Bosonic Symmetry Protected Topological States: Theory, Numerics, And Experimental Platform

Zhen Bi

University of California, Santa Barbara

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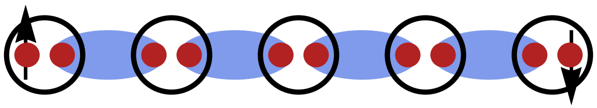
"Oversimplified" Introduction to SPT States

- Bosonic Symmetry Protected Topological (SPT) States
  - Generalization of TI/TSC to spin/boson systems
  - Bulk: gapped and non-degenerate; Boundary: gapless
  - Always require strong interactions
- Example: 1d Haldane phase of spin-1 chain

Haldane 1983



Affleck, Kennedy, Lieb, Tasaki 1987

$$H = \sum_{\langle ij\rangle} \boldsymbol{S}_i \cdot \boldsymbol{S}_j + \frac{1}{3}\left(\boldsymbol{S}_i \cdot \boldsymbol{S}_j\right)^2$$

  - Field theory: O(3) NLSM + Θ term ($\pi_2[S^2] = \mathbb{Z}$)

$$S = \int dx\, d\tau\, \frac{1}{g}(\partial_\mu \boldsymbol{n})^2 + \frac{i\,\Theta}{4\,\pi}\boldsymbol{n}\cdot\partial_t\boldsymbol{n}\times\partial_x\boldsymbol{n} \qquad \Theta = 2\,\pi$$

build with Neél order parameter $\boldsymbol{n} \sim (-)^i\, \boldsymbol{S}_i$

Haldane 1988, Ng 1994, Coleman 1976

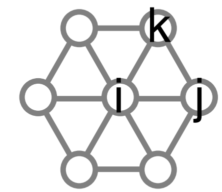
"Oversimplified" Introduction to SPT States

- Higher dimensional bosonic SPT states are much more complicated, they can be classified mathematically.

  Chen, Gu, Liu, Wen 2011; Kapustin 2014; Wen 2014; Kitaev …

- What about lattice model/Hamiltonian?
  - Levin-Gu model

$$H_{\mathrm{LG}} = -\sum_i \tilde{X}_i, \quad \tilde{X}_i = -i\, X_i \prod_{\langle jk \rangle \in \hexagon} \exp\left(\frac{i\,\pi}{4} Z_j\, Z_k\right)$$

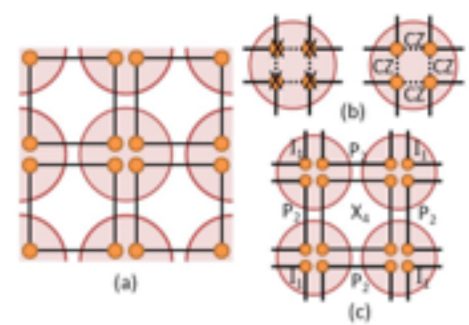


  - CZX model

$$H_{p_i} = -X_4 \otimes P_2^u \otimes P_2^d \otimes P_2^l \otimes P_2^r$$

$$X_4 = |0000\rangle\langle 1111| + |1111\rangle\langle 0000|$$

$$P_2 = |00\rangle\langle 00| + |11\rangle\langle 11|$$



  - The boundary is gapless assuming the Z2 symmetry.

Levin, Gu 2012

Chen, Liu, Wen 2012

"Oversimplified" Introduction to SPT States

- **More generic properties:**
  - The boundary of many 2d bosonic SPT states can be thought of as 1+1d O(4) WZW CFT with anisotropies:

$$\mathcal{L} = \int dx d\tau \frac{1}{g}(\partial_\mu \vec{n})^2 + \int_0^1 du \frac{i2\pi}{\Omega_3} \epsilon_{abcd} n^a \partial_x n^b \partial_\tau n^c \partial_u n^d$$

