^0 \begin{pmatrix} J_{xx} & 0 & 0 \\ 0 & J_{yy} & J_{yz} \\ 0 & J_{zy} & J_{zz} \end{pmatrix} \mathbf{S}_2^0$$

$$J_{\pm\pm} = (J_{xx} - J_{yy})/4$$

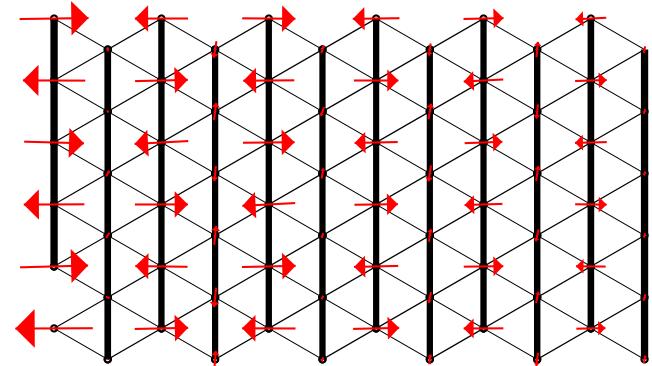
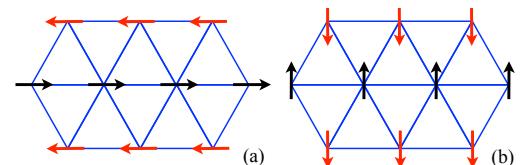


- 

DMRG  $\Rightarrow$  stripe clusters, or **stripe-superposed states**, yielding  $S(\mathbf{q})$  similar to experiments

- subleading  $J_{\pm\pm}$  terms are a (small) difference of the diagonal terms in the exchange matrix
- disorder in the latter (20%, consistent with experiments)  
 $\Rightarrow$  **random pinning fields for stripes**

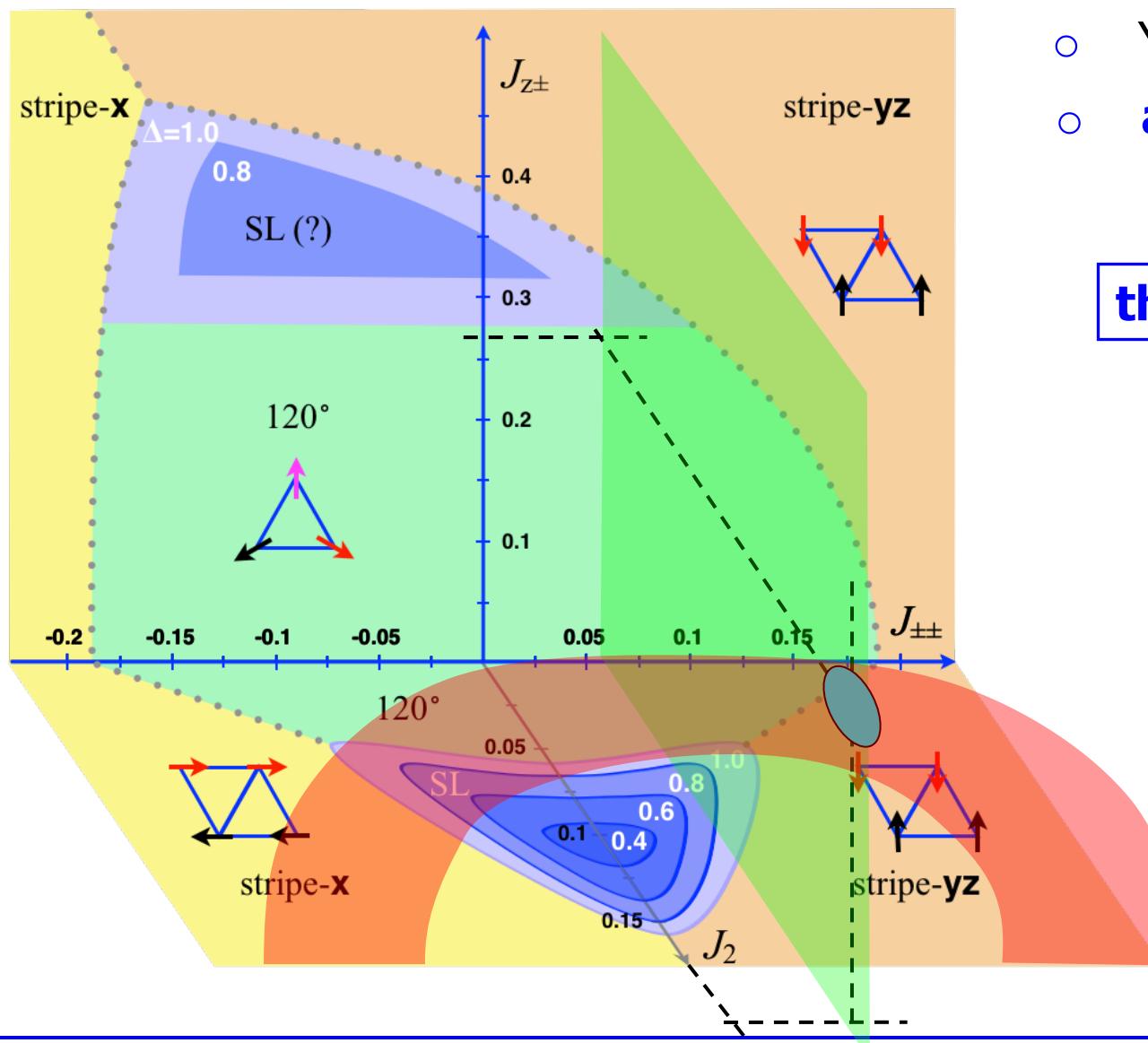
Y. Li *et al*, PRL 118, 107202 (2017).



see also E. Parker and L. Balents, PRB 97, 184413 (2018);  
Z. Ma *et al*, PRL 120, 087201 (2018).