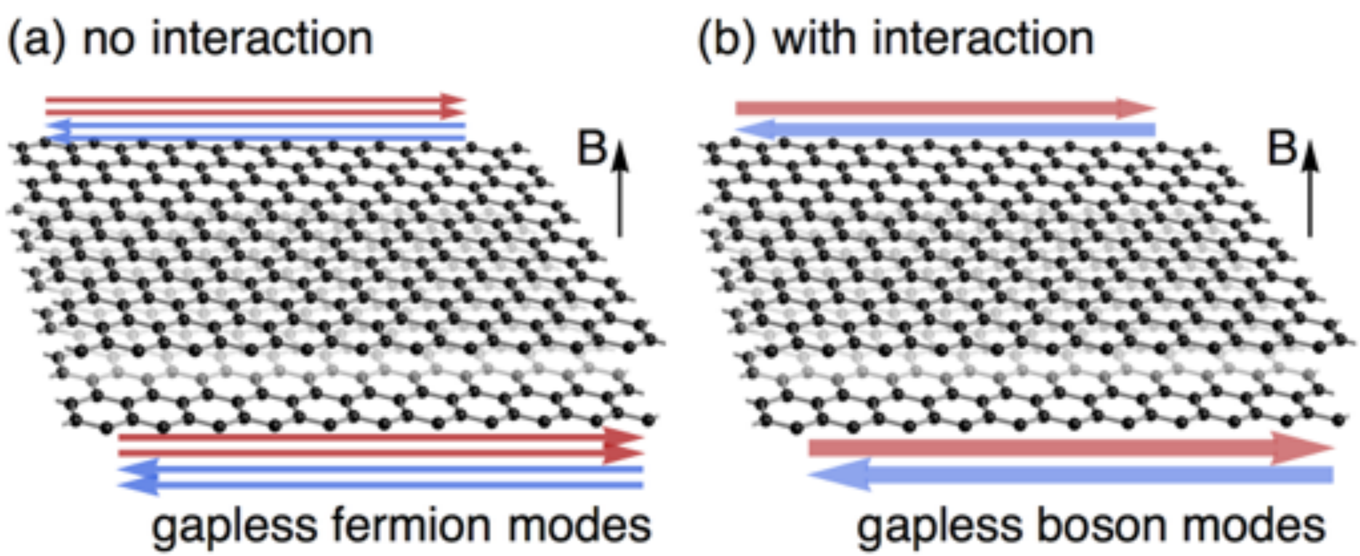
  - *SO(4)~SU(2)*$_L$ *x SU(2)*$_R$.
  - Example: The boundary of bosonic integer quantum Hall state, corresponds to breaking the *SU(2)*$_L$ symmetry completely, but break the *SU(2)*$_R$ symmetry to *U(1)* charge conservation symmetry. Senthil, Levin 2012

- **Goal**: To find a realistic condensed matter system to realize/ mimic bosonic SPT state in 2d.

Realize 2d Bosonic SPT States in Bilayer Graphene

- **Proposal**:
  Bilayer graphene under (strong) magnetic field can be driven into a "bosonic" SPT state with U(1)×U(1) symmetry by Coulomb interaction.

  Bi, Zhang, You, Young, Balents, Liu, Xu (2016)



- **Meaning**:
  - Boundary: gapless boson modes with U(1)×U(1) symmetry, fermion modes gapped out by interaction.

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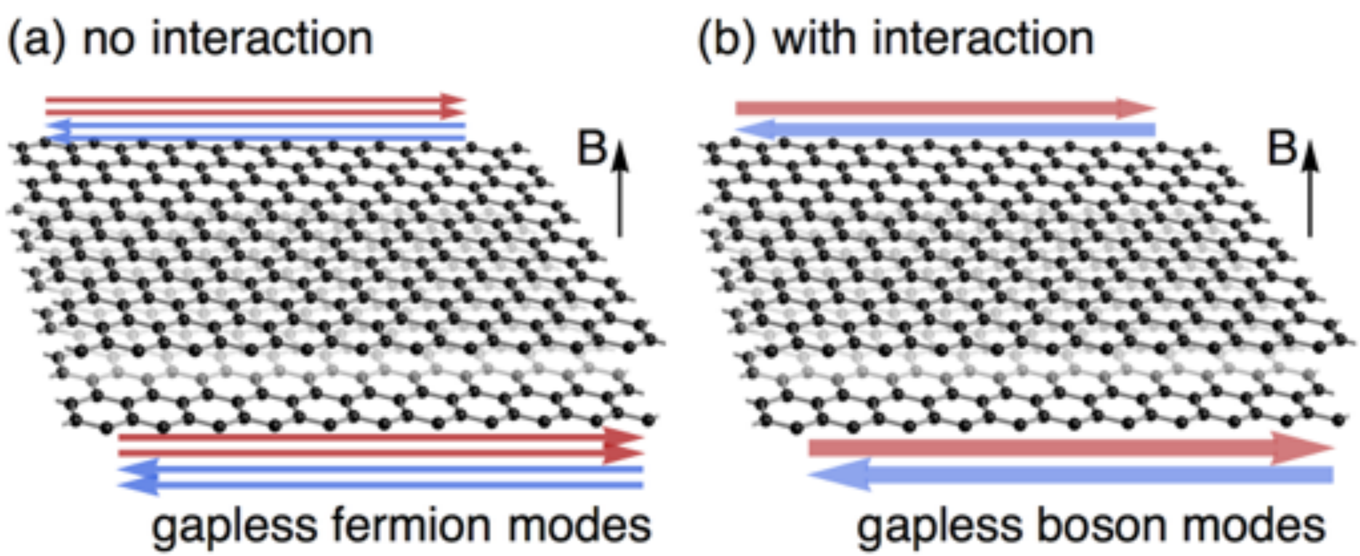
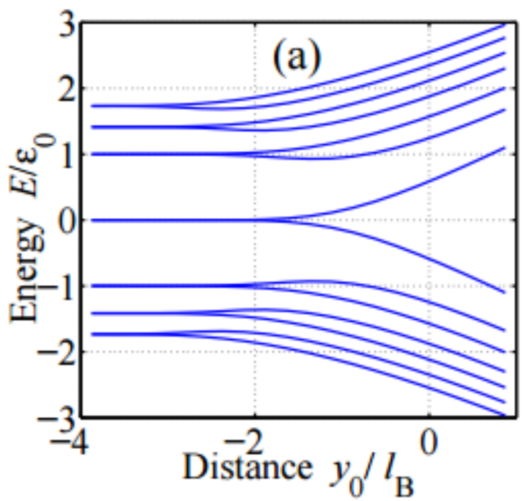


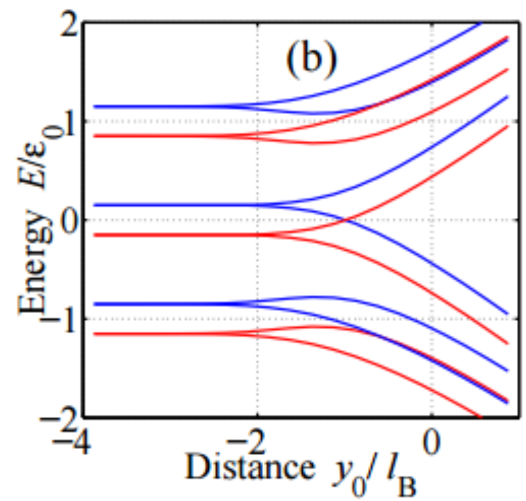
- **Meaning**:
  - Bulk: quantum phase transition between BSPT and trivial state only closes boson gap, fermions remain gapped.

Boundary Analysis

- Boundary: fermion modes gapped out under interaction, remaining gapless boson modes with U(1)×U(1) symmetry.
  - Single layer graphene under perpendicular magnetic field without interactions.



Abanin, Lee, Levitov 2006, Fertig, Brey 2006



Young et.al. 2014

- Helical edge mode: a pair of counter-propagating fermion modes (c=1 CFT)

Boundary Analysis

- Bilayer graphene
  - Noninteracting: QSH×2, two helical edge modes (c=2)
  - Coulomb interaction is relevant → gaps out all the fermion modes → only a pair of gapless counter-propagating boson modes (c=1 CFT)
  - Bosonization

$$H_0 = \int dx \sum_{l=1}^{2} \bar{\psi}_{l,L} iv\partial_x \psi_{l,L} - \bar{\psi}_{l,R} iv\partial_x \psi_{l,R}$$

⇨ $$H_0 = \int dx \frac{v}{2\pi} \sum_{l=1}^{2} \frac{1}{K}(\partial_x \theta_l)^2 + K(\partial_x \phi_l)^2 \qquad \psi_{l,L/R} \sim e^{i(\theta_l \pm \phi_l)}$$