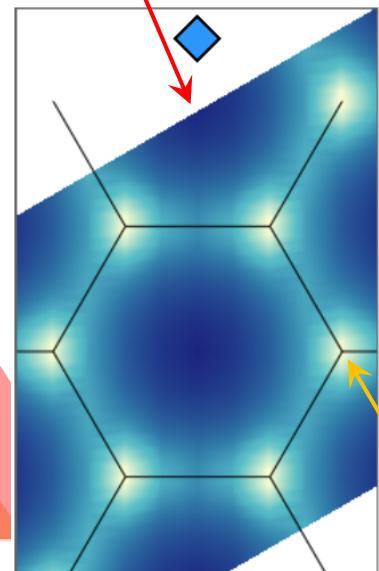
**YMGO** is in a stripe phase with large ordered moment;  
disorder makes it look like a spin-liquid

# “where is disorder-free YMGO?”

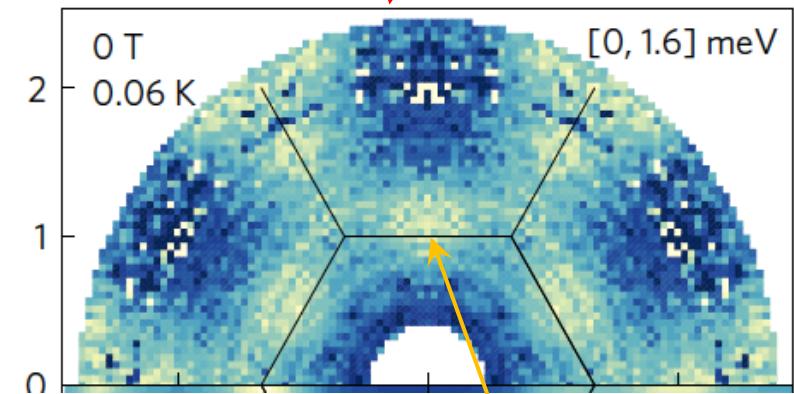


- YMGO?: well outside SL#1, in a stripe phase
- also: “wrong SL”! [ $S(\mathbf{q})$  different]

**theory SL**



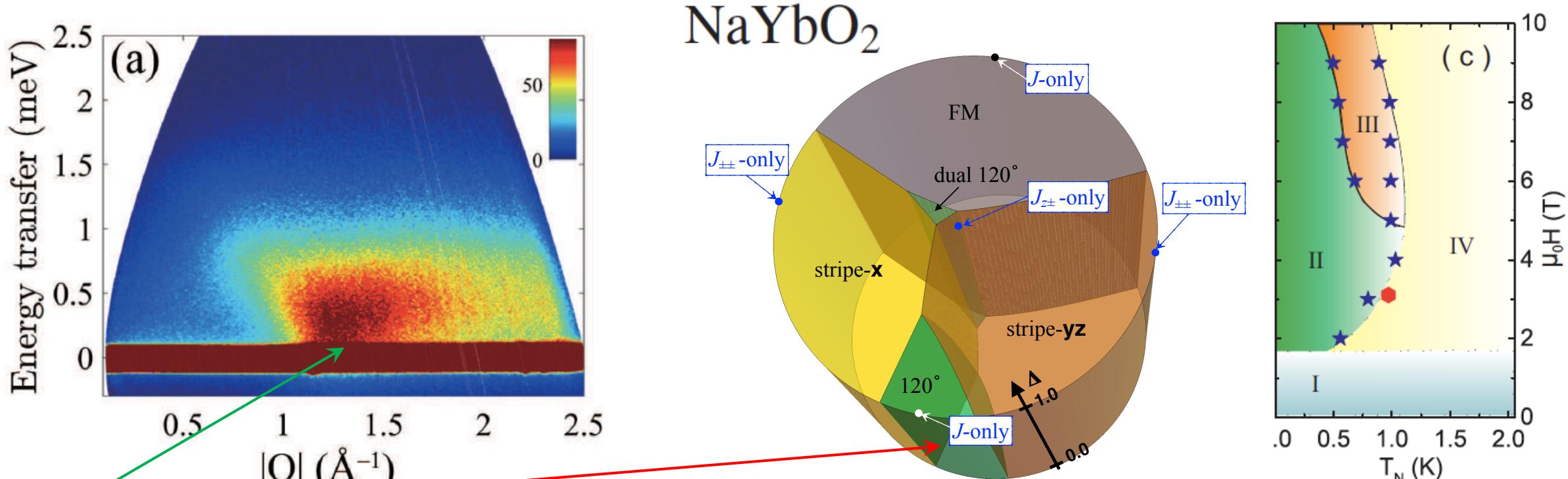
**observations**



**K-points**

**M-points**

# new materials?



K-point

- possibly smaller  $XXZ$  anisotropy ( $\Delta \approx 1$ ), small other terms
- no ordering (**disordered 120° ? or SL#1?**); max intensity at the K-point
- consistent with being in 120° phase (and disordered)
- orders in small fields
- $\text{NaYbS}_2$ :  $\Delta \approx 0.3$ , also not ordered

# conclusions

- ✓ phase diagram of the nearest-neighbor anisotropic-exchange triangular lattice model: **two spin liquids [non-Kitaev]**
- ✓ framework for many triangular-lattice based systems