Coulomb $H_v \sim \cos(2(\phi_1 - \phi_2)) \sim \psi^\dagger_{1,L}\psi_{1,R}\psi^\dagger_{2,R}\psi_{2,L}$

⇨ $$\tilde{H} = \int dx \frac{v}{2\pi} \left(\frac{1}{\tilde{K}}(\partial_x \theta_+)^2 + \tilde{K}(\partial_x \phi_+)^2 \right)$$

Boundary Analysis

- Boundary theory

$$\tilde{H} = \int dx \frac{v}{2\pi}\left(\frac{1}{\tilde{K}}(\partial_x\theta_+)^2 + \tilde{K}(\partial_x\phi_+)^2\right)$$

- Boundary collective modes:

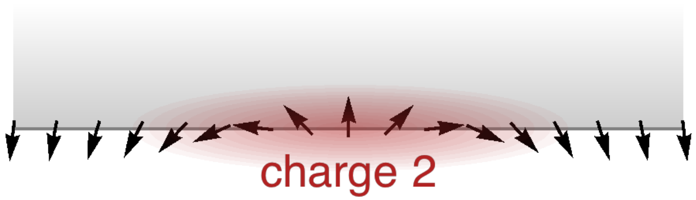
  - SC $\quad n_1 + in_2 \sim e^{i\theta_+} \sim \epsilon_{\alpha\beta}\psi_{1,\alpha}\psi_{2,\beta}$
  - XY SDW $\quad n_3 + in_4 \sim e^{i2\phi_+} \sim \sum_l (-1)^l \psi_l^\dagger \sigma^+ \psi_l$

- We can derive the boundary effective theory. It's an O(4) WZW model at level-1 (with anisotropy)

$$\mathcal{L} = \int dx d\tau \frac{1}{g}(\partial_\mu \vec{n})^2 + \int_0^1 du \frac{i2\pi}{\Omega_3}\epsilon_{abcd} n^a \partial_x n^b \partial_\tau n^c \partial_u n^d$$

Boundary Analysis

- Boundary: fermion modes gapped out under interaction, remaining gapless boson modes with U(1)×U(1) symmetry.
  - Naïve picture for why the boundary must be gapless: spin defect carries charge, charge defect carries spin



  - Edge current is transported by the bosonic edge modes (charge 2e Cooper pairs) → shot noise measurement
  - Tunneling from a normal metal → single particle gap
  - Such purely bosonic gapless boundary cannot occur with only one layer of QSH insulator

Wu, Bernevig, Zhang (2005)
Xu, Moore (2005)

Boundary Analysis

- Bulk wave function can be derived from boundary CFT correlation according to the bulk-boundary correspondence. Moore, Read (1991)

$$\langle e^{i\theta_+(z,\bar{z})} e^{-i\theta_+(0)} \rangle = |z|^{-\tilde{K}/2}$$

$$\langle e^{i\theta_+(z,\bar{z})} e^{-i2\phi_+(0)} \rangle = \bar{z}/|z|$$

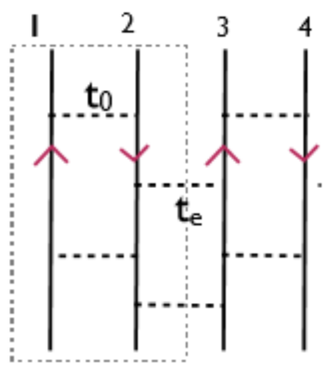
$$\langle e^{i2\phi_+(z,\bar{z})} e^{-i2\phi_+(0)} \rangle = |z|^{-2/\tilde{K}}$$

$$\begin{aligned}\Psi[w_i, z_j] &= \langle \prod_i e^{i\theta_+(w_i)} \prod_j e^{i2\phi_+(z_j)} \mathcal{O}_{bg} \rangle \\ &= F(|z_i - z_j|, |w_i - w_j|, |z_i - w_j|, \tilde{K}) \prod_{i,j} (z_i - w_j) e^{-\frac{1}{4}\sum_i (|w_i|^2 + |z_i|^2)}\end{aligned}$$

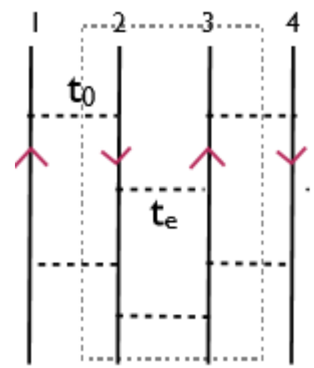
- The last factor encodes the essential physics that the spin and charge view each other as flux. Consistent with the flux attachment picture of Senthil & Levin 2012. Senthil, Levin (2012)

Bulk Analysis

- Bulk: quantum phase transition between BSPT and trivial state only closes boson gap, fermions remain gapped.
  - Bulk theory can be build from boundary with a Chalker-Coddington / coupled-wire type of model



$t_0 > t_e$, trivial



$t_0 < t_e$, Chern insulator

  - Example: Chern insulator & trivial insulator. We can build the bulk with coupled chiral fermions. The quantum critical point between Chern insulator and trivial insulator is precisely a 2+1d Dirac fermion.

Bulk Analysis

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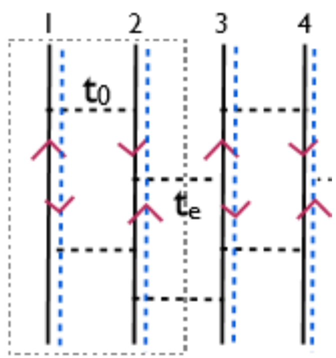
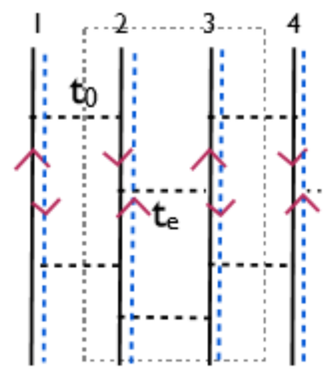


$t_o > t_e$, trivial



$t_o < t_e$, BSPT

  - Boundary theory only has gapless bosons (at low energy) → expect (and supported by numerics) that bulk transition is also "bosonic" → mimic a bosonic SPT-trivial transition.

Sign Problem Free Lattice Model

- **The spirit: spherical chicken**

  Leonard Hofstadter from the Big Bang Theory:

  There's this farmer, and he has these chickens, but they won't lay any eggs. So, he calls a physicist to help. The physicist then does some calculations, and he says, um, I have a solution, but it only works with spherical chickens in a vacuum!

- Topological state, is a chicken that can be thought of as a sphere, so seemingly different chickens can behave exactly the same.

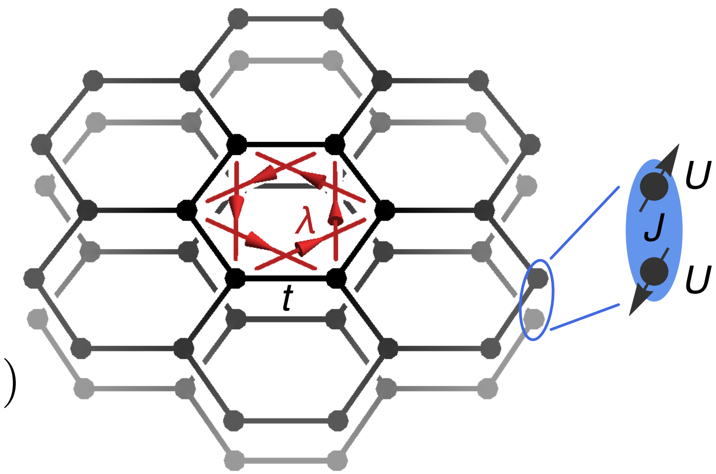
Sign Problem Free Lattice Model

- We designed a lattice model with all the key physics and with no sign problem

$$H = H_{\text{band}} + H_{\text{int}}$$

$$H_{\text{band}} = -t \sum_{\langle ij \rangle, \ell} c^\dagger_{i\ell}\, c_{j\ell} + \sum_{\langle\langle ij \rangle\rangle, \ell} i\, \lambda_{ij}\, c^\dagger_{i\ell}\, \sigma^z\, c_{j\ell}$$

$$H_{int} = J \sum_i (\mathbf{S}_{i1} \cdot \mathbf{S}_{i2} + \frac{1}{4}(n_{i1} - 1)(n_{i2} - 1))$$



- Simple limits of this model:
  - Free limit: bilayer QSH, $\sigma_{sH} = \pm 2$ (depending on λ)
  - Strong *J*-interacting limit: trivial Mott, $\sigma_{sH} = 0$

$$|\Psi\rangle = \prod_i \left(c^\dagger_{1\uparrow}\, c^\dagger_{2\downarrow} - c^\dagger_{1\downarrow}\, c^\dagger_{2\uparrow}\right) |0\rangle$$ rung singlet product state

Slagle, You, Xu (2014)

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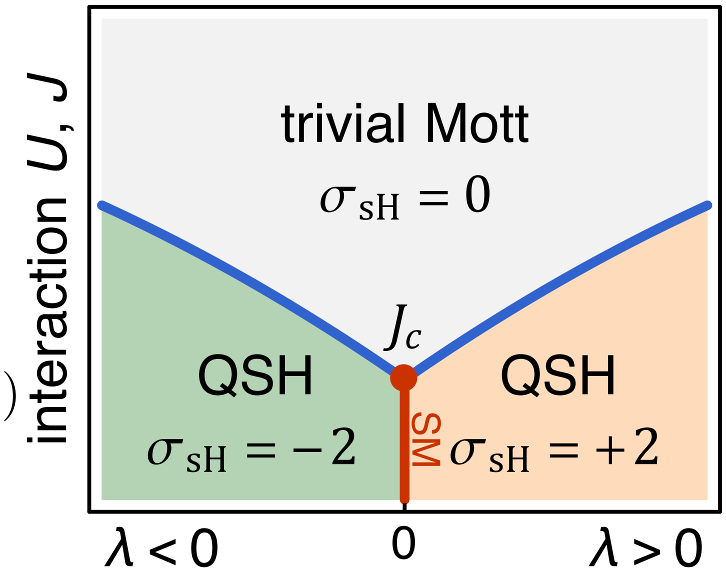

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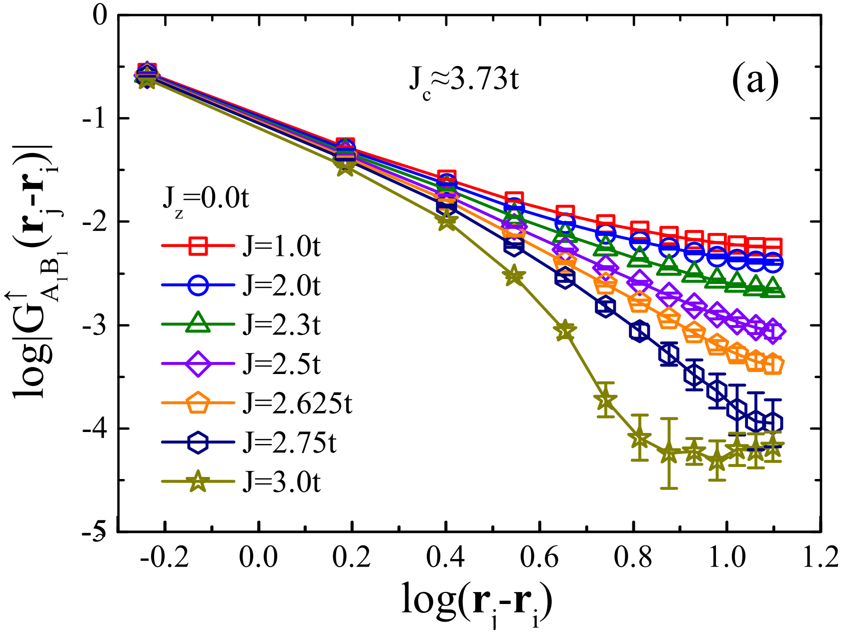
Slagle, You, Xu (2014)

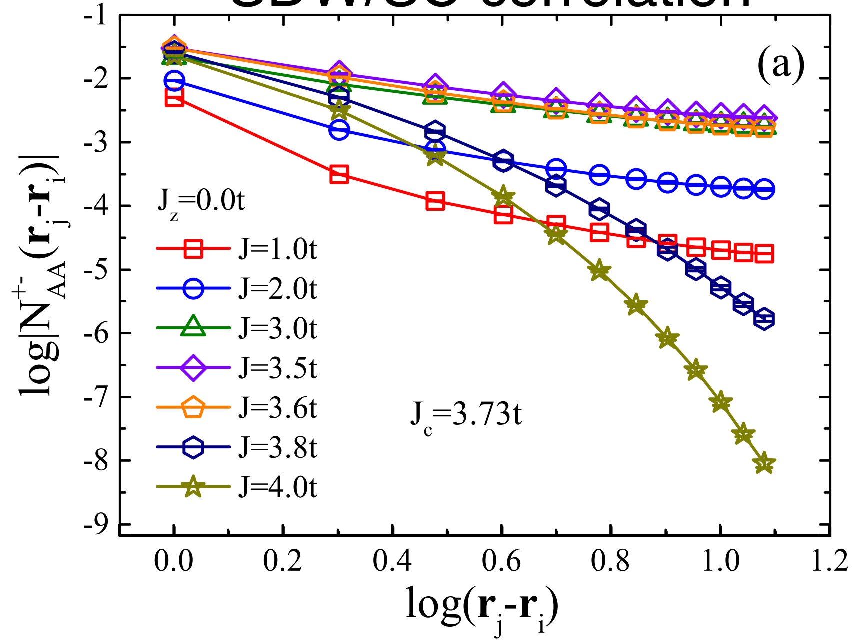
Sign Problem Free Lattice Model

- Determinant QMC (Edge)

fermion correlation



SDW/SC correlation

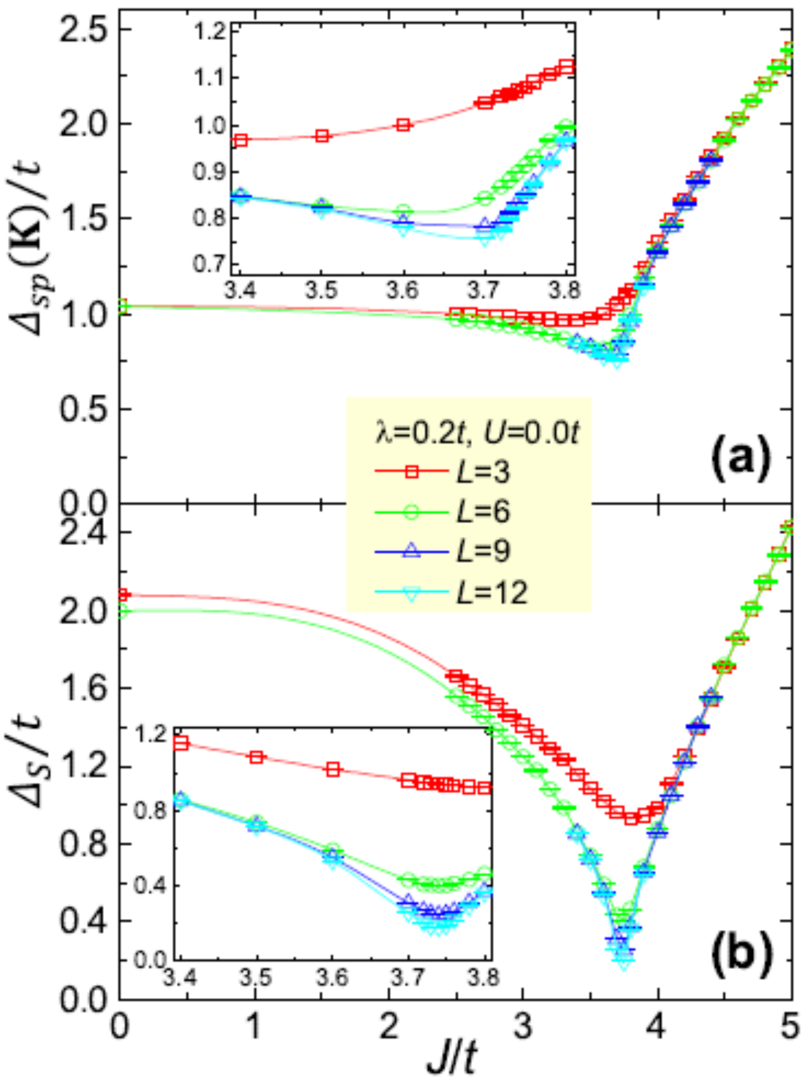


- When the fermion Green's function already decays exponentially at the boundary, bosonic modes still have power law correlation, until the system hits the bulk transition into the trivial Mott phase.

Wu et al. 2016

Sign Problem Free Lattice Model

- Determinant QMC (Bulk)
  - Fermion gap always finite.
  - Bosonic modes become gapless at the SPT-trivial critical point.
  - Fundamentally different from free fermion QSH transition.
- Because the fermionic degrees of freedom never show up at either the boundary or the bulk quantum transition, the whole system can be viewed as a bosonic SPT state.



He et al. 2015

Sign Problem Free Lattice Model

- There's a family of 2d Sign Problem Free lattice model for Bosonic SPTs with Sp(N)xSp(N) symmetry. And the boundary realizes $Sp(N)_1 \times Sp(N)_{-1}$ CFT.

  - Take 2N identical copies of QSH insulators. Free boundary theory:

$$H_{bdy} = \sum_{l=1}^{2N} \int dx \left(\psi^\dagger_{l,L} i\partial_x \psi_{l,L} - \psi^\dagger_{l,R} i\partial_x \psi_{l,R} \right) \sim U(2N)_1 \times U(2N)_{-1}$$

  - CFT decomposition $U(2N)_1 = Sp(N)_1 + SU(2)_N$
  - Interactions gap out SU(2) sector and all the fermions, while leaves the Sp(N) sector intact (**bosonic**).

$$H_{int} \sim -J^a_{SU(2)_L} J^a_{SU(2)_R}$$

  - An effective time reversal symmetry guarantees the model is sign problem free.

You, Bi, Mao, Xu (2015)
Barkeshli, Wen (2010)

A Theory for the Bulk Transition

- A conjectured field theory for the bulk transition:

$$\mathcal{L} = \sum_{j=1}^{2} \bar{\psi}_j \gamma_\mu (\partial_\mu - i a_\mu) \psi_j + i A_\mu \bar{\psi} \gamma_\mu \tau^z \psi + \frac{i}{2\pi} a \wedge dB$$

$$+ m \bar{\psi}_j \psi_j + \frac{i}{4\pi} A \wedge dA - \frac{i}{4\pi} B \wedge dB$$

  - *m* is the tuning parameter of the transition. Across the transition, the Hall conductivity of external field *A* (*B*) changes by +2 (-2) → consistent with the BSPT physics

    Senthil, Fisher 2005

  - It has the desired SO(4) ~ SU(2)xSU(2) symmetry, because of the self-dual structure

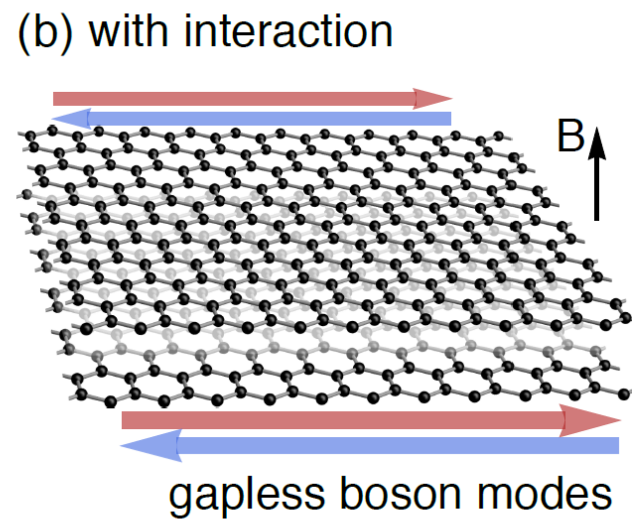
    Xu, You 2015; Karch, Tong 2016; Hsin, Seiberg, 2016

  - Besides our numerical results, other numerics studies also suggest that $N_f = 2$ QED_3 (at m = 0) is indeed a CFT.

    Karthik, Narayanan 2016

Summary

- **Predictions**:
    - The boundary is a conductor with single particle gap
    - Contrast between transport and tunneling



- The competition between magnetic and electric field in the bulk may lead to a purely bosonic quantum phase transition, with gapped electron but gapless bosonic collective modes;

- **Other possible systems:**
    - Topological mirror insulator

Zhang, Xu, Liu (2014)